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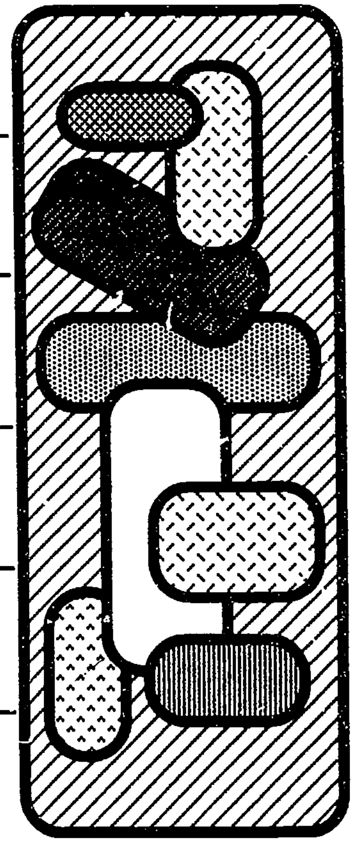
ABSTRACT

This document is designed to assist teachers and other school personnel in the planning and teaching of the seventh grade mathematics course. Contents include: (1) Overview of Grade 7 Mathematics (mission statement, purpose and philosophy, goals, National Council of Teachers of Mathematics' Professional Standards for Teaching Mathematics, and uses of technology and manipulatives); (2) Essential Elements of Instruction and Learning Objectives; (3) Texas Assessment of Academic Skills (TAAS) (focus, domains, objectives, targets, and instructional strategies); (4) Sample Lessons for Teaching Grade 7 Mathematics; and (5) Evaluation (philosophy, types of evaluation, samples, test-taking strategies, grading, and homework). TAAS features three domains: concepts, operations, and problem solving. The Essential Elements are: problem solving; patterns, relations, and functions; number and numeration concepts; operations and computation; measurement; geometry; and probability, statistics, and graphing. "Beyond the Procedures: Extending Students' Experiences with Computational Problems," a paper by Anne L. Madsen, is appended. Contains 10 references. (MKR)

GUIDELINES FOR TEACHING GRADE 7 MATHEMATICS



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Texas Education Agency
Austin, Texas

GUIDELINES FOR TEACHING GRADE 7 MATHEMATICS

Texas Education Agency
1701 North Congress Avenue
Austin, Texas 78701

FOREWORD

Guidelines for Teaching Grade 7 Mathematics is designed to help teachers and other school district personnel plan and teach seventh grade mathematics. The publication presents the philosophy and intent of the course and discusses the required essential elements, TAAS instructional targets, instructional strategies, and the use of technology and manipulatives. Also included are sample objectives and activities to illustrate how the essential elements for seventh grade mathematics can be taught. School district personnel may want to use these suggestions to develop their own curriculum documents for the course.

We hope these guidelines will be useful in planning and teaching mathematics for Grade 7 and in equipping the mathematics classroom.

Lionel R. Meno
Commissioner of Education

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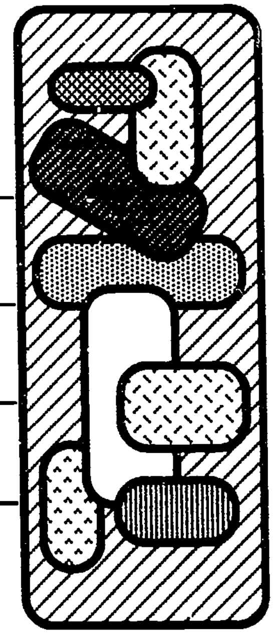
Overview of Grade 7 Mathematics



Mission Statement

Guidelines for Teaching Grade 7 Mathematics is designed to assist teachers and other school personnel in the planning and teaching of the seventh grade mathematics course. This document discusses the philosophy and intent of the course, the goals, essential elements, and sample objectives, activities, and lesson plans. Also included is a discussion of the uses of technology and manipulatives, as well as Texas Assessment of Academic Skills (TAAS) domains, objectives, and targets; evaluation techniques and samples; and suggested resources and references. School personnel may want to use these suggestions to develop their own curriculum documents for seventh grade mathematics. These guidelines will be useful to district personnel in planning curriculum, teaching mathematics, and equipping classrooms.

When students are allowed to experience the numerous perspectives of mathematics, they will be more successful. This document has been developed with that in mind. A teacher's understanding of the content matter taught is most important. A teacher's understanding of how a student can achieve mastery of concepts; how the mistakes made by students and a teacher's readiness to reduce or eliminate these occurrences; and how a teacher uses books, resources, and materials sets the stage for successful experiences in learning and teaching mathematics.

The seventh grade essential elements and TAAS objectives were used as a basis for the lesson activities in these guidelines. Some of the activities may be challenging for students. Challenging or advanced topics should be taught because they demonstrate to students what they are capable of learning, understanding, and accomplishing in mathematics. This document is designed to inspire teachers to inspire students to want to learn mathematics. Only when educators begin to expand their horizons in this way will a significant impact be made upon students.

Purpose and Philosophy

Mathematics is useful, exciting, and creative and can be enjoyed by all middle school students. Problem-solving skills and logical reasoning are developed while students explore and make sense of their world through the study of mathematics. Unfortunately, mathematics has been viewed by many students as boring, irrelevant, and routine and as externally dictated by a rigid system of rules governed by standards of speed, accuracy, and memory. In the past, computational facility has been emphasized instead of a broad, integrated view of mathematics. While computational skills are important, learner characteristics and the vitality of mathematics itself cannot be overlooked. Mathematics in Grade 7 should be broad based and concept driven and should reflect relevant mathematics and its interrelationships with technology.

Middle school students are in a transitional period, forming lifelong values and skills. Decisions about what students will study and how they will learn can dramatically affect their futures, and failure to study mathematics can result in a loss of opportunities. Attitudes that affect the decisions are often formed during middle school years, so the curriculum must be useful, interesting, and relevant, and it must foster a positive disposition toward mathematics.

The middle school mathematics curriculum should expand students' knowledge of numbers, computation, estimation, measurement, geometry, statistics, probability, patterns and functions, and the fundamental concepts of algebra. Traditionally, textbooks have offered the same topics, approach, and presentation grade after grade, with little change from Grade 5 through Grade 8. Chapters on new material have been included in the last half of textbooks in sections not covered because of lack of time. Thus the result has been a reexamination of materials students have already seen, which promotes a negative image of mathematics and an inadequate background for secondary school mathematics.

A broad-based and flexible view of the middle school curriculum is important for several reasons. First of all, basic skills for the 1990s and beyond mean far more than computational facility. With calculators readily available, the need for tedious paper and pencil proficiency is obsolete, and topics such as geometry, probability, and statistics have become more important through technology. Secondly, if students have been unable to master basic computational skills in elementary school, they are likely to be unsuccessful with the same techniques during the middle school years. Thirdly, many mathematical topics currently omitted actually can assist students in arithmetic concepts and skills through a fresh approach. The seventh grade curriculum should include the following:

- problem situations that establish the need for new ideas and motivate students
- communications with and about mathematics
- mathematical reasoning
- a broad range of topics, including number concepts, computation, estimation, functions, algebra, statistics, probability, geometry, and measurement
- topics taught as an integrated whole
- technology, including calculators, computers, and videos

Goals

According to the *Curriculum and Evaluation Standards for School Mathematics* (the *Standards*) developed by the National Council of Teachers of Mathematics (NCTM), the five overall curriculum goals for students are:

- learning to value mathematics
- becoming confident in their ability to do mathematics
- becoming mathematical problem solvers
- learning to communicate mathematically
- learning to reason mathematically

Moreover, the educational system of today demands new societal goals for education:

- mathematically literate workers
- lifelong learning
- opportunity for all
- an informed electorate

Specifically, teaching the mathematics curriculum to middle school students must be related to the characteristics of the learners and their needs today and in the future. *Everybody Counts* (National Research Council, 1989) posits that "self-confidence built on success is the most important objective of the mathematics curriculum" (page 45). Individuals must be able to cope with mathematics in their later lives—as employees, parents, and citizens. Ability to do so depends on attitudes toward mathematics conveyed in school. The mathematics curricula must not leave a legacy of misunderstandings and apprehension.

National Council of Teachers of Mathematics: Professional Standards for Teaching Mathematics

The *Professional Standards for Teaching Mathematics (NCTM, 1991)* are based on four assumptions about the practice of teaching. These assumptions are abbreviated versions of the more extensive ones found in the original document (NCTM, 1991, pages 21-22).

(1) The goal of teaching mathematics is to help all students develop mathematical power. Teachers must help every student develop conceptual and procedural understandings of number, operations, geometry, measurement, statistics, probability, functions, and algebra and the connections among ideas. They must engage all students in formulating and solving a wide variety of problems, making conjectures and constructing arguments, validating solutions, and evaluating the reasonableness of mathematical claims.

(2) What students learn is fundamentally connected with how they learn it. Students' opportunities to learn mathematics are a function of the setting and the kinds of tasks and discourse in which they participate.

(3) All students can learn to think mathematically. The goals such as learning to make conjectures, to argue about mathematics using mathematical evidence, to formulate and solve problems, and to make sense of mathematical ideas are not just for some group thought to be "bright" or "mathematically able."

(4) Teaching is a complex practice and hence not reducible to reci es or prescriptions. First of all, teaching mathematics draws on knowledge from several domains: knowledge of mathematics, of diverse learners, of how students learn mathematics, of the contexts of the classroom, school, and society. Good teaching depends on a host of considerations and understandings. Good teaching demands that teachers reason about pedagogy in professionally defensible ways within partic lar contexts of their own work.

The *Professional Standards for Teaching Mathematics* identifies a particular set of instructio standards for the effective teaching of mathematics. These standards describe the nature of the tasks, patterns of communication and the learning environment. More specifically, five of these standards focus on instructional strategies. They are:

STANDARD 1: WORTHWHILE MATHEMATICAL TASKS

The teacher of mathematics should pose tasks that are based on:

- sound and significant mathematics;
- knowledge of students' understandings, interests, and experiences;
- knowledge of the range of ways that diverse students learn mathematics;

nd that

- engage students' interests;
- develop students' mathematical understandings and skills;
- stimulate students to make connections and develop a coherent framework for mathematical ideas;
- call for problem formulation, problem solving, and mathematical reasoning;
- promote communication about mathematics;
- represent mathematics as an ongoing human activity;
- display sensitivity to, and draw on, students' diverse background experiences and dispositions;
- promote the development of all students' dispositions to do mathematics.

STANDARD 2: THE TEACHER'S ROLE IN DISCOURSE

The teacher of mathematics should orchestrate discourse by:

- posing questions and tasks that elicit, engage, and challenge each student's thinking ability;
- listening carefully to students' ideas;
- asking students to clarify and justify their ideas orally and in writing;
- deciding what to pursue in depth from among the ideas that students bring up during a discussion;
- deciding when and how to attach mathematical notation and language to students' ideas;

- deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;
- monitoring students' participation in discussions and deciding when and how to encourage each student to participate.

STANDARD 3: STUDENTS' ROLE IN DISCOURSE

The teacher of mathematics should promote classroom discourse in which students:

- listen to, respond to, and question the teacher and one another;
- use a variety of tools to reason, make connections, solve problems, and communicate;
- initiate problems and questions;
- make conjectures and present solutions;
- explore examples and counterexamples to investigate a conjecture;
- try to convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers;
- rely on mathematical evidence and argument to determine validity.

STANDARD 4: TOOLS FOR ENHANCING DISCOURSE

The teacher of mathematics in order to enhance discourse, should encourage and accept the use of:

- computers, calculators, and other technology;
- concrete materials used as models;
- pictures, diagrams, tables, and graphs;
- invented and conventional terms and symbols;
- metaphors, analogies, and stories;
- written hypotheses, explanations, and arguments;
- oral presentations and dramatizations.

STANDARD 5: LEARNING ENVIRONMENT

The teacher of mathematics should create a learning environment that fosters the development of each student's mathematical power by:

- providing and structuring the time necessary to explore sound mathematics and grapple with significant ideas and problems;
- using the physical space and materials in ways that facilitate students' learning of mathematics;
- providing a context that encourages the development of mathematical skill and proficiency;
- respecting and valuing students' ideas, ways of thinking, and mathematical dispositions;

and by consistently expecting and encouraging students to:

- work independently or collaboratively to make sense of mathematics;
- take intellectual risks by raising questions and formulating conjectures;
- display a sense of mathematical competence by validating and supporting ideas with mathematical argument.

Uses of Technology and Manipulatives

Calculators and computers are tapped for important roles in mathematics education at all levels and across topics. Changes in technology and the broadening of the areas in which mathematics is applied have resulted in growth and changes in the discipline of mathematics itself. The new technology has altered the very nature of the problems important to mathematics and the methods mathematicians use to investigate them.

The *Standards* call for the following regarding technology in the classroom:

- appropriate calculators for all students at all times
- a computer for every classroom for demonstration
- access to a computer for individual and group work
- students learning to use the computer as a tool for processing information and performing calculations to solve problems

Calculators and computers offer teachers and students a rich learning aid. Their potential is great and as yet untapped both in developing concepts and in developing positive attitudes and persistence in problem solving.

Computers can be utilized in a variety of ways in the mathematics classroom, and the appropriateness of a particular approach depends on the goals. Three qualitatively different methods suggested by R. Taylor in *The Computer in the School: Tutor, Tool, Tutee* are:

- as a sophisticated teaching machine
- to be programmed (or taught) by the student
- as a mode for applications in research and development through software that displays graphs, manipulates symbols, analyzes data, and performs mathematical procedures. Applications such as spreadsheets, word processing, data bases, and communication packages have the appeal of matching the classroom's use of technology with that of society's.

Calculator use is not for the purpose of replacing paper-and-pencil computations but to reinforce them. According to N. Kober in *What We Know About Mathematics Teaching and Learning*, calculator use is apt to sustain independent thought, not replace it. For example, students can be challenged to invent calculator algorithms to replace procedures taught in textbooks. The students explain why their procedures work and debate the advantages and disadvantages of their procedures over others. Calculators are programmable, produce graphics, and work in fractional and algebraic notation. Teachers need to be innovative, experiment, and share ideas.

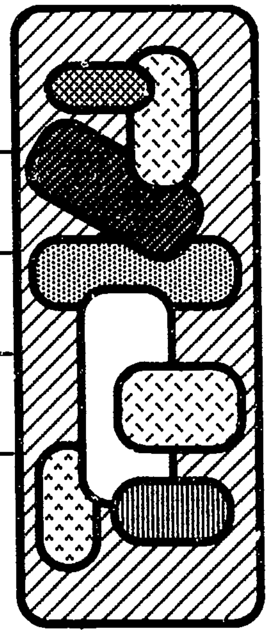
Furthermore, manipulatives offer an excellent way to enable students to connect between mathematical ideas. Learning is enhanced when students are exposed to a concept in a variety of manipulative contexts. As an example, fractions represented with colored cubes, pattern blocks, fraction bars, fraction circles, and Cuisenaire rods help students understand the concept of fractions independent of the physical representation. In addition to using manipulatives for new concepts, activities should be oriented to help students connect between concrete, pictorial, and abstract representations of ideas.

The primary goal in teaching mathemaiics is to develop conceptual understanding of mathematical ideas. Technology and manipulatives are tools to achieve this goal if implemented in classroom activities.

I hear and I forget. I see and I remember. I do and I understand.

Chinese proverb

Essential Elements of Instruction and Learning Objectives



Essential Elements of Grade 7 Mathematics

The State Board of Education in 1989 revised the essential elements of instruction for mathematics, Grades 1-8. These revised essential elements follow closely the recommendations made by the National Council of Teachers of Mathematics in its nationally recognized *Curriculum and Evaluation Standards for School Mathematics*. According to the Texas Education Agency (1989), "The mathematics curriculum review committee and the Agency [TEA] have tried to be sensitive to a balance between changes expected of teachers and improvements necessary to help students learn mathematics more effectively." Some of these major changes include:

- narrowing the spiral of the curriculum—beginning some topics later and finishing some topics sooner in the curriculum to eliminate some of the redundance
- revising the role of review in the curriculum so that the majority of each grade level is new material and so that review is placed in relevant contexts
- emphasizing the development of problem-solving skills in relevant and interesting situations
- incorporating calculators and computers throughout all grades as problem-solving tools
- adding an essential element on patterns, relations, and functions
- separating the teaching of operations and computation so that all students learn the meaning of the operations
- strengthening the areas of probability, statistics, and geometry
- emphasizing the importance of communication in mathematics

- building on a sound foundation of concepts rather than on rote procedures
- putting mathematics into meaningful contexts

The revised essential elements for Grade 7 are as follows:

(1) **Problem Solving.** Experience in solving problems designed to systematically develop students' problem-solving abilities through a variety of strategies and approaches. The student shall be provided opportunities to engage in the following types of activities:

(A) develop an organized approach to solving application and nonroutine problems appropriate for Grade 7;

(B) analyze problems by identifying relationships, discriminating relevant from irrelevant information, sequencing, observing patterns, prioritizing, and questioning;

(C) communicate an understanding of a problem by describing and discussing the problem and recording the relevant information;

(D) select appropriate strategies from a variety of approaches;

(E) select appropriate materials and methods for solutions; and

(F) generate and extend problems.

(2) **Patterns, Relations, and Functions.** Use of models and patterns to develop the algebraic concepts of relations and functions. The student shall be provided opportunities to:

(A) investigate patterns generated by repeating and terminating decimals;

(B) build a concrete model of a functional relationship and describe the relationship using function notation;

(C) introduce the concept of using letters to represent variables;

(D) develop the concept of operations with variables using concrete materials such as models;

(E) investigate solutions to simple open sentences (equalities and inequalities);

(F) evaluate algebraic expressions using mental calculations and calculators when appropriate; and

(G) formulate a possible problem when given a simple equation.

(3) **Number and Numeration Concepts.** Concepts and skills associated with the understanding of numbers and the place value system. The student shall be provided opportunities to:

(A) convert between fractions, decimals, whole numbers, and percents mentally, on paper, and with a calculator;

(B) compare and order integers;

(C) explore the absolute value of an integer;

(D) simplify expressions involving exponents using a calculator when appropriate;

(E) develop the meaning of squares and square roots using geometric models; and

(F) express numbers in scientific notation including numbers less than one using a calculator when appropriate.

(4) **Operations and Computation.** Use of manipulatives to develop the concepts of basic operations on numbers and to apply these concepts to the computational algorithms. The student shall be provided opportunities to:

(A) select an appropriate operation and/or strategy to solve a problem and justify the selection;

(B) use the order of operations to solve multi-step problems using a calculator when appropriate;

(C) add, subtract, multiply, and divide fractions and mixed numbers resulting from problem situations;

(D) add, subtract, multiply, and divide integers resulting from problem situations using models and connecting to rules;

(E) write and solve a simple inequality resulting from a problem situation and graph the solution on a number line;

(F) write and solve simple linear equations from problem situations and check the reasonableness of the results;

(G) use proportions to solve a variety of problems; and

(H) estimate solutions to problems using decimals and percent.

(5) **Measurement.** Concepts and skills using metric and customary units. The student shall be provided opportunities to:

(A) investigate the relationship between the perimeter and area of a polygon;

(B) develop the concept of volume for prisms/cylinders as the product of the area of the base and the height using models;

(C) develop the concept of volume for cones/pyramids as one-third the product of the area of the base and the height using models;

(D) explore surface area of three-dimensional figures using concrete models and graphing technology when appropriate;

(E) estimate and solve application and nonroutine problems involving volume, and

(F) explore the relationships between the dimensions and the volumes of similar solids by changing one of the dimensions.

(6) **Geometry.** Properties and relationships of geometric shapes and their applications. The student shall be provided opportunities to:

(A) identify parts and characteristics of common geometric figures;

(B) develop the concept of the Pythagorean Theorem using several different approaches;

(C) classify triangles and quadrilaterals by sides and angles;

(D) construct an angle bisector, the bisector of a segment, perpendicular lines, parallel lines, and triangles;

(E) construct a model of a three-dimensional figure when given the top, side, and front views;

(F) use the properties and relationships to two- and three-dimensional figures to solve problems; and

(G) apply geometry to such areas as art, architecture, construction, etc.

(7) **Probability, Statistics, and Graphing.** Use of probability and statistics to collect and interpret data. The student shall be provided opportunities to:

(A) compare different graphic representations of the same data to determine the appropriateness of the graph;

(B) use box and whisker graphs, stem and leaf plots, and histograms to display information in ways that illustrate the appropriate uses of mean, median, and mode;

(C) draw inferences and construct convincing arguments based on data analysis;

(D) investigate and recognize misuses of statistical or numeric information;

(E) construct sample spaces by listing, tree diagrams, and frequency distribution tables;

(F) find the probability of simple events; and

(G) use permutations and combinations in application problems.

Sample Learning Objectives

The following are sample objectives that mathematics educators could use to teach the essential elements for Grade 7.

ESSENTIAL ELEMENT 1: Problem Solving

(A) Use a problem solving hueristic (Understand, Plan, Solve, and Look Back) to solve application and nonroutine problems.

(B) Recognize problems having superfluous or insufficient information. Order the events in a problem. Describe patterns observed in problems.

(C) Identify the question asked in a problem. Rewrite a word problem to describe the question, facts, and key ideas.

(D) Select and use a variety of problem solving strategies (e.g., guess and check, find a pattern, make a list, use a model, make a table, eliminate, simplify, choose and operation, work backwards) to solve word problems.

(E) Use concrete models (base ten blocks, color tiles, pattern blocks) to solve problems.

(F) Write word problems for each of the problem solving strategies listed in Objective (D) above.

ESSENTIAL ELEMENT 2: Patterns, Relations, and Functions

(A) Rename fractions as decimals and determine which are repeating or terminating.

(B) Use an illustration or concrete model and functional notation to describe functional relationships.

(C) Identify and describe the variable in an equation, then represent it with a letter.

(D) Use Algebra Tiles to solve operations with variables.

(E) Identify and describe solution sets for simple open sentences for equalities and inequalities.

(F) Use mental arithmetic strategies to solve algebraic expressions. Use calculators to solve algebraic expressions.

(G) Given simple algebraic equations, write word problems that represent them.

ESSENTIAL ELEMENT 3: Number and Numeration Concepts

(A) Convert between fractions, decimals, whole numbers and percents using mental arithmetic strategies, paper and pencil, and calculators.

(B) Rewrite a given set of integers in order from largest to smallest or smallest to largest. Use a number line to order a given set of integers.

(C) Use a number line to identify the absolute value of integers.

(D) Use a calculator to solve exponential expressions.

(E) Use Algebra Tiles to illustrate the meaning if squares and square roots of numbers.

(F) Rewrite numbers greater and less than 1 using scientific notation.

ESSENTIAL ELEMENT 4: Operations and Computation

(A) Given a mathematical word problem, identify an appropriate operation(s) and problem solving strategy needed to solve it, justify the selection.

(B) Use a calculator and the order of operations to solve mulit-step problems.

(C) Solve word problems using the aithmetic operations with fractions and/or mixed numbers.

(D) Use the rules for addition, subtraction, multiplication, and division and Two-Color Counters (from the Algebra Tiles materials) as concrete models to solve word problems with integers.

(E) Write a description of an inequality from a problem situation and use a number line to graph the solution.

(F) Write and solve linear equations from problem situations. Check the results for reasonableness.

(G) Set up proportions to solve problem situations.

(H) Use the estimation strategies of front-end, clustering, rounding, and compatible numbers, to solve decimal and percent problems.

ESSENTIAL ELEMENT 5: Measurement

(A) Use concrete manipulatives such as Color Tiles to investigate the relationship between the perimeter and area of a polygon.

(B) Use concrete manipulatives and models to understand the concept and formula for the volume of prisms and cylinders. Fill prisms and cylinders with unit cubes (from the Base Ten Blocks) to determine volume.

(C) Use concrete manipulatives and models to understand the concept and formula for the volume of cones and pyramids.

(D) Use grid paper to cover three-dimensional figures to find their surface area.

(E) Estimate the volume of objects.

(F) Investigate the relationship between the dimension of similar solids when one dimension is changed.

ESSENTIAL ELEMENT 6: Geometry

(A) Write a description, create a model, and illustrate the parts and characteristics of common geometric figures such as points, lines, planes, rays, line segments, sides, angles, parallel and perpendicular lines.

(B) Use concrete objects (such as Color Tiles), isometric grid paper, or tangrams to develop the concept of the Pythagorean Theorem.

(C) Create models and use illustrations to make and classify triangles and quadrilaterals.

(D) Use a compass and straight edge to construct an angle bisector, line segment bisector, perpendicular and parallel lines, and triangles.

(E) Use concrete objects (such as Multilink Cubes or Color Cubes) to build three-dimensional figures from plans containing the top, side, and front views.

(F) Solve problems involving two- and three-dimensional figures using various properties and relationships such as similarity, congruence, perimeter, area, surface area, volume, etc.

(G) Apply the geometric ideas of transformations (reflections, rotations, translations), tessellations, symmetry, similarity, congruence to areas such as art, architecture and construction.

ESSENTIAL ELEMENT 7: Probability, Statistics, and Graphing

(A) Compare different graphs (a circle graph, bar graph, line graph and histogram, or scatter plot) using the same data to determine the appropriateness of each of the graphs in representing the data in a clear and accurate way.

(B) Collect a set of data, determine the mean, mode and median, and use box and whisker graphs, stem and leaf plots, and histograms to display information that illustrates the appropriate uses of the statistics.

(C) Draw inferences and construct convincing arguments based on data analysis; investigate and recognize misuses of statistical or numeric information; construct sample spaces by listing, tree diagrams, and frequency distribution tables; find the probability of simple events; and use permutations and combinations in application problems.

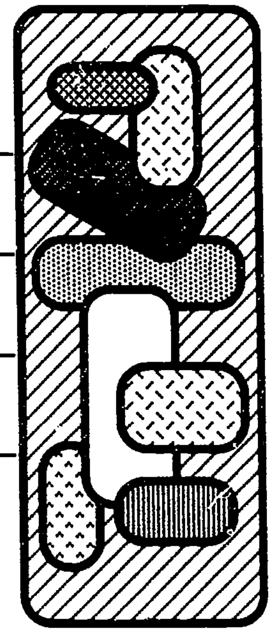
(D) Use research methods to answer a problem or question. Define the problem or question, decide on the data needed to answer the problem or question, describe the method for collecting and analyzing the data, collect the data, analyze the data, and report the results.

(E) Use data from newspapers, periodicals, or other sources to investigate and recognize misuses of the information.

(F) Conduct experiments involving the probabilities of an event(s) and compare experimental results with the theoretical probabilities of the event(s).

(G) Solve application problems involving permutations and combinations.

Texas Assessment of Academic Skills



Focus

The Texas Education Agency implemented the Texas Assessment of Academic Skills (TAAS) testing program in October, 1990. The program is in effect for the 1990-1995 period. The purpose of the assessment program is to provide Texas schools with an accurate measure of student achievement. The scope of content of the TAAS includes more of the instructional targets delineated in the essential elements. Every section of the TAAS test contains a certain number of broad objectives. These objectives remain constant from grade to grade, because they represent the core concepts that form the basis for a sound instructional progression from Grade 1 through Grade 12. What will differ from grade to grade are the instructional targets - or essential elements - which comprise each objective. A portion of this extended set of instructional targets is selected for assessment annually, but not every target is tested every year.

The broadened scope of the TAAS assessment program allows for a different focus, one which addresses the academic requirements of the 1990s. Those skill areas which demand little more than rote memorization are deemphasized, while those areas which improve a student's abilty to think independently, read critically, write clearly, and solve problems logically receive increased emphasis. This emphasis is in keeping with current national trends in education, which stress the importance, and necessity, of teaching students higher-order thinking skills.

Domains, Objectives, Targets

There are three domains in the TAAS - concepts, operations and problem solving. Each domain contains objectives which are derived from the essential elements. For every objective there are instructional targets which describe the kinds of mathematical experiences to reflect that objective. Each instructional target was taken for the most part directly from the essential elements as delineated in the *State Board of Education Rules for Curriculum*. Each target is defined in behavioral terms appropriate for pencil-and-paper testing.

DOMAIN: Concepts

Objective 1: The stu lent will demonstrate an understanding of number concepts.

(a) Compare and order nonnegative rational numbers, excluding whole numbers
(b) Round whole numbers and decimals (to nearest tenth, one, ten, or hundred)
(c) Determine relationships between and among fractions (denominators of 2, 3, 4, 5, 6, 8, and 10), decimals, and percents
(d) Use exponential notation to represent whole number expressions
(e) Factor whole numbers
(f) Find the least common multiple and the greatest common factor

Objective 2: The student will demonstrate an understanding of mathematical relations, functions, and other algebraic concepts.

(a) Use nonnegative rational number properties and inverse operations
(b) Determine missing elements in patterns
(c) Find relationships between ratios
(d) Solve simple linear equations
(e) Identify ordered pairs on a coordinate plane
(f) Use number line representations of fractions and decimals

Objective 3: The student will demonstrate an understanding of geometric properties and relationships.

(a) Recognize properties of two- and three-dimensional figures
(b) Identify translations, reflections, rotations, and their applications
(c) Recognize similarity, congruence, and symmetry

Objective 4: The student will demonstrate an understanding of measurement concepts using metric and customary units.

(a) Use metric and customary units
(b) Convert within the metric system
(c) Convert within the customary system
(d) Find perimeter and circumference
(e) Determine area (with and without grids) and volume

Objective 5: The student will demonstrate an understanding of probability and statistics.

(a) Use counting arrangements
(b) Use sample spaces to find fractional probability
(c) Predict possible outcomes from a sample
(d) Analyze data and interpret graphs
(e) Find means (averages)

DOMAIN: Operations

Objective 6: The student will use the operation of addition to solve problems.

(a) Add whole numbers, fractions, and decimals

Objective 7: **The student will use the operation of subtraction to solve problems.**

(a) Subtract whole numbers, fractions, and decimals

Objective 8: **The student will use the operation of multiplication to solve problems.**

(a) Multiply whole numbers, fractions, and decimals

Objective 9: **The student will use the operation of division to solve problems.**

(a) Divide whole numbers, fractions, and decimals

DOMAIN: Problem Solving

Objective 10: **The student will estimate solutions to a problem situation.**

(a) Estimate with whole numbers and decimals

Objective 11: **The student will determine solution strategies and will analyze or solve problems.**

(a) Formulate strategies or solve problems using basic operations with whole numbers, fractions, and decimals
(b) Determine strategies for solving or solve problems requiring the use of geometric concepts
(c) Analyze or solve problems through the use of similarity, congruence, and symmetry
(d) Analyze data and interpret graphs

Objective 12: **The student will express or solve problems using mathematical representaiton.**

(a) Formulate solution sentences
(b) Analyze or interpret graphs and charts and use the information derived to solve problems

Objective 13: **The student will evaluate the reasonableness of a solution to a problem situation.**

(a) Evaluate reasonableness

Instructional Strategies

A 50-Minute Lesson Format

1. Eye Opener Activity/Magic Tricks—3 minutes
2. Vocabulary and Facts—10 minutes
3. Main Lesson—30 minutes
4. Mental Math—5 minutes
5. Closure/Review—2 minutes

Eye Opener:
In the school library are magic books. Finding a trick that may or may not be math related will enhance your students' interests. Students will look forward to the eye opener. The eye opener may also be some type of activity that is quick and to the point. This may be teaching the students how to multiply by nine using their fingers.

Vocabulary and Facts:
This is essential for the students. Ask students at the beginning what is the product of 2 and 3? Most will probably not know if they have not previously studied math vocabulary. Note however that most all the students recognize the number form of 2 x 3 = .

Main Lesson:
Having done the above, students are ready for the main lesson.

Mental Math:
End the math class by teaching students a mental math skill. For example, how to multiply by 25 mentally.

Closure/Review:
Summarize the lesson's activities and concepts.

Following Directions

Administer this test to the students at the beginning of the school year to stress the importance of following directions. Tell students this is a timed test. They have one minute to work on it. Place the test face down on their desks. Say "start" for students to begin. Observe how many students did not read directions. You may be surprised!

DIRECTIONS:

- This is a timed test.
- Work quickly and carefully.
- Write the answers in the blanks provided.
- Work only problems three and ten.

1. 11 + 4 =
2. 6 x 3 =
3. Your name is
4. Raise your right hand!
5. 13 - 3 =
6. Stand up, then sit down!
7. 20 x 0 =
8. Walk quickly to the wall closest to you, touch the wall, then go back to your seat and sit down!
9. 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 =
10. Come to the front and turn in your paper!

Twelve Suggestions that Work

1. The eye opener activity at the beginning of every session helps to ensure that students are attentive and focused prior to the presentations of mathematics materials.

2. The facts and vocabulary activity provides students an opportunity to be successful (during reviews); to strengthen their knowledge of mathematics (facts/vocabulary); to respond openly in a group setting.

3. The quick analysis that follows each session improves presentations that follow.

4. Providing instructions/expectations prior to each activity improves the student's opportunities for success.

5. Rewarding students with praise or small gifts after exemplary performances or active participation reinforces the importance of the learning process.

6. Activities that involve each individual in the group make use of peer pressure which improves participation by students who tend to be passive learners (chanting, spelling vocabulary words, walking among students, and insisting on participation by reluctant participants proves to be motivational).

7. Treating the taking of notes as a following instructions activity helps in getting students to focus in activities.

8. Mental games (Number Sense and quick answers to problems involving a series of operations) prove challenging and a change of pace to the regular classroom routine.

9. The rapid move from the eye opener activity to the facts vocabulary activity to the main concept activity to the mental math activity to the review activity has students constantly engaged.

10. The daily handouts that are provided for student use serve to reinforce facts, vocabulary, and concepts introduced during the sessions.

11. Constantly reminding students of the importance of listening, following directions, being observant, and taking an active role in the session's different activities reinforces expectations set forth at the beginning.

12. The diagnostic test administered at the beginning of the program helps to determine group needs.

Fifty Two-Card Deck: A Teaching Tool for Basic Facts

Consider using a standard deck of playing cards to introduce or review certain facts with your students. Prior to introducing the use of playing cards, familiarize students with the following information (if possible, use an actual deck of cards).

1. Each deck consists of four suits:

A) black cards	B) red cards
1. spades	1. hearts
2. clubs	2. diamonds

   Tell students that the four suits can represent the four seasons in a year (spring, summer, fall, and winter).

2. Each suit contains 13 cards: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King. The Jack, Queen, and King are called *face cards*. If you let the Ace = 1, Jack = 11, Queen = 12, and King = 13, the sum of all the playing cards is 364. Since the deck contains 2 Jokers, you can make the following observation: The 52 playing cards and Joker (Joker = 1), when added, equals 365 (the number of days in a year). The extra Joker (Joker = 1), when added, give 366 (the number of days in a leap year).

   The standard deck has 12 face cards (NOTE: Each suit has 3 face cards. Since there are 4 suits, there are 4 x 3 or 12 face cards). The 12 face cards can represent the number of months in one year.

3. Each deck has 52 regular playing cards and 2 Jokers. The 52 regular playing cards can represent the number of weeks in a year.

4. A year measures the time it takes the Earth to revolve once around the sun. Since it takes the Earth 365 1/4 days to revolve once around the sun, then one year = 365 1/4 days. For convenience, it was agreed to let 1 year = 365 days with the understanding that every four years, the accumulated 1/4's (1/4 + 1/4 + 1/4 + 1/4 = 1) would require a leap year consisting of 366 days.

<u>Resource</u>
McAllen ISD's *The Magic of Math* handbook for teachers.

Thinking Mathematically: Extra Time Fillers

Using Number Concepts

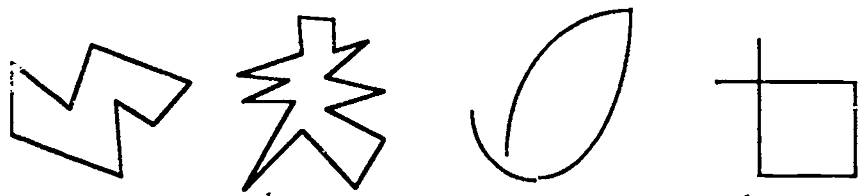
1. Two to four players, two sets of number cards numbered 0 to 9, paper and pencil. Draw a game board, one per player, that contains four rectangles. Shuffle the cards and place them face down. The first player takes the top card and writes the digit in any of his or her rectangles. The next player draws the next card and writes the digit in any of his or her rectangles. Continue until each player has all four digits filled. The player with the largest number wins. The game may also be played so that the winner is the player with the smallest number. Guide the students into realizing that there is a method of placing the numbers in the rectangles.

2. Draw a grid of squares, 3 by 3. Fill in the squares with the numbers 2 through 10 so that each row, column, and diagonal adds up to 18.

Visual Reasoning

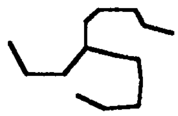
3. Think of shapes being stretched and bent. Imagine that they are made of rubber bands. These figures are both closed loops.

Which of these figures are equivalent to those above? (a and b) (Both are closed figures.)

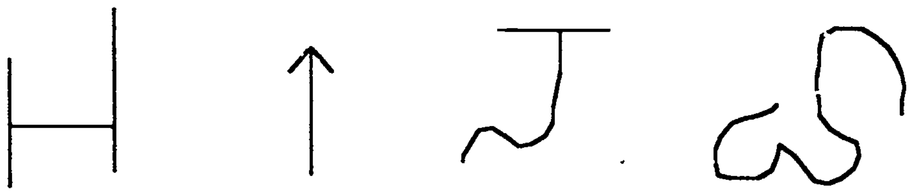


a. b. c. d.

Look at this figure.



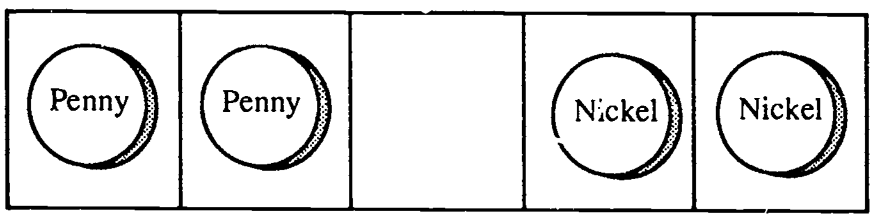
Which of the figures below could be made from it? (b and c) (Both are made up of three line segments with a common connecting point.)



b. c. d.

Investigating Patterns

4. Draw a five square board. Place two pennies on the left and two nickels on the right. The object of the game is to switch the order of the coins. Coins may move one square into an open square or jump over one coin. Pennies can move only right and nickels only may move to the left. The first move is always with a penny.



Resource

Mathematics in Action. Macmillan/McGraw-Hill, 1991, New York, NY.

Mathematics Shortcuts

Here are a few quick mathematics shortcuts which students enjoy learning and using:

When multiplying by 25, divide the other number by 4. If there are no remainders, add two 0's to the answer. If there is a remainder of 3, write a 75 at the end. If there is a remainder of 2, write a 50 at the end. (3/4 = 75%, 2/4 = 1/2 = 50%)

When multiplying by 50, divide the other number by 2. If there is no remainder, write two 0's at the end of the answer. If there is a remainder of one, then write 50 at the end instead of the two 0's.

When multiplying a two-digit number by 101, take the two-digit number and write it twice: 23 x 101 = 2323.

When multiplying a number by 100, take the decimal point of the other number and move it two times to the right.

When multiplying a number by 1000, take the decimal point of the other number and move it three times to the right.

When multiplying a number by 10000, take the decimal point of the other number and move it four times to the right.

When dividing a number by 10, take the decimal point of the other number and move it once to the left.

When dividing a number by 100, take the decimal point of the other number and move it to the left two times.

When dividing a number by 1000, take the decimal point of the other number and move it three times to the left.

Squaring two-digit numbers ending in 5:

Always write 25 for the last part of your answer. Add one to the other digit. Take this answer and multiply it to that first digit:

$\underline{35}^2$

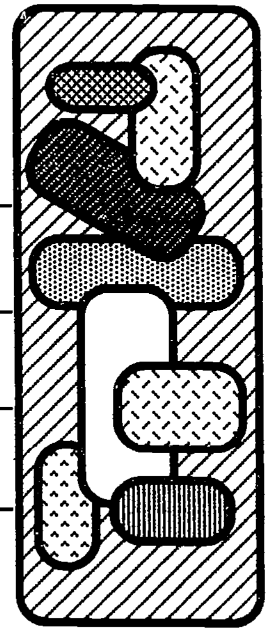
$5^2 = \underline{\ 25\ }$ Always write 25 for the last part of your answer.

$4 \times 3 = \underline{\ 12\ }$ Add one more to the 3, you get 4. Multiply 3 x 4 and you get 12 which is the first part of the answer.

Therefore: $\underline{35}^2 = \underline{1225}$

Always write a 25 for the last part of your answer. To get the first part, always add one more to the tens place digit and multiply these two numbers.

Sample Lessons for Teaching Grade 7 Mathematics



The following lessons represent the kind of mathematical experiences recommended for students in seventh grade mathematics classes. Several different manipulatives are included in these activities. It is important that students use these manipulatives as they work through the activities. Manipulatives and concrete objects enable middle school students to better understand the mathematical problems and concepts they so often struggle to learn. Students' experiences with manipulatives are recommended in the essential elements, TAAS's instructional targets, and NCTM's *Curriculum and Evaluation Standards for School Mathematics*.

Many activities in this section also recommend that students work together in pairs or small groups. Working together in cooperative groupings promotes communication, mathematical confidence, and students' problem-solving abilities.

Objective Students use the problem-solving guide and the strategy of finding a pattern to solve the problems posed in the activity.

Activity Ants in a Line

Materials Calculators, worksheet

Resources *The Problem Solver 6,* Creative Publications

Procedure

1. Have students read the problem and tell what they are asked to find.
2. Let the students work in pairs to solve the problem.
3. When most of the student pairs have solved the problem, ask them how they solved it.

   Example: (A) Recreated the pattern and counted.
   (B) Sum of black ants (1 + 2 + . . . + 14) and white ants (15).

4. Show students how to find the sum of consecutive numbers using Gauss's Gimmic.

   $\frac{n(n+1)}{2}$ So, $\frac{14(15)}{2} = 105$ [number of blak ants]

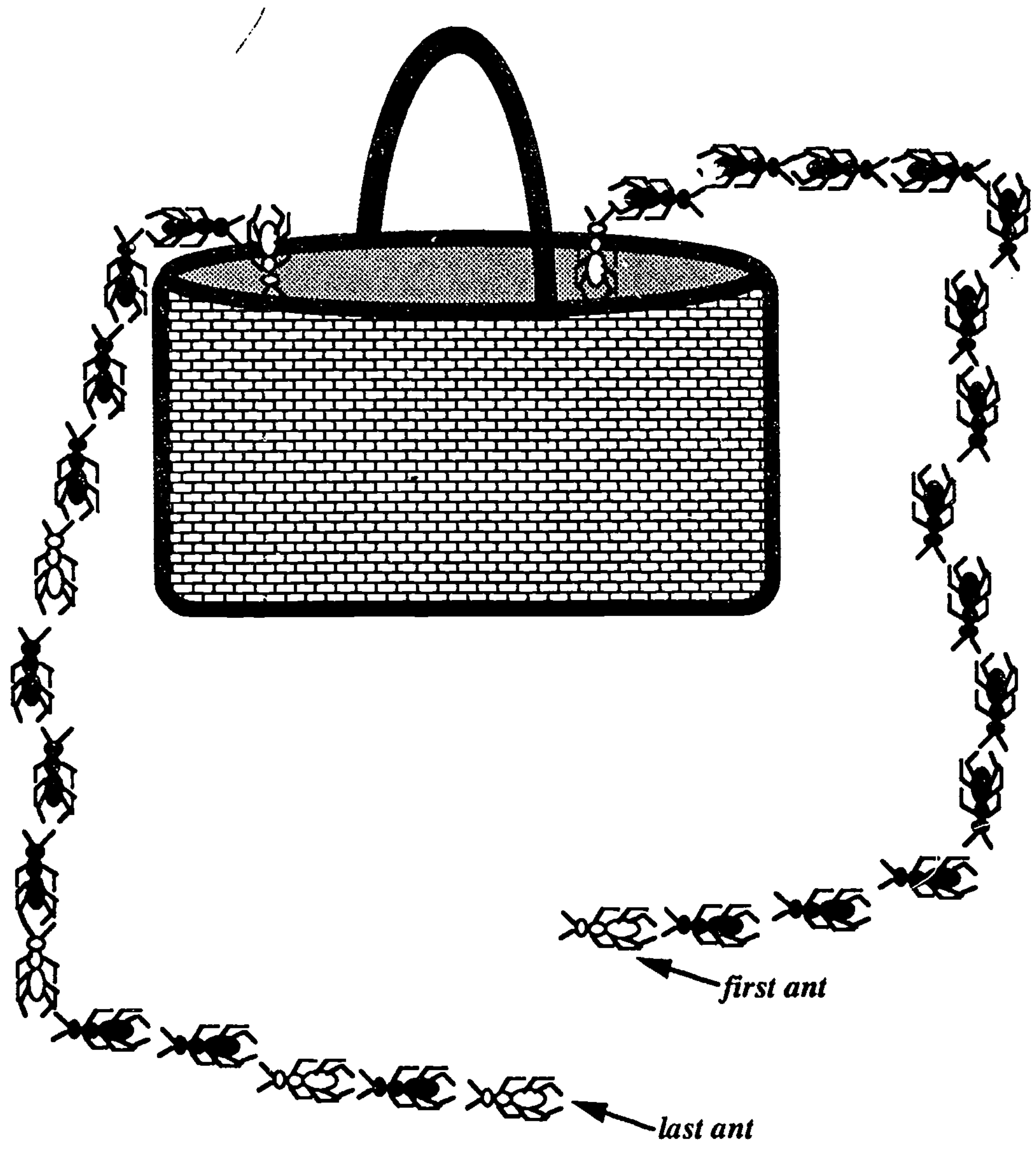
Evaluation

Have students (in their original groups) develop a similar activity. The activities will be traded among the groups for problem-solving purposes. They will share their strategies with the class. The teacher will not only check solutions, but will also look at the construction of the problem.

Extension Find the sum of numbers from 1 to 100, from 1 to 500, 1 to 1,000.

Ants in a Line

A continuous line of black and white ants
marches through a picnic basket.
How many ants are in the line in all?



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Objective Throughout the year, students will use the seven different tasks for computational problems in addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals.

Activity Learning Goals for Computational Problems

Materials Whole number problems —calculators, base ten blocks; fraction problems—TI-Math Explorer, fraction bars; decimal problems—TI-Math Explorer, decimal squares

Procedure

1. For lessons in whole number, fraction, and decimal operations, refer to the article in Grade 7 Curriculum Guideline's Appendix for a description of the seven tasks.

2. Give examples using the following problems:

Wholes:	$18 + 6 = ?$	$18 - 6 = ?$	$18 \times 6 = ?$	$18 \div 6 = ?$
Fractions:	$\frac{3}{4} + \frac{1}{4} = ?$	$\frac{3}{4} - \frac{1}{4} = ?$	$\frac{3}{4} \times \frac{1}{4} = ?$	$\frac{3}{4} \div \frac{1}{4} = ?$
Decimals:	$.90 + .30 = ?$	$.90 - .30 = ?$	$.90 \times .30 = ?$	$.90 \div .30 = ?$

3. Have students make up their own problems and do the seven tasks.

Evaluation

Since this is an ongoing activity, teacher observation may be the best evaluation. The teacher should listen, as students estimate, for the reasonableness of the prediction. The teacher should also make certain that the students are entering the correct information as they work; incorrect information can throw off the entire process.

Extension Students practice writing and illustrating word problems.

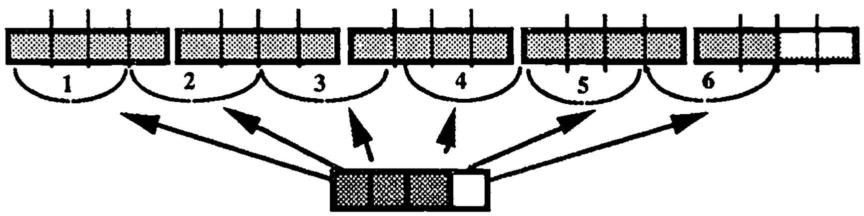
Reference Madsen, Anne L. (1992). Preparing elementary teacher candidates to teach mathematics nontraditionally. *Teacher Education and Practice, Vol. 8*(1), p. 83. Reprinted by permission of the author.

Learning Goals for Computational Problems

• Understand the concepts.

The concepts of rational number and division.
How many 3/4ths are there in 4 1/2?"

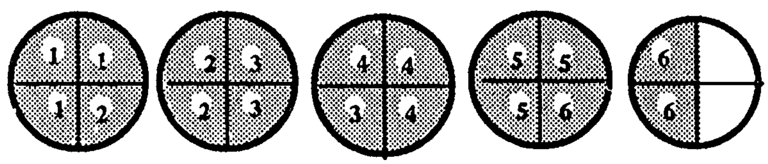
• Illustrate the operation.



• Estimate the answer.

$$4\frac{1}{2} \div \frac{3}{4} = ? \quad (5) \div (1) = (5)$$

• Use concrete models.



• Write a word problem.

"How many pieces of ribbon that are three-fourths of a yard can I cut from a ribbon that is four and a half yards long?"

• Use a calculator.

TI-Math Explorer:

4 [Unit] 1 [/] 2 [÷] 3 [/] 4 [=] [Simp] [=] [Simp] [=]

• Use the algorithm to calculate the answer.

$$4\frac{1}{2} \div \frac{3}{4} = \frac{9}{2} \text{ X } \frac{4}{3} = \frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{2}}} \text{ X } \frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{3}}} = 6$$

Objective Students will solve missing addends problems using an appropriate problem-solving strategy (such as guess and check or find a pattern.

Activity Some Sums

Materials Worksheet, calculators, 35 discs on which to write numbers (if needed)

Procedure

1. Discuss the directions with the students.
2. Have students work in pairs on Problem 1.
3. Have students work in pairs on Problems 2 - 5.
4. Discuss the results and the strategies the students used to solve the problems.

Evaluation

Give the students one more to do on their own—using the numbers 2, 4, 8, 10, 12, 16, 18. As they look for a solution, they write the steps (and/or strategies) they use to find the solution. The teacher collects and checks for understanding.

Extension Have students make up similar problems to share with the class. (Be sure to have them include the answers.)

Reference *Problem Solving in Mathematics: Algebra.* Lane County Mathematics Project, Dale Seymour Publications.
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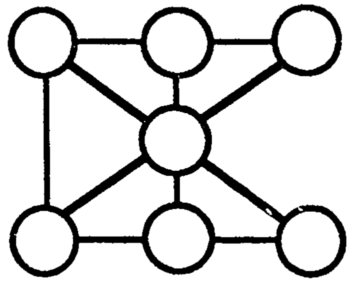
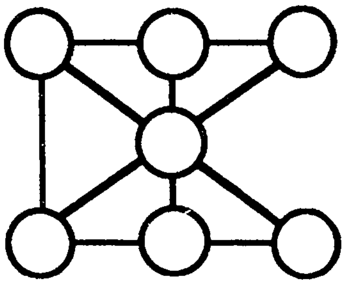
Some Sums

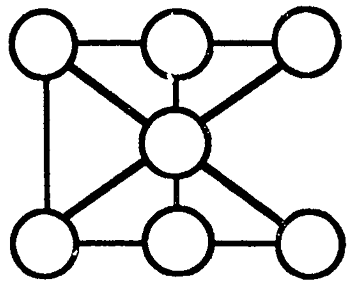
- Place the digits 1, 2, 3, 4, 5, 6, and 7 in the circles so each line of connected circles has the same sum.



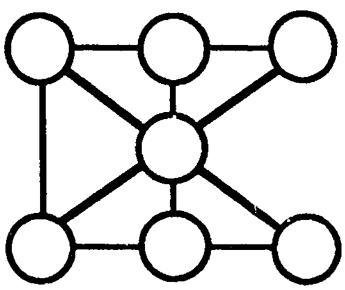
- Repeat using 2, 3, 4, 5, 6, 7, 8.



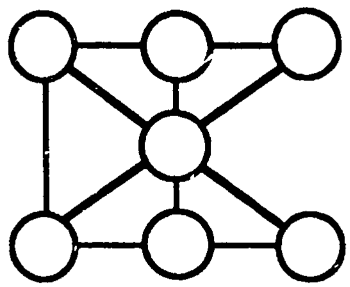
- Repeat using 2, 4, 6, 8, 10, 12, 14.



- Repeat using -4, -6, -8, -10, -12, -14, -16.



- Repeat using 1, 3, 5, 7, 9, 11, 13.



- Find other numbers that could be used to solve the puzzle. On the back of this page show where to place the numbers.

Objective Students will solve mathematics problems following a set of directions. They will then write an algebraic axpression for each direction using a variable.

Activity Math Magic

Materials Worksheet, calculators (if needed)

Procedure

1. Have students solve the first problem, discuss the result.
2. Have students discuss how they would write the same set of directions algebraically using a variable.
3. Have students work problems 2 - 4 alone then check the results with their partners.
4. Discuss the results with the class.

Evaluation

Provide the following problem: Think of a number. Multiply by 2. Add 18 to the product. Divide by 2. Subtract the original number. Have the students not only solve the problem, but also write the directions algebraically using a variable, to discern whether or not students understood the lesson.

Extension Have students make up a problem similar to the ones on the worksheet.

Reference *Problem Solving in Mathematics: Algebra.* Lane County Mathematics Project, Dale Seymour Publications.
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Math-Magic

First work through the following Math-Magic problems using numbers. Then use a variable to show why the magic works.

1. Pick a number.
   Add 5.
   Multiply by 3.
   Substract 15.
   Divide by 3.

   (You should end up with your original number.)

   n ______
   ______
   ______
   ______
   ______

   (Remember, after you've tried some numbers, use a variable to show why the trick works.)

2. Pick a number.
   Multiply by 12.
   Add the number of inches in a yard.
   Subtract the number of eggs in a dozen.
   Divide by the number of inches in a foot.
   Subtract your original number.

   (Your answer should be 2.)

   n ______
   ______
   ______
   ______
   ______
   ______

3. Pick a number between 1 and 10.
   Multiply by 99.
   Add your original number.
   Divide by 100.

   (Your answer should be the same as your original number.)

   n ______
   ______
   ______
   ______

4. Write down the number of the month in which you were born.
   Multiply by 2.
   Add 5.
   Multiply by 50.
   Add your age.
   Subtract 365.
   Add 115.

   (Your age will be the last 2 digits.
   The first digits will be the month in which you were born.)

   n ______
   ______
   ______
   ______
   ______
   ______
   ______

41

Objective Students will use the problem-solving strategy of finding a pattern to solve the problems. They will also write algebraic expressions for the problems with variables.

Activity An Odd Happening

Materials Worksheet, calculators

Procedure

1. Have students discuss the directions for the worksheet.
2. Students will work in pairs to solve the problems.
3. Review the answers and discuss the strategies they students used in solving the problems.

Evaluation

After a discussion, the teacher should ask the students to write answers to the following questions: How many numbers in the 9th row? What would the middle number be? What is the last number in the 8th row? What numbers would be included in the 11th row?

Reference *Problem Solving in Mathematics: Algebra.* Lane County Mathematics Project, Dale Seymour Publications.
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An Odd Happening

1

3 5

7 9 11

13 15 17 19

21 23 25 27 29

__ __ __ __ __ __

__ __ __ __ __ __ __

1. Write down the next two rows.

2. How many numbers in the
   a. 6th row? c. 100th row?
   b. 10th row? d. nth row?

3. Why does the 50th row not have a middle number?

4. What is the middle number in the
   a. 3rd row? c. 7th row? e. nth row?
   b. 5th row? d. 25th row?

5. What is the difference in the first and last number of the
   a. 4th row? c. 6th row? e. nth row?
   b. 5th row? d. 40th row?

6. What is the last number in the
   a. 5th row? c. 40th row?
   b. 6th row? d. nth row?

7. What is the sum of the numbers in the
   a. 30th row?
   b. nth row?

EE: 1A, 1B, 1C, 1D, 1E, 1F, 7G *TAAS Objectives: 5A, 8A, 11A, 13A*

Objective Students will solve the problem by making a list of the different combinations of clothes. They will then explore the idea of permutations.

Activity Jenny's Outfit

Materials Worksheet, calculators, scissors (if needed)

Procedure

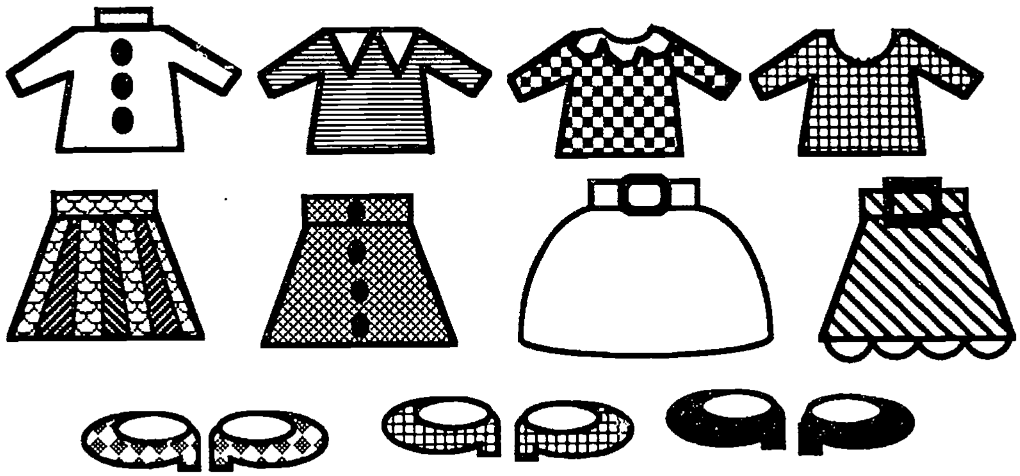
1. Discuss the problem-solving strategy of making a list. Use an example like: "How many different ice-cream sundaes can i make if I have three kinds of ice cream (chocolate, vanilla, butter-pecan) and two kinds of toppings (strawberry and fudge)?"
2. Have students work together on this problem and then discuss the results.
3. Have students work in pairs on the problem of Jenny's Outfits.
4. When the students are finished, have them discuss the results and tell how they made their lists.
5. Have students make up similar problems and trade them with their partners.
6. Discuss permutations, the characteristics of permutation problems, and how they are solved.

Evaluation

Give students one more wardrobe . . . Jim's clothes: four pairs of slackes—white, blue, khaki, and black; three shirts—white, yellow, and pink; three pairs of shoes—sneakers, loafers, and bucks. They will solve the problem independently. The teacher checks.

Jenny's Outfits

Jenny has 4 blouses, 4 skirts and 3 pairs of shoes.
How many different outfits does she have?



Objective Students will use pattern blocks to understand and solve computational problems with fractions.

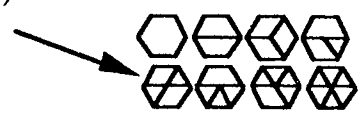
Activity Fractions and Pattern Blocks

Materials Pattern blocks, crayons, scissors, envelopes (to store pattern block pieces), isometric paper, worksheet

Procedure

1. Have students color and cut out pattern block pieces.

2. Start the class with the problem, "How many ways can you make pattern blocks equal in size and shape to your yellow hexagon?" (Show the students that one way would be two green triangles and two blue parallelograms.)



3. Discuss the results.

4. Have students use their pattern blocks to solve the problems on their worksheet. Their answers need to be expressed using one letter. (See the example on the worksheet.)

5. Go over the results.

Evaluation

On the overhead, write: Y - B = _____, 2Y + R + B + _____, Y + R - 3B = _____. While they solve the problems, the teacher can walk around and observe, interact, then collect papers.

Extension Have students write the fractional equivalents for the problems.

Reference *Pattern Blocks Activities* Dale Seymour Publications.
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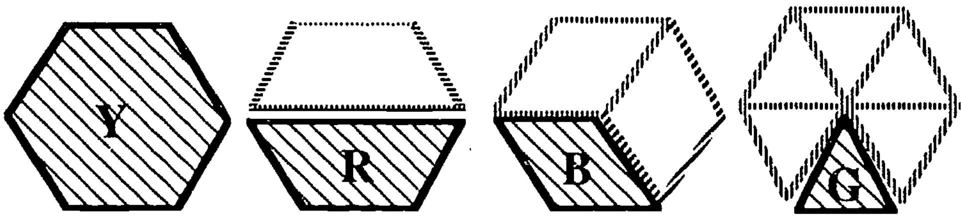
Fractions and Pattern Blocks



Use your pattern blocks to find the answers to the following problems
Write each answer using one letter.

Example: **3R + B + G = 2Y**

R + R = ____ **G + G = ____** **B + G = ____**

Y - R = ____ **B - G = ____** **R - G = ____**

G + G + G = ____ **B + B + B = ____**

R + G = ____ **Y - G = ____** **5G - B = ____**

Y + R = ____ **2B + G = ____** **5G - R = ____**

R + 2B = ____ **R + 5G = ____**

2B + 2G = ____ **R + B + G = ____**

47

Objective Students will find missing addends to a fraction magic square problem.

Activity Fraction Magic Square

Materials Worksheet, TI-Math Explorer (if needed)

Procedure

1. Discuss the directions for the magic square. Be sure students know that the sums must be equal to *8 1/2* in every column, row, and diagonal.

2. Have students work alone, then share their results.

Evaluation

Give students a 3 x 3 square, the sums will equal 6 1/3. Place the following numbers in the corners: 3, 1 1/3, 1 2/3, and 2 2/3. They fill in the blanks, discuss as a class or the teacher collects.

Extension Have students make up their own fraction magic square problems to give to the class.

Fraction Magic Square

Directions: The magic sum is $8\frac{1}{2}$

Fill in the blank spaces with fractions such that each row, column, and diagonal (corner to corner) will have the magic sum.

4		$\frac{3}{4}$	
	$\frac{3}{4}$		2
	$1\frac{3}{4}$		3
1			$\frac{1}{4}$

Objective Students will identify and classify triangles based on their attributes.

Activity Geometry-1

Materials Worksheet, scissors (if needed), tape or glue, paper

Procedure

This activity should be used prior to and at the end of students' work in geometry.

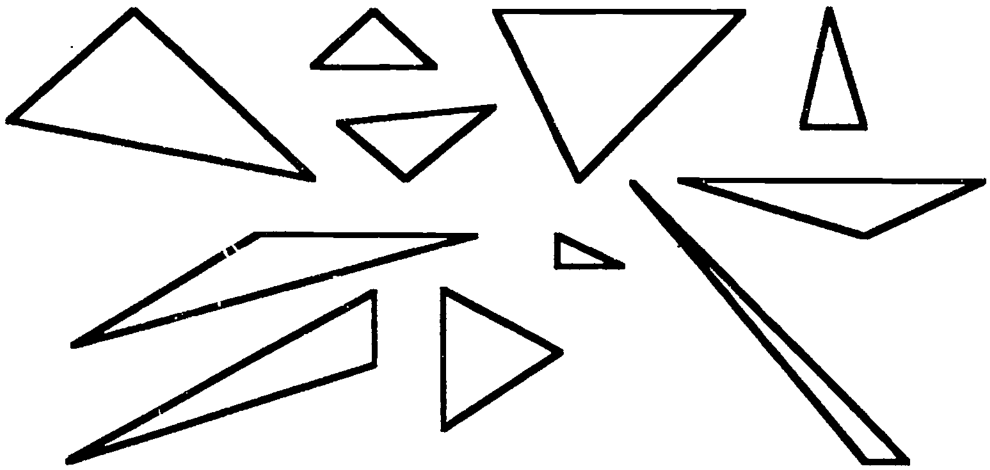
1. Prior to the Geometry Unit:
   Give students the worksheet and have them write answers to Question 1 and 2. They should be encouraged to describe and organize these triangles in any way they want to. Tell students they will discuss these answers at the end of the unit.

2. At the end of the Geometry Unit:
   Return the original worksheets to the students and have them answer Question 3. Ask students if their answers to Questions 1 and 2 had changed and why.

Evaluation

The teacher can study the responses of the post-tests for each of the activities. Through observation the teacher can discern students' thoughts and considerations of the attributes of the shapes.

Geometry-1



PRETEST

1. What, if anything, do these shapes have in common?

2. How you would put these shapes into groups?

POSTTEST

3. What changes would you make from your first responses to Questions 1 and 2.

51

Objective Students will identify and classify triangles based on their attributes.

Activity Geometry-2

Materials Worksheet, scissors (if needed), tape or glue, paper

Procedure

This activity should be used prior to and at the end of students' work in geometry.

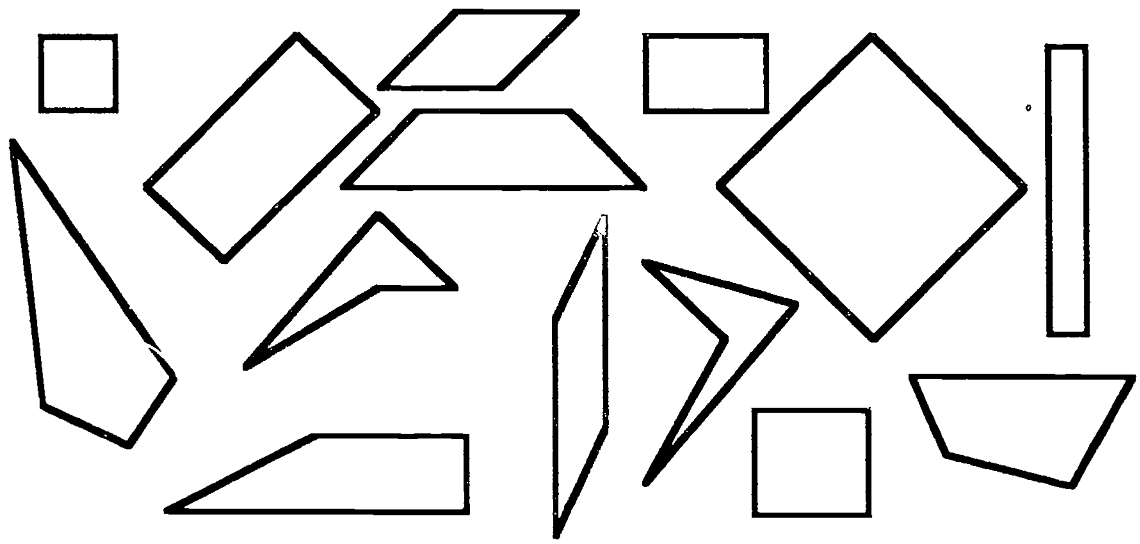
1. Prior to the Geometry Unit:
   Give students the worksheet and have them write answers to Question 1 and 2. They should be encouraged to describe and organize these quadrilaterals in any way they want to. Tell students they will discuss these answers at the end of the unit.

2. At the end of the Geometry Unit:
   Return the original worksheets to the students and have them answer Question 3. Ask students if their answers to Questions 1 and 2 had changed and why.

Evaluation

The teacher can study the responses of the post-tests for each of the activities. Through observation the teacher can discern students' thoughts and considerations of the attributes of the shapes.

Geometry-2



PRETEST

1. What, if anything, do these shapes have in common?

2. How you would put these shapes into groups?

POSTTEST

3. What changes would you make from your first responses to Questions 1 and 2?

53

Objective Students will identify and classify closed curves based on their attributes.

Activity Geometry-3

Materials Worksheet, scissors (if needed), tape or glue, paper

Procedure

This activity should be used prior to and at the end of students' work in geometry.

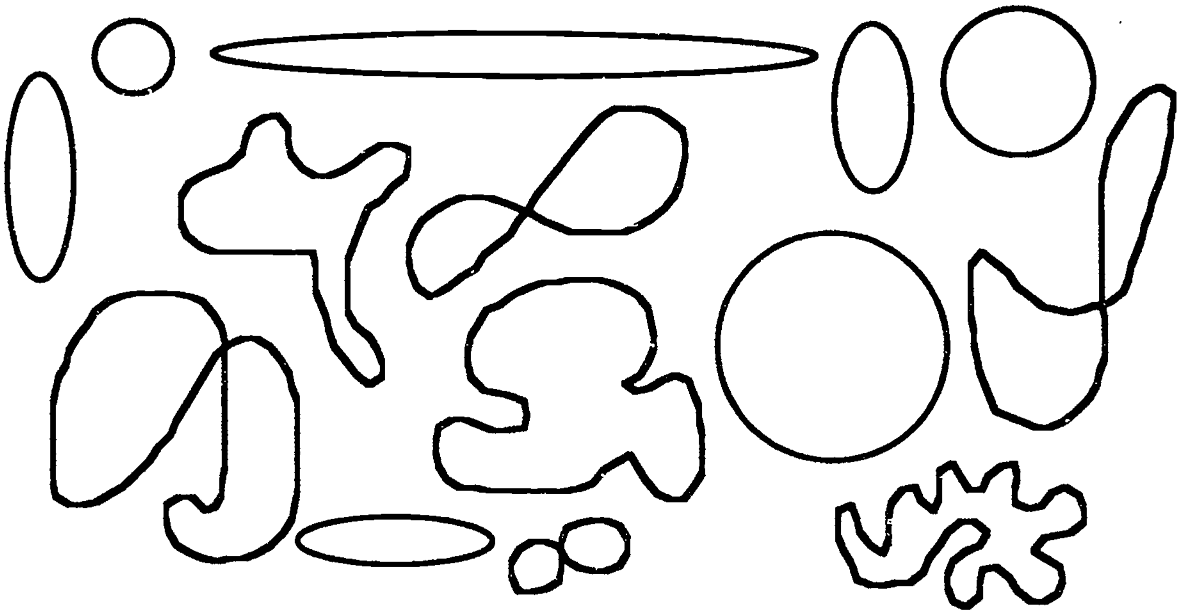
1. Prior to the Geometry Unit:
   Give students the worksheet and have them write answers to Questions 1 and 2. They should be encouraged to describe and organize these closed curves in any way they want to. Tell students they will discuss these answers at the end of the unit.

2. At the end of the Geometry Unit:
   Return the original worksheets to the students and have them answer Question 3. Ask students if their answers to Questions 1 and 2 had changed and why.

Evaluation

The teacher can study the responses of the post-tests for each of the activities. Through observation the teacher can discern students' thoughts and considerations of the attributes of the shapes.

Geometry-3



PRETEST

1. What, if anything, do these shapes have in common?

____

2. How you would put these shapes into groups?

____

____

____

POSTTEST

3. What changes would you make from your first responses to Questions 1 and 2?

____

____

____

55

Objective Students will identify and classify polygons based on their attributes.

Activity Geometry-4

Materials Worksheet, scissors (if needed), tape or glue, paper

Procedure

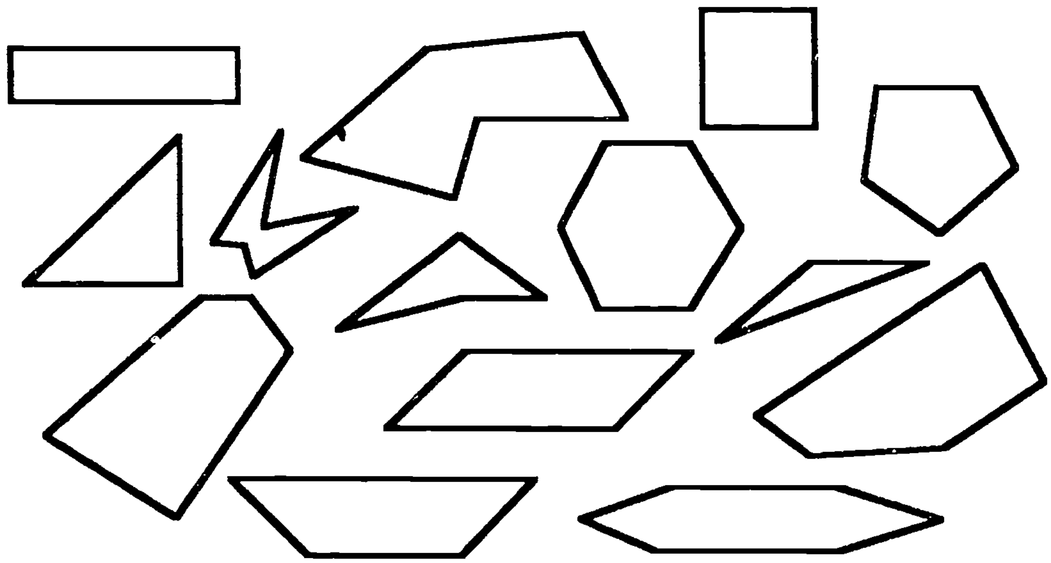
This activity should be used prior to and at the end of students' work in geometry.

1. Prior to the Geometry Unit:
   Give students the worksheet and have them write answers to Questions 1 and 2. They should be encouraged to describe and organize these polygons in any way they want to. Tell students they will discuss these answers at the end of the unit.

2. At the end of the Geometry Unit:
   Return the original worksheets to the students and have them answer Question 3. Ask students if their answers to Questions 1 and 2 had changed and why.

Evaluation The teacher can study the responses of the post-tests for each of the activities. Through observation the teacher can discern students' thoughts and considerations of the attributes of the shapes.

Geometry-4



PRETEST

1. What, if anything, do these shapes have in common?

____

2. How you would put these shapes into groups?

____

____

____

POSTTEST

3. What changes would you make from your first responses to Questions 1 and 2?

____

____

____

Objective Students will make three-dimensional soma puzzle pieces from two-dimensional drawings. They will construct a 3 x 3 x 3 soma cube using all of the puzzle pieces.

Activity Soma Cube Puzzle

Materials 27 multilink cubes or 27 sugar cubes (and glue) per student, worksheet

Procedure

Using multilink cubes:

1. Have students follow directions on the worksheet and make their puzzle pieces. Note: 6 of the pieces require 4 cubes to make, 1 piece requires only 3 cubes.

2. Have students build the soma cube.

Using sugar cubes and glue:

1. Have students make the soma pieces by gluing the sugar cubes together. Have them run a thin line of glue along the edges to secure the pieces. Let the pieces dry over night.

2. The following day have students build the soma cube.

Evaluation

This activity can be evaluated by observing students as they work as well as by examining the constructed cubes.

Reference Cox, Phillip. (1986). *Geometry: An Informal Approach Teacher's Guide.* Allyn and Bacon, (pp. 184-185).
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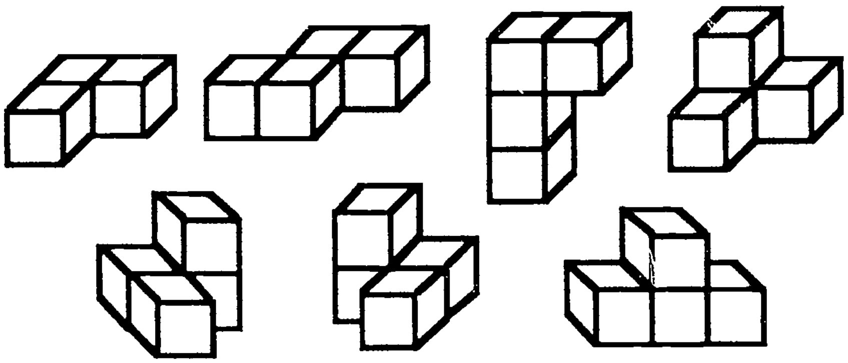
Soma Cube Puzzle

Build these 7 puzzle pieces with 27 multilink cubes.
Fit the 7 pieces together to make a 3 by 3 by 3 cubes.



Objective The students will use two soma puzzle pieces to solve three-dimensional building problems.

Activity Two-Piece Puzzle

Materials Soma puzzle pieces, worksheet, isometric paper

Procedure

1. Have students use the two soma pieces shown on the worksheet.
2. Students are to fit the two pieces together in ways that make the 6 buildings on the worksheet.
3. After they make a building, they are to color in one of the pieces to show how they fit together.
4. When the students finish the worksheet, they are to take two other soma pieces and fit them together in ways that make new buildings. For each new building, the students are to draw it on the isometric paper and shade in one of the pieces.

Evaluation The teacher can check the worksheet to see how the students shaded the cubes.

Extension Have students use three soma pieces to build a building and draw it on the isometric paper. They will then color each of the three pieces in the building using different colors.

Reference *Middle Grades Mathematics Project: Spatial Visualization.* Addison-Wesley Publishing Co., (pp. 111-112).
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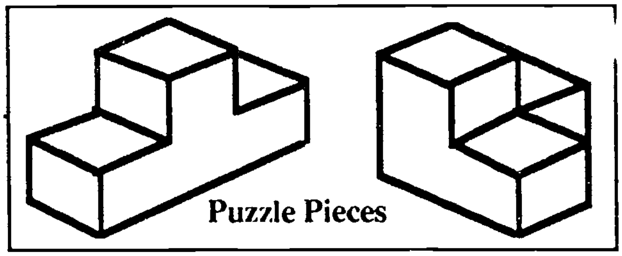
Two-Piece Puzzle

- Build two soma puzzle pieces from multilink cubes.
- Use the puzzle pieces to build each building.
- Show how you built them by shading in one of the puzzle pieces on each drawing.



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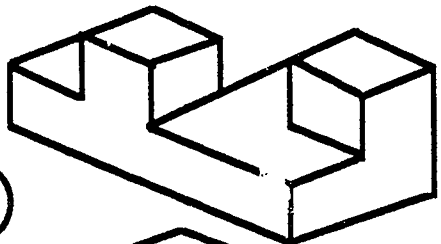

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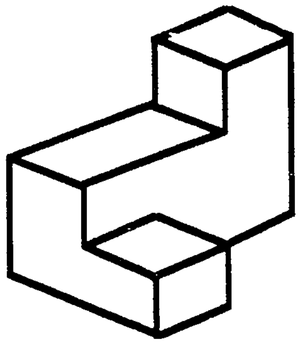

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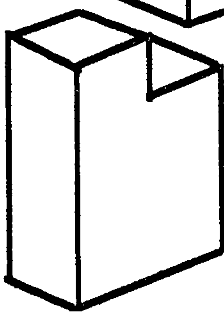

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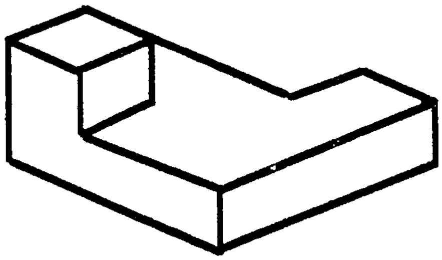

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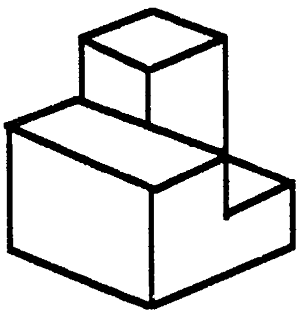

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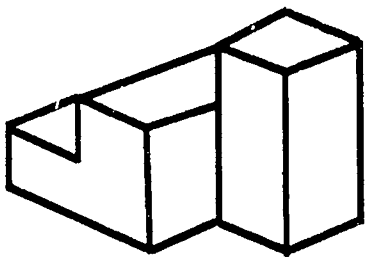


Objective Students will find the perimeters of polygons and generate an algebraic expression to represent a pattern they have found.

Activity Perimeter-1

Materials Tracing paper, worksheet, calculators

Procedure

1. Go over the directions and have students work on the examples at the top of the worksheet together.

2. Do the first problem with the students.

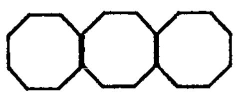
3. Have students do the remaining problems in pairs. Students may want to trace each figure and count the sides to determine the perimeter of each new shape.

4. Discuss the results.

Evaluation

Teacher observation: The student should be able to fill in the blanks with the correct numbers, and produce a generalization for *n*.

Extension Have students find the pattern for making polygons from octagons.



Number of Octagons	1	2	3	4	5	6	10	n
Perimeter								

Reference *Problem Solving in Mathematics: Algebra.* Lane County Mathematics Project, Dale Seymour Publications.
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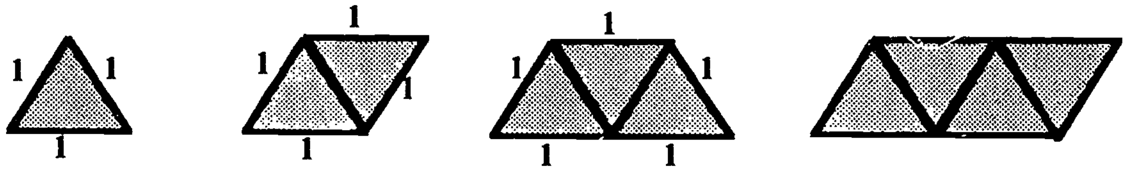
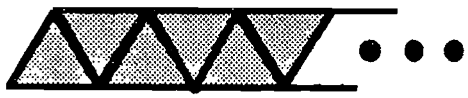
Perimeter-1

The perimeter is the distance around. Study these examples.

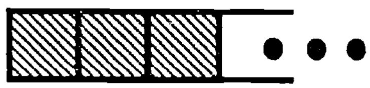


p = 3   p = 4   p = 5   p = 6

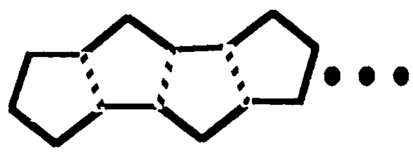
Complete the tables below.



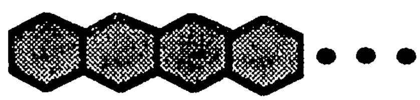
Number of Triangles	1	2	3	4	5	6	10		n
Perimeter	3	4	5					40	



Number of Squares	1	2	3	4	5	6	10		n
Perimeter	4	6						62	



Number of Pentagons	1	2	3	4	5	6	10		n
Perimeter	5	8						92	



Number of Hexagons	1	2	3	4	5	6	10		n
Perimeter	6							102	

Objective Students will find the perimeters of polygons and generate algebraic expressions to represent a patterns of the perimeters they found.

Activity Perimeter-2

Materials Color tiles, crayons, scissors, envelope, calculators, worksheet

Procedure

1. Have students make a set of color tiles. Have students make the first figure on the worksheet using 3 tiles.

2. Discuss the perimeter of the first figure (8). Have the students make the second figure using 6 tiles. Discuss the perimeter of the second figure (14).

3. Have students build the third and fourth figures and find the perimeters of the polygons in the rest of the table. Discuss the results.

4. Students should work in pairs on the second, third, and fourth sets of figures.

5. Discuss the results when the students are finished.

Evaluation

Teacher observation: The student should be able to fill in the blanks with the correct numbers, and produce a generalization for *n*.

Reference *Problem Solving in Mathematics: Algebra.* Lane County Mathematics Project, Dale Seymour Publications.
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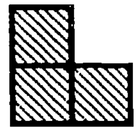
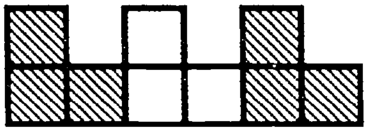
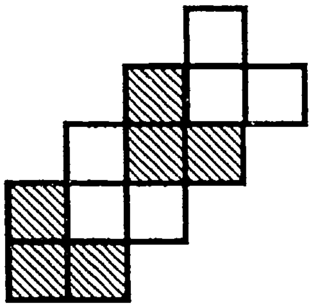
Perimeter-2

In these problems, the figures look like this →



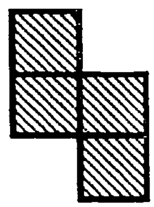


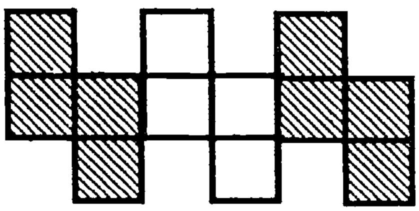
Number of Figures	1	2	3	4	5	10		n
Perimeter	8	14	20				92	



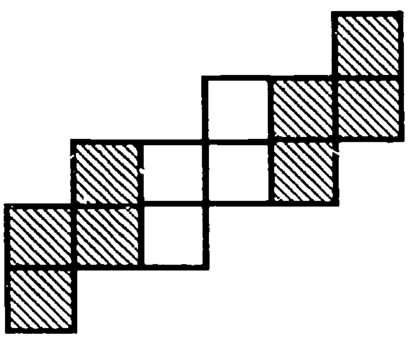
Number of Figures	1	2	3	4	5	10		n
Perimeter	8						104	

In these problems, the figures look like this →





Number of Figures	1	2	3	4	5	10		n
Perimeter	10	18					162	



Number of Figures	1	2	3	4	5	10		n
Perimeter	10						148	

65

Objective Students will subdivide polygons into two pieces by making one straight line cut and will fit the two pieces together to make squares.

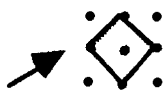
Activity Square Search

Materials Worksheet, tracing or waxed paper—one sheet per student

Procedure

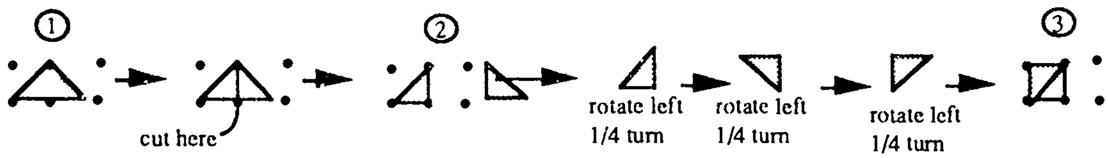
1. Read the directions with the students. Use the example to show them how to subdivide the figures.

2. The students can use the tracing/waxed paper to trace the piece they would cut out and check if it fits with the remaining piece to make a square. Let sutdents work in pairs.

3. IMPORTANT: Not all squares on the worksheet are built like this  .. Some squares can look like this  . IMPORTANT: Not all straight line cuts are made from one point to another in the figures. Some cuts are made between points.

4. When students are finished, discuss the kinds of moves that were needed for the cut pieces to make the square.

Example:

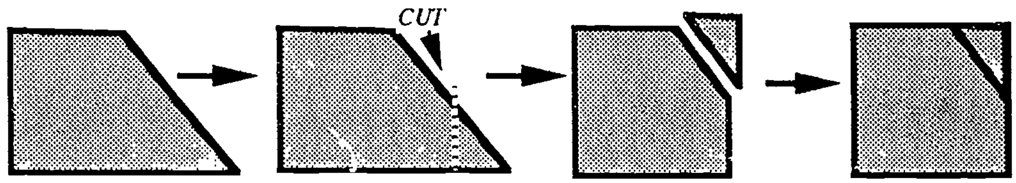


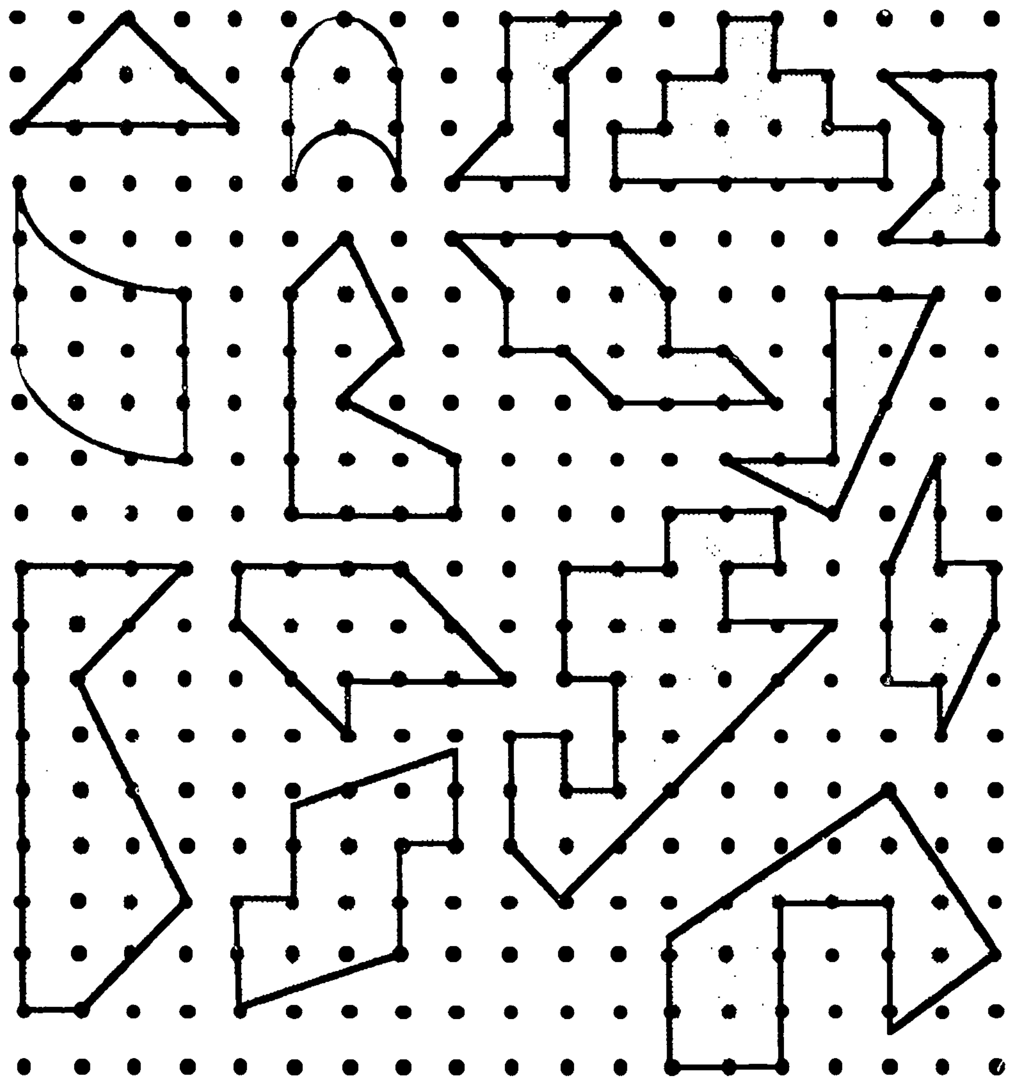
Evaluation

Teacher observation: Students will test their thinking using wax paper, then they will make the appropriate cuts in the provided worksheet and make the squares using tape to keep the pieces together.

Square Search

Draw one straight line so the two pieces make a square.





Objective Students will subdivide polygons into two pieces by making a cut (curved, jagged, or straight) so that the two remaining pieces are congruent.

Activity Congruent Halves

Materials Worksheet, tracing or waxed paper—one sheet per student

Procedure

1. Review the directions with the students. This activity is different from Square Search in that the students do not have to make only straight line cuts.
2. Have the students work together, if they wish, to solve these problems.
3. When the students are finished, discuss the answers. Some problems have more than one way to make congruent pieces.

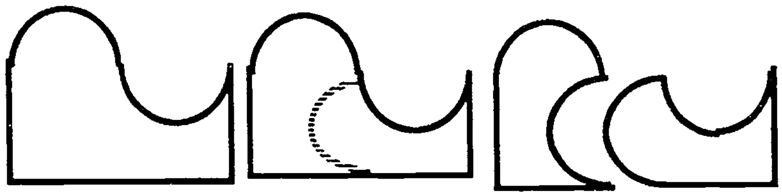
Evaluation

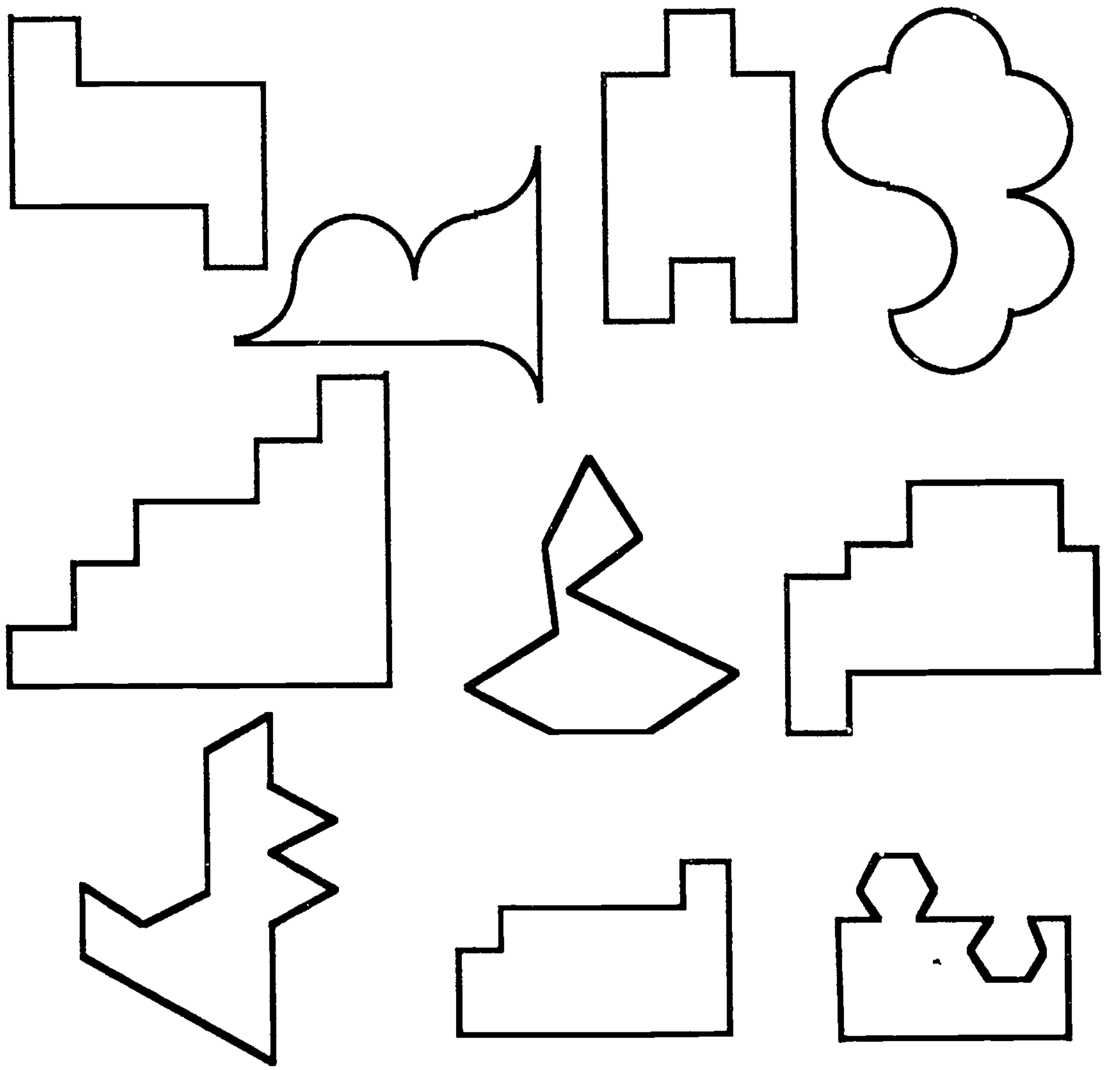
Teacher observation: Students will use wax paper, make the appropriate cuts in the provided worksheet, and make congruent pieces.

Extension Have the students make their own puzzles to share with the class.

Congruent Halves

Pick a problem. Make one cut that creates two congruent halves.
(The cut doesn't have to be straight.)





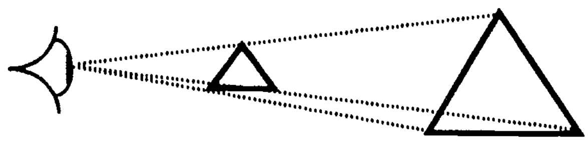
Objective Students will use tests for similarity to determine which triangles in the set are similar to each other.

Activity Similar Shapes

Materials Worksheet, scissors, tape or glue, plain paper (1/2 sheet per student)

Procedure

1. Review with the students the idea of similarity in geometric figures. Tests for similarity would be equal angies, proportional sides, and sighting one triangle with another.



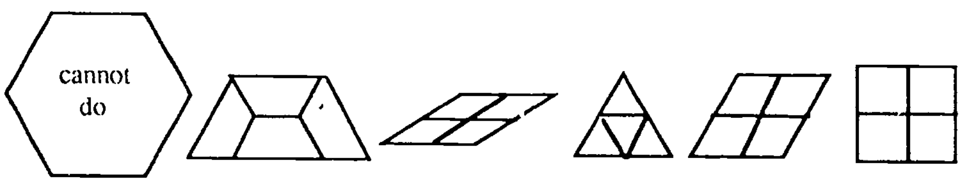
2. Have the students cut out the triangles nad use one or more tests to determine which are similar. They can glue/tape similar triangles together on a black piece of paper.

3. When students are finished, have them discuss the method they used to determine similarity.

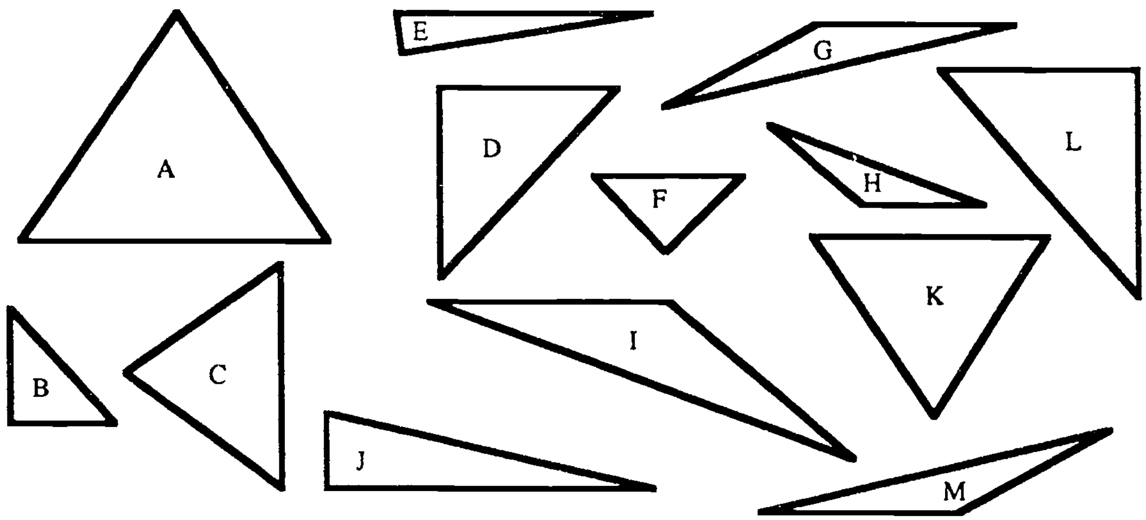
Evaluation

Have students devise a similar activity using four-sided polygons. They draw the shapes, write the questions, then answer the questions (on another sheet of paper, as if they were constructing a teacher's key).

Extension Have the students use their pattern blocks (four of each block) to make a larger block that is similar. (Only the hexagon cannot be made into a larger similar shape.)



Similar Shapes



Cut out these triangles.

Which triangles are similar?

____

How do you know?

____

____

71

EE: 1A, 1B, 1C, 1D, 1E, 1F, 2C, 2F, 4A, 5E, 6G   TAAS Objectives: 2B, 2D, 3A, 11A, 11B, 12A

Objective Sutdents will find the number of cubes of a given size that will fit into a cube of a given size. They will then write a general algebraic expression for the number of cubes (size $n \times n \times n$) that would fit into a 1^3, 2^3, 3^3, 4^3, 5^3, n^3 cube.

Activity How Many Cubes?

Materials Multilink cubes (64 single cubes per two students), worksheet

Procedure

1. Have students use their multilink cubes to build a 3 by 3 by 3 cube. Go over Question 1 at the top of the worksheet with the students. Complete the chart for the 3 by 3 by 3 cube.

2. Let the students work in pairs to find the answers for the other cubes.

3. When they are finished, discuss the answers for the cubes and the algebraic expressions they write.

Evaluation The teacher can evaluate student performance as students complete the worksheet.

Reference *Problem Solving in Mathematics: Algebra.* Lane County Mathematics Project, Dale Seymour Publications.
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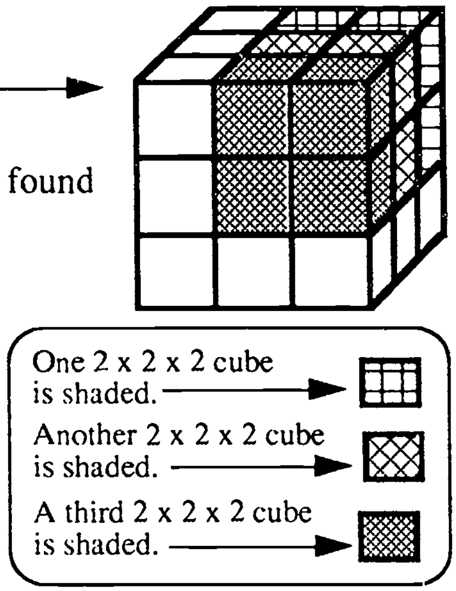
How Many Cubes?

This cube is 3 by 3 by 3.



1. How many of these cubes can be found in the 3 by 3 by 3 cube?
   a. 1 by 1 by 1 _____
   b. 2 by 2 by 2 _____
   c. 3 by 3 by 3 _____

2. Complete this table. Look for patterns.

Cube Size	Number of Cubes					
	1 x 1 x 1	2 x 2 x 2	3 x 3 x 3	4 x 4 x 4	5 x 5 x 5	n x n x n
1 x 1 x 1						
2 x 2 x 2						
3 x 3 x 3						
4 x 4 x 4						
5 x 5 x 5						
n x n x n						

Objective Students will understand the concept of a normal distribution by conducting an experiment and analyzing individual and group results.

Activity The Famous Ant Drob

Materials Worksheets 1-6, ants (100 per student—either plastic toy ants or cut from a transparency), scissors, tape, envelope (to store the ants), calculator

Procedure

1. Discuss the idea of normal distributions in statistics. Ask students:

   "If I held a handful of sand about two feet above the ground and let it fall slowly to the ground, what shape would the sand make?"

   "If you turn an hour glass upside down, what shape does the sand make when it falls through the narrow center?"

2. Tell the students they are going to do an experiment to find the distribution of ants that fall on a mat.

3. Have students make their Ant Drop Mat and cut out their ants.

4. Follow the directions on the worksheet.

5. For the group results, have students (in twos, threes, or fours) combine their individual totals.

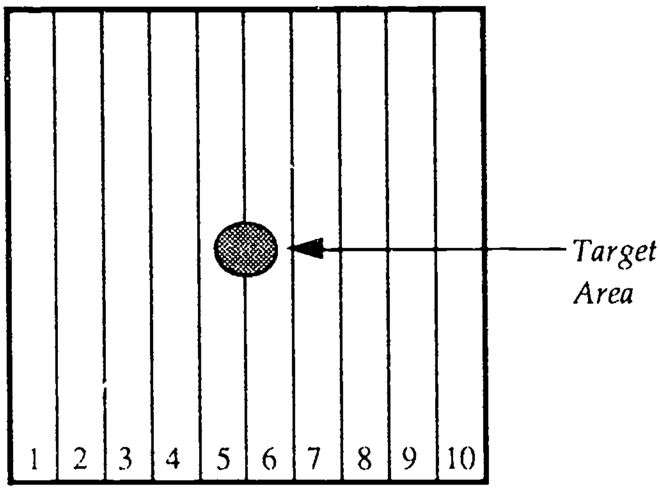
6. Discuss the shape of the graph and what it means.

Evaluation

The teacher will observe students as they work and interact. The teacher will listen as they interact as a class. The teacher will want the class to discuss the individual results and how the individual results compare to the group results. How do the group results compare to the class results? What would happen if the ants were dropped ten more times? One hundred more times? Will students be able to make generalizations?

The Famous Ant Drop

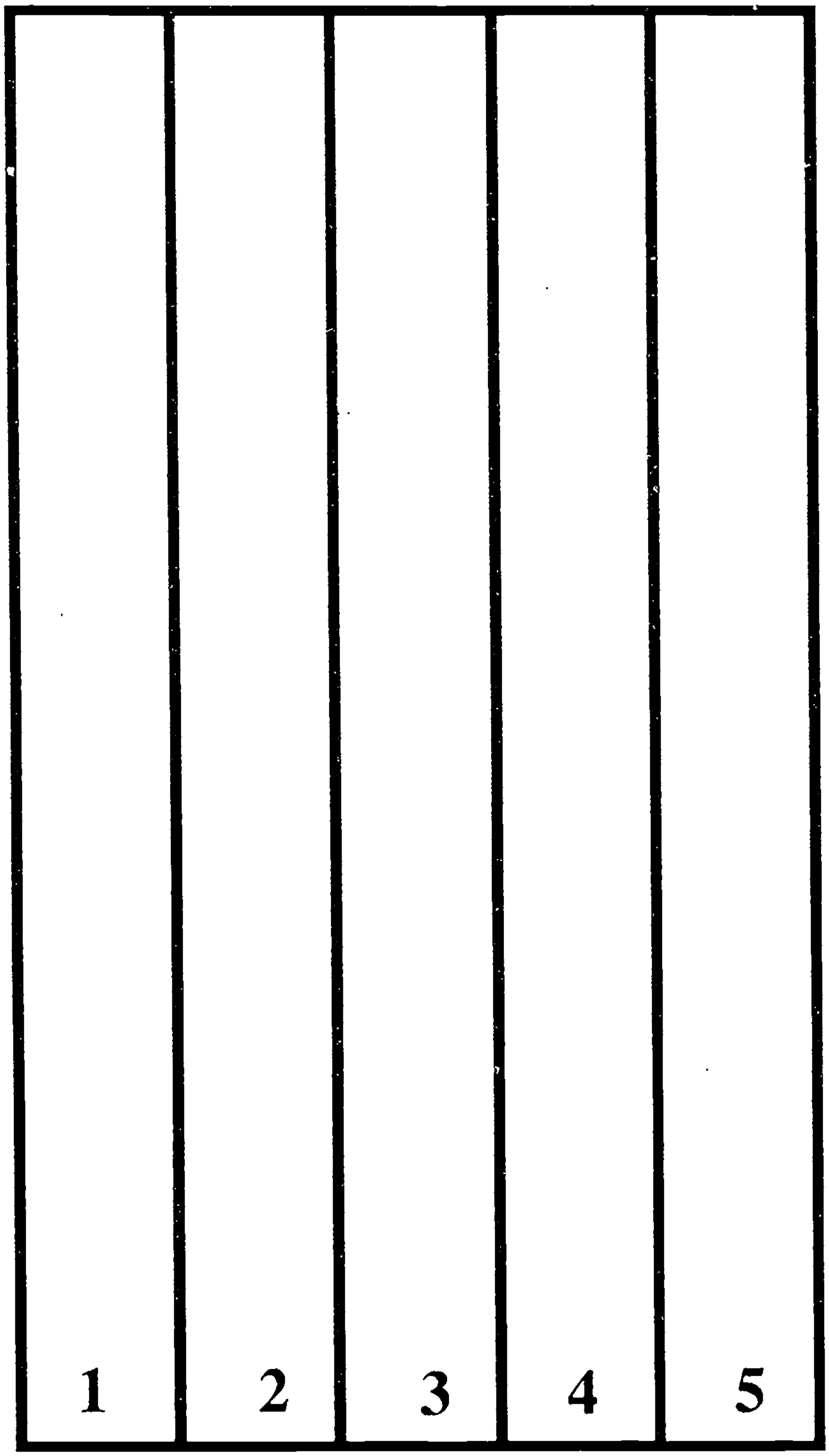
1. Construct an ant drop mat by taping the two halves of the mat (pages 2 & 3) together as pictured below.



2. Drop your ants (page 4) a couple at a time from an arms length above the mat. Always aim at the target (●).

3. Record the number of ants that dropped in each region (1 to 10) in the chart below.

REGION	1	2	3	4	5	6	7	8	9	10
NUMBER OF ANTS										



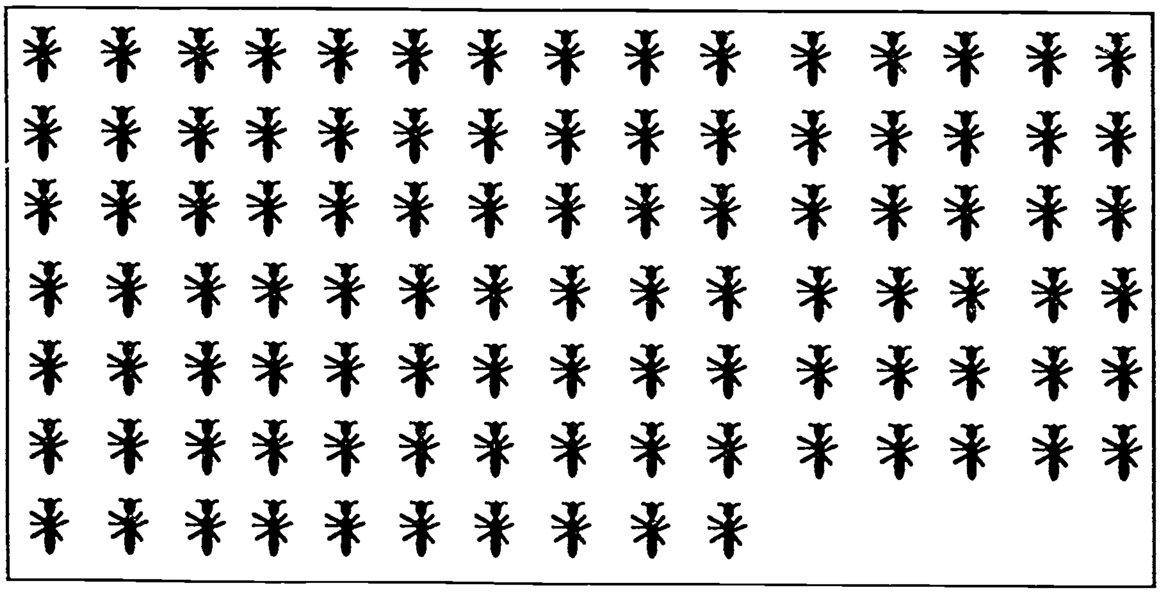
Ant Drop Activity—page 2

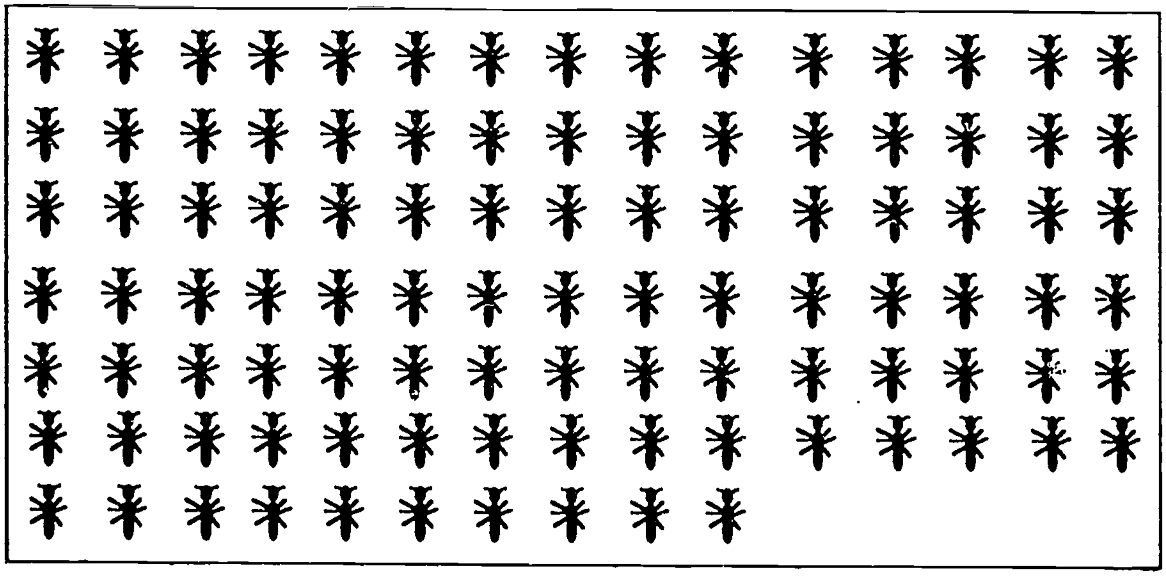
6	7	8	9	10

Ant Drop Activity—page 3

Ants for the Famous Ant Drop

1. Make a transparency of the ants.
2. Each student should have about 100 ants. (There are two sets of 100 ants on this page.)
3. Have the students cut out each ant.
4. Students can store ants in envelopes.





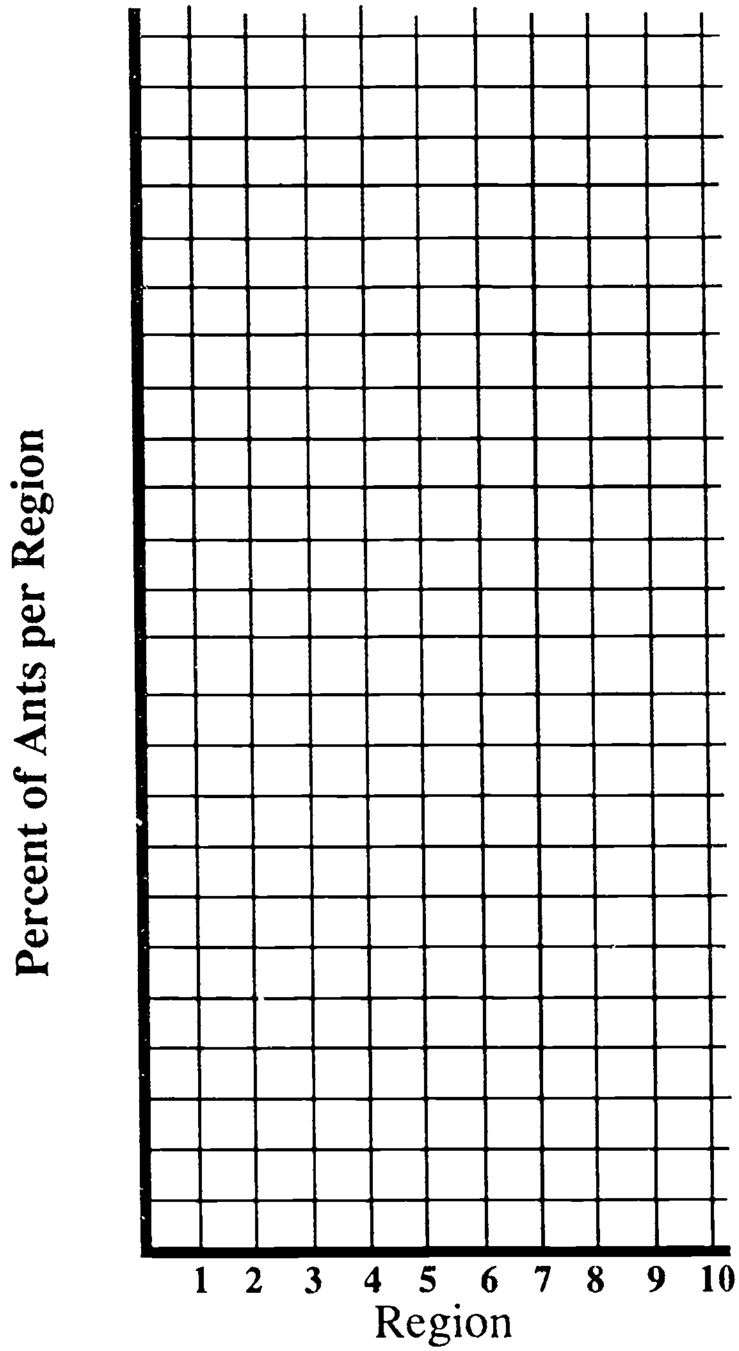
Ant Drop Activity - -page 4

Class Data

Group	Number of Ants per Region										
	1	2	3	4	5	6	7	8	9	10	Grand Total
Totals											
As Decimals											
As Percents											

Class Data

Region	% of Ants
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	



Ant Drop Activity—page 6

Objective The students will build an understanding of pattern generalization leading to algebraic statements, kinds of variations, variables, and functions.

Activity Discovery with Cubes

Materials Worksheets, multilink cubes, calculators

Procedure

1. Have the students build a 2 by 2 by 2 cube. Go over the questions in Activity 1.
2. Have them work in pairs to find the answers to the problems for Activities 2-6.
3. For Activity 6, the students will generalize the number of painted sides for a 7 by 7 by 7 and a 10 by 10 by 10 cube from their previous results.
4. Work with the students to build the generalization for the number of painted sides for an n by n by n cube. (Suggestion: If students have difficulty in writing the generalization, have them describe in writing the pattern they notice and then rewrite their statements using variables.)

Evaluation Monitor students as they work. Check their recorded information in activity six.

Reference Reys, Robert. (1974). Discovery with cubes. *The Mathematics Teacher, Vol. 67*(1), (pp. 47-50).
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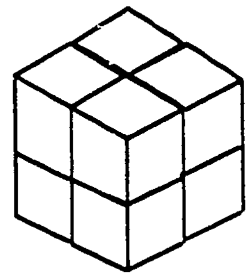
Discovery With Cubes

Small cubes have been stacked and glued together to form these larger cubes.

ACTIVITY 1

a. How many small cubes are in the large cube? _____

If this large cube were dropped into a bucket of paint and completely submerged:

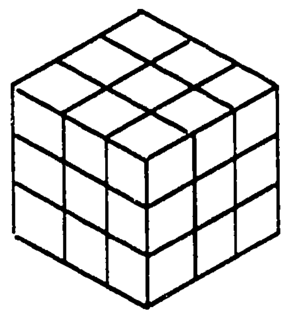


b. How many of the smaller cubes would be painted on three sides? _____
c. How many on only two sides? _____
d. How many on only one side? _____
e. How many on zero sides? _____
f. What is the sum of your answers in b, c, d, and e? _____
g. How does your answer to f compare to a? _____

ACTIVITY 2

a. How many small cubes are in the large cube? _____

If this large cube were dropped into a bucket of paint and completely submerged:

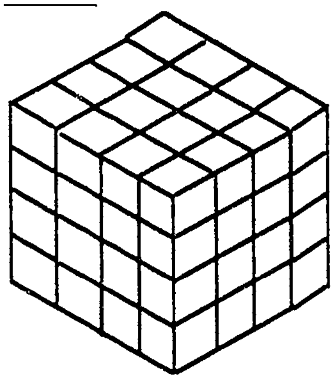


b. How many of the smaller cubes would be painted on three sides? _____
c. How many on only two sides? _____
d. How many on only one side? _____
e. How many on zero sides? _____
f. What is the sum of your answers in b, c, d, and e? _____
g. How does your answer to f compare to a? _____

ACTIVITY 3

a. How many small cubes are in the large cube? _____

If this large cube were dropped into a bucket of paint and completely submerged:

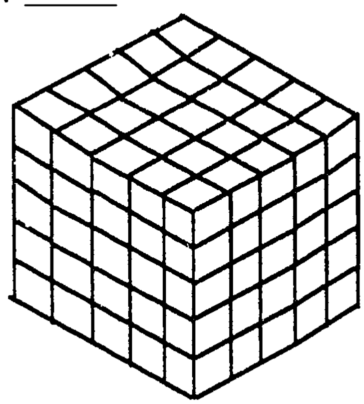


b. How many of the smaller cubes would be painted on three sides? _____
c. How many on only two sides? _____
d. How many on only one side? _____
e. How many on zero sides? _____
f. What is the sum of your answers in b, c, d, and e? _____
g. How does your answer to f compare to a? _____

ACTIVITY 4

a. How many small cubes are in the large cube? _____

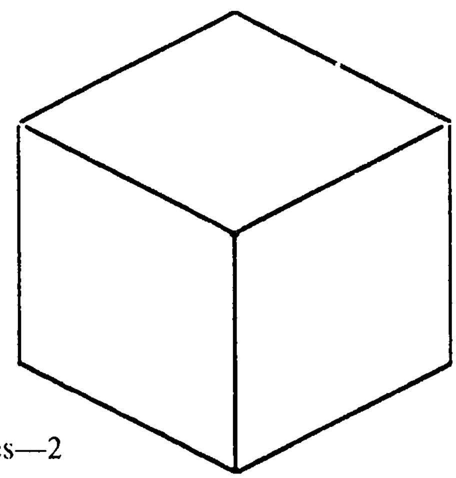
If this large cube were dropped into a bucket of paint and completely submerged:



b. How many of the smaller cubes would be painted on three sides? _____
c. How many on only two sides? _____
d. How many on only one side? _____
e. How many on zero sides? _____
f. What is the sum of your answers in b, c, d, and e? _____
g. How does your answer to f compare to a? _____

ACTIVITY 5

- Suppose your cube were 6 by 6 by 6.
- Complete this model by sketching a 6 by 6 by 6 cube.
- Use it to determine the total number of cubes as well as the number of faces with zero, one, two, three, and four sides painted.



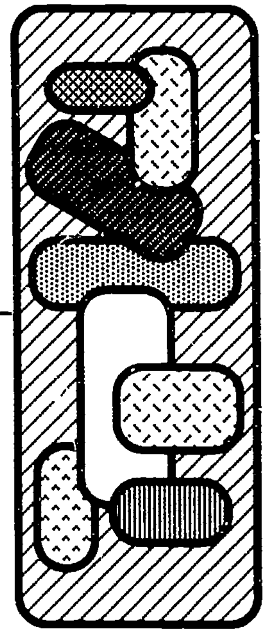
ACTIVITY 6

Now that you have solved several problems with the cubes, record the information in the table below:

Length of Side of Cube	Number of Painted Sides					Total Number of Cubes
	0	1	2	3	4	
2						
3						
4						
5						
6						
↓						
7						
↓						
10						
↓						
n						

Discover with Cubes—page 3

Evaluation



Philosophy

NCTM's *Professional Standards for Teaching Mathematics* and *Curriculum and Evaluation Standards for School Mathematics* (the *Standards*) emphasize the connection between assessment of students and analysis of instruction. In other words, mathematics teachers should monitor students' learning (both formatively and summatively) in order to assess and adjust teaching. Teachers must observe and listen in order to tailor teaching strategies. Information about what students are understanding should be used to revise and adapt short- and long-range plans, and students' understandings should guide teachers in shaping the learning environment. Also, teachers are responsible for describing students' learning to administrators, parents, and students themselves.

Students' mathematical power depends on various understandings, skills, and dispositions. The development of students' abilities to reason mathematically—to conjecture, justify, and revise based on evidence and to analyze and solve problems must be assessed. Students' dispositions toward mathematics (confidence, interest, perseverance, etc.) are also a key dimension that teachers should monitor.

The importance of using assessment to improve instruction is crucial. Information should be gathered from multiple sources using numerous assessment techniques and modes that are aligned with the curriculum. Assessment techniques must reflect the diversity of instructional methods implied in the *Standards* and the various ways students learn and process information. Instructional decisions should be based on this convergence of information from different sources.

Types of Evaluation

While paper and pencil tests are one useful medium for judging aspects of students' mathematical knowledge, teachers need information gathered in a variety of ways and using a range of sources. Observing, interviewing, and closely watching and listening to students are all important means of assessment. While monitoring students, teachers can evaluate the learning environment, tasks, and discourse that have been taking place. Using a variety of strategies, teachers should assess students' capacities and inclinations to analyze situations, frame and solve problems, and make sense of concepts and procedures. Such information should be used to assess how students are doing, as well as how well the tasks, discourse, and environment are fostering students' mathematical power and then to adapt instruction in response.

Principles relevant for judging assessment instruments at all levels also apply to program evaluation. Inherent in the *Standards* is an assumption that all evaluation processes should use multiple assessment techniques aligned with the curriculum and consider the purpose of assessment. Mathematics education described in the *Standards* places new demands on instruction and forces a reassessment of the manner and method by which students' progress is charted. Testing instruments must reflect the scope and intent of the instructional program to have students solve problems, reason, and communicate, and they must enable teachers to understand students' perceptions of mathematical ideas and processes and their abilities to function in mathematical contexts. Also, the testing instruments must be sensitive enough to help teachers identify individual areas of difficulty in order to improve instruction.

Available assessment techniques suggested in the *Standards* include multiple-choice, short-answer, discussion, or open-ended questions; interviews; homework; projects; journals; essays; portfolios; presentations; and dramatizations. These techniques are appropriate for students in whole-class settings, small groups, or individually, with the mode of assessment written, oral, or computer oriented.

Samples

The purpose of an assessment should dictate the questions asked, methods employed, and uses of resulting information. Methods for gathering information should be appropriate to the developmental level and maturity of the students.

The following samples are excerpted and adapted from the NCTM *Curriculum and Evaluation Standards for School Mathematics.*

Sample 1

Individuals' pulse rates vary. What is the normal pulse rate for students in your class? Consider various characteristics and conditions (such as exercise). How do they relate to pulse rate?

Evaluation should focus on the reasonableness of students' questions, representations of data, verification of results, and generalizations. This exercise is appropriate for group work and can extend over several days. A score can be given to each group, and specific segments of work can be scored separately. Calculators and computers should be employed.

Sample 2

Three of five doctors interviewed recommend Dinosaur vitamins. Write a question to go with this statement to make a problem and solve it.

This problem is appropriate for large-group discussion. Assessment should focus on the choice of questions which must be logically connected to the statement. Solving one's own problem gives an indication of ability to solve and encourages the formulation of reasonable questions.

Sample 3

Seven students have test scores of 57, 96, 73, 86, 92, 75, and 89. Find the average score. How much is the average score increased if each student's score is increased by

a. 5 points?
b. x points?

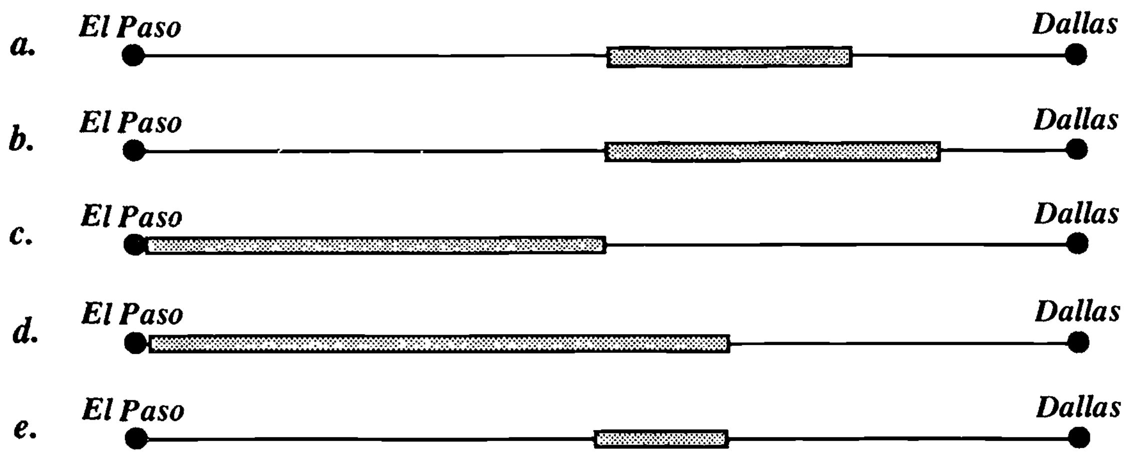
Write a statement about how much the average score is increased if each individual score is increased by x points. Convince another student that the statement is true.

Students can work in small groups. A computer is useful for investigating other situations or to consider the scores of an entire class. This sample focuses on students' ability to generalize from specific cases.

Sample 4

On a trip from El Paso to Dallas, Sara fell asleep after half the trip. When she awoke, she still had to travel half the distance she traveled while sleeping. For what part of the entire trip had she been asleep? Assuming that the shaded part in each diagram shows when Sara was asleep, which diagram best depicts the answer?

This sample assesses whether students can interpret the problem and identify the correct representation of the solution. Fractions must be considered in relation to different units. Students must think of the trip as a whole and then consider the portion of the total trip during which Sara was asleep.



Sample 5

Find the least common multiple of the following numbers:

a. 7 and 49
b. 8 and 9
c. 1 and 5
d. 5 and 6 and 20

Assessment should focus on whether students can arrive at the correct answer with facility. Note that in selection "a" one number is a multiple of the other; in selection "b" the numbers are relatively prime; in answer "c" one number is 1; and "d" might require a modification of procedures previously used.

Test-Taking Strategies

1. If the score is based on the number of questions you answer correctly, you should answer every question (even if you have to guess). Do this before going to the next problem.

2. You should put a light mark next to questions that you may be able to work but that will require more time. If time permits, you can then try to work these. Erase the light marks before turning in your answer sheet.

3. Make sure you scan all of the answer choices. Some of the choices are those that are obtained when a common error is made.

4. Sometimes the answer choices may give you an idea as to how to work a problem.

5. If you do not know how to solve a problem using standard methods you may try working it backwards--that is trying each answer and seeing which one works. As your first test, pick a middle-sized answer and plug it in. Is it to little? If so, then you need answers with bigger numbers. If this middle-sized answer is too big, then test only the answer choices with smaller numbers.

6. Estimating the result of a certain calculation can reduce the amount of time needed to obtain a solution. If the answers are spread apart from each other, estimating is easy. If they are close to each other, you may not be able to estimate effectively.

7. Sometimes answer choices can be eliminated because you can see without computation that they cannot possibly be right.

8. Make sure that the question you answer is the question that was asked.

9. When first approaching a word problem, put down your pencil, read the problem over quickly to get the general idea, identify the desired unkown, and scan quickly over the answer choices.

10. Some problems contain too much information. Weed out that which is unnecessary as you set up equations, and don't worry if you do not use all the information given.

11. Remember that an answer can be represented in more than one way. If you're fairly sure you did a problem correctly, check to see if the answer you got is the same as one of the answer choices given for the problem but given in a different form.

12. When you can eliminate one answer choice or more, try to make an educated guess and record an answer.

13. Make full use of the time alloted. If you have attempted all of the problems and have some time left, take the time to check your work.

14. If you are allowed to use scratch paper, use it. Do not recopy a problem from a test to the scratch paper if it is unnecessary.

15. Avoid spending too much time on any one question.

16. Make sure that the answer sheet is filled correctly. Be careful when erasing; take the time necessary to ensure that answers to problems worked are placed correctly on the answer sheet.

Resource; McAllen ISD: *The Magic of Math* handbook for teachers.

Grading

Different purposes are served by assessing students' knowledge in the mathematics classroom. One of these purposes is the grading of students. Teachers are faced with a number of decisions that they must make concerning timing, form, rigorousness, and usability of assessment. Any single form of assessment is too limiting to describe fully a student's mathematical knowledge. Thus a variety of methods over time would best evaluate the different aspects of mathematical knowledge.

Standard 1 in the Evaluation Section of the *Standards* is titled "Alignment." Standard 1 calls for the methods and tasks for assessing students' learning to be aligned with the curriculum's:

1. Goals, objectives, and mathematical content;

2. Relative emphases given to various topics and processes and their relationships;

3. Instructional approaches and activities, including the use of calculators, computers, and manipulatives.

Teachers might begin changes in classroom assessment following five steps suggested by Lester and Kroll (1991).

Step 1: Start small.

Step 2: Incorporate assessment into the class routine.

Step 3: Set up an easy and efficient record-keeping system.

Step 4: Establish an assessment plan.

Step 5: Personalize the assessment plan.

In assessing students, teachers need to remember that giving a correct answer does not mean a student knows a concept. Students may give a correct solution for the wrong reason. Thus information on reasoning must come from open-ended questions and verbal interactions. Effective assessment of critical thinking depends to a great extent on how well teachers facilitate the communication of evidence of students' understandings, critical thinking, and reasoning.

Finally, portfolios have become increasingly used in mathematics assessment. The article by Jean Stenmark in Appendix, Grade 8, "Math Portfolios: A New Form of Assessment," discusses the purpose and uses of portfolios to assess student learning. Included in a portfolio might be: written descriptions of the results of mathematical investigations; extended analyses of problem situations; descriptions and diagrams; statistics and graphics; reports; responses to open-ended questions or homework; group reports; copies of awards; photographs of projects; and other ideas. Dates should be placed on all entries. Portfolios can provide essential evidence of performance and ability beyond factual knowledge.

Homework

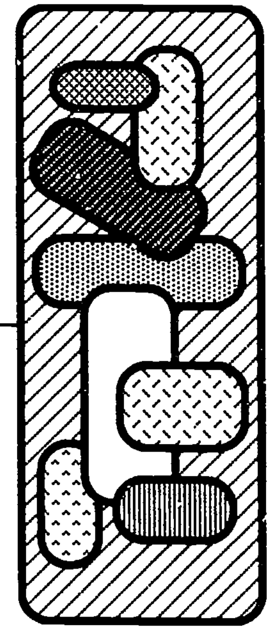
Assignments may be given for completion in class or at home. Two purposes of assignments might be to train students in meeting responsibility and to provide practice after a skill-oriented lesson. The assignments may be scored by the students themselves or by the teacher. Starting assignments during class time provides students the opportunity to enlist needed help from the teacher or peers. Homework provides an opportunity for parental involvement.

David Johnson in *Every Minute Counts* offers some possible starts to a class period that are more effective than roll call. For example, as soon as students enter the classroom, they may take out paper and pencil and solve a typical problem chosen by the teacher from the previous night's assignment. If only a few students are able to solve the exercise, reteaching is in order. Also, answers to homework assignments may be written on transparencies before students enter for checking work at the start of class. A walk around the class to view scores on each page indicates how well each student is mastering objectives. Collecting homework immediately after checking inhibits learning from mistakes. If students are not provided opportunities to see and correct errors, there is a good chance they will make the same mistakes again.

A few basic guidelines for assigning homework suggested by David Johnson are listed below.

- Know what is being assigned.
- Know what the students can do.
- Know the importance of timing. (Don't give homework too early in the class or just as the students are dismissed.)
- Make sure the students understand the assignment.
- Be careful in assigning reading in the mathematics textbook.
- Be positive about homework.
- Don't spend precious planning time correcting papers.
- Think about valuing homework scores in grading.

Appendix



Beyond the Procedures: Extending Students'Experiences with Computational Problems

by Anne L. Madsen

10 + 5 = ?	10 - 5 = ?	10 x 5 = ?	10 ÷ 5 = ?
$\frac{5}{6} + \frac{1}{3} = ?$	$\frac{5}{6} - \frac{1}{3} = ?$	$\frac{5}{6} \times \frac{1}{3} = ?$	$\frac{5}{6} \div \frac{1}{3} = ?$
.78 + .3 = ?	.78 - .3 = ?	.78 x .3 = ?	.78 ÷ .3 = ?

What should students be able to do with these problems?
What should they understand about these problems?
What are reasonable mathematical goals for these problems?

Students' experiences with computational problems are frequently limited to applying algorithms to problems in routines of drill and practice. It is not surprising they find these routines tedious and dull and often forget algorithms learned only through drill and practice. While computational facility is certainly a justifiable and worthy mathematical goal, other goals should be incorporated into computational exercises with whole and rational numbers. In NCTM's Curriculum and Evaluation Standards (1989, p. 44), whole number computation in grades K-4 include opportunities for students to: (a) model, explain and develop reasonable proficiency with basic facts and algorithms; (b) use mental computation and estimation techniques; (c) use calculators; (d) develop their thinking and reasoning abilities; and, (e) select and use computational techniques appropriate to specific problems.

Learning Goals for Computational Problems

Students need to experience a variety of ways to solve computational problems with whole and rational numbers. Their experiences solving problems in different ways enhances mathematical thinking, encourages the development of mathematical reasoning, and promotes retention of the algorithms. This article suggests learning goals and experiences for mathematical problems involving computations of whole and rational numbers. The operations in the four fraction expressions presented at the beginning of the article serve as an example for each learning goal.

Understand the Concepts of the Operations

The most critical part of instruction in fractions is the careful development of sound concepts of fractions and decimals prior to any computational work.

(Payne & Towsley, 1990, p. 24)

Frequently students do not have the opportunities that would help them understand the meanings of the arithmetic operations. Consequently, they do not make connections between the whole and rational number arithmetic operations. The algorithms for addition, subtraction, multiplication, and division of whole numbers are viewed by students as being very different from those used for fraction and decimal operations. Erlwanger (1973) described Benny, a sixth grader, who believed there were over a hundred different rules for fractions. It is not surprising that Benny thought this way - - many students have reported the same thoughts. Students need opportunities to understand how the meanings of whole number operations are connected to the operations of fractions and decimals. Language experiences can help students relate the meanings of the operations from the whole numbers to the rationals. By using the same phrases to express the operations, the students can begin to relate concepts of the operations of whole numbers to those of fractions and decimals. For example, relating operations using ten and five to those using five-sixths and one-third.

$10 + 5 = ?$	$\frac{5}{6} + \frac{1}{3} = ?$
What is the sum of ten and five?	What is the sum of five-sixths and one-third?
$10 - 5 = ?$	$\frac{5}{6} - \frac{1}{3} = ?$
What is five from ten?	What is one-third from five-sixths?
$10 \times 5 = ?$	$\frac{5}{6} \times \frac{1}{3} = ?$
What are ten groups of five?	What is one-third of five-sixths?
$10 \div 5 = ?$	$\frac{5}{6} + \frac{1}{3} = ?$
How many fives are there in ten?	How many one-thirds are there in five-sixths?

Students should have opportunities to write and discuss the meanings of the whole number and rational number operations. For example, at the end of a lesson they could write in their mathematics journals what it means to add, subtract, multiply or divide whole numbers, fractions, and decimals. Given whole number computations, students could discuss and write the meanings of the problems in their own words, instead of computing the answers. This exercise should be repeated for fractions and decimals. Language and writing enables students to link the meanings of the operations of whole numbers to rational number operations.

Use Concrete Models and Illustrations

Students need to use concrete models and illustrations to show the operations. In the guidelines for instruction of fractions and decimals, Payne and Towsley (1990, p. 24) suggest that teachers,

1. Allocate more instructional time to the development of the concepts of the numbers, including more time using concrete materials at all levels.

2. Allocate more time to initial instruction in operations, making good connections between models and algorithms.

3. Make major adaptations to textbooks. No current textbook includes sufficient developmental work on concepts.

Although some recent textbooks contain more examples of the operation concepts, opportunities for active student exploration of these concepts is limited. There are many very good commercially prepared materials that provide students with concrete experiences which help students develop an understanding of the fraction and decimal operations. Whether commercially prepared or not, it is important that concrete manipulatives are available and used by students. It is equally as important that they illustrate what was done with the concrete materials. Students may resist using manipulatives and illustrations, however, they become less resistant after realizing how the manipulatives help them understand the meaning of the operations.

Students in ninth grade general mathematics class made sets of fraction circles to use in studying fraction concepts and operations. At first they resisted using the fraction circles, however by the end of the 8-week unit they were quite comfortable with using them. The following observation portrays three ninth grade students solving a fraction problem: Melanie remembered the algorithm and solved the problem correctly; Kenneth applied his own algorithm and was incorrect; and, Randy remembered his experience with the manipulatives and solved it through an illustration. The following selection shows the teacher, Ms Kaye, and her students adding fractions (Madsen-Nason, 1986, pp.74-76).

Ms Kaye: Suppose you combined one-half and one-third and one-sixth?
Kenneth: Oh, I know!
Ms Kaye: I want you to tell me how much that would be.
Kenneth: Three-twelfths. No, no, that's not right. Wait, I can redeem myself.
[Ms Kaye writes the following on the chalkboard.]

$$\begin{array}{r} \frac{1}{2} \\ \frac{1}{3} \\ +\ \frac{1}{6} \\ \hline \end{array}$$

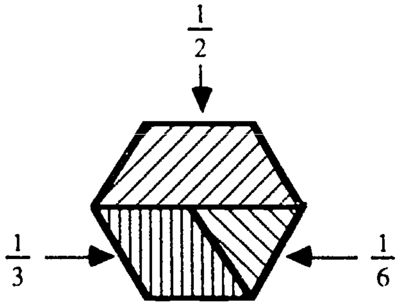
Ms Kaye: All right, I'll give you a chance to redeem yourself.
Kenneth: It's three-elevenths . . . or . . . one whole.
Ms Kaye: Kenneth, we just said you can't add numerators and denominators.
Kenneth: Well then, how can you add that?
Randy: Because it's one-third plus one-sixth which equals one-half, and one-half and one-half equals one whole!
Melanie: I did it differently. I put one-half, one-third, and one-sixth in a line and I got a common denominator and I added them.
[Ms Kaye writes on the chalkboard what Melanie told her.]

$$\begin{array}{r} \frac{1}{2} = \frac{3}{6} \\ \frac{1}{3} = \frac{2}{6} \\ +\ \frac{1}{6} = \frac{1}{6} \\ \hline \end{array}$$

Ms Kaye: All right, so you did the arithmetic way. You are correct, but can anyone do it with a picture?

Randy: I can!

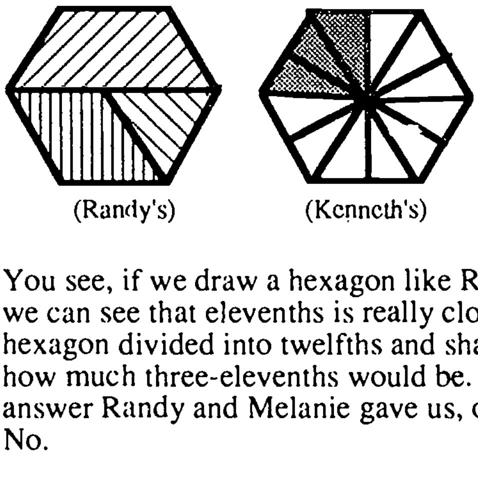
[Randy goes to the chalkboard and draws the following picture.]



Randy: Here is the one-half, and here is the one-third, and here is the one-sixth. You can see they are equal to one whole!

Ms Kaye: Yes, you could do it that way also. I don't care if you do it this way with the arithmetic or not. I do care that you do it with a mental picture. Because if you can draw a picture you may avoid adding across the numerators and denominators. If you added across you would get three-elevenths.

[Ms Kaye shows the students the following on the chalkboard.]

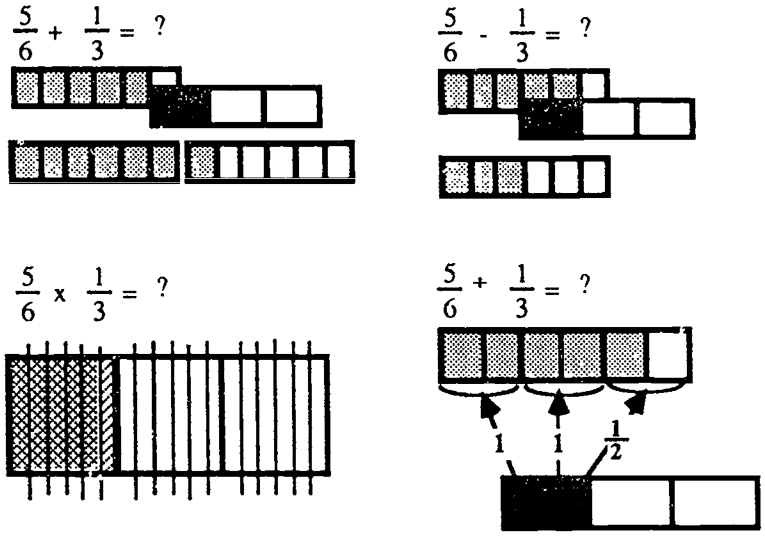


(Randy's) (Kenneth's)

Ms Kaye: You see, if we draw a hexagon like Randy's and divide it into twelfths we can see that elevenths is really close to twelfths. If we took this hexagon divided into twelfths and shaded three of those this is about how much three-elevenths would be. Does that even look close to the answer Randy and Melanie gave us, one whole?

Students: No.

Concrete models (such as fraction bars and fraction circles) and illustrations provide students with another way to think about computational operations. It enables them to make connections between the concepts of the operations and the algorithms. The following are illustrations of the operations with the five-sixths and one-third. In this case fraction bars served as the concrete models.



Estimate Answers

The study of estimation should be integrated with the study of concepts underlying whole numbers, fractions, decimals, and rational numbers so that these concepts can be constructed meaningfully by the learner. The exploration of a wide range of student-generated estimation strategies is recommended. The use of rounding to estimate is singled out for less attention in the Standards. Research and common sense clearly document that traditional rounding rules are often inappropriate and inefficient when estimating. (Reys & Reys, 1990, p. 23)

Students' experiences with estimation lessons in textbooks are often limited to operations with whole numbers. While some newer textbooks include estimation with fraction and decimal operations, the students need many more opportunities to apply these strategies. The strategies of front-end, compatible numbers, adjusting, flexible rounding, and clustering, to name a few, should be discussed and practiced by students on a regular and frequent basis. In addition, the estimation strategies need to be connected to those used in whole number operations. In working with fractions and decimals, students should decide whether the fraction is close to 1, 0, or 1/2 (0.5). Given the four operations on five-sixths and one-third, deciding whether the fractions are closer to 1, 0 or 1/2 allows students to solve the problems quickly in their heads.

$5/6 + 1/3 = (1) + (1/2) = (1\ 1/2)$ $\qquad$ $5/6 - 1/3 = (1) - (1/2) = (1/2)$

$5/6 \times 1/3 = (1) \times (1/2) = (1/2)$ $\qquad$ $5/6 \div 1/3 = (1) \div (1/2) = (2)$

Students should be given opportunities to estimate answers with whole numbers, fractions and decimals instead of always calculating the exact answers. They should work with fractions and decimals greater than one as well as those less than one. The students should be encouraged to discuss which strategy they used to solve a problem and why they chose it. One teaching suggestion is:

Given a set of 10 to 20 textbook computational problems:

Suggestion for a whole class assignment:

- Let half the students estimate the answers to the even numbered problems and calculate the answers to the odd numbered problems.
- Let half the students estimate the answers to the odd numbered problems and calculate the answers to the even numbered problems.
- When finished, discuss the answers and have students talk about the estimation strategies they used.

Suggestion for a group work assignment:

- Divide the problems so that each student in the group calculates part of the problems and estimates another part.
- When the group is finished the members should agree on their answers and hand in one paper.

Since estimation is frequently used in the world outside the classroom, students could find out the way people use estimation and when they estimate. There are many application problems that involve estimation strategies. There are several good commercially prepared materials available to supplement the estimation content which may be covered insufficiently in the textbook.

Write Word Problems

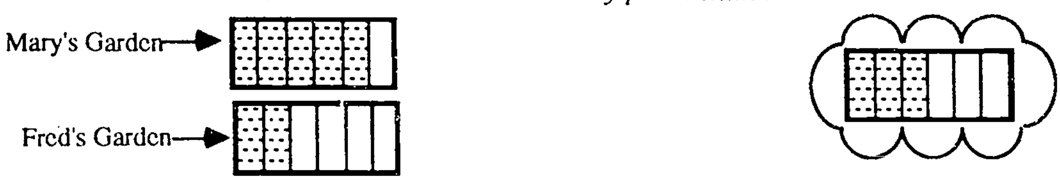
problem solving ... should be a cornerstone of mathematics curriculum and instruction, fostering the development of mathematical knowledge and a chance to apply and connect previously constructed mathematical understandings.

(Campbell & Bamberger, p. 14)

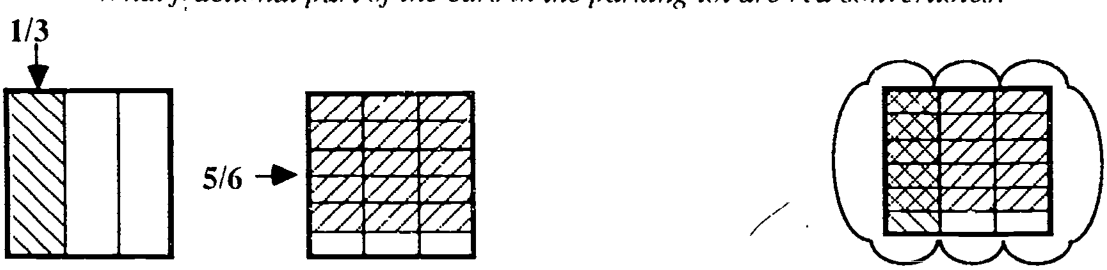
When students write word problems they have the opportunity to apply the operations in a problem solving framework. They should be encouraged to think about writing problems using several models of the part-whole fraction interpretation, these models include: region; area; linear; and, set. Students could work in pairs or triads to write and illustrate word problems for a given set of exercises. These problems could then be shared with the whole class. Student groups could also write word problems for operations on fractions of their choice. The student groups could trade problems and answer the questions. Examples of word problems and illustrations (solutions are circled) of the operations for the fractions five-sixths and one-third are:

Sue ran five-sixths of a kilometer and walked one-third of a kilometer.
How far did she walk in all?

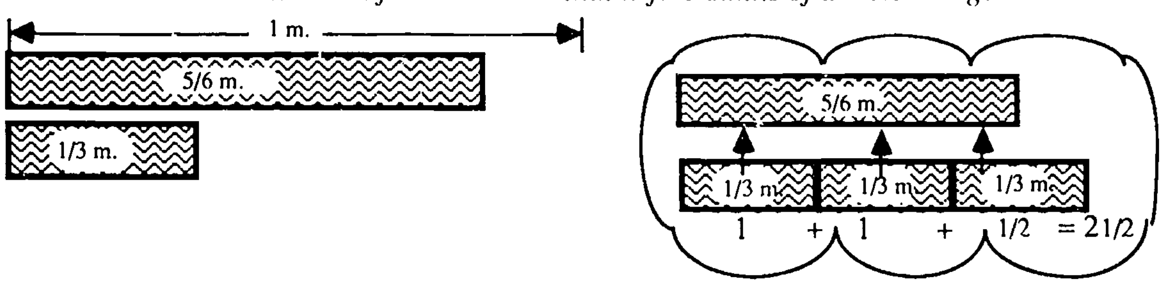
Mary planted corn on five-sixths of her one acre of land.
Fred planted corn on one-third of his one acre of land.
How much more corn did Mary plant than Fred?



One-third of the cars in the parking lot are red.
Five-sixths of the red cars are convertibles.
What fractional part of the cars in the parking lot are red convertibles?



How many ribbons one-third of a meter long
can be cut from a ribbon that is five-sixths of a meter long?



For the addition and division problems a linear model was used to represent the word problems. A region model was used for the subtraction and multiplication problems. It is important that students generate their own word problems and illustrations. Students working in groups encourages communication about the concepts and problems and offers support to students who might feel hesitant to begin this task.

Use a Calculator for Computational Problems

Calculators should routinely be available to students in all activities associated with mathematics learning, including testing. Students should be taught to distinguish situations in which calculators are appropriate aids to computation from situations in which mental operations or paper-and-pencil computations are more appropriate. (NCTM, 1984, p. 2)

Students should become familiar with using a calculator to find answers to whole and rational number problems. The TI-Mathematics Explorer calculator enables students to solve problems with fractions by displaying the familiar fraction form. Students should be encouraged to use fraction calculators when writing and solving word problems in activities like those described in the previous section. In addition, they should be encouraged to use fraction calculators to solve fraction problems that require extensive hand calculations. For example, the problem 17/19 + 21/23 = ? could be solved quickly with a fraction calculator and estimation. Computing answers to problems in which students gather data becomes less tedious when calculators are used. For example, calcuiators are appropriate for an activity where students are asked to measure (in standard units) and find the areas of different objects.

Apply the Algorithm.

Students should finally be expected to use the algorithm correctly in solving computational problems. In the past, students learned computational algorithms by watching the teacher demonstrate the procedure and then practicing the procedure on a set of similar problems. It was assumed that students would master the procedures through routines of drill and practice. If this assumption was true, then why are there so many students who continue to have difficulty remembering computational procedures at the middle and high school levels. Clearly, routines of drill and practice are not sufficient to insure students success in learning the algorithms for whole and rational number operations. Students should be asked to write in their own words the procedure they use for adding, subtracting, multiplying, and dividing, whole numbers, fractions, decimals. For example the students could be given the following task:

In your own words:

Tell a friend how to solve the following problems?

$\frac{5}{6} + \frac{1}{3} = ?$ $\quad \frac{5}{6} - \frac{1}{3} = ?$ $\quad \frac{5}{6} \times \frac{1}{3} = ?$ $\quad \frac{5}{6} \div \frac{1}{3} = ?$

This activity encourages students to clarify their thinking about applying a procedure to a problem. It also enables the teacher to assess what each student knows about a procedure. After students finish the task they could form groups and decide on the best description of a procedure for each of the four problems and share them with the class. This activity could be used for whole number, fraction, decimal, and percent problems.

Summary of the Learning Goals for Computational Problems

What should students be able to do with computational problems?
What should they understand about these problems?
What are reasonable mathematical goals for computational problems?

The questions posed at the beginning suggest that in addition to procedural goals there are also conceptual goals for computational problems. The NCTM's Professional Standards for Teaching Mathematics (1991) addressed these questions in a discussion of the importance of selecting tasks which promoted mathematical learning. In "Standard 1: Worthwhile Mathematical Tasks", the following are six of the eleven criteria suggested in identifying tasks which are selected for students.

Mathematical tasks should:

- engage students' intellect;
- develop students' mathematical understandings and skills;
- stimulate students to make connections and develop a coherent framework for mathematical ideas;
- call for problem formulation, problem solving, and mathematical reasoning;
- promote communication about mathematics;
- represent mathematics as an ongoing human activity. (p.25)

When this criteria is used to select tasks for computational problems, routine drill and practice activities alone are not sufficient to provide students with the opportunities to develop mathematical understanding, interest and competence. Tasks that are selected for computational problems should include those which engage students in a variety of mathematical explorations in many different ways. When students study the arithmetic operations, they should be provided with activities in the following mathematical strands.

MATHEMATICAL STRANDS → MATHEMATICAL CONTENT ↓	Concepts	Problem Solving	Algorithms	Estimation & Mental Arithmetic	Technology
Whole Number Operations					
Rational Number Operations Fractions, Decimals, Percents					

When students are engaged in activities in all the strands they have the opportunity to gain a better understanding of the operations and the procedures. In closing, a last suggestion - - when students are given a page of 25 computational problems (whether whole numbers, fractions, decimals, or a combination thereof) have them:

- choose any 5 and illustrate the problems and solutions;
- choose any 5 and estimate the answers;
- choose any 5 and use a calculator to find the answer;
- choose any 5 and write and illustrate a word problem;
- choose and 5 and calculate the answer.

Hopefully the students would choose the most difficult problems to solve by using estimation or a calculator and choose the easiest problems to calculate the exact answer. In making such choices they should be reminded that this is, after all, what is often done when mathematics problems are solved in real life situations outside of the textbook.

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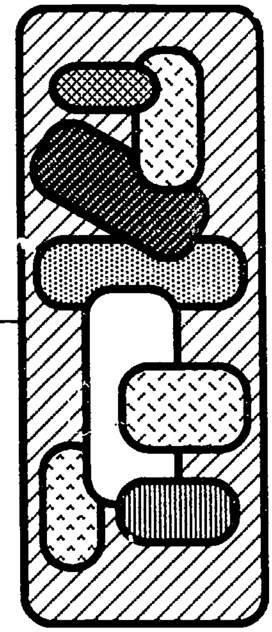

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(1) acceptance policies on student transfers from other school districts;

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(3) nondiscrimination in extracurricular activities and the use of school facilities;

(4) nondiscriminatory practices in the hiring, assigning, promoting, paying, demoting, reassigning, or dismissing of faculty and staff members who work with children;

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