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ABSTRACT

Modeling, simulation, and discrete mathematics have all been identified by professional mathematics education organizations as important areas for secondary school study. This classroom study focused on the components and tools for modeling and how students use these tools to construct their understanding of contextual problems in the content area of vectors and forces. The study was conducted in an integrated algebra, trigonometry, and physics class at an alternative public school with 17 students in grades 9 through 12. Computer models used by students were Interactive Physics and Function Probe. Results showed the emergence of four major themes related to student model building: (1) diversity in student approaches, (2) challenging of conventional notions of closure and completeness, (3) integration of the simulation environment as access to an expert's model, and (4) progressive complexity in student models. Contains 35 references. (MKR)

An Integrated Approach to Mathematical Modeling: A Classroom Study

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A paper presented at the Annual Meeting of the American Educational Research Association, April 18, 1995, San Francisco, CA.

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Introduction

Modeling, simulation and discrete mathematics have all been identified by the National Council of Teachers of Mathematics (1989), by the Mathematical Sciences Education Board [MSEB] (1989, 1990), and by other professional mathematics education organizations as important areas for secondary school study. As is noted by the MSEB (1989), the interplay between mathematics and computer technology opens up new areas for investigation and study: "Mathematics provides abstract models for natural phenomena as well as algorithms for implementing these models in computer languages. Applications, computers and mathematics form a tightly coupled system producing results never before possible and ideas never before imagined" (p. 36). This classroom study focuses on the components and the tools for modeling and on how students use these tools to construct their understandings of contextual problems in the content area of vectors and forces.

The specific content area selected for this study is the motion of an object down an inclined plane. Students' beliefs about force and motion are not only largely incompatible with Newtonian concepts, but are also resistant to change through conventional physics instruction (Brown & Clement, 1989; Hestenes, Wells, & Swackhamer, 1992; McCloskey, 1983; Niedderer, Schecker, & Bethge, 1991; Viennot, 1979). Thus far, the major application of computer technology that has emerged from that body of research on students' beliefs is the use of microcomputer based laboratories to enhance students' conceptual understanding of physics (Laws, 1991; Thornton, 1987). This study elucidates ways that the established research tradition in physics education dealing with student conceptions can be linked to the potentially powerful possibilities of student-based modeling activities, and seeks to further the development of a theory of modeling, informed by the analysis of the practice in a particular classroom setting.

Typical use of computer modeling in the secondary curriculum involves the manipulation of a previously built model (an expert's model) within some set of parameters. What is less well understood is the potential effectiveness of engaging students in the actual process of building models using computer-based tools. In this study, three significant components of the modeling process are explored: the action of building representations and relationships from physical phenomena, the use of a simulation environment to explore conjectures, and the iterative process of developing and validating a solution through the use of a multi-representational analytic t[illegible]ning with a physical setting, students gather data and conjecture potential relationships between quantities. Through systematic inquiry and the coordination of multiple representations including those in a simulation environment, the students explore, refine and validate solutions to the posed problems. This study is designed to understand how these components are interrelated.

This paper presents the theoretical framework for the model building approach that was implemented and examined in this study, with particular attention to the components and tools of the modeling process and their interrelationships. The research methodology, the classroom setting, the pedagogical strategy and the curricular design are described. Four major themes related to student model building emerged from the results of this study; each theme is discussed in turn. Finally, some of the implications of these emergent themes for instruction and curriculum are discussed.

Theoretical Framework

The distinction between model *exploration* (or running a pre-built model) and model *building* has been identified by many researchers (Clauset, Rawley, & Bodeker, 1987; Moar et al., 1992; Roberts & Barclay, 1988; Webb & Hassell, 1988; Whitfield, 1988). This distinction has also been described as the difference between exploratory models and expressive models (Bliss & Ogborn, 1989; Bliss et al., 1992). Exploratory models are those models which are constructed by experts to represent knowledge in some content domain. Learners typically explore consequences of their actions within the boundaries of these content domain models. These models are in essence microworlds that provide the student with a set of simulated, idealized worlds that embody, for example, the Newtonian laws of motion while allowing the student to explore the consequences of changes in the simulation's parameters. The Alternative Reality Kit (Hennessy et al., 1990; Moar et al., 1992), the dynaturtle environment (diSessa, 1982), the NEWTON microworld (Teodoro, 1992), and the ThinkerTools simulation (White & Horwitz, 1987) are examples of this type of model. Such exploratory models provide a way of asking if learners can understand an expert's way of thinking about a problem.

Model building (or expressive models), on the other hand, provides learners with the opportunity to express their own concepts and to learn through the process of representing their concepts, defining relationships, and exploring the consequences of those relationships. Tools such as spreadsheets, Function Probe, Matlab, STELLA and Interactive Physics are examples of the kinds of software that can be used in expressing and developing student-conceived models of physical phenomena. Such expressive models provide a way of asking if learners can understand their own way of thinking about a problem. This is an important shift in perspective from the activity of exploring a pre-built model, which necessarily embodies the concepts and structures of an expert. As Coon (1988) points out, the process of model building forces students to make explicit their own ideas about the relationships among variables and to examine the consequences of their ideas. The Computers in the Curriculum Project at King's College, London, developed some 150 computer simulations to foster guided discovery learning (or exploration). As a result of that experience, the researchers concluded that "students, too, would learn more or understand better if they researched and developed their own computer models" (Riley, 1990, p. 255). Other researchers (Feurzeig, 1988; Roth, 1992; Tinker, 1993; Webb & Hassell, 1988) similarly argue that model building is a potentially powerful way for students to act and reflect on contextual problems.

There is some research evidence that suggests the potential for a modeling approach that begins with simple concepts and progressively builds on them. Clement (1993) claims that computer modeling can provide a bridge between "anchoring concepts," those concepts about which students have strong, correct intuitions, and the concepts of Newtonian physics. Linn, diSessa, Pea, and Songer (1994) argue that a modeling approach to instruction should build on the sense-making efforts and first-hand observations of students. The notion that a model is built over time, beginning with the commonsense knowledge of students, is examined by Roschelle (1991) who claims that the development of students' conceptual knowledge should be viewed as "achieving scientific understanding through incremental reformulation of commonsense knowledge" (p. 3). These results suggest a model building process that begins with first-hand, simple events and increases in complexity in order to explain other data or to include more variables.

A theoretical framework for modeling must also include the components and the supporting tools of the modeling process and the nature of the relationships among the component activities. In many classroom problem-solving activities, students move from a physical phenomenon to some sort of a mental or conceptual model of that phenomenon to a mathematical representation. The mathematical representation is generally limited to algebraic equations with a single right answer. This highly linear problem-solving heuristic is often unduly limited to the symbolic representations of algebra and the subsequent manipulation of those symbols. The modeling process is often described as iterations of this linear problem-solving approach: understand the particular phenomenon to be modeled; define the context and constraints; identify the key variables; explicitly define the relationships among the variables; translate those relationships to an appropriate computer implementation; analyze and interpret the results; and then refine the model and one's understanding through an iterative process by repeating the above steps (Edwards & Hansom, 1989).

An alternative to this linear approach is that proposed by Bell (1993) as a basis for a K-6 mathematics curriculum, developed at the University of Chicago, that is structured around the "nodes" of a modeling paradigm. The diagram in Figure 1 showing the nodes of the modeling process is based on a variant of Bell's (1993, p. 4) paradigm and that given by the National Council of Teachers of Mathematics (1989, p. 138). Bell (1993) observes that in the modeling process one might "visit any or all of the nodes in this diagram, in no particular order and often with considerable bouncing around and recycling -- one may go anywhere in the diagram from anywhere" (p. 4). Bell argues that the mathematics curriculum should provide ample experiences for students at each of the nodes and that the process of modeling involves many moves between the various nodes.

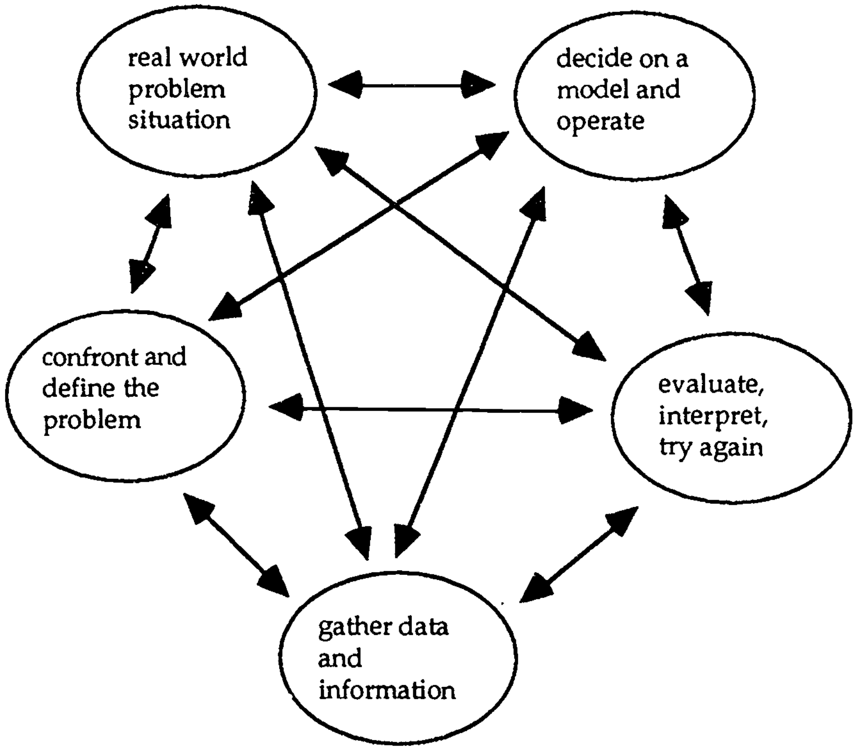

Figure 1 The Nodes of the Modeling Process

In an earlier study, Lesh, Surber, and Zawojewski (1983) also reject the linearity of the Polya-type problem-solving stages that proceed in a uni-directional process from givens to goals. These researchers argue for a non-linear progression through different phases of the modeling process: interpretation, integration/differentiation, and verification. They observed that in their study students spent an overwhelming amount of time in the first phase, refining their understandings about the problem. They go on to argue, however, that these phases do not necessarily occur in any given order and that within each phase students map their perceptions to their cognitive models, transforming their models and mapping back to the perceived problem situation.

This classroom case study examined a model building process based on three key components: experimentation with physical phenomena, the exploration of possible alternatives through a simulation environment, and the iterative process of developing a solution with a multi-representational analysis tool. These components of the modeling process are represented in Figure 2.

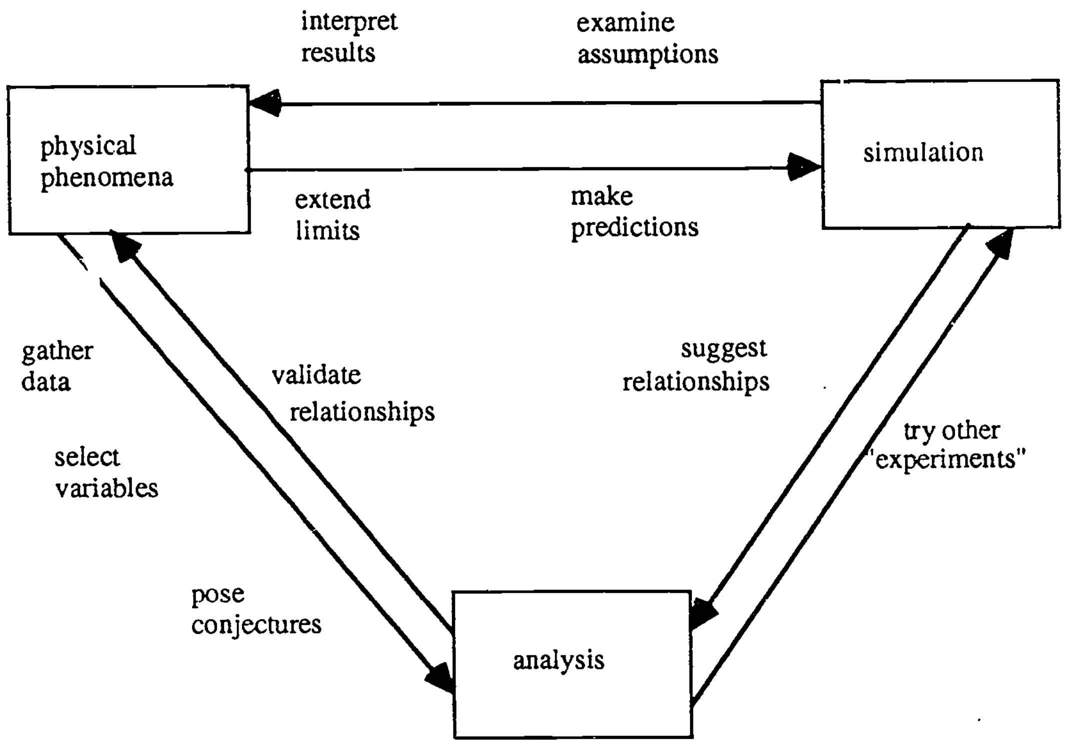

Figure 2 A Comprehensive Modeling Process

Beginning with an experiment, students must make explicit their own representations of a physical event and must choose variables and pose relationships among them. The simulation environment supports the exploration of possible alternative conjectures and the use of multiple representations. Approximate solutions are developed and refined through the use of a multi-representational analysis tool and through critical evaluation and reflection by the students. This process of iteration is not necessarily linear, but rather students spend time at each of the nodes and move back and forth between the nodes as they develop their understanding. Thus, a model is not a solution to a given problem but rather a developing tool that a student can use and re-use to find solutions.

Methodology

This study was conducted in an integrated algebra, trigonometry, and physics class at an alternative public school with 17 students in grades nine through twelve. The class was team taught by an experienced mathematics teacher and an experienced physics teacher, who were familiar with the computer technology. The overall curricular unit was designed to integrate three components: the gathering of data from a physical experiment, the development and exploration of a computer simulation, and the mathematical (symbolic, graphical, tabular and geometric) analysis of the data. The second and third components were supported through Interactive Physics© (Baszucki, 1992) and Function Probe© (Confrey, 1992), respectively. The study was designed to investigate student activities taking place at three levels: the whole class, small groups within the class, and at an individual level. The instructional approach emphasized small group learning and interactions. Hence, the primary focus of the analysis was on one small group selected from within the larger classroom setting. The activities of the focus group were observed and analyzed as the students interpreted the posed problems, generated and negotiated their conjectures, used varying strategies for analyzing the data and confirming the sense of one or more conjectures, interacted in their investigation of the posed problems, and used the tools and data. Four indiyidual students were selected for teaching interviews. This provided an opportunity to explore individual student understandings and problem-solving approaches as they solved contextual problems using physical apparatus and data, simulation tools, and analytic tools.

To motivate and guide the model building, an essential question was posed for the unit: "How will an object behave if it is traveling down an inclined plane? How can you predict the behavior of such an object for any randomly chosen angle of incline?" To answer this question, the students investigated four major sub-problems: (1) the resolution of a vector into its horizontal and vertical components; (2) the effect of multiple forces acting on an object on an inclined plane; (3) the relationship between force, mass, and acceleration; and (4) the role of friction as it affects the motion of an object down an inclined plane. These four sub-problems were then integrated to answer the essential question as the students analyzed the behavior of an object moving down an inclined plane.

For each of the four sub-problems, there was a corresponding experimental set-up. The physical experiments were done with basic equipment and everyday objects: strings and pulleys, spring scales, weights, protractors, and strip timers. The experiments and the related questions for investigation were:

(1) A five-Newton weight was attached to a pulley on a string. The string was attached to a fixed stand at one end and a spring scale at the other. The students were asked to investigate the relationships among the weight, the angle of pull on the string, and the magnitude of the force recorded on the spring scale.

(2) A known weight was suspended just above an inclined plane set at an angle which could be varied from 0 to 90 degrees. Using two spring scales and some ropes, the force parallel to the plane and the force perpendicular to the plane were measured for various angles. Again, the students were asked to investigate the relationships among the two measured forces, the angle and the weight.

(3) A frictionless cart of known mass experienced an acceleration that could be varied by increasing the weight of sand in a coffee can that was

suspended by a string attached to the cart over a pulley at the end of a board. A strip of paper at the other end of the cart passed through a timer that generated tock marks on the paper at a fixed frequency. The students were asked to conjecture a relationship between the force as measured by the weight of sand and the can and the acceleration of the cart, as calculated from the second differences with the strip timer data.

(4) Using a spring scale, a block of known weight was pulled at a constant velocity across a given surface. The weight of the block and the velocity were varied; then, for a given block, the angle of inclination of the surface was varied and the force for pulling the block was measured. The students were asked to qualitatively describe the force that was being measured and from that to find a relationship between the force of friction and the angle of incline.

For the first and third experiments, the data was collected by each small group within the classroom. For the other experiments, the data was collected in the whole class setting. Following the data collection and discussion of the posed questions, the students were free to use the analysis tool and the simulation environment, as they chose.

The analysis tool, Function Probe, is an interactive, multi-representational software package designed to foster the investigation of functional relationships through the coordinated use of tables, graphs, and a calculator. As such, it was an ideal tool for encouraging the expression of student-generated relationships among variables and for providing a flexible and powerful environment for investigating alternative relationships and possible conjectures.

Interactive Physics is a simulation environment that allows the student to create a wide variety of kinematics experiments. The user interface consists of a palette of tools that allows the student to create a variety of mass shapes, connected by springs, ropes, and pulley systems. The physical characteristics of an object, such as position, initial velocity, and coefficient of friction, can be directly controlled. The environment provided the students with an extensive tool kit with which they could create any number of experiments to run in the simulated environment.

Data Sources

In order to develop an understanding of the model building processes in which the focus group of the study engaged, data were gathered at multiple levels and from multiple sources. The three levels for the data sources were the whole class, the small focus group, and four individuals within the class. Data were gathered from the following sources: (1) video and audio tapes of all class sessions; (2) video and audio tapes of all activities of the focus group; (3) observation notes taken during classes, including small group work; (4) copies of the computer work of all the small groups; (5) copies of student final assessments; (6) pre-test and post-test results for the curricular unit; (7) video and audio tapes of individual student interviews and (8) copies of student pencil and paper work and computer work during the individual interviews.

Results

Four major themes related to student model building emerged from the results of this study. First, students pursued problems with far more diversity in approaches than the problem itself might have initially suggested. The first sub-problem began with an inquiry into the relationship between the magnitude of a force pulling at a varying angle and a known weight suspended on a pulley. Through both small group and whole class discussion, the students decided that they needed to gather evidence for the existence and magnitude of a horizontal force. Each of the five small groups within the class, including the focus group of this study, elected to use the simulation environment to pursue the investigation of this question. The simulation environment provided the students with a flexible set of tools with which they could build a representation of the physical event that made sense to them. The groups took four distinct approaches to making sense of a this event; see Figure 3, for example, for simulation devised by the focus group. The analysis of these contrasting approaches has been reported elsewhere (Doerr & Confrey, 1994). This variation in student representation of phenomena is in sharp contrast to the approach of structured series of microworlds.

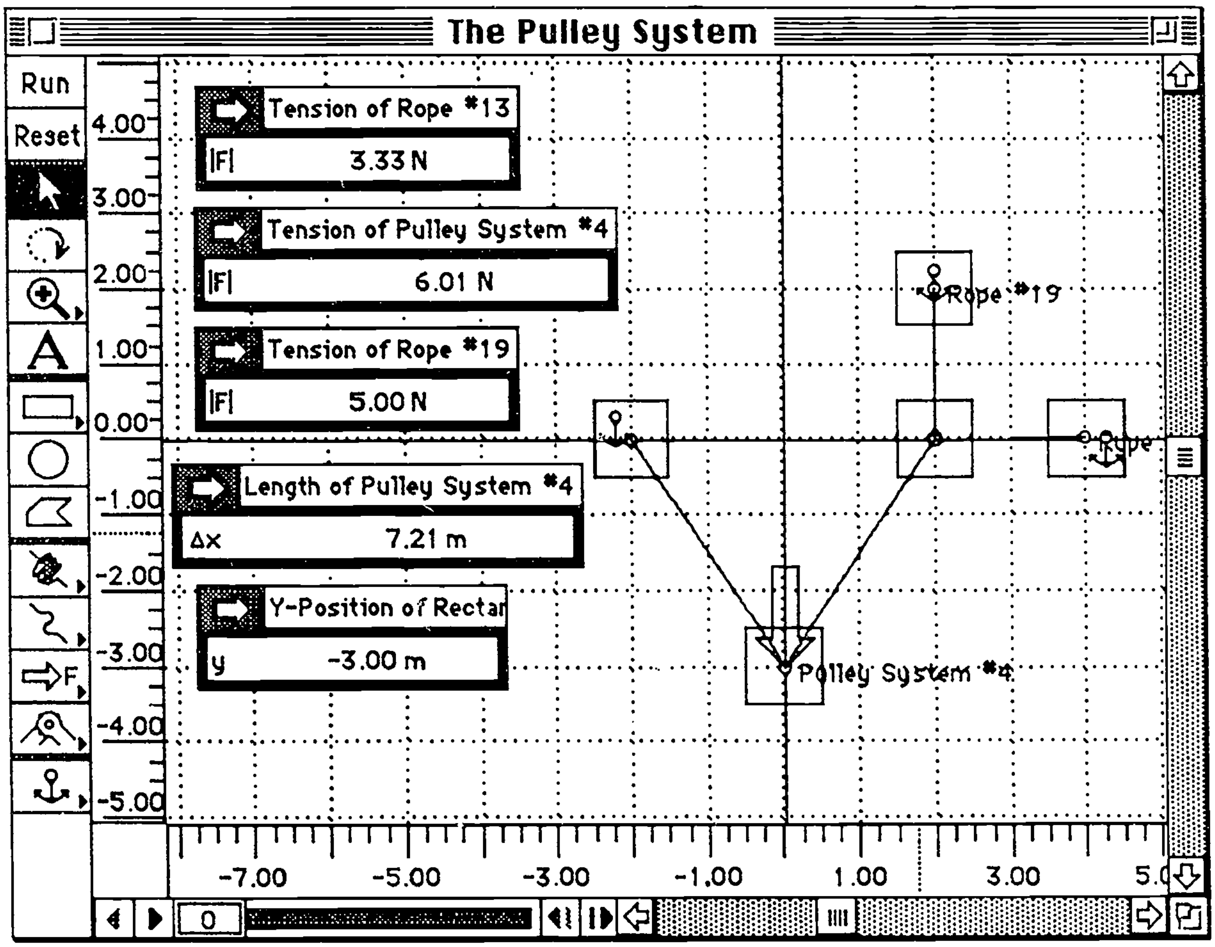

Figure 3. A Simulation of a Weight on a Pulley

These results suggest that an environment that provides students with a palette of tools to use will provide an opportunity for many solution paths for many different students. The grounding of the investigation in an inquiry about a physical event was the crucial starting point. Beginning with a hands-on event, the students generated their own representations of the phenomena at hand. Each group carefuliy verified that their simulation model gave results that they were convinced were true in the physical world: that at a 45 degree angle, the vertical and horizontal components of a force must be equal. The representations that they chose were many and varied; moreover, the students were able to argue persuasively how their representations were related to the physical phenomena under investigation. This diversity in representation was a reflection of the diverse understandings and approaches that the students brought to the inquiry.

A second theme that emerged from this study is a challenge to conventional notions of closure and completeness. During the second sub-problem, the focus group attempted to find a relationship between the normal force, the weight of an object and the component of the weight that is parallel to the inclined plane (see Figure 4).

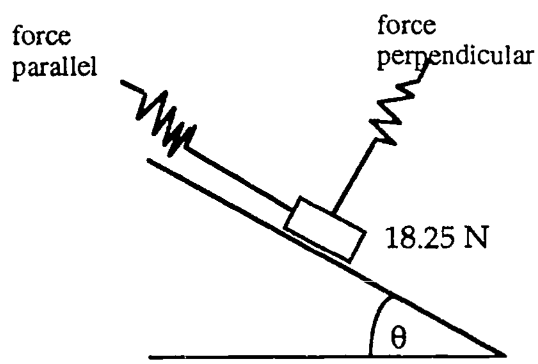

Figure 4 Forces Acting Along an Incline

The focus group had already established that a force acting at an angle can be thought of as having a vertical and a horizontal component and that these components are related trigonometrically. Using two spring scales and an object of a given weight, the force parallel and the force perpendicular were measured for various settings of the angle theta between zero and 90 degrees. The students entered this data into a Function Probe table (see Figure 5) and created two graphs for force versus angle.

Table

a	p	P
angle	parallel	perp.
0.00	0.00	18.25
10.00	3.75	18.00
20.00	7.25	17.25
30.00	9.25	15.75
40.00	11.00	14.00
45.00	12.75	13.00
50.00	13.75	12.00
60.00	15.75	10.25
72.00	17.50	6.50
82.00	18.10	4.50
90.00	18.25	0.00

Figure 5 Experimental Data

At this point, the focus group turned their attention to the geometry of the situation and began to analyze the role of the vertical and horizontal components of the parallel and perpendicular forces. They clearly established a qualitative argument that the sum of the vertical components of the parallel and perpendicular forces must equal the weight of the object. However, they were unable to successfully create a symbolic expression in the table window or to make sense of that expression in terms of the data that they had collected. It is tempting to suggest that this would be an ideal point for the teacher to intervene with the group of students and give them some of the next steps that would allow this path to be brought to closure. However, if we are to take seriously the goal of independent problem-solving skills by students, then the ability to persist independently in exploring alternative solutions will need to be more highly valued in classrooms.

Later in this episode, the teacher directed the students' attention back to their original graphs and pointed out that they could read the relationship between the angle and the force from th~ graph. The students then algebraically fit a sine and a cosine curve to their data and, at the very close of class, established that the magnitude of the perpendicular force is given by the weight of the object times the cosine of the angle of incline, and similarly for the parallel force. However, the group did not bring any closure to the central idea that *they* were working on, namely, that the sum of the vertical components of the perpendicular force and the parallel force must equal the weight of the object.

When class began the next day, the teacher pointed out to the students that the quantitative relationships that they had established were created on the basis of fitting a curve to empirical data and that they didn't have a visual, pictorial argument to support the fact that they came up with sines and cosines. The teacher then directed the students to develop a convincing argument for the relationships of the forces acting on an object on an inclined plane based on the geometry of the force diagram. The focus group was able to swiftly use the geometric basis of their previous analysis to convince themselves that the cosine relationship from the curve fit must also hold from the geometry of the force diagram.

The evidence from the second sub-problem suggests that the time spent on the partial and incomplete model had tremendous pay-off in the subsequent lesson. The partial analysis that the students spent a great deal of time on, without coming to any closure, was exactly the right groundwork for a convincing geometric argument for *why* the graph of the empirical relationship that they found was trigonometric. The closure achieved by the focus group tightly wove their reasoning about the physical situation, a graphical and an algebraic representation of data, and an argument from geometry into a coherent whole. This process of building a model, therefore, allows for the pieces of the model to be brought into place over time and implies that, along the way, students are likely to have partial and incomplete models.

The third theme that emerged from this study was the integration of the simulation environment as access to an expert's model which could be used as the students built their own model of the phenomena being investigated. Thus, rather than polarizing the use of expert models with student-built models, the approach in this study was to use the expert model to help the students in building a small piece of their own model. When the analysis of the data from their experimental set-up did not yield the precise analytic relationship that they needed, the students turned to the simulation environment. The small groups used Interactive Physics to apply known forces to a given mass. The focus group visually observed the acceleration that the mass experienced and noted the recording given by the acceleration "meter." They were able to run through a series of simulated experiments fairly quickly. They systematically selected a range of forces that allowed them to conclude that as the force doubled, the acceleration doubled; as they divided the force by five, the acceleration was divided by five. They did not initially translate this proportional reasoning into a symbolic or a graphical relationship among force, mass and acceleration.

In the next lesson, the teacher posed the following question: given that they had already established F=ma, "With a given force acting on a variety of masses, what would be the relationship between those masses and the accelerations that they experience?" The group began by using their simulation from the previous day, setting a constant force of 15 newtons and then systematically varying the mass across a wide range of values and observing the resulting acceleration. From this, they established a table of values for force, mass and acceleration and they were then convinced that indeed F=ma. One student in the group later pointed out in the discussion that what they had really used was F/m=a but that this was equivalent.

The first use of the simulation environment in this episode provided a mechanism for the students to access the expert knowledge (i.e. the Newtonian laws of physics) that was built into an existing environment. The group used the simulation environment to set up neater and cleaner experiments than they were able to in the classroom. They were also able to run many more trials of the experiment. I believe that a key point to recognize is that the experimental set-up in the simulation environment was motivated by the messiness of the experimental data that they had collected and then analyzed through tables and graphs in the Function Probe environment. The class turned to the simulation environment when the analysis of their experimental results was inconclusive. However, unlike the first sub-problem, where each group used the tools of the environment to express its own representation of the phenomena being investigated, in this situation, the groups used the simulation environment in an information-gathering approach. The students were initially inclined to create a more complex simulation that would be analogous to the experiment that they did in class. However, at the suggestion of the teachers, they were encouraged to simplify their simulations as much as possible. This approach then became similar to looking up the law or formula in a textbook, with the simulation environment having the advantage of dynamic representations and multiple trials. The teachers pointed out that in their previous class work they had been careful to validate that the simulation environment yielded results that were consistent with experimental data, and thus they could have confidence in the results from a simulation. However, unlike in the first sub-problem, in this case, the gap between their experimental set-up with the carts and the simulation experiment was considerable. Thus, it is less clear that the students were convinced that the results from the simulation represented the same quantitative relationships they were seeking in their experimental set-up.

The next time the focus group used the simulation environment, however, the whole class had already accepted the equation F=ma. This second use of the simulation provided the students with an alternative way of generating a data table. In addition to providing the numeric values from the display meters, the simulation also provided an animated display of the information: the mass objects accelerated across the screen as forces were applied. This display supported their proportional reasoning about the event (i.e. for a given force, when the mass increased, the acceleration decreased) and through the construction of a data table, they confirmed the relationship that F/m=a. This use of the expert's model followed the validation of the results of that environment by comparison with an experiment for which the group had collected data. The expert model allowed them to continue to make progress in the larger development of their own overall model of the phenomena under investigation.

The fourth theme is that of progressive complexity in the student model, though not necessarily completeness or closure, but rather a structure that was built over an extended period of time and was well grounded in physical phenomena. Over the course of the unit, the focus group went through a process of developing various components of the overall model, integrating them with previous components and extending them to account for more complex situations. At the same time, throughout the process, the students refined and modified their conjectures, as well as validated components of their model with physical experimentation. The process began with an investigation of the relationships among the horizontal and vertical components of a force acting at an angle. The force component relationships were then used to analyze the force relationships that exist when three forces act on a stationary object.

However, in order to address the essential question, the students needed to transfer their understanding and analysis of vertical and horizontal force components to the situation where an object is stationary on an inclined plane. That is to say, the frame of reference becomes rotated through an arbitrary angle of inclination. The focus group unsuccessfully attempted to establish an algebraic relationship for the parallel and perpendicular (normal) components of the force due to the weight of an object at rest on an inclined plane. However, building on the results of discussion of the whole class, and their own graphical representations of the data they had collected, the group developed a compelling geometric argument to explain why the relationship between the parallel and perpendicular (normal) forces and the weight was trigonometric.

The students next needed to understand how objects behave when forces are applied. They began their analysis by examining the behavior of objects on a horizontal, frictionless plane as a force was applied to an object of known mass. The focus group ultimately confirmed their tentative conjecture about the data through the use of the simulation environment. The focus group then extended their notion of equilibrium, previously limited to stationary objects, to include objects moving at constant velocity. Following this, they quickly established the relationship between the force of friction and the weight of an object, for an object moving at constant velocity on a horizontal surface. A whole class discussion revealed many of the assumptions about their model of friction. The focus group then had to integrate their previous analysis of the force components on the inclined plane with their new understanding of friction on a horizontal surface to account for the data they collected for an object moving at constant velocity up an inclined plane at various angles.

This progressive process simultaneously builds both a practical and a theoretical understanding of the behavior of an object on an inclined plane. Thus, rather than starting with a posed problem to which physical laws expressed as abstract principles are to be applied through the constraint equations of motion, this modeling process simultaneously intertwined the construction of the appropriate laws as they are needed, integrated with existing student knowledge, and the solution of particular posed problems.

The students moved from an initial event of a force acting at an angle to an integrated model of the motion of an object moving up or down an inclined plane with or without friction, at any arbitrary angle. At the end of the unit, their model incorporated an object of any mass, any initial velocity, any coefficient of friction, and any angle of incline. This is represented in Figure 6.

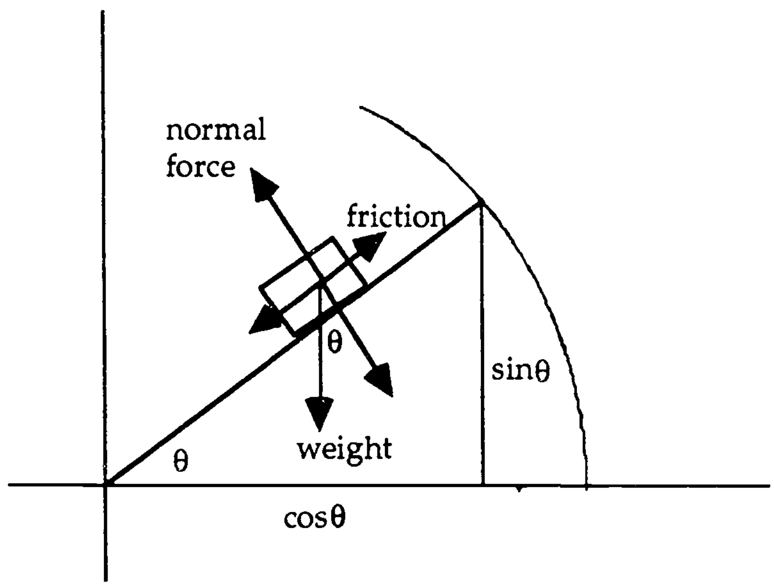

Figure 6 A Model of Forces Acting on an Object on an Inclined Plane at an Arbitrary Angle

The compactness and elegance of this model was very useful to the students as they applied it to a variety of situations in the final group work done at the end of the unit. For a given set of conditions, the students were able to determine the appropriate component forces, perform the relevant vector additions, and describe the overall behavior of the object. In solving these final problems, the students relied heavily on algebraic equations, geometry, and qualitative arguments about force vectors to develop their solutions. The students did make use of the analytic tool (Function Probe) in solving a final problem that began with a set of data rather than with the boundary conditions for the equations for linear motion. The students did not use the simulation environment as a mechanism for finding, validating or expanding the solutions to the final problems at the end of the unit. This, however, was not entirely surprising. Throughout the unit, the teachers both encouraged and used the simulation environment as a way of finding and validating solutions to problems, but the solutions to the final problems of the unit were addressed through algebraic equations and geometry. Thus, the students did not use the simulation environment as a way of expanding or exploring the multiple possibilities within the final problems, but rather, they used the compactness of the algebraic equations and the geometry to develop their solutions.

Implications for Instruction

Engaging students in a model building process is likely to foster a diversity, creativity, and richness of responses far beyond that encountered in most mathematics classrooms, especially those where a premium is placed on following a correct mathematical procedure. This classroom environment was one which actively encouraged students to pursue their own line of reasoning in investigating a problem; diversity of approaches was valued by both the teachers and the students. Nonetheless, managing such diversity in a classroom setting while fostering coherence and progress is not a trivial task. This study showed the potential effectiveness of integrating a curriculum structured around an essential question with an instructional strategy that blends whole class discussion with small group investigations. The small group provided the setting wherein students generated hypotheses, tested alternative conjectures, examined multiple representations, created their own representations and experiments, and developed and validated relationships among variables. The open-ended inquiries were supported by a suite of tools for building simulations and probing functional relationships. The whole class discussion framed the question for the student inquiry and provided a forum where the varying results and approaches of the small groups were shared and where conclusions were summarized and crystallized. Thus, this blend of whole class and small group activity served to focus the students on the questions and bring them back to the essential problem while fostering and encouraging diversity and student autonomy.

The technology tools available to the students provided and supported many opportunities for diverse approaches to problems. This diversity, however, does bring with it the challenge inherent in the difficulty of identifying and exploring the unanticipated mathematics as it is uncovered through student activities. For example, the approach taken by the focus group in finding the relationship between the vertical and horizontal components and the given force (investigated in the first sub-problem) suggests that further work with explicitly comparing the relationships between data columns might have been useful. Similarly, the relationship between the measured force, the force of friction and the parallel component of the weight (investigated in the fourth sub-problem) suggested a further possible investigation of the nature of the sum of a sine and a cosine curve. Indeed, an examination of this would have led to an alternative confirmation of one student's initial hypothesis about the relationship. However, it should be noted that seeing these opportunities was, at least in part, facilitated by the presence of two teachers and an observer in the room. The teachers in this study took careful note of some such opportunities, with the intention of taking advantage of them in the following year.

The model building approach that was central to this instructional unit provides a significant challenge in understanding how to nurture, accommodate, and respond to the partial and incomplete models that students are likely to build. The partial model built by the focus group in their analysis of the forces acting on a object on an inclined plane laid the groundwork for the geometric explanation about the relationship among the component forces that they did complete later. The focus group appeared to substantially benefit from their earlier work, even though that earlier work did not initially bring them to the same set of relationships as were established through the whole class discussion. Thus, there is some evidence to support the conclusion that students show later benefit from their earlier partial models and hence for confidence in a model building process that takes place over an extended period of time.

However, this does not entirely address the issue. In the first sub-problem (when the focus group investigated the relationships between the angle at which a force was exerted and its component forces) the students had taken their simulation data and constructed an elegant set of relationships in the table window of Function Probe. They did not, however, create expressions or graphs for the sine and cosine relationships for the force components from their data, nor did they go back and confirm those relationships after the results of other small groups had been reported. While the focus group showed high confidence in their simulation model and they were able to express to the whole class the results of the model qualitatively, it is not clear that they recognized the validity and value of their table-based analysis and its relationship to the symbolic relationships given by the other small groups. There are two possible instructional approaches to address this lack of closure: first, if the results had been printed and assembled into a lab report, then the teachers could have confirmed what the students had accomplished. This is likely to be a very time-consuming and possibly difficult task for a teacher, especially in light of analyzing approaches that might be unfamiliar to the teacher. Second, the students could be given the opportunity to go back to their work and assess it in light of the classroom discussion. This suggests a more student-centric, reflective, self-assessing approach that could lead students to identify what they felt worked well in their investigation, what parts were not as useful, and what parts they still needed to make sense of. Research is currently underway (Haarer, in progress) that investigates the potential effectiveness of a built-in recorder of student actions in Function Probe for supporting student reflection and self-assessment. This extension to the software could facilitate the students' creation of lab reports or write-ups of their procedures, results, and their own assessment of the same. More explicit instructional strategies for having the students reflect on and assess their own work as they proceed could serve to validate and value their partial work and possibly help to further their own thinking.

Implications for Curriculum

There are two major implications of the results of this study for curriculum. First, the model building approach examined in this study requires a curriculum that allows for a significant amount of time to be given over to student exploration and the pursuit of student methods. This directly changes the activities that students spend time on. In addition, this challenges the notion of broad content coverage in traditional curricula and suggests that students will acquire problem-solving skills while focusing on in-depth investigations of fewer concepts. The students engaged in this curriculum spent a significant amount of time investigating open-ended questions. The students were encouraged to make conjectures, to investigate alternative relationships, and to argue for relationships based on physical evidence and geometry. The students proceeded without cook-book instructions, spending considerable time discussing and deciding how to proceed to answer a particular question. The students spent little time in class reviewing homework, solving routine textbook exercises, memorizing formulas or solution methods, or listening to lectures.

As the focus group developed its methods for analyzing the component forces, the forces acting on an object on an inclined plane, and the role of friction, the process took considerably longer than if the students had simply been told formulas or given a vector-based analysis. However, it is well established that this latter process does little to change student conceptions of force and motion (Hestenes et al., 1992; Niedderer et al., 1991; Thornton, 1992). The physical experimentation was an important grounding activity from which the students began their inquiry and to which they ultimately returned in making arguments for symbolic and graphical relationships among variables. The technology tools, both the simulation environment and the multi-representational analytic tool, provided support for diverse approaches to the sustained investigation of conjectures and validation of hypotheses. A modeling-based curriculum will need to provide ample time for students to discuss, conjecture and validate, while spending less time on drill and practice, memorization, and lecturing.

The amount of content that was covered in the 35 instructional days for this unit would have been covered in about 10-12 days in a traditional classroom setting. The conventional notions of broad content coverage will have to give way to a re-conceived curriculum that builds on longer-term investigations of fewer concepts and engages students in sustained problem-solving activities. As Lesh (1981) has observed, multi-stage project-type problems are time consuming, but "the time invested in major conceptual models is worthwhile because they contribute to the acquisition of other ideas" (p. 245). The results of this study, shown in the students' achievement on the Force Concept Inventory (Hestenes et al., 1992) reported elsewhere (Doerr, 1994), suggests that the gains in student conceptual knowledge can be significant.

The second implication for curriculum is that questions for a modeling-based curriculum should be central notions in mathematics and/or science while sustaining student interest and inviting exploration. The model building approach in this study centered on the investigation of an essential question that addressed student concepts of force and motion. The modeling was not simply an add-on activity to the curriculum, but rather a more fundamental reformulation of the curriculum that gave primacy to students' construction of content knowledge through an inquiry process of experimentation, simulation, and analysis. The inquiry was guided by an essential question that in turn generated several sub-problems. These questions and a rich set of activities provided a focus for the inquiry and sustained student interest throughout the investigation.

As discussed above, there are several difficulties associated with following multiple paths in a classroom setting. In this respect, the interplay between the small group investigations and the whole class discussion and activities seemed crucial. This interplay brought coherence to the overall model building process and served as the larger framework for enabling a progressively complex model to be built while weaving together the multiple paths chosen by the small groups. There were many partial threads developed along the way. Physical experiments provided powerful beginning points for the modeling activity. The students in the focus group and those involved in the teaching interviews made extensive use of the physical experiment in making sense of their simulations and analysis. The sense-making points, at the boundary conditions and at the midpoints, as well as their qualitative arguments, were grounded in the physical event. Further, the students demonstrated an ability to reason from their representations, whether in the simulation environment, the graph or table windows, the symbolic algebraic expressions, or a vector diagram, back to the original event under investigation.

Seemingly simple problems with familiar everyday apparatus provided rich conceptual territory for students to explore and such exploration was greatly enhanced by technology tools for both simulation and analysis. The essential question for this curriculum unit was a seemingly simple problem that provided rich territory for investigating vector components, the effect of multiple forces on an object, the relationship between force, mass and acceleration, and the role of friction as an object moved down an inclined plane. The simulation environment and the analytic tool provided the students with an extensive palette of tools for constructing their own simulation experiments and for analyzing data from physical experiments and from the simulation. These tools were powerful and flexible in ways that enhanced student investigations of student-generated hypotheses and conjectures.

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