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ABSTRACT

If it can be shown that the Scholastic Aptitude Test mathematics test (SAT-M) is a reasonably good predictor of success in particular mathematics courses, it may have a role as a measure of prerequisite skills. The predictive validity of the SAT-M was studied by collecting grades from freshman mathematics courses at 10 colleges (3,499 students). Compared to tests that were specifically designed for placement purposes, the SAT-M score was a relatively poor predictor of grades in most courses. Even after correcting for the considerable range restriction that may occur when within-course scores are analyzed, coefficients were typically only in the mid 0.30s (compared to corrected coefficients for a local placement test that ranged from the high 0.40s to the low 0.60s). Nevertheless, the SAT-M significantly improved predictions from high school grade point average (GPA) alone, especially for calculus courses. In courses at all levels, grades of males and females were similar, but SAT-M scores of males were significantly above the scores of females. Gender differences could be reduced or eliminated by considering high school GPA with SAT-M scores. For algebra versus calculus placement decisions, a weighted composite of SAT-M and high school grades predicted 64% to 71% of actual placements. For precalculus versus calculus, about 80% of the actual placements could be predicted. Appendixes describe the colleges in the study, show grade predictability in three tables, and give sample items from the SAT-M. (Contains 6 figures, 21 tables, and 12 references.) (SLD)

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ABSTRACT

Because the Scholastic Aptitude Test Mathematical Score (SAT-M) is a measure of general mathematical aptitude that is not closely linked to the specific skills that are prerequisites for a particular course or series of courses, it should not be the only information considered in making placement decisions. Nevertheless, if it can be shown that the SAT-M score is a reasonably good predictor of success in particular mathematics courses, then it may have a role to play as one of the pieces of information to be considered. The question of the predictive validity of SAT-M performance was addressed by collecting grades in freshman mathematics courses from 10 colleges. Compared to tests that were specifically designed for placement purposes, the SAT-M score was a relatively poor predictor of grades in most courses. Even after correcting for the considerable range restriction that may occur when within-course scores are analyzed, coefficients were typically only in the mid-.30s (compared to corrected coefficients for a local placement test that ranged from the high .40s to the low .60s). Nevertheless, the SAT-M score significantly improved predictions from high school grade-point average alone, especially for calculus courses.

In courses at all levels, grades of males and females were very similar, but the SAT-M scores of males were significantly above the scores of females. Gender differences for predicting grades in particular mathematics courses generally could be eliminated or greatly reduced by considering high school GPA together with SAT-M scores. Three subscores (Algebra, Insight, and Routine) were generated from the November 1985 SAT-M test but they had no differential utility in prediction. No practically significant trait-treatment interactions were demonstrated.

For algebra versus precalculus course placement decisions, a weighted composite of SAT-M score and self-reported high school grades and courses taken correctly predicted 64 percent to 71 percent of the actual placements. For precalculus versus calculus decisions, about 80 percent of the actual placements could be predicted.

INTRODUCTION

Although the primary purpose of the Scholastic Aptitude Test (SAT) is the selection of students for admission to colleges and universities, its ready availability on student transcripts has led some institutions to use it for course placement of admitted students, also. The two course sequences in which placement decisions must be made most frequently are English and mathematics. The Test of Standard Written English (which is a standard part of regular SAT administrations) was specifically designed to aid in English placement decisions and its validity for placement purposes in conjunction with the SAT-Verbal (SAT-V) score has been studied previously (Breland, Conlan, and Rogosa 1976; Breland 1977). However, there is no comparable mathematics placement test included as part of SAT administrations and the validity of SAT-Mathematical (SAT-M) scores for mathematics placement decisions is unclear.

Because it was designed as a general aptitude test, the SAT-M lacks several characteristics that placement tests exhibit. SAT-M content is not closely linked with the content of any course and the test does not attempt to sample broadly from the typical high school curriculum. Indeed, the computational procedures required to answer the questions are limited to those taught only through the first year of high school algebra. Nevertheless, because many of the computationally simple items require a high level of mathematical reasoning ability, the test has an adequate ceiling even for students who have taken four years of high school mathematics. Given this conceptual orientation, the test would not be expected to be (and in fact is not) subject to large gains from a relatively short-term (i.e., one- or two-semester) instructional intervention (Messick 1980).

Given this limited content coverage, SAT-M scores should not be used to certify a particular level of mathematics achievement. Nevertheless, SAT-M scores might still be useful for placement if they demonstrate adequate predictive validity. Predictive validity is a relevant component of placement validity if the assumption is made that students should not be placed in courses which they are likely to fail. An even stronger case for placement validity can be made if it can be shown that these failures would more likely be successes if they were first placed in a lower-level course, while students with high scores on the placement test would not benefit, or would benefit very little, from taking the lower-level course. In more technical terms, the regression line of mathematics achievement (as measured by grades at the end of the second course in the sequence) on aptitude (as measured by the placement test) should be steeper for students who were placed directly into the second course than for students who first took the previous course in the sequence. Note, however, that this conception of a trait-treatment interaction assumes that performance in the second course should be maximized for all students. In practice, this is frequently not the case. Students who could not initially succeed in calculus, for example, might enroll in programs that would never require them to take calculus rather than taking enough remedial courses to allow them eventually to succeed in calculus. Thus, the placement test that identified potential calculus failures could be useful even without demonstrating a trait-treatment interaction.

This report focuses on the predictive validity of the SAT-M performance in comparison with other predictors [such as school-specific placement tests and the Admissions Testing Program (ATP) Mathematics

Achievement Tests], as well as its ability to demonstrate trait-treatment interactions. Schrader (1971) found ". . . substantial evidence of the superiority of the Mathematics Achievement Test scores over SAT-M scores in predicting mathematics grades" (p. 139), but Ramist (1984) noted no such superiority in a small sample of more recent studies conducted by the Validity Study Service. It may be necessary to look much more closely at the nature of specific courses for which predictions are being made. Bridgeman (1980) indicated that SAT-M scores are of very little value for making placement decisions for lower-level remedial courses. They cannot distinguish students who need help with general mathematics from those having difficulty with elementary algebra. However, Bridgeman also noted that for more advanced courses SAT-M scores appeared to be much better predictors of courses success, possibly even as good as especially constructed placement tests. Sample sizes for the advanced courses were small and these findings need to be replicated.

Not only may there be difficulty making effective predictions for different types of courses but different item types on the SAT may also be differentially useful. In general, placement test items should reflect the specific skills necessary for success in the course in which students are to be placed. For example, geometry-based SAT items may be less valuable in predicting algebra course success than are algebra-based items, although it is by no means certain that this would be true given the high correlations among the different types of items. Another distinction that may be useful is between items requiring insight and essentially routine computational items. For the insight items, the examinee must first decide what to do and then do it, while for the computational items the examinee need only complete the indicated computation. Although the SAT-M test puts more emphasis on insight items than most achievement or placement tests, both item types are represented. An example of an insight SAT-M item (from the November 1985 SAT) is: "Gina has exactly $34 in $10, $5, and $1 bills in her purse. If she has more $1 than $5 bills and more $5 than $10 bills, how many different combinations containing at least one of each type could she have?" An example of routine computational item from the same SAT is: "$(-2a)^3=$". A finding of differential predictive efficiency would have implications for the creation of placement subscores for the current SAT or for a future SAT redesigned to yield more placement information.

The next section describes the sample, section three presents general predictive validity information, section four details subscore validity, section five investigates trait-treatment interactions, section six describes the ability of SAT-M scores and Student Descriptive Questionnaire data to replicate actual placement decisions, and section seven presents the conclusions.

SAMPLE

Although the institutions invited to participate in this study did not form a true random sample of colleges, they did represent both public and private institutions from a range of geographical areas. From a set of 30 institutions initially contacted (with the assistance of College Board field staff), 10 eventually sent usable data. Five were major state universities, two were private institutes of technology, and the remaining three were private universities. All were at least moderately selective. See Appendix A for a brief description of each institution in the sample.

We had hoped to locate several colleges that currently did no placement testing, but only one such institution was included in the final sample. Of the 10 colleges in the sample, three used placement tests from the Mathematics Association of America, five used their own locally developed tests, and one college used the SAT in conjunction with information on intended major and the student's request for course placement. In most cases strict cutoffs on the tests were not used. Rather, advisers used the tests along with information on high school courses to guide students in the most appropriate placement.

Four of the state universities sent data tapes with complete information on first- and second-semester mathematics grades for all freshman students who began their studies in the fall of 1986. For the other institutions, only grades in first semester mathematics courses were provided on data sheets filled out by individual instructors. Thus at these latter institutions no second-semester grades were available and there were missing data from faculty members who failed to return the sheets. Institutions also were asked to send scores on the placement test used locally, but this information was supplied by only four institutions.

Once the data arrived at Educational Testing Service (ETS), they were merged with information from the SAT history files, including information from the Student Descriptive Questionnaire (SDQ) and, when available, scores on the Mathematics Achievement Test (Level I or Level II). For students who had taken the November 1985 SAT administration (the largest administration for students who were freshmen in the fall of 1986), subscores based on responses to particular item clusters were generated (see the section on Subscore Validity). Because of strict data release regulations at College 1, this institution could not send personally identifiable grade information. College 1 sent SAT scores and high school grade-point averages from its files, but SDQ scores and the subscores from the November 1985 SAT-M administration could not be included for this institution.

Grades were coded on a 0 to 4 scale except for those colleges that reported letter grades with pluses or

Table 1. Means and Standard Deviations for SAT and SDQ Scores

Sex	*N*		*SAT-M*	*SAT-V*	*Experience*	*Math Average*	*High School GPA*
Male	1,941	*M*	573.0	477.7	3.18	3.27	3.31
		SD	94.5	89.5	.70	.69	.55
Female	1,558	*M*	518.5*	476.4	2.99*	3.25	3.43*
		SD	91.4	85.6	.74	.70	.50

*Significant sex difference ($p < .01$)

minuses. A plus added a third of a point (e.g., B+ = 3.33) while a minus subtracted a third of a point (e.g., B− = 2.67).

PREDICTIVE VALIDITY

Scores

In addition to SAT-M scores, several other scores were included in the predictive validity analyses for comparison purposes. Three scores were created from the student responses to the SDQ. The first score, "experience," was a five-point scale reflecting the highest level mathematics course taken in high school (4 = calculus, 3 = trigonometry or precalculus, 2 = more than one year of algebra, 1 = 1 year of algebra and 1 year of geometry, 0 = less than 1 year of algebra and 1 year of geometry). The second score indicated the average grade in all mathematics courses taken in high school (0 to 4). The third score represented high school grade-point average. Students could indicate plus and minus in addition to the letter grade, so the scale had 13 possible points ranging from 0 for an F to 4.33 for an A+. Scores on locally administered placement tests were available for Colleges 1, 6, 8, and 9. SAT-V scores were included as a general ability measure that has no mathematics content.

Means and standard deviations of these scores combined across all institutions (except College 1, which had no SDQ data) are presented in Table 1.[1] Males and females had nearly identical SAT-V scores, but the SAT-M scores of males were more than half a standard deviation above the scores for females. On the average males had taken slightly more advanced courses in high school, but the average high school mathematics grades of males and females were essentially equivalent. High school grade-point averages were about one quarter of a standard deviation higher for females. Correlations among these scores are presented in Table 2. Patterns for males and females were very similar. As expected, SAT-M scores correlated more highly with the average grade in high school mathematics courses than did SAT-V scores. Less expected was the superiority of SAT-M over SAT-V scores for predicting (actually postdicting) high school grade-point averages (for males, $t = 4.98$ (1,938), $p < .01$; for females, $t = 2.27$ (1,555), $p < .05$).

Table 2. Correlations of SAT and SDQ Scores

	1	*2*	*3*	*4*	*5*
(1) SAT-M		.53	.45	.43	.40
(2) SAT-V	.54		.24	.15	.30
(3) Experience	.44	.24		.34	.35
(4) Math Average	.44	.18	.37		.63
(5) GPA	.40	.35	.34	.62	

Note: Male above diagonal, female below.

Results for Algebra Courses

The algebra courses analyzed in this section were categorized as intermediate-level courses. They were typically the lowest level algebra courses offered for college credit, although some institutions may have offered lower level, noncredit remedial courses. One year of high-school-level algebra was the prerequisite for these courses, but most students had additional high school courses in mathematics.

The results of the predictive analyses in six colleges, by sex, are summarized in Table 3, while results for combined sexes are presented in Appendix B. (For this and all subsequent analyses, only courses with at least 30 students were included in the data base. Thus in any particular table one or more colleges may be missing.) In general, the correlation of SAT-M scores with grades was quite low. Some authors prefer unstandardized regression weights to correlation coefficients when summarizing over several samples (Messick 1988). For some analyses we will use regression weights, but we prefer correlations here for two reasons. First, they facilitate comparisons across measures that are on differ-

1. Results at College 1 generally paralleled those in the rest of the sample. The SAT-M score mean for the 5,153 male students was 545.7 ($SD = 99.5$), while the mean for the 3,482 female students was 505.5 ($SD = 98.6$), for a difference of .43 in standard deviation units. The SAT-V score means were 468.6 ($SD = 90.4$) for males and 467.2 ($SD = 91.0$) for females. The high school grade-point average (actual, not self-reported) was 2.93 ($SD = 63$) for males and 3.14 ($SD = .57$) for females for a difference of .34 in standard deviation units. The correlation of high school GPA with SAT-M scores was .41 for males and .48 for females; the correlation of GPA with SAT-V scores was .32 for males and .38 for females. The correlation of SAT-V with SAT-M scores was .55 for males and .59 for females.

Table 3. Prediction of Algebra Grades

College	Sex	N	Statistic	Grade	Local Placement Test	SAT-M	SAT-V	SDQ		
								Experience	*Math Average*	*GPA*
1	M	489	*M*	2.41	19.45	449.2	426.9	—	—	2.51*
			SD	1.25	5.34	75.2	79.7			.50
			r		.33	.07	−.14			.31
	F	383	*M*	2.60	19.29	415.7	418.1			2.77
			SD	1.20	5.46	74.5	77.7			.44
			r		*.34*	*.19*	*.09*			*.20*
2	M	67	*M*	2.62		509.1	467.8	2.88	2.85	3.06
			SD	1.14		86.0	86.0	.54	.76	.39
			r			−.02	−.21	−.14	.19	.15
	F	52	*M*	3.31		495.2	476.0	2.86	3.25	3.43
			SD	.91		77.7	88.0	.63	.74	.46
			r			.03	−.20	.05	.25	.15
3	M	99	*M*	1.91		480.6	418.1	2.51	2.86	3.04
			SD	1.86		85.2	78.8	.76	.74	.41
			r			.09	.02	.14	.18	.22
	F	173	*M*	1.75		452.3	444.7	2.46	2.83	3.18
			SD	1.81		70.6	75.1	.77	.73	.44
			r			.10	−.06	.13	.24	.05
4	M	27	*M*	1.85		477.0	447.4	2.56	2.55	2.84
			SD	1.21		51.1	70.5	.80	.70	.40
			r			.13	.10	−.07	.45	.47
	F	19	*M*	1.67		431.6	444.2	2.68	2.95	3.00
			SD	1.27		53.4	57.1	.58	.52	.37
			r			.45	−.34	.15	.31	−.08
5	M	19	*M*	1.95		462.1	442.1	2.58	2.84	3.11
			SD	1.18		51.4	63.2	.61	.50	.45
			r			.24	−.33	.20	.55	.08
	F	29	*M*	2.03		447.6	429.3	2.76	2.97	3.40
			SD	1.32		67.2	70.8	.64	.63	.44
			r			.44	−.12	.14	.43	.24
6	M	25	*M*	2.36	13.46	554.0	463.2	2.96	3.40	3.59
			SD	.67	5.62	71.6	80.3	.61	.65	.34
			r		.46	.19	.01	.27	.33	.42
	F	33	*M*	2.53	11.64	514.2	481.8	2.85	3.33	3.56
			SD	.91	4.83	75.8	71.6	.57	.54	.36
			r		.45	.67	−.06	.32	.16	.04
			Colleges							
Weighted average	M	514	(1 and 6)		.34	.08	−.13			.32
		726	(all)			.07	−.12			.29
		237	(2–6)			.09	−.07	.06	.26	.24
Weighted average	F	416	(1 and 6)		.35	.23	−.08			.19
		689	(all)			.20	−.00			.14
		306	(2–6)			.20	−.11	.14	.26	.08

*At College 1 only, GPA is that reported to the college by the high school rather than from the SDQ; *r* is correlation of predictor with grades.

ent scales (e.g., high school GPA, local placement test scores, and SAT-M scores). Second, the criterion scores (i.e., grades) from different colleges appear to be on the same 0–4 scale but because of different grading practices may not be. Comparing unstandardized regression weights treats college-to-college variation in grade standard deviations as if it represented true differences in group variability and not just an artifact of different grading practices. If this were correct, one would expect an institution with a relatively high standard deviation of mathematics grades to have a similarly high standard deviation on some other measure of mathematics ability (e.g., SAT-M scores.). To test this hypothesis, the standard deviation of mathematics grades in each gender group in each institution was paired with the corresponding SAT-M score standard deviation and the correlation computed. For the 12 pairs in the algebra sample, the correlation was .08, for the 13 pairs in the precalculus sample it was .05, and for the 21 pairs in the calculus sample it was .11. Thus, the hypothesis was not supported and the standardization provided by a correlational approach appeared to be justified.

For males, the highest correlation was .24 and the median was .11. For females, the correlation always exceeded that for males, but there was considerably more variation among institutions. In three colleges the correlation was .44 or higher while in the other three the correlation was .20 or lower. This difference cannot be attributed to differential restriction in the range of the SAT-M scores. The correlation was only .03 in the college with the largest standard deviation and .45 in the college with the smallest standard deviation. Note, however, that the correlations were lowest in the three largest samples. It might be argued that because of highly efficient placement procedures (and/or self-selection), it is impossible for any test to predict variation within a course. However, note that the local placement tests at Colleges 1[2] and 6 correlated at least .33 with grades even though they had been used as explicit selection instruments. The contrast is particularly striking in the large sample of males at College 1 for whom SAT-M scores correlated only .07 with grades while the local test correlated .33. This is consistent with previous findings that highly targeted placement tests (such as the Descriptive Tests of Mathematics Skills) provide better predictions of grades in algebra courses than SAT-M scores (Bridgeman 1980).

The within-course mean differences in grades and SAT-M scores are also noteworthy. Without exception, SAT-M score means for males were above score means for females, usually substantially. In each course, an effect size for the gender difference was computed by subtracting the SAT-M score mean for females from the SAT-M score mean for males and dividing by the pooled standard deviation. These effect sizes were then averaged over the six algebra courses. The effect size mean was .44 (median = .41). Yet, grade means for females were higher in four out of the six samples, and in the remaining two samples the grade difference (in standard deviation units) was much smaller than the SAT-M score difference (.09 versus .37 at College 3 and .15 versus .87 at College 4). In the large sample at College 1, grades of females were significantly above grades of males (t = 2.29 (890), $p < .05$), while SAT-M scores of males were significantly above scores of females (t = 6.63 (890), $p < .01$). Scores on the local placement test were nearly identical in the two groups. High school grade-point averages of females were higher than averages of males in all but one college (College 6, where they were essentially identical).

At the bottom of Table 3 weighted averages are presented. The weighted average was computed by multiplying the correlation by the N on which it was based, adding these products, and dividing by the total N. Because different scores were missing at different institutions, three sets of weighted averages are presented so that all comparisons across columns may be based on the same N. Although the very large samples at College 1 necessarily have a major impact on the weighted average, note that the weighted average that excludes College 1 is similar to the weighted average for all colleges.

Results for Precalculus Courses

Courses in this category either were given this explicit label by the institution or were college algebra courses one step above the courses in the preceding category. Results for these courses are presented in Table 4. The overall pattern of results closely matches that found for the more elementary algebra courses. Course grades of females were higher than those of males in five out of the seven courses, with nearly identical grades in one of the remaining two institutions (College 4). Only for the small sample at College 8 did grades appear to be substantially higher for males, though this difference was not statistically significant (t = 1.40 (51), ns). SAT-M scores of males were higher (generally substantially higher) than scores of females in each course. The effect size mean was .36; the effect size median over the seven courses was also .36. The high school grade-point average of females was higher in each course. These relationships for the nonengineering precalculus course at College 1 can be seen graphically in the regression lines plotted in Figure 1. For any given SAT-M score (within the 300 to 700 range), the predicted course grade for a female is above the predicted grade for a male. Figure 1 also shows the relatively weak relationship between SAT-M scores and grades. The predicted grade for students with a SAT-M score of 600 is less than one letter grade above the predicted grade for students with a score of 400.

Because of the similarities in both course content and results for the algebra and precalculus course categories, they were combined for a graphic presentation of the validity coefficients. A frequency distribution of SAT-M scores with course grade validity coefficients for the 13 courses in the combined algebra/precalculus category is presented in Figure 2. The median coefficient for males was .13 and for females .23, but there

2. At College 1 three placement tests were administered: a 25-item basic mathematics test, a 32-item algebra test, and a 15-item trigonometry test. For courses below calculus, the placement score reported here is the sum of the basic mathematics and algebra scores (maximum = 57); for calculus courses the score reported is the sum of the algebra and trigonometry scores (maximum = 47).

Table 4. Prediction of Precalculus Grades

College	Sex	N	Statistic	Grade	Local Placement Test	SAT-M	SAT-V	SDQ Experience	SDQ Math Average	GPA
1N*	M	703	M	2.24	27.61	486.1	443.7			2.67
			SD	1.24	6.22	74.7	80.9			.49
			r		.34	.15	−.04			.32
	F	737	M	2.46	28.71	459.0	440.1			2.91
			SD	1.21	7.50	75.1	80.7			.51
			r		.37	.23	.04			.29
1E*	M	678	M	1.92	27.76	499.4	442.7			2.74
			SD	1.31	6.24	74.4	80.8			.50
			r		.28	.13	−.08			.30
	F	218	M	2.13	29.28	471.7	434.0			2.97
			SD	1.29	6.62	70.9	79.0			.61
			r		.31	.24	.06			.30
2	M	70	M	2.50		595.0	480.4	3.19	3.28	3.33
			SD	1.08		65.0	65.4	.46	.57	.46
			r			.09	−.16	−.04	.23	.30
	F	48	M	2.56		550.0	495.4	3.08	3.42	3.49
			SD	1.09		83.1	97.1	.58	.50	.43
			r			−.05	−.25	−.10	.45	.17
3	M	73	M	2.15		518.2	428.4	2.74	3.00	3.12
			SD	1.90		86.5	71.1	.67	.75	.55
			r			.22	−.02	.15	.27	.31
	F	92	M	2.20		490.8	462.2	2.78	3.02	3.28
			SD	1.82		70.3	72.4	.68	.70	.47
			r			−.01	−.10	.22	.32	.11
4	M	84	M	1.69		512.1	455.0	2.93	2.98	3.00
			SD	1.31		76.1	75.4	.58	.69	.44
			r			.34	.25	.18	.35	.41
	F	66	M	1.68		483.9	448.0	2.79	3.06	3.13
			SD	1.12		73.2	75.4	.71	.63	.43
			r			.30	.42	.34	.24	.35
5	M	86	M	2.73		553.5	476.5	3.09	3.28	3.40
			SD	1.01		71.5	80.1	.63	.64	.56
			r			.13	.17	.37	.35	.33
	F	97	M	2.79		542.1	488.1	3.27	3.56	3.57
			SD	1.25		68.1	80.0	.53	.52	.46
			r			.41	.24	.13	.29	.29
8	M	15	M	3.00		529.3	461.3	3.00	3.20	3.49
			SD	.93		88.1	101.3	.38	.68	.42
			r			−.02	.02	−.41	.57	.43
	F	38	M	2.58		505.3	446.8	2.68	3.37	3.58
			SD	1.00		77.6	62.4	.62	.63	.44
			r			.13	.20	.17	.21	.34
			Colleges							
Weighted average	M	1,381	(1)		.31	.14	−.06			.31
		1,709	(all)			.15	−.03			.32
		328	(2,3,4,5,8)			.19	.07	.15	.32	.34
Weighted average	F	955	(1)		.36	.23	.04			.29
		1,296	(all)			.22	.06			.28
		341	(2,3,4,5,8)			.18	.11	.17	.30	.24

*At College 1, two precalculus courses were offered. "E" is a 5-credit course called Algebra, Trigonometry, and Analytic Geometry and is a prerequisite for the engineering calculus course. "N" is a 3-credit course called College Algebra and is a prerequisite for the nonengineering calculus course.

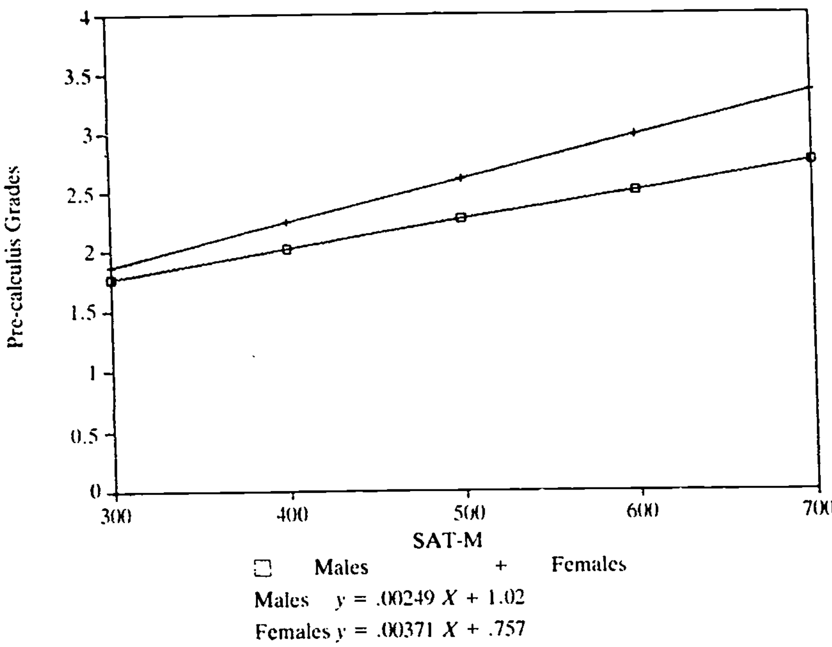

Figure 1. Regression of Precalculus Grades on SAT-M Scores for College 1.

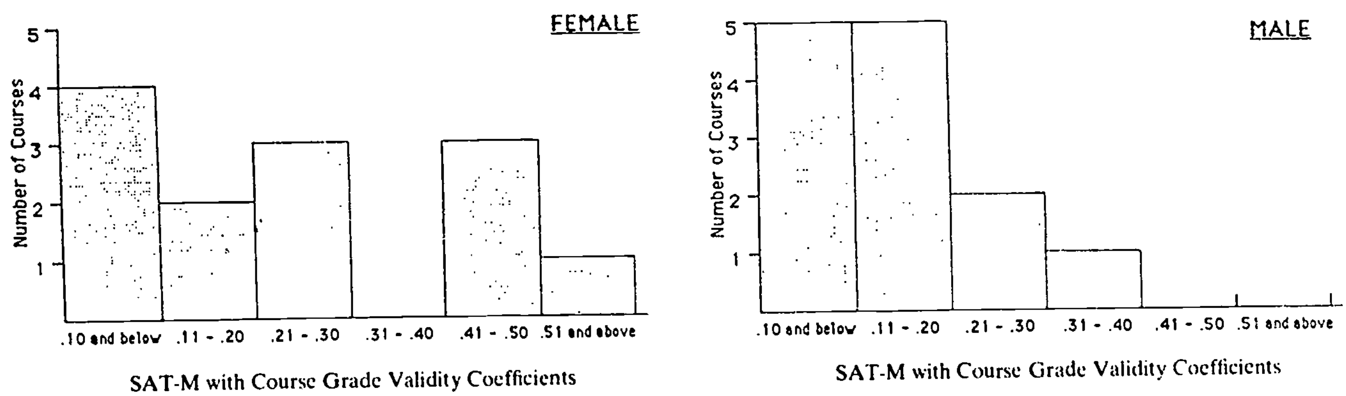

Figure 2. Frequency of SAT-M Predictive Validity Coefficients for Algebra/Precalculus Courses.

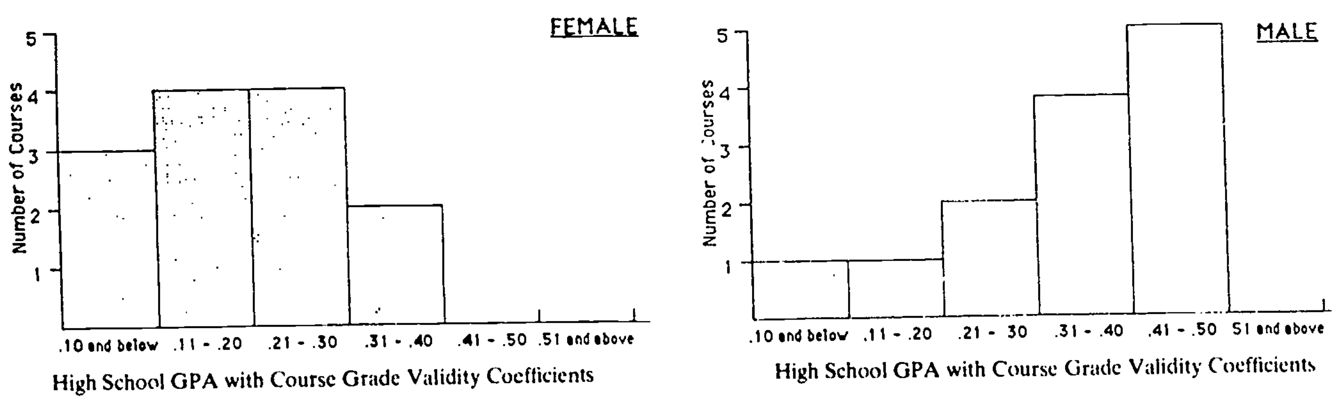

Figure 3. Frequency of GPA Predictive Validity Coefficients for Algebra/Precalculus Courses.

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Table 5. Prediction of Calculus Grades

College	Sex	N	Statistic	Grade	Local Placement Test	SAT-M	SAT-V	SDQ Experience	SDQ Math Average	GPA
1N*	M	477	*M*	2.46	27.04	569.1	476.7			3.04
			SD	1.23	6.20	78.9	81.2			.62
			r		.42	.21	.07			.32
	F	573	*M*	2.46	27.35	546.6	490.1			3.35
			SD	1.16	5.83	74.5	79.2			.43
			r		.36	.20	.05			.21
1E*	M	1,662	*M*	2.32	31.79	608.4	491.9			3.21
			SD	1.27	6.35	74.2	85.0			.60
			r		.34	.12	-.03			.23
	F	631	*M*	2.21	31.81	585.2	505.2			3.50
			SD	1.21	5.96	69.5	86.2			.46
			r		.33	.20	.01			.12
2N*	M	70	*M*	2.36		608.1	511.4	3.40	3.37	3.39
			SD	1.22		63.0	82.5	.60	.59	.60
			r			.18	.26	.15	.31	.43
	F	36	*M*	2.94		613.9	505.8	3.50	3.69	3.70
			SD	.79		63.1	87.5	.56	.52	.43
			r			.39	-.12	.51	.00	.17
2E*	M	164	*M*	2.40		646.4	512.3	3.52	3.68	3.63
			SD	1.13		63.9	85.5	.65	.48	.49
			r			.25	-.07	.21	.26	.28
	F	50	*M*	2.42		616.4	545.0	3.50	3.74	3.96
			SD	1.03		69.8	69.3	.70	.44	.33
			r			.29	.07	.13	.24	.31
3	M	54	*M*	2.59		615.7	486.9	3.46	3.67	3.54
			SD	1.27		75.9	72.3	.54	.51	.49
			r			.32	.38	.31	.28	.41
	F	18	*M*	2.78		593.3	488.3	3.28	3.72	3.69
			SD	1.00		57.5	73.8	.57	.46	.42
			r			.22	.10	.52	.49	.38
4	M	138	*M*	1.96		598.9	482.4	3.56	3.37	3.27
			SD	1.36		71.2	86.6	.51	.57	.41
			r			.40	.13	.30	.37	.38
	F	46	*M*	2.13		561.3	504.1	3.27	3.54	3.66
			SD	1.33		74.5	62.7	.70	.62	.38
			r			.36	.05	.34	.46	.44

*N is the nonengineering calculus course; E is the engineering science calculus course.

was more variability in the coefficients among females. For comparison purposes, the distribution of high school GPA with course grade validity coefficients is presented in Figure 3. For males, the distribution, with the mode in the .41 to .50 range, looks almost like the mirror image of the distribution of SAT-M score validity coefficients. The distribution of coefficients for females is noticeably shifted to the left relative to the male distribution. The median correlation of high school GPA with grades was .32 in the male samples and .19 in the female samples. SAT-V scores did not predict grades in algebra/precalculus courses. The median correlation for both males and females was .00 and only 3 out of 26 coefficients were greater than .20.

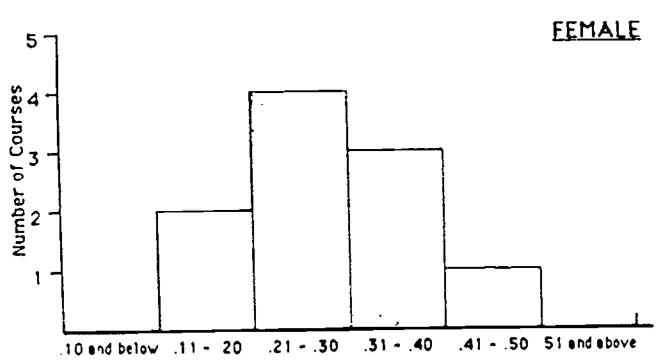
Results for Calculus Courses

Results for the 11 calculus courses are presented in Table 5. Colleges 1 and 2 had two separate calculus courses, one for engineers and the other for nonengineers. There is no female sample for College 10 because it is an all-male institution. Once again SAT-M scores for males were higher than for females except in the non-engineering course at College 2 (where means differed by less than 7 points) and in the small sample at College 9 (a technical institute in the South). The effect size mean was .33 (median = .39). The course at College 9 was one of only two where grades of males were higher than those of females. The other was the engineering course at College 1 where males were .09 SD units higher in grades and .32 SD units higher on SAT-

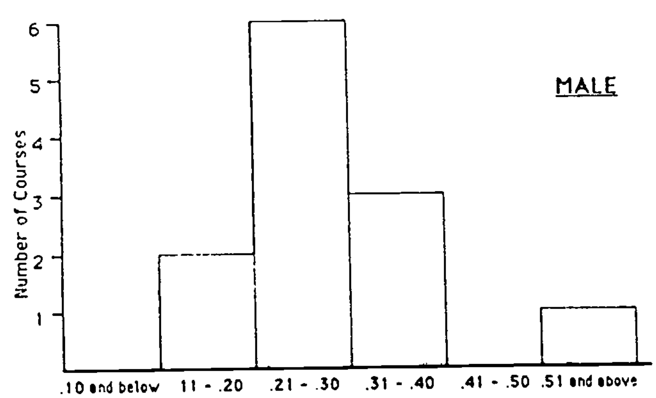
Table 5. Prediction of Calculus Grades (*continued*)

College	*Sex*	*N*	*Statistic*	*Grade*	*Local Placement Test*	*SAT-M*	*SAT-V*	*SDQ* *Experience*	*SDQ* *Math Average*	*SDQ* *GPA*
6	M	76	*M*	2.76	21.41	625.5	494.1	3.49	3.64	3.65
			SD	.86	2.72	71.5	92.4	.66	.48	.39
			r		.63	.52	.02	.27	.27	.43
	F	53	*M*	2.91	22.00	587.9	525.1	3.45	3.64	3.81
			SD	.85	2.91	60.7	80.0	.54	.48	.41
			r		.56	.39	.01	.26	.13	.21
7	M	20	*M*	2.57		611.5	536.5	3.65	3.65	3.58
			SD	.93		63.0	59.9	.49	.49	.37
			r			.25	.39	.11	.69	.46
	F	34	*M*	3.07		560.9	517.9	3.41	3.53	3.64
			SD	.88		82.9	85.3	.70	.66	.53
			r			.22	.07	.02	.35	.24
8	M	30	*M*	2.66	15.43	600.67	497.3	3.33	3.63	3.73
			SD	1.31	3.14	76.9	93.6	.61	.61	.46
			r		.34	.28	.15	.34	.58	.60
	F	31	*M*	2.73	15.54	550.3	506.8	3.45	3.68	3.75
			SD	1.26	3.65	80.5	82.8	.51	.48	.36
			r		.57	.44	.46	.53	.15	.32
9	M	38	*M*	2.66	17.29	598.2	470.8	3.58	3.42	3.34
			SD	1.21	3.34	84.8	94.5	.55	.64	.58
			r		.50	.22	.09	.51	.74	.48
	F	15	*M*	2.47	18.54	626.7	557.3	3.47	3.73	3.67
			SD	1.36	3.67	39.6	79.4	.52	.46	.36
			r		.80	.28	.24	.79	.10	.35
10	M	83	*M*	2.69		660.4	541.0	3.54	3.77	3.73
			SD	1.18		70.9	85.1	.52	.42	.47
			r			.21	.02	.00	.39	.43
			Colleges							
Weighted average	M	2,283	(1,6,8,9)		.37	.16	.00			.26
		2,812	(all)			.18	−.02			.29
		673	(all but 1)			.33	.10	.26	.41	.45
Weighted average	F	1,303	(1,6,8,9)		.36	.21	.04			.17
		1,487	(all)			.23	.03			.19
		283	(all but 1)			.34	.06	.33	.24	.29

*N is the nonengineering calculus course; E is the engineering science calculus course.



SAT-M with Course Grade Validity Coefficients



SAT-M with Course Grade Validity Coefficients

Figure 4. Frequency of SAT-M Predictive Validity Coefficients for Calculus Courses.

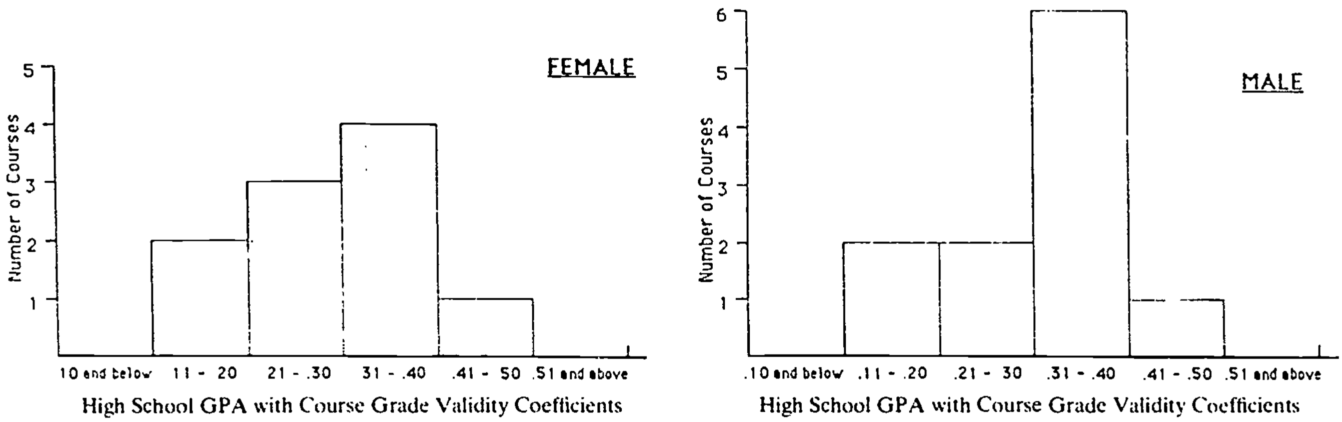


Figure 5. Frequency of GPA Predictive Validity Coefficients for Calculus Courses.

Table 6. Comparison of Grade Predictions from College 1 Placement Test and SAT-M Scores in Selected and Unselected Samples

		Placement Test		*SAT-M Scores*	
Course	*Sex*	*Selected Group*	*Unselected Group*	*Selected Group*	*Unselected Group*
Algebra	M	.33	.61	.07	.29
	F	.34	.61	.19	.44
Precalculus (nonengineering)	M	.34	.56	.15	.33
	F	.37	.53	.23	.37
Precalculus (engineering)	M	.28	.48	.13	.28
	F	.31	.50	.24	.38
Calculus (nonengineering)	M	.42	.58	.21	.37
	F	.36	.51	.20	.35
Calculus (engineering)	M	.34	.48	.12	.28
	F	.33	.47	.20	.34

M scores. In the 10 courses with both male and female samples, high school grade-point averages were higher for female students.

Predictive validity coefficients for SAT-M scores are summarized in Figure 4 and may be compared to the validity coefficients for high school grade-point average summarized in Figure 5. Going back to Figure 2 and comparing it with Figure 4, it is clear that SAT-M performance is a better predictor of calculus grades than it is of algebra/precalculus grades.

For five of the calculus courses, scores on a local placement test were available. For each of the 10 (5 courses × 2 gender classifications) comparisons, the local placement test was a better predictor of course grades than was SAT-M performance.

Note that the weighted average for all males is depressed by the low correlation in the engineering course at College 1. The weighted average jumps from .18 to .33 when College 1 is excluded from the sample. A similar, though smaller, effect can also be seen in the female sample.

Correction for Sample Selection

Validity coefficients in a selected sample (e.g., all persons enrolled in a particular mathematics course) will be lower than those in an unselected (or less selected) group (e.g., all freshmen in the college). Furthermore, if the validity of two tests is to be compared in the selected group, and one of those two tests was used in the selection process, the validity of the test used for selection typically will be underestimated relative to the other test. Thus, the within-course validity coefficients reported in this study are underestimates of what the coefficients would be in the entire freshman class at a college, but the SAT-M score coefficients are not as severely underestimated as the coefficients for the local placement tests.

If the variances and correlations among the variables (explicit selector [local placement test], incidental selector [SAT-M score], and grades) in the selected group are known and the variance of the explicit selector in the unselected group is also known, then it is possible to estimate how well both tests would function as predictors of grades in the unselected group. (See Gulliksen 1950 for a complete explanation of this procedure.) This information was available at College 1. For courses below calculus, the local placement test was a 57-item test of basic mathematics skills and algebra, and for the calculus courses it was a 47-item algebra and trigonometry test. The results are presented in Table 6. They show the anticipated increases in validity for the local placement test and the slightly smaller validity increases for SAT-M scores. Given the similarity in course placement procedures and within-course SAT-M score standard deviations between College 1 and other colleges in the sample, validity increases of about the same magnitude could be expected at those institutions.

Discussion of Correlation Results

For courses below calculus, the SAT-M score appears to be of little value for predicting grades in college mathematics courses, especially when compared to available placement test scores or high school grade-point averages. For calculus courses, the predictive validity of SAT-M scores is a little better, but generally still not as good as that of local placement tests or high school GPA. An additional problem with the SAT-M score is that it shows fairly large gender differences even within courses in which women do as well or better than men. Thus it appears that the SAT-M score, *if used as a sole predictor*, would make course assignments that were not only relatively poor but that would result in underprediction of course grades for women. But the SAT-M score would not have to be used by itself. A more reasonable question is whether the SAT contributes to what could have been predicted from high school grades alone. Because high school grade-point averages tend to be higher for women, the use of both predictors combined might not only result in higher validity coefficients but also lessen or eliminate any underprediction of women's course grades. This question is addressed in the next section.

Regression Analyses

For all courses containing at least 100 students, regression analyses predicting course grades were run entering high school GPA first, SAT-M scores second, SAT-V (Verbal) scores third, and sex (male = 1, female = −1) last. The multiple correlation coefficient (R) is presented as each variable is added, and the standardized and unstandardized (b) weights for the four predictor models are presented. SAT scores were divided by 200 in order to put them on a 1 to 4 scale that is more comparable to the GPA scale. (This has no impact on the standardized regression weights, but makes it possible to report the unstandardized weights with fewer decimal places.) For comparison purposes, the zero-order correlation of SAT-M scores with course grade is presented (i.e., step one in the multiple regression with SAT-M score entered first) and a footnote indicates if sex would be a significant factor if entered right after SAT-M scores.

Algebra and Precalculus Courses

Results for the nine algebra/precalculus courses are presented in Table 7. At Colleges 4 and 5, zero-order predictions from SAT-M scores were relatively high and SAT-M scores also significantly improved predictions from GPA alone, but at the other colleges SAT-M scores resulted in only a minimal increase in the multiple correlation. In the very large courses at College 1 these increases were statistically significant ($p < .05$), but the largest increase in R was only .03. SAT-V scores had a negative weight in seven of the nine courses (i.e., higher grades were associated with lower SAT-V scores) and a very low positive weight at College 5. Only at College 4 did SAT-V scores have a significant positive weight and even there they raised the multiple correlation by only .02.

Considering gender of the student in addition to SAT-M score by itself improved predictions in four of the courses, but in three of the courses gender could be eliminated as a predictor if GPA were considered in addition to SAT-M score (i.e., using both SAT-M score and GPA as predictors, a single regression line will make unbiased predictions for both sexes). If the slopes of the regression lines for males and females differed, two regression lines might still be needed. However, with both SAT-M score and high school GPA in the equation, slopes for males and females did not significantly differ ($p > .05$). Although the table shows gender entered after SAT-V score, the same nonsignificant effects were observed with gender entered immediately after GPA or after GPA + SAT-M score. Gender was a significant predictor when entered after GPA and SAT (M and V) scores, only for the first course at College 2 where course grade means for females were substantially above those for males (3.3 versus 2.6).

For comparison purposes, Table 7 shows three additional correlations with grades: the multiple correlation with GPA, SAT-M score, and the mathematics experience score from the SDQ as predictors; the correlation of the placement test at College 1; and the multiple correlation with GPA and the College 1 placement test as predictors. Adding the experience score to GPA and SAT-M score raised R by from .01 to .05 points. The GPA + SAT-M score R was about as high as the r

Table 7. Regressions for Algebra/Precalculus Courses

College	*N*	*SAT-M r*	*Independent Variables*	*Standardized Weight*	*b*	*R*	*GPA+ SAT-M+ Experience R*	*Placement Test r*	*GPA+ Placement Test R*
1A	872	.12*	GPA	.26'	.72	.26			
			SAT-M	.21'	.69	.29		.33	.41
			SAT-V	−.16'	−.51	.32			
			Sex	−.05	−.06	.32			
1B	1,440	.17*	GPA	.31'	.78	.33			
			SAT-M	.20'	.64	.35		.36	.43
			SAT-V	−.15'	−.44	.37			
			Sex	−.05	−.06	.38			
1C	896	.15*	GPA	.32'	.82	.31			
			SAT-M	.23'	.82	.34		.29	.38
			SAT-V	−.19'	−.60	.38			
			Sex	−.03	−.05	.38			
2A	126	−.07*	GPA	.20'	.49	.25			
			SAT-M	.04	.11	.26			
			SAT-V	−.24'	−.62	.33	.27		
			Sex	−.23'	−.25	.39			
2B	120	.03	GPA	.25'	.61	.24			
			SAT-M	.19	.53	.25			
			SAT-V	−.27'	−.73	.34	.27		
			Sex	−.05	−.06	.35			
3A	281	.10	GPA	.13'	.53	.12			
			SAT-M	.14'	.64	.15			
			SAT-V	−.10	−.47	.18	.18		
			Sex	.00	.01	.18			
3B	169	.13	GPA	.20'	.71	.21			
			SAT-M	.19'	.87	.23			
			SAT-V	−.17	−.84	.27	.28		
			Sex	−.04	−.07	.27			
4	157	.31	GPA	.33'	.92	.37			
			SAT-M	.19'	.62	.47			
			SAT-V	.19'	.61	.49	.48		
			Sex	.01	.02	.49			
5	187	.29	GPA	.24'	.53	.30			
			SAT-M	.21'	.70	.38			
			SAT-V	.06	.18	.38	.41		
			Sex	.00	.01	.38			

Note: Within a college, course A is the least advanced course, followed by course B, etc.
*If entered next, sex would be statistically significant ($p < .05$) and have a standardized weight of at least −.10.
'Regression weight statistically significant ($p < .05$)

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for the placement test, but not as high as the GPA + placement test *R*.

Calculus Courses

Results for the six calculus courses are presented in Table 8. Without exception, SAT-M scores made a statistically significant improvement in *R;* in five of the courses the improvement in the multiple correlation was at least .05. SAT-V scores had a negative weight in five of the courses and a nonsignificant positive weight in the sixth.

Gender was a significant predictor when entered immediately after SAT-M scores in courses 2N, 4, and 6, but only at College 4 did the gender effect disappear with GPA and SAT-M and SAT-V scores in the model. However, for course 2N inclusion of high school GPA did appear to lessen the gender effect. When entered immediately after SAT-M scores, sex had a standardized weight of −.26 and raised the multiple correlation from .22 to .34, but when entered right after GPA sex had a weight of −.18 and raised the multiple correlation by only .04 (from .39 to .43). For the engineering calculus course at College 1, sex had a small but statistically significant positive weight when both GPA and SAT-M scores were in the model.

The experience score raised the GPA + SAT-M *R* by .02 to .04 points. The GPA + SAT-M composite was slightly poorer as a predictor of grades than the placement tests at Colleges 1 and 6. For courses at this level, adding GPA to the placement test score made only a slight improvement in prediction (increase in *R* of .02 to .04 points).

Mathematics Achievement Tests as Alternative Predictors

The College Board Mathematics Achievement Tests (Level I and Level II) are designed to be used for admissions and for placement into college-level mathematics courses. Mathematics Achievement Tests were not required by any of the colleges sampled, but a number of students had taken the Level I test and some had taken the Level II test. Because the Level I and Level II test scores are placed on approximately the same scale (Cook, Eignor, and Mazzeo 1986), students presenting

Table 8. Regressions for Calculus Courses

College	*N*	*SAT-M r*	*Independent Variables*	*Standardized Weight*	*b*	*R*	*GPA + SAT-M + Experience R*	*Placement Test r*	*GPA + Placement Test R*
1N	1,050	.20	GPA	.31†	.75	.29			
			SAT-M	.20†	.61	.34		.39	.43
			SAT-V	−.11†	−.32	.35			
			Sex	.05	.06	.35			
1E	2,293	.15	GPA	.24†	.59	.22			
			SAT-M	.17†	.58	.24		.34	.36
			SAT-V	−.15†	−.45	.28			
			Sex	.06†	.08	.29			
2N	108	.22*	GPA	.34†	.67	.39			
			SAT-M	.18	.67	.45			
			SAT-V	.08	.21	.45	.48		
			Sex	−.18†	−.21	.48			
2E	217	.25	GPA	.28†	.65	.27			
			SAT-M	.33†	1.10	.36			
			SAT-V	−.25†	−.66	.42	.38		
			Sex	−.03	−.04	.43			
4	185	.37*	GPA	.37†	1.15	.39			
			SAT-M	.40†	1.43	.51			
			SAT-V	−.13	−.42	.52	.55		
			Sex	−.02	−.03	.52			
6	130	.44*	GPA	.26	.56	.35			
			SAT-M	.49†	1.21	.52	.55	.60	64
			SAT-V	−.18†	−.34	.53			
			Sex	−.20†	−.18	.56			

*If entered next, sex would be statistically significant ($p < .05$) and have a standardized weight of at least .10.
†Regression weight statistically significant ($p < .05$).

Table 9. Prediction of Precalculus and Calculus Grades from SAT-M and Mathematics Achievement Test Scores

College	*N*	*Independent Variables*	*r*	*Independent Variables*	*R*
Precalculus Course					
4	68	GPA	.40	GPA + SAT-M	.44
		SAT-M	.25	GPA + MAT	.46
		MAT	.28		
Calculus Courses					
2	88	GPA	.28	GPA + SAT-M	.45
		SAT-M	.37	GPA + MAT	.38
		MAT	.28		
4	113	GPA	.35	GPA + SAT-M	.47
		SAT-M	.34	GPA + MAT	.51
		MAT	.41		
6	90	GPA	.37	GPA + SAT-M	.51
		SAT-M	.42	GPA + MAT	.54
		MAT	.47		

either score were combined for analysis. For the few students who took both levels, the scores were averaged.

Four courses were identified in which at least 50 students had a score on the Mathematics Achievement Test (MAT). Results for these courses are presented in Table 9. In terms of both the zero-order prediction of course grades and the incremental validity given GPA, MAT scores were slightly superior to SAT-M scores, except at College 2. At Colleges 4 and 6 once GPA and MAT scores were in the equation, SAT-M scores added practically nothing. In both groups at College 4 the weight on SAT-M scores was negative, and at College 6 the standardized weight was an insignificant .02. At College 2 the tests reversed roles and the standardized weight on MAT scores dropped to an insignificant −.02 with SAT-M scores in the equation. With GPA and either of the tests in the equation there were no gender differences. Given the closer match with specific prerequisite skills provided by the MAT, it may be surprising that it is not a significantly better predictor. If the MAT were the sole basis of deciding who could take a course it might be more predictive, but remember that students in these courses were placed using both test scores and knowledge of high school course experience. Theoretically, a student with excellent conceptual ability in mathematics but no experience beyond the first year of high school algebra could have a very high SAT-M and a very low MAT score. But in the real world few such students exist, and those who do would probably not be placed in a college calculus course. The value of the MAT score also may have been attenuated in the current study because it was not a required test. Students who were not confident in their mathematics ability did not have to take the MAT.

Discussion of Regression Results

For courses at all levels the combination of SAT-M scores and GPA appears to be significantly better than either predictor alone. In many, but not all, cases, inclusion of both SAT-M scores and GPA permitted unbiased predictions to be made without regard to the sex of the student. Although inclusion of SAT-V scores with a negative weight might improve predictions, it seems unfair to penalize students who get high scores. Thus a safer choice is simply not to use SAT-V scores for any mathematics placement decisions.

Because the SAT-M score is not a measure of mathematics skills taught in courses beyond the first year of high school algebra and the high school GPA is only a partial measure of such skills, it would make little sense to place students into advanced mathematics courses based on these measures alone. Nevertheless, the results presented here suggest that consideration of these measures along with the mathematics courses on the high school transcript may help colleges identify which students are likely to have the most difficulty in mathematics courses.

SUBSCORE ANALYSES

Are there subscores that could be created from the SAT-M test that would be differentially effective in predicting success in mathematics courses?

Subscore Definition

For students in the current sample (i.e., freshmen in the fall of 1986), the largest single SAT administration was in November 1985. From the mathematics items administered at that time, three subscores were created. For each subscore, formula scores of the type used for the full SAT-M test were computed (i.e., scores were corrected for guessing by subtracting a fraction of the number incorrect from the number correct).

The first subscore consisted of 25 algebra items, each of which came from one of the three lowest ability classifications on a five-category classification scheme defined by the item developers (1 = perform mathematical manipulations, 2 = solve routine problems, 3 = demonstrate comprehension of mathematical ideas and concepts, 4 = solve nonroutine problems requiring insight or ingenuity, and 5 = apply higher mental processes to mathematics). Although this "Algebra" scale requires no geometry skills and avoids the most conceptually difficult items from the two highest ability classifications, it still requires a much higher level of abstract thinking than

Table 10. SAT-M Subscore Means and Standard Deviations

			Subscores			
Sex	*N*	*Statistic*	*Algebra*	*Insight*	*Routine*	*SAT-M*
M	964	M	16.79	4.51	10.91	572.4
		SD	4.83	2.20	2.52	94.5
F	811	M	14.71	3.61	9.91	519.4
		SD	4.84	2.07	2.68	92.3
Sex Difference in *SD* Units			.43	.43	.39	.56

the average algebra achievement test. See Appendix C for a list of the items on this and the other two scales.

The second subscore, labeled "Insight," consisted of 11 items in ability category 4. Although most of the items on this scale were very difficult, note that two relatively easy items (items 1 and 3 in section 1) were included. The third subscore consisted of 14 items from the two lowest ability classifications (perform mathematical manipulations and solve routine problems). This scale was labeled "Routine." The Insight scale did not overlap with either of the other two scales, but some items on the Algebra scale also appear on the Routine scale.

The coefficient alpha reliability for the Algebra scale was .77 for males and .74 for females. The reliability of the considerably shorter Insight scale was .51 for males and .46 for females, and the reliability of the Routine scale was .61 for males and .58 for females. The reliability of the total SAT-M score in this sample was .88 for males and .86 for females.

Subscore Results

Means and standard deviations for the three subscores and the total SAT-M score are presented in Table 10. As expected, the Algebra scale was moderately easy, with the average student getting more than half of the items correct. The Insight scale was quite difficult, with the average student getting fewer than half of the items correct (for females, fewer than a third of the items were answered correctly). The Routine scale was quite easy, with the average student getting over two-thirds of the items correct. Table 10 also shows that the large gender difference apparent for the total score is equally apparent in each of the subscores. The slightly lower magnitude of the gender difference in the subscores compared to the difference for the full score is to be expected given the difference in reliabilities. The magnitude of these gender differences can be compared to gender differences on other measures of mathematical ability.

As can be seen in Table 11, gender differences in the locally developed algebra placement tests at Colleges 1 and 6 are considerably smaller than the SAT-M gender differences, even though these placement tests are actually slightly longer than the Algebra scale on the SAT-M test. However, the placement tests differ from the Algebra subscore in two important respects. They focus on the instructional content of mathematics courses and they have much more generous time limits than the SAT-M test.

The College Board Achievement Tests provide another useful comparison. These tests are much more closely linked to course content than the SAT, but they also have fairly strict time limits. The results for both the Level I and Level II Mathematics Achievement Tests are presented in Table 12. Although the contrasts with SAT-M scores are less dramatic than they were for the local placement tests, the same overall pattern of reduced gender differences for the Achievement Tests is evident, especially for the Level I test. Even on the difficult Advanced Placement Calculus Test, the difference between males and females on the 45-item multiple-choice section was only .16 standard deviation units

Table 11. Means and Standard Deviations of Algebra Placement Test and SAT-M Scores

College	*Sex*	*N*	*Statistic*	*Placement Test*	*SAT-M*
1	M	5,153	*M*	17.10	545.7
			SD	7.39	99.5
	F	3,482	*M*	16.22	503.5
			SD	7.17	98.6
			Difference in *SD* Units	.12	.43
6	M	95	*M*	19.38	606.5
			SD	6.56	78.4
	F	86	*M*	18.02	559.7
			SD	7.01	75.6
			Difference in *SD* Units	.20	.61

Note: The placement test used at College 1 contained 32 items; the test at College 6 contained 33 items.

Table 12. Means and Standard Deviations of Mathematics Achievement Tests, Level I and Level II, and SAT-M Scores

Sex	*N*	*Statistic*	*Mathematics Achievement Test*	*SAT-M*
			Level I	
M	501	*M*	545.6	581.3
		SD	79.3	79.7
F	430	*M*	523.3	542.8
		SD	81.0	86.8
		Difference in *SD* Units	.28	.46
			Level II	
M	219	*M*	649.7	651.3
		SD	83.5	76.9
F	89	*M*	610.8	599.7
		SD	79.3	82.4
		Difference in *SD* Units	.47	.66

(based on data from the 28,140 males and 19,419 females who took the test in 1987). The SAT-M score difference for these students was not available but could be computed in a follow-up study.

Table 13. Correlations of SAT-M Subscores

	1	*2*	*3*	*4*	*5*
(1) Algebra		.66	.83	.84	.45
(2) Insight	.60		.55	.78	.46
(3) Routine	.81	.55		.68	.36
(4) SAT-M	.87	.75	.74		.53
(5) SAT-V	.51	.40	.42	.54	

Note: Male above diagonal; female below

The correlations among the SAT-M subscores are presented in Table 13. Note that the correlation between the Algebra and Routine scales is inflated because they share nine items while the Routine and Insight scales are independent. When corrected for unreliability, the correlation between the Insight and Routine scales was .99 for males and 1.06 for females. The corrected correlation between the Algebra and Insight scales was .96 for males and 1.13 for females. Thus, these data give no reason to question the assertion that there is a single factor in the SAT-M test. Even in a single factor test it is conceivable that a set of very difficult items might predict performance in a particular course better than a set of very easy items. However, this was not the case in this sample as can be seen in Tables 14, 15, and 16. Even though score means for the Routine scale in calculus courses (Table 16) were within 3 points of the maximum

Table 14. Prediction of Algebra Grades from SAT-M Subscores

College	*Sex*	*N*	*Statistic*	*College Grade*	*SAT Scores* *Algebra*	*Insight*	*Routine*	*SAT-M*
2	M	50	*M*	2.50	14.94	4.08	9.67	529.4
			SD	1.07	4.68	1.86	2.96	87.9
			r	—	−.22	−.21	−.17	−.07
	F	36	*M*	3.14	14.44	3.56	9.99	516.1
			SD	1.06	4.49	1.97	2.28	78.5
			r	—	−.01	.14	.07	.03
3	M	61	*M*	2.10	12.50	2.99	9.05	486.4
			SD	1.86	4.96	2.14	2.96	88.0
			r	—	.01	.06	.04	.06
	F	95	*M*	1.86	11.80	2.72	8.67	461.3
			SD	1.80	4.01	1.87	2.76	68.8
			r	—	.03	−.06	.01	.06
4	M	19	*M*	1.54	11.72	2.89	8.73	460.5
			SD	1.45	2.79	1.63	2.31	64.2
			r	—	.17	−.02	−.08	.09
	F	17	*M*	1.53	11.01	2.34	7.41	445.9
			SD	1.20	4.36	1.97	2.57	63.2
			r	—	.47	.09	.14	.38
Weighted average	M	130			.05	−.06	−.06	.01
	F	148			.07	.01	.01	.09
	Total	278			.01	−.02	−.02	.05

Table 15. Prediction of Precalculus Grades from SAT-M Subscores

College	Sex	N	Statistic	College Grade	SAT Scores: Algebra	Insight	Routine	SAT-M
2	M	41	*M*	2.45	17.84	4.91	11.50	595.1
			SD	1.13	3.27	1.93	1.62	66.2
			r	—	.51	.08	.52	.44
	F	32	*M*	2.57	15.09	3.59	10.40	544.7
			SD	1.09	4.99	2.14	2.71	78.8
			r	—	.13	.09	.03	.13
3	M	39	*M*	2.59	13.45	3.76	9.97	508.7
			SD	1.79	4.40	1.86	2.39	82.8
			r	—	.38	.33	.35	.44
	F	46	*M*	2.30	13.09	3.47	9.65	499.1
			SD	1.84	4.36	1.84	2.86	75.9
			r	—	−.14	.01	.09	.00
4	M	66	*M*	1.68	13.76	3.41	9.75	518.9
			SD	1.32	5.04	2.08	3.04	75.5
			r	—	.18	.14	.13	.38
	F	44	*M*	1.89	12.10	3.24	8.93	480.2
			SD	1.09	4.52	1.62	2.77	68.0
			r	—	.17	.00	.20	.27
5	M	23	*M*	2.70	15.01	3.73	9.81	542.2
			SD	.93	4.56	1.66	3.05	63.1
			r	—	−.14	.24	−.22	.06
	F	32	*M*	3.13	16.69	4.20	10.92	555.3
			SD	1.24	3.88	1.94	1.92	69.2
			r	—	.36	.39	.29	.38
Weighted average	M	169			.26	.18	.23	.36
	F	154			.11	.10	.15	.18
	Total	323			.19	.14	.19	.28

possible, that score still predicted performance as well as the Insight scale, in which means were closer to the scale midpoint.

Discussion

The creation of subscores from the current SAT-M test does not appear to be justified. However, the current data from local placement tests suggest that a relatively short algebra placement test can predict performance in specific courses better than the full SAT-M score. If the SAT-M test is to be made more useful for placement, the addition of a short section of algebra achievement items has much more promise than any rescoring of the existing test.

The magnitude of the gender differences in SAT-M performance, or even in sets of relatively easy items from the SAT-M, deserve further study, especially because these gender differences are not apparent in grades in mathematics courses and are notably smaller in other measures of mathematics ability that focus more on computational skills as opposed to the mathematical reasoning skills stressed in the SAT-M test. Further research should focus on differences in item content as well as on differences in speededness between the SAT-M and other mathematics tests.

TRAIT-TREATMENT INTERACTIONS

A trait-treatment interaction exists if students with low scores on a predictor are more successful in an advanced course if first placed into a less advanced course (long sequence), while students with high scores on the predictor do better (or at least no worse) in the more advanced course if placed directly into it (short sequence). Trait-treatment interactions for placement are typically very difficult to demonstrate. The reasons for this are partly

Table 16. Prediction of Calculus Grades from SAT-M Subscores

College	Sex	N	Statistic	College Grade	SAT Scores: Algebra	Insight	Routine	SAT-M
2	M	101	M	2.35	20.02	6.01	11.94	648.3
			SD	1.12	3.47	2.04	1.98	63.2
			r	—	.20	.20	.22	.23
	F	27	M	2.44	18.78	5.08	11.61	611.9
			SD	1.05	3.71	1.94	2.12	74.8
			r	—	.23	.28	.22	.32
3	M	31	M	2.61	19.16	5.22	11.69	620.6
			SD	1.17	5.02	2.37	2.88	81.4
			r	—	.19	.07	.23	.37
	F	11	M	3.00	18.41	4.83	11.88	590.9
			SD	1.00	2.93	1.65	1.18	67.9
			r	—	.00	−.06	.27	04
4	M	78	M	1.80	17.76	4.53	11.22	598.1
			SD	1.36	4.36	2.17	2.61	68.6
			r	—	.39	.35	.23	.43
	F	29	M	1.89	15.79	3.76	10.66	551.7
			SD	1.32	3.64	1.54	1.32	68.6
			r	—	.40	.30	.03	.47
6	M	24	M	2.79	18.55	5.41	11.85	612.1
			SD	.87	3.72	2.27	1.90	82.5
			r	—	.35	.12	.21	.44
	F	22	M	2.73	18.26	5.03	11.27	593.6
			SD	.91	4.00	1.81	2.03	57.2
			r	—	.33	.00	.42	.40
7	M	19	M	2.84	19.43	4.57	11.81	607.4
			SD	.80	3.76	1.07	1.78	63.6
			r	—	.36	.15	.40	.34
	F	26	M	3.01	17.61	4.44	11.26	577.3
			SD	.77	3.93	1.88	1.81	82.5
			r	—	.19	.23	.11	.27
8	M	15	M	2.51	19.90	5.63	12.66	635.3
			SD	1.38	3.04	1.90	1.35	67 0
			r	—	.11	.38	.27	.36
	F	17	M	3.00	16.06	3.74	9.87	543.5
			SD	1.12	3.19	1.68	1.69	76.1
			r	–	.55	.66	.60	.63
10	M	47	M	2 49	20.40	6.35	12.23	666.4
			SD	1.25	4.25	1.97	2.47	67.4
			r	—	.22	.21	.24	.24
Weighted average	M	315			.27	.23	.24	.32
	F	132			.30	.25	.24	.37
	Total	447			.28	.23	.24	.34
Medians	M		(7 Samples)		.22	.20	.23	.36
	F		(6 Samples)		.29	.26	.25	.37
	Total		(13 Samples)		.23	.24	.23	.36

statistical (because of the large standard errors associated with assessing differences in regression slopes, large samples are required); partly the result of ineffective remedial courses; and partly the result of course enrollment patterns. (Students most in need of remedial courses may be properly placed into those courses and benefit from them but never take a more advanced course and thus be lost to the trait-treatment interaction study that looks at performance in the advanced course as the criterion.) It is also difficult to demonstrate a trait-treatment interaction if the institution is already making good placement decisions. The students with low scores on the placement test who would probably fail if placed directly into the more advanced course (thus contributing to a relatively steep regression line in the short-sequence group) are not permitted to enroll immediately in the advanced course.

From an evaluation perspective, at least 100 students should be randomly assigned to each sequence (Cronbach and Snow 1977). But from a practical perspective random assignment into mathematics courses does not occur, and a quasi-experiment using existing placement procedures must be substituted for a true experiment. Futhermore, few institutions have at least 100 students in a short sequence and 100 students in a long sequence. In the current sample, only three courses at College 1 met the 100 students in each group criterion (the algebra and precalculus sequence, the precalculus and calculus sequence for nonengineers, and the precalculus and calculus sequence for engineers). Relaxing the criterion to include sequences with at least 40 students in each group permitted the addition of the precalculus and calculus sequences at College 2.

The criterion score is the grade in the most advanced course (e.g., the grade in calculus whether taken first [short sequence] or after precalculus [long sequence]). Ideally, students in both the short and long sequences would be mixed together in the same calculus course. But in practice students in the short sequence take calculus in the fall while students in the long sequence take it in the spring. It is then necessary to assume that the fall and spring courses are identical in terms of the material covered and the grading standards applied. In some courses this assumption may be untenable, but it is probably reasonable for large freshman sections of a mathematics course that uses a standard textbook from one semester to the next and relatively standard objective examinations.

Three sets of test scores were used as predictors. First, the SAT-M score was used as the sole predictor. The second predictor was the equally weighted composite of GPA and SAT-M/200 that the previous regressions suggested would be better than either predictor by itself and would not be biased by gender. The third predictor was the local placement test used at College 1.[3] Regressions were performed in which the test score (or test+GPA composite) was entered first, followed by a dummy variable indicating whether the person was in the long sequence (coded +1) or the short sequence (coded −1), followed by the product of the dummy times the score to test for the significance of the interaction (i.e., whether the regression slopes were different in the short and long sequences).

Results and Discussion

Table 17 presents the N for each of the groups in the trait-treatment interactions and the multiple correlation (R) as each term is entered into the equation. Although statistically significant ($p < .05$) interactions were found in two courses, these interactions were of little practical significance. The largest significant interaction increased R by only .009.

The interaction for the precalculus criterion and the composite SAT-M and GPA predictor is presented in Figure 6. In this group, scores of the long-sequence students ranged from 3.6 to 6.5 and scores of the short-sequence students ranged from 3.6 to 7.0. Overall, 95 percent of the scores were between 3.9 and 6.4 and it is thus apparent on the graph that both the long- and short-course sequences were about equally effective for students in this range.

Note also that the point of intersection of the lines (4.8 on the composite score scale) is below the midpoint of the range, so that the harmful effect of taking algebra first for high-scoring students appears to be greater than the beneficial effect for low-scoring students. This paradoxical situation could be explained by the nonrandom ways in which students end up in either the long or the short sequence. Two students (call them Q and R) with equal, and relatively high, scores on the composite score scale could attend the first one or two precalculus classes in the term and Q might discover that she or he lacked knowledge of some basic skills that were unmeasured by the composite score. Q would then drop out of the precalculus course and instead take the long sequence. If the basic skill deficit were partially, but not fully, remediated by the algebra course, Q's precalculus grade might still be lower than R's grade. An observer who saw only that Q and R initially had the same composite score might wrongly conclude that the long sequence is harmful. If the initial composite score had also assessed the critical basic skill, there might have been no interaction

3. Three placement tests were used, one in basic mathematics, one in algebra, and one in trigonometry. For predictions of precalculus grades a composite of the basic mathematics and algebra tests was used (total of 57 items); for predictions of calculus grades a composite of the algebra and trigonometry tests was used (total of 45 items).

Table 17. Trait-Treatment Interactions

College	Sequence	N		R		R		R
1	Short = precalculus	1,465	SAT-M	.151	SAT-M+GPA	.316	PLACE	.319
	Long = algebra and precalculus		sequence	.158	sequence	.316	sequence	.330
		471	interaction	.167	interaction	.321*	interaction	.332
1	Short = calculus for engineers	2,321	SAT-M	.217	SAT-M+GPA	.316	PLACE	.366
	Long = precalculus and calculus for engineers		sequence	.248	sequence	.321	sequence	.371
		519	interaction	.249	interaction	.321	interaction	.372
1	Short = calculus for nonengineers	1,093	SAT-M	.238	SAT-M+GPA	.369	PLACE	.363
	Long = precalculus and calculus for nonengineers		sequence	.262	sequence	.370	sequence	.368
		483	interaction	.267*	interaction	.371	interaction	.375*
2	Short = calculus for engineers	108	SAT-M	.179	SAT-M+GPA	.435		
	Long = precalculus and calculus for engineers		sequence	.236	sequence	.467		
		43	interaction	.251	interaction	.467		
2	Short = calculus for nonengineers	47	SAT-M	.191	SAT-M+GPA	.336		
	Long = precalculus and calculus for nonengineers		sequence	.279	sequence	.394		
		146	interaction	.290	interaction	.411		

*$p < .05$

PLACE is the local placement test at College 1

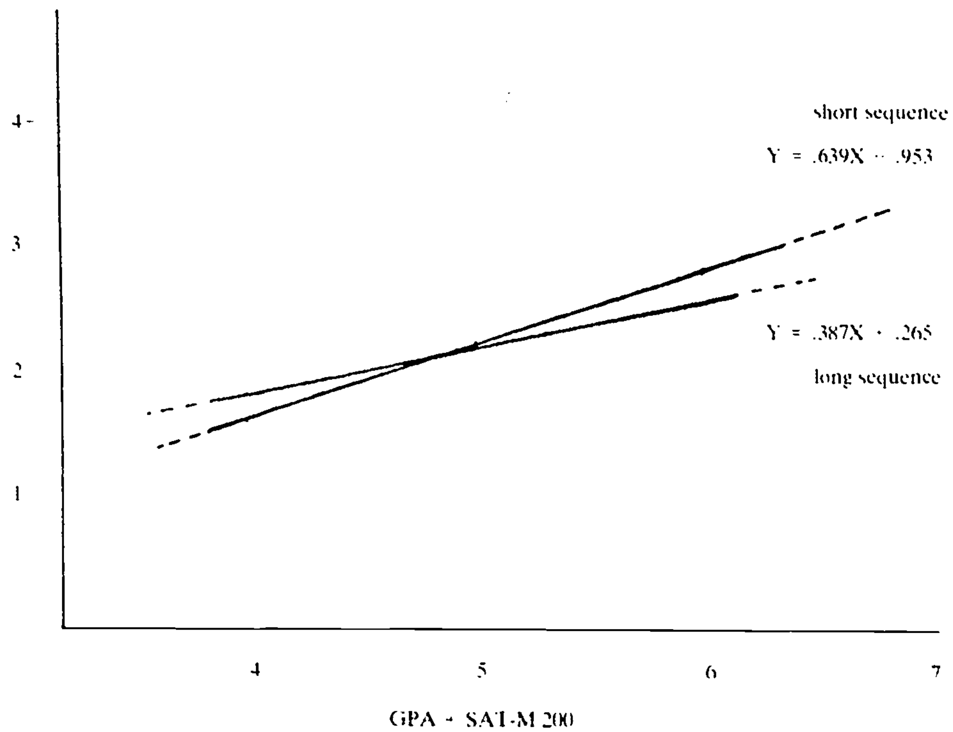

Figure 6. Trait-Treatment Interaction in College 1 Nonengineering Precalculus Course.

demonstrated. Thus, in the absence of random assignment, it would be incorrect to conclude that a placement test that demonstrated a trait-treatment interaction was necessarily superior to another placement test that did not demonstrate such an interaction.

Conclusion

Although no practically significant interactions were demonstrated, the lack of random assignment and the apparent efficiency of the placement procedures already in place at the institutions studied mitigated against finding interactions. Even with random initial assignments, interactions might be difficult to find if students could adjust their class placements after the first few class meetings.

REPLICATION OF PLACEMENT DECISIONS

This section analyzes the extent to which placements actually made at the colleges (using local placement test scores, faculty recommendations, and student self-selection) could be replicated using only SAT-M scores and information from the SDQ (i.e., mathematics experience [as defined in the section on Predictive Validity], high school mathematics grade-point average [M-GPA], and high school grade-point average [GPA]). The SAS Logist procedure (Harrell 1983) was used to compute logistic regressions with the aforementioned scores as the independent variables and actual class placement as the dependent variable. Two sets of dichotomous placement decisions were analyzed; one for algebra versus precalculus decisions and one for precalculus versus calculus decisions.

Results and Discussion

Statistics for the algebra versus precalculus decisions are presented in Table 18. For each student in the sample, the model uses the SAT-M and SDQ scores to predict the probability that the student is in the precalculus class. *C* is one way of assessing the predictive ability of the model. Suppose all possible pairs of observations were considered in which one member of the pair was in algebra and the other in precalculus. Then count the number of pairs in which the predicted probability is higher for the member in precalculus (i.e., the number of pairs in which the predicted probabilities are concordant with actual placements). The proportion of all pairs that are concordant is *C*. Somers' *D* is the rank correlation between predicted probabilities and actual placement (coded 0 or 1).

For each college, the SAT-M score was a significant predictor of actual placements with a chi-square at least twice as large as any other predictor. Experience (i.e., index of highest level mathematics course taken in high school) was a significant predictor in two out of the three courses where it was available. GPA was a significant predictor only at College 1 and mathematics GPA

Table 18. Logistic Regressions for Algebra Versus Precalculus Decisions

College	*N*	*Variable*	*Beta*	*Chi-square*	*p*	*C*ᵃ	*D*ᵇ
1	3,358	SAT-M	.006	88.3	.01		
		GPA	.631	37.1	.01		
		Full Model		147.0	.01	.66	.32
2	237	SAT-M	.011	29.4	.01		
		Experience	.605	4.0	.05		
		M-GPA	.263	1.0	.32		
		GPA	.622	2.8	.09		
		Full Model		59.9	.01	.77	.55
3	392	SAT-M	.006	17.5	.01		
		Experience	.356	4.8	.03		
		M-GPA	.189	1.1	.30		
		GPA	.084	.1	.76		
		Full Model		39.5	.01	.69	.37
4	196	SAT-M	.007	7.6	.01		
		Experience	.383	2.1	.15		
		M-GPA	.425	1.7	.19		
		GPA	.403	.7	.40		
		Full Model		19.5	.01	.70	.41

ᵃ*C* = proportion of concordant pairs
ᵇ*D* = Somers' *Dyx*

Table 19. Actual Versus Predicted Class Placements for Algebra Versus Precalculus Decisions

College			*Actual Placement* −	*Actual Placement* +		*Grade*	*Percent Agree*	*Random Percent Agree*
1	Predicted Placement	−	467	425	34%	A or B		
					38%	D or F		
		+	425	1,041	57%	A or B		
					19%	D or F		
			892	1,466			64	53
2	Predicted Placement	−	83	36	36%	A or B		
					14%	D or F		
		+	36	82	45%	A or B		
					11%	D or F		
			119	118			70	50
3	Predicted Placement	−	161	66	50%	A or B		
					47%	D or F		
		+	66	99	62%	A or B		
					36%	D or F		
			227	165			66	51
4	Predicted Placement	−	18	28	14%	A or B		
					68%	D or F		
		+	28	122	31%	A or B		
					33%	D or F		
			46	150			71	64

Note: − = algebra; + = precalculus.

was not a significant predictor in any course. Although GPA is a good indicator of success within courses at this level (see Table 8), it is a relatively poor indicator of placements between courses.

Table 19 displays an alternative way of summarizing these data. It shows how well the predicted placements agree with the actual placements. Predicted placements were generated by arranging the probabilities from the logistic regression in order from highest to lowest. The top *n* scores were then selected with *n* equal to the number of students who were actually placed in the precalculus course. Note that this procedure guarantees that the number of false positives will equal the number of false negatives. Percent agreement is percent of the total group for which the predicted and actual placements were in agreement. Random percent agreement indicates what the percent agreement would be if students were placed randomly into the courses with fixed sizes. Random percent agreement is 50 percent if both courses are about equal in size but can be substantially above 50 percent if one course is much larger than the other (as at College 4). For students who took precalculus and were either false negatives (i.e., SAT/SDQ indicated placement into algebra but actual placement was precalculus) or true positives (i.e., SAT/SDQ agreed with actual placement), the table also shows the percent of students who got A's or B's and the percent who got D's or F's.

Because there is no guarantee that the actual placements were optimal, the percent agreement data should be considered together with the grade data. For example, at College 1 the agreement was only 64 percent, but 38 percent of the false negatives actually did very poorly (i.e., D or F) in the course compared with only 19 percent of the true positives who did poorly. A similar situation existed at College 4 where 68 percent of the false positives did very poorly. Compare these numbers to College 2 where the percent of students doing poorly was almost the same for the false negatives and the true positives.

Table 20 presents the logistic regressions for precalculus versus calculus decisions. Again, for each course, the largest chi-square was associated with the SAT-M score. But GPA (or mathematics GPA) was also a significant predictor for four out of five courses. For the three courses with the necessary data, the expe-

Table 20. Logistic Regressions for Precalculus Versus Calculus Decisions

College	*N*	*Variable*	*Beta*	*Chi-square*	*p* <	*C**	*D*'
1N	2,560	SAT-M	.012	277.3	.01		
		GPA	1.301	155.9	.01		
		Full Model		690.9	.01	.81	.62
1E	3,238	SAT-M	.017	475.6	.01		
		GPA	1.406	197.0	.01		
		Full Model		1188.1	.01	.88	.76
2	224	SAT-M	.006	6.9	.01		
		Experience	.856	9.2	.01		
		M-GPA	.408	1.8	.18		
		GPA	.036	0.0	.92		
		Full Model		25.9	.01	.69	.37
3	137	SAT-M	.016	25.9	.01		
		Experience	1.456	16.0	.01		
		M-GPA	1.113	7.8	.01		
		GPA	.016	0.0	.97		
		Full Model		120.4	.01	.90	.80
4	334	SAT-M	.012	35.9	.01		
		Experience	1.248	23.7	.01		
		M-GPA	.099	.1	.72		
		GPA	1.150	8.5	.01		
		Full Model		151.7	.01	.86	.72

**C* = *proportion of concordant pairs*
'*D* = Somers' *Dyx*

rience score was a significant predictor. As indicated in Table 21, the percent agreements were quite high. Where the percent agreement was relatively low (College 2), the grade data suggest that the fault may lie more with the college's procedures than with the SAT/SDQ predictor set. Among the false negatives 23 percent did very poorly compared with only 9 percent of the true positives and only 23 percent of the false negatives got A's or B's compared with 68 percent of the true positives.

The ability of the SAT/SDQ to replicate placement decisions is particularly remarkable given the timing of the tests. Because most students take the SAT (and provide SDQ information) at the end of their junior year or near the beginning of their senior year in high school, mathematics course work taken during the senior year is not reflected in the SAT scores or SDQ reports.[4] Thus, colleges could use the SAT/SDQ as a good first approximation for placement decisions but should make some provision for follow-up testing (or review of high school transcripts) of students who were just below the cut score and who were enrolled in mathematics courses during their senior year. Even if the colleges choose to do their own placement testing of all incoming students, the early availability of a SAT/SDQ-based placement score could be useful for planning needed section sizes in freshman-level mathematics courses.

CONCLUSIONS

The SAT-M score by itself is a relatively poor predictor of success in college mathematics courses when compared to tests specifically designed for placement purposes. Because the SAT-M score does not assess many important skills taught in courses beyond the first year of high school algebra, it should not be the only consideration for course placement. It may, however, provide useful supplementary information. Indeed, when SAT-M scores were combined with information from the high school record, about 80 percent of the precalculus versus calculus decisions could be accurately predicted. Although the within-course grade correlations are severely attenuated by the restricted range within a single course, these range restrictions have much less impact on between-course placement decisions.

Use of SAT-M scores alone could result in overestimating grades of males and underestimating grades of females. SAT-M scores of males were generally one-third of a standard deviation or more higher than

4. Although the SDQ does ask students to indicate the courses that they *plan* to take during their senior year, there is no assurance that the courses were satisfactorily completed and the course grades obviously cannot be reported in advance.

Table 21. Actual Versus Predicted Class Placements for Precalculus Versus Calculus Decisions

College			*Actual Placement* −	*Actual Placement* +		*Grade*	*Percent Agree*	*Random Percent Agree*
1N	Predicted Placement	−	1,151	315	40%	A or B		
					32%	D or F		
		+	315	779	59%	A or B		
					17%	D or F		
			1,466	1,094			75	51
1E	Predicted Placement	−	634	276	34%	A or B		
					39%	D or F		
		+	276	2,052	45%	A or B		
					22%	D or F		
			910	2,328			83	60
2	Predicted Placement	−	78	40	23%	A or B		
					23%	D or F		
		+	40	66	68%	A or B		
					9%	D or F		
			118	106			64	50
3	Predicted Placement	−	146	19	32%	A or B		
					26%	D or F		
		+	19	53	62%	A or B		
					13%	D or F		
			165	72			84	58
4	Predicted Placement	−	114	36	17%	A or B		
					64%	D or F		
		+	36	148	45%	A or B		
					26%	D or F		
			150	184			78	50

Note: − = algebra; + = precalculus.

scores of females in courses where females had equal or higher grades. Gender differences in SAT-M performance appear to be larger than for the placement tests studied here. Combining SAT-M scores with high school GPA not only raised validity coefficients but also tended to eliminate any underprediction of women's course grades.

Based on a much more limited sample, the College Board Mathematics Achievement Tests predicted grades about as well as or slightly better than SAT-M scores.

Subscores generated from the current SAT-M test do not appear to be useful for placement. However, the relatively high correlations noted for short, locally developed placement tests suggest that creating a new SAT-M subscore by adding similar algebra achievement test items may substantially enhance the placement utility of the SAT-M test. The timing of the SAT, because it is often administered before high school mathematics course work is completed, would remain a problem regardless of any additions to item content.

Considering the relatively low correlations, the content coverage (no advanced algebra or trigonometry), and the timing of the test (often administered near or before the beginning of the senior year in high school), the most reasonable use of SAT scores for placement may be as a preliminary screening instrument. High-scoring students could be exempted from basic mathematics courses, but students scoring below the cutoff should be given another opportunity to demonstrate their competence.

REFERENCES

Breland, H. M. 1977. *A Study of College English Placement and the Test of Standard Written English*. College Entrance Examination Board Research and Development Reports (RDR 76–77, No. 4).

Breland, H. M., G. C. Conlan, and D. Rogosa. 1976. *A Preliminary Study of the Test of Standard Written English*. Princeton, N.J.: Educational Testing Service.

Bridgeman, B. 1980. *Validation of the Descriptive Tests of Mathematics Skills*. College Entrance Examination Board Research and Development Reports (RDR 79–80, No. 2).

College Board. 1987. *The College Handbook*. New York: College Entrance Examination Board.

Cook, L., D. Eignor, and J. Mazzeo. 1986. *A Study of the Comparability of Scores Obtained on the ATP Mathematics Level I and Level II Tests*. Princeton, N.J.: Educational Testing Service.

Cronbach, L. J., and R. E. Snow. 1977. *Aptitudes and Instructional Methods: A Handbook for Research on Interactions*. New York: American Council on Education.

Gulliksen, H. 1950. *Theory of Mental Tests*. New York: John Wiley and Sons.

Harrell, F. E. 1983. In *SUGI Supplemental Library User's Guide 1983 Edition*. Cary, North Carolina: SAS Institute.

Messick, S. 1980. *The Effectiveness of Coaching for the SAT: Review and Analysis of Research from the Fifties to the FTC*. Princeton, N.J.: Educational Testing Service.

Messick, S. 1988. "Validity." In *Educational Measurement*, 3d ed. edited by R. L. Linn. New York: Macmillan.

Ramist, L. 1984. "Predictive Validity of the ATP Tests." In T. Donlan, *The College Board Technical Handbook for the Scholastic Aptitude Test and Achievement Tests*. New York: College Entrance Examination Board.

Schrader, W. B. 1971. "The Predictive Validity of College Board Admissions Tests." In *The College Board Admissions Testing Program: A Technical Report on Research and Development Activities Relating to the Scholastic Aptitude Test and Achievement Tests*, edited by William H. Angoff. New York: College Entrance Examination Board.

Appendix A. College Descriptions

College	*Description*	*Enrollment**	*% Top Tenth*†	*Placement*
1	Eastern state university	27,000	45	Locally developed exams in basic mathematics, algebra, and trigonometry
2	Rocky Mountain state university	17,000	31	Locally developed exam
3	Rocky Mountain state university	14,000	Mid 50% SAT-V: 410–520 SAT-M: 430–590	Locally developed exam
4	New England state university	7,000	21 Mid 50% SAT-V: 450–550 SAT-M: 480–620	Locally developed exam
5	Southern state university	18,000	Mid 50% SAT-V: 430–530 SAT-M: 480–580	Mathematics Association of America (MAA) exam
6	Western private (church affiliated) university	4,000	39 Mid 50% SAT-V: 470–580 SAT-M: 520–640	Locally developed exam
7	New England private university	3,000	38 Mid 50% SAT-V: 470–580 SAT-M: 520–640	SAT
8	Southwestern private university	5,000	28 Mid 50% SAT-V: 460–580 SAT-M: 500–600	Mathematics Association of America (MAA) exam
9	Southern institute of technology	3,000 (74% men 26% women)	30 Mid 50% SAT-V: 450–520 SAT-M: 540–600	Mathematics Association of America (MAA) exam
10	Midwestern institute of technology	1,000 (all men)	60 Mid 50% SAT-V: 500–610 SAT-M: 620–730	None

Note: Data are from *The College Handbook* (College Board, 1987), except placement data which were provided in questionnaires completed by the colleges.

*Enrollment, in this appendix, is number of full-time undergraduates rounded to the nearest thousand.

†"% top tenth" is the percent of the freshman class ranking in the top tenth of their high school classes.

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APPENDIX B. Prediction of Grades by Course

Table B-1. Prediction of Algebra Grades

College	*N*	*Statistic*	*Grade*	*Local Placement Test*	*SAT-M*	*SAT-V*	*Experience*	*Math Average*	*GPA*
1	872	*M*	2.49	19.40	434.9	423.0			2.64
		SD	1.23	5.39	76.3	78.9			.44
		r		.33	.10	−.05			.26
2	119	*M*	2.92		503.0	471.3	2.87	3.03	3.22
		SD	1.10		82.4	86.9	.58	.78	.46
		r			−.03	−.18	−.06	.28	.25
3	272	*M*	1.81		462.6	435.0	2.48	2.84	3.13
		SD	1.83		77.3	77.4	.76	.73	.43
		r			.10	−.04	.14	.22	.10
4	46	*M*	1.78		458.3	446.1	2.61	2.72	2.91
		SD	1.22		56.2	64.7	.71	.66	.39
		r			.27	−.06	.00	.35	.22
5	48	*M*	2.00		453.3	434.4	2.69	2.92	3.28
		SD	1.25		61.2	67.5	.62	.58	.46
		r			.37	−.19	.16	.47	.18
6	58	*M*	2.45	12.40	531.4	473.8	2.90	3.36	3.57
		SD	.81	5.21	76.0	75.4	.58	.58	.35
		r		.42	.46	−.02	.29	.21	.17
		Colleges							
Weighted	930	(1 and 6)		.36	.12	−.05			.25
average	1,415	(all)			.12	−.06			.22
	543	(2–6)			.15	−.08	.10	.27	.16

Table B-2. Prediction of Precalculus Grades

College	*N*	*Statistic*	*Grade*	*Local Placement Test*	*SAT-M*	*SAT-V*	*Experience*	*Math Average*	*GPA*
1N	1,440	*M*	2.35	28.18	471.8	441.9			2.81
		SD	1.29	6.93	75.7	80.8			.50
		r		.36	.17	.00			.34
1E	896	*M*		28.13	492.0	440.6			2.80
		SD		6.36	74.3	80.4			.52
		r		.29	.14	−.05			.30
2	118	*M*	2.53		576.7	486.5	3.14	3.34	3.40
		SD	1.08		75.9	79.8	.51	.54	1.08
		r			.01	−.20	−.07	.31	.25
3	165	*M*	2.18		502.9	447.2	2.76	3.01	3.21
		SD	1.85		78.8	75.8	.67	.72	.51
		r			.10	−.06	.19	.29	.21
4	150	*M*	1.69		499.7	451.9	2.87	3.01	3.06
		SD	1.23		75.9	75.2	.64	.67	.44
		r			.32	.32	.25	.31	.39
5	183	*M*	2.77		547.4	482.7	3.19	3.43	3.49
		SD	1.14		69.8	80.0	.58	.60	.52
		r			.28	.21	.24	.31	.31
8	53	*M*	2.69		512.1	450.9	2.77	3.32	3.55
		SD	.99		80.6	74.7	.58	.64	.43
		r			.11	.14	.11	.28	.34
		Colleges							
Weighted average	2,336	(1)		.33	.16	−.02			.32
	3,005	(all)			.16	.01			.32
	669	(2,3,4,5,8)			.18	.09	.16	.30	.30

Table B-3. Prediction of Calculus Grades

College	N	Statistic	Grade	Local Placement Test	SAT-M	SAT-V	Experience	Math Average	GPA
1N	1,050	M	2.46	27.21	557.8	483.9			3.24
		SD	1.20	6.00	77.4	80.4			.48
		r		.39	.20	.06			.30
1E	2,293	M	2.29	31.80	602.2	495.6			3.32
		SD	1.26	6.24	73.5	85.5			.51
		r		.34	.14	-.02			.22
2N	106	M	2.56		610.1	509.5	3.43	3.48	3.50
		SD	1.13		62.8	83.8	.59	.59	.56
		r			.23	.15	.25	.29	.42
2E	214	M	2.41		639.4	520.0	3.51	3.69	3.71
		SD	1.11		66.4	83.0	.66	.47	.48
		r			.26	-.07	.19	.26	.27
3	72	M	2.64		610.1	487.2	3.42	3.68	3.58
		SD	1.20		72.0	72.2	.55	.50	.47
		r			.29	.32	.34	.32	.41
4	184	M	2.00		589.5	487.8	3.50	3.41	3.37
		SD	1.35		73.7	81.6	.57	.58	.43
		r			.37	.12	.29	.40	.39
6	129	M	2.82	21.67	610.7	506.8	3.47	3.64	3.72
		SD	.85	5.31	69.5	88.5	.61	.48	.40
		r		.60	.43	.03	.26	.21	.35
7	54	M	2.88		579.6	524.8	3.50	3.57	3.62
		SD	.92		79.4	76.8	.64	.60	.48
		r			.12	.12	-.01	.41	.30
8	61	M	2.69	14.56	575.1	502.1	3.39	3.66	3.74
		SD	1.28	6.03	82.1	82.7	.56	.54	.41
		r		.46	.34	.29	.42	.39	.47
9	53	M	2.60	14.78	606.2	495.3	3.54	3.51	3.43
		SD	1.24	5.27	75.5	98.0	.54	.61	.55
		r		.60	.21	.09	.59	.55	.40
10	83	M	2.69		660.4	54.0	3.54	3.77	3.73
		SD	1.18		70.9	85.1	.52	.42	.47
		r			.21	.02	.00	.39	.43
Weighted average	N	Colleges							
	3,586	(1,6,8,9)		.37	.17	.01			.26
	4,299	(all)			.19	.02			.27
	956	(all but 1)			.29	.08	25	.33	.37

SECTION 1 | **Time—30 minutes 25 Questions** | In this section solve each problem, using any available space on the page for scratchwork. Then decide which is the best of the choices given and blacken the corresponding space on the answer sheet.

The following information is for your reference in solving some of the problems.

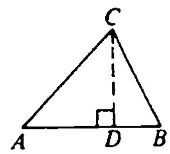
Circle of radius r: Area $= \pi r^2$; Circumference $= 2\pi r$

The number of degrees of arc in a circle is 360.

The measure in degrees of a straight angle is 180.

Definitions of symbols:

- $=$ is equal to
- $\neq$ is unequal to
- $<$ is less than
- $>$ is greater than
- $\leqq$ is less than or equal to
- $\geqq$ is greater than or equal to
- $\parallel$ is parallel to
- $\perp$ is perpendicular to



Triangle: The sum of the measures in degrees of the angles of a triangle is 180.

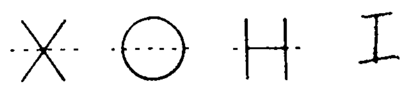
If $\angle CDA$ is a right angle, then

(1) area of $\triangle ABC = \frac{AB \times CD}{2}$

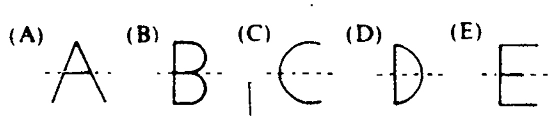
(2) $AC^2 = AD^2 + DC^2$

Note: Figures that accompany problems in this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible EXCEPT when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated. All numbers used are real numbers.

1. A letter is defined as being "foldable" if it is symmetric with respect to its horizontal midline. For example, the three letters below are "foldable" because they are symmetric with respect to the midlines shown.



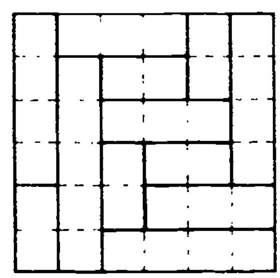
Each of the following letters is "foldable" EXCEPT



$$x + y + z = 6$$
$$2y + z = 7$$

2. In the equations above, if $z = 1$, then $x =$

(A) -1 (B) 1 (C) 2 (D) 3 (E) 5



3. In the figure above, twelve rectangles (outlined by solid lines) are arranged to form a 6-by-6 square as shown. If the same rectangles were connected, without overlapping, to form a strip of width 1 unit, the strip would be how many units long?

(A) 12 (B) 18 (C) 24 (D) 30 (E) 36

4. $\frac{1}{1{,}000} + \frac{3}{10} + \frac{5}{100} =$

(A) 0.135 (B) 0.153 (C) 0.315 (D) 0.351 (E) 0.531

5. If $n = 10$, which of the following has the <u>least</u> value?

(A) $2 - n$ (B) $n - 2$ (C) $\frac{2}{n}$ (D) $\frac{n}{2}$ (E) $\frac{n}{2} - 2$

GO ON TO THE NEXT PAGE

6. A certain company makes a total of 96 ski vests a day in sizes small, medium, and large. If the number of vests in the small and large sizes combined is equal to the number of vests in the medium size, what is the daily production of vests in the medium size?

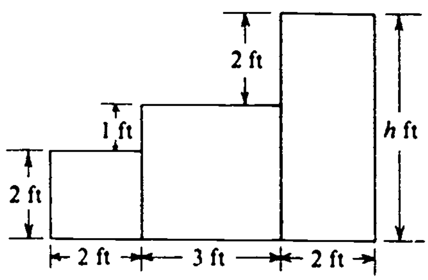
(A) 23
(B) 24
(C) 28
(D) 32
(E) 48

A

7. On a rectangular coordinate graph, which of the following points would be the same distance from the origin as (2, 0)?

I. (0, 2)
II. (−2, 0)
III. (1, 1)

(A) I only
(B) III only
(C) I and II only
(D) I and III only
(E) I, II, and III



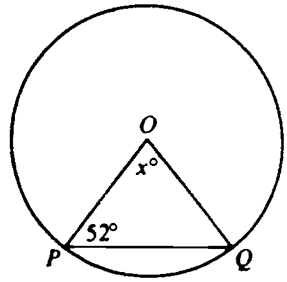
8. The rectangles above represent three cut tree trunks arranged with trunks perpendicular to the ground to form "stairs." What is the value of h, in feet?

(A) 5 (B) 6 (C) 7 (D) 10 (E) 12

R

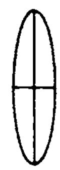
9. Of the 35 students in a certain homeroom, 20 joined the math club. Of these 20 students, $\frac{3}{5}$ were females. What was the number of female students in the homeroom?

(A) 12
(B) 15
(C) 18
(D) 21
(E) It cannot be determined from the information given.



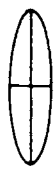
10. In the figure above, if O is the center of the circle, then $x =$

(A) 70 (B) 72 (C) 74 (D) 76 (E) 78

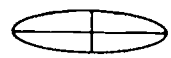


11. Each of the line segments in the figure above is the perpendicular bisector of the other. If the figure is rotated in the same plane 180° clockwise about the point of intersection of the segments, the resulting figure will look like which of the following?

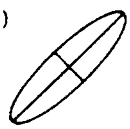
(A)



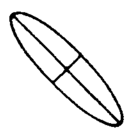
(B)



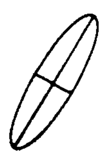
(C)



(D)



(E)



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12. $(-2a)^3 =$

(A) $-8a$ (B) $8a^3$ (C) $-2a^3$
(D) $-8(-a)^3$ (E) $-8a^3$

13. Which of the following can be expressed as the product of two consecutive even integers?

(A) 24 (B) 36 (C) 42 (D) 60 (E) 72

14. Martin and Alice buy newspapers for $0.20 each and sell them for $0.25 each. If, at the end of one week, Martin made a profit of $12.60 and Alice made a profit of $18.75, how many more papers did Alice sell than Martin?

(A) 125
(B) 123
(C) 63
(D) 62
(E) 43

15. The volume of solid X is $\frac{4}{3}\pi w^3$ and the volume of solid Y is $2\pi w^3$. The volume of solid X is what percent of the volume of solid Y?

(A) $33\frac{1}{3}\%$
(B) 50%
(C) $66\frac{2}{3}\%$
(D) 75%
(E) $133\frac{1}{3}\%$

16. The operation Δ is defined for particular values a, b, and c by the equations $a \Delta b = 2$ and $2 \Delta c = 5$. For these values of a, b, and c, the expression $(a \Delta b) \Delta c$ is equal to

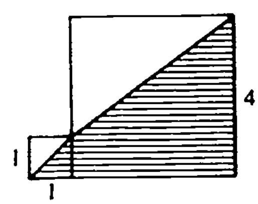
(A) 3
(B) 4
(C) 5
(D) 7
(E) 10

17. There are 5 locked doors and 3 keys. Each key opens one and only one door, and no two keys open the same door. Sam chooses a key, and tries it on different doors until he opens one. Leaving that door open, he repeats this process with the next key, and then the next, until three doors are open. If x is the total number of attempts, both successful and unsuccessful, to open these doors, what are the minimum and maximum possible values of x?

(A) 3 and 5
(B) 3 and 12
(C) 3 and 15
(D) 5 and 12
(E) 5 and 15

18. If x and y are integers and $\frac{x}{y} = \frac{2}{7}$, then $(x + y)$ could equal each of the following EXCEPT

(A) -9 (B) 27 (C) 45
(D) 56 (E) 117



19. In the figure above, a square with side of length 1 and a square with side of length 4 are placed as shown. What is the area of the shaded region?

(A) $8\frac{1}{2}$ (B) 10 (C) $10\frac{1}{4}$ (D) $10\frac{1}{2}$ (E) 12

20. If x is an integer and the product $x(x + 1)(x + 2)$ is negative, then the greatest possible value for x is

(A) -4 (B) -3 (C) -2 (D) -1 (E) 1

GO ON TO THE NEXT PAGE

21. The first two terms in a sequence are 2 and 4. The third term and all successive terms are generated by taking the average (arithmetic mean) of <u>all</u> the preceding terms. For example, the third term is 3, which is the average of 2 and 4. What is the sixth term of the sequence?

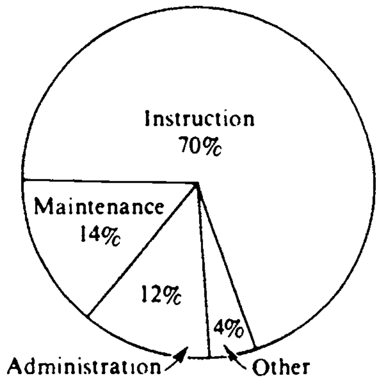
(A) $7\frac{1}{2}$

(B) 6

(C) $3\frac{1}{2}$

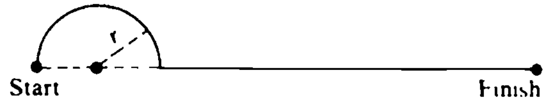
(D) 3

(E) $2\frac{1}{2}$



22. The graph above gives the breakdown of expenditures in school district X last year. If the cost of maintenance had been 50 percent less and the amount saved had been applied to instruction, then the increase in the expenditures for instruction would be what percent of the actual expenditures for instruction?

(A) 3.5% (B) 7% (C) 10%
(D) 20% (E) 50%



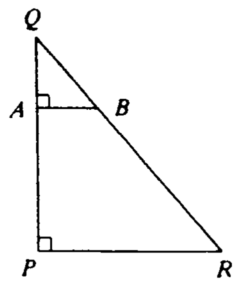
Note: Figure not drawn to scale.

23. In the figure above, the solid line shows the aerial view of a 15-mile path on which a race is run. If the radius r of the semicircular path is $\frac{2}{\pi}$ miles, how many miles long is the straight portion of the path?

(A) 11 (B) 13 (C) $15 - 2\pi$
(D) $15 - \frac{2}{\pi}$ (E) $15 - \frac{8}{\pi}$

24. If $n = (11^7 \times 2) + (11^7 \times 4) + (11^7 \times 6)$, which of the following is NOT a whole number?

(A) $\frac{n}{2}$ (B) $\frac{n}{3}$ (C) $\frac{n}{2} - \frac{n}{3}$
(D) $\frac{n^2}{6}$ (E) $\frac{n+6}{12}$



25. In the figure above, the area of $\triangle PQR = 54$. If $AQ = \frac{1}{3}PQ$ and $AB = \frac{1}{3}PR$, what is the area of $\triangle AQB$?

(A) 6 (B) 9 (C) 17 (D) 18
(E) It cannot be determined from the information given.

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THE TEST. STOP

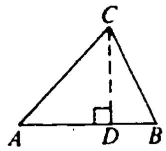
SECTION 3	Time—30 minutes 35 Questions	In this section solve each problem, using any available space on the page for scratchwork. Then decide which is the best of the choices given and blacken the corresponding space on the answer sheet.

The following information is for your reference in solving some of the problems.

Circle of radius r: Area = πr^2; Circumference = $2\pi r$
The number of degrees of arc in a circle is 360.
The measure in degrees of a straight angle is 180.

Definitions of symbols:

$=$ is equal to	$\leq$ is less than or equal to
$\neq$ is unequal to	$\geq$ is greater than or equal to
$<$ is less than	$\parallel$ is parallel to
$>$ is greater than	$\perp$ is perpendicular to



Triangle: The sum of the measures in degrees of the angles of a triangle is 180.
If $\angle CDA$ is a right angle, then

(1) area of $\triangle ABC = \dfrac{AB \times CD}{2}$

(2) $AC^2 = AD^2 + DC^2$

Note: Figures that accompany problems in this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible EXCEPT when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated. All numbers used are real numbers.

1. $2(9 + 6) - 4(10 \div 2) =$

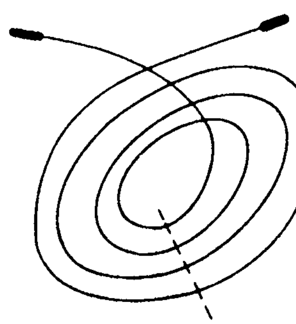
(A) 10 (B) 5 (C) 4 (D) −5 (E) −10

2. How many revolutions are made in 5 minutes by a fan that makes 30 revolutions per second?

(A) 900 (B) 1,500 (C) 1,800
(D) 5,400 (E) 9,000

3. A baker's dozen contains 13 items. A normal dozen contains 12 items. How many normal dozens contain as many items as 12 baker's dozens?

(A) 12 (B) $12\frac{1}{2}$ (C) 13
(D) 14 (E) $15\frac{1}{3}$



4. A rope is coiled 4 times as shown in the figure above. If a cut is made through the rope along the dotted line segment shown, into how many pieces will the rope be cut?

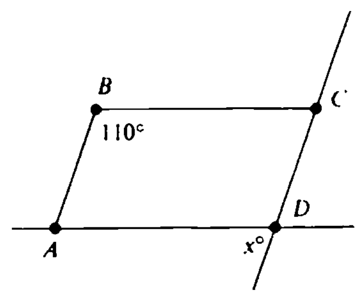
(A) Two (B) Four (C) Five
(D) Six (E) Eight

5. If the average (arithmetic mean) of four numbers is 37 and the average of two of these numbers is 33, what is the average of the other two numbers?

(A) 35
(B) 39
(C) 40
(D) 41
(E) 43

6. Which of the following is NOT equal to $\frac{1}{2}$ of an integer?

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$ (E) 2



7. In the figure above, $ABCD$ is a parallelogram. What is the value of x?

(A) 50
(B) 60
(C) 70
(D) 80
(E) 110

GO ON TO THE NEXT PAGE

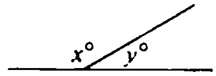
Questions 8-27 each consist of two quantities, one in Column A and one in Column B. You are to compare the two quantities and on the answer sheet blacken space

- A if the quantity in Column A is greater;
- B if the quantity in Column B is greater;
- C if the two quantities are equal;
- D if the relationship cannot be determined from the information given.

AN E RESPONSE WILL NOT BE SCORED.

EXAMPLES

	Column A	Column B	Answers
E1.	2×6	$2 + 6$	● Ⓑ Ⓒ Ⓓ Ⓔ
E2.	$180 - x$	y	Ⓐ Ⓑ ● Ⓓ Ⓔ
E3.	$p - q$	$q - p$	Ⓐ Ⓑ Ⓒ ● Ⓔ



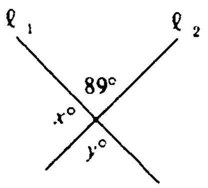
Notes:

1. In certain questions, information concerning one or both of the quantities to be compared is centered above the two columns.
2. In a given question, a symbol that appears in both columns represents the same thing in Column A as it does in Column B.
3. Letters such as x, n, and k stand for real numbers.

	Column A	Column B
8.	The price of 1 dozen apples	The price of a 3 pound bag of apples

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

	Column A	Column B
9.	The product of the even integers in S	The product of the odd integers in S



	Column A	Column B
10.	x	y

There are 22 boys and 21 girls in a class of 43 students.

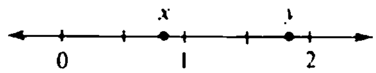
	Column A	Column B
11.	The ratio of the number of girls to the number of boys	The ratio of the number of boys to the number of girls

Questions 12-13 refer to the following definition of $[a, b, c]$ for all real numbers a, b, and c.

$$[a, b, c] = (a - b)(a - c)(b - c)$$

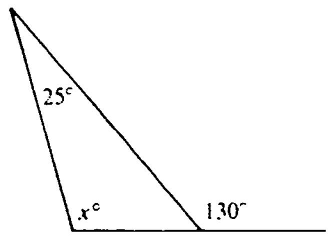
	Column A	Column B
12.	$[1, 1, 1]$	$[2, 2, 2]$
13.	$[a, 2, 1]$	$a^2 - 3a - 2$

Column A | Column B

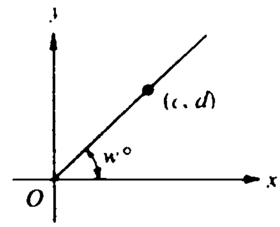


x and y are points on the number line.

	Column A	Column B
14.	xy	2



	Column A	Column B
15.	x	110



Note: Figure not drawn to scale

$c \neq d$

	Column A	Column B
16.	w	45

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SUMMARY DIRECTIONS FOR COMPARISON QUESTIONS

Answer:
- A if the quantity in Column A is greater;
- B if the quantity in Column B is greater;
- C if the two quantities are equal;
- D if the relationship cannot be determined from the information given.

AN E RESPONSE WILL NOT BE SCORED.

	Column A	Column B
17.	$(3+5)^2$	3^2+5^2

A,R

×	P	Q	R
P	1	6	
Q	6		18
R			

An incomplete multiplication table for positive integers P, Q, and R is shown.

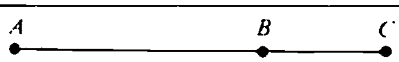
18. The number of blank squares for which products can be calculated — 5

I

If a certain number n is reduced by 3, the result equals $\frac{1}{2}$ the value of n.

19. n — 5

A



Segment AC represents a ribbon $15\frac{3}{4}$ inches long. The length of AB equals $\frac{2}{3}$ of the length of AC.

20. The length of BC — 5 inches

R

$xy = 8$ and $yz = 12$.

21. $\frac{8}{x}$ — $\frac{12}{z}$

A,R

22. The volume of a cube if the area of one of its faces is 36 square inches — The volume of a rectangular solid if its edges are 3 inches, 7 inches, and 10 inches

	Column A	Column B

$x + (4 - y) = 10$

23. $x + y$ — $x - y$

The fraction $\frac{1}{7}$ can be written as the repeating decimal 0.142857142857 . . . where the block of digits 142857 repeats.

24. The 610th digit following the decimal point — 5

A

a and b are positive numbers.

25. $a \cdot b$ — $\frac{a}{b}$

$b + c = a$
$b + a = d$
$c + c = d$

26. $2b$ — $c + 1$

I

$y = 4 - (2 + x)^2$

27. The greatest possible value of y — 4

A

GO ON TO THE NEXT PAGE

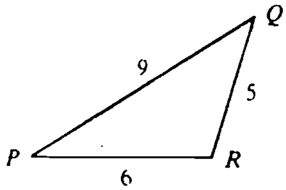
Solve each of the remaining problems in this section using any available space for scratchwork. Then decide which is the best of the choices given and blacken the corresponding space on the answer sheet.

28. If x and y are positive integers and $x - \frac{1}{y} = \frac{7}{2}$, then $x =$

(A) 3 (B) 4 (C) 6 (D) 7 (E) 8

29. Mary has $(m + 17)$ apples, Scott has $(m + 8)$ apples, and John has $(m + 5)$ apples. All of these apples are put into 3 empty boxes so that each box contains exactly x apples. What is the value of x in terms of m?

(A) $m + 10$
(B) $3m + 10$
(C) $3m + 30$
(D) $10m$
(E) $11m$

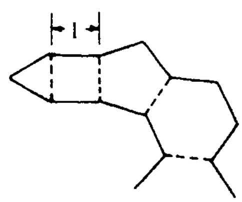


30. In $\triangle PQR$ above, if point S (not shown) is on QR between Q and R, then the length of segment PS could be any number between

(A) 7 and 10 (B) 6 and 9 (C) 5 and 11
(D) 1 and 9 (E) 1 and 15

31. 5 percent of 6 percent is

(A) 0.11% (B) 0.3% (C) 3%
(D) 11% (E) 30%



32. When the figure above is completed, it will consist of six regular polygons, each having sides of length 1 and each having one side in common with each of its adjacent polygons. If the two polygons that complete the figure have 8 and 10 sides, respectively, what will be the perimeter of the completed figure?

(A) 24 (B) 26 (C) 30 (D) 31 (E) 36

33. If a bus averages 55 miles per hour on a certain trip, it will arrive at the terminal 2 hours early. If the bus averages 35 miles per hour, it will arrive 2 hours late. What is the length, in miles, of the trip?

(A) 9 (B) 90 (C) 125
(D) 385 (E) 550

34. Gina has exactly \$39 in \$10, \$5, and \$1 bills in her purse. If she has more \$1 than \$5 bills and more \$5 than \$10 bills, how many different combinations containing at least one of each type could she have?

(A) Two
(B) Three
(C) Four
(D) Five
(E) Six

35. If a rectangular piece of paper is cut into exactly three pieces by making two straight cuts, which of the following could be the total number of edges on the three pieces?

I. 9
II. 11
III. 12

(A) I only (B) III only (C) I and III only
(D) II and III only (E) I, II, and III

STOP

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THE TEST.