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ABSTRACT

This document is designed to assist teachers and other school personnel in the planning and teaching of the fifth grade mathematics course. Contents include: (1) Overview of Grade 5 Mathematics (mission statement, purpose and philosophy, goals, National Council of Teachers of Mathematics' Professional Standards for Teaching Mathematics, and uses of technology and manipulatives); (2) Essential Elements of Instruction with sample learning objectives and sample clarifying activities; (3) Texas Assessment of Academic Skills (TAAS) (focus, domains, objectives, and targets); (4) Sample Lessons for Teaching Grade 5 Mathematics; and (5) Evaluation (philosophy and types of evaluation). TAAS features three domains: concepts, operations, and problem solving. The Essential Elements are: problem solving; patterns, relations, and functions; number and numeration concepts; operations and computation; measurement; geometry; and probability, statistics, and graphing. Suggested resources include children's trade books, software, and suggested manipulatives. Contains 22 references. (MKR)

ED 384 497

GUIDELINES FOR TEACHING GRADE 5 MATHEMATICS

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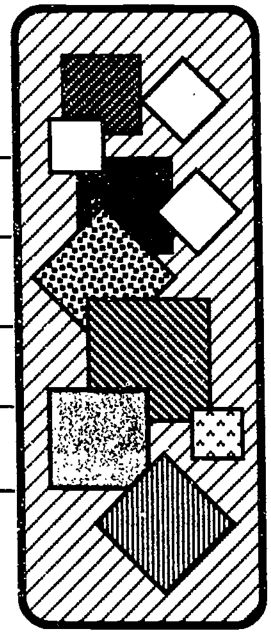
Texas Education Agency
Austin, Texas
Fall 1994

Guidelines for Teaching Grade 5 Mathematics



Texas Education Agency
1701 North Congress Avenue
Austin, Texas 78701

FOREWORD

Guidelines for Teaching Grade 5 Mathematics is designed to help teachers and other school district personnel plan and teach fifth grade mathematics. The publication presents the philosophy and intent of the course and discusses the required essential elements, TAAS instructional targets, instructional strategies, and the use of technology and manipulatives. Also included are sample objectives and activities to illustrate how the essential elements for fifth grade mathematics can be taught. School district personnel may want to use these suggestions to develop their own curriculum documents for the course.

We hope these guidelines will be useful in planning and teaching mathematics in Grade 5 and in equipping the mathematics classroom.

Lionel R. Meno
Commissioner of Education

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This publication was developed by Agency staff members in the Office of the Deputy Commissioner for Curriculum, Assessment, and Professional Development.

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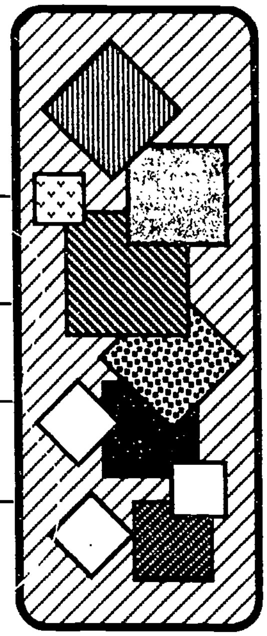
Overview of Grade 5 Mathematics



Mission Statement

Guidelines for Teaching Grade 5 Mathematics is one in a series of five documents for the first through the fifth grades designed to assist teachers and other school personnel in the planning and teaching of elementary mathematics. The discussions of philosophy, goals, instructional strategies, uses of technology and manipulatives, and aspects of evaluation are provided as starting points for districts to begin the process of developing their own curriculum documents. The essential elements of instruction for each grade level are supported with sample learning objectives, sample clarifying activities, and complete sample lessons. These guidelines should prove useful to district personnel in: (1) planning curriculum, (2) planning instruction, and (3) equipping classrooms for mathematics teaching and learning.

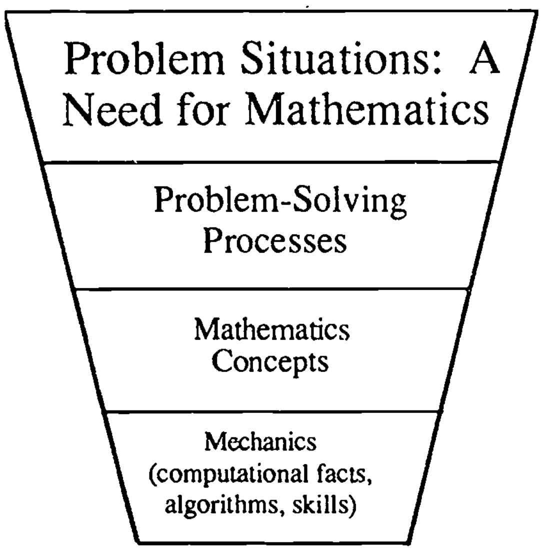
Purpose and Philosophy

Mathematics is useful, exciting, and creative and can be enjoyed by all elementary school students. Problem-solving skills and logical reasoning are developed while students explore and make sense of their world through rich, worthwhile mathematical experiences. Unfortunately, mathematics has been viewed by many students as boring, irrelevant, and routine and as externally dictated by a rigid system of rules governed by standards of speed, accuracy, and memory. In the past, computational facility has been emphasized instead of a broad, integrated view of mathematics. While computational skills are important, learner characteristics and the vitality of mathematics itself cannot be overlooked. Mathematics in the elementary grades should be broad-based and concept driven and should reflect relevant mathematics and connections between mathematics concepts and between these concepts and other disciplines.

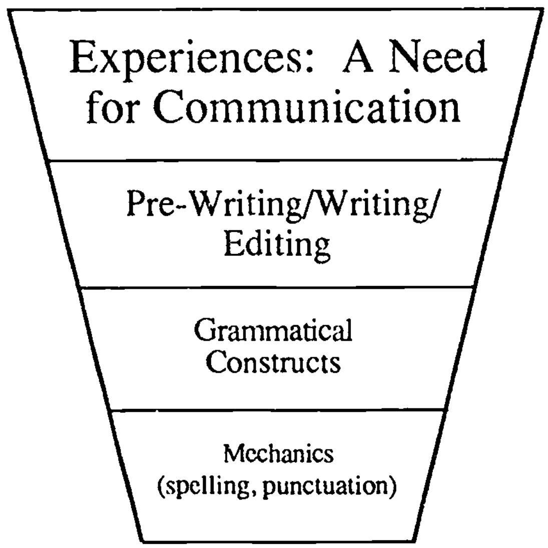
Children enter elementary school with a natural curiosity and enthusiasm for learning. Mathematics experiences at the elementary level should tap into these characteristics for children to begin developing mathematical power—the ability to think and communicate, drawing on mathematical ideas and using mathematical tools and techniques. The attitudes students form in elementary school toward mathematics will determine the choices they make of future mathematics coursework and consequently the availability or loss of educational and career opportunities.

The elementary school mathematics curriculum should emphasize the processes of problem solving, reasoning, communication, and making connections within the contexts of investigating geometry, measurement, probability, statistics, graphing, patterns, and functions, as well as with number, numeration, and operation concepts. Problem solving should be the focus of instruction with skills and concepts being introduced, developed, and applied through meaningful problem situations. Mathematics instruction needs to begin with meaning and purpose in much the same way as elementary teachers present language arts instruction, as reflected in the following graphic illustration:

MATHEMATICS



LANGUAGE ARTS



All students need rich and relevant problem-solving experiences with appropriate teacher guidance and questioning. Such experiences will empower students to build meaning for the mathematics they encounter today and to strengthen reasoning skills needed for the mathematics of tomorrow.

Goals

According to *Curriculum and Evaluati ı Standards for School Mathematics* developed by the National Council fo Teachers of Mathematics (NCTM), the five overall curriculum goals for students are:

- learning to value mathematics
- becoming confident in their ability
- becoming mathematical problem solvers
- learning to communicate mathematically

- learning to reason mathematically

Moreover, the educational system of today demands new societal goals for education:

- mathematically literate workers
- lifelong learning
- opportunity for all
- an informed electorate

Specifically, teaching the mathematics curriculum to elementary school students must be related to the characteristics of the learners and their needs today and in the future.

Everybody Counts (National Research Council, 1989) posits that "self-confidence built on success is the most important objective of the mathematics curriculum" (p. 45). Individuals must be able to use mathematics in their later lives—as employees, parents, and citizens. Ability and disposition to do so often depends on attitudes toward mathematics developed in school.

Through the use of worthwhile mathematical activities investigated in cooperative, group environments, teachers of elementary mathematics can empower their students with strong mathematical understanding and disposition.

National Council of Teachers of Mathematics: Professional Standards for Teaching Mathematics

The *Professional Standards for Teaching Mathematics (NCTM, 1991)* are based on four assumptions about the practice of teaching. These assumptions are abbreviated versions of the more extensive ones found in the original document (NCTM, 1991, pages 21-22).

(1) The goal of teaching mathematics is to help all students develop m· .nematical power. Teachers must help every student develop conceptual and procedural understandings of number, operations, geometry, measurement, statistics, probability, functions, and algebra and the connections among ideas. They must engage all students in formulating and solving a wide variety of problems, making conjectures and constructing arguments, validating solutions, and evaluating the reasonableness of mathemati..l claims.

(2) What students learn is fundamentally connected with how they learn it. Students' opportunities to learn mathematics are a function of the setting and the kinds of tasks and discourse in which they participate.

(3) All students can learn to think mathematically. The goals such as learning to make conjectures, to argue about mathematics using mathematical evidence, to formulate and solve problems, and to make sense of mathematical ideas are not just for some group thought to be "bright" or "mathematically able."

(4) Teaching is a complex practice and hence not reducible to recipes or prescriptions. First of all, teaching mathematics draws on knowledge from several domains: knowledge of mathematics, of diverse learners, of how students learn mathematics, of the contexts of the classroom, school, and society. Good teaching depends on a host of considerations and understandings. Good teaching demands that teachers reason about pedagogy in professionally defensible ways within particular contexts of their own work.

The *Professional Standards for Teaching Mathematics* identifies a particular set of instructional standards for the effective teaching of mathematics. The standards describe the nature of the tasks, patterns of communication, the learning environment, and the analysis of instruction. More specifically, five of these standards focus on instructional strategies. They are:

STANDARD 1: WORTHWHILE MATHEMATICAL TASKS

The teacher of mathematics should pose tasks that are based on:

- sound and significant mathematics;
- knowledge of students' understandings, interests, and experiences;
- knowledge of the range of ways that diverse students learn mathematics;

and that

- engage students' interests;
- develop students' mathematical understandings and skills;
- stimulate students to make connections and develop a coherent framework for mathematical ideas;
- call for problem formulation, problem solving, and mathematical reasoning;

- promote communication about mathematics;
- represent mathematics as an ongoing human activity;
- display sensitivity to, and draw on, students' diverse background experiences and dispositions;
- promote the development of all students' dispositions to do mathematics.

STANDARD 2: THE TEACHER'S ROLE IN DISCOURSE

The teacher of mathematics should orchestrate discourse by:

- posing questions and tasks that elicit, engage, and challenge each student's thinking ability;
- listening carefully to students' ideas;
- asking students to clarify and justify their ideas orally and in writing;
- deciding what to pursue in depth from among the ideas that students bring up during a discussion;
- deciding when and how to attach mathematical notation and language to students' ideas;
- deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;
- monitoring students' participation in discussions and deciding when and how to encourage each student to participate.

STANDARD 3: STUDENTS' ROLE IN DISCOURSE

The teacher of mathematics should promote classroom discourse in which students:

- listen to, respond to, and question the teacher and one another;
- use a variety of tools to reason, make connections, solve problems, and communicate;
- initiate problems and questions;
- make conjectures and present solutions;
- explore examples and counterexamples to investigate a conjecture;
- try to convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers;
- rely on mathematical evidence and argument to determine validity.

STANDARD 4: TOOLS FOR ENHANCING DISCOURSE

The teacher of mathematics in order to enhance discourse, should encourage and accept the use of:
- computers, calculators, and other technology;
- concrete materials used as models;
- pictures, diagrams, tables, and graphs;
- invented and conventional terms and symbols;
- metaphors, analogies, and stories;
- written hypotheses, explanations, and arguments;
- oral presentations and dramatizations.

STANDARD 5: LEARNING ENVIRONMENT

The teacher of mathematics should create a learning environment that fosters the development of each student's mathematical power by:

- providing and structuring the time necessary to explore sound mathematics and grapple with significant ideas and problems;

- using the physical space and materials in ways that facilitate students' learning of mathematics;
- providing a context that encourages the development of mathematical skill and proficiency;
- respecting and valuing students' ideas, ways of thinking, and mathematical dispositions;

and by consistently expecting and encouraging students to:

- work independently or collaboratively to make sense of mathematics;
- take intellectual risks by raising questions and formulating conjectures;
- display a sense of mathematical competence by validating and supporting ideas with mathematical argument.

STANDARD 6: ANALYSIS OF TEACHING AND LEARNING

The teacher of mathematics should engage in ongoing analysis of teaching and learning by:

- observing, listening to, and gathering other information about students to assess what they are learning;
- examining effects of the tasks, discourse, and learning environment on students' mathematical knowledge, skills, and dispositions;

in order to:

- ensure that every student is learning sound and significant mathematics and is developing a positive disposition toward mathematics;
- challenge and extend students' ideas;
- adapt or change activities while teaching;
- make plans both short- and long-range;
- describe and comment on each student's learning to parents and administrators, as well as to the students themselves.

The movement toward this vision of instruction for mathematical empowerment of all students is strongly dependent upon the environment of the classroom, an environment governed in a large part by the decision-making role of the classroom teacher. The NCTM teaching standards identify five major components necessary in the instructional environment for the mathematics classroom and tie these components directly to teachers asking, and encouraging students to ask, appropriate and stimulating questions. The five major instructional components and suggestions for questions are (NCTM, 1991, pp. 3-4):

- **Helping students work together to make sense of mathematics**
  "What do others think about what Janine said?"
  "Do you agree? Disagree?"
  "Does anyone have the same answer but a different way to explain it?"
  "Would you ask the rest of the class that question?"
  "Do you understand what they are saying?"
  "Can you convince the rest of us that that makes sense?"

- **Helping students to rely more on themselves to determine whethe something is mathematically correct**
  "Why do you think that?"
  "Why is that true?"

"How did you reach that conclusion?"
"Does that make sense?"
"Can you make a model to show that?"

- **Helping students learn to reason mathematically**
  "Does that always work?"
  "Is that true for all cases?"
  "Can you think of a counterexample?"
  "How could you prove that?"
  "What assumptions are you making?"

- **Helping students learn to conjecture, invent, and solve problems**
  "What would happen if . . . ." What if not?"
  "Do you see a pattern?"
  "What are some possibilities here?"
  "Can you predict the next one? What about the last one?"
  "How did you think about the problem?
  "What decision do you think he should make?"
  "What is alike and what is different about your method of solution and hers?"

- **Helping students to connect mathematics, its ideas, and its applications**
  "How does this relate to . . .?"
  "What ideas that we have learned before were useful in solving this problem?"
  "Have we ever solved a problem like this one before?"
  "What uses of mathematics did you find in the newspaper last night?"
  "Can you give me an example of . . .?"

Instructional Strategies

The following diagrams are examples of one teacher's planning efforts to connect measurement and geometry concepts to life, earth, and physical science units:

Measurement

Weather
Measure, record
- temperature °C °F
- volume of rainfall
- feet of snow

Ecology
Buy an acre of rainforest. Measure and mark off an acre of land.

Dental Health
- Measure time spent brushing teeth
- Measure time spent in dentist office

Plants
Students keep a record of plant growth, using inches, cm. Meaure weight of various seeds. Record volume of water used to feed plants.

Human Body
Estimate and measure body parts in inches. Draw a picture of yourself to show results. Estimate the weight of classmates. Organize/sequence from least to most weight. Weigh on scales, collect data, check predictions.

Insects
Students measure wings or length of various insects. Measure length of ant food trails—distance to mound.

Food Groups
Use foods to help measure capacity of various containers. Weigh and compare fresh fruit vs. dried fruit.

Dinosaurs
Use feet to measure length of dinosaurs. Compare capacity; e.g., 1 dinosaure = 1 bus. How do their masses compare?

Animals
Bring in stuffed animals to create a classroom zoo. Sequence animals by height, weight, longest tail, arms, legs, wings, etc. Use string and inches to verify and graph results.

Simple Machines
Use small cars (wheels) and ramps (inclined planes). Race cars off ramps, measure distance in inches, feet. Does incline affect distance?

Earth science
Predict the weight sequence of a set of rocks. Use scales to verify the sequence. Weigh different soil types. Do a cup of sand and a cup of top soil weigh the same?

Geometry

Weather
Critical Thinking—Write a paragraph. What if . . . hail were shaped like cones, cubes, pyramids, etc. Create unique cloud shapes using precut construction paper shapes.

Dental Health
Discuss various teeth and their jobs. what shape (3-D) do these teeth resemble? Critical thinking: What if our teeth were all shaped like cones, spheres? Etc.

Animals
Use measurement info on animals to graph results. Use pattern blocks to create unique animals.

Space
Use three-dimensional shapes to make a rocket ship. Use construction paper to create a space-shape monster. Write data identifying the monster's various shape/body parts. Identify planets as spheres.

Human Body
Bilateral symmetry—Art project. Draw one side of body; fold, trace other side. Plot measurements of student height or weight on line graphs.

Simple Machines
Explore angles of inclined planes for racing activities. Does incline affect the distance the car travels? Rotational symmetry of various wheels and gears create efficiency of machines. Use toy wheels, gears, etc. to create a unique machine. What job does it perform?

Insects
Identify symmetry in a variety of insects. Use art to help with this concept; i.e., making butterflies with paint on one side of paper, folding to create mirror image. Look at rotational symmetry in bee hives and at the shape of cells in hives.

Food Groups
Bring in a variety of foods. Have students identify foods that closely match geometric solids: cube, cylinder, cone, pyramid, and sphere.

Plants
Identify the shapes found in real plants; e.g., leaves, petals, stems. Use pattern blocks to create a unique plant. Identify shapes. Plot growth of plants to create a line graph.

Appropriate questioning techniques and meaningful problem-solving situations are two major strategies for effective mathematics instruction.

Uses of Technology and Manipulatives

Calculators and computers are tapped for important roles in mathematics at all levels and across topics. Changes in technology and the broadening of the areas in which mathematics is applied have resulted in growth and changes in the discipline of mathematics itself. The new technology has altered the very nature of the problems important to mathematics and the methods mathematicians use to investigate them.

The NCTM *Curriculum Standards* (1989) call for the following regarding technology in the classroom:

- appropriate calculators for all students at all times
- a computer for every classroom for demonstration
- access to a computer for individual and group work
- students learning to use the computer as a tool for processing information and performing calculations to solve problems

Calculators and computers offer teachers and students an important learning aid. Their potential is great and as yet untapped both in developing concepts and in developing positive attitudes and persistence in problem solving.

Computers can be utilized in a variety of ways in the mathematics classroom, and the appropriateness of a particular approach depends on the goals. Three qualitatively different methods suggested by R. Taylor in *The Computer in the School: Tutor, Tool, Tutee* are:

- as a sophisticated teaching machine
- to be programmed (or taught) by the student
- as a mode for applications in research and development through software that displays graphs, manipulates symbols, analyzes data, and performs mathematical procedures. Applications such as spreadsheets, word processing, data bases, and communication packages have the appeal of matching the classroom's use of technology with that of society's.

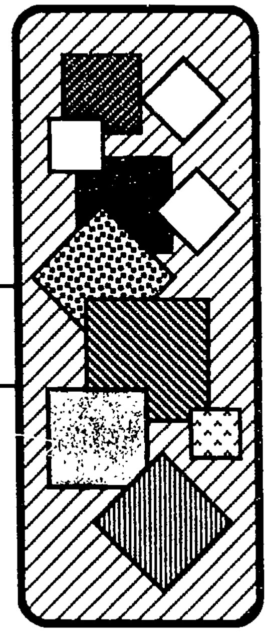
Calculator use is not for the purpose of replacing paper-and-pencil computations but to reinforce them. According to N. Kober in *Ed Talk: What We Know About Mathematics Tecching and Learning*, calculator use is apt to sustain independent thought, not replace it. For example, students can be challenged to invent calculator algorithms to replace procedures taught in textbooks. The students explain why their procedures work and debate the advantages and disadvantages of their procedures over others. Calculators are programmable, produce graphics, and work in fractional and algebraic notation. Teachers need to be innovative; they need to experiment and share ideas.

Furthermore, manipulatives offer an excellent way to enable students to connect between mathematical ideas. Learning is enhanced when students are exposed to a concept in a variety of manipulative contexts. As an example, fractions represented with pattern blocks, fraction bars, fraction circles, and Cuisenaire rods help students understand the concept of fraction independent of the physical representation. In addition to using manipulatives for new concepts, activities should be oriented to help students connect between concrete, pictorial, and abstract representations of ideas.

However, the use of manipulatives should not become an end in itself. Learning the motions of modeling addition and subtraction with Cuisenaire rods does not guarantee understanding of the mathematical relationship between these inverse operations. It is important not only that the concrete manipulation of materials closely matches the mathematical concept being developed but that the actions are accompanied with appropriate questioning by the teacher and reflection by the student.

In the instructional uses of both technology and manipulatives, the goal is to enhance mathematical thinking. Again, the teacher's role as questioner and decision maker influences the effectiveness of the incorporation of these tools.

Essential Elements of Instruction



Essential Elements of Grade 5 Mathematics with Sample Learning Objectives and Sample Clarifying Activities

The State Board of Education in 1989 revised the essential elements of instruction for mathematics, Grades 1-8. These revised essential elements follow closely the recommendations made by the National Council of Teachers of Mathematics in its nationally recognized *Curriculum and Evaluation Standards for School Mathematics*. According to the Texas Education Agency (1989), "The mathematics curriculum review committee and the Agency [TEA] have tried to be sensitive to a balance between changes expected of teachers and improvements necessary to help students learn mathematics more effectively." Some of these major changes include:

- narrowing the spiral of the curriculum—beginning some topics later and finishing some topics sooner in the curriculum to eliminate some of the redundance
- revising the role of review in the curriculum so that the majority of each grade level is new material and so that review is placed in relevant contexts
- emphasizing the development of problem-solving skills in relevant and interesting situations
- incorporating calculators and computers throughout all grades as problem-solving tools
- adding an essential element on patterns, relations, and functions
- separating the teaching of operations and computation so that all students learn the meaning of the operations
- strengthening the areas of probability, statistics, and geometry
- emphasizing the importance of communication in mathematics

- building on a sound foundation of concepts rather than on rote procedures

- putting mathematics into meaningful contexts

The following essential elements and descriptors for fifth grade mathematics are annotated with sample learning objectives and sample clarifying activities (except for EE1: Problem-Solving). The learning objectives give a brief look at the developmental components of the concept(s) in the descriptor. The sample clarifying activities are addressed to the teacher and provide a glimpse of what student engagement with this concept might look like in the classroom.

Each set of sample learning objectives and sample clarifying activities is meant to be viewed as an integrated whole (not necessarily matched one-to-one) to clarify the descriptors and to identify connections among them, as well as connections to meaningful problem situations. The Problem-Solving strand therefore is annotated only with sample learning objectives and is connected to the other strands through the language and situations used in their sample clarifying activities.

Many of the activities involve the use of manipulatives and common materials such as hundreds charts or grid paper. A list of these manipulatives can be found in the References and Resources. Also in the References and Resources are lists of the children's trade books, teacher resource books, and software cited in the activities as examples of instructional materials.

The revised essential elements, sample learning objectives, and sample clarifying activities for Grade 5 are:

(1) **Problem Solving**. Experience in solving problems designed to systematically develop students' problem-solving abilities through a variety of strategies and approaches. The student shall be provided opportunities to engage in the following types of activities:

(A) develop an organized approach to solving application and nonroutine problems appropriate for Grade 5;

Sample Learning Objective

Involving patterns, relations, and functions;
number and numeration concepts;
operations and computation; measurement;
geometry; probability, statistics, and
graphing

(B) analyze problems by identifying relationships, discriminating relevant from irrelevant information, sequencing, observing patterns, prioritizing, and questioning;

Sample Learning Objective

Make inferences and predictions

(C) communicate an understanding of a problem by describing and discussing the problem and recording the relevant information;

Sample Learning Objectives

Demonstrate creative thinking through fluency, flexibility, elaboration, creation of new ideas, and spontaneity

Estimate outcomes including appropriate units for outcomes

- *reflect on the problem-solving process and solution of a problem by evaluating outcomes for reasonableness (including appropriateness of units), make revisions as needed, describe and discuss the process and solution, and make a decision based on the solution*

(F) generate and extend problems.

(2) **Patterns, Relations, and Functions.** Use of models and patterns to develop the concepts of relations and functions. The student shall be provided opportunities to:

(A) investigate patterns that occur when changing numerators and denominators of fractions beginning with concrete models and extending to calculator investigations;

Sample Clarifying Activities

Have students build "trains" with Cuisenaire rods to illustrate equivalent fractional parts. Students can record the fractions that describe equivalent parts and describe the patterns in the numerators and denominators of these fractions.

Have students work in groups of four on the following problem: "Divide a bag of 4 marbles among 4 people. Record the fraction of the bag of marbles each person received (1/4). Now divide a bag of 8 marbles among 4 people and record the fraction of the bag each person received (2/8). Continue this with 12, 16, 20, etc. marbles in the bag. Describe any pattern you see in the fractions you used to record how much of the bag each person received. Did each of the four people always receive 1/4 of the bag of marbles?"

(B) use patterns to explore the rules for divisibility;

Sample Learning Objectives

Explore patterns of factors.

Explore patterns of multiples.

From the patterns of factors and multiples, develop rules for divisibility and test them.

Sample Clarifying Activities

Give each group of students a hundreds chart and have them place a blue cube on each of the multiples of 2, a green cube on each multiple of 3, and a yellow cube on each multiple of 4. Have students repeat with a different color for each factor through 10 and stack the cubes on each number into a tower for that number. (Some towers will have only one story.) Students can look for patterns in the towers.

Have students make a chart listing the multiples of 2, multiples of 3, multiples of 4, etc. and look for patterns in each group of numbers. Students can write statements describing a characteristic of all numbers divisible by 2, divisible by 3, etc. Have students test their descriptions on a number given to them by a partner and use the calculator to verify whether it is divisible by a certain factor or not.

(C) investigate patterns of powers of 10 (exponents and expanded notation) using a calculator or computer when appropriate.

Sample Clarifying Activities

Have students use the exponent key on the calculator to connect 10^1, 10^2, and 10^3 to the long, flat, and block pieces for 10, 100, and 1000 in the place value materials.

Have students use only the 1, 0, +, and = keys to investigate different ways to display their telephone numbers on the calculator. Students can compare their results with other students' and generate a pattern (e.g., expanded notation) for using the least number of addends to display a telephone number:

1,000,000 + 1,000,000 + 100,000 + . . . = 2163111.

Have students translate the multiples of 10 in their patterns to exponential notation:

$10^6 + 10^6 + 10^5 + 10^4 + \ldots = 2163111.$

(3) **Number and Numeration Concepts.** Concepts and skills associated with the understanding of numbers and the place value system. The student shall be provided opportunities to:

(A) find common factors of a set of numbers;

Sample Learning Objectives	*Sample Clarifying Activities*
Use concrete models to demonstrate common factors.	Have students use chips, squares, or grid paper to build rectangular arrays for a set of numbers to determine their common factors.
Identify patterns and make generalizations about how to find common factors.	Have students use calculators to divide given numbers by 1, 2, 3, etc. to find their factors (when the remainder is 0).
	Have students represent given numbers with beans or other counters. Students can separate the sets into groups of 1, 2, 3, etc. to find common factors of the given numbers.
	Have students work in groups to address problems such as, "Mr. and Mrs. Brown invited the math club to their house for apples and cookies. Each child got the same number of apples. Each child got the same number of cookies. Mr. and Mrs. Brown served 30 applies and 40 cookies altogether. What could the number of children have been?

(B) use factors and multiples to write equivalent fractions;

Sample Learning Objectives	*Sample Clarifying Activities*
Use concrete models to identify patterns and make generalizations about using multiples to write equivalent fractions.	Have students use the constant key on a calculator to generate lists of multiples for the numerator and denominator of a given fraction and arrange the list in a chart like the following: Numerator: 3 6 9 12 . . . Denominator: 4 8 12 16 . . .
Use concrete models to identify patterns and make generalizations about using factors to simplify fractions.	Have students use fraction bars or pattern blocks to model the fractions in the chart and justify their equivalence (an example of the Fundamental Law of Fractions).
	Have students fold rectangles to model the fractions in the chart and justify their equivalence.

(C) write the common denominator of two or more fractions;

Sample Learning Objectives	*Sample Clarifying Activities*
Use concrete models to demonstrate the need for common denominators.	Have students use fraction bars to model several equivalent forms of two given fractions until a common denominator is found.
Use concrete models to investigate patterns in common denominators. Use the patterns in common denominators to make generalizations about using factors and common multiples to find common denominators.	Have students use tiles to build a rectangle that is 1/2 red, 1/3 yellow, 1/6 blue and record the rectangle on grid paper. Students can build and record two other rectangles that fit this description but use a different number of total tiles. Ask students follow-up questions such as, "Try 1/4 yellow, 3/4 green; or 1/2 blue, 3/8 green, 1/8 yellow. How did you determine the number of tiles to use each time? What can you write about your discoveries?

(D) compare and order fractions in both standard and decimal form;

Sample Learning Objectives	*Sample Clarifying Activities*
Use concrete models to compare fractions and write sentences using the symbols <, >, or =.	Have students use fraction models to compare and order fractions.
Use concrete models to order fractions from greatest to least and least to greatest.	Have students make a multilabeled number line showing halves, thirds, fourths, sixths, eighths, ninths, and twelfths. Students can make observations about the relative values of different fractions in relation to their positions on the number line. For example, 2/3 = 6/9 = 8/12 because they all label the same point on the number line, and 2/3 is less than 3/4 because it is to the left of 3/4 on the number line.
Generalize a strategy for using common denominators to order fractions.	See TEA Module 5, Grades 3-6, Numeration.
Use concrete models to compare decimals and write sentences using the symbols <, >, or =.	Have students use decimal squares (10 x 10 grids) or base ten materials to compare and order decimals.
Use concrete models to order decimals from greatest to least and least to greatest.	Have students explore sale ads to compare situations such as 1/2 or 1/3 off the regular price. Students can also explore "25% off" in relation to the fraction.
	Students can use a fraction calculator to change fractions to decimals to compare and order them. Have students keep a record of the equivalent pairs of fractions and decimals and look for patterns.
Generalize a strategy for ordering decimals and make connections to using common denominators with fractions.	Have students play the game Dicey Decimals. Students roll a die and place the number on the die in one of the blanks (_._ _ _) on a game sheet. Once a number has been placed, it cannot be moved. The player with the largest (or smallest) decimal number receives one point. Students must use mathematical reasoning to justify which decimal is largest or smallest.

(E) identify the prime factors of a number;

Sample Learning Objectives

Use concrete models to make generalizations about prime numbers.

Use concrete models to determine prime factors of a number.

Use patterns in numbers to determine prime factors.

Sample Clarifying Activities

Have students build with tiles all possible rectangular arrays for the number 1-24, inclusive, and cut models of the arrays from graph paper. (This work can be divided up among the groups in the class.) Prepare a class chart displaying all rectangular arrays for each of the numbers. Students can look for patterns, classify numbers in groups with similar characteristics (numbers that have exactly two arrays, etc.), and develop definitions for prime and composite numbers.

Have students use factor trees to identify the prime factors of a number and discuss the fact that no matter what order the factor tree is developed for some number, its prime factors are always the same (the Fundamental Law of Arithmetic).

(F) read, write, and represent decimals;

Sample Learning Objectives

Use decimal symbols to record the amount represented by a decimal model.

Read, write, and represent decimals less than and greater than 1 (including hundredths) in meaningful situations.

Sample Clarifying Activities

Have students use a decimal square (10 x 10 grid) to represent a unit and use this unit to model decimal numbers.

Have students use a place value chart to read and write decimal numbers.

Have students identify real-world uses of decimals and discuss when the representation of a piece of a unit is important in real life. Examples could include money, interest, measurement, sporting events, etc.

Divide the class into two groups and hold a debate on the use of fractions versus the use of decimals. One side can present the advantages of standard fraction notation and the drawbacks to decimals, while the other side presents the advantages of decimal notation as opposed to common fractions.

(G) round whole, fractional, and decimal representati... of numbers;

Sample Learning Objectives	*Sample Clarifying Activities*
Identify and describe situations in which rounding numbers would be useful.	Have students use a number's position on the number line to determine its value when rounded to the nearest 10, tenth, etc.
Round whole numbers to the nearest 10 or 100 unit.	Have students use grocery advertisements and fliers to estimate the cost of groceries on a list.
Round decimals to the nearest tenth or whole unit.	Have students use a catalog to select 10 items, not to exceed a total cost of $100, by mentally rounding each item to the nearest $10. Students can use calculators to compute the exact costs and compare to their estimates.

(H) develop the concept of ratio using models.

Sample Learning Objectives	*Sample Clarifying Activities*
Use concrete models or visual representations to investigate the meaning of ratio.	Have students use concrete models and appropriate language to develop the meaning of ratio. For example, give each student in the class 4 counters. As one student comes to the front of the room, the ratio recorded is "4 counters: 1 student" or "4/1." When a second student comes to the front of the room, the ratio recorded is 8 counters: 2 students of 8/2. Have students discuss that the ratios are the same because there are still 4 counters for each student and compare the patterns to equivalent fractions.
Write ratios to describe the relationship between two pieces of information.	Have students discuss everyday ratios such as students to teachers, boys to girls in the class, cost to cartons of milk, erasers to blackboards, etc.
	Have students investigate sports averages as examples of ratios.
	Have students use ratios to double or halve recipes (2 cups per 1 batch = 4 cups for 2 batches.

(4) Operations and Computation. Use of manipulatives to develop the concepts of basic operations on numbers and to apply these concepts to the computational algorithms. The student shall be provided opportunities to:

(A) use concrete models to estimate answers to problems involving addition and subtraction of fractions;

Sample Clarifying Activities

Have students use pattern blocks and work with partners to create and model addition and subtraction problems involving fractions. For example, let a yellow hexagon represent a whole pizza. Gary has 2-1/2 pizzas left over from a party. He eats 2/3 of a pizza for lunch. How much pizza is still left?

Have students use a ruler or number line with the units and half-units marked as reference points to estimate sums and differences of fractions as being more or less than 1/2, more or less than 1, etc. For example, 5/6 - 2/6 would be less than 1; 3/6 +4/5 would be between 1 and 1-1/2.

(B) select an appropriate operation and/or strategy to solve a problem and justify the selection;

Sample Learning Objectives

Analyze a problem by identifying relationships and discriminating relevant from irrelevant information.

Connect the results of the analysis to the meaning of operations by selecting a strategy that will solve the problem, then justify the selection of that strategy.

Sample Clarifying Activities

When students are sharing solution strategies to an appropriate problem, point out that some students used addition to solve it while others used multiplication. Have students discuss the relationships between the two operations.

Have students average their grades and explain the process they used.

Have students draw a diagram for a more efficient arrangement in the mathematics classroom. Ask them to explain why the arrangement is more efficient.

Have students work in groups on problems such as, "Friendly Freddy likes to share his pencils with his friends. In his first class, he gave 1/2 of his pencils to Judy and 2 more to a new student. In his next class, he gave 1/2 of his remaining pencils to Eric and 2 more to a new student. He continued to follow this pattern in each of his four other classes that day. At the end of the day, Friendly Freddy had only two pencils left. His mother asked him how many he had brought to school, but he couldn't remember. How could he figure it out?"

(C) solve problems involving addition, subtraction, and multiplication;

Sample Learning Objectives	*Sample Clarifying Activities*
Use addition, subtraction, and multiplication in problem-solving situations.	Have students check the total of an order in a restaurant.
Select whether to solve problems by estimation only, mental calculation, paper and pencil calculation, and/or calculator and justify the selection.	Have students plan a week's worth of meals for the family and calculate the cost.
Use estimation and the context of the problems to check the reasonableness of the solutions.	Have students work in groups on problems such as, "Rebecca bought 4 rabbits. At the end of the first month, Rebecca had twice as many rabbits plus two more than she had at the beginning of the first month. At the end of the second month, she had twice as many rabbits plus 2 more than she had at the beginning of the second month. If this pattern continues, how many rabbits will Rebecca have at the end of the year?"
	Have groups of students form construction companies for the purpose of building toothpick bridges according to a building code and within a specified budget accounting for construction costs (from *Building Toothpick Bridges* by Jeanne Pollard). This 10-day project incorporates estimating spatial configurations, estimating dollar amounts in the thousands, adding and subtracting 7-digit numbers, multiplying and dividing numbers of at least two digits, and measuring.

(D) solve division problems with divisors that are less than 10 or multiples of 10 using the division algorithm.

Sample Learning Objectives	*Sample Clarifying Activities*
Analyze a problem by acting it out or using manipulatives to demonstrate the actions described in the problem.	Have students calculate the fuel efficiency of the family's car.
Estimate the solutions to the problems.	
Connect the division algorithm to the actions with the physical materials.	Have students discuss the use of division in careers. For example, the principal of I. M. Smart Elementary School must decide upon how many teachers to hire. If there are 700 students enrolled, and every teacher is to have no more than 20 students, what is the least number of teachers the principal should hire? How many fewer teachers would be needed if the class size was raised to 30?
Check the reasonableness of the solutions based on the estimates and the context of the problems.	

(E) add and subtract decimals;

Sample Learning Objectives

Estimate the solutions to the problems.

Use concrete models to add and subtract decimals less than and greater than 1 (including hundredths).

Use decimal symbols and the addition and subtraction algorithms to solve meaningful problems.

Use the estimates and the context of the problems to check for reasonableness of the solutions.

Sample Clarifying Activities

Have students balance a checkbook.

Have students spend $100 (or $1000 or $1,000,000) by using catalogs or newspaper advertisements to select items. Students must purchase at least four items and may not purchase more than one of any item. See activities if TEA Module 6, Grades 3-6, Computation and Error Analysis.

Have students discuss the different strategies for computational estimation involving rounding. For example, 2.43 + 4.79 may be estimated as a little over 7 because 2 + 4 = 6 and 0.43 + 0.79 is greater than 1. Or 2.43 + 4.79 may be estimated as less than 7.50 because 2.43 is little less than 2.5 and 4.79 is a little less than 5. Each student selects a strategy that he or she prefers to use, finds another student who prefers a different strategy, and compares results as they do whole number and decimal calculations.

(F) solve application problems involving multiplication.

Sample Clarifying Activities

Have students spend $100 (or $1000 or $1,000,000) by using catalogs or newspaper advertisements to select items. Students must purchase at least four items and may purchase more than one of any item.

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(5) **Measurement.** Concepts and skills using metric and customary unit. The student shall be provided opportunities to:

(A) use models to develop and apply formulas for the area of a square, rectangle, triangle, and parallelogram;

Sample Clarifying Activities

Have students construct all of the rectangular arrays that can be built with 12 tiles and cut models of the arrays from grid paper. Students can record in a chart the bottom edge (B), side edge (S), area (A), and perimeter (P) for each array and look for patterns. Have students repeat the activity with 24 tiles. Students can discover and apply rules for measuring the area of a rectangle: A = B x S; they can make conjectures about how the shapes of rectangles with the same area affect their perimeters. (For example, the longer and skinnier the rectangle, the greater its perimeter compared to the others with the same area.) See *Mouse and Elephant: Measuring Growth* developed by the Middle School Mathematics Project.

Have students measure and record the lengths of the bases and heights of several nonrectangular parallelograms. Students can draw a dotted line (perpendicular to opposite sides) to represent the height of each parallelogram, cut along each dotted line, and place the two pieces of the parallelogram together in a different way to form a rectangle. Students then can measure and record the lengths of the base and height of each rectangle, compare the base and height of each parallelogram to the corresponding rectangle.

Have students cut out two identical triangles and measure and record the base and height of the triangles. Students can fit the two triangles together for form a parallelogram and measure and record the base and height of the parallelogram. Have students address questions such as, "What fraction of the parallelogram is one triangle? What is the relationship between the area of the parallelogram and the area of one triangle?"

(B) develop and apply formulas for the circumference of a circle, using estimation when appropriate.

Sample Clarifying Activities

Have students measure the diameters of several wheels (furniture, lawnmower, bicycle, etc.) and compare how far they roll in one complete rotation. Students can explore the ratio of the distance of one rotation (the perimeter or circumference of the wheel) to wheel diameter. Introduce the term *pi* as a name for this constant ratio. (See TEA Module 7, Grades 3-6, Geometry.)

Have students investigate problems such as "Gregory likes birthday pizzas instead of birthday cakes. His friends want to put candles at one inch intervals around the outside edge of the pizza they ordered that has a diameter of 12 inches. How many candles do they need?"

(C) estimate and solve application and nonroutine problems involving perimeter and area.

Sample Clarifying Activities

Have students construct with tiles all of the rectangular arrays that can be built with a specified perimeter (say 12 or 24 units) and cut models of the arrays from grid paper. Students can record in a chart the bottom edge (B), side edge (S), area (A), and perimeter (P) for each array and look for patterns. Students can discover and apply rules for measuring the area and perimeter of a rectangle: $A = B \times S$; $P = (B + S) \times 2$ or $(B \times 2) + (S + 2)$, and make conjectures about how the shapes of rectangles with the same perimeters affect their areas. (For example, the longer and skinnier the rectangle, the smaller its area compared to the others with the same perimeter.) See *Mouse and Elephant: Measuring Growth* developed by the Middle School Mathematics Project.

Have students trace around their feet on centimeter grid paper and figure the areas of their feet. Each student can cut a piece of string equal to the perimeter of his or her foot and tape the string on the centimeter paper in the shape of a square. Have students compare the area of the square to the area of its corresponding foot, explain why they are different, and make the conjectures about the relationships between the area of a square and a foot with the same perimeters.

Have students design a rectangular dog pen using 64 feet of fencing. Students can make some scale drawings of rectangles that would work. Have students justify their choice of which rectangle would be the best to build.

Have students pretend that a rock band has hired them to design the stage for their next concert. The stage should be rectangular and have an area of 1000 square feet. It will have a security rope on both sides and across the front. Students can make some scale drawings of different rectangles that have 1000 square feet of area. Have students determine how many feet of security rope are needed for each one and justify their choice of rectangle to use.

(D) measure volume using nonstandard units;

Sample Clarifying Activities

Have students fill a liter container with various substances (marbles, beans, rice, sand) and discuss which substance gives the closest approximation to the volume of the container.

Tell students that they can each have two handfuls of jelly beans. Have students discuss whether this is fair or not and why.

(E) identify and use concrete models that approximate volume units;

Sample Clarifying Activities

Have students look for objects in their environment that approximate a cubic centimeter, a cubic inch, a cubic foot, a cubic meter. Post a wall chart for each of the units on which students can record their findings.

Have students look for containers that approximate different capacities (one cup, one liter, one ounce, one milliliter). Students can set up a table of displays for each size of container for the class to examine.

(F) estimate volume and check the estimate by actual measurement;

Sample Clarifying Activities

Have students choose the appropriate unit from centimeter cubes, inch cubes, or student-constructed meter cubes to estimate, then measure, the volume of various containers, such as a suitcase, pencil box, classroom, etc.

Have students estimate the amount of liquid in a nonstandard container such as a paper cup. Students can use a graduated cylinder to measure the liquid and compare the measurement to their estimates.

Have students predict whether or not they could fit inside a cubic meter, then build a model to check.

(G) describe the relationship between volume units in the metric system such as cubic decimeter and liter, cubic centimeter and milliliter.

Sample Clarifying Activities

Have students construct acetate models of a cubic decimeter and a cubic centimeter. Students can fill the models with water or sand and use a graduated cylinder to measure the sand or water held in milliliters or liters.

Have students stack centimeter cubes in a hollow cubic decimeter model and predict how many it will hold. Students can relate the number of centimeter cubes required to fill the container to the number of milliliters in a liter.

(H) apply measurement concepts and rounding techniques to application problems involving length, weight, capacity, and volume;

Sample Clarifying Activities

Have students estimate, then measure, how far they can jump, throw a cotton ball, and throw a paper plate like a Frisbee. Students can select which unit would be most appropriate to use for making each measurement and discuss the effect of the size of the unit on rounding partial units to whole numbers of units.

Have students determine whether milliliters or liters would be the best for estimating the amount of water in a bathtub, milk in a baby's bottle, juice in a glass at breakfast, gasoline in a car's fuel tank, and liquid in a test tube.

Have students design a box that would hold their mathematics text. Students can round each dimension of the box to the nearest inch or centimeter and determine the volume of the box.

(I) use the relationship between units to convert measures within the same measurement system.

Sample Clarifying Activities

Have students measure six objects or distances in the classroom. Students can measure each object using each of these units: millimeters, centimeters, decimeters, and meters. Have students make a chart listing each measurement and describe any patterns found in the chart.

Have students solve problems such as, "You want to make a friendship bracelet for your best friend and you need 130 inches of red string. However, the string you want is sold only in yards. How many yards of red string should you purchase? Explain your solution. How many yards of string would you need to buy if you wanted to make several bracelets and didn't want any string left over? How many bracelets would it make?"

Have students explore the patterns in metric measures (875g = .875kg) and their relationship to the base ten place value chart.

(6) Geometry. Properties and relationships of geometric shapes and their applications. The student shall be provided opportunities to:

(A) construct examples of symmetric and congruent figures;

Sample Learning Objectives	*Sample Clarifying Activities*
Use concrete and pictorial models to explore symmetric and congruent figures.	Have students examine models of a square, rectangle, parallelogram, right triangle, hexagon, and circle and predict the lines of symmetry in each. Students can cut out paper models of each shape and check their predictions by folding.
Generalize a definition for symmetry and congruence.	Have students explore the lines of symmetry in the letters of the alphabet. Have students draw a figure on paper and a congruent figure on transparency film. Students can place the second figure over the first one to test for congruence.
	Have students trace congruent figures in Escher tessellation prints.
	Have students describe and draw real-world examples of congruent and symmetric figures such as commercial logos for banks, companies, and car manufacturers; quilt patterns; fabric and wallpaper patterns; flag designs; and works of art.

(B) construct examples of reflections, rotations, and translations;

Sample Learning Objectives	*Sample Clarifying Activities*
Use concrete and pictorial models to compare and contrast examples of reflections, rotations, and translations.	Have students make observations of real-world examples of flips, slides, and turns. Examples: reflections in a mirror, slides flipped over in a slide projector, reverse lettering of the word ambulance on emergency vehicles, movement of a child's body down a slide, a Ferris wheel, a carousel, etc.
Identify what information is important to be able to construct each of the transformations-a reflection, a rotation, or a translation.	Have students draw mirror images (reflections across an imaginary mirror line). Students can use a Mira to verify the reflection. Have students draw a figure, then draw the figure as it would appear after a slide, flip or turn. Students should discuss what properties such as shape, lengths of line segments, sizes of angle, and parallelism are preserved and that the transformed figure is congruent to the original figure. Have students create tessellations and discuss how reflections, rotations, and translations are used in their creation. Students can investigate real-world examples of tessellations such as fabric designs, bricklaying patterns, works of art, and manufacturer's packaging design decisions. Students can discuss the role that reflection, rotation, and translation play in the computer game Tetris.

(C) construct circles and identify the radius, diameter, chord, center, and circumference of circles;

Sample Clarifying Activities

Have students model a circle on a circular geoboard and identify diameters, radii, and other chords of the circle.

Have students suppose that the class may make a field trip anywhere within 40 miles of the school. Ask students to show on a map the entire area that could be considered for the trip. Ask questions such as, "What is the shape of the area? What is the school's location in relation to this shape? If we went from our school straight to a location exactly 40 miles away, what would our path be called? What is the outermost edge of the region called? What kind of trip would we make if we traveled on a diameter? On some other kinds of chord?"

(D) measure and draw angles using a protractor;

Sample Clarifying Activities

Have students draw a picture or collect an assortment of pictures containing representations of various angles. Students can measure and label the angles with their measurements.

Have students cut out models of two rays and connect them with a paper fastener to form a model of an angle. Students can use the model to make and measure a variety of angles.

Have students use the angle model to form the angles related to certain times on a traditional clock: a 3:00 angle, a 2:00 angle, a 2:15 angle, a 6:00 angle, an 8:00 angle, a 10:00 angle. Students can sketch the angles and label them. (See *Geometry in the Middle Grades* by Dorothy Geddes.)

(E) estimate answers and solve problems using geometric concepts;

Sample Clarifying Activity

Have students find three angles in the classroom, one that appears to be less than 90 degrees, one about 90 degrees, and one greater than 90 degrees. Students can write a description or draw a sketch of each.

(F) measure the angles in a triangle and draw conclusions about angle measures in triangles;

Sample Clarifying Activities

Have students cut out a large triangle, put a dot in the triangle near each vertex, and cut or tear off each vertex of the triangle. Students can tape the dotted vertices together into a straight edge and make conjectures about the angle measures of a triangle.

Have students use a protractor to measure the angles of a variety of triangles (isosceles, equilateral, and scalene). Students can find the sum of the angles in each triangle and make a conjecture about other triangles. (See TEA Module 14, Grades 3-6, Measurement.)

(G) construct a three-dimensional model using blocks or other manipulatives and describe it from different perspectives.

Sample Learning Objectives

Identify faces, edges, and vertices of three-dimensional models

Describe three-dimensional models one attribute at a time.

Describe three-dimensional models from different perspectives.

Sample Clarifying Activity

Have students explore perspective with activities such as those in *Spatial Visualization* developed by the Middle School Mathematics Project.

(7) **Probability, Statistics, and Graphing.** Use of a probability and statistics to collect and interpret data. The student shall be provided opportunities to:

(A) collect, organize, and interpret data to solve application problems;

Sample Learning Objectives	*Sample Clarifying Activities*
Collect, organize, and use information from a survey to make a decision.	Have students survey the class to find out what are their favorite soft drinks (discrete data). Students can graph the results of the survey and use the results to plan a class party.
Collect and graph experimental data to make a prediction or draw a conclusion.	Have students plant five pinto beans in identical containers of dirt at the same time. Students can put 0 teaspoons of fertilizer into the first container, 1 teaspoon into the second container, 2 teaspoons into the third container, 3 teaspoons into the fourth container, and 4 teaspoons into the fifth container. Plants should receive identical amounts of water and sunlight. Have students collect and graph the information on daily growth (continuous data) to determine the effects of different amounts of fertilizer.
	Have students collect and organize data about their resting pulse rates. (Introduce stem-and-leaf plots for organizing data with a large range.) Students can describe the shape of the data and determine the typical resting pulse rate for the class. Then have students exercise and take their post-exercise pulse rates, organize, and display these data. Students can compare the results with the resting pulse rates.

(B) explain the decisions that need to be made before constructing a graph;

Sample Learning Objectives	*Sample Clarifying Activities*
Discuss the purpose for the graph.	Have students analyze the steps they took to construct the soft drink graph described above. What decisions did they make and why?
Based on the purposes discussed, determine the type of graph best suited for survey or experimental data.	Have students compare the graphs of bean growth made by different groups. Students can discuss the decisions each group made in designing their graphs: Why did they choose the type of graph they did? How did they decide how to organize the data?
	Have students discuss reasons for choosing a stem-and-leaf plot instead of a line plot for organizing the pulse-rate data.

(C) recognize measures of central tendency as ways of summarizing a set of data.

Sample Learning Objectives	*Sample Clarifying Activities*
Use mean, median, and mode to describe data from surveys or experiments.	Have students investigate the use of the mean, median, and mode to describe the class's general preference of soft drink. Students can discuss why the mean and the median are not very useful for summarizing these data.
Compare and contrast the three different measures of central tendency.	Have students use the mean, median, and mode to describe how much the bean plants grew each day.
	Have students discuss the finding that children ages 10 through 13 get an average of 10 hours of sleep each night. Students can work in groups to write brief reports that include several different sketch graphs of how the data behind this average might look, a description of each sketch, and an explanation of which of the graphs they think are more likely to be close to the actual shape of the data.

(D) use averaging in problem-solving situations;

Sample Learning Objectives	*Sample Clarifying Activities*
Determine which measure of central tendency should be used to make a decision.	Have students pretend they own a shoe store and have collected data concerning the shoe sizes sold for a year. Students can discuss the advantages and disadvantages of using each measure of central tendency to decide what to buy for the next year.
Investigate the impact that changing one piece of datum would have on each of the measures of central tendency.	Have students figure their average mathematics grades for the six weeks and discuss what impact a zero on a homework paper would have on their mathematics average (mean).
	Have students figure the average number of hours of sleep students in their class get each night, the average number of hours of television watched each day, and the average amount of time spent on homework each day.

(E) predict the number of arrangements of a given set of objects and experimentally verify the predictions;

Sample Clarifying Activities

Have students pretend they are giving a party and have three favorite tapes they want to play. In how many different orders can they be played? Students can predict and verify the different ways with a list or chart or picture. Have students predict how many different orders there would be if they bought one new tape.

If the school had three cheerleaders, in how many different orders could they face the crowd to lead cheers? Students can discuss the problem, predict the number of ways, and verify their reasoning with a list or chart or picture. Have students predict how many different orders there would be if one new cheerleader were elected.

(F) list all possible outcomes of an experiment;

Samples Learning Objectives	*Sample Clarifying Activities*
List all the possible outcomes of single-stage and multi-stage experiments.	Have students list the possible outcomes of tossing two dice (a multi-stage experiment). Students can discuss whether the outcomes should be described as the sums of the two dice or the pairs of addends shown on the dice. Ask questions such as, "Which type of outcome would allow you to generate probabilities to describe the likelihood of different events occurring? Which list consists of outcomes that are equally likely?" If students use dice of two colors, the need to include both 3 + 5 and 5 + 3 as outcomes becomes clear.)
Compare and contrast lists of outcomes of different experiments.	Have students list all the possible outcomes of tossing two coins, drawing two marbles from a bag of red and blue marbles, or spinning a spinner (1/2 red and 1/2 blue) twice (multi-stage experiments). Students can look for patterns among the different lists of outcomes and determine which lists consist of equally likely outcomes. For example, in tossing two coins, the list could be constructed three ways: TT, HH, TH, TH (equally likely); or "two heads, two tails, one of each kind" (not equally likely); or "same, different" (equally likely). Have students discuss the advantages of using a list of equally likely outcomes.

(G) use a fraction to describe the probability of a given event;

Sample Clarifying Activities

Have students use the list of equally likely outcomes for rolling two dice to generate fractions to describe the probabilities of rolling sums of 1, 12, and 7.

Have students use the list of equally likely outcomes for tossing two coins to generate fractions to describe the probabilities of tossing exactly two heads, tossing exactly two tails, tossing at least one head, etc.

(H) make and refine predictions based on exploration of different sample sizes within experiments;

Sample Learning Objectives	*Sample Clarifying Activities*
Make and refine predictions based on several different sample sizes.	Have students predict the results of tossing two coins. Students can toss two coins and record what comes up each time. After 10 tosses, students can compare their experimental results with their predictions and revise their predictions, if necessary. Have students compare their results with their revised predictions after 50 tosses, after 100 tosses. Students can compare their experimental results with the fractional probabilities generated from the list of equally likely outcomes.
Make a generalization about the effects of sample size on the ability to make predictions.	Have students use the results from the soft drink survey to predict what the favorite soft drink would be for the whole school. Students can extend the survey to include one class from each grade level in the school. Have students compare these results and revise their predictions, as necessary. Students can discuss whether it is necessary to survey the entire school. What is a representative sample?

(I) plot points on a coordinate plane that represent ordered pairs of whole numbers, arising from application problems.

Sample Learning Objectives	*Sample Clarifying Activities*
Make a table representing the relationship of two sets of data.	Have students make a table representing the relationship of the length of the side of a square to its area. Students can graph the ordered pairs generated in the table.
Translate the information in the table into ordered pairs.	Have students make a table representing the relationship of the length of the side of a square to its perimeter. Students can graph the ordered pairs generated in the table.
Use the ordered pairs to represent the relationship on a graph.	Have students use the above graphs to compare/contrast the two relationships involving sides of squares and their areas of perimeters.

Texas Assessment of Academic Skills

Focus

The Texas Education Agency implemented the Texas Assessment of Academic Skills (TAAS) testing program in 1990. The program is in effect for the 1990-1995 period. The purpose of the assessment program is to provide Texas schools with an accurate measure of student achievement. The scope of content of the TAAS includes more of the instructional targets delineated in the essential elements than previous state assessments. Every section of the TAAS test contains a certain number of broad objectives. These objectives remain constant from grade to grade because they represent the core concepts that form the basis for a sound instructional progression from Grade 1 through Grade 12. What will differ from grade to grade are the instructional targets—or essential elements that comprise each objective. A portion of this extended set of instructional targets is selected for assessment annually, but not every target is tested every year.

The broadened scope of the TAAS assessment program allows for a different focus, one that addresses the academic requirements of the 1990s. Skill areas that demand little more than rote memorization are de-emphasized, while areas that improve a student's abilty to think independently, read critically, write clearly, and solve problems logically receive increased emphasis. This emphasis is in keeping with current national trends in education, which stress the importance and necessity of teaching students higher order thinking skills.

Domains, Objectives, and Targets

The TAAS features three domains—concepts, operations, and problem solving. Each domain contains objectives that are derived from the essential elements. For every objective, there are instructional targets that describe the kinds of mathematical experiences that will reflect that objective. Each instructional target was taken for the most part directly from the essential elements as delineated in the *State Board of Education Rules for Curriculum*. Each target is defined in behavioral terms appropriate for pencil-and-paper testing.

DOMAIN: Concepts

Objective 1: The student will demonstrate an understanding of number concepts.

(a) Compare and order fractions and decimals
(b) Round whole numbers and decimals (to nearest tenth, one, ten, or hundred)
(c) Determine relationships between and among fractions (denominators of 2, 3, 4, 5, 6, 8, and 10)
(d) Recognize and compare fractions using patterns and pictorial models
(e) Factor whole numbers

Objective 2: The student will demonstrate an understanding of mathematical relations, functions, and other algebraic concepts.

(a) Use nonnegative rational number properties and inverse operations (whole numbers, fractions, and decimals)
(b) Determine missing elements in patterns
(c) Use number line representations of fractions and decimals
(d) Identify ordered pairs on a coordinate plane

Objective 3: The student will demonstrate an understanding of geometric properties and relationships.

(a) Recognize properties of two- and three-dimensional figures
(b) Recognize congruence and symmetry
(c) Identify translations, reflections, and rotations

Objective 4: The students will demonstrate an understanding of measurement concepts using metric and customary units.

(a) Solve problems with metric and customary units
(b) Find perimeter and circumference
(c) Determine area (with and without grids) and volume using models
(d) Convert within the metric system
(e) Convert within the customary system

Objective 5: The student will demonstrate an understanding of probability and statistics.

(a) Determine possible outcomes in a given situation
(b) Analyze data and interpret graphs
(c) Use counting arrangements
(d) Use a fraction to describe the probability of an event
(e) Find means (averages)

DOMAIN: Operations

Objective 6: The student will use the operation of addition to solve problems.

(a) Add whole numbers and decimals (tenths and hundredths)

Objective 7: The student will use the operation of subtraction to solve problems.

(a) Subtract whole numbers and decimals (tenths and hundredths)

Objective 8: The student will use the operation of multiplication to solve problems.

(a) Multiply whole numbers

Objective 9: The student will use the operation of division to solve problems.

(a) Divide whole numbers (divisors less than 10 or multiples of 10)

DOMAIN: Problem Solving

Objective 10: The student will estimate solutions to a problem situation.

(a) Estimate with whole numbers and decimals (decimals: addition and subtraction)

Objective 11: The student will determine solution strategies and will analyze or solve problems.

(a) Formulate strategies or solve problems using basic operations with whole numbers and decimals (decimals: addition and subtraction)
(b) Determine strategies for solving or solve problems requiring the use of geometric concepts
(c) Analyze or solve problems through the use of congruence and symmetry
(d) Analyze or solve problems using probability and statistics concepts

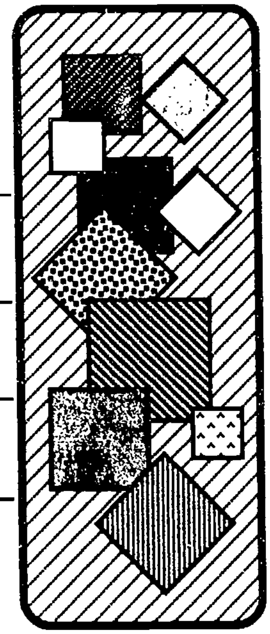
Objective 12: The student will express or solve problems using mathematical representation.

(a) Formulate solution sentences
(b) Analyze or interpret graphs and charts and use the information derived to solve problems (no circle graphs)

Objective 13: The student will evaluate the reasonableness of a solution to a problem situation.

(a) Evaluate reasonableness

Sample Lessons for Teaching Grade 5 Mathematics



The following sample lessons represent the kind of mathematical experiences recommended for students in fifth grade mathematics classes. These expanded sample activities include ideas for motivaional introductions, exploratory questions to ask during activities, summary questions for reflection after exploring the concept, and ideas for extension and assessment. They are included as examples of significant, mathematical tasks that address the state's curriculum requirements in light of the national recommendations. Note that each sample activity involves several essential element descriptors, as well as several objectives from the Texas Assessment of Academic Skills (TAAS). Several different manipulatives are included in these activities. It is important that students use these manipulatives as they work through the activities. Manipulatives and concrete objects enable elementary school students to better understand the mathematical problems and concepts they so often struggle to learn. Students' experiences with manipulatives are recommended in the essential elements, TAAS's instructional targets, and NCTM's *Curriculum and Evaluation Standards for School Mathematics*.

Many activities in this section also recommend that students work together in pairs or small groups. Working together in cooperative groups promotes communication and mathematical confidence and enhances students' problem-solving abilities.

EE: 2B *Related EEs: 1A, 1B, 1C, 1D, 1E, 1F, 3A, 3E, 4D* *TAAS Objectives: 1, 8, 9, 11, 12, 13*

Objective The student will explore place value and number sense and how they lead to the rules for divisibility.

Activity Divisibility Rules

Materials Calculators (one per pair of students); newsprint; markers; base ten materials

Procedure

Introduction:

1. List numbers on the board. Ask students to tell you which numbers from the list are divisible by 2. Ask how they know which numbers have 2 as a factor.
2. Ask students if they know any other rules for divisibility (such as divisible by 5 if it ends in 0 or 5 and divisible by 10 if it ends in 0).
3. Give students a hundreds chart and ask them to find all the numbers that are divisible by 2. Students should then find the numbers that are divisible by 3. Students should then find the multiples up to 100 of four through 12. (A calculator could be used for this.) Encourage students to find original ways of recording their data.
4. Share the results of the students. Ask for any patterns they have found.
5. Ask students if they can find rules to explain what numbers are divisible by 2 - 12. (Students will probably generate rules for dividing by 2, 5, and 10.)
6. Give students a number such as 354 that they have not considered and ask if it is divisible by 2. Ask students how they know it is divisible by 2. Ask students why they only need to look at the ones digit. (The ones are the only place that could make a difference because all groups of 10 are divisible by 2, all groups of 100 are divisible by 2, etc.) Let students model with base 10 blocks and discuss this with several examples.
7. Repeat this process with 5 and 10 to discover why we only look at the ones place to test for divisibility with these numbers.
8. Tell students the rule for divisibility by three: An integer is divisible by 3 if and only if the sum of its digits is divisible by 3. Let students test this with several numbers such as 621 and 802. (Students could use calculators to help with this.) Ask students why this rule works. Let them explore.
9. After students have explored the divisibility test for 3 awhile, ask them to model a number such as 234 with base ten materials.
10. Ask students to take the hundreds one at a time and do the exchanges necessary to divide each hundred into three equal groups. How much is left over out of each hundred? (one unit) Next tell students to take each ten one at a time and divide each ten into three groups. How much is left out of each ten? (one unit) The ones place has 4 leftover ones which will not be divided up.
11. In looking at the leftovers, we have two leftovers from the hundreds, three leftovers from the tens, and four leftovers in the ones place. Since all of 234 has been divided into three equal groups, except the leftovers, we must see if the leftovers can be divided by 3. If we add up the digits of the number 234 (which match the number of leftovers) and this sum is divisible by 3, then the whole number is divisible by 3.
12. In contrast, 235 is not divisible by 3 because $2 + 3 + 5 = 10$ which is not divisible by three. Remember—the 2, the 3, and the 5 now represent the leftovers.
13. Once students have had ample time to explore the divisibility rule for 3 and really understand why it works, you could introduce the divisibility rule for 9: An integer is divisible by 9 if and only if the sum of its digits is divisible by 9. For example, 738 is divisible by 9 because $7 + 3 + 8 = 18$ and 18 is divisible by 9. The reason that this

works is the same as why the divisibility rule for 3 works. Each hundred and each ten will give you one leftover when divided by 9.

Exploration:
- How do you know which numbers have 2 as a factor?
- What other rules for divisibility do you know?
- How did you decide to record your data?
- Is there another way to organize your data?
- What patterns did you notice as you found the multiples of 2 - 12 from your hundreds chart?
- Why do you only need to look at the ones digit to see if a number is divisible by 2? 5? 10?
- Can you state the divisibility rule for 2? 5? 10?
- What is the divisibility rule for 3?
- Why does the divisibility rule for 3 work?
- What do the digits that you are adding represent?
- When you're modeling the rule of divisibility for 3, why do you divide one hundred at a time and one ten at a time?
- Why don't you divide the ones when you are modeling the divisibility for 3?
- What is the divisibility rule for 9?
- Why does the divisibility rule for 9 work?

Extension:
- Explore the divisibility rules for 4,6,7,8,11, and 12.
- Explore the relationship between the divisibility rule for 10 and the divisibility rules for its factors.

Summary:
- Why would knowing the divisibility rules be helpful?
- What is the divisibility rule for 2? 3? 5? 10? 9?
- Why does the divisibility rule for 3 work?
- Why does the divisibility rule for 2 work?
- Why does the divisibility rule for 5 work?
- Why does the divisibility rule for 10 work?
- Why does the divisibility rule for 9 work?

Assessment

Questions:
(See summary questions.)

Observations:
- Did students recognize the multiples of 2 - 12 on the hundreds chart?
- Were students organizing data?
- Were students using base ten materials correctly?

Tasks:
- Brainstorm a list of situations when knowing the divisibility rules would be helpful.
- Write in your journal what the divisibility rules are and why they work.

EE: 2C *Related EEs: 1A, 1B, 1C, 1D, 1E, 1F, 4B, 4C, 7A* *TAAS Objectives: 1, 2, 6, 7, 8, 11, 12*

Objective The student will use the calculator to develop number sense through a better understanding of the patterns in place value.

Activity Telling Phone Numbers

Materials Calculators (one per pair of students); overhead calculator; paper/pencil; telephone directories (optional)

Procedure

Introduction:

1. Explain the problem: A calculator is broken. Only 1, 0, +, -, x, ÷, and = work. Can you display your phone number?
2. Allow time for exploration.
3. Stop groups and share strategies up to this point. Model ways to record strategies and encourage students to keep records of what they are doing. Discuss advantages and disadvantages to each strategy.
4. Ask for ideas of new strategies after students have shared beginning ideas.
5. Allow more time for exploration with record keeping.

Exploration:

- How can you get the numbers in the positions you want them?
- What do the different positions represent?
- What operations have you tried?
- What difficulties have you encountered? How did you solve them?
- Can you find any patterns that may help you to develop a strategy?
- What strategy seems the best to you so far? Why? (quicker, easier to remember, etc.)
- How many steps did it take to display your phone number? Could you do it in fewer steps?

Extension:

- Make a chart or display of the strategy that would require the fewest number of steps.
- What phone number would require the fewest steps? (It must be a legitimate phone number.)
- What phone number would require the most steps?
- Try your strategy on another type of number (zip code, etc.). Does it work? Or do you need to change it?
- What if only the 1 and 2 and operation keys worked? How would your strategies change?

Summary:

- How many steps did it take to display your phone number?
- How did you shorten the number of steps you needed?
- How did place value help you optimize your strategy?
- What are the advantages and disadvantages of __________ (strategy)?
- How did you deal with 0 in a phone number?
- Compare the steps needed in an addition strategy to those in a subtraction strategy. Are they the same? Different? Why? Which do you think is easier? Why?
- How could (or did) you use multiplication in a strategy?
- How did you use patterns in developing your strategy?

Assessment

Questions:

- What changes in your strategies were made due to place value?
- How does place value contribute to the strategies?
- Why did you use (+, -, x, ÷)?

(See also summary questions.)

Observations:

- Were students keeping coherent written records of strategies?
- Did students change their strategies when they didn't work?
- Did students look for ways to improve the strategies that did work?
- Were students talking about place value with their partners?

Tasks:

- Write a summary of your findings about the use of place value to display your phone number.
- Design a poster or display to explain your best strategy.

EEs: 3B, 3D, 3F *Related EEs: 1A, 1B, 1C, 1D, 2A, 5H, 5I* *TAAS Objectives: 1, 2, 4, 10, 11, 12, 13*

Objective The student will investigate patterns that occur in numerators and denominators of equivalent fractions; model and write equivalent fractions; explore the relationship of fractions and decimals; estimate whole number, fraction, and decimal calculations; and use fractions in measurement with metric units of mass.

Activity Fruity Fractions

Materials Balances with metric masses (one per group); worksheets; one apple per group; paper plates; paper towels; paring knife; cutting board; toothpicks; plastic bags; student-made fraction kits; newsprint; markers; fraction calculators; overhead calculator

Resource *Fun With Foods*, AIMS, 1987.

Procedure

Introduction: ACTIVITY 1

1. Inform students that they will be using fruit to discover patterns in fractions. First they must collect some data to be ready for the next lesson.
2. Distribute Worksheet 1 and an apple per group. Look at the first column—"Mass of Whole Apple in Grams." Have students estimate the total mass of an apple and record it on Worksheet 1. (Students can each have a recording sheet.)
3. Have students measure the apple's mass on a metric balance scale and record the actual mass under the actual column.
4. Discuss student findings. Stress that the apple is whole, by using the word *whole*.

ACTIVITY 2

1. Demonstrate for the class how a whole apple is cut into fractional parts. Inform the students that the apple is going to be divided approximately equally among two students. Discuss what is the fairest way to divide the apple into two parts.


or


2. Distribute fraction kits. Have students represent one whole with the fraction kit.

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3. Cut the apple in half. Have students model the parts with their fraction kits.

1/2	1 / 2

4. Taking each "half" of the apple, have students predict how many parts there would be if each of the halves was divided into two equal parts. Cut one of the parts in half. Have students model the parts with their fraction kits.

1/4	1/4	1 / 4	1 / 4

5. Have students predict how many parts there would be if each fourth was divided into two equal parts. Cut one of the fourths in half. Have students model the parts with their fraction kits.

1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8

6. Taking each eighth, have students predict how many parts there would be if each eighth was divided into two equal parts. Cut one of the eighths in half. Have students model the parts with their fraction kits.

1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16

7. Discuss the fractional parts of the apple.
8. Ask students to name an equivalent fraction for 1/2. Using their fraction kits, have groups find and record as many pairs of equivalent fractions as they can. Students should organize their data on newsprint.
9. Share and discuss student findings.
10. Have students complete Worksheet 2 with their partners.

ACTIVITY 3*

1. Review findings from Worksheet 2.
2. Distribute plastic bags with cut apples and calculators.
3. Discuss the column heading "Fractional Part = Decimal" from Worksheet 1. If possible, use an overhead fraction calculator to show students how to find the decimal equivalent for a fraction using the F-D key or division and review reading the decimals. Do an example: 1/2 = 0.5. Record under column "Fractional Part = Decimal." Have students complete the column.
4. Have students estimate the mass of each fractional part of the apple and record on Worksheet 1.
5. Have students write a fraction for the estimated mass of the part to the estimated mass of the whole apple and record on Worksheet 1.
6. Students can use the F-D key or division on a calculator to find the decimal equivalent for each fraction.
7. Have students measure the fractional parts of the apple on a metric balance scale and record the actual weight on Worksheet 1.
8. Have students write a fraction for the actual mass of the part to the actual mass of the whole apple and record on Worksheet 1.
9. Students can use the F-D key or division to find the decimal equivalent for each fraction.

*As a culmination of the activity, let students enjoy a whole apple!

Exploration:

ACTIVITY 1

- How accurate were your estimates?
- What unit did you use?
- How would the mass of the whole apple differ if the apple was peeled?

ACTIVITY 2

- What is the fairest way to cut up the apple so that each part is as close to half as possible?
- Can you represent the "whole" apple with the fraction kit?

- Can you represent the parts of the apple after the first cut using the fraction kit?
- Can you represent the parts of the apple after the second cut using the fraction kit?
- Can you represent the parts of the apple after the third cut? Fourth cut?
- If the pieces of the apple were put back together, what would you have?
- What equivalent pairs of fractions did your group find?
- How do you know they are equivalent?
- Did you see a pattern?
- What strategies did you use?
- What do the numerator and denominator represent?

ACTIVITY 3
- How accurate were your estimates?
- How did you make your estimates?
- Did you use any information from the chart in making your estimates? How?
- Compare the decimal equivalents for the fractional parts of the apple, the fractions of the estimated masses, and the fractions of the actual masses.
- Did you see any patterns or relationships?
- Did the decimal equivalents differ for the fraction, the fraction of estimated mass, and the fraction of actual mass? If so, why do you think they did?

For example,

Fractional = Decimal Equivalent	Estimated Mass of Part/ Actual Mass of Whole	Decimal Equivalent	Actual Mass of Part/ Actual Mass of Whole	Decimal Equivalent
1/2=0.5	23/50	0.46	22/46	approx. 0.48

Extension:
- Make thirds and sixths for your fraction kit.
- Add a few rows for thirds and sixths on your chart and make predictions for the masses of each piece.
- Figure out a way to test your predictions for thirds and sixths without actually cutting the apple pieces.

Summary:
ACTIVITY 1
- Compare the estimate and actual mass of the apple.
- After measuring the mass of the apple, would you have changed your estimate? Why or why not?
- How can making estimates help you? How does making measurements help later estimates?

ACTIVITY 2
- Does it make a difference in the way you cut the apple?
- What is the fairest way to cut up the apple? Why?
- Could there be any error? Can you ever measure the apple exactly?
- Can you represent the parts of the apple using your fraction kit?
- What is a fraction?

- If you put the parts of the apple back together, what would you have?
- What relationships did you find with your fraction kit?
- What is an equivalent fraction? How do you know they're equivalent?
- What do the parts of a fraction represent?
- Did the way you organize your data give you new information?

ACTIVITY 3
- Compare the estimates and actual masses of the fractional parts of the apple. How would you account for the differences?
- Was it helpful to use information from the chart to make your estimates?
- What patterns and relationships did you see among the three decimal equivalents for each fractional part of the apple?
- Were the decimal equivalents close?
- If the decimal equivalents differed, what could be the cause?

Assessment

Questions:
(See summary questions.)

Observations:
ACTIVITY 1
- Were students accurately measuring mass using the balance scale?
- Were students discussing techniques of estimating mass?

ACTIVITY 2
- Did students accurately represent the parts of the apple using their fraction kits?
- Did students accurately find equivalent fractions?
- Did students organize their data?
- Did students understand what a fraction (numerator/denominator) represents?

ACTIVITY 3
- Were students accurately measuring mass using the balance scale?
- Did students use information from the chart to make their estimates?
- Did students express relationships between the decimal equivalents for the fractional parts of the apple?

Tasks:
ACTIVITY 2
Complete questions on Worksheet 2.

ACTIVITY 3
- Have the students compare the fractional parts obtained if the apple had first been divided into thirds.

  1/3 1/6 1/12

- Students could also complete a worksheet that has them find decimal equivalents for other fractions using a calculator.
- They could also find other equivalent fractions by using a pattern.

Fruity Fractions

	Mass of Whole Apple in Grams		Mass of Fractional Parts in Grams		Fractional = Decimal Equivalent	Estimated Mass of Part/ Actual Mass of Whole	Decimal Equivalent	Actual Mass of Part/ Actual Mass of Whole	Decimal Equivalent
	Estimate	Actual	Estimate	Actual					
one-half apple					1/2 = 0.5				
one-fourth apple					1/4 =				
one-eighth apple					1/8 =				
one-sixteenth apple					1/16 =				

Worksheet 2
FRUITY FRACTIONS

Name(s) ____

1. Before the apple is cut, it is equivalent to____.

2. After the first cut is made, we have ________ parts. Each part is equivalent to _______ of the whole.

3. After one of the halves is cut, we have ________. Each smaller part is equivalent to ________ of the whole.

4. If we divide each of the smallest pieces into two equal parts, we would have ________ parts. Each part would be equivalent to _______ of the whole.

5. If we divided each of the smallest pieces again into two equal parts, we would have _______ parts. Each part would be equivalent to ________ of the whole.

Equivalent Fractions

1 = ________ = ________ = ________ = ________

1/2 = ________ = ________ = ________

1/4 = ________ = ________

1/8 = ________

6. What patterns do you see in the equivalent fractions chart?

7. What does the top number in a fraction represent? What is it called?

8. What does the bottom number in a fraction represent? What is it called?

9. What are equivalent fractions?

10. How do you know they are equivalent?

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EE: 3H | Related EEs: 1A, 1B, 1C, 1D, 1E, 1F, 4B, 5C, 5H, 6E | TAAS Objectives: 1, 2, 3, 4, 8, 9, 10, 11, 12, 13

Objective The student will use nonstandard and standard units of measure to draw a proportional representation of his or her own body.

Activity Put Yourself in Proportion

Materials String, scissors, butcher paper or newsprint, measuring tapes, markers, calculators, calipers

Procedure

Introduction:

1. Read excerpts from *Gulliver's Travels*, *The Littles* series, or *Honey, I Shrunk the Kids*.
2. Have students brainstorm ways they might figure out how to draw their bodies to fit on a given sheet of newsprint. Discuss the fact that everything would shrink and you would look the same shape, but smaller.
3. Have students work with a partner to draw themselves.

Exploration:

- What relationship (or ratio) are you using to fit yourself on the paper? How did you determine that ratio? What did you compare?
- How are you going to represent a three-dimensional measurement (like around your waist) as a two-dimensional picture?
- If your picture doesn't look right, what might be the cause? What changes in procedure might you make? What part does visual perception play?

Extension:

- Have students predict sizes other objects would need to be to fit in their smaller world.
- Have students draw a full-sized outline of themselves and compare it to their smaller drawing.

Summary:

- What method did you and your partner decide to use? Why? Could you make any improvements on your method after trying it out?
- How detailed did you decide to make your body? Why?
- How many different body measurements did you decide to make? Why? How did you decide what to omit?
- What shapes did you use to represent your body parts? Do you like the image it presents? What could you do differently to improve it?
- Does your drawing look proportional? If not, how could you improve your method?
- How might error in measurement affect the results?
- Did you predict some measurements without having to actually make them? How? If you knew someone's suit size, hat size, and shoe size, could you draw their body?
- If you had used some other ratio (1/3 instead of 1/2, for example) how would it have changed your pictures?

Assessment

Questions:

- How would you make a drawing of yourself on the 5" x 8" card?
- What does it mean for your drawing to be "in proportion" to your body?
- What are some different ways that we can express a ratio?
- (See also summary questions.)

Observations:

- Were students using appropriate methods of measurement (nonstandard or standard)?
- Were students using appropriate vocabulary to describe ratio and proportion: "1/2 of something, twice as much, etc."
- Were students able to consider and try appropriate alternatives when the drawing appeared out of proportion?

Tasks:

- Have students make a proportional drawing of something they are interested in.
- Have students write a summary paragraph describing their procedure in the investigation.
- Have students make a proportional drawing of themselves on a 5" x 8" card.

EEs: 4B, 4C *Related EEs: 1A, 1B, 1C, 1D, 1E, 1F* *TAAS Objectives: 1, 6, 7, 10, 11, 12, 13*

Objective The student will select the proper operations to solve a problem and justify the selection.

Activity The Dog Problem

Materials Play money

Resource *A Collection of Math Lessons From Grades 3 Through 6.* by Marilyn Burns

Procedure

Introduction:

1. Present this problem to the students: "A woman bought a dog for $50. Then she sold it for $60. She bought it back for $70. Finally, she sold it for $80." Ask the students what the financial outcome of these transactions is, ignoring possible costs of food, kennels, etc.
2. Ask the students for their solutions and list them on the board, but don't allow them to explain their reasoning until later.
3. Designate a different place in the room for each answer and tell people who are undecided that they may switch groups if they change their minds once the solutions are discussed.
4. When students reach their places, ask persons from each group to explain how they arrived at their answers. Do not ask them to reach a group decision; there may be more than one way to reach the same solution. Encourage students to explain not only why they feel their answers are correct, but why other answers are incorrect.
5. After discussing the problem, send students back to their seats. Ask volunteers to act out the problem for the class using play money.
6. Do not tell the students the "right" answer at any time. This forces the students to rely on their own judgment and gives them a reason to keep thinking about the problem.

Exploration:

- What do you think is the correct answer?
- How did you arrive at your answer?
- What operations did you use to arrive at your answer?
- Why do you think another answer is incorrect?
- What convinced you to switch to another group?
- What could you say to convince someone to switch to your group?
- How many people will we need to act out this problem?
- What will their roles be?
- How much money do we need to act out this problem?
- What should we do to act out this problem?
- When person A gives money to person B, what operation occurs from person A's point of view? Person B's point of view?

Extension:

- Explore any patterns that you see in these transactions. If this pattern persisted and three more transactions occurred, what would be the financial outcome then?
- Brainstorm reasons that someone would buy a dog, sell it, buy it back, and then sell it again.
- If this problem were translated for French students, what currency would be used? What would be equivalent to $50 in this currency? $60? $70? $80?

Summary:

- What do you think is the financial outcome of these transactions?
- Why do you think this?
- Why do you think other answers that were proposed are wrong?
- If you switched groups, what convinced you to do this?
- Did watching others act out this problem help you decide what the solution is? Why or why not? If it did help, how did it help?
- Can you think of another time when you would go through this process?
- What operation(s) did you use to solve this problem? What helped you to decide which operation(s) to use?

Assessment:

Questions:

- What will you remember about this activity? What will help you remember it?
- (See also summary questions.)

Observations:

- Did students propose a solution to the problem?
- Were students able to explain why they supported a solution?
- Were students able to explain why they disagreed with a solution?

Tasks:

Have students write in their mathematics journal about their solution strategies.

Objective The student will use a concrete model to "explore" the addition and subtraction of fractions.

Activity The Fraction Expedition

Materials 3" x 18" strips of construction paper (one each of five different colors per student); wooden, cubes labeled 1/2, 1/4, 1/8, 1/16, 1/16; scissors; plastic bags for storing fraction kit

Resource *About Teaching Mathematics*, by Marilyn Burns

Procedure

Introduction:

1. Tell the students to imagine they are on an expedition with Columbus (or an explorer that you are studying). Columbus must distribute the last loaf of bread to his remaining crew. Tear some "bread" into unequal pieces and distribute to the students (crew). Ask the students if they think the bread was divided fairly. What was wrong with the way it was divided? This question should bring the discussion around to talking about equal shares and fractional parts. In this lesson, they will explore addition and subtraction of fractions
2. Give each student one strip of paper and have them label it "one whole."
3. Have students fold a second strip in half and cut it into two pieces, labeling each piece "1/2" and "one half."
4. Have students continue folding, cutting, and labeling three other strips, one each for fourths, eighths, and sixteenths.
5. Have students work in pairs to play the following fraction game:
   - Use your "whole" strip as a game board and the cube marked with fractions.
   - Partners take turns rolling the cube to determine what size fraction piece to place on their game board.
   - The first person to completely cover the whole strip without going over wins.

Exploration:

- What are some combinations of fractions you have had on your game board that make 1 (a whole)?
- What information do the pieces on your gameboard give you about how much you have and how much you need to win?
- How could you use number sentences to record what has happened in the game?
- Your gameboard shows 1/8 + 1/8 + 1/8 + 1/8 + 1/2 = 1. How could you trade your fraction pieces to make a shorter sentence? (1/2 + 1/2 = 1).
- How could you trade your fraction pieces to have a number sentence with all the same denominators? (1/8 + 1/8 + 1/8 + 1/8 + (1/8 + 1/8 + 1/8 + 1/8) = 1, so 1/2 = 4/8).
- Why would you want a number sentence with all the same denominators?

Extension:

- If each student gets 1/4 loaf of bread, how many loaves will we need to feed our class?
- If every student needs 1/3 quart of milk, how many quarts of milk will the class need? (The fraction kit does not contain 1/3. How will you solve this?)
- Play the fraction game in reverse. Cover the game board (the "whole" strip) with the two half-pieces. Roll the die to tell you what to take off (trades may have to be made). The first person to take off the exact amount of fraction pieces (to remove a total of 1) is the winner.

- Make a fraction kit with paper circles, paper equilateral triangles, interlocking cubes. Describe how the process is alike or different from using the rectangular strips of paper.

Summary:
Ask questions that lead to conclusions such as:
- Fractions can be added and subtracted.
- If the denominators are the same in two fractions, you add or subtract the numerators and keep the same denominator to find the sum or difference.
- It is possible to add fractions that have different denominators by finding equivalent fractions that have the same denominators.
- You can estimate the sum or difference of two fractions if you know their relative values and their values in relation to 1.

Assessment
- Teacher observation of student participation during activities and discussion.
- Students' writings that assess their understanding of fraction concepts.
- Students working in pairs to decide which is larger, 2/3 or 3/4, and explaining their reasoning.

EE: 5B | Related EEs: 1A, 1B, 1D, 1E, 2A, 3D, 3G, 3H, 4B, 6C, 6E | TAAS Objectives: 1, 2, 3, 4, 9, 10, 11, 12, 13

Objective The student will investigate the relationship of the circumference of a circle to its diameter.

Activity $C/d = \pi$

Materials String; chalk; calculators; rulers (customary and metric); measuring tapes (customary and metric); assorted circular objects, such as tops of cylinders, lids, rims of pans and bowls and cups

Resource *Math + Science = Solution* from AIMS Education, 1987.

Procedu
In uction:

lave students find the perimeter of various objects around the room.
)iscuss student findings. Ask if students had difficulty finding the perimeter of any objects or shapes (circles, for instance).
-lave students brainstorm strategies for finding the "perimeter" of a circle. (Use string, roll the object, etc.)
Inform students that the distance around a circle is called the circumference.
Have students brainstorm circular objects (pizza, cookie, plate, etc.). Have students decide what would be the fairest way to divide a circle. For example, what is the fairest way to slice pizza? (Through the center.)

6. Identify the cut through the center of a circle as the diameter.
7. Have students cut a length of string approximately equal to the diameter of a circle found in the room. Ask them how many strings of this length it would take to go around the circumference of the circle. Have students test their predictions and record their results. Have each group measure several circles around the room in this way.
8. Share and discuss student results, emphasizing the consistency there seems to be in the relationship of circumference to diameter (a little more than 3 to 1).
9. With their partners, have students explore the relationship between the circumference and diameter of a circle, $C/d = \pi$. Encourage students to measure using both customary and metric units.
10. Have partners record their information in a chart with headings such as:

object	diameter	circumference	ratio = C/d	decimal value $C \div d$

11. Share and discuss student findings from their charts. Tell students that the ratio they have explored, C/d, actually has a constant value called *pi* that is a little greater than 3. Give its symbol (π).

Exploration:

- What is the perimeter of an object?
- What is circumference? Diameter?
- How many diameters did you predict it would take to go around the circumference of the circle? How did you decide on your prediction?
- Are you seeing any relationship between the diameter and circumference of a circle? Can you describe it?

- If you found a circle that does not seem to fit the pattern, why do you think this happened? (possible errors in measurement)
- Does it take more diameters to go around a larger circle than a smaller circle? Why or why not?
- Does it work with any circle?
- Do you see any differences in the ratio of circumference to diameter when you change the unit of measure?

Extension:
- Introduce three unknown circles: A, B, and C. Give the diameters of A and B and the circumference of C. "Can you find the missing dimensions of the circles?"
- Have students organize and record their data into a table. They may wish to draw a picture or cartoon also.
- Explore the approximations of π (22/7, 3.14, etc.).
- Graph the results from the worksheet.
- Explore the relationship between the perimeter of a square and the length of its diagonal.

Summary:
- What is perimeter? Diameter? Circumference?
- How does the circumference of any circle compare with its diameter?
- Why would you want to know the relationship between the diameter and circumference of a circle?
- Does the size of the circle affect the relationship between the diameter and circumference? Why or why not?
- Why do you think this relationship exists?
- If you know the circumference of a circle, can you find the diameter? How?
- Can you find the circumference if you know the diameter? How?
- What if you put the diameters and the circumferences measured into a graph? What kind of graph would you use? What would it look like?
- Does the unit of measure affect the relationship between the diameter and circumference of a circle?
- What is pi? Can you give a numerical representation of pi?
- How will you determine the circumference of a circle now? Why?
- What do you think is the easiest way to find the circumference of a circle?

Assessment

Questions:
(See summary questions.)

Observations:
- Were students accurately measuring the diameter of a circle? Its circumference?
- Were students making predictions as they worked?
- Were students able to describe the relationship between the circumference and diameter of a circle?
- Did students understand that the size of the circle does not affect the ratio of circumference to diameter? Also that the kind of units used do not affect the ratio?
- Did students see that using this relationship allows them to find the circumference of a circle by only measuring the diameter?

Tasks:
Write what you discovered about diameter and circumference in your journals. This could be done in a paragraph, chart, drawing, cartoon, etc.

EE: 5H *Related EEs: 1A, 1B, 1C, 1D, 1E, 4B, 5C, 5D, 5E, 5F, 7A, 7D* *TAAS Objectives: 1, 4, 5, 11, 12*

Objective The student will develop a method to collect and organize information to determine which brand of paper towel is the best and why.

Activity Custodian's Helper

Materials Three to five different brands of paper towels; water; aluminum pans; trays; glue; scissors; measuring devices, such as balances and masses, measuring cups, beakers, graduated cylinders, eye droppers, cubes, counters, rulers, tape measures, yarn; timing devices, such as clocks, stopwatches, minute timers; materials for recording, such as chart paper, graph paper, construction paper, posterboard, pencils, crayons, and markers

Procedure

Introduction:

1. Set up a supply table with measurement devices and recording materials for students to select from as necessary.
2. Tell students that they have just been chosen to be the custodian's helper in the cafeteria. The job will be to carry a roll of paper towels to wipe up milk that has been spilled. Display the choices of paper towels and ask them which paper towel they would choose.
3. Have students brainstorm (in the large group or in small groups) the characteristics of paper towels they might consider when making a decision about which is the best. (Why might they choose one paper towel over any other?) List on the board or overhead transparency the characteristics they identify; e.g., size, strength, cost, ability to soak up liquid (absorbency).
4. Discuss possible ways of comparing the paper towels according to the characteristics identified. For example, strength could be tested by timing how long it takes a paper towel to tear when two students are pulling on it in different directions. The discussion should include consideration of whether the test really measures what you want it to measure and if there are any other variables involved. For example, how can the students know they are pulling the same amount on each of the paper towels?
5. Let each group of four decide which characteristic(s) they want to use in picking the best paper towel. Each group will design a test for the characteristic(s) chosen, carry out the test, collect data from the results of each of the tests, and organize and record the data in a way that allows them to use the data to make a decision on the best paper towel.
6. Have each group present their data and conclusions to the whole class.

Exploration:

- What jobs do you expect the paper towel to be used for?
- What characteristic(s) of the paper towel do you think would be most important to you?
- How can you tell if the paper towel has that characteristic?
- What measuring tools could you use to help you test the paper towels?
- How do you think the test will turn out? Why?
- What measurements or observations are you going to make?
- How are you going to record these measurements or observations?
- After trying the test once, did you decide to make some changes and try it again? Why?
- If you tested a paper towel more than once, did you get exactly the same results? Why or why not?
- Do you see a pattern in your data?
- How might you arrange your data in a different way?
- How can you present your data to the rest of the class so that it makes sense?

- What conclusions can you make from your data?

Extension:
- Try a tissue, cotton balls, and construction paper and compare your results.
- Select another characteristic to test and compare your results.
- Repeat your tests to see if you get the same results.
- What if you repeated your tests with only one-half sheet of each paper towel? Would the results be the same?
- Design another method to test the same characteristic and compare your results.
- Design a plan to decide which bubble gum is the best to chew or which type of paper is the best to use for drawing.

Summary:
- What characteristic(s) did your group decide to investigate? Why?
- What techniques did your group use to compare paper towels?
- How did the measuring tools help you?
- How did your group decide to represent the data? Why?
- How did your group decide which paper towel was best? Did everyone agree?
- How is your group's data different from the data of another group?
- Are the data from any of the groups alike?
- How are your group's conclusions alike or different from those of other groups?
- What conclusions can you make from the combined data of the whole class? How are these conclusions alike or different from those from your group's data?
- What questions can't be answered from the data?
- What things would you like to know how to do better before doing an activity like this again?

Assessment

Questions:
- Which paper towel would you choose? Why?
- What things were important to consider when designing a way to compare the paper towels?
- How were the ways the groups presented the data alike?
- How were they different?
- If you were buying paper towels to use as napkins in the kitchen, would you choose the same one or a different one? Why?
- (See also summary questions.)

Observations:
- Were students actively participating?
- Were groups working together to solve the problem?
- Were students using the measuring tools appropriately?
- Were students discussing alternative approaches to comparing the towels?
- Did students develop appropriate strategies for solving the problem or were they making random decisions?

Tasks:
- Design an advertisement for your choice of paper towel.
- Draw a picture in your journal of your group of scientists at work.
- Given a product (noodle soup, fruit chews, computer games), make a list of important characteristics for choosing your favorite brand of that product.
- Describe a way to select the best of some kind of product.

Objective The student will explore the concept and process of tessellating or the repeated use of polygons and other closed figures to completely fill a plane without gaps or overlapping.

Activity Tessellations

Materials Overhead projector; pattern blocks; tagboard; pens and markers; tape; overhead pattern blocks; scissors

Resources "The Art of Tessellation" by P. Giganti, Jr. and M. J. Cittadino in *Arithmetic Teacher*, March 1990. Also, books about Islamic art, M. C. Escher, and Leonardo da Vinci; quilts; wallpaper.

Procedure

Introduction:

1. Show designs of M. C. Escher and ask students to discuss what they notice.
2. Ask students if they have seen this type of pattern in their environment (e.g., bricks on a patio, certain wallpapers, tiles in the bathroom and kitchen).
3. Tell students that a tessellation is a tiling made up of the repeated use of simple, closed curves to completely fill a plane without gaps or overlapping, just like the tiles on a kitchen or bathroom floor.
4. Give students several of one shape from the pattern blocks. Have students explore to see how they can arrange their figures to cover a surface completely. Ask them to share arrangements and discuss patterns they see.
5. Ask students to try to find shapes that do not cover the entire surface without gaps or overlapping. Share ideas why these shapes and arrangements do not work.
6. Give each student just one pattern block. Have each student place this polygon in the middle of a blank piece of paper and trace around it. Have students trace their polygons again, lining them up with any side of the traced image. Students should repeat this process until they have covered the entire page. Teachers can also demonstrate their arrangements using pattern blocks on an overhead projector.
7. Let students create their own geometric shapes. Have students trace and cut out 15 congruent shapes. (Could be done as homework.) Students should test their shapes to see if them will tessellate. Make a "will work/won't work" wall chart on which students can paste up examples of figures that will tessellate and ones that will not.
8. See "The 'Nibble' Technique" on p. 9 of the *Arithmetic Teacher* article to explain how to alter polygons and create irregularly shaped figures that also tessellate. Give students many opportunities to try all the techniques of making a shape that tessellates.
9. Students should create a poster-sized tessellation using shapes they like. See "The Art in Tessellations" on pp. 11-12 of the article for lesson suggestions.

Exploration:

- What is a tessellation?
- What is a polygon? What are some polygons that tessellate?
- What do the polygons that tessellate have in common?
- What do you notice about polygons that don't tessellate?
- What is a rotation? A translation? A reflection?
- How are rotations, translations, and reflections related to tessellations?
- How did you go about creating the irregularly shaped polygon?
- What does the shape that you just created look like to you?
- Do all squares tessellate?

- Do all triangles tessellate? Which ones do? Which ones don't?
- Why do you think that some triangles do not tessellate?
- Are you leaving any gaps or overlapping any shapes?

Extension:
- Write a summary of how you can tell if a shape will tessellate or not.
- Make a display of your tessellation for the mathematics/science fair.
- Make a scrapbook of various tessellations found in the world around us. For example, you can include pictures from magazines or snapshots of examples.
- Make a shape similar to one of the states in the U.S. that will tessellate.

Summary:
- What is this activity really about?
- How can you tell if a shape will tessellate or not?
- Why don't some shapes tessellate?
- What do the shapes that tessellate have in common?
- What do the shapes that don't tessellate have in common?
- What is a polygon?
- How can you tell if polygons are congruent?
- Why do tessellations work?
- If you altered a square to make an irregular shape and then halfway through covering your poster with a tessellation you notice that your tessellation isn't working, what could have caused the problem?
- Why do you think someone noticed tessellations?
- How are rotations, translations, and reflections related to tessellations?

Assessment

Questions:
(See summary questions.)

Observations:
- Did students understand the need to conserve the area of shapes they alter for tessellation?
- Did students correctly alter polygons to create tessellations?
- Can students describe some attributes that make it possible for a shape to tessellate?
- Can students define tessellation?
- Can students successfully manipulate shapes to create tessellations?
- Did students appreciate the artistic quality of tessellations?

Tasks:
- Create a poster-sized tessellation.
- Write what you have discovered about tessellations.

EEs: 7A, 7H *Related EEs: 1A, 1B, 1C, 1D, 1E, 1F , 4B, 4C* *TAAS Objectives: 1, 5, 6, 10, 11, 12, 13*

Objective The student will explore the frequency with which the letters of the alphabet are used in the English language.

Activity Alphabet Frequency

Materials Craft paper; markers; calculators; assorted books and magazines; a list of the alphabet, in order of usage, written on adding machine tape and rolled up

Resource *A Collection of Math Lessons From Grades 3 Through 6* by Marilyn Burns.

Procedure

Introduction:

ACTIVITY 1

1. Ask students to write individually their predictions of the five letters used most frequently in the English language.
2. Students should then discuss individual predictions in their groups and prepare a group prediction.
3. Brainstorm situations when it is important to know how frequently letters are used. (Scrabble, Wheel of Fortune, packages of punch-out or rub-off letters, cryptograms, and keyboard design.) Investigate the Dvorak keyboard as opposed to the traditional keyboard designed by Christopher Sholes.
4. Show students the rolled up list of the alphabet in order of usage and tell them you already know which letters the experts say are used most often. The class will conduct its own investigation by taking statistical samples. Students will then compare their results to the predictions and to the experts results.
5. Ask students to take an individual statistical sample by picking a sentence with at least five words out of a book or magazine and writing it on a sheet of paper.
6. Each student will then record how many times each letter of the alphabet appears in the sentence.
7. Tell students that they will use this information to create a larger statistical sample the next day.

ACTIVITY 2

1. Review the predictions made in Activity 1 and why letter frequency is important.
2. Ask students to take out the statistical sample they made individually during Activity 1.
3. Ask students to use a calculator and to compile their individual samples into group totals. Tell them to enter each group's totals into a class chart. (Create this chart of craft paper.)

	a	b	c	d	e	f . . . z
Group 1						
Group 2						
.						
.						
.						
Group n						
Totals						

Use three different colors of markers (for instance red, green, and black), alternating colors (red, green, black, red, green, black, etc.) for the letters in the headings of the chart. Ask students to record their totals in the same color that the letter it pertains to is written in. (This makes the chart much easier to read from a distance.)

4. To avoid traffic jams, ask half of the groups to start at "a" and work toward "z" and half of the groups to start at "z" and work toward "a."
5. As groups finish, assign letters to them so they can find class totals. Discuss the need for more than one person to do the totals to make sure they are accurate.
6. After all the totals have been calculated and entered, have students use the chart to identify the letters in order of frequency and record the order on adding machine tape.
7. Unroll the list compiled by the experts and compare it to the lists compiled by the students. Discuss the similarities and differences in the lists.

Exploration:

ACTIVITY 1

- What five letters do you think are used most frequently?
- What influenced your prediction?
- Why do you think those are the five most frequently used letters?
- How did your group go about arriving at a group prediction?
- Are you happy with your group's predictions? Why or why not?
- When would it be important to know how frequently letters are used?
- Why did you choose the sentence that you did?
- Does your sentence have at least five words in it?
- Are you recording how many times each letter appears in your sentence?
- What strategy are you using to record how many times each letter appears in your sentence?

ACTIVITY 2

- What five letters did your group predict are used most frequently?
- When would it be important to know how frequently letters are used?
- How did you go about recording your individual statistical sample?
- Did your individual sample come out the way you expected?
- How is your group going to come up with your group totals? How did you oganize yourselves?
- How do you know if your group has come up with the correct totals for your group?
- Are you recording your totals on the chart in the correct color?
- Which end of the chart did your group start from?
- Do your group totals seem to be consistent with your individual samples?
- How do you know which letter occurs most frequently?
- How should we indicate letters that occurred equally frequently?

- Do the class totals seem consistent with your individual results? Your group results?

Extension:

- Discuss what is unusual about these four sentences:

  This is odd. Do you know why? Try to find out. Do you know how?

- Play the "You Can't Say *N*" game. One person starts to talk, trying not to say any word with the letter *n* in it. Others in the group listen. When the person talking uses a word with an *n*, someone else tries.

Summary:

ACTIVITY 1

- When would you need to know how frequently letters are used?
- Can you think of another instance in which someone would need to know how frequently different outcomes occur?
- Why were you asked to pick a sentence with at least five words?
- What is your statistical sample in this experiment?
- How did you record your individual data?
- Why did you choose to record your data this way?
- Tell me why you would or would not use this method of recording data again.
- What could have changed the outcome of your data?
- Did your data come out as you expected? Why or why not?
- What is the purpose of this activity?

ACTIVITY 2

- When would you need to know how frequently letters are used?
- Can you think of another instance in which someone would need to know how frequently different outcomes occur?
- Why did we combine your individual results into group totals?
- Why did we combine your group totals into class totals?
- What is a statistical sample?
- What operation did you perform to arrive at the totals?
- Was your group's results consistent with your individual results?
- Was your group's results consistent with the class results?
- Do the final results surprise you? Why or why not?
- How accurate were your predictions? How do you know?
- Why do you think the letters that occurred most frequently are used most often?
- What could we have done that would have significantly changed our class results?
- How could we have improved the accuracy of our results?
- Are you satisfied with the results that we came up with? Why or why not?
- For what other situations could you use the technique we used in this experiment?

Assessment

Questions:

- How do you think your group worked together?
- How could your group have worked together more effectively?
- What will you remember about this activity? What will help you remember it?
- (See summary questions.)

Observations:

- Did students accurately record the number of times each letter occurred?
- Did students organize data effectively?
- Could students discuss the purpose of this activity?

- Could students discuss other uses for this technique?
- Did students recognize that enlarging the statistical sample improved the accuracy of the data?

Tasks:
- Use the results of this experiment and design a new keyboard.
- Compare your keyboard to the Dvorak and/or Sholes keyboard.
- Investigate the game of Scrabble. List the letters in the game two ways—in order of their values and in order of how many there are of each. See how Scrabble relates to our findings. Describe your thoughts as to why you do or do not think the Scrabble scoring or the quantity of any letter should be changed.
- Do a statistical sample for another language and compare the results with our findings for English.
- Use the data from our experiment to solve a cryptogram.

EEs: 7A, 7B, 7C, 7D, 7H | *Related EEs: 1A, 1B, 1C, 1D, 1E, 1F, 3F, 3G, 4B* | *TAAS Objectives: 1, 5, 6, 9, 10, 11, 12*

Objective The student will collect, organize, and interpret data to solve application probiems; explain the decisions that need to be made before constructing a graph; recognize measures of central tendency as ways of summarizing a set of data; use averaging in problem-solving situations; and make and refine predictions based on exploration of different sample sizes within experiments.

Activity How Average Are You?

Materials Large newsprint, construction paper, crayons/markers, scissors, glue, yardsticks/metersticks(for teachers)

Resources *In One Day* by Tom Parker

Procedure

Introduction:

1. To begin, discuss with the whole class the purpose of gathering information through surveys. "What if I owned a sporting goods store and we wanted to find out how many people in the United States were left- or right-handed so I could purchase baseball gloves to sell. How could we go about it? Since it would be impractical to ask every person in the country which hand he or she prefers using, how could we do it? How many people would we have to ask before we were able to make a fairly accurate prediction concerning handedness in the United States?" Introduce the idea of sample size and how it affects the accuracy of information we gather and the decisions we make as a result.
2. Brainstorm a list of information students would like to know about other students their age. Examples might include: color of hair, number of brothers/sisters, favorite color, favorite rock group, birth month, favorite night to watch television, bedtime on school nights, the way they get to and from school, favorite the way to spend leisure time, favorite soft drink.
3. "If we wanted to find out what the average fifth grader is like, how many fifth graders would we have to sample? Do you think our class is representative of all the fifth graders in our town? In the United States? In the world? Why or why not? Since it is impractical to survey every fifth grader in our town, how many fifth graders should we sample to get a pretty good idea what the average fifth grader is like?"
4. Select about 10 to 15 questions from the brainstormed list to use to develop a survey. Distribute the survey and collect information from the sample suggested by the class in the discussion above.
5. By cutting the survey into strips with one question and response on each strip, have students assemble data from the surveys. Have each group select two to three questions from the survey. The groups will design an original graph to represent the data from each of their survey questions. One graph depicting the data from each of the questions on the survey will be constructed and posted in the classroom or out in the hall.

Exploration:

While the groups are designing and producing their graphs, ask the following questions:

- How did you decide what kind of graphs to make?
- How did your group decide to organize your data?
- What organizational strategies are you using?
- Are you satisfied with your decisions? Why?
- Does everyone in your group understand your decision-making process? If I asked you to, could you explain your decision to someone from another group?

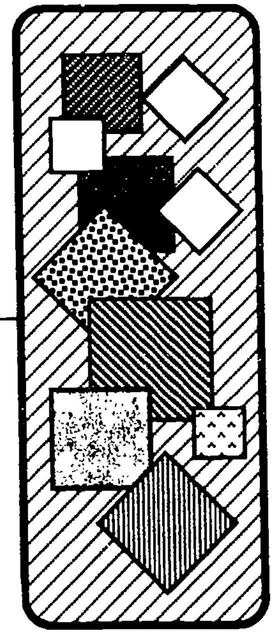
- How could you use the information on your graph?

Extension:
- Use appropriate data analysis software to generate graphs on a computer.
- How do you think your results would change if you sampled fifth graders from another school? Another town? Make a prediction, collect more data, and compare your results.
- How would your results change if you sampled students from a lower grade level? Students from middle school?
- How would your description of the average student change in each of the above situations?

Summary:
- How did you organize as a group?
- How did you decide what kind of graph to make to display your information? What other decisions did you have to make?
- Are you satisfied with your decisions or would you change them if you had to repeat this assignment? Why?
- Tell me some things you notice about the graphs we have posted?
- As a group, select three of the graphs. Write one question that can be answered from each graph. Write one question that cannot be answered from each graph. Write one true statement about each of the three graphs you have selected. Post near the graphs.
- Suppose we had to use the information from our graphs to describe the average fifth grader. What measure of central tendency (mean, median, or mode) should we use to determine the average fifth grader?
- Use information from our graphs to write a paragraph describing the average fifth grader.
- How do you compare with the average fifth grader you described in your paragraph?
- What does the word *average* mean?

Assessment
- Teacher observation of student participation during exploration and discussions.
- Group graphs evaluated for completeness and appropriateness of organization.
- Group explanations of the decisions students made while designing their graphs.
- Questions and truth statements written by groups evaluated for completeness and evidence of mathematical reasoning.
- Paragraph describing the average fifth grader assessed for understanding of measures of central tendency and their implications in this situation.

Evaluation



Philosophy

NCTM's *Professional Standards for Teaching Mathematics* and *Curriculum and Evaluation Standards for School Mathematics* (the *Standards*) emphasize the connection between assessment of students and analysis of instruction. In other words, mathematics teachers should monitor students' learning (both formatively and summatively) in order to assess and adjust teaching. Teachers must observe and listen in order to tailor teaching strategies. Information about what students are understanding should be used to revise and adapt short- and long-range plans, and students' understandings should guide teachers in shaping the learning environment. Also, teachers are responsible for describing students' learning to administrators, parents, and students themselves.

Students' mathematical power depends on various understandings, skills, and dispositions. The development of students' abilities to reason mathematically—to conjecture, justify, and revise based on evidence and to analyze and solve problems—must be assessed. A student's disposition toward mathematics (confidence, interest, perseverance, etc.) is also a key dimension that teachers should monitor.

The importance of using assessment to improve instruction is crucial. Information should be gathered from multiple sources using numerous assessment techniques and modes that are aligned with the curriculum. Assessment techniques must reflect the diversity of instructional methods implied in the *Standards* and the various ways students learn and process information. Instructional decisions should be based on this convergence of information from different sources.

In summary, the following aspects of students assessment and program evaluation should receive increased and decreased attention (NCTM, 1989):

Increased Attention	Decreased Attention
• Assessing what students know and how they think about mathematics	• Assessing what students do not know
• Having assessment be an integral part of teaching	• Having assessment be simply counting correct answers on tests for the sole purpose of assigning grades
• Focusing on a broad range of mathematical tasks and taking a holistic view of mathematics	• Focusing on a large number of specific and isolated skills organized by a content-behavior matrix
• Developing problem situations that require the applications of a number of mathematical ideas	• Using exercises or word problems requiring only one or two skills
• Using multiple assessment techniques, including written, oral, and demonstration formats	• Using only written tests
• Using calculators, computers, and manipulatives in assessment	• Excluding calculators, computers, and manipulatives from the assessment process
• Evaluating the program by systematicallycollecting information on outcomes, curriculum, and instruction	• Evaluating the program only on the basis of test scores
• Using standardized achievement tests as only one of many indicators of program outcomes	• Using standardized achievement tests as the only indicator of program outcomes

Types of Evaluation

While paper and pencil tests are one useful medium for judging aspects of students' mathematical knowledge, teachers need information gathered in a variety of ways and using a range of sources. Observing, interviewing, and closely watching and listening to students are all important means of assessment. While monitoring students, teachers can evaluate the learning environment, tasks, and discourse that have been taking place. Using a variety of strategies, teachers should assess students' capacities and inclinations to analyze situations, frame and solve problems, and make sense of concepts and procedures. Such information should be used to assess how students are doing, as well as how well the tasks, discourse, and environment are fostering students' mathematical power and then to adapt instruction in response.

Assessment instruments and techniques should be properly aligned with the curriculum to enable educators to draw conclusions about instructional needs, progress in achieving the goals of the curriculum, and the effectiveness of a mathematics program. That is, the content, processes, and skills assessed must reflect the goals, objectives, and breadth of topics specified in the curriculum. The particular emphases of the assessment should reflect the emphases of instruction. For example, primary children, whose understanding of fractions is closely tied to the use of physical materials, should be encouraged to use such materials to demonstarte their conceptual knowledge. Assessment items need to be structured around the central ideas of the curriculum and need to provide opportunities for students to demonstrate their understanding of the connections among major concepts. In addition, assessment must reflect the relative emphasis placed on technology during instruction; to the extent that calculators and computers have been important during instruction, they should also be available during assessment.

Assessment techniques suggested in the *Standards* include multiple-choice, short-answer, discussion, and open-ended questions; interviews; homework; projects; journals; essays; portfolios; presentations; and dramatizations. Assessment can occur during and after whole-group explorations, during whole-group discussions, in sharing sessions, during individual conferences, during small-group conferences, while students are working on projects, after completion of projects, when students are engaged in self-evaluation tasks, and continually while students are explaining, justifying, debating, and questioning ideas and concepts.

Using Portfolios. Student portfolios are becoming more prevalent as a means of keeping a record of student progress in mathematics. Teachers have always kept folders of students' work, but portfolios should have more focus and be more important for assessment. An assessment portfolio is a planned selection of a student's work collected throughout the school year. Teachers as well as students should be allowed to choose the items to be included in portfolios, since it gives a good indication of what is valued in the work the students do throughout the school year. A portfolio might include samples of student-produced written descriptions of the results of practical or mathematical investigations; pictures and dictated reports from younger students; extended analyses of problem situations and investigations; descriptions and diagrams of problem-solving processes; statistical and graphic representations; responses to open-ended questions or homework problems; group reports and photographs of student projects; copies of awards or prizes; video, audio, and computer-generated examples of student work; and other material based on project ideas developed with colleagues.

Using Writing. Communication in mathematics has become important as we move into an era of a thinking curriculum. Journals, logs, problem-solving notebooks, explanations, justifications, and reflections are ways to include writing in the matheamtics curriculum. Sutdents should be urged to discuss ideas with each other, and to ask questions, to diagram and graph problem situations for clarity. Writing in mathematics classes, once rare, is now vital. In particular, mathematics journals can include the following:

- vocabulary definitions written in the student's own words along with explanations of how the terms are used in mathematics

- rules or procedures written as if explained to a friend in a letter or to another student who was absent during the instruction

- free writing, including what students think they will learn in an experience, descriptions of accomplishments, how students can use what they've learned, what isn't fully understood or is causing difficulty, examples in the real world related to the mathematics learned, a discovery made or additional ideas and conjectures related to the topic, and what else students might want to learn about.

These writing experiences are also important matheamtics learning experiences in that they:

- help students become more active in their own learning
- help students internalize what they are learning to make it more meaningful
- allow students to express their feelings and attitudes toward mathematics
- give students a source they can use for studying
- allow students to reflect upon and clarify their own thinking
- give students the opportunity to share with each other what they are learning, also allowing them to learn from one another
- allow students to go beyond what they are learning in class and to make conjectures and connections
- give students the opportunity to think of mathematics as existing outside the classroom
- give students the opportunity to communicate with the teacher in an informal setting
- give the teacher an idea of how students are thinking
- allow the teacher to informally assess student learning (whether it be pre- or post-assessment)

Using Teacher Observations. Teacher observations can be broken down into two levels: Formal and informal. Formal observations include checklists, comment cards, and summaries. Informal observations include mental notes. Students should be observed both individually and as they work in groups. When using observations, a teacher should look for students' learning styles, students' ideas, communication techniques, cooperation strategies, and use of manipulatives. Some possible questions that can guide observations of students doing matheamtcs are:

- Does the student consistently work alone or with others?
- Does the student try to explain organizational and mathematical ideas?
- Does the student synthesize and summarize his or her own or a group's thinking?

- Individually or within the group, does the student choose and use appropriate manipulatives?

Using Questioning. Asking the right question is an art to be cultivated by all educators. Low-level quizzes that ask for recall or simple computations are over used and over done. Using good, high-level open-ended questions that give students a chance to think are one of the goals of mathematics assessment. These questions might be used as teaching or leading questions as well as for assessment purposes. Both questions and responses may be oral, written, or demonstrated by actions taken. When using oral questions, the teacher can prepare a list of possible questions ahead of time. (For examples, see the sample activities in the previous section.) The teacher should allow for plenty of wait time. The teacher may keep a written record of observations during the questioning time to use for formal assessment. Questioning for assessment should occur in several places during instruction:

- during introductory activities to assess students' prior knowledge and experience
- during exploration to focus students' attention on important concepts and connections
- after instruction, in order for students to summarize results, reflect on their experience, and clarify their thoughts

Using Student Presentations. Student presentations can take many forms, including oral explanations, oral presentations, and projects. One of the best ways to assure the connection between instruction and assessment is to embed assessment into instruction. When students become involved in projects or investigations, assessment becomes natural and invisible. Student presentations may be related to connections within mathematics and connections outside mathematics. When evaluating student presentations, the teacher should look for whether the student can identify and define the problem; make a plan; collect needed information; organize the information and look for patterns; discuss, review, revise, and explain results; and produce a quality product or report.

Using Performance Assessment. Performance assessment involves giving a group of students, or an individual student, a mathematical task that may take from half an hour to several days to complete or solve. The object of this form of assessment is to look at how students are working, as well as at the completed tasks or products. Performance assessment requires the teacher to look at how students solve a problem. Performance activities may be videotaped, tape recorded, or recorded in writing. The task might be from any mathematical content area and might include some connections such as with science, social studies, language arts, or fine arts. Performance assessment is an excellent place for students to use manipulatives to demonstrate understanding of mathematics content. Information from performance assessment can be recorded using rubrics that assign point values to important aspects of the problem-solving process. For example, the following assessment criteria could be used during observation or based on written work to judge a student's involvement in problem solving:

1. Understanding the Problem:

0 points	Does not understand the problem
1 point	Misunderstands part of the problem
2 points	Completely understands the problem

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2. Choosing and Implementing a Solution Strategy

0 points	Makes no attempt or uses a totally inappropriate strategy
1 point	Chooses a partly correct strategy based on interpreting part of the problem incorrectly
2 points	Chooses a strategy that could lead to a correct solution if used without error

3. Getting the Answer

0 points	Gets no answer or a wrong answer basd on an inappropriate solution strategy
1 point	Makes a copying error or computational error; gets partial answer for a problem with multiple answers; or labels answer incorrectly
2 points	Gets correct solution

Some excellent resources on assessment, in addition to the NCTM *Curriculum and Evaluation Standards*, include *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions* (1991) and *Assessment alternatives in Mathematics* (Stenmark, 1989).

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Parker, T. (1984). *In one day.* Boston, MA: Houghton Mifflin.

Pollard, J. (1985). *Building toothpick bridges.* Palo Alto, CA: Dale Seymour Publications.

Popping with power. (1987). Fresno, CA: AIMS Education Foundation.

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Wahl, J. and Wahl, S. (1976). *I can count the petals of a flower.* Reston, VA: National Council of Teachers of Mathematics.

Children's Trade Books

Numerous children's books have the potential for motivating rich mathematics activities. This short list gives the bibliographic information of the books mentioned in the activities in this series of curriculum guides for elementary mathematics (Grades 1 - 5).

Anno, M. (1982). *Anno's counting house.* New York: Philomel Books.

Anno, M. (1983). *Anno's mysterious multiplying jar.* New York: Philomel Books.

Aardema, V. (1976). *Why mosquitoes buzz in people's ears.* New York: Dial Books for Young Readers.

Bemelmans, L. (1960). *Madeline.* New York: Viking Children's Books.

Bishop, C. H. (1938). *Five Chinese brothers.* New York: Coward.

Brier, C. (1983). *The shoemaker and the elves.* New York: Lothrop, Lee & Shepard.

Carle, E. (1977). *The grouchy ladybug.* New York: Scholastic, Inc.

Carle, E. (1989). *The very hungry caterpillar.* New York: Putman.

Dahl, R. (1964). *Charlie and the chocolate factory.* New York: Alfred Knopf.

de Paola, T. (1978). *The popcorn book.* New York: Holiday House.

Ehlert, L. (1990). *Fish eyes: A book you can count on.* San Diego: Harcourt, Brace, Jovanovich.

Faucher, E. (1989). *Honey, I shrunk the kids.* New York: Scholastic, Inc.

Flournoy, V. (1985). *The patchwork quilt.* New York: Dial Books for Young Readers.

Freeman, D. (1968). *Corduroy.* New York: Viking Press.

Giganti, Jr., P. (1992). *Each orange had 8 slices.* New York: The Trumpet Club.

Haley, G. E. (1971). *Noah's ark.* New York: Atheneum.

Hogrigan, N. (1966). *Always room for one more.* New York: Holt.

Hulme, J. N. (1991). *Sea squares.* New York: Hyperion Books.

Hutchins, P. (1987). *The doorbell rang.* New York: Scholastic, Inc

Lobel, A. (1983). *Frog and toad are friends.* New York: Harper.

Mathews, L. (1978). *Bunches and bunches of bunnies.* New York: Scholastic, Inc.

McMillan, B. (1991). *Eating fractions.* New York: Scholastic, Inc.

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Price, L. (1990). *Aida.* New York: Harcourt, Brace, Javonovich.

Sendak, M. (1962). *Chicken soup with rice.* New York: Harper and Row.

Silverstein, S. (1981). *A light in the attic.* New York: Harper and Row.

Silverstein, S. (1974). *Where the sidewalk ends.* New York: Harper and Row.

Singer, M. (1991). *Nine o'clock lullaby.* New York: HarperTrophy.

Steptoe, J. (1980). *Daddy is a monster . . . sometimes.* New York: HarperTrophy.

Ward, C. (1988). *Cookie's week.* New York: Putnam.

West, C. (1987). *Ten little crocodiles.* New York: Barron's.

Yolen, J. (1987). *Owl moon.* New York: Philomel Books.

Software

The following list contains bibliographic information for the software packages mentioned in this series of curriculum guides for elementary mathematics (Grades 1 - 5). Other appropriate software may be obtained from these and other companies.

Blockers and Finders from WINGS for learning/Sunburst Communications, 1600 Green Hills Road, P.O. Box 660002, Scotts Valley, CA 95067-0002.

Geometric preSupposer from WINGS for learning/Sunburst Communications, 1600 Green Hills Road, P.O. Box 660002, Scotts Valley, CA 95067-0002.

Hands-On Math: Volumes 1, 2, and 3 from Ventura Educational Systems, 3440 Brokenhill Street, Newbury Park, CA 91320.

Suggested Manipulatives

The following is a list of the manipulative materials used in the activities in this series of curriculum guides for elementary mathematics (Grades 1 - 5):

Calculators
Base ten blocks
Coins and bills (play or real money)
Interlocking cubes
Colored tiles
Pattern blocks
Cuisenaire rods

Graphing floor mat
Polyhedral dice (including the regular cube)
Colored chips
Two-color counters

Attribute blocks
Geoblocks
Geoboards
Tangrams
Plastic mirrors
Wooden or plastic models of geometric solids

Balance scales and masses (customary and metric)
Spring scales
Tape measures (customary and metric)
Rulers (customary and metric)
Meter sticks and yardsticks
Trundle wheels
Graduated cylinders
Measuring cups and spoons
Stopwatches

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