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ABSTRACT

This study examined the differential effects of two instructional strategies, an explicit schema-based strategy and a general cognitive strategy, on the acquisition, maintenance, and generalization of mathematical word problem solving by students with mild disabilities and at-risk students. Twenty-three second, third, fourth, and fifth graders were randomly assigned to each of the 2 treatment conditions (schema and traditional). Results indicated that both groups' performance increased from pretest to posttest. Generally, students in both groups maintained their use of word problem solving skills. However, word problem solving scores from the posttest to delayed posttest showed a slight decrease for the traditional group, while an increase in scores was noted for the schema group. The performance of the schema group (84%) surpassed that of their normally achieving peers (82%). In addition, generalization of word problem solving skills to novel word problems occurred for both groups. Contains 41 references. (Author)

Running Head: Word Problem Solving

The Differential Effects of Two Strategies on the Acquisition, Maintenance, and Generalization of Mathematical Word Problem Solving by Students with Mild Disabilities and At-Risk Students

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Abstract

This study examined the differential effects of two instructional strategies, an explicit schema-based strategy and a general cognitive strategy, on the acquisition, maintenance, and generalization of mathematical word problem solving by students with mild disabilities and at-risk students. Twenty three second, third, fourth, and fifth graders were randomly assigned to each of the two treatment conditions (schema and traditional). Results indicated that both groups' performance increased from pretest to posttest. Generally, students in both groups maintained their use of word problem solving skills. However, word problem solving scores from the posttest to delayed posttest showed a slight decrease for the traditional group, while an increase in scores was noted for the schema group. The performance of the schema group (84%) surpassed that of their normally achieving peers (82%). In addition, generalization of word problem solving skills to novel word problems occurred for bc th groups.

The Differential Effects of Two Strategies on the Acquisition, Maintenance, and Generalization of Mathematical Word Problem Solving by Students with Mild Disabilities and At-Risk Students

Recent results of the 1992 National Assessment of Educational Progress (NAEP) mathematics assessment suggested that American children and youth are seriously deficient in their mathematical problem solving ability. Anrig and LaPointe (1989) noted that only 16% of U.S. eighth-grade students have mastered the content of a typical eighth-grade mathematics textbook. According to Mayer, Tajika and Stanley (1991), the performance of U.S. students in mathematics was relatively low when compared to students in other industrialized countries. Specifically, American 13-year-olds placed last in mathematics in comparison to students in four other countries and four Canadian provinces (LaPointe, Mead, & Phillips, 1989; Anrig & LaPointe, 1989). Empirical investigations support the findings that international differences in students' mathematical performance may be a function of the amount and kind of exposure to mathematics instruction rather than real differences in abilities (Stevenson, 1992; Stevenson, Chuansheng, & Lee, 1993; Stevenson, Lee, Chen, Stigler, Hsu, & Kitamura, 1990; Stevenson & Stigler, 1992; Stigler, Lee, & Stevenson, 1990).

Problems of mathematics underachievement and underinstruction appear to be particularly severe for students with disabilities and at risk for mathematical failure (Nuzum, 1987; Zentall & Ferkis, 1993; Carnine, Jones, & Dixon, 1994). For example, Cawley and Miller (1989) reported that the mathematical performance of 8- and 9-year-old students with learning disabilities was equivalent to the first-grade level, especially on applied mathematical problems. Similarly, the mathematics performance of 16- and 17-year-old students with learning disabilities was at about the fifth-grade level. According to Carnine et al. (1994), "fewer than 25% of the students with learning disabilities demonstrated mastery of measurement skills" (p. 407). Carpenter (1985) noted that students with learning disabilities spend more than one-third of their resource room time studying mathematics.

Based on the National Assessment of Educational Progress (1992), it appears that mathematical word problem solving is especially difficult for students of all ages and ability levels.

Specifically, students with disabilities and at risk who have difficulties with reading, computation, or both are likely to encounter difficulties with word problem solving (Dunlap, 1989). The importance of providing quality instruction for students with disabilities and those at risk for mathematical failure is unmistakably clear. In fact, since the National Council for Teachers of Mathematics (NCTM) identified problem solving as its number one priority in its *An Agenda for Action* (NCTM, 1980, cited in Moore & Carnine, 1989), this area has received considerable attention in the recent literature (Miller & Mercer, 1993).

Given the poor performance of students with learning problems and the call for mathematics curricula to better address problem solving skills, the purpose of this study was to investigate the differential effects of a schema-based strategy and a traditional basal strategy on the acquisition of simple one-step addition and subtraction word problems by students with mild disabilities and at-risk students. A second purpose of the study was to assess the maintenance of the instructional strategies over time and their transfer to a new context. In the following section, we address the importance of fostering schema knowledge and strategic instruction needed for solving mathematical word problems.

Schema Knowledge and Strategic Instruction

Instruction in solving mathematics word problems should adequately emphasize activities that teach both conceptual understanding and efficient execution of processes and strategies (Goldman, 1989). Various models to understand and assess children's solution of addition and subtraction word problems have been postulated. Most have emphasized the importance of the problem's semantic characteristics in contributing to the student's ability to solve them. Current models of word problem solving are generally derived from and influenced by schema theories of cognitive psychology (Briars & Larkin, 1984; Carpenter & Moser, 1984; Fennema, Carpenter, & Peterson, 1989; Kintsch & Greeno, 1985; Riley, Greeno, & Heller, 1983). The importance of schemata in word problem solving is summed up by Marshall (1990) as follows: "The individual's choice of schema to govern the problem solving is at least as important as the particular solution obtained" (p. 157). In the present article, we gleaned from the models of word problem solving postulated by Marshall (1990) and Riley et al. (1983) because they provide an explicit framework of schemata for understanding word problem solving. The essential elements of these models are categorized as problem schemata, action schemata, and strategic knowledge.

Essentially, three steps lead to the solution of a word problem. The first step involves processing the problem schemata or the "definitive characteristics, features, and facts" (Marshall, 1990, p. 158) necessary to recognize and represent the situation described in a problem. Specifically, the step is associated with the identification of the basic semantic relations underlying word problems. Using diagrams and explanations, schema identification instruction makes explicit the semantic relations within the problem context by highlighting a "logical structure that serves to organize the information in the problem" (Marshall et al., 1987, p. 15). Once the learner recognizes that the underlying situation depicts a particular schema relation, he or she can examine the components that make up the schema relation and plan to solve the problem using the second step, action schemata.

Action schemata requires the selection of action procedures (e.g., counting, adding, subtracting) that correspond with the representation of the problem identified in the first step. To activate the action schemata, necessary information (i.e., prerequisites and consequences) must be present in the problem to elucidate the problem schemata and invoke the appropriate action procedures. Therefore, this step requires planning to show the sequence of steps in a way that permits a top down approach to problem solution. However, before the learner can decide on the specific operation to be used, the information represented in the problem may need to be further interpreted. This would entail selecting an arithmetic operation based on which part of the problem situation is unknown and which of the critical elements in the problem structure represents the total.

The third step, strategic knowledge, comprises a set of procedures, rules, or algorithms that can be effectively executed to reach the solution (Marshall, 1990, 1993). In summary, effective word problem solving requires the learner "to create a representation of the problem that mediates solution" (Goldman, 1989, p. 45). There are several ways in which such a representation might be attained: "make tacit processes explicit; get students talking about processes; provide guided practice; ensure that component procedures are well learned; emphasize both qualitative understanding and specific procedures; and test for understanding and reasoning processes" (Heller & Huntgate, 1985, p. 45). Because deficits in memory and attention may constrain the effectiveness of strategy instruction for students with learning disabilities, providing extensive practice in each of the components of word problem solving is an important means of enhancing word problem solving (Goldman, 1989; Pressley, 1986).

For purposes of this study, the three problem types, change, group, and compare, that characterize most addition and subtraction word problems presented in commercial basal mathematics programs were examined (Marshall, Pribe, & Smith, 1987; Marshall, Barthuli, Brewer, & Rose, 1989; Marshall, 1990; Riley et al., 1983; Silbert. Carnine, & Stein, 1990). Change problems usually start with a beginning state in which the objects to be manipulated are specified. Then a change action occurs at some later time that increases or decreases the beginning amount, and the change is permanent resulting in a recognizable ending state. In the change situation, the beginning and ending object identities (e.g., marbles) remain the same. In contrast, group relation problems involve two distinct groups (i.e., subordinate or smaller categories) that are considered in combination to form a new group (i.e., superordinate or larger category), usually having a new identity. A group problem situation is static and requires understanding part-whole relationships and knowing that the whole (i.e., superordinate) is equal to the sum of the parts (i.e., subordinates). A compare problem situation consists of two objects (i.e., compared and referent) that have contrasting values and the difference between the two quantities is determined. Three different questions can be posed for all three problem types (see Table 1).

Table 1 About Here

This study incorporated the use of two instructional strategies, an explicit schema-based strategy and a general cognitive strategy that was derived from a basal mathematics program used in one of the target school districts. The two approaches represented contrasting strategies to teaching mathematics in that the instructional sequence of the schema strategy utilized a domain-specific procedure, while the general strategy utilized a guided discovery or general strategy to solve all word problem types. The general strategy is the most commonly used strategy employed by many basal textbooks (Dixon, Carnine, & Kameenui, 1991). The present study hypothesized that students instructed in a traditional basal strategy would perform less well than those instructed in a schema-based strategy on the posttest, delayed posttest, and generalization test.

Method

Overview

The study consisted of two phases. In the first phase, the effects of schema-based direct instruction on the acquisition of simple one-step word problems were assessed by contrasting the performance of two groups of students with mild disabilities and at-risk for mathematical failure. These students were placed into pairs based on prerequisite computational skills test scores and randomly assigned to each of the two treatment conditions (schema-based and traditional). In the second phase, maintenance and generalization of the two instructional strategies were assessed. Dependent measures included word problem solving scores on criterion tests prepared by the first author.

Subjects

Twenty three second, third, fourth, and fifth graders from elementary schools in a northeastern and a southeastern school district participated in the study. Of the potential sample of 23 students, 14 were classified as having mild disabilities (LD, EMR, or SED), while the remaining nine students included low performing (at-risk) students who were experiencing difficulty in mathematics. Students were initially selected on the basis of teacher judgments to possess adequate addition and subtraction computational skills, but to be poor word problem solvers.

All students were required to meet three additional criteria. First, as a measure of their computational skills, students were required to successfully complete addition and subtraction computational problems necessary for solving word problems with 90% accuracy. Second, students had to solve simple action problems, a necessary prerequisite to word problem solving, at or above 90% accuracy. Third, students' performance on a criterion test of word problem solving was below 60%. Table 2 describes the characteristics of the two treatment samples. The groups appeared equivalent on sex, grade, age, ethnicity, and special education classification. In addition, there were no statistically significant differences between groups on IQ and achievement scores.

Table 2 About Here

Materials

Instructional Materials. Sets of simple one-step situations and word problems were developed to teach students each of the steps in the schema-based strategy described in the instructional procedures section. Each strategy step focused on different types of addition/subtraction word problems (e.g., change, combine, and compare). In addition to the word problems that were used to teach the specific problem types, additional sets of one-step word problems were used to teach students to apply the schemata rules in concert. Probes for each strategy step were also designed to assess students' mastery in the use of the strategy steps.

Testing Materials. Three forms of a 15-item problem-solving instrument were designed for the pretest, posttest, and delayed posttest. Each form contained five instances of three problem types based on the earlier categorization scheme (i.e., change, group, and compare). Forms 1-3 were parallel forms; the only difference in the three parallel sets were the specific numerical values used, which were randomly generated using two-digit and three digit numbers. In addition, to assess generalization of the strategy to word problems different from the controlled ones used during training, a separate form that consisted of a set of 15 one-step addition and subtraction word problems was developed. The generalization set utilized word problems from grades 4 and 5 selected from a basal mathematics program not used in the present study. To determine reliability of the instruments, Forms 1-3 were administered to 24 third graders. The parallel form reliability coefficients of the three forms ranged from .77 to .88. To demonstrate equivalency of the four tests, mean scores were calculated. The scores on the pretest, posttest, and delayed posttest averaged 83%, 81%, and 82% correct respectively.

Student Interviews

Interview protocols developed by the first author were used to gather information about what students liked most and least about each strategy. In addition, students were asked how often they would use the strategy in their class, if they would recommend the strategy to a friend, and how they would rate the strategy on a scale of 1 to 5, with 5 being the most useful and 1 representing the least useful.

Fidelity of Intervention Implementation

Checklists of steps during intervention were developed for both groups to assess agreement for the instructor's delivery of the independent variable. The presentations of instructors in the two conditions were judged on the presence or absence of nine features that corresponded with the critical scripted or planned behaviors. Independent variable agreement was computed as agreements/agreements + disagreements x 100. Approximately 30% of the lessons were observed for treatment fidelity. Analysis of this data for both groups indicated that the instructors observed the instructional procedures 100% of the time.

Procedures

Testing Procedures

Each student was given a packet containing the set of word problems and pencils. On both the pretest and posttest, students were instructed to read the word problems and solve them. No feedback regarding the accuracy of the solution was provided during this time. The number of correct responses was noted. All students were also tested for generalization of the strategy using the same procedures as those for the pretest and posttest. In addition, students in the two instructional strategy groups were tested a week or two later on Form 3 to assess maintenance of the word problem solving strategy.

Instructional Procedures

All Conditions. Forty to 45-minute training sessions were conducted with small groups of 3 to 4 students each in the resource room. To decrease the threat to the integrity of the independent variable, scripted formats were used for each instructional condition. Although both groups were taught with different instructional methods, all students were engaged in instruction for the same amount of time and received identical sets of word problems. To control for the two groups' time with instructors during the second phase of intervention, think math sheets were provided until the allocated time expired. At the end of the intervention session, students were informed about the usefulness of the strategy based on their successful performance and encouraged to use the taught strategy in the future to solve word problems.

Schema-based Training Condition. To attribute changes in word problem solving to instruction in schema based direct instruction, we ensured that students learned each step of the strategy using a mastery-learning paradigm (Bloom, 1976). Students received instruction and guided practice in solving the different word problems using the schema-based strategy which, in this study, was taught in two discrete steps. Students were taught to (a) recognize the features of the semantic relations in the problem (i.e., problem schema) and check that the problem had the salient elements of the chosen problem schema and (b) design a solution strategy (i.e., action schema) and select and execute the correct arithmetic operation (strategic knowledge) (see Marshall, Barthuli, Brewer, & Rose, 1989, for details). In addition, training was based on the principles of explicit or direct instruction (Rosenshine, 1986; Rosenshine & Stevens, 1984; Silbert et al., 1990;): explicit explanation of the rules, modeling the strategy, guided practice in controlled materials, monitoring and corrective feedback, and independent practice.

According to Marshall et al. (1989), instruction is best when all the schemata (change, group, and compare) are discussed together. Therefore, each step of the strategy was taught in concert for the three different semantic relations. During the first phase of training, students were provided with worksheets that included story situations only with problem schemata diagrams (see

Figure 1) and practiced identifying the different problem types and mapping the features of the situation onto the schema diagrams. The problem schemata instruction employed teacher-led demonstration and modeling along with frequent student exchanges to identify the critical elements or constraints of the problem.

Figure 1 About Here

The second phase of training also began with a review of the problem schemata. The only difference was that problems rather than story situations were presented. Students read the word problem orally and the instructor explained the constraints that governed the problem schema. Instruction demonstrated how critical elements of the specific problem are mapped onto the schemata diagrams, while the missing element is flagged using a question mark. This instruction also included teacher-led demonstration and modeling with frequent student exchanges to identify underlying semantic features of the problem. Student misconceptions about problem schema constraints were consistently addressed with explicit feedback and additional modeling.

Next, instruction in the remaining step (action schema and strategic knowledge) of the strategy was presented. The instructor explained how to determine the operation to be used by focusing on the specific information provided in the verbal text and the part of the situation that was unknown. At the same time, students were provided with feedback on their ability to synthesize the steps of the strategy. During the practice trials, students were guided to complete additional problems using the same procedures that were modeled by the examiner. If any errors were noted in students' performance in a given session, corrective feedback was provided.

At the end of each training session, students completed a worksheet containing word problems with problem schemata diagrams. Initially, students worked on only one type of problem. Later, when they had completed instruction in the combined use of the strategy steps for all problem types, worksheets with word problems that included all three problem types were presented. Upon completion, the worksheet was checked and appropriate feedback was provided.

Traditional Condition. Instruction in this condition was derived from the Addison-Wesley Mathematics (Eicholz, O'Daffer, & Fleenor, 1985) basal mathematics program. During the first phase of training, students were involved in solving think math (e.g., logical reasoning, discovering patterns, number puzzles, number relationships, money and place value, etc.) sheets from the basal program. The second phase of training entailed instruction in word problem solving using a five-step checklist procedure. The checklist consisted of (a) understanding the question by having students focus on the question, (b) finding the needed data given in the problem, (c) planning what to do (e.g., add, subtract) by guessing and checking, (d) finding the answer by computing using the operation determined in the previous step, and (e) checking back by rereading the problem to decide whether or not the answer is reasonable.

Interscorer Reliability

A doctoral student scored all tests using answer keys. Reliability was assessed by having a research assistant score 25% of each test. Interscorer reliability was computed by dividing the number of agreements by the number of agreements and disagreements and multiplying by 100. Reliability across the tests was 100%.

Results

Acquisition and Maintenance Effects of Training on Word Problem Solving

Pretest and posttest scores on the word problem solving tests are displayed in Table 3. Pretest scores were 49% and 51% correct for traditional and schema group respectively. An analysis of variance (ANOVA) of pretest data yielded no significant differences, $F(1, 21) = .20$, $p > .65$, indicating that the two groups' pretest scores did not differ significantly prior to the training. Performances of the two groups on word problem tests were analyzed using a 2 (Groups: traditional, schema) x 2 (Test time: pretest, posttest, and delayed posttest) analysis of variance (ANOVA) with repeated measures. A statistically significant main effect was obtained for time of test, $F(2, 42) = 40,15$, $p = .0001$, but there was no significant main effect for group.

Post hoc comparisons on the main effect of time indicated statistically significant differences on all comparisons. The traditional group and schema group showed an increase in scores from the pretest to posttest (22% and 26% respectively). In addition, students in both groups maintained their use of word problem solving skills. However, the scores from posttest to delayed posttest showed a decrease (2%) for the traditional group, while a increase in scores (7%) was noted for the schema group. The performance of the schema group (84%) surpassed that of their normally achieving peers (82%).

Table 3 About Here

Generalization of Instruction

A repeated measures 2 (Groups) x 2 (Test time) ANOVA was used to assess the observed difference between the two groups from the pretest to the generalization test. The results indicated no significant differences between groups on word problem solving scores (Table 3), but generalization of word problem solving skills to the new word problems occurred for both groups, $\underline{F}(1, 21) = 69.2$, $\underline{p} = .0001$.

Student Interviews

Following the intervention, all students were individually interviewed. When the schema group was asked what they liked the most about the strategy, a common answer was learning to solve the problems. For example, one student reported that "it's easier to understand, it gave me clues." When asked what they liked the least, most noted having to compute (adding or subtracting), three students answered "nothing," two indicated having to read the problems or figuring out where the question mark is placed in the diagram. When asked if they would recommend this strategy to a friend, all but one answer was positive and the responses ranged from "very strongly" (N = 3) to "quite strongly" (N = 5) to "somewhat strongly" (N = 1). Student responses about how often they would use the strategy in their classroom were equally divided in terms of "very often" and "often" to "at times" and "never." Ratings regarding the benefits of the strategy, from the most useful (5) to the least useful (1), averaged 4.36 (SD = .9).

In contrast, when the traditional group was asked what they liked the most about the strategy, responses were varied. Student noted the practice provided in solving word problems, planning to find the answer, working in groups, computing, and checking answers. When asked what they liked the least, three students did not respond, while the remaining responses varied from doing word problems, checking back the answer, finding the needed data, understanding the question, and reading the problems. When asked if they would recommend this strategy to a friend, all but three responses were positive and the responses ranged from "very strongly" (N = 2) to "quite strongly" (N = 3) to "somewhat strongly" (N = 3) to "Not Strongly" (N = 2). Student responses about how often they would use the strategy in their classroom were similar to the schema group's responses. Ratings regarding the benefits of the strategy, from the most useful (5) to the least useful (1), averaged 4.4 (SD = .62).

Discussion

In general, our results indicate that when elementary school students with learning problems are instructed using either a schema-based or traditional approach to teaching math word problems, their performance on measures of acquisition, maintenance, and generalization is comparable. Based on the findings of our earlier study (Jitendra & Hoff, in press), we hypothesized that the schema-based strategy would not only produce differential effects across the two methods of instruction but also bolster students scores to a level commensurate with the normative sample. However, at immediate posttesting, for example, the combined student performance level in the traditional and schema-based groups was at 71% and 77% respectively, compared to the 81% achieved by the nondisabled third graders who did not have the benefit of instruction. Despite statistically significant increases attained by both groups from pre- to post-testing, the results are discouraging. Plausible explanations for the study findings and implications for further research follow.

A common struggle for researchers conducting applied research in the classroom is controlling for extraneous variables that may pose threats to the validity of the experiment (Gay, 1987). In our case, many of the students assigned to the traditional approach had received additional word problem solving strategy instruction from their teachers prior to the study. These students were provided with instruction that supplemented the strategy delineated in the basal mathematics program that, in turn, may have artificially inflated their overall performance on the posttests to produce nonsignificant findings when comparisons were made.

Another difficulty that occurs when conducting classroom research, particularly when working with special education students, is finding enough of them to run an experimental study. Roscoe (1975) recommends a minimum of 15 subjects per group, if tight experimental controls are incorporated into the study. Despite Herculean efforts to involve students with mild disabilities, we conducted the study with very few subjects per group (10 in one, and 13 in the other) and not all of these students had identified disabilities (some were labeled "at-risk"). Given the small sample size, detectable differences between groups may not have been possible.

Nevertheless, the schema-based strategy examined in this study has merit and, we believe, deserves further investigation. The strategy was carefully designed to incorporate the elements of problem schemata, action schemata, and strategic knowledge that characterize the models of word problem solving postulated by Marshall (1990) and Riley, Greeno, and Heller (1983). Many mathematical word problem solving strategies emphasize the use of key words (i.e., "left" means subtract) and cause students to react to the word at a surface level of analysis. In contrast, the schema-based strategy instruction used in this study focuses on a deep-structure analysis of the interrelationships among the words in the problem and the context in which it is embedded. The use of a visual representation (or schemata diagram) during instruction reinforces further the semantic relations in the problems (Goldman, 1989). In theory, the strategy is instructionally sound. When used with a larger number of students, differences across groups and the strength of schema-based instruction in practice, may be revealed.

Word problem solving instruction in mathematics for students with mild disabilities has shifted from an emphasis on instruction in key words and computational skills to a focus on instruction in problem comprehension (Englert, Culatta, & Horn, 1987) and cognitive and metacognitive processing (e.g., Montague, 1992). The schema-based problem solving instruction examined in this study has the potential to respond, not only to this change in the way we think about instruction, but also to the needs of students with learning problems. However, further research is indicated before confident recommendations can be made.

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Table 1

Classification of Addition/Subtraction Word Problems

Problem Type	Example
Change	
Ending Quantity Unknown	Jane had 3 marbles. Then Ted gave her 5 more marbles. How many marbles does Jane have now?
Change Quantity Unknown	Jane had 3 marbles. Then Ted gave her some more marbles. Now Jane has 8 marbles. How many marbles did Ted give her?
Beginning Quantity Unknown	Jane had some marbles. Then Ted gave her 5 more marbles. Now Jane has 8 marbles. How many marbles did Jane have in the beginning?
Group	
Whole (Superordinate or Larger Group) Quantity Unknown	Jane has 3 marbles. Ted has 5 marbles. How many marbles do they have?
Part (s) (Subordinate or Smaller Groups) Quantity Unknown	1. Jane and Ted have 8 marbles altogether. Jane has 3 marbles. How many marbles does Ted have? 2. Jane and Ted have 8 marbles altogether. Ted has 5 marbles. How many marbles does Jane have?
Compare	
Difference Quantity Unknown	Jane has 8 marbles. Ted has 5 marbles. How many more marbles does Jane have than Ted?
Compared Quantity Unknown	Jane has 3 marbles. Ted has 5 more marbles than Jane. How many marbles does Ted have?
Referent Quantity Unknown	Jane has 8 marbles. She has 5 more than Ted. How many marbles does Ted have?

Note: Representation adapted from "Development of children's problem-solving ability in mathematics" by M.S. Riley, J. G., Greeno, and J. I. Heller, 1983, in The development of mathematical thinking (p. 160) by H. P. Ginsburg (Ed.), Academic Press. Copyright 1983 by Academic Press, Inc.

Table 2

Demographic Information

	Traditional Group[a]	Schema Group[b]	Total[c]
Sex			
Male	4	6	10
Female	6	7	13
Grade			
2	0	1	1
3	3	3	6
4	4	5	9
5	3	4	7
Age in months			
M	121	122.62	121.91
SD	11.80	13.27	12.39
Ethnicity			
Caucasian	9	11	20
African American	0	2	2
Hispanic	1	0	1
Classification			
LD	3	4	7
EMR	2	2	4
SED	1	1	2
ND	4	6	10

	Traditional Group		Schema Group		Total	
	M (SD)	n	M (SD)	n	M (SD)	n
IQ[d]						
Verbal	80 (9.3)	3	89 (14.6)	7	86 (13.5)	10
Performance	73 (11.4)	3	85 (13.6)	7	81 (13.6)	10
Full Scale	74 (9.1)	3	86 (12.8)	7	82 (13.4)	10
Math[e, f]						
Computation	2.66 (.99)	5	2.96 (.83)	5	2.81 (.87)	10
Con and Appl.	2.26 (.60)	5	3.14 (1.2)	5	2.70 (.99)	10
Total	2.48 (.78)	5	3.02 (.92)	5	2.75 (.85)	10

Note. LD = Learning Disabled; EMR = Educably Mentally Retarded; SED = Seriously Emotionally Disturbed; ND = Nondisabled; [a]n = 10; [b]n = 13; [c]n = 23. [d]Test scores were obtained from the following tests: Weschler Intelligence Scale for Children-Revised, Weschler Intelligence Scale for Children-Third Edition, and Kaufman Assessment Battery for Children. [e]Test scores were obtained from the California Achievement Test. [f]GE = grade equivalent scores.

Table 3

Means and Standard Deviations on Word Problem Solving Tests for Traditional and Schema Groups

	Traditional Group (n = 10)	Schema Group (n = 13)	Total[a] (n = 23)
Pretest			
M	7.30	7.69	7.52
SD	2.54	1.65	2.04
Immediate Posttest			
M	10.70	11.62	11.22
SD	2.95	3.04	2.97
Delayed Posttest			
M	10.40	12.62	11.35
SD	3.66	2.43	3.34
Generalization Test			
M	11.60	12.08	12.17
SD	3.27	3.01	2.81

[a]Based on 15 possible points.

Figure 1. Schemata diagrams for change, group, and compare problem types.