

DOCUMENT RESUME

ED 395 795

SE 058 383

AUTHOR Spadano, Joseph W.  
 TITLE Developing Problem Solving Behaviors In Secondary Mathematics Education through Homework.  
 PUB DATE 29 Apr 96  
 NOTE 33p.; Paper presented at the Annual Conference of the New England Educational Research Organization (April 29, 1996).  
 PUB TYPE Reports - Research/Technical (143) -- Speeches/Conference Papers (150)  
 EDRS PRICE MF01/PC02 Plus Postage.  
 DESCRIPTORS \*College Students; \*Educational Responsibility; Higher Education; \*Homework; \*Problem Solving; Secondary Education; \*Secondary School Mathematics; Student Responsibility

ABSTRACT

This qualitative research investigated the effect of a homework model designed to develop students' problem solving behaviors by advancing their ownership of understanding and responsibility. Students were expected to be active learners by isolating and communicating points of homework confusion. The teachers in this study provided feedback through semi-structured personal conversations, group meetings, memos, journals, and reflective writings. Results suggested that the homework model placed a reasonable share of the work to gain ownership of understanding on the student while developing students' problem solving behaviors, provided an opportunity for the students to act responsibly, increased mathematical communication, decreased the amount of class time spent going over questions on the homework, and provided teachers with a detailed assessment of student needs, without radically altering curriculum or pedagogy. Contains 40 references. (Author/MKR)

\*\*\*\*\*  
 \* Reproductions supplied by EDRS are the best that can be made \*  
 \* from the original document. \*  
 \*\*\*\*\*

ED 395 795

DEVELOPING PROBLEM SOLVING BEHAVIORS IN SECONDARY  
MATHEMATICS EDUCATION THROUGH HOMEWORK

PAPER PRESENTED AT  
NEW ENGLAND EDUCATIONAL RESEARCH  
ORGANIZATION

1996 ANNUAL CONFERENCE

BY

JOSEPH W. SPADANO

B.S., FITCHBURG STATE COLLEGE (1977)

M.Ed., UNIVERSITY OF MASSACHUSETTS-LOWELL (1992)

Ed.D., UNIVERSITY OF MASSACHUSETTS-LOWELL (1996)

Joseph W. Spadano is a graduate of the Mathematics and  
Science Doctoral Program at the University of  
Massachusetts-Lowell and a teacher at Westford Academy,  
Westford, Massachusetts. His specializations include  
mathematics education, professional development, and  
problem solving.

26 Forge Village Road, Westford, MA 01886

(508) 692-1941

email: spadanoj@woods.uml.edu

*Joseph W. Spadano*

U.S. DEPARTMENT OF EDUCATION  
Office of Educational Research and Improvement  
EDUCATIONAL RESOURCES INFORMATION  
CENTER (ERIC)

This document has been reproduced as  
received from the person or organization  
originating it.

Minor changes have been made to improve  
reproduction quality.

• Points of view or opinions stated in this docu-  
ment do not necessarily represent official  
OERI position or policy.

ERIC  
Full Text Provided by ERIC

BEST COPY AVAILABLE

# DEVELOPING PROBLEM SOLVING BEHAVIORS IN SECONDARY MATHEMATICS EDUCATION THROUGH HOMEWORK

## ABSTRACT

This qualitative research investigated the effect of a homework model designed to develop students' problem solving behaviors by advancing their ownership of understanding and responsibility. Since responsibility is predicated by autonomy, the researcher capitalized on learner autonomy. Nowhere is the mathematics learner more autonomous than during homework. The focus of inquiry was the homework model, adapted from problem solving theory, which expected students to be active learners by their isolating and communicating points of homework confusion. The teachers in this study were research assistants and provided feedback through semi-structured personal conversations, group meetings, memos, journals, and reflective writings. Classroom observations, noting nonverbal cues, were also transcribed to further search for details surrounding the phenomenological experiences. The strength of the phenomenological descriptions involved the specific experiences as conveyed by the teachers. The transcriptions were analyzed for common themes involved in the implementation, application, and evaluation of the homework model. The research suggests that the homework model placed a reasonable share of the work to gain ownership of understanding on the student while developing students' problem solving behaviors. provided an opportunity for the students to act responsibly. increased mathematical communication, decreased the amount of class time spent going over questions on the homework. and provided teachers with a detailed assessment of student needs, without radically altering curriculum or pedagogy. The homework model appeared to have more of an educational significance with honors level students than with intermediate level students.

# DEVELOPING PROBLEM SOLVING BEHAVIORS IN SECONDARY MATHEMATICS EDUCATION THROUGH HOMEWORK

## Introduction

One of the configurations of education is schooling, which addresses prescribed curricula by providing formal instruction or training, pedagogically, through specific techniques and experiences. An element of schooling is homework (see figure 1.1).

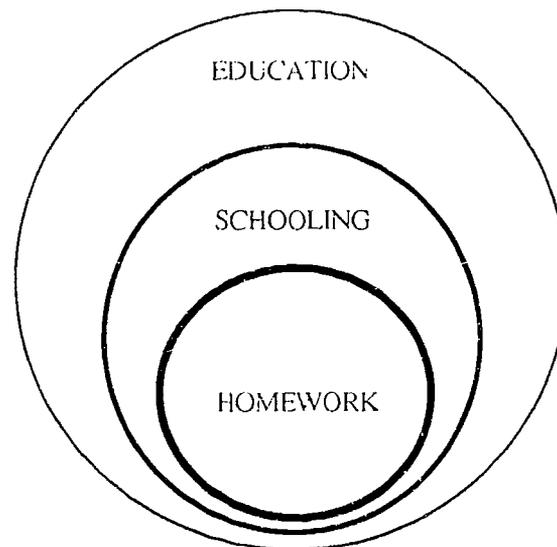


Figure 1.1: Understanding Homework's Place in Education.

Homework's ubiquitous presence as an educative product is a proclamation of its importance to the educative process. From this it follows that credit or blame for student achievement may be due, in part, to homework.

The objective of this qualitative research was to investigate the educational significance of a high school mathematics homework model as a means of developing students' problem solving behaviors by advancing their ownership of understanding and responsibility. The problem solving behaviors include those associated with the problem solving process and are those influenced by instruction. The rationale of this research was to implement the researcher's Homework Model, developed from problem solving theory, and determine its effect on advancing learners' ownership of understanding and

to implement the researcher's Homework Model, developed from problem solving theory, and determine its effect on advancing learners' ownership of understanding and responsibility through teacher perceptions and researcher observations. In order to examine the educational significance of the Homework Model, two events had to occur, 1) students had to become active agents, central to the learning process by isolating and communicating points of homework confusion (ownership of understanding is advanced), and, 2) students had to be autonomous and accept a reasonable share of the work to solve the problem (responsibility is advanced). The purpose of this research was to make educationally significant judgments by exposing the phenomena surrounding the implementation, application, and evaluation of the Homework Model within the contexts of Vygotskian education psychology and developmental theory (see figure 1.2).

### **The Presentation of the Problem**

The deteriorating problem solving skills of high school students are tragically at risk. Not only do students have virtually no understanding of problems they solve, they are confounded by a system in which they become dependent upon their teachers for presenting and verifying solutions, depriving the students of the responsibility for understanding. Students in contemporary mathematics classrooms mechanically apply the "algorithm of the day" to routine exercises while typical instructional practices provide little opportunity for the students to communicate the breadth and depth of their misunderstandings. The students' dependence on the teacher for presenting and confirming solutions to problems interferes with the development of students' mathematical problem solving behaviors, denying students the opportunity to establish mathematical belief systems. The failure to recognize the importance of developing problem solving behaviors in mathematics classrooms will contribute to the further decline of students' understanding in solving problems. This research investigated the teachers' perception of whether their students began to sense that when "understanding" became their

Investigating the phenomena surrounding the homework model

<p>Implementation</p>	<p>How do teachers present the homework model?</p> <p>What are the students' reaction when introduced to the homework model?</p> <p>How do teachers train students in the use of the homework model?</p> <p>How much of a change in routine does the implementation of the homework model represent?</p> <ul style="list-style-type: none"> <li>- from the teachers' perspective?</li> <li>- from the students' perspective? (as perceived by their teacher)</li> </ul>
<p>Application</p>	<p>Are students active agents, central to the learning process? (Are students able to isolate and communicate points of confusion?)</p> <ul style="list-style-type: none"> <li>- How successful are students at isolating and communicating points of confusion?</li> <li>- How successful are teachers at designing learning experiences to clarify the confusion?</li> </ul> <p>Do students accept a reasonable share of the work to solve the problem? (Are students willing to isolate and communicate points of confusion?)</p> <p>Do students make reference to improving or refining the homework model?</p>
<p>Evaluation</p>	<p>Do teachers make reference to improving or refining the homework model?</p> <p>How successful is the teacher in collecting and analyzing data through the use of the teacher-research assistant forms?</p>

Figure 1.2: The Phenomena Surrounding the Homework Model

responsibility, they became learners, actively taking control of the problem solving process.

## **The Background of the Problem**

Recent education reform initiatives (AAAS Project 2061, 1990; Massachusetts Department of Education's Curriculum Frameworks, 1995; NCTM Standards, 1989, 1991, 1992, 1995) suggest an educational emphasis on problem solving skills. Problem solving skills include those behaviors required to understand and solve the task. Research (Carpenter, Lindquist, Matthews & Silver, 1983; National Assessment of Educational Progress, 1982; Schoenfeld, 1985) reveals that students may not understand some of the problems they solve. In routine problems, students tend to use all the numbers given in the problem to mechanically apply a computational algorithm and do not need to understand the problem. When they are given non-routine problems, answers reveal failure to understand the situation and the nature of the unknown due to field dependence, which is using a problem solving strategy because of previous success with that strategy. "In some cases, students survived (often with good grades!) by implementing well-learned mechanical procedures, in domains about which they understood virtually nothing" (Schoenfeld 1985, p. 13).

From these reported research findings and the universal reform demand for improving students' problem solving abilities, it seems necessary and important to examine problem solving behaviors in mathematics. Cognitive psychology and mathematics education research (Case & Bereiter, 1984; Cobb & Steffe, 1983; Davis, 1984; Hiebert, 1986; Lampert, 1986; Lesh & Landau, 1983; Schoenfeld, 1987) suggests that learning occurs when students actively collect new information and construct their own meanings. In mathematical terms this translates to mean actively connecting prior knowledge to new knowledge, experientially, developing a mathematical belief system. The Homework Model placed a reasonable share of the work and the responsibility to understand the problem on the student. Students were expected to be actively involved in the problem solving process by isolating and communicating their points of homework confusion. Empowering homework as a possible means of developing and advancing "problem

solving" behaviors was an alternative to the wholesale restructuring of content and instruction (see figure 1.3).

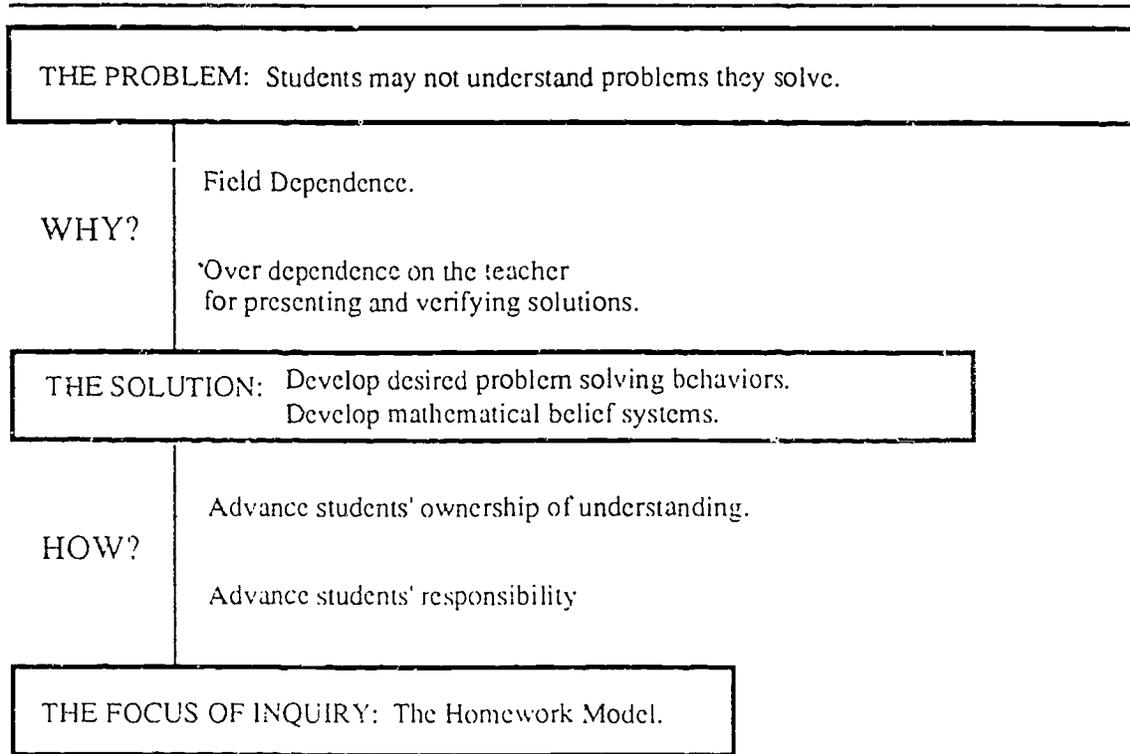


Figure 1.3: Understanding the Context of the Research.

Demand for education reform is one thing, however, its practical possibility is quite another. Given the fixed temporal constraint of the "school calendar" (the fixed length of class periods, school days, or the school year), the extent to which "problem solving" content can be integrated into the curriculum is directly related to that content which can be removed. This suggests that teachers, administrators, or policymakers must decide what implementing "problem solving" into the curriculum will replace. In changing curriculum, "The problem for curriculum developers, therefore, is much less what to add than what to eliminate" (AAAS, p. xix). The issue may be change itself. Research literature (Hall & Loukes, 1979; Jackson, 1971; Orlich, 1989; Schlechy, 1990; Zaltman, Floris, & Sikorski, 1977) abounds with issues of change, uncovering the difficulty, reluctance, and fear of change. Certainly, a more practical solution to change in education would be one which did not radically alter curriculum and pedagogy. This was the rationale behind using the

Homework Model in the service of education reform. It provided a means for unobtrusively implementing problem solving reform initiatives without radical change in curriculum or instructional practices. From this we can sense the significance of means used and ends reached and the frustration of intriguing theories seldom practiced. Is there a means by which we can reach desired ends? If so, can the theory be put into practice?

Figure 1.4 illustrates an intriguing possibility. When students become active agents,

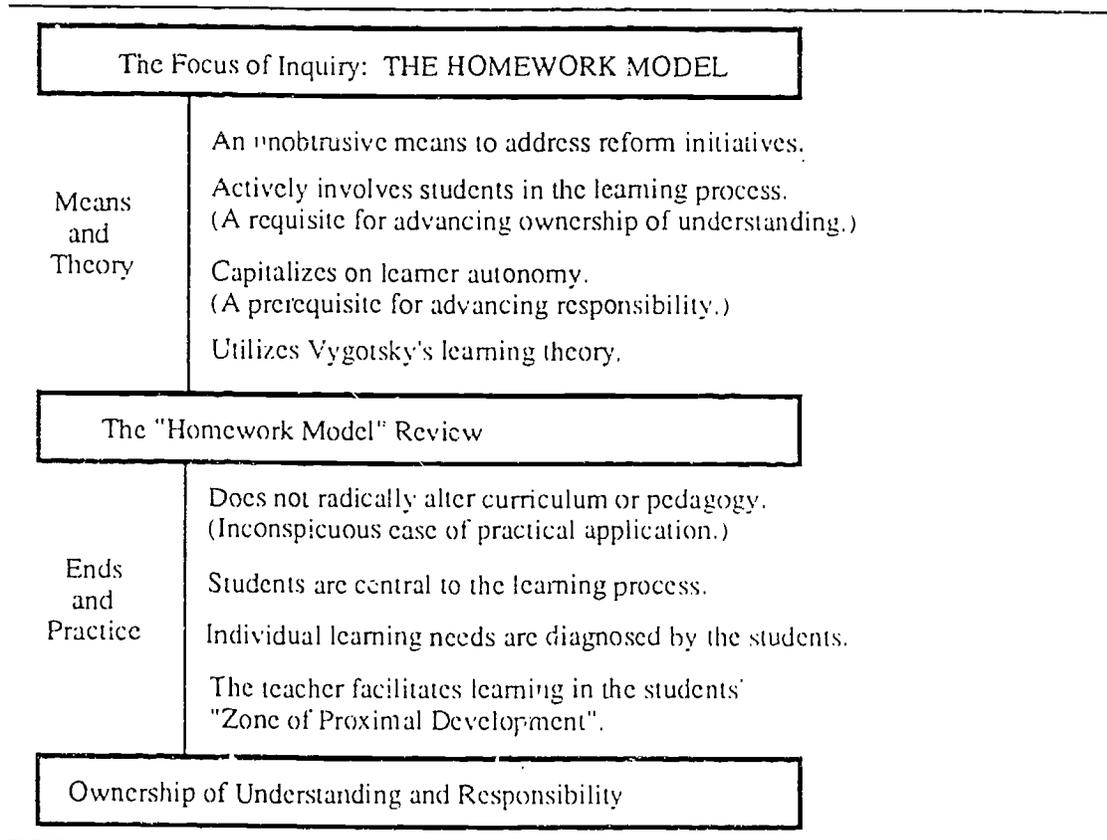


Figure 1.4: Developing Outcomes of Educational Significance

central to the learning process, they become learners. When students are able to diagnose confusion, teachers can design learning experiences to clarify the confusion and advance understanding. When students accept a reasonable share of the independent, self-governing work to solve the problem by isolating and communicating points of homework confusion, responsibility is advanced.

If the demand for reform is accepted and there is a perceived need its accomplishment, the question which must be addressed is, "Who is responsible for correcting the worsening problem solving situation?". If the likely case is that teachers would be responsible for effecting change, they must ask two questions, "Is there a theoretical framework of problem solving that can provide a model for advancing the recommended mathematical behavior?", and, "Is there a theoretical framework of developmental learning that is consistent with problem-solving theory?". The answers to these two questions provided the theoretical support for the proposed practices (implementation, application, and evaluation of the Homework Model) aimed at addressing and improving the problem solving behaviors of high school mathematics students by advancing learner responsibility and ownership of understanding within the contexts of Vygotskian education psychology.

The research examined a **homework model** intended to advance student **ownership of understanding** and **responsibility** within the framework of **problem solving theory** and **Vygotsky's theory of development**. The homework model is an adaptation of Polya's (1988) plan for solving problems. By adapting and applying Polya's problem solving plan to the homework, the teacher places a reasonable share of work to solve the problem on the student as well as the responsibility of understanding the problems.

The homework model identifies four salient phases in solving homework problems. Students' homework should have demonstrated that, 1) they have understood the problems, 2) devised plans for solving the problems, 3) engaged those plans to effect their solutions, and 4) checked their solutions. If the students had confusion with their homework, they were asked to communicate their confusion using the four step, problem solving plan. This placed a reasonable share of the work to solve the problem and the burden of responsibility for understanding with the student, making them active participants in the learning process.

**BEST COPY AVAILABLE**

Ownership of understanding initiates the students' mathematical belief systems. Understanding is more than learning, it implies a much more complete or educationally significant outcome. Bloom (1981) suggests that learning should progress through six levels; knowledge, comprehension, application, analysis, synthesis and evaluation. Understanding is developed when learners demonstrate the ability to not only apply comprehended knowledge, but also analyze, synthesize, and evaluate their knowledge. Understanding, as an educational outcome, develops evidentiary belief systems critical to fostering mathematical problem solving behaviors. When students isolate and communicate their points of confusion they are analyzing, synthesizing, and evaluating their knowledge and are demonstrating an ownership of understanding. Through student conversations which reveal understanding, insights into problem solving behaviors become evident. It is the researcher's contention that ownership of understanding is advanced when students are active agents and central to the learning process. The shift from teacher-verifying to student-realization is important to ownership of understanding. When students move from mechanically applying "the algorithm of the day" to inferencing, conjecturing, and reasoning about problem solving, understanding is undeniably advanced (Resnick, 1987).

Responsibility is a behavioral term, it involves the students' independence and self-governance (autonomy). Without autonomy students cannot become responsible. Self-discipline is an affective means of developing responsibility. According to Etzioni (1984), the self-discipline aspect of homework is more important than student achievement because it "inculcates habits of self-control and a capacity for deferred gratification that prepare the student for the adult working world" (p. 24). Responsibility is often inappropriately used synonymously with accountability. The former can be the source or cause of behavior whereas the latter explains behavior and cannot be the source or cause of the behavior, it must rely on a mediating factor (often in education, grades) which associates a causal relation. When the students assume a "reasonable share" of the work to solve problems

they are becoming responsible learners, actively involved in the learning process. Their active involvement in developing understanding advances responsibility.

Polya (1988) and Schoenfeld (1985) problem solving methodologies provide a framework for promoting the mathematical behavior necessary for solving problems (see figure 1.5). Polya's first step in solving problems requires an "understanding of the

---

## JUXTAPOSITION OF PROBLEM SOLVING METHODOLOGIES

---

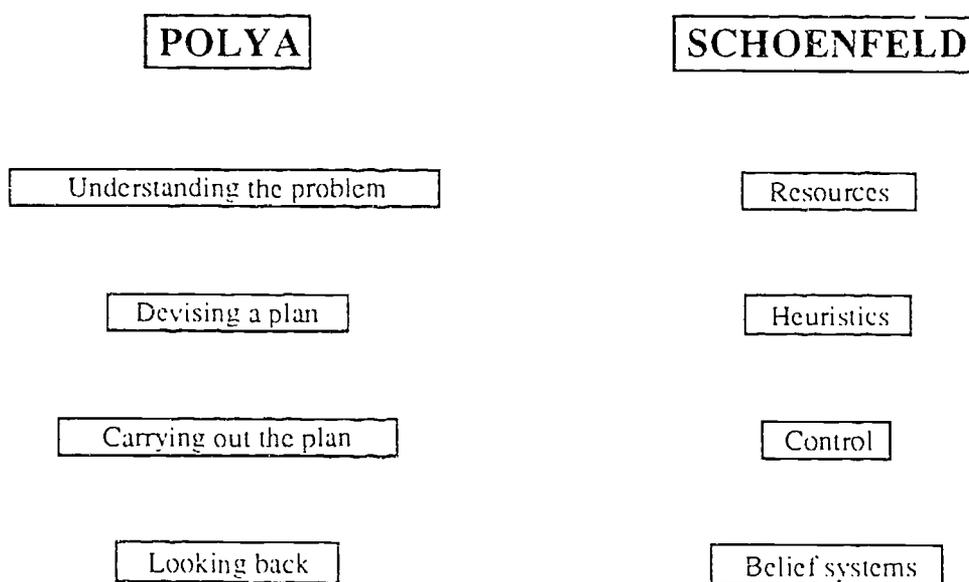


Figure 1.5: Juxtaposition of Problem Solving Methodologies

problem"; what is the given, what is the unknown, what are the conditions. Schoenfeld's "resources" step is the actual mathematical knowledge owned by students: basic skills, experience, or ability to "understand the problem". Their "devising a plan" and "heuristics" steps elaborate the strategies used in solving problems. "Carrying out the plan" and "control" steps exercise understandings and strategies to solve the problem. These are the students' mathematical gymnastics. It represents one part of their active involvement in solving problems. The final steps of "looking back" and "belief systems" involve examining the problem's solution; checking results and applying results to other problems.

developing a cognitive mathematical understanding. Problem solving, in educational terms, requires students to work from a "problem state" to a "solution state" by applying heuristics (Mayer, 1992). If students have difficulty solving problems, areas of confusion could be in their interpretation of the problem, or possibly an unfamiliarity with problem solving strategies, or in carrying out their strategies to effect a solution. The students' final problem solving step, of checking their results, is where mathematical belief systems are developed.

Working from a problem state to a solution state requires that students understand the problem and have the **resources** necessary for its solution. This immediately addresses the students' actual developmental level. Working to the solution state of an unfamiliar problem is the goal of problem solving instruction. If the student is developmentally ready for solving the problem, the teacher may guide or manage the learning processes by providing facilitative instruction during the "homework model" review, addressing the students' potential to solve the problem. When teachers work from what students understand to what students are able to understand, they are working in Vygotsky's (1978) "Zone of Proximal Development". The zone of proximal development is defined as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (p. 86). The actual developmental level defines functions which have matured and the zone of proximal development represents those functions in the process of maturing.

Vygotsky (1978) believed that behavior included the "specification of the societal context in which the behavior developed" (p. 6). This idea of "society" is an extremely important perspective in Vygotskian theory. In the mathematics classroom "society", when the teacher poses a problem to be solved, both the teacher and student work in the zone of proximal development. In a Vygotskian "school", learning in the zone of proximal development is advanced by helping the student to understand decontextualized concepts

within a discipline (Zeuli, 1986). The zone of proximal development is a phase in development where a person, unable to perform a task independently, can eventually accomplish and internalize it with the didactic assistance of someone more experienced. Vygotsky's key ingredient for learning in the zone of proximal development is the structured interaction of the more experienced person leading the learner through the task. This structured interaction is where understanding is developed.

Working from the problem state to the solution state involves a transition through the zone of proximal development in four levels [adapted from Wertsch, (1979)]. The transition through these four levels represents an educationally significant connection between Polya's (1988) problem solving theory to Vygotsky's (1978) theory of development.

Level I: The students are unable to understand the problem or the task situation and they are unable to begin or the task.

Level II: The students understand the problem but are unable to solve it because they do not have a plan or strategy (heuristic). The students may not be able to respond to specific directives or commands of the teacher in connection with the specific task because of a particular inexperience with the necessary problem solving strategy.

Level III: The students understand the problem or task situation and have a plan for its solution. The students have taken on a significant share of the strategic responsibility for the task, yet is unable to effect the correct solution. They respond to specific directives but are unable to carry out the plan. At this level, a computational error or other control factors are possibly blocking the correct solution.

Level IV: The problem-solving activity progresses from the interpsychological to the intrapsychological plane. In the process of working from the problem state to the solution state the students develop mathematical problem solving behaviors and establish mathematical belief systems. The student successfully carries out the plan without any strategic assistance from the teacher.

Determining students' actual (mental) development can be accomplished through homework and the students' ability (or inability) to solve problems independently. "In evaluating mental development, consideration is given only to those solutions to test problems which the child reaches without the assistance of others, without demonstrations, and without leading questions" (Vygotsky, 1978, p. 88). The students' actual level of development represents their prior knowledge; the knowledge necessary to make connections with new knowledge. The homework problems students are able to solve without any strategic assistance from the teacher represent student resources. Once the student's actual level of development has been determined, the teaching and learning of solving unfamiliar problems can begin. This orients learning toward students' development, linking prior knowledge to new knowledge. The zone of proximal development more accurately demarcates the learner's potential level of development. This is a higher cognitive level of development than the actual mental level of development. An application of Vygotsky's (1978) theory is in the "homework model" review, where didactic, facilitative teaching represents the means through which understanding is advanced.

### **Mathematical Pedagogy**

For the results of this research to be transferable, contemporary mathematical pedagogy must be understood so that the reader will develop a vicarious perspective of educational orientations. Two educational orientations, teacher-centered (social behaviorist) and student-centered (experientialist), which complement efficient and effective learning can be juxtaposed within the context of understanding (Efficient learning is skillfully mastering the use of algorithms and effective learning is conceptual understanding) (see figure 1.6). This juxtaposition raises the question, "Who owns the understanding?". In teacher-centered classrooms, the ownership of understanding belongs to the teachers and is shared with their students. In these classrooms, the students learn by accepting this transfer of knowledge, in a non-cognitive sense (faith), and rely on the

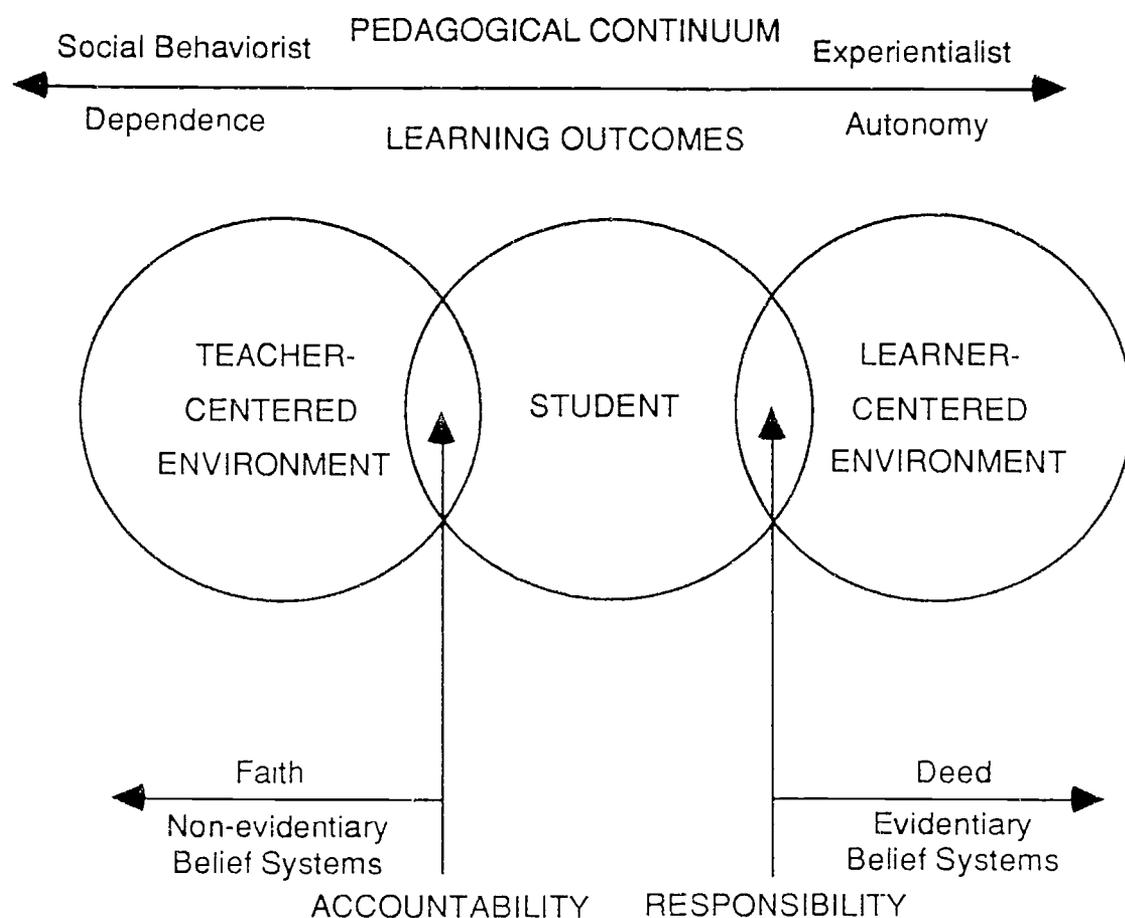


Figure 1.6: Autonomy and Responsibility (Spadano & Zeidler, 1996)

teacher for verifying their resultant belief systems. In the learner-centered classroom, the learner is an active agent, central to the learning process and gains ownership of understanding, in a cognitive sense (deed or reason) (National Research Council, 1989). Recent education reform initiatives (AAAS Project 2061, 1990; Massachusetts Department of Education's Curriculum Frameworks, 1995; NCTM Standards, 1989, 1991, 1992, 1995) suggest a shift from the traditional teacher-centered classroom to one which is student-centered. Capitalizing on learner autonomy through the use of the homework model, the learner is expected to become actively involved in "understanding" homework problems, satisfying suggested education reform initiatives. Consequently, the learner's resultant mathematical problem solving behaviors may be less dependent on the teacher for

BEST COPY AVAILABLE

verification. When contrasted pedagogical means are different, the examination of ends, in this case mathematical belief systems, developed through ownership of understanding and responsibility, are of interest. The debates between the teacher-centered, social behaviorists (with their adherence to measurement, precision, efficiency, and mechanical technique) and the student-centered, experientialists (with their child-centered, democratic, effective, problem-solving orientation) pit formal science against investigative science. The social behaviorist seeks controls through generalized knowledge by credentialed experts while the experientialist explores through inquiry with others, for insights into experiences that may not yet be known (Schubert, 1986). Homework has its place in each educational orientation (see figure 1.7). For the social behaviorist, homework serves as drill and reinforcement of the prescribed criteria, helping students gain mastery of behavioral objectives. Dewey (1938) referred to this as assign-study-recite and labeled it static instruction. The student often asks the teacher, "Is this right?", verifying the control of the teacher, as the owner and authority of knowledge. This creates students' dependence on the teacher for confirmation of understanding. The experientialist uses homework to initiate and develop independence, helping students gain ownership of understanding. The student develops a belief system that is compared, not only to the teacher's, but also to their peers. Often associated with the teacher-centered (social behaviorist) and student-centered (experientialist) orientations of education is the debate of whether teachers should teach the algorithms of concepts or have students derive algorithms with their understanding of the concept (Resnick, 1987). This debate is often triggered by their varied interpretations of learning, the learners' role in learning, how learning occurs, and what results from learning (see figure 1.8). In teacher-centered classrooms, the efficient use of time is often the deciding factor in giving the student the algorithm and understanding the concept is something that will develop if the student is able and genuinely interested in learning its derivation. This efficient use of time allows students to be introduced to many other algorithms and indoctrination to more algorithms

# JUXTAPOSITION OF EDUCATIONAL ORIENTATIONS WITHIN THE CONTEXT OF HOMEWORK

The juxtaposition of these educational orientations is important only to the extent as to how they relate to homework. Homework has a meaningful place in each orientation.

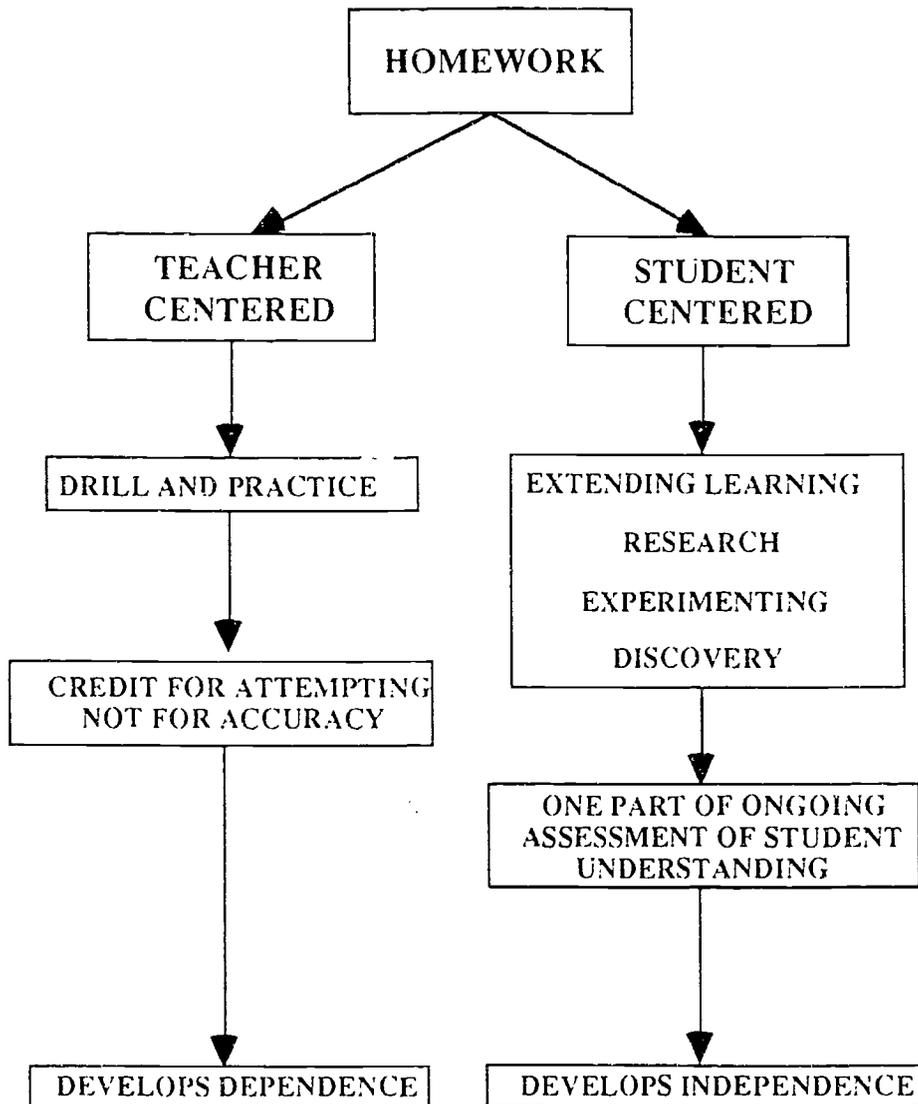


Figure 1.7: Homework's Role in Educational Orientations

---

## JUXTAPOSITION OF EDUCATION ORIENTATIONS

---

### SOCIAL BEHAVIORIST

Learner is passive and reacts to environment.

Learning occurs through associations among or between stimuli and responses. Grades act as mediating factors.

Knowledge consists of associations which have been learned.

Learning is acquiring new associations.

Education consists of arranging stimuli so desired associations are developed.

### EXPERIENTIALIST

Learner is active and masters environment.

Learning occurs as a result of learner trying to understand the activity. Assessment is embedded holistically.

Knowledge consists of cognitive structures developed experientially.

Learning is constructed by new experiences.

Education consists of allowing/encouraging exploration of complex environments.

Figure 1.8: Juxtaposition of Education Orientations

Adapted (Phye & Andre, 1986, p. 2)

is more knowledge. I hesitate to call this teaching and would distinguish it as indoctrination. This is a term that Green (1971) contrasts against teaching, in the context of beliefs. Indoctrination develops beliefs non-evidentially (faith) while teaching does so evidentially (deed or reason). The difference is not necessarily in the ends of beliefs, but rather the means by which they are developed. "In short, even though the beliefs one holds

are true, one cannot be said to know they are true, if they are believed in this non-evidential fashion" (Green, 1971, p. 299).

In student-centered classrooms, students explore a concept, discover a pattern or relationship, and derive a corresponding algorithm, developing their own understanding which may be more time-consuming but is justified by the belief that it is "better" (effective) learning. "To focus simply upon securing a right solution without understanding the nature of mathematical operations is the mathematical equivalent of indoctrination" (p. 303). The inefficiency of the experiential model, or the student-centered classroom, is often considered to be the enormous quantity of mistakes students make in constructing understanding. However, it is precisely during mistakes where proponents of the experiential model claim that students (in fact, people in general) actually begin to understand. Borasi (1987; 1989) suggests that student errors can foster a deeper and more complete understanding of mathematical content if used as starting points for mathematical explorations in problem solving. This is active learning. It is experiential understanding constructed by the learner's deed or reason. Learning in this evidentiary sense is cognitive. The experientialist may admit to the "time" factor but is adamant that effective, genuine understanding is more important than efficiency.

The literature tells us very little about homework in an experiential setting. The primary focus of homework appears to be in a social behaviorist setting, reinforcing content through drill-and-practice. There is minor, if any, attention given to homework as a vehicle for developing autonomous learners. Therefore it is quite clear, from the lack of literature, that there is virtually no homework model which addresses the development of student responsibility. In fact, it is obvious that just the opposite is true. The bulk of the literature is devoted to how the teacher should go over, or manage going over, the homework, not the student. The current state of homework does not develop student responsibility, it develops student dependence. The student learns very quickly in the mathematics classroom that attempting to do homework is sufficient, since credit is given

for effort, not understanding, and whatever effort is made the teacher will go over the homework questions in class. This allows students to attempt just enough to get credit for doing the homework and it teaches them nothing about their self-governing control in gaining ownership of understanding. The student also learns that homework is likely not to be collected and evaluated in contemporary teacher-centered classrooms, removing purpose and importance from doing the homework.

### **Research Methodology**

The methodology of this qualitative research was interactive and data, generated by the implementation, application, and evaluation of the Homework Model, were collected and analyzed by two evaluative means (See Figure 1.9). The first was formative evaluation data, which searched for content, themes, and reactions, and the second was summative evaluation data, which elaborated on content and themes through interpretations and judgments. The research's emergent design was not self-reported opinions of information. It involved teachers as researchers and mutually shaped data to guard against deliberate or subconscious distortions. The two data sources and the emergent design attempted to improve and understand the focus of inquiry, a homework model intended to develop students' problem solving behaviors by advancing students' ownership of understanding and responsibility. The grounded theory or "working hypotheses", developed during the research, are the research's outcomes.

### **Formative Evaluation**

In this research, the teacher-research assistants (respondents) were involved with the collection and analysis of data which were arranged and organized to facilitate "researcher-respondent" discussion. "The very requirement of an emergent design, in which succeeding methodological steps are based on the results of steps already taken, implies the presence of a continuously interacting and interpreting investigator" (Lincoln & Guba, 1985, p. 102). This research's constructed reality was supported by evaluative data, both formative and summative. During the course of collecting formative evaluation

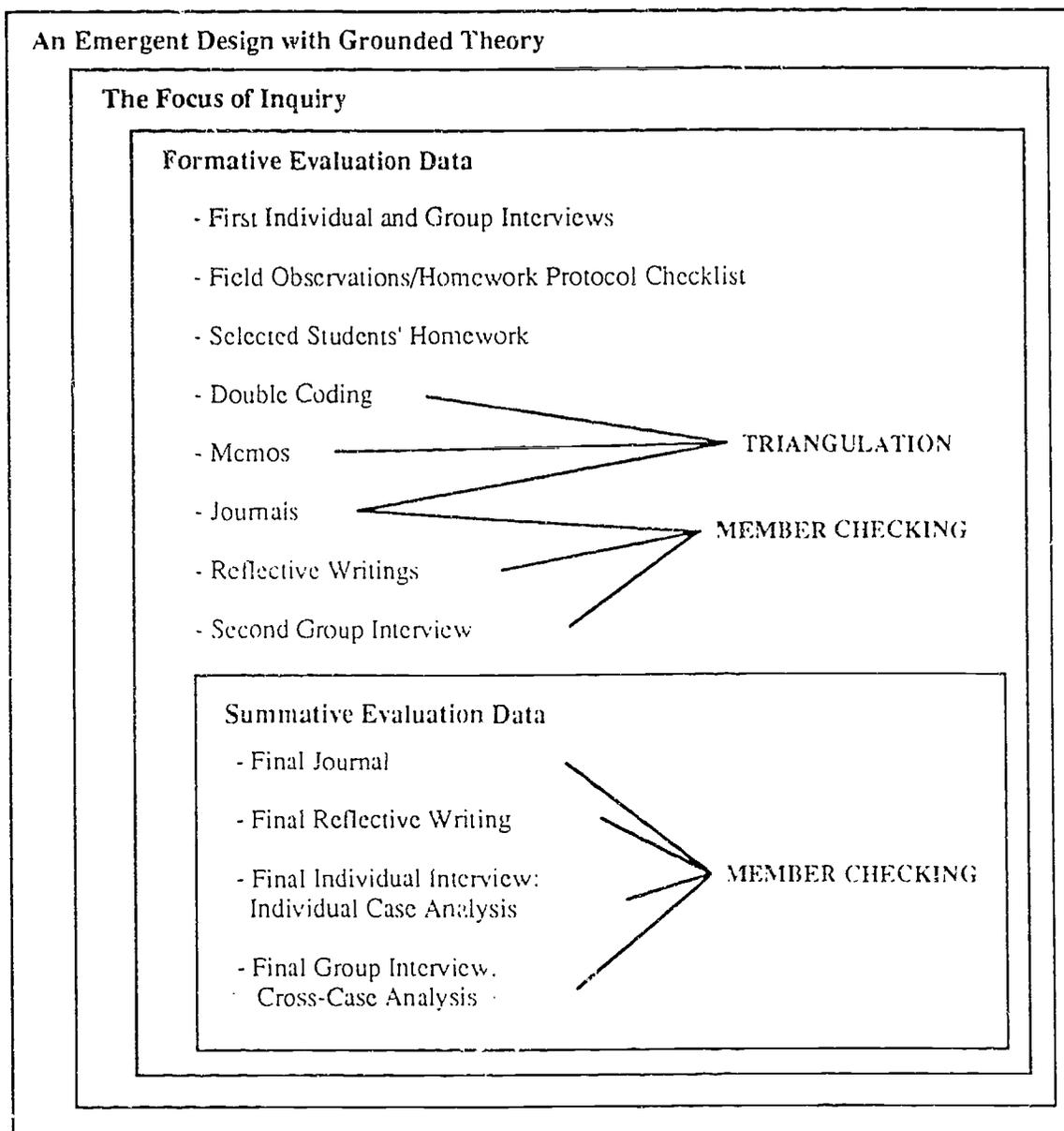


Figure 1.9: Research Methodology Legend

data, the researcher and teacher-research assistants evaluated the phenomena surrounding the homework model. The evaluation involved many modes and was designed to refine and understand the evaluand.

### Procedures and Explanations

The individual, audio-taped, semi-structured interviews, provided ethical inquiry with flexible probing. The interviews preceded the training sessions and determined each

teacher's policy regarding homework as well as their perceptions of the importance and relevance of homework to student understanding and responsibility. The data collected from these interviews were individually and cross-cased analyzed for common themes. The first group interview was audio-taped and followed the training sessions. During this interview, the teacher-research assistants discussed their understanding of the theory, application, and evaluation of the homework model. Both the individual and group interviews were referenced in summative evaluation data to provide insights into the behavioral and procedural changes regarding homework as a means of advancing student understanding and responsibility.

The researcher observed every class involved in the study and audio-taped and transcribed each class' homework review. The purpose of conducting the field observations was two-fold. First, the researcher determined whether the intended theoretical design of the "homework model" review was being implemented. If the design or methodological procedure was not following its intended path, the researcher scheduled time to discuss the homework model's application with the teacher-research assistants. Second, the researcher gained tacit knowledge, searching for and noting non-verbal data that added insight to the transcribed audio tapes. These data included, classroom management, time spent reviewing homework, classroom activities, and gestures that emerged as the research took place. Data from field notes were organized, analyzed, and reduced by a categoric system where entries from observational notes were arranged according to categories and continua in protocol checklists to further search for details involved in the phenomenological experiences related to the use of the homework model.

A non-random sample of selected students' homework was analyzed by the teacher-research assistants and the researcher. The teacher-research assistants provided input as to which students would be considered for selection. The analysis of the selected students' homework determined the degree to which the students applied the homework model, as evidenced by salient parts of the problem solving process and isolated points of confusion.

Reference to judgments involving selected students' homework was made in the summative evaluation data.

The researcher and teacher-research assistants coded and compared transcripts of the homework model review on a daily basis. During the week of training and throughout the study, the teacher-research assistants and the researcher developed the labels for coding information units which related to problem solving behaviors, ownership of understanding and responsibility. The coding of individual information units provided vocabulary and generated perspectives by which the researcher and research assistant could effectively communicate. The researcher adapted the double-coding method explained by Miles and Huberman (1984) in the analysis of data where the researcher and the research assistant independently coded transcripts of the homework model review, reviewed and compared their coding, and then discrepancies in coding were clarified. Unreconciled interpretations were noted, however, "the investigator is not bound to honor all of the criticisms [disagreements] that are mounted, but ... is bound to hear them and weigh their meaningfulness" (Lincoln & Guba, 1985, p. 315). Discussion times were mutually arranged and formally scheduled to reconcile discrepancies.

The memo was intended to be a brief reaction to phenomena surrounding the implementation of the homework model and was to be completed during or at the end of reviewing each previous night's homework by the teacher-research assistants so that the researchers could use the memos when making specific reference to transcriptions. The memo reduced the data by analyzing areas of student success with isolating points of confusion and areas of teacher success with questioning, prompting, and providing hints or suggestions to clarify the students' homework confusion to allow the student to continue solving the problem. The researcher triangulated this data with the teacher-research assistants by using the coded homework review transcripts.

The journal was written weekly by the teacher-research assistant and was intended to cull the week's daily class activities into the growth of a cumulative experience. In the

journal, the teacher-research assistants were asked to summarize the data from their memos by uncovering and explicating covert perceptions and feelings or reactions to the phenomena surrounding the implementation of the homework model. In addition, the teachers were asked to comment on specific observations which supported their perceptions about their students' mathematical behavior and responsibility as well as any other categories they felt were necessary. The researcher confirmed this data through triangulation and member checking.

The reflective writings were one or two pages of teacher-research assistant reactions, interpretations, and judgments of the overall effects of the phenomena surrounding the implementation of the homework model and were written at three times during the study: once after the training sessions, the second at the halfway point of the study, and the third at the conclusion of the study. The first two reflective writings were aimed at impressions regarding the homework model's benefits or limitations that were considered themes of the research. However, the teacher-research assistants were not limited to any particular feature of the phenomena and could personalize their reflections. It was the researcher's responsibility to juxtapose the teacher-research assistants' reflections for summative evaluation data.

### **Summative Evaluation Data**

The strength of the phenomenological descriptions involved the specific experiences as conveyed by the teachers. The transcriptions of their descriptions were analyzed for common themes about the phenomena generated by the use of the homework model. With reference to themselves and their students, the teachers spoke and wrote with conviction. The analyses of the teachers' experiences, governed by triangulation and member checking, contributed to the trustworthiness and credibility of their testimony. "Making sense" of collected data went beyond individual testimony to include group input, as it began to address the issue of "transferability" (Lincoln & Guba, 1985).

Summative evaluation data were collected and analyzed at the conclusion of the research's field observations. This data analyzed the descriptions, interpretations, and judgments of issues and themes that followed from the formative evaluation data and provided the means for explaining the phenomenological experiences surrounding the use of the homework model. Summative evaluations were shared with the teacher-research assistants for the purpose of their critique. This research's constructed reality was mutually shaped and both the researcher and teacher-research assistant evaluated the overall effects of the phenomena surrounding the homework model. The researcher confirmed this data through member checking.

### **Procedures and Explanations**

The final journals and final reflective writings included judgments directed to the overall effects of the homework model and were prepared by the teacher-research assistant at the conclusion of the research's field observations. It was the researcher's responsibility to juxtapose the teacher-research assistants' reflections as they contributed to summative evaluations.

The researcher conducted a final interview with each teacher-research assistant to discuss each Individual Case Analysis (Miles & Huberman, 1984) to report individual findings. In this Analysis, data were examined, a first draft was written by the researcher, which contained a summarization of content and themes of each individual case and then an interpretation of results. These results were shared with the teacher-research assistants during the final individual interviews for the purpose of collecting feedback for the Analyses' interpretations and judgments. Actual quotations were used whenever possible to support themes and add credibility through testimony.

The researcher conducted a final group interview with the teacher-research assistants to discuss research outcomes through the Cross-Case Analysis (Miles & Huberman, 1984). In the cross-case analysis, the researcher elaborated on the content, themes, reactions, and judgments of the phenomena surrounding the homework model

study. The cross-case analysis looked for the replication of patterns identified within and across individual case analyses. The principal elements examined in the cross-case analysis were content (the coded homework reviews and experiential feedback), form (the application and refinement of the homework model), and context (the effectiveness of the homework model as a means of advancing ownership of understanding and responsibility). The teacher-research assistants offered feedback through an assessment of the report's overall adequacy during this final group meeting. In both the Individual and Cross-Case Analyses, the researcher reconstructed recognizable representations and provided the teacher-research assistant with the opportunity "to correct errors of fact and challenge what were perceived to be wrong interpretations" (Lincoln & Guba, 1985, p. 314).

### **Research Outcomes**

The findings from this research pertain to particular events for a specific time, place, and situation. The contexts of the idiography which facilitated the continuing unfolding of this research's emergent design must be explicated and understood in order to make sense of the data and transfer its reported findings (Spadano, 1996). The outcomes of this inquiry, that may be transferable, are "working hypotheses" useful for educational considerations. The educationally significant outcomes of this research involve how teachers teach and learners learn. Having the opportunity for a prolonged, intensive reflection on their homework practices, the teacher-research assistants became "students", learning many important aspects of how they manage their classrooms. Most frequently, comments centered around the importance of isolating points of confusion in designing learning experiences to clarify the confusion. Having the opportunity to isolate their points of confusion, the students were perceived to become independent, self-governing "teachers", as they solved homework problems and developed their problem solving behaviors. To some degree, from this, we can imply that teachers became students and students became teachers, unusual outcomes with educationally intriguing importance. Students that accepted a reasonable share of the work to solve the problems advanced their

responsibility. Students that were able to isolate and communicate their points of homework confusion, so that teachers could design learning experiences to clarify the confusion, advanced their ownership of understanding.

Embedded in the Homework Model's educationally significant outcomes are two major "working hypotheses". 1) The homework model increased mathematical communication (internal and external) and familiarized students with a structured, problem solving framework which they referenced when doing homework, particularly when they did not get correct answers. In order for students to convey their isolated points of confusion with homework problems, they had to communicate mathematically. The transcriptions of the homework model reviews overwhelmingly evidence teachers and students thinking and speaking mathematically within the problem solving framework. This stands in stark contrast to contemporary mathematics classrooms where just the teachers do the thinking and speaking. 2) The homework model review provided a means to efficiently and effectively provide individualized instruction. Instead of doing the entire problem that a student requests, isolated points of confusion were the focus of homework review. Homework problems done in class were done to provide learning experiences to address the specifically diagnosed needs of students with attention given to the isolated point of confusion. The mathematical didactic dialogue of the homework model review, as evidenced by transcripts, illustrates the active involvement of students in analyzing and diagnosing their problem solving behaviors and needs, so that the teacher could facilitate and provide learning experiences to address those needs. These two outcomes are in direct alignment with Vygotsky's (1978) principles of learning development; interpsychological to intrapsychological activity and the zone of proximal development. The teacher-research assistants recognized the importance of developing students' problem solving behaviors and adapted and applied Polya's ideas not only during homework but also to their mathematics teaching practices. Their experiences with the homework model helped them create an instructional model for their classrooms. In this subtle sense, the homework

model initiated experience and contributed to the creation and refinement of a mathematics environment which incorporated problem solving theory.

## LIST OF REFERENCES

- AAAS, (1990). Science for all Americans: Project 2061. New York: Oxford.  
[American Association for the Advancement of Science].
- Bloom, B. S. (1981). Evaluation to improve learning. New York: McGraw-Hill Book Co.
- Borasi, R. (1987). Exploring mathematics through the analysis of errors. For the Learning of Mathematics: An International Journal of Mathematics Education, 7, (3), 2-8.
- Borasi, R. (1989, March). Students' constructive uses of mathematical errors: A taxonomy. Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco, CA.
- Carpenter, T. P., Lindquist, M. M., Matthews, W. and Silver, E. A. (1983). Results of the third NAEP mathematics assessment: Secondary school. Mathematics Teacher, 76, (9), 652-659.
- Case, R. and Bereiter, C. (1984). From behaviorism to cognitive development. Instructional Science, 13, 141-158.
- Cobb, P. and Steffe, L. P. (1983). The constructivist researcher as teacher and model builder. Journal for Research in Mathematics Education, 14, 83-94.
- Davis, R. B. (1984). Learning mathematics: The cognitive science approach to mathematics education. Norwood, N. J.: Ablex.
- Dewey, J. (1938). Experience and education. New York: Macmillan.
- Etzioni, A. (1984). Self-discipline, schools and the business community. Washington, D. C.: National Chamber Foundation.
- Green, T. F. (1971). Studies in philosophy and education: A topology of the teaching concept. New York: Syracuse University.

Hall, G. E. and Loukes, S. (1979). Teacher concerns as a basis for facilitating and personalizing staff development. In A. Lieberman and L. Miller (Eds.), Staff development: New demands, new realities, new perspectives. New York: Teachers College Press.

Hiebert, J. (Ed.). (1986). Conceptual and procedural knowledge: The case of mathematics. Hillsdale, N. J.: Lawrence Erlbaum Associates.

Jackson, P. W. (1971). Old dogs and new tricks: Observations on the continuing education of teachers. In L. J. Rubin (Ed.), Improving in-service education: Proposals and procedures for change. Boston: Allyn and Bacon, p. 19-36.

Lampert, M. (1986). Knowing, doing, and teaching mathematics. Cognition and Instruction, 3, 305-342.

Lesh, R. and Landau, L. (Eds.) (1983). Acquisition of mathematics concepts and processes. New York: Academic Press.

Lincoln, Y. S. and Guba, E. G. (1985). Naturalistic Inquiry. Newbury Park, CA.: Sage.

Massachusetts Department of Education (1995). In Coneys, M. (Ed.) Curriculum Frameworks, Boston.

Mayer, R. E. (1992). Thinking, problem solving, cognition. New York: Freeman

Miles, M. B. and Huberman, A. M. (1984). Qualitative data analysis. Beverly Hills: Sage.

NAEP (1982). The third national assessment of educational progress (NAEP) mathematics assessment. Washington, D. C.

NCTM (1989). Curriculum and evaluation standards for school mathematics. Reston, VA.

NCTM (1991). Professional standards for teaching mathematics. Reston, VA.

NCTM (1992). Curriculum and evaluation standards for school mathematics: Data analysis and statistics across the curriculum. Reston, VA.

- NCTM (1995). Assessment standards for school mathematics. Reston, VA.
- National Research Council, (1989). Everybody counts: A report on the future of mathematics education. Washington, D. C.: National Academy Press.
- Orlich, D. C. (1989). Staff development: Enhancing human potential. Needham Heights, MA: Allyn and Bacon.
- Phye, G. D. and Andre, T. (Eds.). (1986). Cognitive classroom learning: Understanding, thinking, and problem solving. San Diego: Academic Press.
- Polya, G. (1988). How to solve it: A new aspect of mathematical method. Princeton, N.J.: Princeton University Press.
- Resnick, L. B. (1987). Education and learning to think. Washington, D. C.: National Academy Press.
- Schlechty, P. C. (1990). Schools for the twenty-first century. San Francisco: Jossey-Bass.
- Schoenfeld, A. H. (1985). Mathematical problem solving. San Diego, California: Academic Press, Inc.
- Schoenfeld, A. H. (1987). Cognitive science and mathematics education. Hillsdale, N. J.: Lawrence Erlbaum Associates.
- Schubert, W. H. (1986). Curriculum: Perspective, paradigm, and possibility. New York: Macmillan Publishing Company.
- Spadano, J. W. (1996). Examining a homework model as a means of advancing ownership of understanding and responsibility in secondary mathematics education. Unpublished doctoral dissertation, College of Education, University of Massachusetts-Lowell.
- Spadano, J. W. and Zeidler, D. L. (1996). What's the buzz? Tell me what's happening! Ownership of understanding and responsibility. Manuscript submitted for publication.

Vygotsky, L. S. (1978). Mind in society. Cambridge, MA: Harvard University Press.

Wertsch, J. V. (1979). From social interaction to higher psychological processes: A clarification and application of Vygotsky's theory. Human Development, 22, 1-22.

Zaltman, G., Floris, D. H. and Sikorski, L. A. (1977). Why research is not readily used in educational settings. Dynamic Educational Change. New York: The Free Press.

Zeuli, J. P. (1986). The use of the zone of proximal development in everyday and school contexts: A Vygotskian critique. Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco, California.