DOCUMENT RESUME

ED 389 619 SE 057 264

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| AUTHOR | McClain, Kay; Whitenack, Joy |
| TITLE | An Analysis of the Teacher's Proactive Role in Redescribing and Notating Students' Explanations and Solutions. |
| SPONS AGENCY | National Science Foundation, Washington, D.C. |
| PUB DATE | Oct 95 |
| CONTRACT | RED-9353587 |
| NOTE | 9p.; Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (17th, Columbus, OH, October 21-24, 1995). For entire conference proceedings, see SE 057 177. |
| PUB TYPE | Reports - Research/Technical (143) -- Speeches/Conference Papers (150) |
| EDRS PRICE | MF01/PC01 Plus Postage. |
| DESCRIPTORS | *Cognitive Development; Elementary School Teachers; Grade 1; Mathematics Instruction; *Mathematics Teachers; Primary Education; *Symbols (Mathematics); *Teacher Role |

ABSTRACT

The analysis reported in this paper is part of a year-long first-grade teaching experiment and focuses on the teacher's proactive role in supporting students' mathematical growth in an inquiry mathematics classroom. Within the project classroom, the teacher often redescribed and notated students' responses so that what students had done mathematically might become an explicit topic of conversation. As part of this process, she frequently introduced either informal or conventional notation to record students' explanations of their mathematical activity. The introduction of these notational schemes led to students' development of ways of notating their own reasoning. In this way, the notation emerged from the students' activity while supporting shifts in their mathematical development. (Author)

# An Analysis of the Teacher's Proactive Role in Redescribing and Notating Students' Explanations and Solutions

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Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education

(17th PME-NA, Columbus, OH, October 21-24, 1995)

# AN ANALYSIS OF THE TEACHER'S PROACTIVE ROLE IN REDESCRIBING AND NOTATING STUDENTS' EXPLANATIONS AND SOLUTIONS

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The analysis reported in this paper is part of a year-long, first-grade teaching experiment and focuses on the teacher's proactive role in supporting students' mathematical growth in an inquiry mathematics classroom. Within the project classroom, the teacher often re-described and notated students' responses so that what students had done mathematically might become an explicit topic of conversation. As part of this process, she frequently introduced either informal or conventional notation to record students' explanations of their mathematical activity. The introduction of these notational schemes lead to students' development of ways of notating their own reasoning. In this way, the notation emerged from the students' activity while supporting shifts in their mathematical development.

The purpose of this paper is to document crucial aspects of one effective reform teacher's proactive role in initiating and guiding students' mathematical development. Particular attention will be given to how the teacher redescribed and notated student responses. This activity will be related both to shifts in discourse and to students' development of ways of notating their own reasoning. We will also attempt to clarify how the development of notational schemes became realized in the classroom by developing empirically grounded analyses of the teaching-learning process as it was interactively constituted in the classroom (Cobb, Wood, Yackel, & McNeal, 1992). The intent is, therefore, not to develop a prescriptive list of behaviors that purports to guarantee effective reform teaching. Instead, it is to develop a detailed account of one teacher's practice situated in a specific classroom. The reported analysis should be of more than local interest because it might serve as a paradigmatic case that can both help other teachers develop understandings of their own practice and contribute to the growing research literature on reform teaching.

In the following paragraphs, we first provide a description of the teacher and her classroom and then outline the data corpus. Against this background, we analyze the teacher's proactive role in supporting her students' mathematical growth by redescribing and notating their explanations.

## Ms. Smith's Classroom

The majority of the eleven girls and seven boys in Ms. Smith's first-grade classroom were from middle or upper middle class backgrounds. There were no minority children in the classroom, although a small percentage attended the school.

The research reported in this paper was supported by the National Science Foundation under grant No. RED-9353587. The opinions expressed do not necessarily reflect the views of the Foundation.

The students in the class were representative of the school's general student population. Although not a parochial school, morals and values were considered to be part of the responsibility of schooling and children regularly participated in spiritual activities.

Ms. Smith's classroom is of particular interest because an analysis of videorecorded interviews conducted at the beginning and end of the school year indicates that the students' conceptual development in mathematics was substantial. Students who, at the beginning of the year, did not have a way to begin to solve the most elementary kinds of story problems posed with numbers of five or less had, by the end of the year, developed relatively sophisticated mental computation strategies for solving a wide range of problems posed with two-digit numbers.

The teacher, Ms. Smith, was a highly motivated and very dedicated teacher in her fourth year in the classroom. She had attempted to reform her practice prior to our collaboration and voiced frustration with traditional mathematics textbooks. She explained that her attempts to use a "center" approach left her without the benefits of productive whole class discussions. Although she valued students' ability to communicate, explain, and justify, she indicated that she had previously found it difficult to enact an instructional approach that both met her students' needs and enabled her to achieve her own pedagogical agenda. When we began working with Ms. Smith, it soon became apparent that she constantly assessed both the instructional activities she used and her own practice. In addition, she had a relatively deep understanding of both mathematics and her students' thinking. Ms. Smith was seeking guidance with her reform efforts; we were seeking a teacher with whom to collaborate as we developed sequences of instructional activities.

## Data Corpus

Data were collected during the 1993-94 school year and consist of daily videotape recordings of 103 mathematics lessons from two cameras. During whole class discussions, one camera focused primarily on the teacher and on children who came to the whiteboard to explain their thinking. The second camera focused on the students as they engaged in discussions while sitting on the floor facing the whiteboard. Additional documentation consists of copies of all the children's written work; daily field notes that summarize classroom events; notes from daily debriefing sessions held with Ms. Smith; and videotaped clinical interviews conducted with each student in September, December, January, and May.

A method described by Cobb and Whitenack (in press) for conducting longitudinal analyses of videotape sessions guided the analyses. This method fits with Glaser and Strauss' (1967) constant comparative methods for conducting ethnographic studies. It involves constantly comparing data as they are analyzed with conjectures and speculations generated thus far in the data analysis. As issues arise while viewing classroom videorecordings, they are documented and clarified through a process of conjecture and refutation. Through this process, classroom accounts have been identified which can be used to clarify 1) how Ms. Smith's practices became realized in the classroom, and 2) how these practices contributed to her students' growth. This paper, which is part of a larger investigation will focus specifically on redescribing and notating students' explanations and solutions.

## Classroom Analysis

Lesh, Post, and Behr (1987) observe that many students have "deficient understandings about the models and languages to represent and manipulate mathematical ideas" (p. 37). In our view, it is essential that students experience symbols in relation to their own mathematical activity if they are to develop grounded understanding of their meaning and use. Students might then view symbols and notations as ways of recording and communicating their thinking that they can use as the need arises.

In this particular classroom, Ms. Smith often attempted to initiate shifts in the level of classroom discourse so that what was done mathematically subsequently might become an explicit topic of conversation. As part of this process, she frequently drew pictures or used either informal or conventional notation as she redescribed students' explanations. Ways of notating therefore functioned as protocols of action (Dörfler, 1989) that grew out of the students' activity in a bottom-up manner (cf. Gravemeijer, in press). For example, students often solved an addition task such as 7 + 8 by partitioning the 8 into seven and one and reasoning, seven and seven is 14, and one more is 15. Ms. Smith devised a simple method of notating this activity by using an inverted "V" symbol that came to signify the partitioning of a number. Ms. Smith would typically follow the "V" notation with the number sentences that expressed the result of the partitioning (see figure 1).

```
7 + 8 =
   / \
   7  1

 7 + 7 = 14
14 + 1 = 15
```

*Figure 1.* Notating Decomposition of Numbers

On their own initiative, students often referred to this notation to explain their thinking to other children during whole class discussions. In addition, students began to use the records as a means of comparing solutions, thereby initiating shifts in the discourse such that features of their reasoning became an explicit topic of conversation. We speculate that the students' participation in such discourse supported their reflection on and mathematization of their prior activity. However, it is important to note that students were not obliged to use the notational schemes introduced by Ms. Smith. The children did in fact symbolize their thinking in a variety of different ways when they were asked to make records so that other children might understand how they had solved tasks. This lack of obligation allowed students to develop their own means of representing their reasoning by adapting what had been offered.

As an illustration, consider an incident which occurred on December 7. The task posed by Ms. Smith was a story problem involving a bus — *There are eight people on the bus and six more get on. How many people are on the bus now?* Students were asked to work individually on the task, making a record of their solution process so that others might understand their reasoning. The focus in this part of the lesson was effectively communicating their mathematical thinking — not imitating a given notational system. During the subsequent whole class discussion, Ms. Smith asked students to share their solution methods verbally as she redescribed and notated their activity. In this particular instance, the first offered solution involved using a doubles strategy:

> Kitty: I took one off the eight and I put it on to the six to make 7 plus 7 and I know 7 plus 7 makes 14.

Ms. Smith redescribed and notated (see figure 2).

```
  8 + 6 =
 / \
 7 1

7 + 7 = 14
```

*Figure 2.* Notating Kitty's Solution

After questions and discussion, Ms. Smith asked for a different way. Jane explained that she partitioned the numbers differently.

> Jane: I stayed with the 6 but I broke it up into 3 and 3 and when it had the three it made 11 and three more...it made...uhm...it made...it made 13 and one more is 14.

Again, Ms. Smith redescribed and notated the solution (see figure 3), attempting to clarify to the students how Jane's explanation differed from Kitty's.

```
   8 + 6 =
  / \
  3 3

 8 + 3 = 11
11 + 3 = 14
```

*Figure 3.* Notating Jane's Solution

In addition to Ms. Smith's verbal clarification of the difference in the solutions, symbolizing the two solutions offered opportunities for the other students to compare the two solutions as they attempted to clarify for themselves how these solutions compared to each other and to their own.

In examining the student work from the previously described task, it is important to note that there was diversity in the students' notational schemes. Although students often used elements of the teacher's notation scheme, they did so in original ways as they struggled to communicate their thinking. Even when students' verbal explanations were redescribed and notated by Ms. Smith in a manner consistent with her original scheme, the students worked to devise notational schemes to fit with their thinking that supported their interpretation of the solution process. As a consequence, although students might accept ways of talking about their activity that fit with the teacher's notation scheme, they continued to solve tasks using very different, personally meaningful notation schemes. This in turn made it possible for students to develop interpretations and strategies that reflected their current understanding while experiencing more sophisticated and efficient solutions in class discussions.

8 + 6<br>
/ \\<br>
4 4

6 + 4 = 10<br>
10 + 4 = 14

*Figure 4.*

8 + $\boxed{6}$ 6 + 1 = $\boxed{7}$ + 7 = 14<br>
/ \\<br>
$\boxed{1}$ $\boxed{7}$

*Figure 5.*

6 8<br>
/ \\<br>
8 2 4 10 + 4 = 14

8 + 2 = 10

*Figure 6.*

8 + 6 = 7<br>
/ \\<br>
1 7

7 + 7 = 14

*Figure 7.*

In the sample records shown below (see figures 4 through 7), each child arrived at an answer of 14. However, only the first child's way of notating is consistent with that of the teacher. Although the other three used elements of the teacher's scheme, they adapted them in original ways.

It should be stressed that, from the students' perspective, Ms. Smith appeared to introduce notation almost in passing. In addition, the students were not obliged to follow—and never practiced—particular ways of notating. The notation was offered by Ms. Smith and became taken-as-shared only through a process of mutual negotiation between her and the students.

As a further note, the teacher's proactive role in guiding the development of ways of notating appears to have been critical in supporting her students' mathematical development. The children increasingly notated on their own initiative as they solved problems while working both individually and in groups. These records helped them distance themselves from their ongoing activity and thus reflect on what they were doing. Consequently, the use of notation contributed to the productiveness of whole class discussions by helping to make individual children's contributions explicit topics of conversation that could be compared and contrasted. It was as they participated in these discussions that the teacher guided her students' transition from informal, pragmatic problem solving to more sophisticated yet personally meaningful mathematical activity.

## Conclusion

Throughout this paper, we have attempted to document crucial aspects of Ms. Smith's proactive role in redescribing and notating students' explanations and solutions. For Ms. Smith, the notational schemes emerged from the students' attempts to explain and justify their thinking; they were not predetermined schemes introduced into the classroom in a top-down manner. The analysis of Ms. Smith's role in introducing these schemes indicates that while they became taken-as-shared through a process of mutual negotiation, the teacher played a central role in initiating the development of notational schemes that served as protocols of students' mathematical activity.

The reform movement in mathematics education has emerged as a response to the consequences of traditional mathematics instruction. Often, reform teaching was characterized with reference to traditional teaching, and the emphasis was on what teachers should not do (e.g. funnel students to correct answers and script lesson plans). Although it was generally accepted that reform teachers should actively support their students' mathematics development, this was frequently characterized in vague terms such as facilitate or guide. It is only recently that explicit attention has been given to what specifically effective reform teachers do to support their students' development. This paper contributes to this growing literature by documenting the proactive actions of one teacher as she attempted to reform her practice.

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