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ABSTRACT

Over half the students who select mathematics as a college major eventually switch to other fields. Although equal numbers of males and females start with mathematics majors, females comprise 43% of those completing the undergraduate degree and 20% of those completing the Ph.D. This paper synthesizes findings from two lines of research to shed light on this pattern of participation and persistence in mathematics. First, grades earned in mathematics by males versus females, and persisters versus switchers are examined. Studies of over 39,000 undergraduates indicate that females earn higher mathematics grades than males. Also, switchers earn grades equal to those of persisters, and some of the most talented males and females switch out of mathematics. Second, interview studies of over 1,500 students interested in mathematics and science from over 20 colleges and universities are examined. In every study, undergraduates complain that mathematics courses are designed to weed out students rather than to encourage the best to persist. Switchers, more than persisters, point out the poor quality of undergraduate mathematics instruction compared to instruction in other courses. Low quality of instruction weighs more heavily than do success factors in students' decisions to switch out of mathematics, and more females than males decide to switch. A serious consequence of the perceived low quality of mathematics instruction is the loss of talented students to other majors. (Contains 122 references.) (Author/MKR)

Participation in Mathematics Courses and Careers: Climate, Grades, and Entrance Examination Scores

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Abstract

Over half the students who select mathematics as a college major switch to other fields. Although equal numbers of males and females start with mathematics majors, females comprise 43% of those completing the undergraduate degree and 20% of those completing the Ph.D. We synthesize finding from two lines of research to shed light on this pattern of participation and persistence in mathematics. First, we examine grades earned by males and females as well as persisters and switchers in mathematics. Studies of over 39,000 undergraduates indicate that females earn higher mathematics grades than males. In addition, switchers earn grades equal to those of persisters and some of the most talented males and females switch out of mathematics.

Second we examine interview studies of over 1,500 students interested in mathematics and science from over 20 colleges and universities. In every study undergraduates complain that mathematics courses are designed to "weed out" students rather than to encourage the best to persist. Switchers, more than persisters point out the poor quality of undergraduate mathematics instruction compared to instruction in other courses. This, quality of instruction more than success in mathematics motivates students to switch out of mathematics and more females than males are so motivated. A serious consequence of the perceived low quality of mathematics instruction is the loss of talented students to other majors.

Introduction

Two-thirds of the students who plan to study mathematics in college eventually choose other fields. Additionally, neither grades nor test scores are as predictive of success as is gender. Based on their undergraduate grades in mathematics, women should predominate in mathematics careers, but the reverse is true. Women compared to men earn higher grades in college where they comprise 43% of the mathematics majors. Yet men earn 80% of the PhD's in mathematics and make up 97% of the mathematics faculty at the "top ten" mathematics departments. Of the students entering graduate school in mathematics, 35% are women, yet only about 20% of PhD's in mathematics go to females. To shed light on this perplexing trend we synthesize the results of a broad range of studies reporting on interviews with undergraduates at more than a dozen colleges and universities.

First, we examine the undergraduate mathematics experience, characterizing grades in college courses. Second, we synthesize interview studies to contextualize persistence among undergraduates, delineating why two-thirds of the students who plan mathematics majors switch to other fields. Third, we analyze recent reforms of mathematics instruction, identifying changes congruent with student responses to traditional courses. Fourth, we assess college entrance examinations in mathematics, seeking an explanation for the lower scores earned by females in contrast to their grades. Fifth, we consider the context of mathematics learning, analyzing how stereotypes, beliefs, and expectations distinguish the experiences of males and females. Finally, we suggest how course projects might make undergraduate mathematics instruction more equitable, authentic, and effective for all students.

We draw on recent research on learning and instruction as we review this body of research. We weigh the conceptual and cultural experiences of mathematics students. We analyze the processes students follow in making sense of mathematical ideas. We reflect on how students respond to mathematics learning experiences. We pay particular attention to the views that students develop about their own mathematics learning and about their place in the mathematics community.

Undergraduate Grades Earned by Males and Females

A synthesis of studies at colleges and universities across the United States, demonstrates that female students have higher undergraduate grade point averages than male students.

This trend begins in high school (Kimball, 1989; Linn, 1992) and holds for mathematics, science, and technology courses as well as courses in the humanities (Bridgeman & Wendler, 1991; Johnson, 1993; Leonard & Jiang, 1994; Wainer & Steinberg, 1992). Most faculty would agree that mathematics grades are an excellent indicator of promise for an undergraduate student in mathematics. Grades are awarded in undergraduate courses by knowledgeable members of the mathematics profession who have the authority to set standards for students. Women are meeting these standards proportionally more frequently than men. Yet women are less successful in entering and completing graduate programs than men. We look closely at grades in this section and at impressions of undergraduate courses in the next section.

Grades Earned

Females earn higher grades in general and in mathematics in particular. Significant overall differences in college GPA between males and females are reported in a broad range of studies as shown in Table 1 and Figure 1. In addition, a diverse set of studies show that females perform as well as or better than males in required and advanced mathematics courses (see Table 2 and Figure 2). This pattern holds for courses in college algebra, precalculus, calculus, courses beyond calculus, mathematics majors, and for students in the top 10% of the university, as shown in Table 2 (Boli, Allen, & Payne, 1985; Elliot & Strenta, 1988; Levin & Wyckoff, 1988; Young, 1991). Studies of groups of mathematics courses, as well as studies of all mathematics majors report that women earn equal or higher grades than men in science and mathematics, as shown in Table 3 (Adelman, 1991; Bridgeman & Wendler, 1991; DeBoer, 1984; Leonard & Jiang, 1994; Wainer & Steinberg, 1992).

Examination of the grades earned by males and females in introductory calculus reveals that females earned about half again as many A's as males and males earned about a half again as many C's as females (see Figure 3) (Wainer & Steinberg, 1992). Precollege data reveals the same pattern of female superiority in mathematics grades (Kimball, 1989; Linn, 1992). In general in precollege and college courses, females perform as well as or better than males.

Rates of Participation and Selection Effects

Do more talented females select mathematics, compared to males? Since females earn higher grades than males in all college majors, the selection would most likely occur when students were admitted to colleges and universities. As Leonard and Jiang (1995) suggest, some universities may make selection decisions that underpredict grades for females by relying on mathematics scores on college entrance examinations. MIT has studied this phenomena and adjusted admissions criteria to reduce underprediction for females as discussed further below.

The proportion of males in undergraduate mathematics courses is larger than the proportion in undergraduate majors. 43% of mathematics majors are females but more men than women major in engineering and natural science so undergraduate mathematics courses have more like 40% females and 60% males.

Participation in precollege mathematics follows a similar pattern. For example, the National Center for Educational Statistics (1993a) data reveals that between 1978 and 1990 there has been a slight increase in the number of females participating in advanced mathematics courses, but about 40% of the students are female. Of those taking the AP Calculus exam 43% are females (Blank & Grubel, 1993). In addition, the National Center of Educational Statistics (1993b) reports that the percent of high school seniors reporting taking eight semesters or more of high school mathematics is stable at about 40% of both males and females. Looking at the average number of mathematics courses taken by college-bound students in high school, researchers report that during the four years, females take 3.7 courses on the average, and males take 3.8 courses on the average (White, 1992).

Learning Practices of Males and Females

Why do females earn higher grades in high school and college mathematics courses? Certainly the most straightforward explanation is that females learn more in these courses than males, thereby performing better on assignments and tests. For example, a study of male and female grades on quizzes and tests in several calculus courses revealed that females consistently earned higher scores (Strauss & Subotnik, 1994).

Cognitive analysis of mathematics learning indicates group differences in acquisition of problem-solving methods advocated by the teacher and the textbook. Gallagher (in press) found that females more than males learn the traditional procedures for solving algebra problems in pre-calculus courses. Gallagher argued that learning the methods advocated in the class was obviously appropriate, but generally meant that the best students relied on more time-consuming procedures for solving algebra problems. These time-consuming algorithms taught by algebra teachers resulted in correct solutions to algebra problems but could be replaced by "short cuts" that were not taught (see the example in Linn & Hyde, 1989). Gallagher conjectured that using these traditional procedures but not the short-cuts might account for some of the gender differences in college entrance examinations.

A variety of studies suggest that on average females follow more effective study procedures than males. In several investigations of self-reported study techniques, females prefer practices likely to yield comprehensive and robust understanding of mathematics (e.g., Linn, 1992). In a study of college computer science courses, females report spending more time reflecting on the similarities among problems they study, more time organizing and linking their ideas, and more time planning and reviewing material for problem solutions (Mann, 1992 October 24). Overall, females' self-reported study practices reflect a goal of gaining an integrative view of the field they study.

Persistence in Mathematics

About two-thirds of college students intending to study mathematics or statistics switch to another field. More of these switchers are females than males. Seymour and Hewitt (1994) report on a national sample of over 800,000 students entering four year colleges and universities in 1987. They found that 72% of females and 60% of males who declare math or statistics majors switch to other fields. Furthermore, of those who switch, half the males and two-thirds of the females switch out of math, science, and technology.

Many of the best students in mathematics switch and more of the best switchers are females. For example, in the Seymour and Hewitt study (1994) the mean exit GPA for science and mathematics switchers was 3.3 (range 2.0 - 3.85) and the current GPA for seniors was 3.2 (range 3.0 - 3.95). Similarly, Humphreys and Freeland (1992, p. 5) studied all first-time freshmen entering the College of Engineering at the University of California, Berkeley in the fall semesters of 1985, 1986, and 1987, and found that, "students who persisted and students who switched earned comparable grade point averages (3.10 as compared with 3.07)." This difference was not found to be statistically significant. Engineering switchers were also found to have entered with higher SAT verbal scores than did persisters.

A study at the University of Colorado at Boulder supports this view. For freshmen who entered Science, Mathematics, or Engineering majors between 1980 and 1988, the average predicted GPA for those who persisted was 2.93, only slightly higher than the 2.86 for switchers. More women than men switched and the predicted GPA for females was higher for both switchers (females 2.88, males 2.84) and persisters (females 2.95, males 2.92) (Seymour & Hewitt, 1994, p. 61).

Thus, females earn equal or higher mathematics grades than males suggesting that, if the best students persist, more of these students should be female. In contrast, large numbers of males and females switch, more of the switchers are female, and those with high grades switch as often as those with low grades. To contextualize this situation, we examine the views of students who take undergraduate mathematics courses.

Perceptions of Undergraduate Mathematics Courses

Comments from male and female mathematics students in a broad range of interview studies, help explain both the grades earned by males and females and the reasons many students switch out of mathematics.

Methodology

We report on studies of switchers and persisters that use interviews, questionnaires, and, in some cases, written reflections. In all but one of the studies we examined (see Appendix), the students were especially well qualified (they had high SAT-M scores or high school GPAs, or attended elite colleges) and were interested in or majored in mathematics or science. Structure, frequency, and duration of the surveys and interviews varied from study to study but all address college course experience. Responses were audiotaped and transcribed or recorded in field notes, were coded, and common patterns were sought. Sometimes, particularly in recent large-scale studies such as that of Seymour and Hewitt's 1994 study of 335 undergraduates at 7 different institutions, computer software was used to analyze student responses. The Seymour and Hewitt study is particularly noteworthy: it is the largest of the qualitative studies we report on (the data set is over 600 interview hours) and by far the most extensive (the 544 page final report addresses a broad range of topics). Its findings are consistent with those of earlier studies of science, and also with studies in fields such as computer science (e.g., Spertus, 1991).

These studies describe the full range of student views. Seymour and Hewitt (1994) speak of telling students' stories "as faithfully as they know how" and Holland and Eisenhart

(1990) try to minimize the risk of imposing analyses that their informants might reject by "letting their ideas and feelings come through, in their own words." We too, attempt to convey the views of the many students in these studies by letting them speak in their own words. In these studies, students speak candidly and often their comments sound harsh. At times it is hard to listen to these comments yet they deserve attention as they are, indeed, widely held. Reviewing the range of studies from many different institutions presents a compelling picture. We have selected only comments that are representative of the findings across studies.

An analogy helps explain why we use this approach. Studies of students' learning of graphing (e.g., Moschkovich, Schoenfeld, & Arcavi, 1993; Schoenfeld, Smith, & Arcavi, 1993) have shown that what is salient to students may not be salient to those adept in the domain and vice versa. In one example students noticed the jaggedness of lines displayed on a computer screen but failed to mention that all the lines passed through the origin. In contrast mathematicians would consider the overall pattern far more important.

Students' views of their mathematics experiences also often differ from perceptions held by experts. For example, students report incidents, both in and out of the classroom, that faculty might not notice or remember. Conversely, experiences that mathematicians find enjoyable and interesting, may not impress students in the same way.

The many quotes that follow express students' interpretation of their experiences. The quotes help paint a picture of how students perceive their experiences.

Comments

This section reports on mathematics students' views of study practices, classroom practices, and teaching practices. In addition, we examine switchers and persisters by asking how students perceive academic success, learning environments, career opportunities, and peer and family support.

Study Practices

A study of the reflections of female engineering students from Princeton, provides some insight into the differences in study practices of males and females. One student reports,

> "I did notice, however, that the women tended to work together on homework. This seemed to be because we worked in the same style. The

two or three women I worked with the most usually started the problem sets early, then compared them with each other. This left time for going to see the professor if we were still stuck after consulting each other. It would be interesting to know if the professors notice whether (as a percentage) more women came to them for help. I know that one or all of us were in the professor's office every week. Lots of the men I know started the problem sets the night before they were due and just did their best and handed them in. We (my women friends and I) almost never turned a problem set in that wasn't perfect. I don't know whether these are good generalizations or if I simply attracted people like that around me because I am that way" (Ng & Rexford, 1993, p. 69).

Classroom Practices

In mathematics classes, instructors ask males more questions than females, expect males to answer more difficult questions than females, and give males more time to respond than females. In addition, males ask more questions than females and call out or interrupt more and participate more in discussions (Kimball, 1989; Koehler, 1990; Leder, 1990; Sadker & Sadker, 1986; Sadker & Sadker, 1994; Sandler & Hall, 1982; Wellesley College Center for Research on Women, 1992). As a result, males participate more in the discourse of mathematics than females. More importantly, however, females begin to define themselves as observers and to conclude that the standards of discourse put them at a disadvantage. Females who adopt the strategy of aggressively entering the fray may be labeled "femi-nazis," patronized with instructions to review the material more carefully before speaking, or stereotyped as unworthy of a reply. These discourse practices can reinforce insecurities of female students. For example, one student reports:

> "I am the type of person who feels insecure no matter how well I do in my classes. I am always thinking that I don't know as much about computers as everyone else in the department, no matter what their GPA. I have also convinced myself that my independent work project is not as difficult or significant as other people's. During sophomore year, I was a basket case. I thought I knew nothing and that all those nerdy guys in my classes were geniuses or something. After a while, though, you figure out that they're just as stupid (or smart) as everyone else" (Ng & Rexford, 1993, p. 37).

Females in mathematics classes become more and more silent. Although they comprise 40% of the students in mathematics classes, on average it is common for both males and females to estimate that fewer than 20% of their classmates are females (Keller, 1985). Females learn the material and earn higher grades than males, but males define the rules of discourse and females rarely participate.

Teaching Practices

College and university students often report disappointment in introductory math courses, especially compared to high school calculus experiences. In high school, their calculus teachers viewed themselves as fortunate to instruct the best students and to teach the most advanced material. College calculus instructors view themselves as teaching a broad range of, often unprepared, students and as covering routine material. Recent efforts to reform calculus instruction attest to the past neglect of those courses and offer promise for the future (Cipra, 1993; Cohen, Knobel, Kurtz, & Pengelly, 1994; Douglas, 1986; University of Michigan, 1993).

At most institutions, students report a few examples of college mathematics lecturers who read directly from the textbook. Seymour and Hewitt (1994) found students on all seven campuses they studied who reported this practice. They studied the following institutions: a small private liberal arts college, a private city-based university, a large private university on the West Coast with a highly selective admissions policy, a multi-role public urban university in the Northeast, a large urban public university in the Midwest, a state university in the Southwest, and a large state university considered the "flagship" institution for its southwestern state. They report interviews with 335 mathematics majors, all of whom had SAT-Ms of at least 650. Here we give a flavor of the comments of students from these various institutions. One male engineering student reported,

> "I had one professor who would literally pick up the book and read it to the class. I mean, he would just read. He was actually not bad for the math department, but we had sixty percent of the class drop the course. I counted one day, and out of maybe one hundred eighty students, seventeen showed up for the lecture. I was happy with the course content, and the facilities were wonderful, but the teaching was just a vast disappointment to me" (Seymour & Hewitt, 1994, pp. 215-216).

A female student describes another frustrating situation,

> "They just continuously write. And they're standing in front of what they write, but just don't care. And they'll look over their shoulder now and then, and say, 'Okay, you all are still there.' And then they just keep going. And the number of people that don't go to classes is amazing here, truly" (Seymour & Hewitt, 1994, pp. 215-216).

Females who persist in mathematics often do so for altruistic reasons hoping to change the way mathematics is taught. Thus one female who persisted reports,

> "I am, hopefully, going to teach in a completely different manner from my own math teachers, just because they were all so boring. I want to make my classes fun, so that kids will enjoy math" (Seymour & Hewitt, 1994, p. 86).

Other female persisters say,

> "There's a lot of times I tried to find professors in their designated office hours, and they were just not to be found. And you'd try to catch them after class, and make an appointment with them. It got really aggravating" (Seymour & Hewitt, 1994, p. 183).

> "Part of the problem with the math department, I think, is their attitude. I think they realize they're bad, but they don't really care. It's not their problem that their students are failing their courses. It's the students' problem" (Seymour & Hewitt, 1994, p. 209).

Another study of five campuses with a strong commitment to teaching reports similar comments At all five institutions professors rather than graduate teaching assistants teach basic courses and tenure and promotion depend on effective teaching. Yet, Astin and Astin (1993) report:

> "In spite of this commitment to teaching, very few innovative teaching practices were employed. The vast majority of the courses at all the institutions we observed were taught in a traditional lecture style. The professors typically stood at the front of the room, often behind a table or lectern, and used a chalk board or overhead projector to illustrate a concept or write out a formula. Although all of the professors we

> observed had PhD's and were considered experts in their fields, few of them seemed able to present their knowledge in an interesting or provocative way. Many professors mumbled, avoided eye contact by looking at the floor, and asked rhetorical questions that they quickly answered themselves. The 'energy' was very low in such classes. Many students arrived late and many left early. In essence, students were not engaged in the learning process. Some students expressed disappointment with the 'boring' lectures of certain faculty who taught the same courses year after year. A physicist at Santa Clara, however, defended the traditional lecture method: 'You get it alone by thinking hard...That's what I had to do...That's what they're going to do!' Nevertheless, most students seem to accept traditional teaching methods. We believe that this acquiescence occurred in part because of their lack of exposure to any other teaching style." (Astin & Astin, 1993, pp. 10-21).

In another study, a computer artist comments on a required calculus course:

> "I took calculus because it was required for computer science. I was frustrated by the professor—disappointed by his method of teaching. He mumbled. It seems to be the way that overcrowded departments work. They hire a professor for a different purpose than teaching. They are here to do their masters or are writing a book." (Lyons-Lepke, 1986, pp. 167-8).

Academic Success and Switching

Virtually all studies show that those who switch have grades and preparation comparable to those who remain in mathematics (Seymour & Hewitt, 1994). For example, one student comments,

> "My first year I did really well — like 3.8. And it was a surprise to people, like my mother, that I wanted to change my major — because I was doing so well. And I told her, 'It's not that it's hard. I just don't want to do it.' After that, my grades dropped off a bit, because I wasn't sure what I wanted to do — and that was purely through lack of interest. Last semester, after I had officially changed my major, they went back up to a 4.0" (Seymour & Hewitt, 1994, p. 148).

Learning Environments and Switching

To clarify the decisions students make about switching from mathematics to other fields, we report their comments in their own words (see Figure 4 for further comments). For some students, poor teaching leads to switching but for others it does not. Students describe difficult and often frustrating decisions. Switchers out of mathematics most complain that the learning environment drove them away (Seymour & Hewitt, 1994). They describe discouraging experiences in math classes, non-existent faculty advising, and stereotyped peer pressures. Many of these quotes echo the description of mathematics teaching given earlier. In many cases, efforts to weed out students discourage the wrong students by creating an unrealistic image of mathematics. For example, a student in yet another study commented:

> "In college I started Calculus I. I could understand all of the concepts as it was mostly a review of high school [calculus]. However, this is a known 'weeder' course. I felt, even though I knew the material, that there was no way I could compete with the pre-med students and engineers. I became very stressed. After the first exam I dropped the course, never again to take another math class". . . .
>
> "I think that a real help would have been in college, if there had been less emphasis on 'weeding' people out of math and more on keeping people (especially women) in. My T.A. [teaching assistant] did nothing to keep me from dropping (not even a word). I think if a place like [my university] had all women's sections with women T.A.s, they could make women students feel more comfortable and perhaps more would pursue math and science courses and careers" (Zaslavsky, 1994, pp. 149, 151).

Students in the Seymour and Hewitt study commented:

> "I always liked math, and I was still good at it, so I was thinking maybe I'd integrate it with business. Then, after looking at it some more, I had to confess that I'd lost much of my interest in it, and I changed to geography" (Seymour & Hewitt, 1994, p. 254).
>
> "Their attitude is that they don't expect you to make it through. It's very discouraging. You know that doesn't encourage you to do your best. I

> felt they were telling me, 'No you can't do it. You're not going to make it'" (Seymour & Hewitt, 1994, p. 182).
>
> "I expressed an interest in math when I came here, and I had him as a professor last year, too, but I haven't got to see him as my advisor once this year. And, it's not like he has hundreds of students to advise—the most they have is about twenty-five. You'd really think it wasn't too much to ask to come in once and a while and talk with you a little bit" (Seymour & Hewitt, 1994, p. 197).
>
> "I never knew exactly whether or not I was playing by the correct rules, because it seemed like I talked to one person, and they'd say something; and I'd talk to someone else and they'd say something else... And they definitely need to get their TAs advised about how the system works. I mean, even the dean didn't know what to do" (Seymour & Hewitt, 1994, p. 191).
>
> "I had a horrible math professor freshman year. He didn't speak any English and he was failing everyone. He turned me off. I went to an advisor in the math department and he was of no assistance either" (Lyons-Lepke, 1986, p. 167).

In another study (Tobias, 1990) reports similar comments.

> "Students are influenced in their course choices and their concentration decisions by many people: peers, parents, teachers and advisors. But this advice is often remarkably unhelpful, especially when a student has no way of evaluating it or of putting it to use in an internal process of decision-making. [One student comments:]
>
>> 'I took the math placement and I was on the borderline of Math 1a and Math AR, and they said "Take Math AR," so I did that. [Interviewer: "Who said that?"] The gods from above who sent back the computer readout from the placement test'" (Tobias, 1990, p. 75).

Career Opportunities and Switching

Another aspect of persistence concerns career opportunities as reported by Seymour and Hewitt (1994). Females often mention limited career options for mathematics majors as a problem. For example, one Native American female who switched out of mathematics commented,

> "I talk to some of my friends that are math majors right now, and they're saying, 'What am I gonna do.' I mean they look to grad school because they don't know what else to do.... People say you can do so much with a math major—the problem is finding it" (Seymour & Hewitt, 1994, pp. 260-261).

Other female switchers saw the options as basically dull,

> "You start broad, and now you're just narrowing down your life to this straight path going somewhere. But no one ever gives you a clue about what kinds of things you're supposed to be doing out there" (Seymour & Hewitt, 1994, p. 261).

> "I kinda looked at a couple of jobs, but I realized, no matter what I did with math, it's pretty much going to be a nine-to-five job in an office. Right away, this was my biggest turn-off. It's not so much math; I still love math" (Seymour & Hewitt, 1994, p. 86).

Jackson (1994) reports similar conclusions raised by minority students:

> "For many reasons, including those I just mentioned, minority students are attending medical, business and law schools instead of graduate schools in mathematics. The unfortunate thing is that some of these students actually prefer mathematics. However, to many gifted minority students with several career choices, mathematics is not a good choice. Many view mathematics as having too many roadblocks, an area where they are unwelcome, where there will be limitations. Let me give you an example. I know of a student who came to Hampton University with a very good SAT score and graduated at the top of her class as math major. She did well in every program she was placed in, including a summer program at Ohio State University. At the conclusion of her studies at Hampton, she

> told me quite frankly that she preferred mathematics, but from what she had seen of the mathematics profession, she would be better off in another profession. She is now a medical student at Duke University, exploring the possibilities of somehow satisfying her appetite for mathematics" (Jackson, 1994, p. 448).

Peer and Family Support and Switching

In contrast, a broad range of evidence concerning females who do choose to persist in mathematics reveals the tremendous support that they get from their families and peers, as indicated by comments in Figure 5. For example, one student from Princeton, comments on how her family helped her persist in a traditionally male field:

> "The one thing that I really like about being Chinese-American is the close-knit family ties. I think that played a major role in my being able to get into here. My family was very supportive every step of the way. Even after I graduate they will be behind me 110.2223 percent. I tend to be overprotected, but I know they mean well because they love me. They're ready to let go. A lot of times I'll talk to them about a problem and they'll say, OK, you're old enough to deal with it by yourself" (Ng & Rexford, 1993, p. 40).

Another female who persisted in science reports:

> "My sister's a lawyer, and another sister's a C.P.A., so we're a very career-oriented family. Even going back to my grandfather who worked on the railroads, and you might think would be the opposite. He, too, would say no to women just staying home and caring for children. 'You need to graduate from college,' he would tell us from being little. And so there was no question in my mind from day one that that's where I was going" (Seymour & Hewitt, 1994, p. 378).

Overall, reading these comments from students, it appears that females earn higher grades than males in high school and college mathematics courses because they have better study habits and learn more mathematics. In these courses, however, students learn more than mathematics. They learn to participate or to be silent. They observe the discourse rules of the field. And, they learn that introductory courses are designed to filter out rather than to interest, teach, or nurture students.

Although students who leave mathematics are similar to those who persist in terms of grades, high school preparation, and college entrance examination scores, faculty generally believe that the best students stay and, "people whose mathematical past has caught up with them and should have been flunked out earlier" (Miller, 1990, pg. 7) are the ones who switch. In fact poor teaching and advising may select for the most determined rather than the most talented students. Research universities and institutions with PhD programs appear to discourage more women than liberal arts colleges. Neglect of female students may account for the larger number of female switchers. Courses intended to filter out students may deter the best students and retain the most stubborn.

Patterns of Persistence and Reform of the Curriculum

Although students offer quite congruent views of undergraduate mathematics, institutions vary in the patterns of persistence demonstrated by students. In general, attention to undergraduate education is associated with higher rates of persistence. In this section, we summarize persistence rates for institutions and discuss one aspect of undergraduate mathematics education, Calculus reform.

Institutional Patterns

The pattern of persistence in mathematics for males and females varies by institution (see Table 4). Overall, 43% of BAs in mathematics granted by four year colleges and universities go to females. The top 10 mathematics departments, rated on their research reputations, grant degrees to between 9% females (Princeton University) and 49% females (University of Michigan at Ann Arbor). The average for the top 10 schools is 33%. PhD-granting institutions, in general, award 35% of mathematics BAs to women, in contrast to colleges, where women earn 48% of the BAs in mathematics (Conference Board of the Mathematical Sciences, 1990).

In contrast, 23 of the 57 top research institutions are the best producers of women (US Department of Education/National Center for Education Statistics, 1994). These top-producing schools grant BAs to either 50% or more females, or to 40 or more females per year (see Table 5). The top 10 schools' mathematics departments are all top research institutions, but only the University of Michigan at Ann Arbor is represented among the top producers of women. Thus, schools with the best reputations in research and schools that have PhD programs award fewer undergraduate degrees to women than colleges and less research-oriented institutions.

Women's colleges and liberal arts colleges have as many as 5% mathematics majors compared to an average of 2% at research institutions, but the total number of degrees awarded by these departments is small. Together, these schools produce less than 200 of the over 6,000 mathematics BAs awarded to women annually. Schools that emphasize the undergraduate curriculum have a higher percentage of female mathematics BAs than other institutions (US Department of Education/National Center for Education Statistics, 1994).

Calculus Reform

Both in response to complaints from students and in response to better understanding of mathematics learning, exciting calculus reform efforts are now widespread and respond to many of the concerns raised by students. Many reforms of undergraduate courses and especially calculus emphasize projects that allow students to experience the nature of mathematical thinking rather than the product of mathematical problem solving (Cohen et al., 1994; Schoenfeld, 1994). Traditional calculus courses emphasize routine application of algorithms, fleeting coverage of numerous topics, and piecemeal approaches to complex concepts. As the comments from students suggest, these courses make it difficult to connect and link ideas, recognize fundamental concepts, and see the relevance of mathematics to future work. Helping students develop their own problem solving processes requires analyzing the problem solving process and illustrating techniques for identifying patterns and detecting errors. For example, case studies and course projects can respond to these concerns (Clancy & Linn, 1992; Linn, 1986; Schoenfeld, 1994).

To address the complaint that faculty emphasize the results of problem solving but neglect the complexities, reformers recommend that faculty describe alternative solutions, wrong paths, methods for detecting errors, and recyclable patterns. Typical instruction that consists of telling students the correct algorithms and then assigning problems, leaves a gap between instruction and expectations.

Here we focus on projects as a strategy that has proven successful in several undergraduate programs (e.g., Arizona). Projects are one approach to the "lean and lively" calculus (Cohen, 1994; Douglas, 1986; Steen, 1988; Tucker, 1990), which includes fewer topics discussed in greater depth, and technological tools that make some algorithmic problem-solving unnecessary (Wolfram, 1988).

Creating good project assignments challenges faculty. Students might be asked, for example, to model the rate at which an audience leaves a crowded theater (see other examples in Figure 6). Often these projects require students to work with their peers.

Adding course projects to calculus also necessitates communication to students about the changed goals and activities, and communication to the broader community concerning the expected skills and knowledge of students taking the new courses. Experience introducing projects in several courses has convinced us that a framework is needed to guide this change. Both students and faculty need support to make this new approach effective. Here we describe some of our experiences. These are described in greater detail in other reports (Linn, 1986; Linn, 1995).

Scaffolded Knowledge Integration Framework

Efforts to introduce projects into a middle school science class, Pascal and LISP courses at the university, and an engineering design course at the university resulted in a framework called Scaffolded Knowledge Integration (SKI). The SKI framework starts by defining new goals for instruction that meet student needs and provide an authentic experience of the field. Projects are advocated for calculus to achieve this new goal.

At the universities where these projects have been introduced, obstacles similar to these found in our investigations plagued reformers. The reformers found that students lacked skills necessary for long, complex projects and faculty needed new approaches for communicating about more sustained investigations (Cipra, 1993; Cohen et al., 1994). Students complained about projects, preferring problem sets because they had succeeded on these in the past (Lewis, 1988; Linn, 1980). Students often failed to collaborate when working in groups. These obstacles seem to require several cycles of trial and refinement to improve the course and establish new performance standards.

A major challenge in these reforms is to balance what we call "making thinking visible" with "supporting autonomous learning." To make thinking visible, courses need to illustrate the steps necessary to complete sustained investigations, but if the steps are too constrained, projects look just like problem sets. The goal is to help students learn to solve more complex problems on their own. What sort of assistance makes this possible? First, students benefit from models of the outcome expected in order to understand the assignment that they have been given. Providing students with model calculus projects completed by students in previous semesters or at other institutions can help improve the quality of the projects and the comfort level of the students. In addition, case studies that communicate the process of problem solving not just the solution to the problem can help students identify strategies for solving large, complex problems. Case studies in computer science, engineering, and business help students complete large projects by making some of the steps necessary to complete projects explicit and by illustrating how to organize information in reusable patterns (e.g., Linn, 1995; Linn & Clancy, 1992). Ultimately students need to solve problems on their own, or what we call autonomously. If too much help is given, they get no practice in figuring things out themselves. Yet, without help students may endlessly flounder or produce unsatisfactory work. One success we had was to encourage students to criticize the solutions of others. Those who acted as critics learned both how to solve problems and how to analyze potential solutions, while those who only solved problems were prone to accept incorrect solutions even when they were able to generate correct solutions themselves (Davis, Linn, Mann, & Clancy, 1993).

To help students link and connect ideas while working in groups, the SKI framework also emphasizes providing social support for learners. All learning takes place in a social context, so the goal is to structure social interactions to support all learners. To this end, many mathematics faculty have introduced group work in undergraduate courses. Initially, groups often have difficulty working jointly on problems (Cohen, 1994; Linn & Burbules, 1993). Sometimes one student does all the work. Often students complain about the lack of contributions from their peers. Students who are first participating in groups may reinforce stereotypes concerning who can contribute to a particular field. For example, in engineering, Agogino and Linn (1992 May-June) report on groups that convince women students that their contributions are less valued than those of male students or silence students who could contribute. Mathematics faculty and former undergraduates report similar results (Ng & Rexford, 1993; Selden & Selden, 1993). Group work can help students develop collaborative learning skills when accompanied by effective instruction. Modeling collaborative learning skills and judiciously designing group experiences makes for improved learning.

So far, research suggests some clues about how groups can help each other learn. For example, taking the role of a tutor in a group can help students recognize the limits of their own understanding. Students who tutor others in groups frequently learn more mathematics than students who work on their own (Schoenfeld, 1988; Webb, 1989). Furthermore, when students use each other's ideas they learn how to make sense of disparate information. When they rely on texts to suggest ways to think about problems, students often take the text as the authority but they are likely to analyze ideas from their peers. Sometimes students can describe a problem in words that are understandable to other students when the textbook or faculty fail. This reinterpretation of mathematical ideas helps students make sense of the material (Linn & Burbules, 1993; Songer, 1993).

Overall, calculus projects have the potential to communicate the design and discovery activities of mathematics, and help students link and connect ideas. By adding projects and modifying instruction to help students complete projects, calculus instructors can enhance understanding of the nature of mathematics, since projects allow students opportunities for deep understanding that do not arise in courses with only short problems. Traditional courses may convince students that mathematics is algorithmic and boring, thus discouraging talented students and especially women (Buerk, 1986). Project-based courses that feature appropriate support for students may attract the most creative individuals. These project experiences allow all students to engage in activities common in undergraduate research. Often only the best students, as well as far more men than women, have the opportunity for undergraduate research (Senechal, 1991).

Designing effective courses typically requires several cycles of innovation and revision. Mathematics faculty can speed this process by communicating their own experiences with reform and learning from each others' successes.

Mathematics College Entrance Examinations

If women get higher grades in mathematics, why are they earning lower scores than men on college entrance examinations such as the SAT-M and the ACT mathematics? How are grades in mathematics courses related to scores on mathematics aptitude tests?

General tests of mathematical ability administered to representative samples of precollege students show no gender differences in performance (Chipman, Brush, & Wilson, 1985). For example, Hyde, Fennema, and Lamon (1990), performed a meta-analysis of hundreds of studies conducted in the United States and found an overall effect of about one-tenth of one standard deviation. Internationally, in some countries females out-perform males while in others males out-perform females (Ethington, 1990; Hanna, 1989). However, college entrance examinations reveal a different picture, as discussed below.

We analyze college entrance examinations from the standpoint of their validity as measures of mathematics and their results. Since these tests require students to solve 25 to 35 problems in 30 minutes, there is a premium on efficiency. In contrast, reformulated mathematics courses that include an emphasis on solving complex, ill-posed, and personally-relevant problems place a premium on sustained reasoning. The rapid solutions to problems characteristic of college entrance examinations are not a component of calculus projects. And, ability to refine complex problems and design appropriate solutions goes beyond the skills measured in college entrance examinations. Thus, at least on the face of it, the validity of college entrance examinations for predicting performance in reformed versions of the calculus curriculum is likely to decline.

As discussed in the next section, grades earned in these new courses may help determine which students will succeed in more advanced mathematics courses and change the criteria that govern student decisions to switch out of mathematics. Students taking these reformed courses are likely to be better able to make decisions concerning continuing in mathematics because their experience with mathematics will be more authentic.

Performance on SAT-M and ACT-M

The voluntary sample of college-bound high school students taking the SAT-M or ACT-M shows a consistent gap between male and female performance of about 0.4 standard deviation units (Linn & Hyde, 1989). This gap has narrowed slightly over the years but remains large (Friedman, 1989).

In the 1980's score differences could be statistically eliminated by adjusting for high school course experience (Chipman et al., 1985; Wise, 1985). Examination of recent data from the Educational Testing Service, indicates that for students with the same course experience, the performance gap on SAT is about 0.4 standard deviation units (Grandy, 1987; Linn, 1992). Recently, the gap in course experience has narrowed but the score gap remains (Wainer & Steinberg, 1992). There is little relationship between SAT scores and performance in all levels of college mathematics. For males there is a slightly greater relationship than for females. Females need about 50 *fewer* points on the SAT to perform as well as males in the full range of college courses (see Figure 3).

This pattern holds in studies of high school students as well. For example, Gross (1988) studied more than 4,000 high school students in Montgomery County, Maryland. Girls took the same advanced math courses as boys. These included calculus, pre-calculus, and advanced algebra studied in the same classrooms and with the same teachers. Girls earned higher grades but their SAT scores were lower by 37 to 47 points.

Many studies suggest that the multiple choice, rapid response context of SAT and ACT achievement tests favors males over females. The sustained reasoning and organized communication required for essays seems to favor females (Linn, 1992; National Science Foundation, 1990). Furthermore, mathematics examinations in the Netherlands, England, Australia, and other countries that require solutions to several long problems seem unbiased with regard to gender (de Lange, 1987; Leder, 1994; Murphy, 1982; Stobart, Elwood, & Quinlan, 1992). Thus the context of the multiple-choice achievement test may differentially advantage males.

The speeded nature of the college entrance examinations also probably advantages males who tend to be more confident about their ability to solve problems than females. Studies suggest that males, compared to females, expect to answer correctly, independent of actual performance (Linn, de Benedictis, Delucchi, Harris, & Stage, 1987). This perspective results in less answer checking and reflection, useful skills for speeded tests. Similar findings about speed on spatial reasoning have been reported for males and females (Linn & Petersen, 1985).

Are these achievement tests valid measures of mathematics ability and potential? Example questions that appear on the SAT items are shown in Figure 7. These questions typically distinguish students scoring in the mid-range (500-600). The SAT requires students to solve 35 problems in one 30-minute section and 25 problems in the other 30-minute section. Scoring involves counting about 10 points for each correct answer, minus a percentage of incorrect answers. As a result, a difference of about three to four correct accounts for the gender difference. Thus, students must solve problems quickly, pace themselves appropriately, and maintain concentration for a reasonable period of time. Students report that skills such as eliminating the answers that are known to be wrong, using short-cuts and rules of thumb to solve the problems, and detecting trick questions contribute to high scores (Robinson & Katzman. 1986). Some students view the test as more a measure of trick-detection than a measure of mathematics skill. Females report this as one more reason to conclude that math is frustrating rather than interesting.

Does this test measure mathematical abilities relevant to college work? The most complex items on these tests tap some of the skills that students need to be successful in mathematics yet few students even attempt those items since they are at the end of each section. The concept of a function is beyond the scope of this test yet central to college work. Rapid responses may contribute more to success than the problem-solving skills and mathematical knowledge students need for future courses.

Do these tests predict undergraduate success? The narrow range of performance among students accepted in the most selective colleges makes the relationship between performance on these tests and undergraduate success relatively limited. For example, a study at MIT found that aptitude test scores account for only 5 to 7% of the variability in grade point average by the end of the sophomore year (Johnson, 1993). A new version of the SAT, out this year, addresses some of these concerns but preliminary data show no change in the gender gap (Burton, 1995 April).

Using Tests in College and Scholarship Decisions

Although aptitude tests account for a small percentage of the variance in college grades, their impact is far greater. When selective scholarship and admissions decisions are made primarily on the basis of aptitude test scores, then two-thirds of those selected are men. Conversely, if decisions were made primarily on grade point averages more women would be selected.

Furthermore, scores underpredict the ability of women to succeed. Historically, women have earned higher high school and college grades than expected on the basis of their scores on ability or achievement tests (Wagner & Strabel, 1935). Originally, Thorndike labeled this "over-achievement" (Lavin, 1965; Thorndike, 1963). Research investigated the effect of motivation and study habits on grade performance. More recently, the relationship between scores and grades has been labeled "underprediction" (Reynolds, 1982; Stricker, Rock, & Burton, 1991). Attention has shifted to the psychometric characteristics of these tests, and especially to their validity in explaining this underprediction for females.

For example, Leonard and Jiang (1994) report a study of the cumulative grade point averages at graduation of the approximately 10,000 students who were admitted to the University of California, Berkeley as freshmen between 1986 and 1988. As expected, females have higher grade point averages than males. Some of this difference reflects the majors that males and females choose. But, when field of study is controlled, the SATs continue to underpredict women's cumulative college GPA by a small but significant amount. Had women with lower SATs been selected instead of the men at the cutoff point, the women would have earned higher GPAs than the men who attended the university. The use of SATs in the selection formula resulted in selecting less qualified males than the females who were rejected, based on grades earned in undergraduate major.

Recent examination of data like this has resulted in modification of admissions standards at many colleges and universities. For example, MIT has dramatically reduced score requirements for the SAT mathematics section, resulting in the addition of many women to the MIT class and a narrowing of the gap in GPA between males and females (Johnson, 1993). Similarly, Rutgers University has examined scores and grade point averages of males and females and modified policies to increase fairness (Rosser, 1989). An investigation at Princeton University revealed that SAT scores of the Princeton class of 1990 were higher in mathematics for males than for females. In spite of these different SAT scores, women's grades were higher than men's and the most significant underpredictor of grades for women was the SAT math score (Rosser, 1989).

A large number of universities have reduced or dropped use of entrance examinations like the SAT and ACT altogether (see Table 6). Bates in Maine, Bowdoin in Maine, Middlebury College in Vermont, and Union College in Schenectady, New York have all dropped use of the SAT altogether (Chronicle of Higher Education, 1987; Rosser, 1989; Simpson, 1987). At Penn State the influence of the SAT on college admissions has been reduced in recent years (Petersen & Dubas, 1992). Analysis of applicants who did not submit SAT scores to Bates College reveal that those omitting the SAT scored 80 points lower than applicants who submitted their scores, but did not differ significantly in first year GPA or academic standing (Petersen & Dubas, 1992).

A number of organizations, especially those concerned with equity for women have argued against the use of SAT scores to award scholarships based on high school achievement (Connor & Vargyas, 1992). Legal proceedings arguing against the use of the tests for awarding of scholarships have had some success (State Department of New York, 1989).

A particularly inappropriate use of these college entrance examinations scores is for placement in college mathematics courses. Innovative programs at several colleges use either a repertoire of indicators or rely on students to make sensible decisions themselves.

Overall, aptitude test scores favor males yet the validity of these scores for predicting college performance, for selecting individuals for scholarships, for placement, and for inferring mathematical aptitude is in question. As a result, many colleges and universities across the United States are modifying admissions practices to adjust for the gender gap on college admissions mathematics examinations. This gender gap does not appear to have predictive validity for success in undergraduate programs. And, these tests convey a misleading picture of the nature of mathematics.

Both empirical studies and examination of the validity of the SAT support the decisions of colleges and universities to use entrance examinations as but one indicator of college success. Such tests may help discriminate at a very broad level between students in the eligible and ineligible group. They have limited predictive power for fine-grained selection decisions. College entrance examinations are biased predictors of college grades and should not be used alone.

MIT and other institutions identify a pool of eligible applicants using college entrance examinations, grades, recommendations, and other indicators. They base decisions among those in the eligible pool on indicators other than college entrance examinations. This approach values high school rank in class and breadth of accomplishments. It paves the way for considering original work such as projects or performances.

Discussion

How does the context of mathematical learning contribute to the pattern of grades and scores reported here? Theoretical and empirical work suggests (Eccles et al., 1989; Hyde et al., 1990; Meece, Parsons, Kaczala, & Goff, 1982; Wigfield, Eccles, MacIver, & Reuman, 1991) that women are less confident than males about mathematics and that both males and females stereotype mathematics as a male domain (Hyde et al., 1990). This difference in confidence and in perception of mathematics as a male domain may contribute to different learning practices. Many examples of comments reflecting low confidence appear in Figure 4. Learners who lack confidence in their ability in mathematics rely more on opinions of peers and professors, follow instructions provided by instructors, and have low expectations about their abilities. In general, unconfident learners take test scores and grades more seriously, and interpret the behavior of peers and faculty more negatively. Dweck (1986) describes females as believing they are guilty (likely to fail) until proven innocent.

In spite of the largely overlapping distributions of performance of males and females in mathematics courses and tests, most people expect males to be more successful than females, probably based on the historical predominance of males in mathematics.

Consistent with the normative view that women cannot do mathematics, balance their checkbooks, or handle investments, score differences may reinforce subtle patterns of encouragement for males over females. Rather than considering the distributions of males and females on tests or grades, most expect males in general to outperform females in general (Linn, 1994).

Examples of the kinds of reactions of female students to college course experiences are provided in Figure 8. As can be seen, many females detect a bias against them in their interactions with college faculty. Many feel that they are less encouraged to continue in mathematics than male students. One consequence is that females expect that they will earn lower scores and grades than they actually earn. Males expect to earn higher grades and scores than they earn. These expectations are in line with societal stereotypes about mathematics: males are expected to succeed and females are expected to need help.

Role of Stereotyped Expectations

From the standpoint of a male learning mathematics, success is the modal expectation. This expectation coincides with nonchalance among males concerning success in mathematics. From the standpoint of a female learning mathematics, uncertainty about future performance is the modal expectation. This expectation coincides with concern among females about success in mathematics. Females report lower expectations about their success in mathematics on average than males. Thus, males are more likely than females to view themselves as successful and capable in mathematics. How do these different self-perceptions contribute to course and test performance of males and females?

What happens in mathematics courses to nonchalant and concerned students? Concern may translate into better study habits and more preparation. In classes where teachers encourage good study habits, females report using them more than males (Mann, Linn, & Clancy, 1992). Concerned students may gain a more cohesive and integrated view of the subject. In contrast, nonchalance may translate into less effort exerted on homework and less preparation for classroom tests. Ultimately this could mean less integrated and cohesive understanding of the topic.

In classes where mathematics is routinized and students are encouraged to learn algorithms, it appears that concerned students learn and follow these algorithms more than nonchalant students. This approach is often reinforced in textbooks. As a result, the concerned students may have powerful algorithms but fewer alternative methods and less experience developing personalized algorithms (Leinhardt, Seewald, & Engel, 1979). In these classes, students who conform to the requirements may be more bored than others.

Nonchalant students commonly view introductory courses in mathematics as hurdles. They spend less time studying for these courses and earn lower grades. In contrast, concerned students may conclude from their undergraduate experience that mathematics is an uninteresting field, that the climate does not welcome students at all, especially women students, and that they do not expect the situation to improve.

On mathematics tests, those who are concerned may rely on the algorithms that were taught and may also engage in extra checking of answers to be sure they are right. Nonchalance in contrast may result in more use of short cuts, more variability due to random error, and more confidence in performance on the easiest items. This difference in response style could speed up the performance of the nonchalant and slow the performance of the concerned. It could account for the differences in SAT test performance, if concerned students tend to be females and nonchalant students tend to be males.

To account for gender differences on the SAT-M, it is necessary to explain why females get about three more items wrong, on average, than males. By taking extra time to check answers, concerned students may not have time for the later items that they could perform correctly. By checking easy items, or skipping these, concerned students may allocate time inefficiently. Data from National Assessment of Education Progress items that offer an "I don't know" option shows that females use this response far more than males (Linn et al., 1987). Females also skip easy items more frequently (Lockheed, 1985).

Short cuts and efficient test-taking strategies are among the techniques taught by coaching schools. Schools encourage fast performance on early items, no skipping, and saving time for more difficult items (Robinson & Katzman, 1986). Of course, these brief training programs are unlikely to change practices built up over 12 years of mathematics instruction and reinforced by years of exposure to media and societal expectations. Furthermore, practices followed in informal situations may be abandoned in the high stress situation of a standardized test. A broad range of literature and experience suggests that when individuals are placed in stressful situations, they revert to familiar but often less effective practices. This is especially evident in sports where athletes focus almost as much on being relaxed as they do on technique in order to succeed in high-stress situations. Speeded, high-stakes tests are most likely to distinguish between nonchalant and concerned students.

In summary, the differential effect of the social context of mathematics on males and females could account for observed differences in grades and entrance examination scores. The social context combined with the goals of introductory college courses to "weed out" students could account for more female than male switching. As a result, reforms of mathematics need to pay special attention to the resulting social context for learning.

Implications for Instruction

Perhaps the major implication for instruction from these findings is the importance of communicating to students an accurate view of the field of mathematics. Current courses and college entrance examinations tend to reinforce stereotypes about the field and its participants. They probably often discourage the wrong students. Adding projects that require deeper understanding will communicate a more robust and realistic view of mathematics.

A few small liberal arts colleges produce significant numbers of female majors, graduate students, and PhD's in mathematics (see Tables 4 - 6). Such colleges also offer closer relationships between students and faculty, more opportunities for undergraduate research experience, and a preponderance of female faculty members. Opportunities to participate in discussions with faculty are likely to communicate a deepèr understanding of the nature of mathematics (Senechal, 1991). This may, in turn, counter stereotypes about mathematics and engage more students in the discourse of mathematics.

A second major implication is that nurturing rather than filtering is the key to success. Students in mathematics are excellent at filtering themselves out of mathematics. Nurturing is necessary to sustain the best and brightest students. Programs that nurture students are showing success at St. Olaf and Potsdam (Datta, 1993; Gilmer & Williams, 1989; Poland, 1987; Rogers, 1992; Steen, 1989). Especially when coming from successful high school programs to competitive college experiences, the best students are the most likely to be deterred by traditional courses. If students are filtered out by courses that require only superficial understanding of mathematics, the best students may conclude that mathematics is boring. If filtering is the key focus of the undergraduate curriculum, those determined students who persist may lack creativity as well as research aptitude.

Some competitive colleges address this problem. For example, the Massachusetts Institute of Technology offers no first year grades, clearly articulates to students the relative importance of early courses, and points out that in early courses it will be difficult for them to determine their relative standing compared to other students because of the differential previous experiences that students have. These institutions also help students to understand that often introductory courses are not a good indicator of the kind of work that will be required as they continue in the field. UCLA nurtures students by providing a full time undergraduate advisor dedicated to helping students by, for example, distributing accurate information about jobs for mathematics majors and forming supportive, collaborative groups (Johnson, 1992; University of California at Los Angeles Mathematics Department, 1993).

A third major implication for mathematics instruction suggests the advantage of monitoring behavior and examining activities that may subtly reinforce stereotypes. Faculty techniques that are successful for male students may not work for females. Thus, courses that filter rather than nurture may filter out fewer potentially successful males and a larger proportion of potentially successful females as indicated in the many quotes form students reported earlier.

For example, faculty who display indifference towards all students may have a far greater effect on females than males. Males may simply assume that faculty are disinterested in students all together, where females may take this more personally because of (a) their greater uncertainty about their ability to succeed in the field (Nerad, 1991 Nov 4-5), and (b) their awareness of the general societal view that mathematics is a male domain.

Faculty frequently complain that they cannot monitor their behavior and that students should understand that they are well-meaning. Our examination suggests that this view is short-sighted. All members of society need to pay attention to the subtle messages they communicate. For example, when a professor in mathematics suggests that he and his new male graduate student go to dinner, both will respond favorably. If, in contrast, he invites his new female graduate student for dinner, many would interpret this behavior as inappropriate.

Essentially, in order to make mathematics instruction equitable, it is important to avoid reinforcing and perpetuating stereotypes about women in mathematics and to separate the culture of mathematics from the intellectual practice of mathematics. Many aspects of the culture of mathematics including the discourse style and patterns of interaction are developed within a same-sex group. They are less successful when practiced in a mixed-sex setting.

A fourth major implication from this work concerns the importance of helping the best students recognize their potential. Most students benefit from feedback and guidance concerning their strengths. Female students who have to overcome expectations that their gender does not succeed in mathematics especially need specific feedback to help them make sense of their accomplishments. For example, many female students interpret their grades negatively when in fact they should take them as positive indicators of their progress.

A corollary of this recommendation is that instruction is most successful when it helps all students become autonomous learners and lifelong problem solvers, when it helps all students develop internal standards for success rather than relying primarily on external indicators of success, and when it helps all students interact in a community of mutual respect rather than isolating some students and stereotyping others.

A further corollary of this recommendation is preventing the isolation of students from underrepresented groups. Since mathematics has traditionally been a male domain and since the predominant perception among men is that it continues to be a male domain, it is particularly important to reach out to underrepresented groups, to encourage the organizations that support these individuals, and to engage in community-building that cuts across groups.

Conclusions

Examination of the nature of college entrance examination scores and grades in pre-college and college mathematics suggest that neither are very accurate indicators of success in mathematics careers. Indeed, the best predictor of finishing a PhD in mathematics is neither of these indicators, but rather the gender of the individual. Males are far more likely to earn a PhD than females.

College entrance examination scores are unpredictive of success in mathematics because they sample only a small amount of the skill necessary for this success. College grades are also poor indicators of success in mathematics when undergraduate courses require students to work exercises rather than engage in authentic activities.

Furthermore, the factors contributing to the persistence of males and females in mathematics probably differ. Males are more likely to persist because they want to get to advanced courses, ignoring the features of introductory courses. In contrast, females are more likely to switch because they find introductory courses frustrating, because neither faculty nor peers take them seriously, and because they feel unwelcome in mathematics.

Undergraduate courses require transformation because they fail to communicate the nature of mathematics and may attract just the wrong students to mathematics. Here, females who earn higher grades may be the most influenced. By taking these courses seriously, they are more likely to get the wrong message than students who view them as hurdles to be passed before the real courses start.

Reform of undergraduate courses has already begun. To achieve success, however, these reforms must be refined in real educational settings. Faculty who try these approaches can speed the process by sharing their experiences.

In introductory courses subtle stereotypes about males and females may also deter the best students. Both males and females may be deterred by these experiences but more females than males are likely to leave mathematics as a result. As courses are refined, these subtle stereotypes must be carefully monitored and regularly addressed.

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Table 1: Overall GPAs earned by undergraduate males and females at various institutions

Groups of Institutions	year	N	% Female	Overall College GPA Males	Females
1st year students at 19 colleges and universities [1]	1980	5,897	44	2.69	2.74
1st year students at 41 colleges and universities [2]	1980	68,142	51	2.52	2.68
Single institutions					
1st semester students at large state university [1]	1988	4,307	53	2.52	2.61
1st year students at Rutgers [3]	1985	1,956	51	2.54	2.64*
1st year students at UC Berkeley [4]	1992	3,404	49	2.85	2.94
2nd year students at UC Berkeley [4]	1992	4,243	46	2.97	3.02
3rd year students at UC Berkeley [4]	1992	6,079	48	3.00	3.06
4th year students at UC Berkeley [4]	1992	7,646	46	3.12	3.18
MIT students who completed degree & entered in same year (class 1) [5]		1,000 -1,050	27	4.30	4.30
MIT students who completed degree & entered in same year (class 2) [5]		1,000 -1,050	38	4.30	4.30
Subgroups at Single Institutions					
Undergraduates at American University who took arithmetic or algebra test [6]	1984 -1986	1,692	61	2.88	3.03*
Undergraduates at American University who took arithmetic or algebra test [7]	1987 -1988	2,937	61	2.89	3.01*
Students in two calculus classes, University of Colorado, Boulder[8]	1973	154	27	2.72	3.10*
University of Michigan math majors[9]	1987 -1988	173	26	3.38	3.53*
1st year students at Rutgers in the top 10% of their respective SAT distributions [3]	1985	218	47	2.92	3.18*

* Difference tested and found significant.

1(Stricker et al., 1991, p. 8)
2(Clark & Grandy, 1984, p. 21)
3(Kanarek, 1988 October, p. 5)
4(Office of Institutional Research/Office of Student Research, 1992, p. 43, 50.)
5(Johnson, 1993, p. 75)
6(Sheehan, 1989, p. 79)
7(Sheehan, 1989, p. 84)
8(Struik & Flexer, 1977)
9(Frazier-Kouassi et al., 1992, p. 71)

Table 2:

Grades in mathematics courses earned by undergraduate males and females

			Average Grade		
Course & Institution	N	% Female	Males	Females	d
College algebra [10]	336	52	1.98	2.24 ns	
Algebra and trigonometry [10]	358	39	2.18	2.59*	
Algebra at 6 colleges [11]	1,435	48	2.32	2.39*	0.14
Precalculus at 6 colleges [12]	3,005	43	2.12	2.38*	0.00
Precalculus [13]	3,998	61	2.18	2.32	0.13
Calculus at 8 colleges [14]	4,216	35	2.36	2.39	0.14
Calculus [13]	21,101	42	2.54	2.68	0.14
Calculus at several universities [15]	561	29	2.30	2.77	
Honors mathematics, University of Michigan [9]	173	26	3.22	3.38 ns	
Courses beyond calculus at 51 institutions [16]	3,548	33	2.90	3.00	

* Difference tested and significant.
ns Difference tested and not found significant.

[10](Struik & Flexer, 1984, p. 338)
[11](Bridgeman & Wendler, 1991, p. 279)
[12](Bridgeman & Wendler, 1991, p. 280)
[13](Willingham, Lewis, Morgan, & Ramist, 1990, details supplied by the Educational Testing Service)
[14](Bridgeman & Wendler, 1991, p. 281)
[15](Hughes, 1988, p. 126)
[16](Wainer & Steinberg, 1992, p. 325)

Table 3: Course GPA by Group

	N	% Female		Males	Females
University of Michigan math majors[9]	173	26	Mathematics GPA	3.51	3.55^{ns}
First year students in 41 colleges and universities[2]	165	68	Math and science GPA	2.83	2.98
First year students at Rutgers in the top 10% of their respective SAT distributions, 1985[3]	218	47	Math and science GPA	2.69	2.85^{ns}

* Difference tested and found significant.
^{ns} Difference tested and not found significant.

see Tables 1 & 2 for source data

Table 4: Proportion of females earning BAs in mathematics at all 4 year institutions and at top 10 mathematical research institutions

A. All 4 year institutions

Type of Department	BAs in Mathematics N Men	N Women	% Women
Ph. D. granting departments	3,696	1,970	35%
M. A. granting departments	1,933	1,672	46%
Colleges	2,893	2,663	48%
Total	8,522	6,305	43%

—(Conference Board of the Mathematical Sciences, 1990)

B. Top 10 Mathematical Research Institutions.

Number of mathematics and statistics baccalaureates awarded in 1991 top ten math departments (research).

	BAs in mathematics and statistics N Men	N Women	% Women	% Math BAs	Total BAs all fields	% Women
California Institute of Technology	7	5	42%	7%	183	15%
University of Chicago	40	14	26%	6%	857	41%
Massachusetts Institute of Technology	40	24	38%	6%	1,107	34%
Princeton University	20	2	9%	2%	1,110	40%
Yale University	17	9	35%	2%	1,323	44%
Columbia University	29	5	15%	2%	1,377	40%
Stanford University	13	8	38%	1%	1,470	41%
Harvard University	38	10	21%	3%	1,733	42%
University of Michigan at Ann Arbor	51	46	47%	2%	5,477	49%
University of California–Berkeley	64	35	35%	2%	5,681	47%
Total	319	158	33%	2%	20,318	44%

—(US Department of Education/National Center for Education Statistics, 1994)

Table 5: All Research I institutions† which awarded a high percentage (greater than 50) or number (greater than 40) of baccalaureates in mathematics and statistics to women in 1991.

	BAs in Math and Statistics				Total BAs	
Location/School	Men	Women	% Women	% Math BAs	Total BAs All fields	% Women
Northeast						
Yeshiva University	4	6	60%	3%	393	44%
SUNY at Stony Brook, all campuses	64	43	40%	5%	2,164	52%
Boston University	14	25	64%	1%	3,667	55%
Rutgers, New Brunswick	46	46	50%	2%	5,109	53%
Pennsylvania State University, main campus	88	82	48%	2%	8,293	46%
Midwest						
Indiana University at Bloomington	29	31	52%	1%	5,123	55%
University of Michigan at Ann Arbor*	51	46	47%	2%	5,477	49%
University of Minnesota at Twin Cities	86	47	35%	2%	5,561	51%
University of Wisconsin–Madison	63	43	41%	2%	5,869	53%
University of Illinois at Urbana-Champaign	71	43	38%	2%	6,068	46%
South						
Vanderbilt University	29	38	57%	6%	1,169	51%
Howard University	3	5	63%	1%	1,384	62%
Duke University	9	12	57%	1%	1,566	43%
University of Miami	4	5	56%	0.5%	1,820	46%
University of Virginia, main campus	19	27	59%	2%	2,815	50%
Louisiana State University	7	13	65%	1%	3,057	54%
North Carolina State University at Raleigh	18	27	60%	1%	3,406	40%
University of North Carolina at Chapel Hill	29	31	52%	2%	3,538	59%
Virginia Polytechnic Institute and State University	32	35	52%	2%	3,781	43%
University of Florida	30	34	53%	1%	5,498	50%
Hawaii						
University of Hawaii at Manoa	17	17	50%	1%	2,362	57%
West						
University of California–Los Angeles	97	88	48%	4%	5,105	55%
University of Washington	58	56	49%	2%	5,471	53%

—(US Department of Education/National Center for Education Statistics, 1994)

*Top 10 mathematical research institution

†Research Universities I: These institutions offer a full range of baccalaureate programs, are committed to graduate education through the doctoral degree, and give high priority to research. They receive annually at least $33.5 million in federal support and award at least 50 PhD degrees each year; there were 57 Research Universities I in 1991 (Frazier–Kouassi et al., 1992).

Table 6: Proportion of women earning BAs in mathematics or statistics at selected Women's Colleges and Liberal Arts Colleges

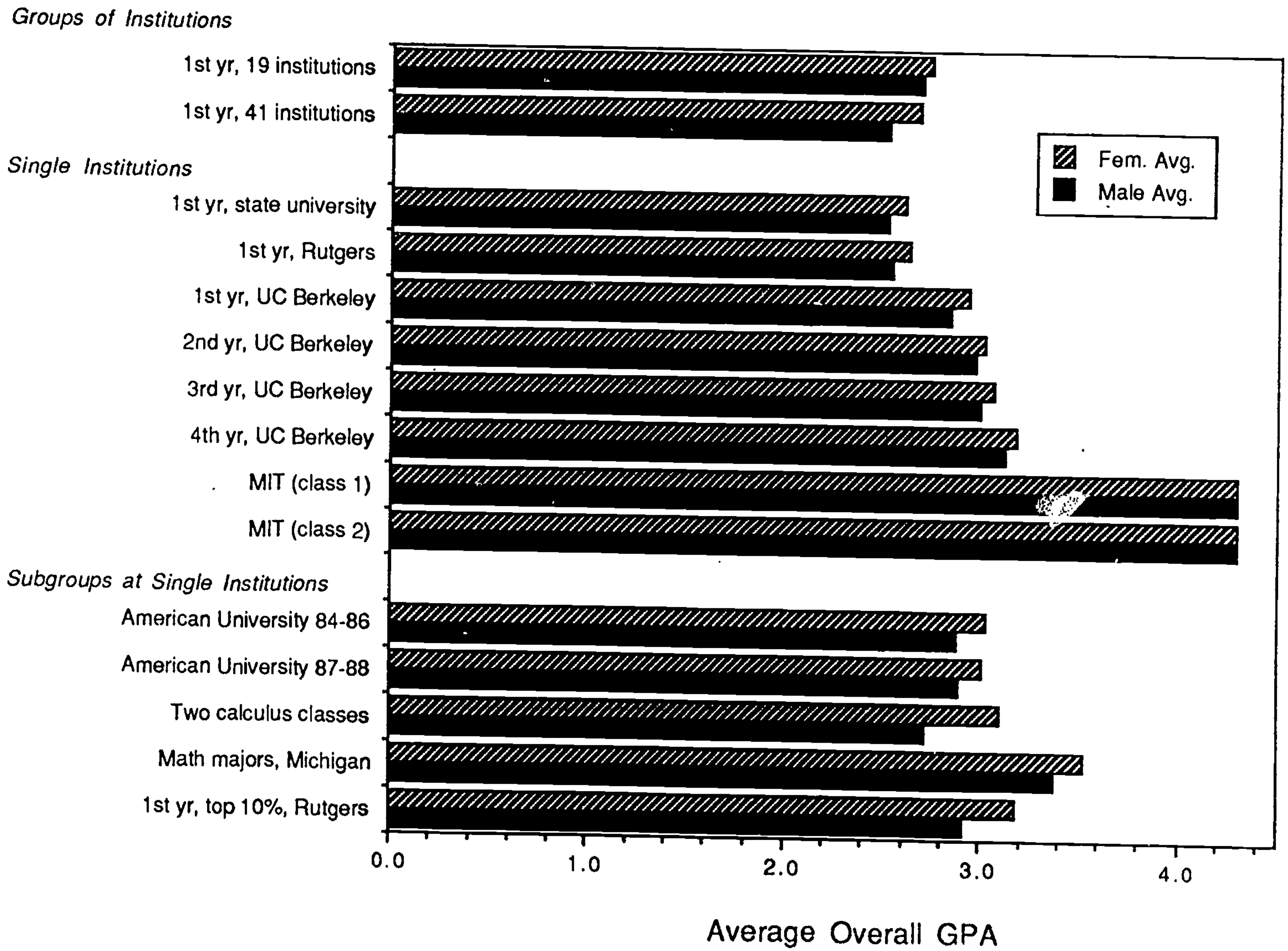
Women's Colleges (1991)	N Men	N Women	% Math BAs	Total BAs all fields
Mills College	-	3	1%	219
Bryn Mawr College	-	15	5%	295
Spelman College	-	23	6%	383
Mount Holyoke College	-	12	2%	526
Barnard College	-	3	1%	528
Wellesley College	-	10	2%	558
Vassar College	3	4	1%	628
Radcliffe College	-	14	2%	643
Smith College	-	19	3%	741
Total	3	103	2%	4,521

Liberal Arts Colleges (1991)	N Men	N Women	% Women	% Math BAs	Total BAs all fields	% Women all fields
Bowdoin College*	6	6	50%	3%	376	47%
Bates College*	3	6	67%	2%	402	54%
Hamilton College	15	10	40%	6%	425	49%
Middlebury College*	6	8	57%	3%	494	49%
Denison College	3	6	67%	2%	528	52%
Union College*	7	12	63%	4%	538	45%
Saint Olaf College	30	28	48%	8%	708	53%
Total	70	76	52%	4%	3,471	50%

—(US Department of Education/National Center for Education Statistics, 1994)

*Dropped SAT as admissions requirement

Figure 1:
Overall GPAs earned by undergraduate males and females at various institutions*



* See Table 1 for details

50

Figure 2: Grades in College Mathematics Courses

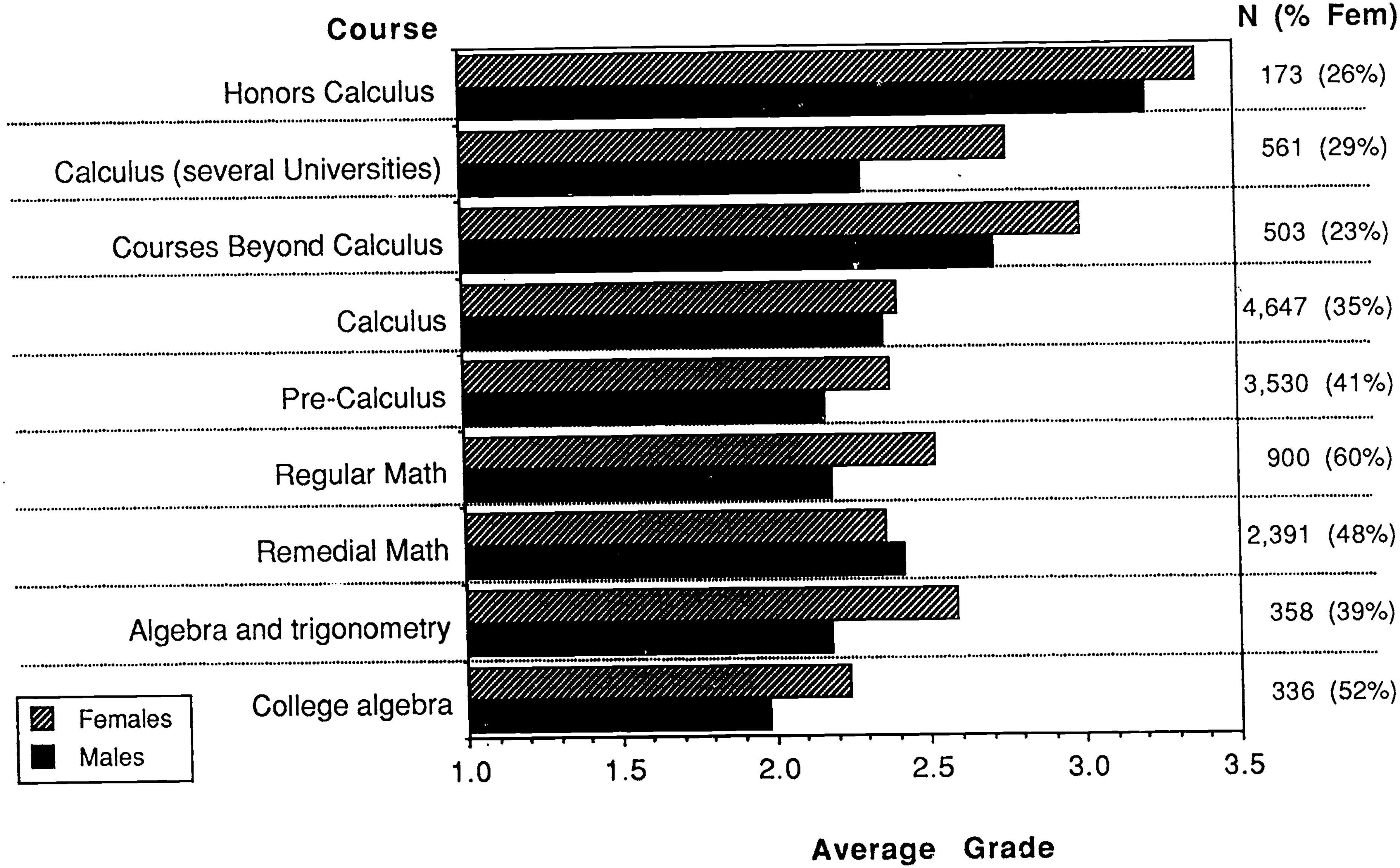
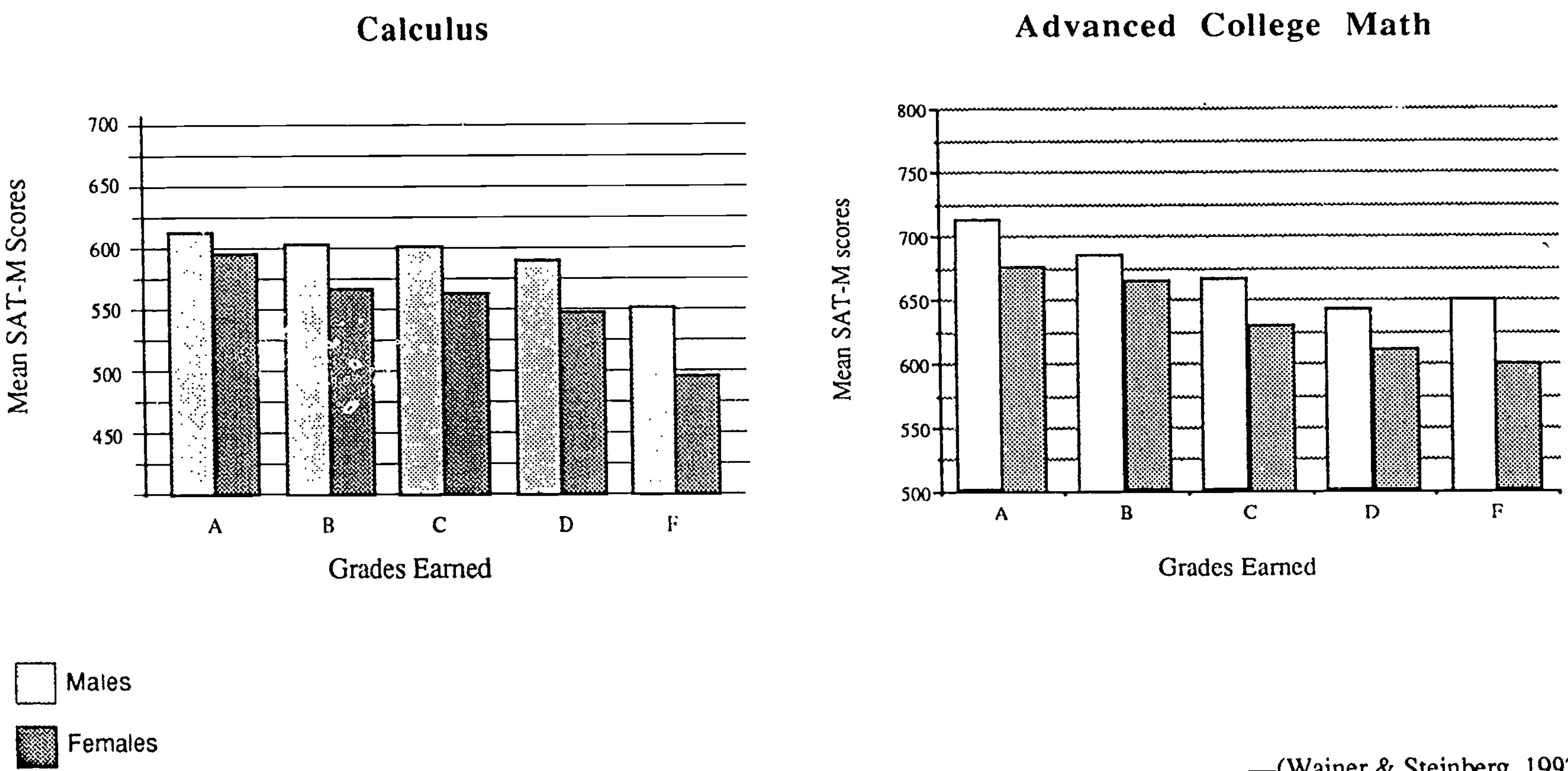


Figure 3: Mean SAT-M scores for males and females earning the same grade in college mathematics courses



—(Wainer & Steinberg, 1992)

Figure 4: Comments of students who switched out of mathematics

I always loved math, but, all of a sudden, I'm coming into sophomore year, and I started thinking about what I was going to do with my life . . . And people start asking what I was going to do with a math major, and where I was going to work, and I had no idea.

—White female, switcher to social science (Hewitt & Seymour, 1991, p. 36)

I always loved math, but, coming into the sophomore year, you know, I started thinking more about what I was going to do with my life. . . . And people kept telling us there were all sorts of things you could do with math, but they never told us what these were.

—Female white mathematics switcher, SAT-M over 650 (Seymour & Hewitt, 1994, p. 260)

I didn't know that Princeton had engineering when I first applied. I was thinking of math but randomly found myself at an electrical engineering orientation meeting. . . . Why I choose EE over, say, mechanical was because I liked math more than physics and the relation of EE seemed stronger to math.

—1993 Princeton engineering graduate (Ng & Rexford, 1993, p. 52)

Originally, I was going to go into mathematics; that's why I chose Princeton over Urbana-Champaign. It's sort of random how I got into electrical engineering: I met a math major who completely intimidated me, and I thought if everyone in the class was like him, I should try something else. I had heard that electrical engineering was very mathematical, so I thought I'd try it and see what it was like.

—1992 Princeton engineering graduate (Ng & Rexford, 1993, p. 38)

I see a lot of people here who have "gone to college" while still in high school, taken courses like this one already. They are math majors because they are two or three years ahead of me. How good a math major am I going to be if I am [already] two years behind?

—Harvard-Radcliffe student (Tobias, 1990, p. 77)

The grapevine has it that people don't major in math or physics here unless they were child prodigies to begin with. This isn't to say that if I had some overriding desire to do math or physics I wouldn't. But it is something to be overcome.

—Harvard-Radcliffe student (Tobias, 1990, p. 78)

Even though I had done really well in the math courses I took here . . . one of my section leaders told me that only people who start out in Math 55 keep taking math.

—Harvard-Radcliffe student (Tobias, 1990, p. 78)

It is really quite a shock when people are so competent. I went into Math 1A first semester, not prepared . . . and I had to drop back because of the kids who are majoring in science.

—Harvard-Radcliffe student (Tobias, 1990, p. 78)

Figure 5: Comments of female mathematics students about peer and family support

My godmother was an engineer from MIT when there were few female graduates from that institution and she encouraged me [to go into engineering]. I had an early interest in math, but decided to study engineering because I thought it was not as "academic" as pure math.

—(Ng & Rexford, 1993, p. 28)

I suppose because I left engineering, I think more of having been an "engineer" rather than being a "woman engineer." However, reflecting back on that time, I think I definitely experienced being a woman engineer. I decided to study engineering because my father was an engineer. In fact, I chose chemical engineering because my father was a chemical engineer; I was always surprised at how many other women engineers had a parent who was an engineer. For women, it always seemed that they had a reason for studying engineering—"I'm good at math and science." For the male students, studying engineering was never even questioned. If they shrugged and said they were studying engineering because they did not know what else to study, this reason was readily accepted. I don't think a woman could have gotten away with that, and that's significant.

—(Ng & Rexford, 1993, p. 42)

You miss someone to have camaraderie with. The guys would all be laughing and talking, and I'd sit there in the corner with my math book on my own. It wasn't until the 400 level classes that there were other girls in the class. And it was much nicer after that. Two or three of us would sit together, and have someone to talk to. Actually, it was out of class too—I mean we'd laugh and joke and entertain each other during class, and help each other with small things we'd missed. And if we didn't understand something the professor said, we'd huddle and check it out with each other.

—Female white science non-switcher (Seymour & Hewitt, 1994, p. 398)

I talked with my advisor a lot, and he would encourage me when I faltered. He'd say, "Look at your good grades. You can do this!" He was backing me up right from my own performance. I saw that he had the same kind of relationship with some of the guys, too—just that kind of mutual respect and support.

—Female white mathematics non-switcher (Seymour & Hewitt, 1994, p. 406)

Figure 6: Examples of calculus projects

If $a_1, a_2, a_3, \ldots$ are the positive integers whose decimal representations do not contain the digit 7, show that

$$\sum_{n=1}^{\infty} \frac{1}{a_n}$$

converges and its sum is less than 90.

Here are some suggestions: Break up the sequence of positive integers into carefully chosen blocks (not necessarily all of the same size), count the number of terms in each block, and see what fraction of them is left after removing those with an offending 7. Then bound the size of the sum of the reciprocals in each block, and try to compare this result to a convergent series.

(Cohen et al., 1994, p. 195)

An ant at the bottom of an empty sugar bowl eats the last few remaining grains. It is now too bloated to climb at a vertical angle as ants usually can; the steepest it can climb is at an angle to the horizontal with a tangent equal to 1. The sugar bowl is shaped like the paraboloid,

$$z = x^2 + y^2 \ (0 \le z \le 4),$$

where the coordinates are in centimeters.

(a) Find the path the ant takes to get to the top of the sugar bowl, assuming it climbs as steeply as possible. Use prior coordinates (r,q) in the xy-plane, and think of the ant's path as parameterized by r; then find a relation between the differentials dq and dr, and integrate this relation to get $\theta(r)$.

(b) What is the length of the ant's path from the bottom to the rim? To answer this, first discover a formula for arc length involving dz, dr, and dq in three dimensions.

(c) Draw a graph of the sugar bowl and the path the ant takes to get out.
HINT: You may want to start with the projection of the path in the rq-plane.

(Cohen et al., 1994, p. 208)

The alternating harmonic series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$

converges to ln 2. Now suppose s is any given number.

You are going to prove the amazing result that the alternating harmonic series can be rearranged (that is, its terms can be written in a different order) so that the resulting series converges and has s as its sum.

You will need to understand the precise definitions of the limit of a sequence and the sum of a series.

HINT: Start by trying to reorder the terms to alternately overshoot and undershoot s with various new partial sums.

(Cohen et al., 1994)

Figure 7: SAT items

372	434	124
62	310	558
496	x	248

10. In the large square above, the sums of the numbers in each row, in each column, and in the two main diagonals all are equal. What is the value of x?

    (A) 181
    (B) 184
    (C) 186
    (D) 188
    (E) 190

11. A certain number x is multiplied by 6. The number that is 5 less than x is also multiplied by 6. How much greater is the first product than the second?

    (A) 5
    (B) 6
    (C) 25
    (D) 30
    (E) It cannot be determined from the information given.

11. When the sum of two numbers r and s is subtracted from twice the difference of r minus s, the result is equivalent to which of the following expressions?

    (A) $2(r - s) - r + s$
    (B) $2r - s - r + s$
    (C) $2r - s - (r + s)$
    (D) $2(r - s) - (r + s)$
    (E) $2r - s + (r - s)$

12. The daylight period is defined as the time between sunrise and sunset of the same day. If sunrise was at 6:39 a.m. and sunset was at 4:47 p.m. on a certain day, at what time was the middle of the daylight period?

    (A) 11:30 a.m.
    (B) 11:43 a.m.
    (C) 12:00 noon
    (D) 12:13 p.m.
    (E) 12:24 p.m.

14. If the product of two consecutive positive integers is 5 more than their sum, then the lesser of the two integers is

    (A) 1
    (B) 2
    (C) 3
    (D) 4
    (E) 6

Figure 8: Comments of female students about the concept of mathematics instruction in undergraduate programs

There weren't many women in our classes, but there were a number of A.B. women in freshman chemistry and physics and organic chemistry. In those large lecture halls, the ratios didn't seem all that terrible. But math classes were another story. Only once did I find myself to be the only woman in a class. . . . It was also the only class I took where the professor obviously disapproved of a woman being there. Maybe he wouldn't have acted the same [way] if there had been other women in the class. How many times I heard him ask if I understood what was being discussed.

—Princeton student (Ng & Rexford, 1993, p. 64)

I found my introductory course in math to be very discouraging after advanced math in high school my junior year, which I excelled in. My intro course dampened my passion for math.

—Wellesley student (Rayman & Brett, 1993, p. 37)

She's teaching a whole different way from the way I learned it. I'm used to taking shortcuts and in her class you cannot take a shortcut. You have to go from one step to the next. If you miss a step, the problem is wrong even if you come up with the right answer.

—Student at southern college (Holland & Eisenhart, 1990, p. 169)

I definitely got frustrated. Freshman year was particularly bad, taking PHY 105 and MATH 203/204. I started in MATH 217 and dropped right down. That was a real confidence-buster. After that, I was used to not understanding things right away. Later, I found I could understand in time. The problem wasn't that the courses were too hard, but the sheer volume was [overwhelming]. I got through it, and I have to say that this year has been better. It's finally starting to pay off.

—Princeton student (Ng & Rexford, 1993, p. 38)

One of my calculus professors; I found him very annoying. When he would say anything and a girl would ask him a question, he would say girls don't have to know because girls don't have to study math. He meant girls in general, not me in particular.

—Student at large northeastern university (Lyons-Lepke, 1986, p. 8)

I had extremely good math teachers in my high school. They could always explain things five different ways, and one of the five usually clicked with someone. I came here and was really discouraged because, although the people teaching math had PhDs, and my teachers probably didn't have more than a master's, and a teaching certificate, I thought they taught a whole lot better . . . Actually, I'm surprised this college retains as many students as they do.

—Student at southwestern university (Hewitt & Seymour, 1991, p. 109)