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AUTHOR Roddick, Cheryl Stitt
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ABSTRACT

This study investigated students' conceptual and procedural understanding of calculus within the context of an engineering mechanics course. Four traditional calculus students were compared with three students from one of the calculus reform projects, Calculus & Mathematica. Task-based interviews were conducted with each participant throughout the course of the 10-week quarter. Results from interviews show a distinct difference in approaches to solving engineering mechanics problems that involve calculus. Calculus & Mathematica students, who learned calculus with a conceptual emphasis, were found to be more likely to solve problems from a conceptual viewpoint than were the traditional students, who were more likely to focus on procedures. (Author)

How Students Use Their Knowledge of Calculus in an Engineering Mechanics Course

Cheryl Stitt Roddick

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HOW STUDENTS USE THEIR KNOWLEDGE OF CALCULUS IN AN ENGINEERING MECHANICS COURSE

Cheryl Stitt Roddick, The Ohio State University

This study investigated students' conceptual and procedural understanding of calculus within the context of an engineering mechanics course. Four traditional calculus students were compared with three students from one of the calculus reform projects, Calculus & Mathematica. Task-based interviews were conducted with each participant throughout the course of the ten-week quarter. Results from interviews show a distinct difference in approaches to solving engineering mechanics problems that involve calculus. Calculus & Mathematica students, who learned calculus with a conceptual emphasis, were found to be more likely to solve problems from a conceptual viewpoint than were the traditional students, who were more likely to focus on procedures.

The introduction of technology in the calculus classroom has been met with mixed emotions. Many enthusiastic supporters have emerged, yet there have also emerged many critics of the quality of learning that occurs. One major criticism has been that students who learn calculus with the help of technology will not have the skills to be successful in later calculus-dependent courses (Krantz, 1993). Others argue that these students have a stronger conceptual understanding and will have an advantage over students who have taken traditional calculus. Supporters of Calculus & *Mathematica*, a calculus reform project which utilizes the computer algebra system *Mathematica* in students' learning of calculus, believe that the use of technology, together with teaching techniques based on constructivist theory, can encourage student ownership of knowledge and a strong conceptual understanding (Davis, Porta & Uhl, 1994).

This study addressed these issues and investigated students from the Calculus & *Mathematica* sequence as they continued their education beyond calculus. Since calculus is a stepping stone to many other courses, success in future calculus-dependent courses may be determined in part by students' experiences in their calculus courses. The focus of the study is a comparison of Calculus & *Mathematica* students with traditional students on their conceptual and procedural understanding of calculus when applied to different situations. An introductory engineering mechanics course was chosen as the course in which to investigate students' understanding of calculus.

Theoretical Framework

Bell, Costello, and Kuchemann (1983) specify five components of mathematical competence: facts, skills, concepts, general strategies, and appreciation. Two of these components — skills and concepts — are the focus of this study. *Skills* are defined to include "any well-established multi-step procedure, whether it involves symbolic expressions, or geometric figures, or neither" (Bell et al., 1983, p. 78). The essential features of skills include actions or transformations that are connected in a linear fashion. *Conceptual understanding* describes "knowledge that

is rich in relationships...[where] all pieces of information are linked to some network" (Hiebert & Lefevre, 1986, p. 3).

The proposed study seeks to explore Calculus & *Mathematica* students' conceptual and procedural understanding of calculus applied to engineering mechanics problems. Shumway (1982) proposes that problem solving can really be investigated by looking at the conceptual and procedural knowledge involved. He observed that the goal of problem solving is to identify a class of problems that can be solved in a similar way. But the process of identifying a class involves conceptual knowledge, whereas determining and carrying out a procedure involves procedural knowledge. So what is really happening during problem solving is that the solver is using conceptual knowledge to reduce a problem to one that can be solved using procedural knowledge. "One could argue that problem solving ends and concept learning begins when one begins looking back, identifying similar problems, and engaging in other post-solution activities" (Shumway, 1982, p. 134).

Silver (1986) believes that it is important not to focus on the distinctions between conceptual and procedural knowledge, but rather to focus on the *relationship* between the two types of knowledge, since problem solving in reasonably complex knowledge domains involves the application of both. Silver suggests we consider the idea that procedural knowledge that is not connected to conceptual knowledge is rather restricted knowledge (Silver, 1981). Thus, a study of the linkages between the two types of knowledge is advised when investigating problem solving.

Mayer and Greeno (1972) found that students who are taught using a conceptual focus produce learning outcomes that are qualitatively different than those produced by students taught with a procedural focus. Their belief is that a conceptual focus encourages the development of a cognitive structure which is more externally connected, or related to other elements in the general structure. This type of cognitive structure would be more useful when faced with problems that may not be familiar to the student. Furthermore, Mayer (1974) examined the resilience of an initial acquired structure and found it to be resistant to change, noting that "an assimilative set is evoked quite early in learning and that content material is structured within the context of the set over the entire course of learning" (p.655). This finding suggests that students who initially learn conceptually will continue to structure new material in the same manner, forming a more externally connected cognitive structure.

This theory correlates closely with the goals of the Calculus & *Mathematica* sequence, one of which is to promote a conceptual emphasis on the process of problem solving. The framework leads to the hypothesis that Calculus & *Mathematica* students will be better able than traditional calculus students to structure new material from engineering mechanics in a conceptual manner, and will have developed stronger links between conceptual and procedural knowledge.

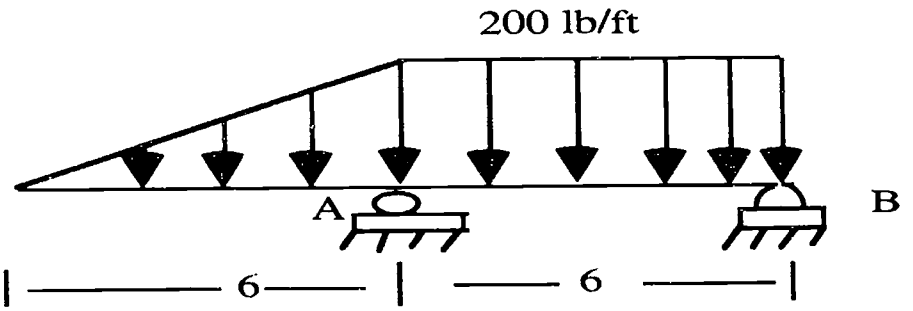
Methodology

Seven engineering mechanics students were chosen to participate in task-based interviews designed to investigate students' use of calculus in their mechanics

course. Three of these students have completed the calculus sequence Calculus & *Mathematica*. Three students have completed the traditional calculus sequence, which employs a lecture-recitation format without the use of technology. The other student has completed an honors section of the traditional course. Two factors were taken into consideration when choosing the engineering mechanics course. Most importantly, the course had to include calculus as one of the prerequisites. This consideration was made to ensure that students had encountered calculus previously and were not learning it for the first time in this course. Secondly, the course had to be one that many students from Calculus & *Mathematica* take. Since a great number of Calculus & *Mathematica* students are engineering majors, the focus was placed on courses that are required for all engineering majors. The engineering mechanics course is an introductory study of statics and mechanics which has a prerequisite of at least three quarters of calculus and one quarter of physics. The main use of calculus in this course is with concepts of differentiation and integration.

Data Analysis and Results

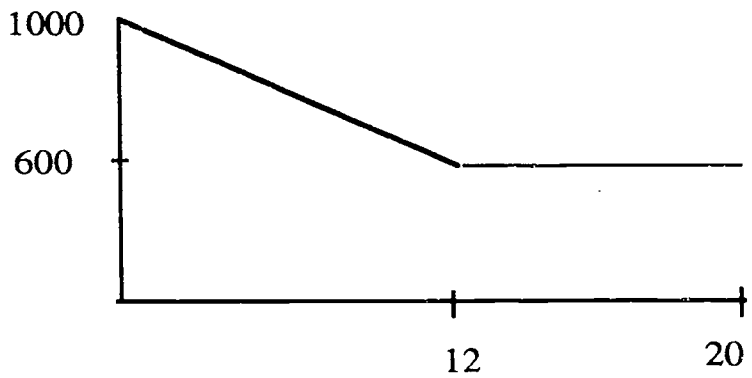
Data from the task-based interviews were used to investigate how Calculus & *Mathematica* students compare with traditional students on their procedural and conceptual understanding of calculus as evidenced by their ability to solve problems in an engineering mechanics course. Several of the problems presented to the students could be solved using either a procedural or conceptual approach. The greatest differences in approach were found in a problem which asked students to sketch shear and moment diagrams for the following load on a beam:



The knowledge necessary for this problem is that the antiderivative of the load function is the negative of the shear function, and the antiderivative of the shear function is the moment function. Workable approaches are 1) to make cuts at key points of the load diagram and use equations of equilibrium to find the shear and moment equations, 2) find the equation of the load and integrate, or 3) use the concepts of slope and area to sketch the shear and moment diagrams from the given load diagram. The first approach does not require knowledge of calculus, while the other two approaches do. All of the Calculus & *Mathematica* students initially approached this problem in a conceptual manner, using their knowledge of slopes and areas in relation to functions and their antiderivatives. If the need

arose, various computations would be used as a supporting method. When asked whether they could confirm their solution in another way, each of the Calculus & *Mathematica* students responded with the procedural approach, finding the equation of the load and integrating. In sharp contrast, the preferred method of the traditional students was procedural. These students employed either the cut method involving equations of equilibrium or the integration method. Some use was made of the concepts of slope and area, but the prevailing method was procedural in nature.

One problem related to shear and moment diagrams was designed to assess students' conceptual knowledge of the relationship between shear and moment. Given the following sketch of the shear diagram and an initial moment value of -15,600, students were asked to sketch the moment diagram. Students were not asked to find the moment functions, but were required to sketch the general shape and to locate the values of the moment for x = 12 and x = 20.



Since the shear function is the derivative of the moment function, students could find the change in moment by determining the area underneath the shear curve. Again, all Calculus & *Mathematica* students approached this problem conceptually and solve it with ease. One student explains his approach: "The shear is the derivative of the moment so it would start at -15,600 and increase to a certain point. The area of this (shear) would be what it (moment) increases to." When asked how he knew the value of the moment was always increasing for x ranging from 0 to 20, he replied, "for it to come back down somewhere this would have to be negative....Since the area is all positive this bottom line will always be increasing." This same student, however, made an initial conjecture that the value of the moment for x = 20 would be 0, because "'it always ends at 0." This belief seemed to be prevalent among the students interviewed. Upon completion of his solution he changed his response.

One of the traditional students (who completed the honors section of calculus) solved the problem conceptually. He explained, " you can just find the areas under the shear diagram and add it to the moments as you go along. So M(12) would be M(0) + Area 1; M(20) = M(0) + Area 1 + Area 2." (Area 1 is the area for x = 0 to 12; Area 2 is the area for x = 12 to 20.) Two traditional students attempted

to find the shear functions and integrate, but both made mistakes and were unable to arrive at the correct answer. Of these two students, one made a small integration mistake, and could not suggest any other way to check his work. The other student insisted that the moment always ends at 0 and neglected to consider the value of the moment at $x = 12$. The fourth traditional student made several different attempts yet failed to arrive at a reasonable answer.

Several problems addressed knowledge of specific procedures. One of the problems that addressed procedural knowledge involved finding the x-coordinate of the centroid of the area bounded by $y^2 = 2x$, $x = 3$, and $y = 0$. All four of the traditional students were able to solve this problem. They remembered the formula and were able to perform the integration without assistance. One student, who was having some difficulty responding to some of the earlier questions, expressed his confidence with this particular task. "Oh, yes, this I can do," he said. The three Calculus & *Mathematica* students were also able to solve the problem. One of the students integrated incorrectly, but corrected himself when asked to check his work.

This integration problem is representative of the difficulty level required for the engineering mechanics course. In fact, students were instructed to deal with more challenging integration by using an integral table. None of the students, traditional or Calculus & *Mathematica*, felt uneasy with the differentiation and integration skills required.

Conclusions

Results from interviews show a distinct difference in approaches to solving engineering mechanics problems that involve calculus. Calculus & *Mathematica* students, who learned calculus with a conceptual emphasis, were found to be more likely to solve problems from a conceptual viewpoint than were the traditional students, who were more likely to focus on procedures. These results are consistent with Mayer's (1974) finding that a student's initial cognitive structure is resistant to change. Furthermore, students who expressed the most confidence in their solution were found to have used a combination of conceptual and procedural knowledge. Calculus & *Mathematica* students demonstrated a stronger ability to discuss all aspects of a problem, including both conceptual and procedural issues, while traditional students expressed more uncertainty in their work and were less comfortable in discussions as to how to use other knowledge to check their solutions.

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