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AUTHOR	Adams, Verna M.; Price, Terry M.
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ABSTRACT

This study examines the role of discourse in the development of students' understandings of a rule for determining if a number is divisible by 8. The rule was suggested by a student in a seventh-grade mathematics class. Its validity was investigated in whole-class discussions that occurred on 3 consecutive days. In addition to making field notes and videotaping classroom interactions, what students learned was investigated through the use of whole class surveys and interviews with 9 out of 29 students. Discourse served to sustain the investigation, to assist students' development of the idea, and to confuse students. Confusion occurred as a result of "failures of context" when the discourse failed to deal with the complexity of the language structure involved in a rule and to discriminate between meanings of words such as even and evenly. (Author)

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Verna M. Adams and Terry M. Price

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THE DEVELOPMENT OF A STUDENT THEORY: THE ROLE OF DISCOURSE

Verna M. Adams, Washington State University
Terry M. Price, Washington State University

This study examines the role of the discourse in the development of students' understandings of a rule for determining if a number is divisible by 8. The rule was suggested by a student in a seventh-grade mathematics class. Its validity was investigated in whole-class discussions that occurred on 3 consecutive days. In addition to making field notes and videotaping classroom interactions, what students learned was investigated through the use of whole class surveys and interviews with 9 out of 29 students. The discourse served to sustain the investigation, to assist students' development of the idea and to confuse students. Confusion occurred as a result of "failures of context" when the discourse failed to deal with the complexity of the language structure involved in a rule and to discriminate meanings of words such as *even* and *evenly*.

> Learning always takes place in everyday activity, whatever that activity might be. (Lave, Smith, and Butler, 1988, p. 79)

Current perspectives on the teaching and learning of mathematics suggest that the everyday activity in mathematics classrooms should include students talking about mathematics. As a result of that talk, students will be introduced to ideas they have not previously developed on their own. It follows then that a student's understanding can be expected to evolve under conditions of systematic cooperation with the teacher and other students. Moreover, because students' mathematical understandings are anchored to the contexts in which they are learned (Lave, 1988), students understandings will be anchored to the classroom discourse. According to Vygotsky (1978), the introduction of a new concept into the discourse in the classroom initiates a long and complex process in which the student eventually appropriates the concept. He argues that the deliberate introduction of new concepts into the dialogue, rather than precluding the spontaneous development of those concepts, charts new paths for their development and "may influence favorably the development of concepts that have been formed by the student himself" (p. 152).

The Study

The students in this study were 29 "average" seventh-graders in one mathematics class; the teacher was the regular classroom teacher. At the beginning of almost every class, 10 to 15 minutes were used for an activity called mental math. The remainder of the 45-minute class consisted of whole-class discussion, individual seat work, or small-group work. This paper focuses on a theory about divisibility by eight that was presented in a whole-class discussion at the end of the

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sixth week of school. We identify the student who presented the theory by the fictitious name of Blayne. Our investigation followed the recommendation of Lave, Smith, and Butler (1988) to focus on the activity of learners, with an emphasis on what is actually occurring and what is being learned. In this report we discuss the role of the discourse in these components of the investigation.

Method

Our method of investigation draws on the grounded theory method of Glaser and Strauss (1967) and Glaser (1978) in which, after identifying a phenomena of interest from the data, additional data is collected and coded. Selection of this new data is determined by the emerging theory in order to maximize the information relevant to the theory.

Initial data collection, consisting of daily videotaping and recording of field notes of the classroom activities and the collection of curriculum materials, began in the fourth week of school before the start of this study and continued beyond the scope of the study to the end of a unit of instruction on number theory that was 4 weeks long. The topic of interest emerged from the data when Blayne suggested a rule for dividing by eight. Although we use the word *rule* in this paper, the teacher, the students, and the researchers during the data collection referred to it as Blayne's theory. Blayne, with the support of the teacher, led the class in an investigation of the validity of the procedure that he proposed.

In order to investigate the extent that Blayne's theory was appropriated by other students, several stages of data collection occurred. Blayne discussed his rule about dividing by eight on three consecutive class days. Videotaping and field notes of these presentations were followed by an initial whole-class survey and then by individual interviews of 9 students. These interviews were evaluated at the end of each day and new or modified hypotheses about student understandings were considered. Upon completion of the interviews, a second whole-class survey was constructed and administered in order to determine if students had incorporated the ideas into their understanding to the point where they would be able to generalize the rule to dividing by 27.

Overview of the Analysis

The key players in this study were (a) Blayne, who presented the theory of divisibility by eight, (b) the teacher and (c) other students who participated in the class discussions. The teacher orchestrated the discussion without telling the students exactly what to think. She made attempts at getting students to explain their understandings of the theory and gave positive feedback to students by telling them that she liked their ideas. She occasionally corrected student statements about language or the procedure.

What Occurred? Discussions of divisibility by eight occurred in this class on 3 different days consisting of 2 minutes of discussion, 13 minutes, and then 17 minutes. Blayne's first explanation was garbled and the teacher asked him to think about it overnight. The next day, she asked him to present his theory at the overhead to the whole class. On the third day, she asked him restate it so the class could determine if the rule would always "work." On the second day, Blayne stated his theory as "Anything that you are trying to find out if eight goes into, you have to divide by 2 three times, and if the answer to those three are even, then it will go into eight." Later his language became "If all three of those answers are even, then you can divide it by eight." and "If the answers come out even." Even was clarified with the help of the teacher to mean that it would not have a decimal in the answer.

In the class discussions, students tried to validate Blayne's theory using their own numbers. The magnitude of the numbers used by Blayne was limited to 2-digit numbers. Other students, however, used numbers with up to 8 digits. The interview data indicated that students generally had a comfort zone with numbers up to about 100. Thus, through the use of numbers outside their comfort zone, students made the discussion into a true investigation aimed at deciding if the procedure would always work. The teacher made three attempts to get students to draw conclusions from their examples and three attempts at closure by making statements such as "it seems to work." Students puzzled over whether or not they were talking about "even numbers" or "dividing evenly." They asked if Blayne's ideas were related to 2^3 and if you could construct similar divisibility rules for other numbers. One student wanted to know if the answers were the same when one divided by 2, three times and when one divided by 8. At the end of the 3-days of discussion, some students appeared to be convinced that Blayne's method of determining whether or not a number was divisible by eight was valid. Others were skeptical, offering hypotheses such as "maybe the theory works if all the digits are even," conclusions such as "I don't think it works, not all even numbers work," and questions such as "Why not just divide by eight to begin with?" The teacher left the discussion open by telling students to go home and try some examples in order to answer some of their questions.

The surveys and interviews revealed that students who did not believe the theory would always work had different reasons for rejecting the idea. One student said "no" because other theories also worked. Two students were skeptical of the justification process. One explained: "I don't know why I think that, I guess because there must be a number out there that can fool him." Two types of justifications were used by students to either accept or reject the theory. *Problem-specific justification* was an explanation based on a particular problem, rather than several examples. Another type of justification, *example-based justification*, was characterized by comments such as "because I have used it a lot," and "because we have worked on problems in class and out of class and it always worked." Some of the students' justifications were either missing or uninterpretable.

What was learned? The classroom discourse revealed gaps in Blayne's understanding of his own theory. For example, when a student asked if the theory had anything to do with 2^3, he answered, "I don't know." An interview with Blayne revealed that he had learned the procedure from his father. In effect, he had accepted the idea but had not developed an understanding of it. By the time of the second survey, Blayne's responses indicated that he had filled in some of the gaps in his understanding. He recognized that "A number is divisible by 8 if it has 3 factors of 2" and "If 2^3 is a factor of a number, the number is divisible by 8." He, however, did not generalize the procedure to create a rule for dividing by 27.

Ten students actively participated in the discussions, interjecting comments, conjectures and questions. These students either (a) indicated that they did not understand the theory, (b) believed that it would work, (c) did not believe it would work, or (d) felt that it was too long or too much trouble. In the initial survey, 11 students spontaneously used Blayne's theory in response to the question: Is 2000 a multiple of 8? Another ten students divided 2000 by 8, one student guessed, and four students based their answers on misconceptions that can be connected to other divisibility rules. One student reasoned: "8 goes into 200, so it should go into 2000." Two students did not complete the survey.

On the second survey, several questions investigated language issues from the class discussions. For example, the words *divided by* were often misused. Only 8 students had an understanding of how to interpret these words. Nearly half (13) divided the small number into the large number, and 3 always interpreted from left to right. Thus, for many students, the information that influenced their interpretation was the size or the order of the numbers. Students also were less secure with the language "3 factors of 2" than with "2 to the third power" as a meaning for the notation "2^3." An examination of the discourse revealed that the language "3 factors of 2" was not explicitly connected to the notation.

Students' interpretation of what it means to say that the rule worked was different from the researchers' interpretation. Students considered the rule to "work" if the number was divisible by 8 and "not work" if the number was not divisible by 8. One student concluded that Blayne's "theory isn't always going to work." We took this to mean that she thought the rule was not valid. In the interview, a different interpretation emerged and we concluded that the student understood the theory, but not the language of the question that had been posed:

Interviewer : Is 86 divisible by 2?

Student: Yes. [She showed that 86 divided by 2 was 43.]

Interviewer: Is 43 divisible by 2?

Student: No.

Interviewer: So what does that tell you?

Student : That it's not divisible by 8.

Other issues related to the students' understanding surfaced in the interviews. Some students, for example, did not connect the theory to what they already knew. One student , explained: "I don't use it. It doesn't make sense and I think its just too long." Yet when she was asked to factor 64, she created a factor tree by dividing by 2 to get 32, and then dividing 32 by two to get 16, and so on. When the interviewer attempted to see if she would connect her technique to Blayne's theory she responded: "I don't know, mine just seems a little bit easier than Blayne's. . . . maybe because I've been doing mine and I haven't been doing his." The student claimed that dividing an even number by two "only works really if you are trying to make a factor tree." Although she used the method in the context of the factor tree, she refused to utilize it in the context of divisibility by eight.

The Role of Discourse

The discourse made a difference in the way the student rule developed. One example of this influence was that the effect of using large numbers in student examples was to push the ideas outside the students' comfort zones to create a problem-solving situation for which they did not have a clear method of solving. The discussions, however, were sprinkled with instances of language errors and procedural errors related to division, factors, and multiples that appeared to create some confusion. These difficulties point to a need to practice "talking mathematics" in order to coordinate thoughts and words in ways that communicate the thoughts. Thus the classroom discourse provided an opportunity for students to develop their skills in using mathematical language.

Overall, the discourse functioned in a positive way for students, especially for Blayne. He began to consider his ideas in new ways, to refine the language that he used, and to connect the rule to concepts such as factors and powers. The discourse also created some confusion for Blayne. On the initial survey he indicated that he did not believe that his own theory was always true. In an interview he explained that this response was a result of one of the examples given in class, but that he had since changed his mind.

Student misunderstandings that occurred seemed to be attributable to a lack of closure; that is, the discourse did not provide a definitive conclusion about what dividing by 2 three times implied. Failures to resolve some of the issues raised in the discussions can be related to what Edwards and Mercer (1987) refer to as learning failures related to a failure of context. They suggest that "'learning failures' are not necessarily attributable to individual children or teachers, but to the inadequacies of the referential framework within which education takes place. In other words, they are failures of context" (p. 167). Failures of context occurred because the discourse failed to adequately differentiate ideas, particularly to differentiate meanings of some of the words. For example, in the class discussions, the word *even* had two different meanings. It was first used to narrow the set of numbers to "even numbers." To be divisible by two a number must be even. It was also used to mean that there was no remainder when one divided. Confusion of these meanings leads to different conclusions only on the third division when testing Blayne's rule for divisibility by eight.

Differentiation of meaning was important for the word *worked* because the rule works in two ways: The rule tells you that a number is divisible by eight or that a number is not divisible by eight. Many of the students used only the first meaning. A complex discourse structure is implicit in the rule; three different if-then statements, for example, are relevant to understanding the rule. In this study, the rule was explained via examples. The complexity of the implicit discourse structures was not part of the discussion. We conclude that more direct attention to language structures is needed in order for students to participate effectively in classroom discourse that is focused on validating mathematical ideas. If students are going to discuss each other's theories and validate the correctness of the mathematics on their own then they need to begin to have some understanding of mathematical language structures.

Finally, as noted by two students, *efficiency* is a weak purpose for the study of divisibility by eight or any other divisibility rule. The context for the discussion of divisibility by eight in this classroom was that of validating the rule, not just using it. Within this context, students could connect ideas to factors and powers of numbers, to generalize the structure of the rule to other numbers, and to develop an understanding of mathematical language. A strong purpose for including divisibility rules in the curriculum is to build a greater understanding of numbers and number relationships.

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