DOCUMENT RESUME

ED 390 661 SE 057 128

AUTHOR	Schifter, Deborah
TITLE	Voicing the New Pedagogy: Teachers Write about Learning and Teaching Mathematics. Center for the Development of Teaching Paper Series.
INSTITUTION	Education Development Center, Inc., Newton, Mass.
SPONS AGENCY	DeWitt Wallace/Reader's Digest Fund, Pleasantville, N.Y. Business Roundtable.; National Science Foundation, Washington, D.C.
PUB DATE	[94]
NOTE	28p.
AVAILABLE FROM	Center for the Development of Teaching, Education Development Center, Inc., 55 Chapel Street, Newton, MA 02158-1060.
PUB TYPE	Reports - Descriptive (141)
EDRS PRICE	MF01/PC02 Plus Postage.
DESCRIPTORS	*Educational Change; Elementary Secondary Education; Higher Education; Journal Writing; *Mathematics Education; *Personal Narratives; Teacher Education
IDENTIFIERS	*Reform Efforts; Teacher Change

ABSTRACT

Detailed descriptions of classroom process are needed in order to ground discussion of the principles animating the mathematics education reform movement. In response to this need, teachers who were already engaged in transforming their mathematics instruction were invited to write reflective narratives about their evolving instructional practice. This paper describes the structure of that project, the Mathematics Process Writing Project, and presents excerpts from some representative narratives. It also considers how writing these narratives contributed to the continuing development of their authors and discusses how reading them affected students in teacher education courses. Finally, it urges that such narratives be seen as a medium through which teachers can become centrally involved in the national conversation about mathematics education reform. (Author)

Center for the Development of Teaching
PAPER SERIES

Voicing the New Pedagogy

Teachers Write about Learning and Teaching Mathematics

Deborah Schifter

EDC

CENTER FOR THE
DEVELOPMENT OF TEACHING
PAPER
SERIES

Voicing the New Pedagogy

Teachers Write about Learning and Teaching Mathematics

Deborah Schifter

EDC

The Center for the Development of Teaching (CDT) is a research and development center, within the Center for Learning, Teaching, and Technology (LTT) at Education Development Center, Inc. (EDC). The goal of the Center for the Development of Teaching is to learn, with teachers, how teachers' practice can be transformed so that it supports students' construction of knowledge. Initially, the Center is working with mathematics teachers, but will expand to include the teaching of science and/or history and language. The Center carries out a coordinated program of research and action projects that address the issues involved in teacher change at three interacting levels: (1) teachers' beliefs and knowledge about their subjects and about learning and teaching; (2) teachers' classroom practice; and (3) the complex social system that extends from the school and school district to the society at large.

This CDT Paper Series is intended as a vehicle for discussion of research on teaching and teacher development as they relate to education reform. Publications in this series will contribute to the community's understanding of the practice and profession of teaching, as well as to the process of change. It is our editorial policy to have papers reviewed by a panel of four readers — three named by the author and one chosen by CDT. People in the education community who would like to contribute to this discussion are invited to submit papers to be considered for publication, along with the names of three potential reviewers. Please write to:

Editor, CDT Paper Series
Center for the Development of Teaching
Education Development Center, Inc.
55 Chapel Street
Newton, MA 02158-1060

Core support for the Center for the Development of Teaching, and for CDT's Paper Series, has been provided by the DeWitt Wallace–Reader's Digest Fund, whose mission is to foster fundamental improvement in the quality of educational and career development opportunities for all school-age youth, and to increase access to these improved services for young people in low-income communities.

If you would like to be in direct contact with the author of this paper, please write to:

Deborah Schifter
Center for the Development of Teaching
Education Development Center, Inc.
55 Chapel Street
Newton, MA 02158-1060
Telephone: 617-969-7100
FAX: 617-965-6325

Voicing the New Pedagogy

Teachers W'rite about Learning and Teaching Mathematics

Deborah Schifter

Detailed descriptions of classroom process are needed in order to ground discussion of the principles animating the mathematics education reform movement. In response to this need, teachers who were already engaged in transforming their mathematics instruction were invited to write reflective narratives about their evolving instructional practice. This paper describes the structure of that project and presents excerpts from some representative narratives. It also considers how writing these narratives contributed to the continuing development of their authors and discusses how reading them affected students in teacher education courses. Finally, it urges that such narratives be seen as a medium through which teachers can become centrally involved in the national conversation about mathematics education reform.

Out of a convergence between changing social needs and two decades of research in cognitive psychology, a new vision of mathematics instruction has emerged in the United States. In contrast to the traditional classroom, in which the teacher presents material that her students then practice individually, the proposed pedagogy involves students working together on authentic problems—posing their own questions, formulating conjectures, and discussing the validity of various solutions. Though codified in the *NCTM Standards* (National Council of Teachers of Mathematics, 1989, 1991) and embraced by influential segments of the education policy community, it has yet to be shown how this vision will appear when translated into the day-to-day life of the mathematics classroom. Constructing a practice that will realize the principles animating the *Standards* has only just begun, and it follows from the very nature of those principles that classroom teachers must be the primary agents of that process.

With increased authority—and responsibility—for shaping mathematics instruction in their respective classrooms, teachers will also need to invent new forms of collegiality. In some schools, and in some in-service programs, teachers meet together regularly to solve pedagogical problems and consider issues that arise as they work to transform their teaching. In general, however, what teachers are learning in their own classrooms is not being communicated to their colleagues or to others with an interest in mathematics education. The success of the movement for reform cannot rest on individual teachers constructing the new practice classroom by classroom, independently of one another and without engaging in many-sided discussions about their efforts. And teacher educators, curriculum developers, and researchers must both encourage and listen in to such discussion if *they* hope to play a consequential role in the reform process. In the end, it is teachers who must concretely demonstrate to one another, as well as to the rest of us, what the new mathematics instruction will look like.

With this need acknowledged, the absence of teacher voices from the ongoing national conversation about mathematics education reform becomes deafening (Cochran-Smith and Lytle, 1990; Lytle and Cochran-Smith, 1990; Miller, 1990). This silence has provoked some teacher educators, myself included, to wonder, What are the forms and the forums through which teachers might share what they are learning as they begin to transform their practice along the lines mandated by the *Standards*?

In recent years, the conviction has been growing that cases or stories may be more helpful than theoretical expostitions to people who need to learn to think in new ways about complex, context-dependent domains like teaching (Barnett, 1991; Carter, 1993; Shulman, 1986; Shulman, 1992; Witherell and Noddings, 1991). Only through the telling of stories about their classrooms can teachers convey the richness and interconnectedness of what they have come to understand—about their students, their schools, and their communities; about subject matter; about both established classroom structures and experimental practices — as they face the stream of challenges that constitutes everyday life in the classroom. The current mathematics education literature provides examples upon which such teacher narratives might be modeled: case studies of classroom teachers written by researchers (e.g., Fennema et al., 1993; Schifter and Fosnot, 1993; Wilcox et al., 1992); case studies conducted by university faculty who also teach mathematics to kindergarten through twelfth grade and make their own teaching the object of their research (Ball, 1993a, 1993b; Borasi, 1992; Lampert, 1988, 1989); and cases written by full-time classroom teachers (Barnett, 1991; Countryman, 1992). Such studies provide rich accounts of classroom process, illustrating the dilemmas that arise in daily instruction and explicating how teachers think about and resolve such dilemmas.

In this paper I describe an experimental project that borrows elements from each of these models. Designed to support teachers as they write about their own mathematics instruction, the Mathematics Process Writing Project (MPWP) was conducted from 1990 to 1993 by SummerMath for Teachers, a K–12 in-service mathematics program. Since 1983, SummerMath for Teachers, which is based at Mount Holyoke College, has offered summer institutes and semester-long mathematics courses based on constructivist perspectives on learning (Schifter, 1993; Schifter and Fosnot, 1993; Simon and Schifter, 1991).

The Project

The purpose of MPWP was to produce detailed, reflective first-person narratives exploring classroom process and instructional goals and decision making for use in teacher education courses. The idea for the project came from the recognition that, although a significant number of SummerMath for Teachers participants—having engaged in summer institutes, semester-long mathematics courses, and/or a year-round classroom supervision program—had made considerable progress in transforming their teaching, many others were unable to move forward. The reasons were varied, but the need for curricular materials was frequently cited.

In trying to address these teachers' needs, the quandary of the staff was that there were so few innovative published materials available and that traditional formats were of little use. For example, a powerful lesson is often launched by a single question. Yet, that same question, baldly stated and lacking context—as is usually the case with traditional materials—may yield no more than a mechanical exercise in computation. The ability to position such questions in the flow of classroom process would clearly be of far greater value.

MPWP was designed to address this issue, taking advantage of the knowledge and experience of teachers who had been working to enact the new mathematics pedagogy. Each year for three years, 14 to 19 teachers (a total of 44 women and 4 men) who had previously attended at least one SummerMath for Teachers offering were invited to become teacher-writers in a one-semester course that met weekly for three hours and twice for additional full-day workshops. Some participants had been working with SummerMath for Teachers for as long as seven years; others had entered the program the previous summer and were just now beginning to work through what it meant to enact a practice based on a constructivist view of learning.

The course comprised two major activities: reading assigned materials and writing. The reading materials were written by teachers about their own teaching—for example, articles by Ball (1993a, 1993b), Heaton (1991), and Lampert (1988, 1989), as well as articles coming out of the current movement to reform the teaching of reading and writing (Atwell, 1985; Hillocks, 1990). In addition, the second and third groups of teacher-writers read papers written by their predecessors. All readings were critically examined for both content and writing style.

The writing component of the course was fashioned after the process-writing model that many of the elementary teachers already used in their own classes. Consistent with the new mathematics pedagogy, process writers work cooperatively to analyze and edit their projects. For the first several weeks of the course, specific assignments were given so that teachers could explore pedagogical issues and experiment with writing styles (e.g., transcribe a classroom dialogue and then write a narrative based on that dialogue about what happened; describe a student who has revealed to you that he/she has learned something that you are trying to teach; write about a student who expresses a mathematical idea that surprises you). Eventually, teachers determined the direction of their own writing and worked on final projects—15- to 40-page reflective narratives on topics of their choosing. Throughout the course, teachers met in both small and large groups to share their efforts and solicit feedback. All work was turned in to me, the project director and instructor, and I responded in writing. Upon request, I met with teachers in class or in my office, or spoke with them over the telephone.

Appendix A is a compilation of reading and writing assignments given in the three courses.

The Products

The 49 papers produced by project participants are quite varied.[1] Some explore particular grade-specific mathematical topics—third graders working on graphs, sixth graders using Logo to discover properties of triangles, high school students constructing meaning for the concept of *variable*. In contrast to traditional classroom presentations of mathematics activities, these papers position activities and problems in the life-process of particular classrooms. The reader learns about the teacher's goals for the lesson, what was happening before a particular problem was posed, what happened afterwards, about the questions students asked and the ideas they suggested, how they interacted with one another and with the teacher, what students learned, what the teacher learned, and more.

For example, in her paper "Third Graders Explore Multiplication," Virginia Brown (in press) describes the session in which she introduces multiplication in order to illustrate to the reader the routine with which she frequently begins

[1] A subset of these papers will appear in an anthology of two volumes (Schifter, in press, a, in press, b). Other papers have been or will be published individually (e.g., Lester, in press, a; Smith, 1992). Appendix B lists all the teacher-writers and the titles of their papers.

her mathematics lessons. She starts by posing a repeated-addition problem about three pencil cases, each of which holds 12 pencils. How many pencils are there?

> I gave the children a considerable amount of time to think and then asked for solutions. Peter offered 34 as his result; Cathy's answer was 24; Kevin's reply was 36; and Mike's 15. The rest agreed with one of these. I asked the class how they felt about these answers, and the following dialogue ensued:
>
> Emily: I don't think 15 could be right.
>
> Teacher: Why?
>
> Emily: Well, the problem says three pencil cases with 12 pencils in each case. That means you have 12 here (pointing to an invisible case on her desk), 12 here, and 12 here. You have three 12s, so the answer couldn't be 15.
>
> Mike (who had originally offered 15 as his answer): I don't like my answer anymore. I was adding 12 and 3, but now I don't think that makes sense.
>
> Emily (smiling): It doesn't make sense because you would be adding 3 pencil cases and 12 pencils. You can't write with a pencil case.
>
> Steve was waving his arm vigorously, anxious to speak. I asked him to tell us what he was thinking.
>
> Steve: I don't think 24 could be right either, because I know 12 and 12 equals 24. That would only be two 12s, but there are three 12s because there are three pencil cases, so 24 can't be right.
>
> As Steve spoke, other children were nodding in assent, including Mike, who had initially arrived at 15 for his answer, and Cathy, whose answer was the one being challenged. She asked to have her answer erased, no longer agreeing with it.

In this classroom, students have become accustomed to a routine in which a problem is posed and then, without relying on paper and pencil, they are expected to think about possible solutions. When they indicate to the teacher that they are ready, she solicits their answers and writes each one on the board without comment. Discussion begins with the question, "How do you feel about these answers?" The children voice their thoughts and listen to one another. They have learned that it is quite acceptable to propose an answer or an idea a ' then, after further reflection, to change one's mind.

Through Brown's example, a reader new to this kind of pedagogy can begin to attach images to such phrases as "emphasis on student thinking as opposed to answers," "classroom discourse," and "mathematical inquiry." We see how young children can think through the logic of competing solutions, listen to classmates' ideas, and trust their own conclusions. We also see how the teacher can step back from the role of "explainer" and "answer giver" to let the children pursue their own lines of thought.

The routine in Brown's class usually continues with students breaking into small groups to explore the initial problem further, or ones similar to it, with manipulatives. Later, they return to the whole group to share their various solution methods or to voice new questions or discoveries made while exploring the problem.

On this day, however, the initial whole-class discussion leads in an unexpected direction, and Brown suspends her original agenda. Having reached consensus that the answer is 36 and having been shown how to represent the problem as multiplication—$3 \times 12 = 36$—the class listens to Jeff's idea:

> Jeff: I did it the same way Joe did, but I just figured out another way. Can I show it on the board? If you break each 12 up into two 6s, you have six 6s, and that equals 36.

Leaving behind the context of pencils and pencil cases, Jeff has seen that if each of the 12s is viewed as the sum of two 6s, you get six 6s, resulting in another multiplication fact: $6 \times 6 = 36$. Although she had had other plans for the day's lesson, Brown recognizes the opportunity to explore flexibility in the children's thinking about numbers. She asks Jeff to explain his discovery again and waits to see what other ideas the children might come up with.

Tom shows the class how you could break each of the 6s into two 3s to come up with $12 \times 3 = 36$, at which Anna exclaims, "Wow, we've found a lot of things that equal 36. Oh, look! This one is backwards of our first one, 3×12." Then seeing the column of twelve 3s which total 36, Steve suggests grouping three 3s into 9s to get $4 \times 9 = 36$. The lesson ends with the following exchange:

> Joe: [Steve] circled three 3s, which equals 9, and he did this four times, so $4 \times 9 = 36$. And so we

can add another one to the list because if $4 \times 9 = 36$ then $9 \times 4 = 36$, too.

Anna: Does that always work? I mean, saying each one backwards will you always get the same answer?

Teacher: Interesting question! What do you think?

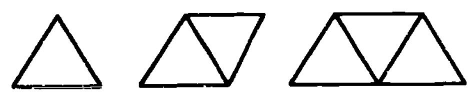
Anna: I'm not sure. It seems to but I can't tell if it would *always* work—I mean for *all* numbers.

Knowing that the recess bell was about to ring, and wanting to encourage their mathematical inquisitiveness, I told them to see if they could figure out a way to prove or disprove Anna's question overnight, and we'd start the next day's math lesson with it.

Not only are these children practicing numerical thinking, but as they come up with various factor pairs of 36 they find patterns—that 12×3 is "the backwards" of 3×12—from which they infer that if $4 \times 9 = 36$, then 9×4 must equal 36, too. However, Anna recognizes that they cannot simply assume that the pattern will always hold for all numbers. Thus, cast in formal language, these third graders have formulated the conjecture that multiplication is commutative, but have decided that they must find proof. And by refraining from telling them the answer, Brown demonstrates her belief that it is important that *the class* think this matter through.

As a set, the papers produced by MPWP span grades kindergarten through twelve in communities that represent a variety of ethnicities and economic levels. In sharp contrast to Brown, who narrates her classroom stories from the perspective of a third-grade teacher in a rural environment, Donna Scanlon teaches algebra in an urban high school with a 70% Hispanic population. As a result of drastic funding cuts, her mathematics department has just suffered a 40% reduction in staff and she is teaching 191 students grouped into six classes. While she acknowledges that "things are not as they should be," she finds that "somehow my students and I are getting pleasure out of working together to make sense out of mathematics. Certainly we hope that this situation will be resolved, but meanwhile we have so much to do, so much to figure out." And though this is not her primary intention, through her story, Scanlon implicitly refutes those who claim that the new mathematics pedagogy is appropriate only for white middle- and upper middle-class children.

In "Algebra is Cool: Reflections on a Changing Pedagogy in an Urban Setting," Scanlon (in press) shares with the reader what she has learned as she "search[es] for tasks that will allow algebra to become a natural and sensible extension of the mathematics my students have previously studied." For example, to give her students "opportunities to see how a variable could be used to say something they would like to say, as in making a rule of generalization," she has them explore a set of problems called "Polygons All in a Row." Given a sequence of equilateral triangles with sides of one unit,



what is the perimeter of a single triangle? of figures formed by abutting 2 triangles, 3 triangles, 4 triangles, 50 triangles, 100 triangles, k triangles? Students are given a table to organize their data and, by the time they complete the table, they are able to put into words the insight that for k triangles, the perimeter will be two more than k, or, in the language of algebra, $k + 2$. In a similar exercise with squares, students create a table, identify a pattern, and generate the expression for the perimeter, $2k + 2$.

Although the class is able to come up with answers to the questions on the worksheet, Scanlon feels uneasy about what they actually understand. By the end of the "squares" exercise, she realizes that her students have used the figures to generate tables and the tables to generate rules, but nobody is looking back to see how the rules connect to the original figures. "Students were not seeing that they were adding the top length and the bottom length of the squares and that was why they were doubling, and that the two end unit lengths were the 'add 2.'" However, before she can pursue that idea, she is faced with another pedagogical issue: many of her students got the same incorrect answer to another problem.

The next day I didn't get anywhere with trying to get someone to connect the formula to the picture, so I gave up for the moment and went

> on to pentagons. . . . This time, students [were given the figures, the first three elements in the sequence, but] were not given a table, and few used one to organize their data. I found as I checked homework that most were not able to decide the perimeter of 9 pentagons or 100 pentagons or k pentagons correctly and yet many students had the same "wrong" answers, especially in reference to 9 pentagons. They thought the perimeter should be 33. I was puzzled by this. I was pretty sure they wouldn't bother copying someone else's answer unless they could explain it because that was such a prevalent underlying current in everything that we did. Students expected to be asked for explanations. Plus it wasn't a few students, it was many who had 33 for the answer. . . .
>
> Since everyone was stuck at about the same place, I asked if anyone had a suggestion as to how we could look at all of our information and try to make sense out of it. Someone suggested a table like our others so I made one on the board. . . . After easily filling in the correct perimeters for 1, 2, and 3 pentagons, students were reluctant to share an answer for 9 pentagons. Eventually, Nathan offered the above-mentioned 33. . . .
>
> "Well, 3 pentagons have a perimeter of 11. So 9 pentagons should have a perimeter that is three times that," he answered quite cheerfully.
>
> "Could anyone explain what Nathan is trying to tell us?" I asked.
>
> Lisette offered, "He's saying that 9 is 3 times as big as 3 so the perimeter should be 3 times bigger and 3 times 11 is 33."
>
> "How can we check to see if Nathan's idea works?" I asked.
>
> "It doesn't work. I drew these [nine pentagons in a row] out and I found out the right answer is 29," said Katie.
>
> "Another way we could check Nathan's theory?" I asked, realizing that this must have been the theory of th others who got that same answer.
>
> "It doesn't work because then it would have to be true for 8 and 2," said Madeline. . . . "The perimeter of 8 [pentagons] is not 4 times the perimeter of 2."

As in Brown's class, Scanlon's students propose answers, explain their thinking, and analyze the problem from various perspectives. Today they decide that the logic that produced the answer 33 is faulty, and that the correct answer is 29. Sarita goes on to volunteer that for 100 pentagons, the answer is 302, "because the perimeter is always equal to 3 times the number of pentagons plus 2 more. So the formula is $3k + 2$."

> I let the class think about this for a little while. Then I noticed a conversation going on between students and was just about to correct them when one of the students, Juan, said, "You add the bottoms, then you add the tops, and then you add the two sides."
>
> As we were all caught up in finding the pattern from the tables, Juan was cutting to the heart of the matter and dealing with the actual diagram. . . . I asked Juan to explain, and to expedite matters he came to the board himself and drew some pentagons:
>
> 
>
> He said, "Bottoms, 4. Tops, 8. Plus the two ends is 14."
>
> I said, "What about if you had 10 pentagons?"
>
> Juan said, "Bottoms, 10. Tops 20. Plus the 2 ends is 32."
>
> The class responded with their attention. This looked like math they could handle.
>
> "What about if you had k pentagons? What would the perimeter be?" I asked.
>
> "Bottoms, k. Tops, 2 times k. Plus 2," said Juan.
>
> I wrote $k + 2k + 2$ on the board with [Sarita's formula] $3k + 2$ underneath. Are they the same? Are they different? Explain your answer. These are the questions students explored for homework.

In this class, Scanlon is trying to help her students see that algebra is not merely a matter of applying rules to meaningless numbers and letters, but is a language they can use to express their own observations of patterns and rules. And she communicates that it is up to the students to figure out basic rules of algebraic equivalence. Scanlon contrasts this approach with her own past teaching:

> Previously, I would have explained this by talking about "combining like terms" and "adding coefficients." I would have justified those procedures with the distributive axiom: $k + 2k = (1 + 2)k$. And students would have plenty of practice combining other like terms.

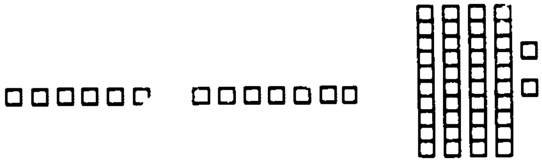
> Although all this "formal mathematics" certainly needs to be addressed in my classroom, it will be more meaningful to my students now that they have had a chance to see how sensible it actually is. They could look at those pentagons and *see* how it makes sense.

Scanlon is working to develop a practice that teaches formal mathematics and conventional algebraic language, but considers the perspectives of her students. She plans her teaching around tasks that involve representations and problems that give her students access to the logic of algebra and organizes lessons to engage the class in discussion of those problems.

Other papers portray issues that classroom teachers face as they engage the new mathematics pedagogy: How does one teach students to listen to one another, work collaboratively, and participate in mathematical inquiry? How might published materials be adapted to meet the particular needs of one's class? How does one reach all students?

In "Beyond Stardom: Challenging Competent Math Students in a Mixed Ability Classroom," Margaret Riddle (in press), a fifth-grade teacher, discusses her concern for those students who come to her after having excelled in traditional mathematics instruction with previous teachers:

> These young math stars. . . are able to figure quickly and accurately in their heads and on paper. Their self-esteem has been enhanced by knowing they are often first with the answer . . . .I believe that it is often their very success as elementary math students that works against these students beginning to develop into truly competent young mathematicians. [They] have a vested interest in maintaining their position at the top of the class, and they certainly don't perceive that they have a problem as math students. The feeling of disequilibrium that comes from wondering and exploring alternative solutions is one with which they have had little experience.
>
> As an example, she cites Scott:
>
> When asked, "What is multiplication?" he glibly replied, "Multiplication is an easier way to add." This was a definition that the students had learned the year before and many of them recited it without appearing to know what it meant. Scott added, more thoughtfully, "6 × 7 means there are 6 groups of 7." When I asked him to use objects to represent what he meant, he arranged them this way:



6 × 7 = 42

> He "read" his picture as "6 sets of 7." However, when I asked him to explain what he meant he said, "Well, here's the 6 and here's the 7 and here's the 42." It appeared that he didn't expect math to make sense in a real way and that he didn't associate computation with a concrete understanding. Although he seemed able to solve real world problems quickly, self-confidently, and accurately, it appeared to me that he was missing out on the relationships and structures of math and that much of what he did was facile and quick but not really conscious.

While Riddle focuses on her most able fifth graders, Nina Koch (in press) writes about high school students who struggle with mathematics and "have missed out on some of the most basic ideas in arithmetic":

> They don't know the difference between additive and multiplicative relationships. They fail to grasp the structure of our number system, especially decimal and fraction numeration. Thus, it's not unusual for a student to add $5 when he intended to add 5%, or to be unable to tell you that 10/3 is a number that's a little more than 3. Most high school teachers have a full portfolio of anecdotes about students who are amazingly uninformed about numbers.

However, rather than confront their arithmetic deficit directly, which students find embarrassing and insulting, Koch advocates "sneak[ing] it into their study of more advanced topics. . . . The study of graphing in a coordinate plane provides a splendid opportunity for sneaking in arithmetic."

This Koch does, in a unit in which her remedial students use "Green Globs," a computer game that sets up a coordinate grid on the screen and scatters a total of 13 "globs" at various locations on the grid. The object of the game is to write an equation whose graph will hit the maximum number of globs. In the lesson Koch describes, her students are using equations of the form $y = A(x - B)^2 + C$—experimenting with different values of A, B, and C to determine how they affect the shape of the graph—to create the parabola that will annihilate the most globs. For Jason, this becomes a lesson in decimals:

> "Okay, Miss Koch, I've got it down to somewhere around point three or point four. But here's the thing—point three is too flat and point four is just a little too steep. See, look. It needs to be in between these two," he motioned at the graphs. "But there aren't any numbers to use."
>
> "You need a number between point three and point four?"
>
> "Right. But there aren't any. I wish I could use halves or something."
>
> Jason had gotten skillful in using the tenths place in his equation, but he was not identifying those numbers as fractions with a denominator of ten. To him, it was just "point this" and "point that" and he knew which ones were higher and which ones were lower. He was not aware that hundredths were available as parts of tenths, because he'd never had a working understanding of those fractions. Most students do have a working understanding of half, if no other fraction, so I decided to pursue that with him. "How would you use the half?"
>
> "Well, you know. Like three and a half is halfway between three and four. I need something like that. But they're not going to let me use point three and a half."
>
> "You couldn't write it that way?"
>
> "No, look. I even tried it already because I knew you'd make me." He gave a sly smile. "Watch what it does." Jason typed in .31/2 as the value of Parameter A. The computer interpreted this as .31 divided by 2, which equals .155. "It comes out way too wide," he reported.

Through Riddle's and Koch's papers, we come to know individual students, their strengths and weaknesses, their understandings and confusions. We also come to see how, in classes structured like theirs, teachers work to challenge and support students of diverse abilities in the construction of mathematical concepts.

Still other papers describe the process of learning to teach mathematics in a new way. In "Pictures at an Exhibition: A Mathphobic Confronts Fear, Loathing, Cosmic Dread, and Thirty Years of Math Education," Lisa Yaffee (in press) begins with scenes—scenes filled with disappointment and humiliation — of her attempts to make sense of the mathematics of her own K–12 schooling. She then describes what happened when she became an elementary-school teacher:

> I stumbled into the least likely of all scenarios, my own classroom! Imagine my surprise, nay HORROR, when I realized on the second day of school (not a moment sooner, mind you) that I would be expected to teach mathematics to 22 fifth and sixth graders who were much smarter than I in every way. Mathphobia—fear, loathing, nausea, clammy palms, rank odors, cold sweat pouring off the brow—these were familiar and therefore comfortable sensations compared to what I felt at that moment of epiphany. What was I going to do? . . . If I had taken math more seriously and tried harder to like it as a student myself, then I wouldn't be imparting my own hatred of the subject to the next generation. I felt angry, guilty, and devastated. How could anyone have allowed me into a classroom to perpetrate the same math crimes of which I had been a victim?

Resolved to do better by her students, she signed up in desperation when a series of workshops entitled "Making Math Learning Stick" was announced in her school. During each of those four sessions, teachers worked in small groups to solve word problems of various kinds and then gathered to verbalize what they learned. At the end of the series, she considered the implications for her own teaching:

> I knew now what had to be done, but didn't know how to do it. Worse yet, I didn't think I could do it. I was a first-year teacher. . . struggling with how to teach writing, reading, health, social studies, science, and study skills to a combined-grade group when there was no set curriculum. I couldn't give up the math text. . . . But, I realized suddenly, I could introduce a "Problem of the Day" with which to begin each class. After the kids solved it and we talked about it, they could go back to working in their books.
>
> The next day I threw the first problem at my students. It was one of those ones where you have a given number of species, heads, and legs, and you have to find out how many of each kind of animal there are. I read the problem out loud to the group, trying to suppress my excite-

> ment. . . . Then I looked up. No one cracked a facial expression or said a word. I could feel the gloomy tension in the air. Finally Max asked quietly, "Why do we have to do this?"
>
> Taken aback, I sputtered, "Well, I thought it would be fun for a change. Everyone is always complaining about how boring math class is, and about how they hate to work out of the book. Here's a chance to do a problem where the answer isn't obvious, nor is the procedure for finding the answer.". . .
>
> "Don't you already know the answer?" demanded a skeptical Amber.
>
> "Well, I have one answer, but I'm not really sure if it's right," I answered truthfully.
>
> Jeff snorted his contempt. "Some teacher!"

We accompany Yaffee through her exhibition, early pictures illustrating the development of her demons, later pictures depicting her confronting them; finally, she describes the process through which she eventually learns to elicit from her students engagement with, and even, at times, excitement over mathematical ideas.

What characterizes all of the papers is their specificity. As the authors tell their stories, they detail students' words and gestures so as to bring us into their classrooms to "see" and "hear" for ourselves. But, unlike videotape, this medium presents scenes from teachers' perspectives, complete with thoughts, doubts, frustrations, and second thoughts. Thus, we come to share the dilemmas they face, the decisions they make, and the satisfactions they experience.

What Did Teacher–Writers Learn?

The process of writing these papers provided the authors with a uniquely powerful way to deepen their own understanding of the new mathematics pedagogy. Participants had already engaged in some kind of reflective writing process. In SummerMath for Teachers' courses taken prior to entering MPWP, they had kept journals that were periodically read by members of the staff, and most teachers had found this kind of writing an essential instrument of their learning. Some used their journals primarily as vehicles to explore mathematical issues; others emphasized analysis of their own learning processes; and still others found the autobiographical record the most useful aspect of journal keeping.

However, their work in MPWP was distinguished in several ways from the writing the teachers had done in the past: 1) they wrote concrete, detailed descriptions of classroom process, often in response to specific assignments; 2) they received regular feedback from their instructor; 3) they also met weekly with peers to discuss one another's work; and 4) the papers were written for a larger and anonymous public.

Teachers wrote concrete, detailed descriptions of classroom process.

The act of narrative-writing allowed teachers to revisit classroom events, and, by viewing these events from a distance, to consider them from new perspectives (van Manen, 1990). Teachers could identify alternatives to the decisions they had made and spot opportunities for learning they had missed. Often students' comments could be seen to have had different and/or greater significance than the teacher had appreciated in the moment.

Initially, many teachers found these new perceptions disconcerting. Reflecting on her experience of doing the weekly writing assignments, one teacher wrote. . .

> . . . the writing process forced a scrutiny of what goes on in my classroom that I have never experienced before—not from having observers in my room, not from being evaluated, and not from writing in a journal. When I first began, it was a painful experience to read what I had written. There were so many incidents and situations that looked different when I read about them that I began to question my teaching skill. Now that I have had a chance to think about the experience for a while, I realize that writing and then reading about what happened puts you and your observations some distance from the situation written about. It allows an objectivity in a more leisurely setting which helps to clarify thinking. Consequently, it makes sense that other options and questions would occur.

Careful descriptions allowed teachers to return to decisions made in the moment in order to analyze their immediate responses and assess their fit to the situation that evoked them:

> Writing for this class helped me to focus not only on activities and responses but on my motivations and expectations. There are times

> when I seem to have settled into a rhythm and I do things instinctively: responding to a student or posing the next question. In writing [my paper], I was forced to examine situations and try to clarify, at least to myself, why the response was appropriate.... Writing made me more aware of students' behaviors and provided a concrete record of my interactions so that I could and can review them by myself and with others to critique my teaching.

Many teachers found that transcribing dialogue—a task assigned early in the course—was particularly effective in highlighting for them the differences between the kind of listening habitual to the tradit onal classroom and that which encourages students to articulate their mathematical ideas. Donna Natowich entitles one section of her paper (1992), "Are You Listening, Mrs. Natowich?" and then writes:

> No. I wasn't listening. At least I wasn't listening in the way I needed to listen. I hadn't been listening for years—I planned lessons instead. I worked hard at planning appropriate motivating lessons. I planned, unaware of a very basic component—what the children actually knew. I made assumptions based on my experience and knowledge of children and the curriculum.
>
> I listened for right answers, confirmation that the students understood what they had been taught. I was accustomed to listening for specific indicators that a student was following my line of thinking. I taught other people's lessons, those deemed appropriate by the experts, and listened for the answers to assessment activities designed by these experts.

And another teacher wrote in her end-of-course evaluation:

> It is almost as if I heard them previously, but I had my next statements already planned. I attempted to adjust their thinking to what I planned to say next, instead of analyzing what they said to determine what I should ask, say, or do next.

Having been assigned to capture their students' words, teachers became more sensitive to what those words were saying, revealing thoughts and understandings—as well as misunderstandings—which frequently surprised them. Being asked to tell stories about classroom interactions had the effect, in one teacher's words, of "changing the lens."

> I know so much more about my kids now. By having to write down exact words, I had to slow down the whole process. I began to take much more seriously each question or look.

Another teacher wrote:

> The transcribing-dialogue assignment pushed me to *listen* to the children and to *think* about what they said. Taking notes during math class, I think, made the single biggest change in my teaching style. I needed to listen, not talk. I needed to slow the conversation down in my head, and therefore was able to process it more.

Furthermore, teachers reported that their own sharpened attention had a profound impact on the atmosphere of the classroom. As students realized how carefully their teachers were listening to their classmates, *they* began to really listen to one another, too.

Teachers received regular feedback from their instructor.

I collected teachers' writings each week and, every two to three weeks, returned their work with extensive feedback. In general, I tried to offer encouragement, pointing out strengths both in the writing and in the teaching it described. Reading with an eye toward how weekly assignments might lead to coherent final papers, I suggested ideas that could be developed, asking for more detail and further explication. My comments frequently urged teachers to write more explicitly about their own decision-making in their analyses of classroom process.

The challenge to be more explicit often led to deeper comprehension of the classroom situation. For example, after reading about a boy described as "learning disabled," I asked the author to provide a more detailed profile: What is this child's learning disability? What are the strengths he can call upon to support his own learning? How does he get along with classmates when they are not doing mathematics together? What are your learning goals for him? As she worked to describe this child more fully, his teacher came to know him better and so was able to define more clearly her goals and expectations.

At times I asked about the mathematics discussions being described: "There is only one girl who spoke up in the discussion. Does this reflect the ratio of boys to girls in your class?" "Perhaps your attempts to protect your students from the embarrassment of their inadequate work interfered with the expectation that they critically analyze what they had done. What do you think?" These questions invited teachers to return to events they had already interpreted, to consider them from yet another vantage point.

When it seemed to me that individual teachers were stuck—in writing or in their classrooms (at times both)—I gave individualized assignments. For example, during the first weeks of the semester, I realized that one teacher, who was just beginning to transform her practice, repeatedly expressed frustration over her class's poor behavior. She was trying to get them to work in pairs, use manipulatives, and talk about their thinking, but they simply were not cooperating. While her frustrations were understandable—and, in fact, quite typical—I felt that her writing did not convey a sense of what any of the individual children in her class were like. There was just a single, obstreperous, amoeba-like organism, the class. Since the group was small (eight ESL students), I asked her to describe each of them — what mathematics does he/she understand? not understand? what else do you want to know about him/her? In the first piece she turned in after I gave her that assignment, this teacher could not analyze her students' understandings, but she did describe their appearance and personalities. In the weeks that followed, her ear became more attuned, and eventually she began to hear her students' mathematical ideas. At the end of the semester, she wrote:

> The one assignment which in retrospect was a turning point for me was the suggestion that I write short narratives about my students as individuals. I had been struggling with the group dynamics and several students who affected the entire atmosphere of the class. After completing the assignment, I felt more connected to the individual students and was able to separate each from what was going on in the group. "Community" still remained an issue but I started to pay increased attention to the mathematics and less [to] "preaching" about how to be a "group."

Teachers met weekly with peers to discuss one another's work.

At each class session, teachers met in groups of three or four for one to two hours to read and discuss their work. While most teachers felt that this was an essential component of the process, for some it was also the most problematic. They found particularly difficult the expectation that they should provide feedback to their peers and receive feedback from them.

Some teachers felt that the feedback from their peers had simply not been helpful. One reported that when she was in need of direction, trying to sort through multiple, often contradictory, suggestions left her confused. Others found reactions from peers who seemed to be operating under different assumptions about teaching and learning quite irrelevant.

Many teachers felt extremely vulnerable and did not want to hear anything that might be construed as criticism of their work. And feeling so exposed and imperfect themselves, they did not believe they "had a right" to be critical of someone else's writing. However, by the end of the course, some of these teachers felt they had lost a valuable opportunity: "Now that it's over, I wish that I had been challenged more." They began to see the value of rethinking their work with a group of colleagues.

Despite these sentiments, the general feeling among MPWP participants was that focused discussions with their colleagues had been critical to producing their papers. It was in their small groups that they could test out their ideas and receive the emotional support that kept them going. Here they could discover what needed to be clarified, what could be deleted, and where they had not communicated what they had intended. And here, too, teachers learned how to look critically at a piece of writing, how to ask clarifying questions, how to analyze what was strong and to suggest ways to make it stronger still.

> At first it was very difficult, but as I began to see [how to correct] weaknesses in my own writing, I felt more comfortable sharing those ideas and suggestions with others.

Perhaps more important were small-group discussions stimulated by the papers' contents.

The writing course became a forum for exploring with colleagues the issues that were at the heart of their teaching. Participants found these discussions helpful in defining the dilemmas they faced and in working through problems in teaching. Meeting with other teachers of the same grade levels had been useful, but they also valued the opportunity—otherwise quite rare—to hear from teachers of all grades, kindergarten through twelve. Karen Schweitzer summarizes the power of this collaborative work in her paper, "The Search for the Perfect Resource" (in press):

> Meeting with a group of teachers each week helped me not only by giving me feedback on my writing but on the math that was happening in my classroom as well. It also gave me a chance to read and hear about what was going on in other classrooms. Developing this habit of reflecting and sharing has been a pivotal part of my change. It has struck me several times that these are pieces that are often missing from teacher education programs and from our daily professional lives, and for me, these were pieces that were essential.

The papers were written for a larger and anonymous public.

Keeping a journal is done mainly for oneself; and writing assignments are targeted at a different, though still singular, reader, the course instructor; but participants in the MPWP were writing for an indefinitely large audience of strangers. Granted most of those were likely to be other teachers, the writers could still not make assumptions about their readers' beliefs about teaching and learning, what their professional development experiences had been like, or the nature of their teaching contexts. Authors were challenged to convey their messages clearly and concretely, and especially to explain themselves without reliance on educational jargon (the word "constructivism" and its derivatives were essentially taboo).

Making descriptions of classroom events understandable to others required these authors to make sense of those events for themselves. One teacher reported, after an extremely frustrating and confusing lesson, that she had spent hours at her journal, trying to sort out what happened. However, once she took it upon herself to write a narrative about that lesson, she had to make comprehensible for others the sequence of events that had thwarted her:

> As I wrote . . . the frustration that I felt cleared. Although I ended saying I was frustrated, I wasn't feeling it as passionately as I was when I started. The writing of it cleared things up for me. I saw learning and a continuity that I didn't (and couldn't) see even after writing in my journal.

A particularly important task for the paper writers was that they analyze their own decision-making processes in the events they described. Early on in the course, they realized that on this score generalities would not suffice; they had to "analyze and explore each detail" in order to explain to the reader why they had done what they did—had to take ideas "at the back of [their] head[s]" and put them "in the forefront."

In writing her paper, "Establishing a Community of Mathematics Learners," Jill Bodner Lester (in press, b) had to face precisely this issue. It had been seven years since she had begun to center her instruction around her students' mathematical thinking, and so, that September, she quite confidently set about turning her latest roomful of second graders into a community of mathematics inquiry. And it was this transformation that she chose to write about. However, though she recorded classroom dialogue in order to track the emergence of a qualitatively different kind of classroom discourse, early versions of her paper conveyed the impression that her students had somehow, magically, learned to engage in the type of mathematical inquiry she was after. Her challenge became to explain to her readers why she had set up her lessons as she did, what particular interventions were intended to achieve, how she interpreted student behavior, and how her interpretations shaped what she next did. At the end of the course, Lester wrote:

> I have a clearer sense of what it means to establish a community of learners. Prior to working on this project, my ideas tended to be nebulous; it was intangible; it couldn't be described. While I still believe that the process is complex, I have more respect for the role of the teacher and the many clearly defined steps that provided a framework for respectful interactions among students.

Throughout the course, I stressed two overriding goals for MPWP: to produce a set of papers directed primarily toward teachers and to support participants' professional development as teachers of mathematics through an examination of their teaching. But, I emphasized, should the two goals conflict, their own learning would take precedence. In fact, it did prove necessary at times to set aside the first objective in order to break through writer's block or to promote greater honesty by reducing fear of exposure. I suggested that teachers first write for themselves and share their writing with me and their small groups. If they chose, they could assume that everything they wrote would remain confidential, within the confines of the class. After their papers were written, they could reconsider and decide to make them public. In the end, however, the two goals supported one another. The teachers exceeded their own expectations and, with the recognition that their thoughts and experiences were of value to others, they now approached their professional activity with greater self-esteem.

What Have Teacher–Readers Learned?

Since the papers were produced, I have distributed them to teachers and teacher educators along with a questionnaire which their authors helped design. In addition, my colleagues and I have used the papers in our own in-service courses.

Responses to the papers have generally been positive. For questionnaire items in which respondents (students in pre-service and in-service courses) rated individual papers from 1 to 5 along four dimensions—not at all interesting/extremely interesting, not at all useful/ extremely useful, not at all readable/extremely readable, not at all applicable to my teaching/ extremely applicable to my teaching—the large majority of papers were rated 4 or 5 in all categories.

Open-ended questionnaire responses, informal discussions with teacher educators who have used the papers, and journal entries from participants in SummerMath for Teachers courses indicate several ways in which pre- and in-service teachers make use of this work.

The papers provide grounding of theoretical principles.

Communications with teacher educators who have used the papers revealed that their vivid descriptions of classroom process provide grounding for theoretical principles where contexts for interpreting these abstractions are lacking. Thus, one colleague who assigned Brown's paper in a pre-service course reported that, although her students had been talking about "discovery" and "discourse" all semester long, it was only after reading that paper that they had an image of what those words might mean, concretely, for an elementary classroom. From then on, these students continually referred back to Brown's paper as they discussed instructional principles and possibilities.

In contrast to decontextualized presentations of the theoretical basis of the new mathematics pedagogy, the classroom situations described in the teachers' papers are instantly recognizable to their peers, helping them to interpret those principles in terms of their own teaching. For example, some readers found in Valerie Penniman's paper (in press) illustration of the principle that "the construction of new understandings is stimulated when established structures of interpretation do not account for novel experiences."

In "Making Graphs Is a Fun Thing to Do," Penniman describes her dismay when she realized that students who had followed her from second grade into third could not answer simple questions about graphs even though they had done a hands-on unit together the year before. As she planned her graphing unit for third grade, she decided to make the task more, rather than less, complex — it had to involve a problem that challenged her students to construct new understandings. Thus, she adapted an activity taken from published materials in which students were to sort and count gummy bears by color and then graph the results. However, since the graph paper the children were to use had only seven squares per column, she made sure that each group had more than seven gummy bears for at least one color. It was up to the children to figure out how to graph their data, given these materials.

After reading this paper, one teacher wrote:

> [The paper] changed the way I always thought about helping children to understand a concept. When the children in Val's class . . . encountered difficulties in graphing, she . . . increased the difficulty of the situation. Prior to reading her paper and seeing the response that her students had, I would naturally have simplified the task. I learned that by increasing the difficulty of some tasks, you can encourage better understanding.

The papers provide teachers with opportunities to encounter mathematics in new ways.

Another conception new to most teachers, and dramatized in the papers, is that mathematics is not a finished body of discrete facts and computational routines, but a dynamic and open-ended domain that involves posing questions, making and proving conjectures, exploring puzzles, solving problems, and debating ideas. Thus, in "Down the Rabbit Hole: On Decimal Multiplication," Rita Horn (1991) leads the reader along the route traced by her students: "We multiplied and got a smaller answer! How can that be?" "I think when you multiply by a decimal, you get an answer smaller than the number you started with." "Not always! Look at when you multiply 3 by 2.7. The answer gets bigger than 3." "So why does it work some of the time, but not all of the time?" And in "Of-ing Fractions," Joanne Moynahan's class of sixth graders is trying to figure out which operation—addition, subtraction, multiplication, or division—fits the following problem: *The Davis family attended a picnic. Their family made up 1/3 of the fifteen people there. How many Davises were at the picnic?* (Moynahan, in press). As readers follow the children's discussion, they are confronted with questions about the meanings of multiplication and division, the relationships between these operations, and the match between a mathematical model and the situation it represents. For some teachers, papers like these provided a first opportunity to think through such conceptual issues for themselves:

> I [was] very interested in the math of Rita's paper. I had never thought about decimal multiplication that way and I was intrigued.

Others found that the papers yield new insights into their students' ways of thinking:

> "Of-ing Fractions" was helpful to me mathematically. I was at a confused state about how and why multiplying fractions was so confusing to kids and hard to teach and that paper really provided the "momentary stay against confusion" I needed at a certain time.

Contrasts between what is portrayed in the papers and readers' own practice provide the opportunity to rethink assumptions.

It is the very concrete and specific nature of the papers that induced teachers to examine—indeed, for many, to formulate for the first time—their own beliefs about deep and important matters. For example,

> I learned that within most children's conversations about mathematics there are some very important ideas, and if I listened in my own class, I would begin hearing them.

Precisely because the papers were narrated from the teacher's perspective, teacher-readers could see themselves in their colleagues' shoes and begin to imagine themselves in similar situations. For some, the contrast between their own classrooms and those depicted in the papers was so great they doubted the truthfulness of what they read. But for others, that contrast pressed them to an examination of their own assumptions and beliefs. One teacher wrote:

> [As I read,] I was constantly having to stop, think about why I do something as I do, or think something, and try to mesh it with what I was reading.

And another reflected:

> I have begun to question my own expectations. I've always said that I have high expectations for my students. However, looking at the second-grade class [in Lester, in press, a] and what they were able to work through, both cognitively and verbally was amazing. I caught myself thinking, "Can my students do this?" Of course, initially it may be difficult. They probably have never been asked to answer questions like, "How do you feel about this answer?" But with much effort—like any other task—we can change our perceptions.

The papers provide ideas for lessons and for techniques to try out.

Teachers extracted specific teaching strategies and lesson ideas from the papers. One reader explained that the papers provided her with pictures of how particular activities could be used to explore a set of mathematical issues:

> When I begin a unit with my class, it helps to have a deeper picture in my head about how one class did it. That picture helps me think about the structure of my lesson even though I know my class will be different.

Others pointed to specific classroom situations and found suggestions for how they might respond in similar circumstances:

> Margie's paper "Beyond Stardom . . ." [Riddle, in press] was very meaningful to me. . . because I recognized the phenomenon of the "star" in my own classes and because her argument of how these stars fit into the "new" type of math class made such good sense to me. It was inspiring to find some understandings and *solutions* that I could readily use in my own class.

Teachers just beginning the process of transforming their mathematics instruction found models for the kind of classroom they ultimately wanted to create and were able to identify places from which to start. For example:

> The paper that had the greatest impact on me was "Is the Algorithm All There Is?"[Lester, in press, a] . . . for the style of teaching and classroom atmosphere. From Jill's paper, I gleaned the importance of working in small groups, posing thought-provoking word problems, questioning the children, listening to their ideas and explanations, and leading group/class discussions. This was very important to me because it gave me a model that I could think about and change to fit my classroom. The knowledge that "someone I know does this and it works" was tremendous inspiration and encouragement. That paper is responsible for BIG CHANGES in my teaching style.

Another teacher wrote:

> I liked the Natowich [1992] paper. It presented some options I could use — listening, wait time, use another question instead of an answer, use of journal instead of plan book.

The papers document the process of transforming a practice.

Perhaps as important as descriptions of the new mathematics pedagogy are the stories teachers tell about their own processes of change. The kind of transformation teachers are being asked to undertake is so profound, the challenge to their professional identity so threatening, that without some larger perspective that contextualizes their difficulties, they will abandon their efforts when the going gets rough—hiding their failures behind the closed doors of the classroom. Thus, one teacher confessed,

> It was so enlightening to hear other people's struggles with their doubts and anxieties. It was a relief to see that other teachers shared some of my frustrations.

Another reported,

> "Learning the Art of Unteaching" [Natowich, 1992] was. . . important to me because it was effective in documenting/painting the process of change a teacher goes through. It helped me to see the change in myself through this same kind of lens.

And still another,

> I learned from Jan's paper [Szymaszek, in press] (as well as from others) that making changes in teaching mathematics is a gradual process. I didn't need to be impatient with myself.

Once teachers realize that their difficulties are not unique and that they do not have to be ashamed if they find themselves struggling, they may be more willing to turn to others for help in finding their way forward.

For one teacher's complex response to her colleague's work, see Appendix C in which Anne Marie O'Reilly describes how, as her own practice changes, her readings of Moynahan's "Of-ing Fractions" paper (in press) alternately inspire, challenge, frustrate, and re-inspire her.

What Might Researchers Learn?

Although MPWP participants wrote with an audience of teachers in mind, the narratives provide a resource for the educational research community as well. As a group, they constitute a set of cases bearing on questions in three broad areas of inquiry: mathematics teaching, mathematics learning, and teacher change. In the previous section, I described how both pre- and in-service teachers have made use of these papers. Though I have not systematically canvassed researchers' reactions in analogous fashion, a range of issues can be identified, relevant to each of these areas, on which education researchers are currently at work and with which the papers intersect.

Mathematics teaching. In many of the narratives, teachers describe and analyze pedagogical dilemmas, report on the decisions they make and the outcomes of these decisions. The narratives also provide accounts of teacher–student interaction, discussions of classroom routine, and descriptions of tasks designed to stimulate exploration of particular mathematical issues. One might analyze these cases to consider such questions as, How can teachers "chart a mathematical course" (Ferrini-Mundy, in press) while encouraging students to follow their own lines of thought? What is the role of teacher telling in a pedagogy deliberately organized to support student construction of mathematical concepts? How do teachers use manipulatives, computers, writing, etc. to promote mathematical understanding? How does social interaction among students promote mathematical understanding? What is the nature of the tasks around which teachers organize their lessons? How do teachers establish communities of inquiry in their classrooms?

Mathematics learning. The narratives also provide accounts of students' mathematical conversations and reveal that, in general, once a community of inquiry is established, children in kindergarten through twelfth grade are interested in exploring mathematical ideas. One might analyze these conversations to address such questions as, What are the mathematical issues that engage children and repeatedly recur in different contexts among different groups? How does the articulation of these ideas shift and develop in different settings and at different ages? Where do we find evidence of early algebraic thinking, and what are the kinds of "algebraic questions that pique children's interest? What constitutes mathematical justification for learners of different ages?

Teacher change. Teachers' accounts of themselves as both mathematics students and mathematics teachers provide a critical perspective on what is involved in transforming one's practice to enact the new pedagogy. How do teachers' past mathematical experiences connect with what they are learning as they work to enact the current reforms? What insights and attitudes are key to developing a practice that supports student construction of mathematical concepts? Which experiences tend to be pivotal in the change process? What are the cognitive, affective, and social components of the change process? How do teachers cope with demands that are inherent in the reform agenda but are also in conflict with one another?

Teacher narratives can be a rich resource for researchers for whom such questions are current. As a set, they provide relevant data from different teachers and classrooms, representing a variety of communities, spanning grades kindergarten through twelve. At the same time, these materials also suggest new sets of questions. For example, in any representation of an event, some aspects are highlighted while others are ignored. What is illuminated and what is eliminated by various communication media? Do these narratives offer something that videotaped classroom scenes accompanied by teacher interviews or journals and/or students' written work cannot?

And again, what is illuminated and what is eliminated by various modes of research? What can be communicated through narrative that cannot be communicated through exposition and vice versa? What can be learned when a teacher tells his own story, as distinct from its being told by a teacher educator or policy researcher, or teacher/researcher team?

Further—and vexing—questions concern the legitimacy of such work: Will the established research community acknowledge the teacher narrative as "research"? There is a growing trend among educators toward acceptance of alternative ways of conducting research and communicating its results. Recognizing that the ways of coming to know are multiple and dependent on the nature of what is to be known, some researchers have become increasingly willing to consider, as an alternative to the tradition of experimental studies, the writings of "reflective practitioners" (Adler, 1993). For example, research journals and academic publishers have begun accepting articles by teacher educators who critically reflect on their own instruction (e.g., Simon, in press; Wineburg, 1991; Wilson, 1990), and by university-based faculty whose K–

12 teaching is the site of their research (e.g., Ball, 1993a, 1993b; Borasi, 1992; Lampert, 1988). And teachers' writings have found their way into the public domain through journals like *Teaching and Change* as well as books like *Inside/Outside* (Cochran-Smith and Lytle, 1993) and *Breaking Ground* (Hansen et al., 1985). Much of this work has grown out of the movement to reform writing instruction and out of projects addressing multicultural education. As of yet, however, few studies of mathematics teaching and learning written by classroom teachers have appeared.

Conclusion

It is teachers themselves who must, collectively, invent the new mathematics pedagogy and, in so doing, discover what the principles that underlie the *NCTM Standards* really mean. They must demonstrate to the rest of us how real children can become the mathematical thinkers envisioned by reformers; they must uncover the dilemmas and contradictions inherent in this ambitious agenda; and they must help us understand the complexities of the process by which a traditional instructional practice is transformed into a new mathematics pedagogy.

The Mathematics Process Writing Project brought together three groups of teachers who had already begun this process of transformation. In reflective, first-person narratives, teacher–authors describe their students and the learning taking place in their classrooms; they tell stories of their own struggles, confusions, insights, failures, and successes as they work to align their practice with their changing beliefs about learning, teaching, and mathematics. In effect, these narratives, which give voice to the new mathematics pedagogy, are a medium through which teachers can become central contributors to the national conversation about mathematics education reform.

However, it was not easy for the teachers who participated in the project to find that voice. As each MPWP course began, its participants were made anxious by the magnitude of the task set for them. They doubted their ability to create a significant piece of work, one that addressed a complex pedagogical issue and honestly represented their teaching. And they were afraid that others would scorn—or, at best, be indifferent to—their work. Thus, I thought that by starting with short assignments, and by providing lots of feedback that pointed out successful writing and identified important ideas, I would help build their confidence. Writing week after week—for 14 weeks—with encouragement and suggestions from me and from their peers, the teachers slowly developed drafts of their final papers. They then had an additional 10 weeks to complete their projects, calling on me and on one another for further support as needed.

Clearly, such writing is time consuming, exposing, and difficult. And in the absence of serious, well-conceived programs designed to encourage it, very few teachers will volunteer to do it. But I hope readers of these papers will be convinced that they represent a form of professional research uniquely suited to the project of reform they are intended to support.

References

Adler, S. A. (1993). "Teacher Education: Research as Reflective Practice." *Teaching and Teacher Education, 9*(2), 159–167.

Atwell, N. (1985). Writing and Reading from the Inside Out. In Hansen, J., Newkirk, R., and Graves., D. (eds.), *Breaking Ground: Teachers Relate Reading and Writing in the Elementary School*, 147–168. Portsmouth, NH: Heinemann Educational Books, Inc.

Ball, D. L. (1993a). "With an Eye on the Mathematical Horizon: Dilemmas of Teaching Elementary School Mathematics. *Elementary School Journal, 93*(4), 373–397.

Ball, D. L. (1993b). "Halves, Pieces, and Twoths: Constructing Representational Contexts in Teaching Fractions." In Carpenter, T. P., Fennema, E., and Romberg, T. (eds.), *Rational Numbers: An Integration of Research*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Ball, D. L. (1992, Summer). "Magical Hopes: Manipulatives and the Reform of Math Education. *American Educator*, 15–18, 46–47.

Barnett, C. (1991). "Building a Case-Based Curriculum to Enhance the Pedagogical Content Knowledge of Mathematics Teachers." *Journal of Teacher Education, 42*(4), 263–272.

Borasi, R. (1992). *Learning mathematics through inquiry*. Portsmouth, NH: Heinemann.

Bridgewater, K. (1991). "Adventures in Math Teaching: Educational reform on a Personal Level." Unpublished manuscript.

Brown, V. (in press). "Third Graders Explore Multiplication." In Schifter, D. (ed.), *What's Happening in Math Class: Issues of Practice in Teacher Narratives from the Mathematics Education Reform Movement.* New York: Teachers College Press.

Carter, K. (1993). "The Place of Story in the Study of Teaching and Teacher Education." *Educational Researcher, 22*(1), 5–12, 18.

Cochran-Smith, M., and Lytle, S. L. (1990). "Research on Teaching and Teacher Research: The Issues That Divide." *Educational Researcher, 12*(2), 2–11.

Cochran-Smith, M., and Lytle, S. L. (1993). *Inside/Outside: Teacher Research and Knowledge.* New York: Teachers College Press.

Countryman, J. (1992). *Writing to Learn Mathematics: Strategies That Work, K–12.* Portsmouth, NH: Heinemann.

Fennema, L., Carpenter, T. P., Franke, M. L., and Carey, D. A. (1993). "Using Children's Mathematical Knowledge in Instruction." *American Educational Research Journal, 30*(3), 555–583.

Ferrini-Mundy, J. (in press). "Mathematical Thought-in-Action: Rich Rewards and Challenging Dilemmas." In Schifter, D. (ed.), *What's Happening in Math Class: Issues of Practice in Teacher Narratives from the Mathematics Education Reform Movement.* New York: Teachers College Press.

Gougeon, H. (1992). "New Beginnings." Unpublished manuscript.

Hammerman, J., and Davidson, E. (1993). "Homogenized is Only Better for Milk." In Cuevas, G., and Driscoll, M. (eds.) *Reaching All Students in Mathematics,* 197–212. Reston, VA: National Council of Teachers of Mathematics.

Hansen, J., Newkirk, T., and Graves, D. (eds.) (1985). *Breaking Ground: Teachers Relate Reading and Writing in the Elementary School.* Portsmouth, NH: Heinemann Educational Books, Inc.

Heaton, R. M. (1991). "Continuity and Connectedness in Teaching and Research: A Self-study of Learning to Teach Mathematics for Understanding." Presented to the University of Pennsylvania Ethnography in Education Research Forum, Philadelphia, PA.

Hiebert, J. (1989, March). "The Struggle to Link Written Symbols with Understanding." *Arithmetic Teacher,* 38–44.

Hillocks, G., Jr. (1990). "Teaching, Reflecting, Researching." In Daiker, D., and Morenberg, M. (eds.), *The Writing Teacher as Researcher: Essays in the Theory and Practice of Class-based Research,* 15–29. Portsmouth, NH: Boynton/Cook Publishers, Heinemann.

Horn, R. (1991). "Down the Rabbit Hole: On Decimal Multiplication." Unpublished manuscript.

Jonsberg, S., and Salgado, M. (in press). "Composing the Multiple Self: Teen Mothers Rewrite Their Roles." In Phelps, L. and Emig, J. (eds.), *Feminine Principles and Women's Experience in American Composition and Rhetoric.* Pittsburgh, PA: University of Pittsburgh Press.

Koch, N. (in press). "One Last Stab: High School Kids and Arithmetic." In Schifter, D. (ed.), *What's Happening in Math Class: Issues of Practice in Teacher Narratives from the Mathematics Education Reform Movement.* New York: Teachers College Press.

Lampert, M. (1988). "The Teacher's Role in Reinventing the Meaning of Mathematics Knowing in the Classroom." In Behr, M. J., Lacampagne, C. B., and Wheeler, M. M. (eds.), *Proceedings of the Tenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education,* 433–480. DeKalb: Northern Illinois University.

Lampert, M. (1989, March). "Arithmetic as Problem Solving." *Arithmetic Teacher,* 34–36.

Lawrence, N. (1991). "The Symphony." Unpublished manuscript.

Lester, J. (in press, a). "Is the algorithm all there is?" In Fosnot, C. T. (ed.), *Constructivism: Foundations, Perspectives, and Practice.* New York: Teachers College Press.

Lester, J. (in press, b). "Establishing a Community of Mathematics Learners." In Schifter, D. (ed.), *What's Happening in Math Class: Issues of Practice in Teacher Narratives from the Mathematics Education Reform Movement.* New York: Teachers College Press.

Lester, J. (1987). "Math Journals: An Individualized Program. *The Constructivist, 2,* (2), 1–7.

Lytle, S. L., and Cochran-Smith, M. (1990). "Learning from Teacher Research: A Working Typology." *Teachers College Record, 92,* 83–103.

Meier, D. (1992, Spring). "School Days: A Journal." *Dissent,* 213–220.

Miller, J. (1990). *Creating Spaces and Finding Voices: Teachers Collaborating for Empowerment.* Albany: SUNY Press.

Moynahan, J. (in press). "Of-ing Fractions." In Schifter, D. (ed.), *What's Happening in Math Class: Issues of Practice in Teacher Narratives from the Mathematics Education Reform Movement.* New York: Teachers College Press.

National Council of Teachers of Mathematics (1989). *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics (1991). *Professional Standards for Teaching Mathematics.* Reston, VA: Author.

Natowich, D. (1992). "Learning the Art of Unteaching." Unpublished manuscript.

Penniman, V. (in press). "Making Graphs is a Fun Thing to Do." In Schifter, D. (ed.), *Constructing a New Practice: Issues of Teacher Development in Teacher Narratives from the Mathematics Education Reform Movement.* New York: Teachers College Press.

Riddle, M. (in press). "Beyond Stardom: Challenging Competent Math Students in a Mixed Ability Classroom. In Schifter, D. (ed.), *What's Happening in Math Class: Issues of Practice in Teacher Narratives from the Mathematics Education Reform Movement*. New York: Teachers College Press.

Scanlon, D. (in press). "Algebra is Cool: Reflections on a Changing Pedagogy in an Urban Setting. In Schifter, D. (ed.), *What's Happening in Math Class: Issues of Practice in Teacher Narratives from the Mathematics Education Reform Movement*. New York: Teachers College Press.

Schifter, D. (ed.) (in press, a). *What's Happening in Math Class: Issues of Practice in Teacher Narratives from the Mathematics Education Reform Movement*. New York: Teachers College Press.

Schifter, D. (ed.) (in press, b). *Constructing a New Practice: Issues of Teacher Development in Teacher Narratives from the Mathematics Education Reform Movement*. New York: Teachers College Press.

Schifter, D. (1993). "Mathematics Process as Mathematics Content: A Course for Teachers." *Journal of Mathematical Behavior, 12*(3), 271–283.

Schifter, D., and Fosnot, C. T. (1993). *Reconstructing Mathematics Education: Stories of Teachers Meeting the Challenge of Reform*. New York: Teachers College Press.

Schweitzer, K. (in press). "The Search for the Perfect Resource." In Schifter, D. (ed.), *Constructing a New Practice: Issues of Teacher Development in Teacher Narratives from the Mathematics Education Reform Movement*. New York: Teachers College Press.

Shulman, J. (ed.) (1992). *Case Methods in Teacher Education*. New York: Teachers College Press.

Shulman, L. (1986). "Those Who Understand: Knowledge Growth in Teaching." *Educational Researcher, 15*(2), 4–14.

Simon, M. (in press). "Reconstructing Mathematics Pedagogy from a Constructivist Perspective." *Journal for Research in Mathematics Education.*

Simon, M. A., and Schifter, D. (1991). "Towards a Constructivist Perspective: An Intervention Study of Mathematics Teacher Development." *Educational Studies in Mathematics, 22*(5), 309 – 331.

Smith, G. (1992). "What Do I Teach Next?" *The Constructivist: Newsletter for the Association for Constructivist Teaching, 7*(4), 1–4

Szymaszek, J. (in press). "A Year of Enquiry." In Schifter, D. (ed.), *Constructing a New Practice: Issues of Teacher Development in Teacher Narratives from the Mathematics Education Reform Movement*. New York: Teachers College Press.

van Manen, M. (1990). *Researching Lived Experience: Human Science for an Action Sensitive Pedagogy*. New York: State University of New York Press.

Wilcox, S., Lanier, P., Schram, P., and Lappan, G. (1992). *Influencing Beginning Teachers' Practice in Mathematics Education: Confronting Constraints of Knowledge, Beliefs, and Context* (Research Report 92-1). East Lansing, MI: Michigan State University, National Center for Research on Teacher Learning.

Wilson, S. (1990). "The Secret Garden of Teacher Education." *Phi Delta Kappan, 72*, 204–209.

Wineburg, S. (1991). "A Case of Pedagogical Failure—My Own." *Journal of Teachers Education, 42*(4), 273–280.

Witherell, C., and Noddings, N. (1991). *The Stories Lives Tell: Narrative and Dialogue in Education*. New York: Teachers College Press.

Yaffe, L. (in press). "Pictures at an Exhibition: A Mathphobic Confronts Fear, Loathing, Cosmic Dread, and Thirty Years of Math Education. In Schifter, D. (ed.), *Constructing a New Practice: Issues of Teacher Development in Teacher Narratives from the Mathematics Education Reform Movement*. New York: Teachers College Press.

Appendix A

Writing and Reading Assignments for MPWP Courses

Writing Assignments

Although I suggest a structure for your weekly writing, it is up to you to redefine assignments, if necessary, so that they are most helpful to you. If you have selected a topic for your final project, you might use the assignment to explore your topic and experiment with ways to address it.

- Frequently, when our students arrive in September, they are used to a traditional approach to mathematics instruction and don't know how to be in a class taught alternatively. What do you do to "acculturate" your students to a new kind of classroom? (What do you say? What activities do you set up? etc.)

- Transcribe a dialogue between you and a student or between two students. Then write a narrative, based on that dialogue, about what happened.

- Rewrite the same story, but now in first person, from the perspective of a student.

- Describe a student who has revealed to you that he/she has learned something that you are trying to teach. What did you want the students to learn? What was the context? What was the student's interaction with you? with other students? What process did the student use? What words and actions indicated learning? How does this learning fit with other things this student has learned? How does it fit with other things yet to be learned? What did you learn from your student?

- Write about a student who expresses a mathematical idea that surprises you. Why did it surprise you? What did you expect? (If the student's idea is not consistent with conventional mathematics, explain why. is there an element of logic in the student's idea?) How did you respond? What did the student learn? What did his/her classmates learn? What did you learn?

- Describe a teaching dilemma that you have faced. Then present two scenarios (at least one of which will be fictional) in which you respond to the dilemma differently. What was the basis of each of the decisions? What were the different outcomes?

- What does it feel like to discover one's own misconception? What is the process of resolving the disequilibrium engendered? You can write from your own experience as a learner or from the perspective of a teacher observing a student.

- Which of the articles I have distributed in this course do you think are particularly effective? Reread these articles to examine the writing. What techniques and strategies does the author employ to make the writing effective? How do these techniques serve to communicate the issues the author addresses? Rewrite one of your pieces (or write a new one) to employ these techniques.

- Select one of your pieces (or work on a new one) that you think could be strengthened by a specific story. Try to tell that story by "bringing the reader into the classroom," allowing the reader to hear and see what's going on.

- Select a piece of your writing that you would like to share with the class for feedback, and bring enough copies for your classmates.

- Write me a statement about your final paper. What topic do you plan to explore with your readers? What ideas do you want to communicate? How do you plan to present your ideas? What will be the flow of the discussion. (You will not be required to follow through on exactly what you say here; you can still change your mind. Even if you're not sure, for the purpose of the assignment pick one of the topics you are considering.)

- Continue work on your project. The focus should be developing by now and your writing should be underway. If you are having trouble figuring out what you are writing about, please schedule a conference with me.

- You should be thinking carefully about the development of your final paper. At this point it won't "just happen" without some thoughtful work. As you continue your writing, I want you to identify the problems you face in trying to create a meaningful and coherent piece and work with me and your classmates to solve them. While the identification of the problems does not have to be a writing task in and of itself, you must be able to articulate them verbally so that others can help you.

- By now all of you are well into your final projects. Most of you are working to 1) clarify for yourselves the points you want to make, 2) make sure that those points are made clearly and strongly in your paper, and 3) make sure that everything that is written in the paper serves the purpose of making those points.

Here are a few minor issues to check:

Have you given the reader information about your class? What grade is it? How many students? What is the ratio of boys to girls? What kind of community does your school serve?

If you use the word "constructivism" (or a related word), does it really serve what you are trying to communicate in your paper? Consider making your points without the "c-word." Instead, say what you mean by it.

If you refer to SummerMath For Teachers, does it really support the points you are trying to make?

Reading Assignments

- "Arithmetic as Problem Solving," by M. Lampert (1989); "The Struggle to Link Written Symbols with Understanding," by J. Hiebert (1989); and "Is the Algorithm All There Is?" by J. Lester (in press, a).

As you read these articles, address the following questions:

> What are the three articles saying about learning and teachin In what ways are the messages the same? In what ways are they different?

What are the differences in writing style? What is the impact of the different techniques? What can be communicated using one style that cannot be communicated using another?

• Three articles on journal-writing: "Math Journals: An Individualized Program," by J. Lester (1987), is about her use of journals in a second-grade class. "Mathematics Process as Mathematics Content: A Course for Teachers," by D. Schifter (1993), uses journal excerpts to illustrate the kind of learning that took place in a mathematics course. "School Days: A Journal," by D. Meier (1992), consists of excerpts from a teacher's own journal, raising issues about education.

How do you respond to the content of these papers? Does anything in these papers spur reflection on your own use of journals? Are there aspects of the style of the papers that you would like to try?

• "A Year of Enquiry," by J. Szymaszek (in press); "Making Graphs is a Fun Thing to Do," by V. Penniman (in press); and early drafts of each of these papers.

How did Szymaszek's and Penniman's work develop? What ideas/issues are they able to communicate in their final papers that don't come through in earlier work? How does the writing change? How does the early work support the final paper?

• "With an Eye on the Mathematical Horizon: Dilemmas of Teaching Elementary School Mathematics," by D. Ball (1993,a); and "New Beginnings," by H. Gougeon (1992).

Review the articles you have read in the course thus far and consider the ones you have found most effective. What are the techniques the author employed to carry the message? Can you identify what makes an article effective? Come to class prepared to discuss this.

• "Down the Rabbit Hole: On Decimal Multiplication," by R. Horn (1991); "Of-ing Fractions," by J. Moynahan (in press); and "One Last Stab," by N. Koch (in press).

These papers continue on the theme of making meaning for mathematics or making mathematics make sense. For some of you, this will be an opportunity for *you* to learn some mathematics. If the issues about mathematics addressed in a paper are a challenge for you, take the time to think it through. Remember that following a line of mathematical thought is not the same as reading prose. It will take you quite a bit longer to sort out the mathematical ideas.

• You have received writing from other members of the class. Read and respond to as many pieces as you feel is a reasonable assignment for the week.

• Two articles by D. Ball: "Magical Hopes: Manipulatives and the Reform of Math Education" (1992) and "Halves, Pieces, and Twoths: Constructing Representational Contexts in Teaching Fractions" (1993b).

What are these papers about? What do you learn from these papers about fractions and about the questions that arise as children engage with fractions? How does Ball use description of classroom events to communicate her points? How does she include her own thinking, analysis, and decision making?

Ball will be in class on Thursday. In the first part of the class, we will observe and discuss a video of her students working on fractions. In the latter part, we will open the discussion to whatever issues you want to raise. Think about what you would like to talk about with her.

• "Teaching, Reflecting, Researching," by G. Hillocks, Jr. (1990); "Writing and Reading from the Inside Out," by Nancie Atwell (1985); and "The Symphony," by N. Lawrence (1991).

Analogies are frequently made these days between language arts and mathematics instruction, and some of you are bringing up such comparisons in your papers. I'd like you to read Hillocks' paper about writing instruction and Atwell's chapter about teaching reading to think about issues of teaching mathematics. Then consider how Lawrence uses her experience as a language arts teacher to inform her work teaching mathematics. In addition, I'd like you to reflect on what makes the papers effective or not.

• "Continuity and Connectedness in Teaching and Research: A Self-Study of Learning to Teach Mathematics for Understanding," by R. Heaton (1991); "Pictures at an Exhibition: A Mathphobic Confronts Fear, Loathing, Cosmic Dread, and Thirty Years of Math Education," by L. Yaffee (in press); and "Adventures in Math Teaching: Educational Reform on a Personal Level," by K. Bridgewater (1991).

I would like you to read these three papers, in which teachers describe their own experiences trying to transform their mathematics teaching.

• "Homogenized is Only Better for Milk," by J. Hammerman and E. Davidson (1993), and "Beyond Stardom: Challenging Competent Math Students in a Mixed Ability Classroom," by M. Riddle (in press).

Consider how the papers are similar and how they differ. Consider both *what* is said and *how* it is said.

• "Building a Case-Based Curriculum to Enhance the Pedagogical Content Knowledge of Mathematics Teachers," by C. Barnett (1991).

Although the basic conception is somewhat different, Barnett also has a project in which teachers write about what happens in their classrooms, and their writings are used for inservice instruction with other teachers. I'm interested in your reactions to what she describes.

• "Composing the Multiple Self: Teen Mothers Rewrite their Roles," by S. Jonsberg with M. Salgado (in press).

Although this paper is not about mathematics, I decided to have you read it because Jonsberg has used a very interesting approach in writing about her teaching (and also addresses important issues). I'd like to hear your reactions to it.

• During these last two weeks of the semester, I would like you to put your efforts into your writing and therefore am not assigning any specific reading. However, sometimes I find that when I am stuck in my writing, it helps to read. In that case, consider rereading some of the articles that you have found particularly helpful, or try reading an article you haven't yet gotten to.

Appendix B

MPWP Papers

Anderson, Christine. Shaping Up.

Appleby, Marie. Petals around the Rose: Building Positive Attitudes about Problem Solving.

Baker, Kathy. Incidental Mathematics: Seizing the Moment.

Bastable, Virginia. A Dialogue about Teaching.

Bridgewater, Kathleen. Adventures in Math Teaching: Educational Reform on a Personal Level.

Brown, Virginia. Third Graders Explore Multiplication.

Buendia, Maria. Integrating Mathematical Thinking and Mathematical Content into the Foreign-Language Curriculum through Children's Literature.

Clark, Elizabeth. Patterns in the Curriculum.

Flynn, Mary. The Challenge of Change.

Gagnon, Allen. Struggling.

Ginsberg, Catherine. Exploring Place Value.

Gougeon, Humilia. New Beginnings.

Graveline, Perrie. The Jou ney: Two Steps Forward, One Step Back.

Gruneiro, Vicky. Journals Insights and Surprises.

Gurdak-Foley, Robin. Math with a Venn-geance.

Hawley, Virginia. First Graders Talk about Learning.

Hendry, Anne. Owning Our First-Grade Math.

Horn, Rita. Down the Rabbit Hole: On Decimal Multiplication.

Isenberg, Caryl. Mainstreaming versus Pull-out in Special Education: The Case of Mathematics.

Koch, Nina. One Last Stab: High School Kids and Arithmetic.

Kostek, Peter. How Do You Expect Us to Do It before You Teach It?

Lawrence, Nancy. The Symphony.

LeBlanc, Doris. Monster Math in Grade One.

Lennon, Donald. Moving Toward the Standards: Multiplication in Third Grade.

Lester, Jill. Establishing a Community of Mathematics Learners.

Lester, Jill. Is the Algorithm All There Is?

Lipinski, Michael. Looking at Math with Stars in My Eyes.

Mather, Michelle. Parents: Friends or Foes of Process Math.

Miller, Barbara Anne. Surplus Baggage and Mathematics Education.

Moynahan, Joanne. Of-ing Fractions.

Natowich, Donna. Learning the Art of Unteaching.

O'Brien, Deborah. Math Journals: First Attempts in Third Grade.

O'Reilly, Anne Marie. Understanding Teaching/Teaching for Understanding.

Penniman, Valerie. Making Graphs is a Fun Thing to Do.

Redman, Jessica. Conversations about Counting.

Riddle, Margaret. Beyond Stardom: Challenging Competent Math Students in a Mixed Ability Classroom.

Rigoletti, Rosemary. Second Graders Discover Square Numbers.

Sajdak, Sherry. Untitled.

Scanlon, Donna. Algebra is Cool: Reflections on a Changing Pedagogy in an Urban Setting.

Schott, Jan. Creating an Environment which Values and Encourages Rich Classroom Discussion: The Process of Breaking the Traditional Algebra Lecture Mold.

Schweitzer, Karen. The Search for the Perfect Resource.

Sheinbach, Alissa. Juggling Math and Mainstreaming.

Signet, Mary. Food for Thought.

Smith, Geri. What Do I Teach Next?

Smith, Susan. Logo: A tool for Exploring Mathematics.

Szymaszek, Janice. A Year of Enquiry.

Toney, Nora. High Teacher Expectations and Honoring Different Cultural Learning Styles: In Relation to Learning and Understanding Mathematics.

Yaffe, Lisa. Pictures at an Exhibition: A Mathphobic Confronts Fear, Loathing, Cosmic Dread, and Thirty Years of Math Education.

Zippe, Joyce. Page Pupa to Process Butterfly.

Appendix C

Journal Entry by Anne Marie O'Reilly

"Of-ing Fractions" brings back memories for me, many of them unpleasant. This is at least the third time that I have read Joanne Moynahan's paper. I am struck by how much more meaning I have taken from it this time, as well as my discovery of just how much more there is to be gleaned from it in future readings. My first reading of the paper left me impressed by all the ideas being exchanged and the apparent investment of the students in the mathematics that was taking place in her classroom. My second reading left me frustrated that I was not having similar experiences with fractions with my own sixth-grade class. (I wanted to blame Joanne for only writing about the positive parts of the experience and not highlighting any of the problems.) Now, a full year later and a little bit wiser, my interest in the paper has changed. I seem to be approaching it in novel ways.

My first experiences with the paper remind me of sitting in a math class, listening to others discussing a math problem without being actively engaged myself. What I hear and see drawn on the board seems to make sense (similar to when a teacher is explaining how to use an algorithm). I'm actually very interested in what is being said. However, my active engagement is limited. So, when I walk away and set to the task of using the algorithm, I'm suddenly confronted with confusion. I don't have any strategies to help me to make sense of it. I feel frustrated. The next time I go to class, I bring my confusion and my questions to the discussion. I will search for answers, ask questions, play a more active role, try to compare my thinking with the thinking of others.

So it was with me the last time I read this paper. I was confronting fractions with sixth graders for the very first time. For some reason, I expected them to react the same way that Joanne's class had. My own limited experience with the math had made it difficult for me to interact with the math in the paper. I had been more passive than I realized on those first two readings, in part because of my limited understanding of the math, in part because I was reading the paper alone. I was not in the company of other learners or a teacher who could give me the creative nudges that I needed in order to actively engage with the math in the paper.

When my own students began reacting differently to fraction problems that were similar to those set up in the paper, I was thrown into a state of confusion. I found it difficult to recognize and interact with the different, but equally valuable, math that was going on in my own classroom. I couldn't acknowledge the math that was taking place in my classroom, because I didn't fully understand it.

I've had many mathematical experiences since then. In the end, I learned a lot from my students. I learned a lot from Deborah Schifter when I brought her my questions and confusion and she gave me some of the creative nudging that I needed. I've also thought a lot over this past year about the fuzzy lines that separate the operations that we perform on numbers. I have some particularly fresh perspectives concerning operations on fractions to bring to my current reading of the paper.

So this time, when I consider Rebecca's reasoning for why 1/3 of 13 should be times, when I look at her line of fifteen 1/3s,

1/3 1/3 1/3 1/3 1/3 1/3 1/3 1/3 1/3 1/3 1/3 1/3 1/3 1/3 1/3,

I'm as hard to convince as her classmates were. I find myself connecting with the thoughts that must have been racing through Joanne's mind as she struggled with the dilemma of what to do when students remove the numbers from the context of a problem and manipulate those numbers to prove their point. This time I did not overlook the times when Joanne said she was surprised by her students' thinking. "I hadn't thought it through like Sally had." This time I connected with Joanne's self questioning. "Is what I have decided to teach really important?" This time, I really began to understand the math and the teaching in Joanne's paper.

Biography of the Author

Deborah Schifter is a project director in the Center for the Development of Teaching at the Education Development Center, Inc., in Newton, MA. She worked with the SummerMath Programs at Mount Holyoke College from their inception in 1982 and directed SummerMath for Teachers from 1988 to 1993. She has also worked as an applied mathematician and has taught elementary, secondary, and college level mathematics. She has a BA in liberal arts from Saint John's College, Annapolis, an MA in applied mathematics from the University of Maryland, and an MS and a PhD in psychology from the University of Massachusetts. She co-authored, with Catherine Twomey Fosnot, *Reconstructing Mathematics Education: Stories of Teachers Meeting the Challenge of Reform*. Dr. Schifter conducted the project described in this paper while Director of SummerMath for Teachers.