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ABSTRACT

This study was based on recent research findings that mathematical activity is fundamentally situated and distributed across physical and social contexts. Interviews, observations, and examination of artifacts were used to explore ways in which a 2nd-grade teacher with seven years of experience understood and used mathematics inside and outside her classroom. The connections she made between school and nonschool mathematics, the ways she taught and learned mathematics in the classroom, and the ways she used mathematics outside the classroom were investigated. Data analysis revealed the following list of categories of mathematical activities within and across contexts: flexibly modifying plans, making sense, using physical objects, stating solutions, measuring and calculating new measures, recognizing multiple solutions, checking one's work, and drawing connections. Contains 35 references. (MKR)

Teachers' Mathematics Inside and Outside Classrooms: A Case Study of Mathematical Activity Across Contexts

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Running Head: TEACHERS DOING MATHEMATICS

I don't know about these things; I didn't go to school. I know about the oysters because we fish; the price of oysters we have to know. If they're selling 5 oysters at 750, then they're selling each one at 150.

—a woman who fishes for oysters

(Nunes, Schliemann, & Carraher, 1993, p. 11)

Many people use mathematics in highly efficient and productive ways at work and in other areas of their daily lives, yet this mathematical activity may seldom be recognized. Some mathematical practices are so embedded in the form and process of other activities that the mathematical properties of the performance go unnoticed (Nunes, Schliemann, & Carraher, 1993). Consequently, people often do not recognize structural equivalences in their mathematical activities across different settings. This case study investigated the mathematical activity of an elementary school teacher in different contexts of her everyday life, inside and outside her classrooms.

This study was based on recent research findings that mathematical activity is fundamentally situated and distributed across physical and social contexts (e.g., Lave, 1985, 1988; Nunes, Schliemann, & Carraher, 1993; Saxe, 1991; Scribner, 1984). Prior research indicates that people often spontaneously invent and use their own mathematical strategies that differ from school-taught algorithms, but that nonetheless lead to accurate and efficient solutions in daily settings. For example, complicated ratio and proportion problems are embedded in calculating best buys at the grocery store (Murtaugh, 1985), adapting recipe sizes (de la Rocha, 1986/1987), buying and selling candy to maximize profit (Saxe, 1991), and reading blueprints (T. N. Carraher, 1986). However, these non-school uses of mathematics are seldom considered "real" mathematics. As Resnick (1987) concluded in a review of this research, school arithmetic and non-school uses of number knowledge do not map well to each other: There are distinct discontinuities of performance between mathematics as taught in classrooms and as used outside classrooms. Students who do well on textbook problems are often unable to apply this school knowledge to interpret actual physical events (Masingila, 1992; Resnick, 1983). Similarly, dairy workers, grocery shoppers, child street vendors, carpenters, street corner bookies, and tailors who are highly accurate (~99%) in solving problems which emerge out of their everyday practices, drop significantly in accuracy on school-like, paper-and-pencil problems (Lave, 1988; Nunes, Schliemann, & Carraher, 1993; Saxe, 1991; Scribner, 1984).

Recent proposals for teaching and learning mathematics in schools have encouraged educators to integrate mathematics with other subjects and children's experiences in out-of-school settings (NCTM, 1989). Forging connections between school and non-school mathematical activity should provide a two-way bridge allowing students to draw on their real-world knowledge in the classroom, and their school knowledge in the real-world, strengthening practice in both arenas. However, given the above cited studies, a yet to be resolved question is, "If people do not make connections between their highly competent mathematical activity in everyday settings and structurally equivalent school problems, what degree of success can schools and teachers achieve in helping students to integrate mathematical experience across diverse settings?" This question provided the focus for the present research study. I explored ways in which one elementary school teacher understood and used mathematics inside and outside her classrooms. I investigated the connections she made between school and non-school mathematics, in the ways she taught and learned mathematics in classrooms, and in the ways she used mathematics outside classrooms. Consistent with previous research in everyday mathematics (e.g., Lave, 1988; Scribner, 1984), I actively participated in her life, trying to describe mathematical concepts and processes engaged in context, using the constant-comparative method to interpret findings (Lincoln & Guba, 1985; Strauss & Corbin, 1990).

Research Design

Participants

To establish a detailed account of participants' relationships to mathematics within and across varied contexts of their lives, it was necessary to limit participation to a small number of participants. I attended one of the first meetings of a university mathematics methods course for elementary school teachers to solicit participation from teachers interested in discussing their own uses of mathematics, inside and outside their classrooms. Two women, Geena and Nora (both pseudonyms) agreed to participate. This paper focuses on Geena's story only.

Geena was a full-time grade two teacher with seven years of teaching experience (six of which were in grade 2 or grade 2/3 classrooms). She had decided to return to university on a part-time basis to complete her degree in English Literature. As one of her elective courses, she decided to enroll in the Designs for Learning—Mathematics (Elementary) course because she "thought [her] math program is kind of boring it needs something to. I want to rewrite it. I want to do something I'm just not sure, I don't know what but it's time to do this course." (GJ 1.3, 102693). As she started getting involved in the course, she found that everything was really coming together for her. From the first assignment, she realized the importance of connections in mathematics, in particular connecting mathematics to the outside world. This realization was the impetus for her participating in this research project. As she explained:

> I had done it [the initial journal assignment] all before I had even thought. That's what triggered me into doing this [agreeing to meet me to find out more about this research project]. And I'm just thinking oh my god this is just too good to be true. It's all, it's all tying in. Math connecting to the outside world, Oh my god, I guess I should do my major project on this because it just seems to be falling into my lap. You know? (GJ 1.1, 931021)

Throughout our conversations, Geena emphasized the connections among her mathematics teaching, her methods course, her non-school mathematics, and her participation in this research project. In the end, she rated the course as excellent and found that both the course and our time together really helped reinforce what she was doing in the classroom.

> I found that any of the work that I had done, all of a sudden it started to tie together. Like when I first went in I thought I want to revise my math program. I want to work at it. This is my goal, here. And what I found is that, I didn't get done what I wanted to do, what I thought I wanted to do. I thought I wanted to revamp everything that I was doing. I was getting bored, blah, blah, blah. And what I found I was doing in that course was I was totally reinforcing that everything that I was doing really was what I really believed in, and perhaps I felt it was boring um because I was unsure of myself. I wasn't really bored, I was just unsure.... And then just all of a sudden noticing, and meeting with you and talking about things really helped me. (GJ 4.8, 940202)

Procedures

Gaining Entry and Securing Participation

After establishing initial contacts through Geena and Nora's mathematics methods course, we set up individual meetings where I could explain the study more fully and ensure that our interests were mutually compatible. During the first session with each participant, I carefully explained my research interests and my plans for the research project. I felt that providing a full description was important because I wanted the research to be conducted openly, with no hidden meanings or agendas. The two teachers both agreed to participate in a series of interviews and observational sessions.

Interviews

During interviews, our conversation smoothly flowed around topics of mutual interest, predominantly those topics related to mathematics, teaching, and learning. I set regular meetings throughout the research with the participants to discuss research questions, emerging analyses, and any issues that arose as the research progressed. Formal interviews loosely follow an interview schedule consisting of a general list of topics or questions to discuss, but we freely introduced new topics and materials as they arose in the setting (Ellen, 1984). Informal interviews involved questions that arose from the situation at hand during field observations. I audiotaped conversations whenever feasible, supplementing this with written notes to help focus interviews.

Research Notebook

I gave each teacher a small notebook to record any further thoughts they had about our interviews. In addition, I asked them to record incidents, processes, concepts, perceptions, and understandings of: (a) school math learning, (b) school math teaching, and (c) non-school math functioning. They were asked to record what mathematical functioning was required, how the problem was solved, how they felt about the problem and its solution, and why they elected to record that problem. I retained copies of all entries made in the research notebooks for document analysis. During subsequent interviews, we discussed and elaborated these entries, providing further data.

Both women made entries in the research notebook during the first few weeks of our interactions. After this point they discontinued using the notebook. Although some interesting data was garnered in this way, I did not pressure the women to continue using the notebooks because I felt that this was a research task, and provided little direct benefit to the women.

Non-School Mathematics

In initial sessions, we discussed how and when each woman used mathematics inside and outside her classrooms (at the university and in the elementary school), and decided together where I could observe this activity. I explained my interest in selecting some mathematical task (or tasks) in which they engaged on a fairly regular basis, and then observing them solving the task "in context." Nora chose comparison shopping and, on two subsequent occasions, I accompanied her as she (a) compared the prices of pet products in pet stores in her neighbourhood; and (b) compared prices of food, clothing, and household items at an American warehouse outlet to prices at local Canadian stores (necessitating a currency exchange). Geena chose baking, so I spent an evening in her kitchen as she prepared two items.

As the women engaged in their selected activities, we, simultaneously, carried on a discussion to elaborate the mathematical concepts and processes involved (cf. de la Rocha, 1986/87). In this way, observations of non-school mathematics were "slightly artificial" because the women were not just engaging in the activity, but engaging in my presence as part of this research project. In this way, the descriptions I provide are not of mathematics in some abstract sense, but of situationally appropriate mathematics. For example, Geena was not just baking cookies, but baking cookies to show me her non-school mathematics. At the same time, the ongoing talk during the baking activity was directed to me in an effort to make her actions rationally accountable and to provide information she felt was necessary in the context of the research project. As a specific example, when Geena was measuring the shortening for her biscuits she explained how she would use volume displacement to measure the shortening if she was doing a lot of baking, but "I'm not going to do that because I've got this [box of shortening with a gauge], but I thought that was an important thing to say (laughs)" (GJ 2.1, 931026). In this way, the situated nature of the women's mathematical activity must be considered.

I documented my observations of the women's non-school mathematics in fieldnotes and with audio and videotapes, as appropriate. I was able to videotape Geena's entire baking session as we worked undisturbed in her kitchen. On the other hand, Nora's shopping expeditions required going to several different stores and it did not seem feasible to audio or videotape in these places of business. Instead, I took fieldnotes in the stores, and then audiotaped our debriefing sessions back at Nora's home. Even taking fieldnotes in the stores was disruptive to the businesses during our cross-border shopping trip. Nora explained the research project to her neighbourhood vendors when we were comparing prices for pet supplies, so I was able to freely take fieldnotes. For our cross-border shopping trip, Nora did not know the vendors, so she provided no explanations. In this way, the vendors were suspicious of my furtive note taking, and in one store we were treated as "corporate spies," that is, the salesclerks thought we were from their Head Office and had come to check their store, so they hovered around us the entire time we were there.

In addition to my observations of Nora's shopping and Geena's baking, other descriptions of non-school mathematics came up during interviews, in the research notebooks, and in course assignments. I did not observe activity in all these contexts, but I did document the women's verbal and written descriptions.

Teaching School Mathematics

I also observed both teachers in their classrooms for one day while they were teaching mathematics and language arts to see how mathematics was integrated into the teaching portion of their everyday lives. I collected lesson plans, materials, assignments, and class planning notes, and wrote field notes during and after my observation periods.

Learning School Mathematics (Pedagogy)

I also collected assignments from their methods course and any other materials they felt were important to their school mathematics. For example, Nora brought me a copy of a Functional Math Program for special needs young adults that she had designed and taught for several years.

Researcher Notes

Throughout the research process from initial planning to final stages of the analyses, I kept a record of my reflections, feelings, reactions, insights, and emerging interpretations. These emerging interpretations and reflections provided an audit trail (Lincoln & Guba, 1985) and formed an integral part of the final analysis.

Data Sources

From the procedures section it is clear that this paper draws from the primary methods of data collection used by ethnographers: participant observation, ethnographic interviewing, artifact examination, and researcher introspection (Eisenhart, 1988). These methods provided various sources of information related to the research questions and allowed multiple perspectives and triangulation of data.

Meetings were conducted on an ongoing basis from October 1993 through February 1994. Overall, I met with Nora eight times for a total of 16 hours, and with Geena six times for a total of 10 hours. Video and audio taping, as well as researcher field notes, were used extensively to document observations and interviews. Audio and video taping provided a permanent record of interactions which could then be reviewed and analyzed many times (Erickson, 1986; Marshall & Rossman, 1989; Merriam, 1988). This was especially helpful when new hypotheses and theories emerged later in the research process, as I was then able to return to earlier segments to check the consistency of my interpretations.

A further benefit of audio and video taping is that the transcripts were available to share with participants and others. As the data analysis proceeded I shared transcript excerpts and my emerging understandings with Dr. Michael Roth and our mathematics thesis group (a group comprised of Dr. Roth, myself, and four mathematics teachers working on Masters theses in mathematics education). This served as *peer debriefing* and provided multiple interpretations and layers of interpretations throughout the research process, thereby strengthening the research design. As Eisenhart (1988) explains "in ethnographic research, the more perspectives represented, the stronger the research design, because each additional perspective contributes to a more complete picture of the scene of interest" (p. 106).

Observations were conducted across various contexts, as participants taught mathematics in their elementary school classroom, learned mathematics at the university, and used mathematics outside classrooms. I recorded observations relevant to the research by taking written notes, drawing diagrams, gathering documents, and video and audio taping, as appropriate.

Formal and informal *interviews* continued throughout the research process. I used this time to discuss the questions from the initial interview schedule and to provide clarification of issues arising during observations, document analysis, transcription, and preliminary stages of the analysis.

Artifact examination has been defined as a "content search of written or graphic materials available on the topic of study" (Eisenhart, 1988, p. 106). I examined (and made copies of) homework and assignments (from university courses and elementary school classes), elementary lesson plans, and all notebook entries, plus Nora's Functional Math Program description.

Researcher introspection involved my reflections on the inquiry activities and context (Eisenhart, 1988). This reflective process is essential to research: researchers must constantly reflect on the self in relation to research because researchers are part of the social world they study (Hammersley & Atkinson, 1983). As described earlier, I kept a record of my reflections, feelings, reactions, insights, and emerging interpretations throughout the research process. These emerging interpretations and reflections guided the emerging research design and formed an integral part of the final analysis.

Data Analysis

Data from observations, conversations, documents, and researcher introspection were analyzed by following examples from previous research (e.g., Lave, 1988; Scribner, 1984) and drawing on principles of the constant-comparative method (Lincoln & Guba, 1985; Strauss & Corbin, 1990). Data analysis followed an inductive process of emerging theory through the processes of unitizing and categorizing (Lincoln & Guba, 1985; Strauss & Corbin, 1990). Data collection and analysis proceeded as ongoing, recursive and dynamic processes (Merriam, 1988). I began analyses with the first notes, continuing in a reflective, dynamic manner. Categories emerged and changed as data collection and analysis proceeded. I continuously compared and revised categories and began early trying to understand Nora and Geena's mathematical activity across varied settings and situations. I used my field notes, audio and videotapes, transcripts, teachers' notebooks, and input from others (participants, as well as other teachers, researchers and students) to try to piece together a holistic picture of the participants' mathematical activity within and across contexts. Throughout the entire process I searched for confirming as well as disconfirming evidence (Erickson, 1986; Lincoln & Guba, 1985).

Consistent with my interest in keeping the research open, I encouraged both women to be involved in all aspects of data collection and analyses, but other contingencies (busy lives, research deadlines, and missed connections) prevented Nora and Geena from engaging in much of the analysis process for the research project.

Once data collection ended, I turned my full attention to data analysis. I read and re-read all transcripts, documents, and field notes accumulated over the course of the data collection stage. After several readings and multiple attempts to make sense of the information I had collected, I arrived at the following list of categories: Flexibly Modifying Plans, Making Sense, Using Physical Objects, Stating Solutions, Measuring and Calculating New Measures, Recognizing Multiple Solutions, Checking Your Work, and Drawing Connections.

Next, it was important to organize these categories and define links among them (see Table 1). *Flexibly Modifying Plans* seemed to frame the self-regulated nature apparent in all of Geena's mathematical activity. Further, *Making Sense* was an overriding concern in all of Geena's mathematical activity. Her efforts to regulate her behaviour (while flexibly modifying plans) were all centered around the importance of Making Sense. Using Physical Objects, Measuring and Calculating New Measures, Recognizing Multiple Solutions, Checking Her Work, and Stating Solutions are more specific mathematical activities in which Geena engaged. These five categories are presented according to the typical chronological order for solving one problem. For example, when Geena was preparing the squares, she used her grater (*Using Physical Objects*) to add "this many scrapes" (*Measuring and Creating New Measures*) of fresh nutmeg. She then explained how she had in other cases used a measuring spoon to measure pre-grated (by her or from the store) nutmeg (*Recognizing Multiple Solutions*) She also explained that she used smell to confirm the appropriateness of the measure of nutmeg (*Checking Her Work*), but that the real test was in the tasting (*Stating Solutions*). Finally, *Drawing Connections* is the category that seemed to summarize all that Geena and I did together, so I use this category to summarize my understandings about Geena's mathematics.

In the following sections, I highlight findings relevant to each of the eight final categories to provide a flavour for Geena's mathematics within and across contexts. I begin with a description of Geena's non-school mathematics, then her school mathematics, followed by some summary descriptions.

Non-School Mathematics

Geena chose baking as a prominent example of her non-school mathematics, so I spent an evening in her kitchen as she prepared two items. First, she baked biscuits by doubling one of her favourite recipes, substituting for some of the ingredients to match her preferences. Next, she attempted to make roll-out cookies from a standard recipe that she had only used once or twice. After mixing all the ingredients together, she realized that the dough was too dry and could not be rolled and cut into cookies. After checking the recipe and hypothesizing the reason for this "failure," she decided to combine the dough with some leftover apples from her classroom to make apple squares. In addition to the evidence gathered during our baking session, Geena also provided verbal and written description of her other uses of non-school mathematics in her research notebook, her course assignments, and in our interviews. The remainder of this section, describes the mathematical activity inherent in Geena's non-school mathematics, focusing on the eight categories.

Flexibly Modifying Plans

The concept of following plans provides an overall framework for Geena's mathematical activity across the various settings observed. In her kitchen, Geena's behaviour was directed by her goals for the activity, which included baking biscuits for herself, baking cookies for teachers in her school, displaying her non-school mathematics, and talking with me about mathematics and baking. These general goals provided a plan for her activity, but this plan evolved over the course of the action. In the context of pursuing these goals (or enacting the plans), other aspects of the environment contributed to the emerging product. Geena used the written recipes as guides to direct her behaviour, but substituted different ingredients, doubled the written measures, approximated measures, and even abandoned the cookies to make squares instead. The artifacts (the biscuits and squares) emerged from the interaction of available tools (measuring cups and spoons, the gauge on the shortening box), materials (the written recipes, baking ingredients specified in the recipe, substitutable ingredients not specified in the recipe, apples left behind by students in her class), the setting (her kitchen), community standards (taste, cookies should not be made with whole wheat flour), and the artifacts themselves (too dry cookie dough). In this sense, Geena started with a general plan, but the product emerged over the course of the baking activity. This evolution of a plan is analogous to the way in which elementary students' bridges and towers evolved from "vague" ideas through interaction with the learning environment (Roth, 1994). A similar process of evolution through flexible plans has been observed in the work of scientists and engineers (Bijker, 1987; Constant; 1989; Suchman, 1987; Starling, 1992). This flexible evolution of plans was central to Geena's baking.

Making Sense

Geena was concerned with making sense of her mathematics, rather than indiscriminately applying algorithms. In her non-school mathematics this was evidenced through her focus on mathematics as a "way of thinking" rather than emphasizing calculations. Evidence of the ways that Geena used mathematics for understanding rather than manipulation is best illustrated in her description of how she could operate within different systems of measurement, but did not convert between them. When she was measuring the brown sugar for her cookies which had by that time been modified to become apple squares, Geena used a metric measurement for the first time (150 mL). When I asked her about this in a later meeting, she explained:

> Oh, I know why. Because I probably measured it just by, I just grabbed a cup that I measure with and it's easier to measure brown sugar with the metal. I think I must have used the metal cup.... And that, I don't have one in cups, it's milliliters. That's why I switched. (laughs) (GJ 4.2-4.3, 940202)

Similarly, Geena described how she used both Fahrenheit and Celsius, rather than converting between the two systems of measurement:

> See I don't convert when it comes to. Same as the milliliters and cups, I don't convert. I use, this is what I have, so this is what I use. This is, I know what eighty degrees is, but I also know what 25 and 30 degrees is in Celsius. I know. I don't convert. Well 80 is minus this and do this. I just go, okay if it's a really hot day, 80 degrees is quite a hot day here in Vancouver. If it's 25 or 30 degrees that's really hot here in Vancouver too. If it's minus, um, um, if it's 32 degrees that's cold, if it's zero that's cold, if it's minus five it's cold.... And if I had, if I was following a recipe that was in the metric then I would be reading the metric stuff. I wouldn't convert. I would just pull out my stuff and I would measure on the milliliter side. (GJ 4.3, 9402020)

In each of these cases, Geena relied on her understandings of mathematics rather than manipulating numbers or using algorithms. Her descriptions and the activities I observed, illustrated a personal, experiential dimension to her knowing akin to Greeno's (1991) notion of number sense. This number sense was at the heart of all of Geena's mathematical activity.

Using Physical Objects

Geena's measuring in the kitchen relied upon and was constituted by the tools available in her environment. To measure the shortening, Geena used the gauge on the package because it was available and because she was only baking a few items, not doing a "marathon baking session." If she was planning to do a lot of baking she would buy a large tub of shortening which would not have a gauge, and then use a measuring cup and the principle of conservation of volume, as she described:

> OK. Now this [shortening] is really easy to measure because it's got a measure, a tape measure right here. When I'm doing something where, if I'm doing a lot of baking quite often as a Christmas present I'll do a lot of baking. And I buy the big huge tub of Crisco. So when I'm measuring Crisco or shortening whatever, what I do. If I need, I would use a bigger measuring cup, but just say I just need a quarter of a cup what I would do is just fill is fill this half mark with cold water then I would scoop out of the tub the shortening until it actually reached the three quarters and then I would have a quarter cup measured. I find that, my Dad taught me that. When you take shortening and put it in a container and then you take a spatula at it, it gets real mucky and messy and sometimes you're not getting as much. (GJ 2.1, 931026)

Similarly, Geena's access to a 1-cup measuring cup marked with 1/4 cup increments may have prompted her repeated addition strategy for doubling the 1 3/4 cups of flour for the baking powder biscuits. Also, when measuring the brown sugar for the cookies, which had by that time been modified to become apple squares, Geena used a metric measurement for the first time (150 mL). "Because I probably measured it just by, I just grabbed a cup that I measure with and it's easier to measure brown sugar with the metal....And that, I don't have one in cups, it's milliliters." (GJ 4.2-4.3, 940202). The available tools shaped Geena's activity by prompting certain actions and preventing other possible actions.

Measuring and Calculating New Measures

Measuring is one of the six fundamental activities that Bishop (1991) argues are "necessary and sufficient for the development of mathematical knowledge" (p. 32). He defines measuring as "quantifying qualities like length and weight for the purposes of comparing and ordering objects" (Bishop, 1991, p. 32). Measuring occurred frequently across the various contexts I observed. The process of measuring in Geena's everyday mathematics is deeply tied to the presence and use of tools, which will be elaborated in the next section. For Geena's activity in the kitchen, measuring typically involved estimation and the use of non-standard measurements, as exemplified when she measured the cheese for her biscuits as described in the following scenario.

> Geena eyed the package of cheese and cut off a chunk. Without prompting, she explained, "I have no idea how much this is, but this is the size of chunk I want." Then she queried whether she still had her scale (a remainder from her Weight Watchers dieting days), but made no attempt to look for it. She quickly changed approaches and told me, "I can figure it out." She held the chunk of cheese she had cut off and marked a smaller chunk with her knife, explaining, "This is one ounce, so I can figure it out." She further explained that her ability to estimate the size of an ounce of cheese was one of the estimation skills refined during her Weight Watchers' experience. She proceeded to mark off further portions of the cheese, counting, "One, two, three, ... eight ounces." After this estimation, Geena then grated the large chunk of cheese and set it aside. (GJ 2.1, 931026)

In this example, Geena made two related estimates. First, she estimated how much cheese to add to the biscuits. This estimate was enacted when she chopped the large chunk from the cheese block. Second, through her fractionating strategy, she estimated the volume of this chunk of cheese, stating "it's about eight ounces." Her estimation was based on a combination of her cooking, tasting, and dieting experiences. She recalled that a chunk of cheese the size she cut was appropriate for her cooking goal to make cheese biscuits. The estimation was influenced by her taste preferences because she knew that she liked biscuits when they had that amount of cheese. Further, her experience in the Weight Watchers dieting program had raised her awareness and increased her ability to estimate food amounts, as de la Rocha's (1986/1987) found.

A further example of Geena's measuring is evidenced in the new measures she created by doubling the biscuit recipe. The first quantity listed in the recipe was 1 3/4 cups of flour, which when doubled would yield 3 1/2 cups of flour. Transcripts of the videotaped observation and field notes recorded that Geena engaged in the following actions:

> *One and three quarter cups*. (Reading the recipe. She fills the 1 cup measure) I'm terrible with math. If I know the, when I go to double a recipe if I can double it really easily then I do. If I'm not sure then I do, (Adds a second full cup) I put two in because I need two full cups and I need two three quarter cups. So, one three quarters of a cup. (She fills the measuring cup to the 3/4 line, shakes it slightly, and checks the level, but shows little concern for the precision of her measure.)

> And then I need one [3/4 cup] more (Fills the measuring cup to the 3/4 line and adds the flour to the mixing bowl). (GJ 2.1, 931026)

Here, Geena translated the multiplicative problem, doubling 1 3/4 cups, into a repeated addition problem. However, her actions showed more than simply repeated addition of the form 1 3/4 cup + 1 3/4 cup. She performed a commutation such that the addition was performed as 1 + 1 + 3/4 + 3/4. This repeated addition was situationally prompted by the tool Geena used to measure the flour, a 1 cup measure with 1/4-cup markings. This representation led to the solution of adding 3 1/2 cups of flour, although Geena's method made no explicit reference to this total amount—her calculation of the new quantity was situationally embedded in the context. As our conversation continued, Geena admitted that she could calculate the answer to this mathematical problem, and had done so in other instances, but had simply elected not to do so at this point:

> I guess it is easier, but it's not that I can't do it....if I sat down and I calculated it out. There are some recipes where I have actually calculated it out and written it right on the recipe. But I don't always double this, and some times I triple it so I haven't written on this recipe. But, for me, sometimes it's faster, especially if I'm tired, and I'm tired. (GJ 2.1, 931026)

Here, it becomes very evident that Geena's goals and actions were influenced by multiple constraints. The particular recipe she worked with was one that she sometimes doubled, sometimes tripled, and sometimes made a single batch. Regardless of which multiple she used, her general approach of using repeated addition would work. Other recipes where she consistently doubles the ingredients she calculates the values and writes those down. Her admission of tiredness reveals another potential impact of my presence and the research needs. After a long day in her classroom, normally she would not bother with baking, let alone explaining each of her actions to an onlooker.

In addition to the algebraic issues of calculating this new quantity, Geena's actions also raised measurement issues. Each time Geena filled the measuring cup, she shook the cup slightly and checked the level, but showed little concern for the precision of her measurements. Approximate measures are close enough to the recipe directions to yield successful performance, therefore validating the situational appropriateness of the level of precision Geena adopted for her measurements (cf. Lynch, 1991). Using the precise measures and controlled experimentation that is expected in physics, Kurti and This-Benckhard (1994) have demonstrated that many modifications can be made to a recipe with very little effect. Geena, however, does not rely on physicists' forays into the kitchen. She knows from her experience that some recipes and some ingredients demand more precision than others. Her baking powder biscuits are flexible and don't require much precision. When it comes to cookies, with which she is less familiar, she takes greater care with her measurement, especially baking powder which "can really make or break cookies." The kinds of precise measures required in physics and other scientific pursuits are not only unnecessary for the success of baked goods, but they would be difficult, if not impossible, to achieve given typical kitchen tools. Measuring cups and spoons are not necessarily precisely calibrated, and actions such as shaking a powdery ingredient influence the settling of contents, thus impacting the precision of a measurement. Such differences would be problematic in physics, but seem not to be so in the kitchen.

These examples of measuring demonstrate Geena's strong reliance on the tools available in her environment—the assorted measuring cups and spoons, the gauge on the shortening box, the cheese grater and other supplies she used in her kitchen. These tools are the topic of the next section.

Multiple Solutions

In her baking, Geena's actions served as one solution to the problem of making the biscuits and cookies/squares. However, as she worked, she frequently explained alternative solutions she could have adopted, or that she had adopted in the past. To measure the shortening, she used the supplied gauge, but described how she could have used volume displacement instead. To double the flour, she enacted a repeated addition strategy (with commutation), but explained that she could have multiplied instead. Throughout our conversations, Geena emphasized the existence of multiple solutions and described some of the constraints affecting her selections among those alternatives.

Checking Work

Geena continually monitored her actions as she baked, checking her work on an ongoing basis. As she mixed the cookie ingredients together she observed, "This seems awfully dry." Her initial reaction to this observation was to confirm the accuracy of her procedure and hypothesize what might have gone wrong.

> *Butter*, no, *egg*, *vanilla*, *sugar*, *baking powder*, *flour*. (Reading the recipe) It's very dry. (Her brow is furrowed) That's interesting. It molds, but I don't know. No, that's going to crumble....Okay, so now what am I going to use it for? Well, isn't that interesting. I wonder why? Flour's old? I wonder if that makes a difference? It shouldn't. I mean, flour sits in flour mills. That's really, it's not going to do anything. Well, let me see. (Molding dough in her hands as she speaks)...I don't understand why. *The dough will be very stiff* (Reading). Well, yes I can see it is very stiff. *Do not chill*. Oh yeah. *Divide it into two balls*. I can't divide it into two balls, I can't get even one. (laughs) *On a floured surface*. Like more flour's going to make this work. Right?! (laughs) Nope....I measured a *cup* [of butter], a *cup* [of sugar], *one large egg*, *teaspoon of vanilla*, *two teaspoons of baking powder*, *3 cups of flour*. (Reading again) But I've done it before. I mean. Well, the flavour will be there. We'll adapt. (GJ 2.6, 931026)

Throughout the interaction, Geena referred to the authority of the recipe, reading the list of ingredients, the directions, and then the amounts, as well as relying on her own knowledge (what cookie dough should look like, where flour comes from, her past success with these cookies). She seemed to be searching her knowledge base for an explanation for this anomaly. After a few minutes, she turned her focus from "I wonder why?" to the question of "what am I going to use this for?" This problem reformulation provided a solution to her dilemma, and she proceeded to make the apple squares.

Stating Solutions

In the kitchen, getting an answer typically does not require stating a solution, rather the answer is realized through embodied actions. As described in the Measuring section, Geena cut a chunk of cheese for the biscuits and then, as an afterthought, attempted to assign a value to this amount of cheese. Initially she said "I have no idea how much this is, but this is the size of chunk I want," but then quickly went on to "figure it out" by marking off smaller chunks and tallying the number of smaller chunks. Assigning a value to the amount of cheese served to rationalize her behaviour to me, an observer in her kitchen, rather than serving an integral function in the act of baking.

Similarly, in doubling the 1 3/4 cups of flour called for in the biscuit recipe, Geena performed repeated addition with commutation, but made no explicit reference to the total amount of flour she added. In the kitchen, an appropriate solution is to add the required amount of flour rather than name the amount added. In the classroom a numerical answer is required, in the kitchen the embodied action of adding the right amount of flour is required. This distinction reveals an underlying difference in goals for the two settings. As Lave (1988) has described, mathematically isomorphic problems presented in different situations may entail different goals requiring different solution processes.

Drawing Connections

In our conversations, Geena recognized the prevalence of mathematics in all of her activities. She continually referred to the fact that mathematics was everywhere, that mathematics was a "way of thinking." She explained this philosophy in her initial journal assignment where she reflected on the article by Corwin (1993):

> Corwin's article has helped to reinforce my beliefs that math is a way of thinking. It's around us all the time in our daily lives. We don't just "learn" Math. We "live" it....Math is more than numbers, equations, and algorithms....Math is a part of everyday life...Mathematics is not just a lesson given at a specific time of day. As we identify that Reading, Spelling and Writing are used constantly in everyday life, so do we experience Mathematics in the same ways. It is because of our mindsets that we may not realize this. We no longer need to set Math aside for manipulation. We need to use Math for understanding. "Mathematics helps (students) make sense of the world (p.339)." (GJ journal.1-3, 931018)

This philosophy was especially evident in our final meeting, when I showed Geena the list of contexts for non-school mathematics that I had created from our conversations and the documents she wrote. I asked her if I should add anything else to the list and she responded that the list could be expanded indefinitely because mathematics is everywhere.

> There is so much of math that is in everything that just happens (snaps her fingers twice) instantaneously and then you move on. You know, that kind of thing. So, it would be like um relating your language arts. You could literally videotape somebody all day long and everything that they do you would pull out, and that's. I think that's why I am, I am, so adamant in that, and I'm sure that Nora is the same way. That math literally is everywhere. So you could take all day long and you could pull out math, all day long, as opposed to a specific situation. Yes you can do specific situations, but I think the specific situations like baking, like banking, those kinds of things, um, the average person probably comes up with, 'Oh yeah that is math, for sure.' It's the things that happen on a regular basis, all the time. The little things that are minutes long, seconds long, whatever. That they're not making those connections with. (GJ 4.11, 940202)

Clearly, Geena recognized that she was doing mathematics all the time, even if she wasn't always consciously aware of this fact. She connected mathematics and her everyday activities.

School Mathematics

To provide a contrast to Geena's non-school mathematics, I also collected data on Geena's school mathematics. I attended her elementary school class one day, and her mathematics methods class one evening. I made copies of her lesson plans and her assignments. We discussed these activities together and I recorded these conversations. I now use the eight categories to describe Geena's school mathematics.

Flexibly Modifying Plans

In her classroom, Geena adopted a flexible scheduling approach, but allowed less flexibility to her students. At the beginning of the year, she wrote a Yearly Preview for her school principal outlining her planned activities, but she freely deviated from this schedule as the situation demanded. For example, she switched the order of classroom activities for January and February. The home mathematics unit plan she designed in her methods course to be implemented in January turned into a few exercises that she sent home in February and March. Similarly, individual classroom activities were flexible enough to allow modifications and redirections. In particular, we discussed how she picked up on mathematical themes whenever they surfaced in the classroom, rather than tying herself to a set time slot to pursue mathematics. For example, during language arts the day I was in her classroom, Geena read a chapter from the current class novel, Mrs. Piggle Wiggle. At one point in the story, the author wrote that the mother spent three hours cleaning the boy's room and only one hour to do all the rest of her housecleaning. Geena paused in her reading to ask how much longer it took the mother to clean the boy's room than to do the rest of her housework. Several students raised their hands to respond, and Geena called on one child who accurately responded "2 hours." I asked Geena about this during our next interview, and she explained that she often took time to ask mathematical questions when they came up in literature:

> Any time it comes up in the book, I pull it out. If I, If I, um, not if I think of it, but if I decide yes, this is worth pursuing right now or if it's not too far above them. If they're ready for it. That kind of thing. Sometimes we can get into real um multiplying, fractions, those kinds of things, depending on the story, what's happening. Those kinds of things. Um, it's way too high for them, especially if it's only in September or October. So, I might not worry too much about it, but I pull it out when I can. (GJ 4.6, 940202)

In this way, classroom activities evolved from interactions in the learning environment, just as Geena's non-school mathematics evolved (cf. Roth, 1994). However, students in Geena's classroom were seldom able to make modifications on their own. For example, during mathematics the day I observed her classroom, Geena had designed a worksheet activity focusing on the concepts of "greater than" and "less than." Students were to draw two handfuls of differently coloured blocks and then create number sentences to specify the relationship between the two handfuls of blocks. The only flexibility students were afforded was in the decision to use manipulatives (some students completed the worksheet by brainstorming number sentences rather than counting handfuls of blocks); all students were required to fill each box on the worksheet with mathematically correct number sentences using greater than and less than signs.

Making Sense of Mathematics

Geena's mathematics background consisted primarily of memorization and rote operations. In contrast, as we have seen, algorithmic approaches to mathematics did not play a great role in her kitchen mathematics. In the introduction to her final project, Geena articulated that her difficulties with mathematics as a child were in making sense of mathematics rather than in manipulating the numbers:

> Math was often a puzzle for me. It was a special secret that the teacher had and only a select few could get in on the secret if they were "smart enough." Sure I could memorize and do the rote equations, but often not without endless fights and tears with my mom. I was told why it was important to know my addition and multiplication tables etc. and I could even relate to some "real life situations," but

> I sure didn't understand what it was, I was memorizing. I just didn't get it! My mom was as upset as I was. She didn't understand why I always had to know "why", and was exasperated with the amount of questions I asked. This often left us both in a state of frustration. (GJ project.1, 931206)

Similarly, when we discussed her understanding of school mathematics, Geena emphasized her efforts to make sense of school mathematics. As she explained during our first meeting, "I was 25 when I really understood, the real understanding of the concept of trading your tens for ones and when you had 4000 take away 2, I crossed off the zeroes and put a bunch of 9s and put a ten at the end because I was told to." (GJ 1.2, 931021). Here, Geena is referring to the standard algorithm for performing subtraction with borrowing.

$$\begin{array}{r} {}^{3}4\,{}^{9}0\,{}^{9}0\,{}^{1}0 \\ -\quad 2 \\ \hline 3\,9\,9\,8 \end{array}$$

Geena's realization of the meaninglessness of this algorithm occurred while she was working with a student who was struggling to implement this algorithm. "And I said 'Why do you do this?' And the kid said, 'My teacher tells me to.' And I'm thinking, 'Right on kid. Me too.'" (GJ 1.2, 931021). Rather than leaving it at this, she made a concerted effort to make sense of this algorithm, for herself and for the student. Using a disk abacus, the two experimented with a "really big question" to see if they could make sense.

Geena did not describe the step-by-step procedure they used nor did she explain what was the "really big question." Figure 1 illustrates the idea behind the use of a disk abacus to solve the equation 4000 - 2, the sample problem Geena mentioned. Through this procedure both Geena and the student with whom she was working came to understand the concept of exchanging in place value. As Geena stated, "all of the sudden it was like this light went on for both of us. It was like wow, I get this. I *really* get this." (GJ 1.2, 931021).

Geena's efforts to make sense of mathematics extended to her classroom where she provided many opportunities for her students to make sense. For example, during Calendar activities on the morning I attended Geena's class, the students were looking for patterns on the posted classroom calendar and one student asked why the month didn't start on Monday. Rather than explaining this (or saying "That's just the way it is," as many people might have), Geena sent this student and another student to the back of the classroom to check the two "real" calendars. These students came back to report that they both started on the same day. As a class, they talked about this finding and why it would be like that.

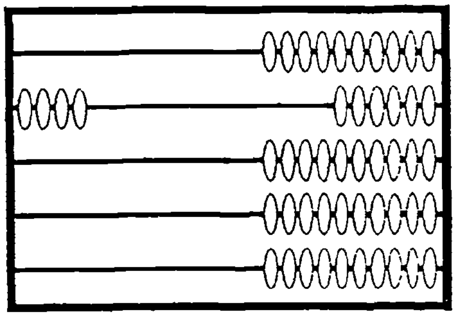
Geena attributed many of the successful sense-making efforts by her and her students to the use of physical representations such as the disk abacus and the published calendar. This provides the focus for the next section.

Using Physical Objects

Geena frequently mentioned the importance of using physical objects to do mathematics. In her classroom, so-called manipulatives were common and students were encouraged to use these to make sense of mathematics. During our first meeting she talked of how the use of these physical objects had improved students' mathematical

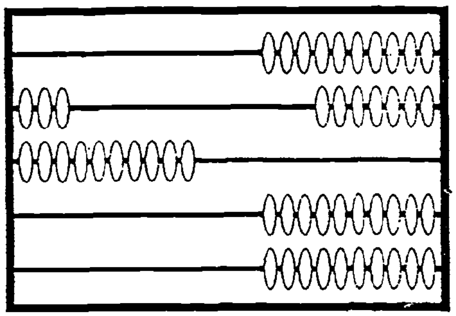
Using the disk abacus to solve the equation 4000 - 2.

4
0
0
0



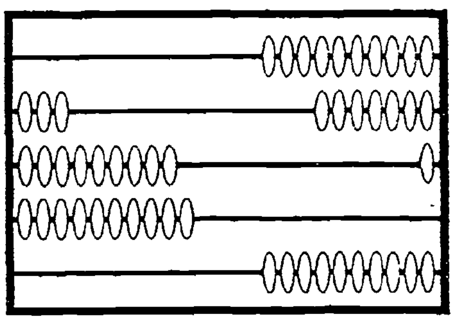
Begin with four disks in the 1000 row.

3
10
0
0



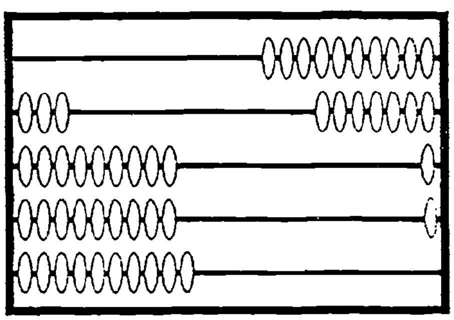
Exchange one disk from the 1000 row for ten disks in the 100 row. This leaves three disks in the 1000 row and ten disks in the 100 row.

3
9
10
0



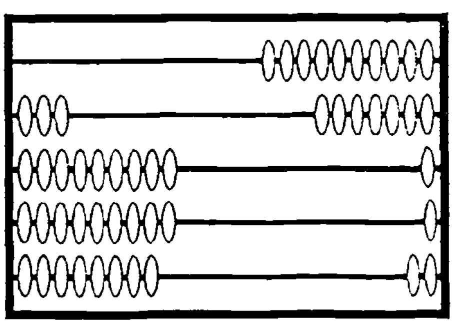
Exchange one disk from the 100 row for ten disks in the 10 row. This leaves three disks in the 1000 row, nine disks in the 100 row, and ten disks in the 10 row.

3
9
9
10



Exchange one disk from the 10 row for ten disks in the 1 row. This leaves three disks in the 1000 row, nine disks in the 100 row, nine disks in the 10 row, and ten disks in the 1 row.

3
9
9
8



Subtract two from the 1 row. This gives the correct answer of 3998.

understandings. As a student herself, she never had access to manipulatives, and she felt that this resulted in limited understandings for her and her peers: "But people my age and older, um, if they didn't understand it, it's because they just never had all this manipulative stuff to work with before." (GJ 1.2, 931021). Arguably, it is not the presence of manipulatives and other "cultural tools" that improve understandings, but the affordances these tools provide within the context of practice (Cobb, 1993; Roth, 1995).

Consistent with Geena's assertions, manipulatives were an integral part of the lesson she gave during the math period I observed (GJ 3, 941116). At that time, the students had been working with inequalities and writing number sentences (e.g., 8 > 2 or 2< 8) for the past few weeks. Geena handed out the worksheet, explaining that students should draw two handfuls of different coloured blocks, count them, and write the two number sentences to represent the relationship. The worksheet had space for eight pairs of number sentences. As I circulated around the room, it was clear that most students were following the procedure and completing the worksheet correctly. There were a few exceptions however. I noticed one boy who appeared to be struggling with the task. He kept mixing up the direction of his greater than and less than signs, including inaccurate sentences such as "5 < 3" or "2 > 9." When I talked with him, he was able to verbally state the correct sentences (5 is greater than 3, and 2 is less than 9), but did not always write them properly. As it turned out, several of his digits were written backwards as well and his difficulty was the result of a visual motor learning disability. He could talk through the answers in words, but was unable to use the mathematical symbolism. Some of the other students, were writing mathematically correct number sentences without the use of the blocks. For example, one child seemed to be experimenting with large numbers and wrote sentences such as "1000 > 2." Obviously, the child had not counted out 1000 blocks, rather she used her "number sense" (Greeno, 1991) and created her own number sentences without the use of physical manipulatives. This shortcut was accepted by Geena as fulfilling the task requirements and indicating conceptual understanding[1].

Examples of the use of manipulatives were common in Geena's mathematics classroom, as was the use of physical objects to make meaning of school mathematics for herself (recall the earlier description of her use of the disk abacus to understand subtraction with borrowing) and as resources in her kitchen mathematics. Similarly, the use of manipulatives was common practice in Geena's methods course. The day I attended her methods course to solicit research participants, class began with the instruction for students to help themselves to the available manipulatives and experiment with them until others arrived.

Measuring and Calculating New Measures

Measuring was also common in Geena's classroom, and, just as in her non-school mathematics, measuring was dependent on the use of physical objects. For example, Geena described a newspaper clipping about a huge snake that one of her students brought for Show and Tell. The article described the length in meters of the snake, and students asked what that length meant. Rather than providing an answer ("Oh, it's about..."), the whole class got out meter sticks and laid them end to end across the classroom to see how long the snake was. Thus, measuring was one of the ways Geena helped students to make sense of mathematics.

[1] It is also possible that the child used this strategy because it was very clear that 1000 was greater than 2, whereas she may not have been able to write complete number sentences had she drawn two handfuls with similar numbers of blocks, e.g., 2 and 3 blocks. The important point in this analysis is that Geena allowed the children the flexibility to modify the worksheet plan as they wished, as long as she felt they demonstrated an ability to use the greater than and less than signs appropriately.

Multiple Solutions

Geena allowed and encouraged students to consider multiple solutions in their mathematical work. As described earlier, they were allowed to complete their worksheet with or without manipulatives. Also, after students completed their worksheets they were given free choice between numerous "math games." Most of these activities focused on a "right" answer (cf. *Stating Solutions* section), but there were numerous ways to arrive at that answer. Two girls worked together with two sets of flash cards and two pictures of a flower with numbered parts. One girl, whose basic number facts seemed more automatic, used subtraction flash cards, and the other girl used addition flash cards. They took turns drawing a flash card and, if they got the correct answer, they covered the appropriate number space on the flower picture. The object of the game was to cover all the numbers on the flower. The flowers each had the same numbers, but one girl used subtraction while the other used addition to complete the task. In a second group, three children asked me to join them for a board game. In this game, players took turns rolling the dice and moving forward the appropriate number of spaces. Some of the squares on the board had additional directions such as, "Go forward 2 spaces," or "Go back 4 spaces." The idea was to move through the maze of the board from the starting point to the end as quickly as possible. This could be accomplished by a combination of high dice rolls, landing on "go forward" squares, avoiding "go back" squares, and landing on the square at the foot of a bridge that provided a short cut. For part of the period, Geena worked with two boys on a math race. Each of them (Geena and the two students) had a mini-chalkboard and chalk, and between them they had a pair of dice. The object of the game was to count to 100. Each took turns rolling the dice, adding together the total and keeping a running tally on their chalkboard. To get started, Geena demonstrated how to play the game (see Figure 2). On her first turn, she rolled a 6 and a 3. She added these together, writing the problem and solution on her chalkboard. She then rolled a second time (2, 5) and explicitly described to the two boys two ways of doing the next calculation. In the first scenario (middle column in Figure 2) she first added 2 to the previous sum of 9 to get a sub-total of 11, to which she added the 5 for a total of 16. She then described an alternate solution. First, add the two dice from the second roll (2, 5) to get 7, then add this to the previous total (9) to get 16 (see right column in Figure 2).

Figure 2. Multiple solutions in a math race.

First role:	6		
Geena rolls 6 and 3	+3		
	9		
Second role:	+2	OR	2
Geena rolls 2, 5	11		+5
	+5		7
	16		+9
			16

In each of these math games, multiple solution paths were available, but only one "right" answer was possible. The worksheet exercise was a goal-free problem whereby students could have picked any number of manipulatives to write any combination of number sentences, and had some flexibility in how they approached the task (i.e., they were not required to use handfuls of blocks). All of these exercises allowed multiple solutions, as was common in Geena's mathematics outside her classroom.

Checking Work

As the students worked on their mathematics, Geena circulated in the classroom checking their performance. When she observed difficulties, she questioned the students and provided hints to help them get back on track. Students were encouraged to monitor their own performance, and when they were finished they were allowed to move on to a math game of their choice. Some students were called back from these games to revise their worksheets.

Stating Solutions

This is the second code that really seemed to discriminate between Geena's school and non-school mathematics. For non-school mathematics, Geena placed very little emphasis on stating a solution, whereas her classroom mathematics consistently involved stating a solution. The blanks on the worksheet had to be filled, the math games had "right" answers, and Quiet Math required the solution of arithmetic problems through accurate hand signals. School mathematics involved rationally accountable practice/products; students were required to do something public and to produce appropriate documents (cf. Lynch, 1991).

Drawing Connections

In her teaching, Geena flexibly weaved together different subjects across the time slots of her day. The Yearly Preview she prepared emphasized cross-curricular themes that integrated subject areas. She planned to discuss pumpkins in math, science, and socials during the month of October to correspond with Halloween. She planned her geometry unit in mathematics to correspond with the space theme for centre-play time, art, and socials. During her unit on Fairy Tales in Language Arts, she drew in the stories of Robert Munsch which have a fairy tale structure. At the same time, her students noticed the prevalence of numbers in these stories, and they discussed this mathematics.

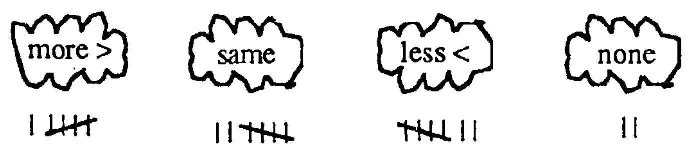
Similarly, Geena explained how she picked up on mathematics issues when they came up in current events (e.g., the snake story) and other times during the school day. As Geena explained, "Whatever's happening, yeah. In the newspaper or in lit, always, any time it [mathematics] comes up in Language Arts or literature." (GJ 4.7, 940202) Even in her bulletin board displays, Geena focused on integrating across subjects, "I put everything from math stuff to a mixture of math and writing, to, to writing to art, a mixture of all three of them." (GJ 4.7, 940202).

Not only did Geena emphasize cross-curricular connections in her teaching, but she also tried to connect to students' home lives. For her final project in the mathematics methods course she designed a geometry unit plan for her classroom that was comprised of activities for students to take home and work on with their families. Her unit grew from an interest in building "home-school relationships" by involving parents. At the same time, Geena explained in her paper that she wanted her students and their parents to recognize the importance of mathematics in their everyday lives.

> Math is everywhere just as reading and writing are. The average person will say that most of the Math they learned in school was a waste of time and the only thing they use on a regular basis is the basic addition, subtraction, and multiplication tables which they use when they go shopping and when they do their banking. They are not aware that Math is so much more and that they use it on a day to day basis. Math doesn't stop at the classroom walls. We, as educators who believe this, need to continue to re-educate the public. Using a Home Math Program is one way to help parents and children realize that Math is everywhere for everyone. (GJ project 3, 931206)

The activities for this unit included games that students could play with their whole family and a series of worksheets where students identified geometric shapes in a picture and then looked for more shapes in their own environments (garden, yard, bathroom, kitchen, etc.). This worksheet series emphasized the second aspect of Geena's two-pronged approach to environmental awareness: becoming more environmentally conscious (composting, recycling, not wasting food) and being more aware of the environment (drawing a map to get from school to home, graphing the difference in the amount of grass in their front and back yards, recognizing geometric shapes in their environment). This focus was also evident on the day I observed in Geena's classroom. When students first entered the classroom, they marked a tally on the board to create a class graph as shown below.

Do you have > or < grass in the front of your house than the back?



This kind of graphing of students' own life experience was also prevalent in the weather and lost tooth graphs posted near the class calendar that became a focus during early morning calendar activities each day. In all of these graphs, students were encouraged to reflect on and graphically represent aspects of their own lives (weather, grass in their yard, infant teeth lost).

Discussing the Connections

Throughout all of our interactions, Geena continually came back to the idea of "connections." During our first phone call to set up an interview, Geena indicated that she was interested in the idea of drawing connections between school and non-school mathematics. This focus was prevalent in Geena's initial journal assignment, the assignment that had convinced her to participate in this research project. For this assignment, Geena was supposed to flip through an issue of the Arithmetic Teacher and write a reflection about one of the articles, "What did it do for us? Did it talk about our experiences? Did it trigger anything?" Geena selected the February 1993 (volume 40, number. 6) issue of the Arithmetic Teacher which was a special issue on "Empowering Students Through Connections." The articles in this issue discussed various connections among mathematical topics, between mathematics and other curriculum areas, between mathematics and students' prior experiences, and between mathematics in the school and in the home. From these articles, Geena selected Corwin's article on creating a mathematical culture. In her response, Geena drew connections to her own prior experience and to events in her classroom. She also described the importance of making connections with colleagues, connections with students' home life, and connections to everyday uses of mathematics. This initial assignment was the taking off point for all of our interactions. During our interactions, Geena constantly drew connections between school and non-school uses of mathematics, mathematics and other subjects, school and home learning, mathematics and art, etc.

The overarching theme of drawing connections can also be seen by analyzing table 1 and the eight dimensions that have framed discussion throughout this paper. Similar themes were prevalent in Geena's mathematics both inside and outside her classrooms, highlighting the connectedness of her mathematics across contexts. Despite, the differences in *Flexibly Modifying Plans* and *Stating Solutions*, Geena was able to draw many connections between school and non-school mathematics. However, one may still argue that the differences between Geena's mathematical activity inside and outside her classrooms represent distinct discontinuities (Resnick, 1987).

Prior research has indicated that the mathematical activity of adults and children differs between classrooms and non-school settings (e.g., Lave, 1988; Nunes, Schliemann, & Carraher, 1993; Saxe, 1991; Scribner, 1984). Consistent with these earlier findings, the present case study suggests that this teacher also displayed mathematical activity in everyday settings that differed from approaches taught in schools, including her own classrooms. Analysis of Geena's non-school mathematical activity revealed a marked contrast from the mathematical activity that is legitimated in classrooms.

It seems plausible that students and teachers bring to school common sense understandings and values about mathematical activities (e.g., measurement, estimation, and calculation) which are valid in everyday settings but different from those taught in schools or universities (Lynch, 1991; Roth & Bowen, 1994). Goals are of a different nature outside the classroom than those in the classroom (see Lave, 1988). Exact calculations and precise measurements are important in the classroom, but creating a tasty product is all that matters in the kitchen. These different understandings and goals lead to different approaches, yet there may be some cause for connecting across these settings as NCTM (1989) documents have suggested. Teaching which emphasizes mathematics used outside classrooms and how this relates to accepted school-taught practices may help to bridge the gap between classrooms and the "real world." This may increase engagement and learning in at least four ways: it may (1) help students to build conceptual understandings by connecting new information with prior knowledge; (2) help students to realize the relevance of curriculum materials to their lives; (3) validate the kinds of informal strategies that students already know and use; and (4) help reduce students' negative reactions to mathematics. Similar positive benefits may also accrue for the teacher who is able to validate his or her own mathematical constructions and may become more comfortable with doing and teaching mathematics. In this way, mathematics education becomes praxis (Fasheh, 1982; Millroy, 1992). Such positive benefits seem to abound in Geena's teaching and in her life, and perhaps her story can serve as a model for others.

References

Bijker, W. E. (1987). The social construction of Bakelite: Toward a theory of invention. In W. E. Bijker, T. P. Hughes, & T. J. Pinch (Eds.), The social construction of technological systems (pp. 159-187). Cambridge, MA: MIT Press.

Bishop, A. J. (1991). Mathematics education in its cultural context. In M. Harris (Ed.), Schools, mathematics and work (pp. 29-41). Bristol, PA: Falmer Press.

Carraher, T. N. (1986). From drawings to buildings; working with mathematical scales. International Journal of Behavioral Development, 9, 527-544.

Cobb, P. (1993, April). Cultural tools. Paper presented at the Annual Meeting of the American Educational Research Association, Atlanta, GA.

Constant, E. W., II. (1989). Cause or consequence: Science, technology, and regulatory change in the oil business in Texas, 1930-1975. Technology and Culture, 30, 426-455.

Corwin, R. (1993). Doing mathematics together: Creating a mathematical culture. Arithmetic Teacher, 40, 338-341.

de la Rocha, O. L. (1987). Problems of sense and of scale: An ethnographic study of arithmetic in everyday life. (Doctoral dissertation, University of California, Irvine, 1986). Dissertation Abstracts International, 47, pp. 4198A-4199A.

Eisenhart, M. (1988). The ethnographic research tradition and mathematics education research. Journal for Research in Mathematics Education, 19, 99-114.

Ellen, R. F. (1984). Ethnographic research: A guide to general conduct. London: Academic Press.

Erickson, F. (1986). Qualitative methods in research on teaching. In M. C. Wittrock (Ed.), Handbook of research on teaching (3rd ed., pp. 119-161). New York, NY: Macmillan.

Fasheh, M. (1982). Mathematics, cultural and authority. For the Learning of Mathematics, 3(2), 2-8.

Greeno, J. G. (1991). Number sense as situated knowing in a conceptual domain. Journal for Research in Mathematics Education, 22, 170-218.

Hammersley, M., & Atkinson, P. (1983). Ethnography: Principles in practice. London: Tavistock.

Kurti, N. & This-Benckhard, H. (1994) Chemistry and physics in the kitchen. Scientific American, 270(4), 66-71.

Lave, J. (1988). Cognition in practice: Mind, mathematics and culture in everyday life. Cambridge: Cambridge University Press.

Lave, J. (Ed.). (1985). The social organization of knowledge and practice: A symposium. [Special Issue]. Anthropology and Education Quarterly, 16(3), 171-213.

Lincoln, Y. S., & Guba, E. G. (1985). Naturalistic inquiry. Newbury Park, CA: Sage.

Lynch, M. (1991). Method: Measurement—ordinary and scientific measurement as ethnomethodological phenomena. In G. Button (Ed.), Ethnomethodology and the human sciences (pp. 77-108). Cambridge: Cambridge University Press.

Marshall, C., & Rossman, G. B. (1989). Designing qualitative research. Newbury Park, CA: Sage.

Masingila, J. O. (1992). Mathematics practice and apprenticeship in carpet laying: Suggestions for mathematics education. Unpublished doctoral dissertation, Indiana University, Bloomington.

Merriam, S. B. (1988). Case study research in education: A qualitative approach. San Francisco: Jossey-Bass.

Millroy, W. L. (1992). An ethnographic study of the mathematical ideas of a group of carpenters. Journal for Research in Mathematics Education, Monograph 5. Reston, VA: NCTM.

Murtaugh, M. F. (1985). A hierarchical decision process model of American grocery shopping. (Doctoral dissertation, University of California, Irvine, 1986). Dissertation Abstracts International, 46, p. 1675A.

National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: Author.

Nunes, T., Schliemann, A. D., & Carraher, D. W. (Eds.). (1993). Street mathematics and school mathematics. Cambridge: Cambridge University Press.

Resnick, L. B. (1983). Mathematics and science learning: A new conception. Science, 220, 477-478.

Resnick, L. B. (1987). The 1987 Presidential Address: Learning in school and out. Educational Researcher, 16, 13-20.

Roth, W.-M. (1994, October). Art and artifact in science classrooms: Studies of work and talk in open-inquiry school science. Paper presented at the Annual Meeting of the Society for the Social Studies of Science, New Orleans.

Roth, W.-M. (1995). Affordances of computers in teacher-student interactions: The case of Interactive Physics™. Journal of Research in Science Teaching, 32, xxx-xxx.

Roth, W. M., & Bowen, G. M. (1994). Mathematization of experience in a grade 8 open-inquiry environment: An introduction to the representational practices of science. Journal of Research in Science Teaching, 31, 293-318.

Saxe, G. B. (1991). Culture and cognitive development: Studies in mathematical understanding. Hillsdale, NJ: Lawrence Erlbaum.

Scribner, S. (Ed.) (1984). Cognitive studies of work [Special Issue]. Quarterly Newsletter of the Laboratory of Comparative Human Cognition, 6(1,2).

Starling, G. (1992). Problem solving as a social process: A theoretical and empirical analysis. The Social Science Journal, 29, 211-225.

Strauss, A., & Corbin, J. (1990). Basics of qualitative research: Grounded theory procedures and techniques. Newbury Park, CA: Sage.

Suchman, L. A. (1987). Plans and situated actions: The problem of human-machine communication. Cambridge: Cambridge University Press.

Table 1. Dimensions of Geena's Mathematics Inside and Outside Her Classrooms

Dimension	Outside Classroom	Inside Classroom
Flexibly Modifying Plans	General goals, but plan evolves. Recipe used as a guide (substitute ingredients, double measures, use approximate measures, abandon cookies to create apple squares).	Designed unit plan, but ended up only using a few exercises. Takes time to discuss mathematics across curriculum (not tied to time frame). Students don't need to use manipulatives to complete their worksheet.
Making Sense	Math is a "way of thinking." She "doesn't convert."	Mathematics was a puzzle for her. Used the disk abacus to make sense for self and her student. Students sent to consult published calendar to find out why month doesn't start same day.
Using Physical Objects	Measuring cups, tools, shortening gauge, etc. in the kitchen.	Worksheet exercise in her class used manipulatives. Using the disk abacus. Emphasis on manipulatives in Educ 475
Measuring and Calculating New Measures	Weight Watchers estimation skills. Conservation of volume to measure shortening. Use commutation to double flour.	Use meter sticks to measure snake.

Multiple Solutions	Alternate ways to measure shortening. Different doubling methods.	Math games with multiple solutions. Open-ended worksheet.
Checking Her Work	When cookies did not work as planned, she checked the recipe step-by-step three times.	Geena checked the students' worksheets and called them back if there were any difficulties
Stating Solutions	Only assigns number to amount of cheese as an afterthought. Doesn't indicate doubled measure of flour.	Worksheet blanks must be filled in. All math games have a "right: answer. Quiet Math requires accurate hand signals.
Drawing Connections	Recognize that "mathematics is everywhere"	Weaving themes across the curriculum. Picking up on mathematical themes whenever they arise. Home mathematics unit plan Graphing information about students' lives and their environment
	Special theme issue on Connections for her journal assignment in Educ 475. Recurring theme in all our discussions Match of eight categories across school and non-school mathematics.	