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ABSTRACT

This paper outlines a theoretical perspective for studying student understandings of the concept of addition. The notion of operational sense is defined as a way to describe the notion of addition as a mathematical object, paving the way for an application of the theory of reification at this level. Previous frameworks relative to problem solving are also incorporated. The report of a year-long investigation in a first-grade classroom is then provided. It was found that understandings of specific aspects of operational sense were beneficial to successful problem-solving strategies on part-unknown action tasks. These understandings were also beneficial to the ability to transfer knowledge of addition to a finite group setting (clock arithmetic). Hence, a connection was found between specific kinds of knowledge of arithmetic and the students' ability to model the actions of a problem. Limitations of the framework and study are also discussed. (Author)

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OPERATIONAL SENSE IN FIRST GRADE ADDITION

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This paper outlines a theoretical perspective for studying student understandings of the concept of addition. The notion of operational sense is defined as a way to describe the notion of addition as a mathematical object, paving the way for an application of the theory of reification at this level. Previous frameworks relative to problem solving are also incorporated. The report of a year-long investigation in a first-grade classroom is then provided. It was found that understandings of specific aspects of operational sense were beneficial to successful problem solving strategies on part-unknown action tasks. These understandings were also beneficial to the ability to transfer knowledge of addition to a finite group setting (clock arithmetic). Hence, a connection was found between specific kinds of knowledge of arithmetic and the students ability to model the actions of a problem. Limitations of the framework and study are also discussed.

In this study, I examine if aspects of the theory of reification (Sfard and Linchevski, 1994) can be used to operationalize student understandings of early arithmetic. Reification involves the transitioning of understandings in line with actions and processes to more permanent understandings in line with mathematical objects. Gray and Tall (1994) provide several nice examples of where reification may occur in a student's mathematical career, including the reification of the counting process into object-oriented conceptions of number.

Operational Sense

Because I was interested in student understanding, and not just problem solving behaviors, I attempted to lay out a theoretical basis that would be 1) consistent with the theory of reification, 2) useful in exploring student understandings of addition, and 3) useful in relating these understandings to existing theories of problem solving (Riley, Greeno, and Heller, 1983; Briars and Larkin, 1984; Carpenter, 1985). I defined *operational sense* in an effort to satisfy these requirements. A base definition of operational sense could involve the ability to use the operation on at least one set of mathematical objects (such as the ability to add positive integers). But this is clearly a minimal conceptualization. I maintain that operational sense which promotes deep understandings of the operation involves various kinds of flexible conceptions which can be interrelated by the learner. From this perspective, operational sense could involve (additive components in parentheses):

1. *A conceptualization of the base components of the process.* (This involves an understanding of the decomposition of addition tasks into uniform counting or, perhaps later, a devised strategy such as: 7+8=15 since 8=3+5, so 7+8=7+3+5=10+5=15).

2. *Familiarity with properties which the operation is able to possess* (commutativity, invertibilty, associativity, existence of an identity).

3. *Relationships with other operations.* (In addition to the relationships the operation of addition has with its inverse (subtraction), the distributive property in any field provides a means of connecting two operations, such as addition and multiplication. Further, multiplication is often initially understood as repeated addition).

4. *An awareness of the various symbol systems associated with the operation* (digits, +, >, etc.).

5. *Familiarity with operational contexts.* (The use of action, comparison, and part-whole situations are familiar contexts for the operation of addition).

6a. *Ability to use the operation on abstract objects.* (This involves the use of addition without a reliance on concrete quantities. Here, the process of addition is being used to act on clearly understood quantities, with no need to rely on the base components of the process, such as counting, or on concrete representations, such as unifix blocks).

6b. *A knowledge of operational facts.*

7. *Ability to relate the use of the operation across different mathematical objects.* (Addition on integers, modular systems of different bases, fractions, decimals, finite groups, variable expressions (symbolic functions), graphs (graphic functions), vectors, and sequences all share a fundamental relationship in regard to the process, even though the mathematical objects are very different. The ability to see connections across these systems can be quite powerful in establishing an advanced operational sense of addition).

8. *Ability to move back and forth between the above conceptions.*

Some of the above dimensions of operational sense have been previously investigated. For example, Cobb and Wheatley (1988) discuss, in considerable detail, the manner in which second grade children use operational sense to solve two-digit addition tasks, as well as how these children were able (or unable) to reify the notion of ten. Much work has been conducted on the effects of situations and contexts on problem solving strategies (e.g., Carpenter, 1985). However, the author knows of no such study which takes the perspective outlined above in collectively investigating first graders' development and use of operational sense. Further, because the evidence at this level suggests that mathematical behaviors will not always fit in to predispositioned frameworks, I expected to modify the above perspective as the data accumulated.

Since the study was conducted at the first grade level, it was obvious that many students would not develop some of the above components of operational sense to any relevant degree. Components 1-4, 6, and 8 were specifically investigated in this study.

Project Design

Data Collection Schedule

This projec. investigated the degree to which an operational sense of addition was achieved by students in a first grade classroom in a small, northwestern city. Videotaped interviews were conducted with 17 students a total of 5 times from October through May. Transcripts of some of the interviews were made. Classroom observations and instructor interviews were conducted approximately once per week to provide descriptions of the instruction.

Interview Data

Each of the student interviews contained a series of tasks designed to elicit the students' operational sense of addition. Number size was in line with the current level of instruction. Student responses to the tasks assisted in designing future interviews. The tasks were analyzed for patterns in student problem solving tendencies and the level of operational sense displayed. Each interview contained an addition story task of each of the forms a+b= _ , a+ _ =c, and _ +b=c. These tasks were designed to provide information regarding computational ability and problem solving strategies. The stories were predominantly action type tasks that included names and objects familiar to each specific child. Probing or clarifying questions were consistently given during and following the working of the tasks in order to elicit more detailed information about the solution strategy and the understandings held by the student. Frequency counts were generated in regard to correctness and classification of solution strategy. Additional analysis was conducted on these tasks in regard to the level of operational sense present in the solution strategies. For example, a counting backward strategy implicates the child in regard to an ability to invert the operation, and counting strategies and invented algorithms provide evidence as to the manner in which addition was understood in regard to its base processes. A variety of other tasks, some of which are described in the discussion below, provided more detailed information in regard to the development of various aspects of operational sense.

Results

Instruction

The instructor was an experienced teacher with a Master's Degree in Elementary Education. The students grew quite close to her throughout the year. In my view, the instructor provided an educational context quite conducive to learning mathematics. This included opportunities for the students to make conjectures, discuss solution strategies, experience mathematics in a variety of situations and representations, and to reflect on the mathematics being discussed. The instructor rarely forced an algorithm or universal way of doing mathematics into the discussion, leaving this to her students. It is worth noting that, despite these qualities, I

was most impressed with her ability to positively stimulate the development of her students as young boys and girls.

The instructor incorporated classroom activities that I would classify as promoting an operational sense of addition. These included numerous counting, estimation, and pattern activities, general addition tasks that explore different ways of adding two numbers to achieve the same result, skip counting, story problems in a variety of contexts, discussions of zero, and tasks which discuss commutativity. In addition, aspects of operational sense were found in student-initiated comments during classroom discussion.

Development of an Operational Sense of Addition

A discussion of the data addressing the students' development of specific aspects of operational sense will now be given. This will be followed by a more inferential reporting of the students' overall development of an operational sense of addition.

Counting. All students demonstrated at least some degree of knowledge regarding the role of counting as a base process for addition. Counting strategies, facts, or heuristics were used by all of the students on result-unknown tasks (a+b=_) during each interview. This was also the case on part-unknown tasks (a+_=c and _+b=c) with the exception of three students who consistently guessed or repeated the total, making no use of their counting abilities. However, this was clearly a result of their inability to understand the problem context rather than a limitation of their ability to relate the counting process to the additive situation.

Properties. Commutativity, invertibility, and the zero identity were additive properties which the tasks were designed to specifically address. Tasks addressing commutativity involved a result-unknown task stated in its two commutative forms. The manner in which the student solved the second task was observed. An immediate answer to the second question followed by discussion that appropriately addressed the order-irrelevant nature of the numbers involved implicated the student on the use of commutativity in the solution process. The use of a counting strategy on the second task suggested that commutativity was not used. Four students consistently recognized commutative situations and applied these understandings in problem solving situations throughout the year. Ten students had difficulty during the initial interview, but showed uses of commutativity the remaining time. Three students held very unstable notions of commutativity, using this property sporadically throughout the year on these tasks. In addition, four students used count-on strategies on initial-unknown tasks, suggesting that commutativity was used to support the count-on procedure.

The identity property was investigated through the use of tasks of the form (a+_=a): "What's your favorite number (FN)? I'm going to figure out my FN by adding a number to your FN. Is there anyway that I could have the same FN as you? (if needed, a specific number was introduced into the wording of the task)." No student was successful on all four of the Identity tasks from Interviews 1,2,4, and 5. Ten of the students expressed knowledge of the zero identity in a very

sporadic manner across the interviews, seemingly forgetting and remembering the zero property from one interview to the next. Two students did not use the property at any time.

A subtraction task immediately following a related addition task (e.g., 3+_=9 and 9-3=_) given during Interviews 3 and 4 were used to analyze the students' ability to invert the operation of addition and relate it to subtraction. No student used invertibility to answer the subtraction task during Interview 3, with most successful strategies involving counting backwards with blocks. Three students made explicit references to the inverse relation among the two related tasks during Interview 4, and three others made comments suggesting some connection was made. Analysis of subtraction strategies on all part-unknown additive tasks were also made, and 6 of the 17 students used subtraction techniques to answer the additive tasks at some point throughout the year.

Relationship to other operations. The ability to relate addition to the operation of subtraction has just been discussed. A multiplication task was given during Interviews 3 (repeated addition) and 5 (array). Five of the 17 students were successful on the repeated addition task, and 9 (including the previous 5) students were successful on the array task. Count-all with blocks and heuristics were the most frequent strategies used.

Knowledge of symbol system. Before the first interview, 10 of the 18 students could count to 100, 2 other students missed one decade, and the remaining five students could not count higher than 30. Only two students were able to correctly write all ten digits, with 5,7, and 9 the most common digits to be written backwards. All but four students could produce the symbol "+". These limitations quickly dissipated and were not a noticeable barrier in student development.

Use on abstract objects. Five students relied on concrete objects when working each task throughout the year. The remaining students showed varying degrees of an ability to perform the process of addition at a more abstract level, including the use of facts and heuristics on additive tasks. However, only one student made consistent use of heuristics throughout the year, with most of the other student uses emerging during Interview 5.

Other tasks provide additional data on this aspect of operational sense. One task during Interview 3 asked to determine which was bigger: 10+3+5 or 13+5. Seven students stated 10+3+5 was larger because "it has more numbers." Of the four students who correctly stated their equivalence, only two explicitly mentioned the equivalence of 10+3 and 13, while the other two found each sum. During this interview the students were also asked if they knew 4+4, and then were immediately asked if they knew 4+5. Of the 12 students who immediately knew 4+4, 2 used a fact to answer 4+5, 5 used a heuristic, and 5 students had to use a counting strategy. The students were also asked to state how many ways they knew to add two numbers together so that they equal nine. During Interview 1, 8 students could think of no number pairs, 6 students stated one number pair (5 of these said 4+5), and two, four, and five number pairs were stated by one student each. But during Interview 4, 6 students gave 8 or more number pairs, and all but one student

gave at least two number pairs, including 6 students who also involved subtraction. Only four students used blocks or fingers on this task during Interview 4.

Overall Analysis. It appears that flexible understandings of addition in regard to base processes are central in the development of other aspects of operational sense. However, some students developed notions such as commutativity before such development. It is hypothesized that these students developed very fragile notions of number as an object which they could apply to a commutative setting, irrespective of counting strategies. All of the investigated additive properties were mentioned during instruction, but commutativity was acquired much more readily than identity or invertibility.

Six of the seven students who exhibited correct strategies on initial-unknown tasks at least 80% of the time also displayed solid understandings of commutativity or invertibility (or both). Because these students either performed a counting back strategy or commuted the number sentence and used a counting on strategy, an understanding of these properties was vital in their solutions. These students also made the greatest use of heuristics. The five students who did not show one correct strategy on the part-unknown tasks throughout the year also showed no understanding of invertibility and little or no use of heuristics. In addition, a transfer task given during Interviews 4 and 5 asked the students to determine what time it would be 5 hours after 9:00 A.M. A picture of a clock was given to those students who initially showed difficulty, and the class had just completed a few lessons on telling time. Though not universal, a pattern in the data was found between success on this task, correct solution strategies, and knowledge of additive properties.

Implications

This study attempted to combine students' ability to reify addition with an analysis of problem solving strategies in studying the mathematical behaviors of first graders encountering addition. Problems did arise from the use of this framework. These included measures of specific understandings of operational sense as well as the narrow scope of a first grade curriculum in relation to the broad definition of operational sense. However, the numerous kinds of data and the connections between specific components of understanding and problem solving allowed for some conclusions to be made.

Riley et al. (1983) suggest that there exists specific knowledge about additive structures that affects problem solving behavior. Others (Briars and Larkin, 1984; Carpenter, 1985) suggest that problem solving behaviors more closely relate to the actions and contexts of the problem. It appears that some connection exists between types of knowledge associated with addition and the types of strategies students use. Knowledge of specific properties allowed students to better model the actions present in the tasks. This study provides both a means of combining the above two perspectives and data to support the compatibility of the theories.

References

Briars, D.J. & Larkin, J.H. (1984). An integrated model of skill in solving elementary word problems. *Cognition and Instruction, 1*, 245-296.

Carpenter, T.P. (1985). Learning to add and subtract: An exercise in problem solving. In E.A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 17-40). Hillsdale, NJ: Erlbaum.

Cobb, P. & Wheatley, G. (1988). Children's initial understanding of ten. *Focus on Learning Problems in Mathematics, 10(3)*, 1-28.

Gray, E.M., & Tall, D.O. (1994). Duality, ambiguity, and flexibility: A "proceptual" view of simple arithmetic. *Journal for Research in Mathematics Education, 25(2)*, 116-140.

Riley, M.S., Greeno, J.G., & Heller, J.I. (1983). Development of children's problem solving ability in arithmetic. In H.P. Ginsburg (Ed.), *The development of mathematical thinking* (153-196). New York: Academic Press.

Sfard, A., & Linchevski, L. (1994). The gains and pitfalls of reification — The case of algebra. *Educational Studies in Mathematics, 26*, 191-228.