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ABSTRACT

The relationship between college students' preferred mode of processing mathematical information--visual or nonvisual--and their performance in calculus classes with and without technology was investigated. Students elected one of three different versions of an introductory differential calculus course: using graphing calculators, using the computer algebra system "Mathematica," or using no technology. A total of 139 students participated in the research. Presmeg's Mathematical Processing Instrument (MPI) was used to determine students' visual processing preference. The interactions of students of different visual processing preferences with the software "Mathematica" were also investigated using task-based interviews. Results from the sections using graphing calculators suggest that appropriate uses of technology may equally benefit students of different cognitive styles. Contains 29 references. (Author)

Visualization and Students' Performance in Technology-Based Calculus

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VISUALIZATION AND STUDENTS' PERFORMANCE IN TECHNOLOGY-BASED CALCULUS

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The relationship between college students' preferred mode of processing mathematical information—visual or nonvisual—and their performance in calculus classes with and without technology was investigated. Students elected one of three different versions of an introductory differential calculus course: using graphing calculators, using the computer algebra system Mathematica®, or using no technology. A total of 139 students participated in the research. Presmeg's Mathematical Processing Instrument (MPI) was used to determine students' visual processing preference. The interactions of students of different visual processing preferences with the software Mathematica were also investigated using task-based interviews. Results from the sections using graphing calculators suggest that appropriate uses of technology may equally benefit students of different cognitive styles.

> Lie was an intuitionalist; this might have been doubted in reading his books, [however] no one could doubt it after talking with him; you saw at once that he thought in pictures. Madame Kovalevski was a logician. Among our students we notice the same differences; some prefer to treat their problems 'by analysis,' others 'by geometry.' *(Poincaré, 1900/1907, p. 17)*

In the midst of calculus reform in the U.S.A. many of the new approaches to the calculus make use of computers or calculators with graphing capabilities. However, individuals vary in their preferences for using visual methods of solution when solving mathematics problems. Among mathematicians, both visualizers and nonvisualizers have made important contributions to the progress of mathematics, as illustrated by the quote from Poincaré (1900/1907). Furthermore, there seems to be agreement on the importance of mental imagery in thinking and in the act of creation (Hadamard, 1945; Koestler, 1967; Shepard, 1978). Among students, according to Krutetskii (1968/1976), the ability to visualize abstract mathematical relationships and the ability for spatial geometric concepts do not determine the extent of mathematical giftedness but only its type. However, several research studies suggest that there is a negative association between students' degree of visuality and their performance in school mathematics (Lean & Clements, 1981; Presmeg, 1986). This previous research has been conducted in classes that use no technology, and the differences in mathematical achievement favoring students who are nonvisualizers have been observed both at the senior high school level, and at the freshman college level. Nevertheless, the author's research on calculus courses suggests that technology, and software with multiple-representation capabilities, can be used to promote conceptual understanding and equally favor both visualizers and students who are nonvisualizers (Galindo, in press, 1994). Some results from this research will be discussed in this paper. Students' interactions with the software Mathematica will also be examined.

Importance and Status of Visualization in Mathematics Education

With an increased emphasis on the study of patterns (National Research Council, 1989; Steen, 1988), visualization is acquiring an important role in mathematical endeavor. Computer–generated graphs are enabling the mathematician to visualize the content of abstract theorems (Pool, 1992), and new conjectures are suggested by the eye (Mandelbrot, 1983). The current status of visualization in mathematics is best summed up by Steen (1990) as follows:

> Thanks to computer graphics, much of the mathematician's search for patterns is now guided by what one can really see with the eye, whereas nineteenth-century mathematical giants like Gauss and Poincaré had to depend more on seeing with their mind's eye.... For centuries the mind has dominated the eye in the hierarchy of mathematical practice; today the balance is being restored as mathematicians find new ways to see patterns, both with the eye and with the mind. *(p.2)*

It seems thus natural to think that if we want students to do mathematics the same way as mathematicians do it, computers and visualization should also have an important role in mathematics education. However, it seems that visualization has had for a long time a low status in school mathematics.

Although it is generally accepted that visual representations offer a powerful introduction to the complex abstractions of mathematics (Bishop, 1989), and that for some subjects such as geometry it is believed that visualization is a necessary tool in concept formation (Hershkowitz, 1989), there are a number of students' difficulties with visualization that have been reported in the mathematics education literature (Clement, 1985, 1989; Goldenberg, 1988, 1991; Yerushalmi & Chazan, 1990). Not only do students have difficulty visualizing concepts and interpreting graphs, but instances of students' reluctance to use visual methods have been reported (Balomenos, Ferrini–Mundy & Dick, 1988; Eisenberg & Dreyfus,1991; Vinner, 1989). Dreyfus (1991) points out that teachers and educators contribute to the low status of visualization in school mathematics:

> The message is that visualization may be a useful and efficient learning aid for many topics in high school and college mathematics, but nevertheless an aid, a crutch, a step, sometimes a necessary and important step, but only a step on the way to the real mathematics. *(p. 34)*

One of the possible consequences of the low status of visual methods in school mathematics is the differential performance in mathematics courses of visualizers and nonvisualizers. Lean and Clements (1981) found that first year engineering students who preferred to process mathematical information by verbal–logical means tended to outperform more visual students on mathematical tests. Presmeg (1986) found that visualizers are seriously under-represented among high mathematical achievers at the senior high school level, and she provides some explanations for this phenomena. The question then arises of whether these differences in mathematical performance in favor of the nonvisualizer student can also be observed in classes that use computers and graphing calculators, that is, technology with multiple-representation capabilities.

Visualization and Mathematical Performance in Technology-Based Calculus Classes

Participants in this study were enrolled in the first course of a three–quarter calculus and analytic geometry sequence for science and engineering majors. The purpose of the course was to provide students with a solid foundation in one–variable differential calculus. Students elected one of three different versions of the course. One approach used graphing calculators and the textbook *Calculus a Graphing Approach* (Finney, Thomas, Demana, & Waits, 1993); another used the computer algebra system Mathematica® and the textbook *Calculus and Mathematica* (Brown, Davis, Porta, & Uhl, 1992); and the last used no technology explicitly and the textbook *Calculus* (Finney & Thomas, 1991). Eighteen out of twenty–six sections of the calculus course participated in the study, with approximately 25 students enrolled in each section. The eight sections using graphing calculators and the eight sections using no technology used the lecture–recitation format. Performance in these classes was evaluated using three midterms, several quizzes, and a final exam. The two sections using Mathematica® had five 48–minute sessions every week in the computer laboratory. The students in these sections were evaluated considering individual and group homework, literacy sheets, participation, 6 in–class quizzes, one midterm and one final exam. The 18-item Mathematical Processing Instrument (Presmeg, 1985, 1986) was used to determine students' visual processing preference.

From the three calculus approaches, a total of 139 students participated in the research. Out of 36 possible points, MPI scores varied from 6 to 29, with a median of 17. It was found that the MPI scores were normally distributed and the cognitive styles of visualizers, nonvisualizers, and students of the harmonic type, were found among students in every calculus approach. One research question investigated the relationship between college students' preferred mode of processing mathematical information and their performance in calculus classes with and without technology. It was found that students who are nonvisualizers obtained significantly better scores than visualizers in the calculus sections using no technology, and in the calculus sections using the software Mathematica. On the other hand, there were no significant differences in the calculus scores obtained by visualizers and nonvisualizers in the sections using graphing calculators. These results and their implications for the use of technology in mathematics education are discussed elsewhere (Galindo, in press).

Students' Interactions With The Software

Another research question investigated the interactions of students of different visual processing preferences with the software Mathematica. Task–based interviews of students of each cognitive style from the sections using the computers were conducted. Students to be interviewed were selected using purposeful sampling, in particular two strategies: theory based, or operational construct, sampling and maximum variation sampling. The theoretical construct used for the selection process was *mathematical visuality*. Two students of each cognitive style were selected for the interviews, thus a total of 6 students were interviewed.

There were two goals for the task–based interviews. The first goal was to gather more information about the student's preferred mode of solving mathematics probiems—visual or nonvisual. The MPI was used early in the course to this end, but it was desired to investigate if students' work in the calculus would give evidence about their visual orientation that was in agreement with the MPI results. A second goal of the task–based interviews was to look at the ways in which students of different degrees of visuality use the software when solving mathematics problems. Mathematica software has different types of tools—tools to graph functions, tools to solve equations symbolically, and tools to do numerical calculations. The course puts great emphasis on graphical methods for solving problems and encourages visual thinking. It was desired to investigate if students would use the software in ways that reflect their visual orientation, or if they would mostly rely on the graphical methods emphasized in class.

The problems solved during the interviews were analyzed and scored using the same point system used in the MPI. A problem was given 2 points if it was solved using visual methods. A problem solved by numerical or symbolic methods was assigned 0 points, and problems solved by a combination of methods or problems where it was hard to tell the method used, were given 1 point. Students' work during the task–based interviews provided further evidence about their visual orientation. After scoring students' solutions to the interview problems using the MPI rubric described above, it was found that the methods of solution used by the students during the task–based interviews reflect the visual orientation indicated by the MPI. Students who obtained a high MPI score tended to use graphic methods of solution and preferred to use the plotting capabilities of the software, whereas students with low MPI scores used numeric and symbolic methods of solution and the corresponding software commands.

As for the interaction of the students with the software, it was found that the tools used by the students did correspond to their visual preferences, with visualiy oriented students preferring to use graphical methods to solve problems, and nonvisualizers preferring to use commands such as Solve, or N[Solve]. If students tend to use software tools that correspond to their visual preference, why is that nonvisualizers seem to outperform visualizers in the Mathematica sections? Some possible explanations are examined in the next section.

Conclusions

The data obtained from both the sections using Mathematica and the sections using no technology provided evidence of a negative relationship between MPI score and total calculus score for the students taking these approaches. These results show that differences in mathematics final scores between visualizers and nonvisualizers prevail at the college level and that they are not easily removed. The fact that no significant differences were found between calculus performance and degree of visuality in the sections using graphing calculators, suggests that appropriate uses of technology may equally benefit students of different cognitive styles. Mathematics education research should seek to investigate the appropriate conditions for this to take place.

It was also found that the students' visual orientation observed during task–based interviews and the software tools they use correspond to the degree of visuality indicated by the MPI. Furthermore, the negative association between course scores and degree of visuality found in the Mathematica sections seems to be the result of the long symbolic sentences that students must enter for Mathematica to plot a graph. The visualizers in these sections needed to go through an analytic expression in order to take advantage of the graphs. Thus, educators and software designers should be aware of the restrictions that Computer Algebra Systems may impose on students of different cognitive styles, as well as of their effects on students' performance in mathematics.

Another important variable that must be considered is the role of the teacher. The present study was repeated for the Mathematica sections during the following quarter, when an experienced Ph.D. in mathematics taught the sections using this calculus approach; no significant differences were found this time in the calculus performance of students of different cognitive style. Furthermore, the interactions between the teacher's cognitive style and the student's visual preference, as well as their effect on student's performance, need to be investigated in computer–based environments that encourage visual thinking.

Presmeg (1985), identified 17 classroom aspects which are reported in the literature to be facilitative of formation and use of visual imagery in school mathematics. Among such aspects we find: conscious teacher attempts to generate imagery in pupils by the use of instruction to form images, and the creation of dynamic situations to think in moving pictures, (b) teacher formation and use of their own imagery, and (c) a pictorial presentation of the topics. The classroom aspects conducive to the students' formation of mental imagery in courses that make use of technology need to be investigated.

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