DOCUMENT RESUME

ED 401 145 SE 059 249

AUTHOR	Secada, Walter G.; And Others
TITLE	Intellectual Content of Reformed Classrooms.
INSTITUTION	National Center for Research in Mathematical Sciences Education, Madison, WI.
SPONS AGENCY	Office of Educational Research and Improvement (ED), Washington, DC.
PUB DATE	95
NOTE	9p.
AVAILABLE FROM	National Center for Research in Mathematical Sciences Education, 1025 W. Johnson Street, Madison, WI 53706.
PUB TYPE	Reports - Research/Technical (143) -- Journal Articles (080)
JOURNAL CIT	National Center for Research in Mathematical Sciences Education Newsletter; v4 n1 p3-8 Win 1995
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	*Concept Formation; Discourse Modes; Educational Improvement; Interdisciplinary Approach; Interviews; *Lesson Observation Criteria; *Observation; *Research Methodology; *Verbal Communication

ABSTRACT

Seventeen schools are part of a National Center for Research in Mathematical Sciences Education (NCRMSE) study of school-level reform that focuses on the intellectual content of instruction. The schools are involved in substantive efforts to reform school mathematics and were selected on the basis of survey data. A description of the scales developed to document classroom observations made by researchers, a vignette taken from observations of an exemplary grade-4 mathematics class, and an analysis of the vignette developed using the scales are provided. The observation scales have specific coding criteria and are either numeric or descriptive. Scales are described that help observers note the mathematical content of lessons, the classroom use of mathematical analysis, depth of knowledge and student understanding, mathematical connections, value beyond the class, mathematical discourse and communication, and locus of mathematical authority. (DDR)

Intellectual Content of Reformed Classrooms

by
Walter G. Secada
Sherian Foster
Lisa Byrd Adajian

Intellectual Content of Reformed Classrooms

by Walter G. Secada, Sherian Foster, Lisa Byrd Adajian

Seventeen schools that are involved in substantive efforts to reform school mathematics are part of a National Center for Research in Mathematical Sciences Education (NCRMSE) study of school-level reform. The schools were selected on the basis of survey data collected from nearly 400 schools in locations throughout the United States and phone interviews conducted with a subset of 50 of the original 400 schools. The quality of student experiences in mathematics will vary within schools, thought the researchers who participate in the NCRMSE Working Group on the Implementation of Reform, but students should encounter mathematics instruction in the selected schools that differs in significant ways from the instruction students receive in schools that use conventional mathematics curricula and teaching practice. Observers, all of them project staff members, will visit each of the schools and observe two or more lessons taught to each of six different classes of students by their respective teachers during the study. This article describes the scales developed to document the classroom observations, a vignette taken from observations of an exemplary fourth grade mathematics class, and an analysis of the vignette developed using the scales.

Intellectual Content of Instruction

Determining the focus of classroom observations is a challenging task. Observers can look at what teachers do in classrooms, at their class management, at instruction with small groups or whole-class groups, and at teachers' interaction patterns with individual students or groups of students. They can focus on student behavior as a whole in a given class, on whether students are paying attention, on the kinds of answers they give, on the students who answer often, or those who dominate the class. Or observers can focus on teacher decision-making, on why teachers act in particular ways at particular times, and on whether their behaviors reveal their beliefs or knowledge about mathematics or teaching.

Researchers in the school-level study considered many of these possibilities, but they chose to focus on the classroom's *intellectual substance.* In order to capture different dimensions of that intellectual substance, NCRMSE staff developed 10 high-inference scales which specify a lesson's content in terms of teacher and student behavior, student engagement, and the shared norms of the class revealed through the interaction patterns of the class. The scales focus on conceptually distinct facets of a classroom. Because the scales are drawn from reform documents and research on the study of teaching, they provide a framework for translating reform recommendations into practice. Each scale is accompanied by specific criteria for coding lessons. For some scales, the scoring is numeric; for others, it is descriptive.

Mathematical Concepts

The *Mathematical Concepts* scale is nonnumeric. It maps lesson content onto one or more important areas of study in school mathematics. Although it is possible to cover an important concept in superficial ways, this qualitative scale does not judge the depth of treatment the concept in question receives. It only determines the mathematical domain or concept that is covered. The content that observers determine to be the focus of the lesson is mapped onto the categories of school mathematics found in *On the Shoulders of Giants* (MSEB, 1991) and the NCTM *Curriculum and Evaluation Standards* (1989). If the lesson involves a mathematical domain that should, based on these documents, be de-emphasized, only the broad domain is mapped. In the case of the development of rote computational skills, for example, only the broad domain of number operations would be selected.

Use of Mathematical Analysis

The *Use of Mathematical Analysis* scale measures the extent to which students engage in mathematical analysis. Mathematical analysis can be thought of as higher order thinking that involves mathematics and goes beyond mechanically recording or reporting mathematical facts, rules, and definitions, or beyond mechanically applying algorithms. It involves searching for mathematical patterns, making mathematical conjectures, and justifying those conjectures. It includes organizing, synthesizing, evaluating, speculating, arguing, hypothesizing, describing patterns, making models or simulations, and inventing original procedures. In all of these, the content of the higher order thinking is mathematics.

Depth of Knowledge and Student Understanding

The *Depth of Knowledge and Student Understanding* scale measures the complexity and depth to which student mathematical knowledge is developed in

a lesson. Instead of reciting fragmented pieces of information, students should dévelop relatively systematic, integrated, or holistic understandings of the mathematical content that was identified on the *Mathematical Concepts* scale. Students may also produce new knowledge by discovering mathematical relationships, solving problems, making conjectures, justifying hypotheses, or drawing conclusions.

Student mathematical knowledge is superficial when important ideas have been treated in a trivial way by the teacher or students. Knowledge is thin when student understanding of important mathematical concepts includes only a surface acquaintance with their meaning. This can be due to instructional strategies that cover large quantities of fragmented ideas and bits of information in ways that do not make connections to other knowledge. Student understanding is shallow when students cannot use knowledge to make clear distinctions or arguments, solve problems, or develop more complex understandings of other related phenomena. It is possible to have a lesson that contains substantively important and deep knowledge. Students may not become engaged or may fail to show an understanding of the complexity or the significance of the ideas. This scale examines the depth to which students pursue lesson content.

Mathematical Connections

The *Mathematical Connections* scale measures the extent to which topics from different mathematical domains, as part of the lesson, are connected to one another. Inquiry that connects mathematical domains is valued because it helps students develop integrated knowledge that is applicable across domains. The use of multiple representations–for example, a graphic representation of a fraction–does not automatically merit a high score on this scale. Because the connections between the domains found in multiple representations are often tacit, scores will be low unless the connections themselves are the focus of study.

Cross-Disciplinary Connections

The *Cross-Disciplinary Connections* scale measures the extent to which mathematics topics are connected to other subject areas. Cross-disciplinary or integrated curricula are valued because students develop integrated knowledge that can be applied across multiple subjects. Mathematical topics can have applications or be otherwise connected to other subjects; but in order for a lesson to receive a high score on this scale, the connections must be made explicit and must be explored by the students.

Mathematics may be used as a tool to develop an understanding of another subject; another subject may be used to provide a setting for the study of mathematics. A balance between the demands of the two disciplines needs to be maintained, and true integration, where the study of one enriches the study of the other, is necessary in order to score well on this scale.

Value Beyond the Class

The *Value Beyond the Class* scale measures the extent to which the mathematics lesson has value and meaning beyond its instructional context. Value and meaning beyond the class are important because students develop an understanding of the real-world importance and applications of the mathematics they are studying. A lesson gains value beyond the class when it relates to the larger social context in which students live. Two areas in which student work makes this connection are (a) public problems in which students confront an actual contemporary issue or problem, such as the use of statistical analysis to prepare a report on the homeless for the city council; (b) students' personal experiences, situations, or aspirations where the lesson builds on those experiences. High scores can be achieved when the lesson includes one or both areas.

A mathematics lesson with little or no value beyond the class has activities that contribute to success in school now or later, but not to any other aspects of life. Student work serves only to certify the level of competence or compliance with the norms and routines of formal schooling.

Mathematical Discourse and Communication

The *Mathematical Discourse and Communication* scale measures the extent to which talking (or sign language, if appropriate) is used to learn and understand mathematics in the classroom. Two things are important: mathematical content and the nature of the conversation.

In classes characterized by high levels of mathematical discourse and communication, there is considerable teacher-student and student-student interaction about the ideas of a topic. The interaction is reciprocal, promotes shared understanding, and has three characteristics. (1) The talk is about mathematics and includes higher order thinking such as making distinctions, applying ideas, forming generalizations, raising questions rather than the reporting of experiences, facts, definitions, or procedures. (2) the conversation involves the sharing of ideas and is not scripted or controlled by one party, as in teacher-led recitation. It allows participants to explain themselves or ask questions in complete sentences, and it permits direct responses to comments of previous speakers. (3) the conversation builds coherently on participants' ideas to promote improved and shared understandings of a mathematical theme or topic and does not require an explicit summary statement. Mathematical discourse and communication resemble the sustained exploration of content characteristic of a seminar where student contributions lead to shared understandings.

Classes in which there is little or no mathematical discourse and communication typically consist of a lecture with recitation, where the teacher deviates very little from delivering a preplanned body of information and set of questions and students give very short answers. Because teacher questions are motivated principally by preplanned lists of questions, facts, and concepts, the discourse is often choppy rather than coherent, and there is little follow-up of student

responses. Such discourse can be viewed as the oral equivalent of fill-in-the-blank or short-answer questions.

The use of mathematical terminology does not ensure the existence of mathematical discourse. Inappropriate use of terminology may actually interfere with the development of collective understandings and shared meanings. When mathematical terms are used, they should be meaningful and appropriate; they should support the conversation.

Locus of Mathematical Authority

The *Locus of Mathematical Authority* scale measures the extent to which a lesson supports a shared sense of authority for validating students' mathematical reasoning. For a lesson to receive a high score on this scale, the teacher and the students should hold each other accountable for convincing themselves and each other that their reasoning is sound and their answers are correct. The scale does not measure student control over the content of a lesson. Teachers still decide what mathematics is worthwhile and which activities are worth exploring in detail. Curricular decisions made by teachers should not preclude the sharing of mathematical authority within a class.

Social Support for Student Achievement

The *Social Support for Student Achievement* scale measures the extent to which the teacher and students support one another by conveying high expectations. These expectations include taking risks and trying to master challenging academic work, learning important knowledge and skills, and creating a climate of mutual respect among all class members. Students with less skill or proficiency in a subject are treated in ways that provide encouragement and value their presence. If disagreement or conflict develops in the classroom, the teacher helps students resolve it in a constructive way. Token acknowledgments of student actions or responses do not constitute evidence of social support by teachers.

Social support can be undermined by teacher or student behavior, comments, or actions that discourage effort, participation, risk taking, or expression of views. Teacher or student comments that belittle a student's answer and efforts by some students to prevent others from taking an assignment seriously undermine support for achievement. Support can also be lacking when overt acts like the above do not occur, but when the overall atmosphere of the class is negative because of previous events.

Student Engagement in Doing Mathematics

The *Student Engagement in Doing Mathematics* scale measures the extent to which students show a serious psychological investment in class work. Student behaviors which show this involvement include attentiveness, doing the assigned work, and showing enthusiasm for the work by raising questions, contributing to group tasks and helping peers. Disengagement is identified by off-task behaviors that reveal boredom or a lack of effort by students. Behaviors include sleeping, daydreaming, talking to peers about nonclass matters, making noise, or disrupting the class. These behaviors indicate that students are not taking the work of the class seriously.

Observations of Ms. Stat's Class

An observer entering Ms. Stat's fourth-grade mathematics class at the beginning of the school day sees a student at the board noting the date and attendance. On the day in question, this student is also collecting her colleague's reports on that morning's temperature in degrees Celsius and how the moon looked the previous night. It is the 10th day of the month. Students decide that 10 is even and composite because 5 and 2 "can divide 10." They decide that the next day, the 11th, is odd and that 11 is prime because "nothing divides it" and "11 is just 11 times 1."

Ms. Stat begins her lesson with an overhead of a chart that shows students' temperature reports:

Temperature	Number of Students
3	10
-2	1
2	5
5	1

She asks students for the "range" of the data. One student explains what the term means and answers "-2 to 5° Celsius." Another student points out that the "top of the range" is 5, and yet another student, that the bottom of the range is -2.

Ms. Stat then puts a sequence of numbers representing temperatures on the board, 5, 4, 3, 2, 1, 0, -1, -2,

and asks, "Which temperature has been reported most often?" Students respond, "3," which she then circles on the board.

Ms. Stat: *I think that -2 is unlikely. Why do I think that?*

Student 1: *Only 1 person got that reading.*

Ms. Stat: *That could be. Any other reasons?*

Student 2: *It can't be -2. That's below freezing, and I can wear shorts and tights today.*

Students 1 and 2 launch into an extended conversation about the day's weather conditions, whether it is freezing or thawing, and whether the temperature is likely to be below 0°C. All the students agree that the one student who reported -2 probably meant +2. The students then use their notebooks to record the modal response of 3°C as the day's temperature.

Ms. Stat takes the previous week's temperatures which had been recorded by students, makes a bar graph with temperatures that range from -6 to +4°C together with the day of the week that the temperature had been reported. She tells the class that they are going to find the average temperature using a graph.

Ms. Stat: *What should we do to fill the chart in evenly?*

Bert: *Move one block from the longest to the shortest bar [Ms. Stat complies].*

Kay: *Take half a block from the 4th day and put it on the 3rd day [Ms. Stat complies].*

Students continue making suggestions on how to smooth out the bar graph, and Ms. Stat acts on them. After the graph has been evened out, she says: "You all know what to do next." One student says they should record -2°C as the "average temperature" for the previous week.

After all students have recorded the previous week's average temperature in their mathematics journals, Ms. Stat moves on to mental mathematics:

Ms. Stat:	*52 + 37?*
Student 3:	*89.*
Ms. Stat:	*What was your strategy? How did you do that?*
Student 3:	*50 + 30 + 2 + 7.*
Student 4:	*52 + 30 + 7.*

Ms. Stat encourages students to practice adding the 10s for this week and to pick the one strategy that is most comfortable to them. After five minutes of mental mathematics, which includes numbers like 524 + 260 (one student's strategy is 524 + 200 + 60), she poses a problem for the students to work on at first individually and then in their respective groups: "What are the five most used letters in the English alphabet?" Ms. Stat moves around the class, encouraging students to make their best guesses and to be sure that they have reasons for their choices.

Students begin to discuss, or to argue about, their choices. Many have chosen the five vowels as the most used letters because "all words have vowels." One student has looked around the room at all the words she can see and picked the letters that she sees most often. Another student "liked the way the letters looked." Three of the six groups of students have added S to their list because "You can put S on any word."

After each group has made its list of best guesses and discussed it, Ms. Stat talks to the students as a class: "Who cares which letters are used most?" With some encouragement, students suggest: "Authors," "Ourselves, if we write a story or a letter," "Ourselves, just to get smarter," and "Of course, teachers." Ms. Stat leads the discussion: "Did anyone drive by a local eatery with a sign that advertises the daily special? They buy their letters one at a time." One student recalls seeing an upside-down 7 in place of an L; another notes that "They have to change those signs sometimes." Ms. Stat passes a full sheet of rub-off letters around the class and asks the students what they notice. Students say there are lots of vowels and quite a few Ss on the sheet. They then discuss how expensive it is to buy letters that are not needed. Ms. Stat asks students about Scrabble: "What letters do you want to get? What letters would you rather get?" Students say they like to get S and H, and would avoid Q and Z. One student calls out, "Wheel of Fortune. It would really be good to know which letters are used most in that game!" Ms. Stat asks students, for homework, to find at least six sentences in print that have five or more words and to count the number of letters used: "You can organize this any way you want. Part of what I want to see is how you organize it."

The next day students file into class. Once again, they note the date and temperature, and discuss the moon's phase. Ms. Stat begins the class with five minutes of mental mathematics, then asks the students when they might hear numbers and need to add them in their heads. Children respond: "If you're a business person." One student suggests "If you work in a factory and something isn't right. You would know because the numbers didn't add up." The comment, "If you buy something at the store," draws the retort, "Yeah, but then you'd have the cash register slip with everything on it." Ms. Stat counts the hands of the 13 students who did last night's homework and says, "Thirteen students did their homework. Twenty-two are here. How many did not do their homework? Do it in your heads." As several students call out "nine," she tells the 9 to pull out a book and do the assignment while she and the other 13 students play a mathematics game that is a bit like Mastermind.

When all the students have counted the number of letters in their six sentences, Ms. Stat points to the front of the class, the location of a chart drawn on a long piece of butcher paper. The letters of the alphabet have been written across the top, and each team's name printed on the side. Each of the groups is to pool it's data and write underneath each of the letters the total number of times that letter has appeared in the group members' sentences. As the groups do so, they discuss why different students get different letter counts. In one group, students recount the letters in a fellow student's passage because they think he made a mistake. Some groups use calculators, but others obtain the total by hand or by mental mathematics. Groups argue over the totals until all of the members are convinced they are correct. After an hour, students are writing their results on the large chart. Ms. Stat makes certain that students enter group numbers in the proper columns.

Ms. Stat asks her students if they want to revise their predictions before they compute the total for each letter for the entire class. Some students make revisions, others add an S because "it is used to make plurals," and others keep only the list of vowels. Each group computes the grand total for several letters. When they have finished, Ms. Stat asks students which letter has been found most frequently, next most frequently, and so forth. She records the answers in that order: E, O, A, T, N.

After the students have looked at their predictions, Ms. Stat asks them whether they would get the same results if they were to repeat the activity. Some say no because they would choose different sentences. One student argues that the kind of book that is used might make a difference because children's books contain different letters than an encyclopedia. Ms. Stat develops the idea: "You asked for the real answer. How do you think professionals really figure out which letters are used most?" Students agreed that they probably do it the same way the students have, except that they probably use a lot more sentences. Ms. Stat notes: "We each counted six sentences with at least five words each. There are 22 of us. We probably did at least 1200 letters."

The students reach the consensus that they would have counted a lot more sentences to determine the real answer. Ms. Stat asks, "Would the real list be very different or just a little different from ours?" The students think that it would be a "little different." They agree that they have counted enough letters to reach a total that will be close to the "real" thing.

At this point, Ms. Stat unrolls a cash register tape, onto which she has recorded the real list, above the students' list of letters:

Real	*E T A O N I S R H L D C U*
	P F M W Y B G V K Q X J Z
Class	*E O A T N I D Y R M L H S*
	W F B U C K G P V J X Z Q

Students are excited to find they have identified the same first six letters that are on the real list. They note that the locations are the same for four of the first six letters. When they compare the two lists further, they note the letters that are "pretty close" and those that are "far off" in ranks. Ms. Stat asks the students to write about the project in their mathematics journals, to describe what they did in class, and to explain why they got their results. She also asks them to write about what they have learned from the activity.

Later that day, Ms. Stat discusses her lesson with a younger colleague in the teachers' lounge. She writes a note in the margin of the book in which she found the activity: "This is a keeper."

Analysis of Observations

Mathematical Concepts: Based on the NCTM *Standards* for Grades 1-4, Ms. Stat's classes focus on mental computation, the use of calculators for complex computations, thinking strategies for computations, and the collection and organization of data.

Use of Mathematical Analysis: In Ms. Stat's class, most students, most of the time, are engaged in mathematical analysis. They are conjecturing, justifying, and inventing original procedures, and, in all cases, the content is mathematics.

Depth of Knowledge and Student Understanding: Students are pursuing many different mathematical topics, but, in all cases, they are developing deep understandings of the topics. For instance, mental computations include the development of specific strategies, and the gathering of temperature data includes checking for possible error. When students are trying to figure out which letters of the alphabet are used most often, they discuss that the "real" answer is probably arrived at in a way similar to their own, how the sample size might affect the results, and how replications might result in different ordering of the letters. These fourth graders demonstrate their understanding of the problematic nature of data gathering by arriving at reasoned and supported conclusions.

Mathematical Connections: When Ms. Stat's class focusses on a single topic, the gathering and display of information to solve a problem, some students use mental mathematics, which has been practiced earlier during the class, to add and to check their sums. This connection is neither recognized nor made explicit.

Cross-Disciplinary Connections: Students discuss why they have selected certain letters of the alphabet: "every word has a vowel" and "You can add an S to get a plural." Students mention that their results are related to the passages and sentences whose letters they have counted. These connections are explored informally.

Value Beyond the Class: Students discuss who would be interested in knowing what the most commonly used letters of the alphabet are. Their conversation also includes an analysis of why these individuals could be interested. After they have figured out which letters occur most frequently, they draw parallels between what they did and how the real answer was arrived at. They explore these connections with the world outside the class in ways that give the mathematics personal meaning and significance, for example, through sampling, data collection, and analysis.

Mathematical Discourse and Communication: Students talk about why numbers are prime or composite, and odd or even; why a temperature might be wrong; and why they have chosen certain letters. There are extended conversations in small groups about the choices as students check their results against their original predictions. These exchanges include higher order thinking, are by no means scripted, and are focused on the creation of collective understandings. Almost every student participates in one or more of the exchanges.

Locus of Mathematical Authority: At the start of each day, one of Ms. Stat's students gathers information and orchestrates a discussion on the nature of the numbers. Ms. Stat voices her opinion, and asks students to guess why she thinks that -2°C is probably wrong. She also structures the lesson so that students can exert this authority. For instance, she provides the chart for gathering and summarizing student data. Authority for mathematical sense-making is shared by the class: In one group, students do not believe one of their peers' answers, and they check his counts of letters.

Social Support for Student Achievement: The class exudes an atmosphere of mutual support and high expectations. Ms. Stat communicates these expectations by turning the issue of undone homework into a mathematics problem that is solved mentally and by telling students that their contributions are important. Students hold themselves and each other to similar standards. Students readily make conjectures and argue about them. They understand that people make mistakes, and when they check another member's work, they do so without hurting his feelings.

Student Engagement in Doing Mathematics: Student engagement in the mathematical substance of Ms. Stat's class is impressive. Almost all of the students are on task almost all of the time. At no time are more than one or two students off-task. These fourth graders remain on task for over two hours on the second day. Clearly, they are interested in how well their list matches the real list.

Concluding Comments

Whether any class could, or even should, score high on all 10 of the scales at the same time or every day is an empirical question. Researchers have

found that the scales help them to focus their classroom observations on the lesson's intellectual content. The scales specify the criteria for a given score and ensure that observers gather data that justify the rating given to a specific lesson. The set of scales is also instrumental in giving researchers an understanding of how teachers such as Ms. Stat orchestrate lessons. Teachers may need to make trade-offs as they plan and present their lessons. A focus on a specific topic, for instance, at the expense of cross-disciplinary connections may enable a teacher to support mathematical analysis and develop greater depth in student understanding. Such trade-offs can be examined through the careful observation and documentation of classroom practice.

The schools in this study include many teachers whose practices resemble those of Ms. Stat. These teachers changed their practices while working in their departments and schools. What organizational features of their schools supported or impeded the efforts of these teachers in changing their practices? This study seeks to understand how schools and mathematics departments can support teachers in learning about, experimenting with, and enhancing their practice. The data that are collected during the study of school-level reform are beginning to provide answers to these and related questions.

Note. The framework and classroom observation scales described in this article rely heavily on the framework, classroom observation scales, and student assessment scales created by the Center on Organization and Restructuring of Schools (CORS) study of restructured schools.

References

National Council of Teachers of Mathematics (1989). *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics (1991). *Professional Standards for Teaching Mathematics.* Reston, VA: Author.

Steen, L. (1990). *On the shoulders of giants: New approaches to numeracy.* Washington, DC: National Academy Press.

NCRMSE RESEARCH REVIEW

The Teaching and Learning of Mathematics

Volume 4, Number 1 Winter 1995

In This Issue:

NCRMSE Research Review: The Teaching and Learning of Mathematics is published by the National Center for Research in Mathematical Sciences Education, Wisconsin Center for Education Research, University of Wisconsin-Madison. The Center is funded primarily by the Office of Educational Research and Improvement, U.S. Department of Education (OERI/ED). The opinions expressed in this publication do not necessarily reflect the position or policy of OERI/ED; no endorsement of OERI or the Department of Education should be inferred. Send correspondence to NCRMSE, 1025 W. Johnson Street, Madison, WI 53706. Telephone (608) 263-7582; FAX (608) 263-3406.