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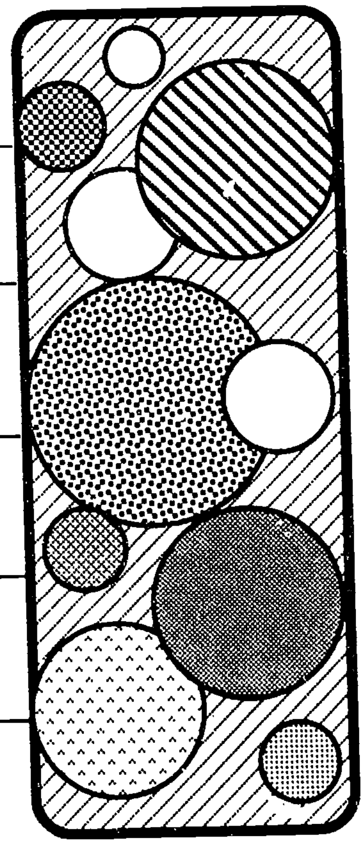
ABSTRACT

This document is designed to assist teachers and other school personnel in the planning and teaching of the second grade mathematics course. Contents include: (1) Overview of Grade 2 Mathematics (mission statement, purpose and philosophy, goals, National Council of Teachers of Mathematics' Professional Standards for Teaching Mathematics, instructional strategies, and uses of technology and manipulatives); (2) Essential Elements of Instruction with sample learning objectives and sample clarifying activities; (3) Texas Assessment of Academic Skills (TAAS) (focus, domains, objectives, and targets); (4) Sample Lessons for Teaching Grade 2 Mathematics; and (5) Evaluation (philosophy and types of evaluation). TAAS features three domains: concepts, operations, and problem solving. The Essential Elements are: problem solving; patterns, relations, and functions; number and numeration concepts; operations and computation; measurement; geometry; and probability, statistics, and graphing. Suggested resources include children's trade books, software, and suggested manipulatives. Contains 22 references. (MKR)

GUIDELINES FOR TEACHING GRADE 2 MATHEMATICS



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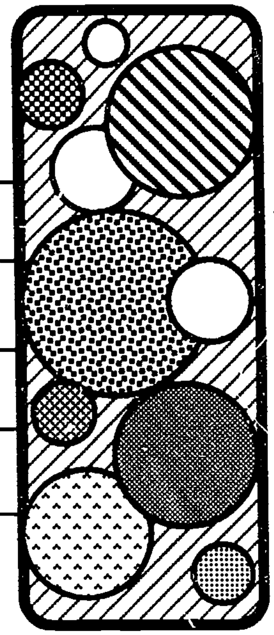

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FOREWORD

Guidelines for Teaching Grade 2 Mathematics is designed to help teachers and other school district personnel plan and teach second grade mathematics. The publication presents the philosophy and intent of the course and discusses the required essential elements, TAAS instructional targets, instructional strategies, and the use of technology and manipulatives. Also included are sample objectives and activities to illustrate how the essential elements for second grade mathematics can be taught. School district personnel may want to use these suggestions to develop their own curriculum documents for the course.

We hope these guidelines will be useful in planning and teaching mathematics in Grade 2 and in equipping the mathematics classroom.

Lionel R. Meno
Commissioner of Education

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CONTENTS

Overview of Grade 2 Mathematics

Mission Statement

Guidelines for Teaching Grade 2 Mathematics is one in a series of eight documents for the first through the eighth grades designed to assist teachers and other school personnel in the planning and teaching of elementary mathematics. The discussions of philosophy, goals, instructional strategies, uses of technology and manipulatives, and aspects of evaluation are provided as starting points for districts to begin the process of developing their own curriculum documents. The essential elements of instruction for each grade level are supported with sample learning objectives, sample clarifying activities, and complete sample lessons. These guidelines should prove useful to district personnel in: (1) planning curriculum, (2) planning instruction, and (3) equipping classrooms for mathematics teaching and learning.

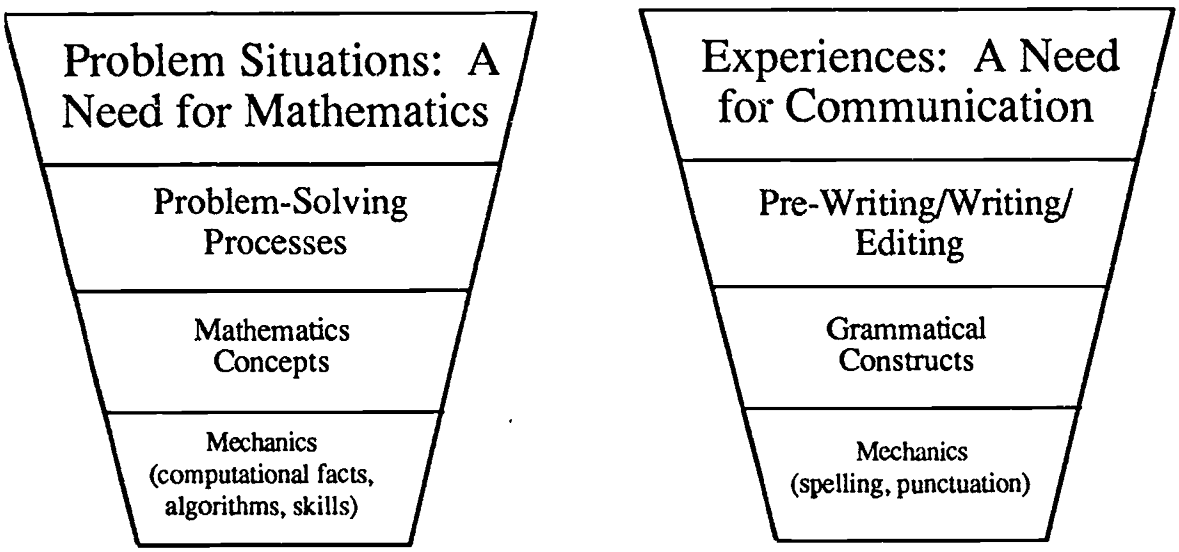
Purpose and Philosophy

Mathematics is useful, exciting, and creative and can be enjoyed by all elementary school students. Problem-solving skills and logical reasoning are developed while students explore and make sense of their world through rich, worthwhile mathematical experiences. Unfortunately, mathematics has been viewed by many students as boring, irrelevant, and routine and as externally dictated by a rigid system of rules governed by standards of speed, accuracy, and memory. In the past, computational facility has been emphasized instead of a broad, integrated view of mathematics. While computational skills are important, learner characteristics and the vitality of mathematics itself cannot be overlooked. Mathematics in the elementary grades should be broad-based and concept driven and should reflect relevant mathematics and connections between mathematics concepts and between these concepts and other disciplines.

Children enter elementary school with a natural curiosity and enthusiasm for learning. Mathematics experiences at the elementary level should tap into these characteristics for children to begin developing mathematical power—the ability to think and communicate, drawing on mathematical ideas and using mathematical tools and techniques. The attitudes students form in elementary school toward mathematics will determine the choices they make of future mathematics coursework and consequently the availability or loss of educational and career opportunities.

The elementary school mathematics curriculum should emphasize the processes of problem solving, reasoning, communication, and making connections within the contexts of investigating geometry, measurement, probability, statistics, graphing, patterns, and functions, as well as with number, numeration, and operation concepts. Problem solving should be the focus of instruction with skills and concepts being introduced, developed, and applied through meaningful problem situations. Mathematics instruction needs to begin with meaning and purpose in much the same way as elementary teachers present language arts instruction, as reflected in the following graphic illustration:

MATHEMATICS　　　　LANGUAGE ARTS



All students need rich and relevant problem-solving experiences with appropriate teacher guidance and questioning. Such experiences will empower students to build meaning for the mathematics they encounter today and to strengthen reasoning skills needed for the mathematics of tomorrow.

Goals

According to *Curriculum and Evaluation Standards for School Mathematics* developed by the National Council fo Teachers of Mathematics (NCTM), the five overall curriculum goals for students are:

- learning to value mathematics
- becoming confident in their ability
- becoming mathematical problem solvers
- learning to communicate mathematically

- learning to reason mathematically

Moreover, the educational system of today demands new societal goals for education:

- mathematically literate workers
- lifelong learning
- opportunity for all
- an informed electorate

Specifically, teaching the mathematics curriculum to elementary school students must be related to the characteristics of the learners and their needs today and in the future.

Everybody Counts (National Research Council, 1989) posits that "self-confidence built on success is the most important objective of the mathematics curriculum" (p. 45). Individuals must be able to use mathematics in their later lives—as employees, parents, and citizens. Ability and disposition to do so often depends on attitudes toward mathematics developed in school.

Through the use of worthwhile mathematical activities investigated in cooperative, group environments, teachers of elementary mathematics can empower their students with strong mathematical understanding and disposition.

National Council of Teachers of Mathematics: Professional Standards for Teaching Mathematics

The *Professional Standards for Teaching Mathematics (NCTM, 1991)* are based on four assumptions about the practice of teaching. These assumptions are abbreviated versions of the more extensive ones found in the original document (NCTM, 1991, pages 21-22).

(1) The goal of teaching mathematics is to help all students develop math ıatical power. Teachers must help every student develop conceptual and procedural understandings of number, operations, geometry, measurement, statistics, probability, functions, and algebra and the connections among ideas. They must engage all students in formulating and solving a wide variety of problems, making conjectures and constructing arguments, validating solutions, and evaluating the reasonableness of mathematical claims.

(2) What students learn is fundamentally connected with how they learn it. Students' opportunities to learn mathematics are a function of the setting and the kinds of tasks and discourse in which they participate.

(3) All students can learn to think mathematically. The goals such as learning to make conjectures, to argue about mathematics using mathematical evidence, to formulate and solve problems, and to make sense of mathematical ideas are not just for some group thought to be "bright" or "mathematically able."

(4) Teaching is a complex practice and hence not reducible to recipes or prescriptions. First of all, teaching mathematics draws on knowledge from several domains: knowledge of mathematics, of diverse learners, of how students learn mathematics, of the contexts of the classroom, school, and society. Good teaching depends on a host of considerations and understandings. Good teaching demands that teachers reason about pedagogy in professionally defensible ways within particular contexts of their own work.

The *Professional Standards for Teaching Mathematics* identifies a particular set of instructional standards for the effective teaching of mathematics. The standards describe the nature of the tasks, patterns of communication, the learning environment, and the analysis of instruction. More specifically, five of these standards focus on instructional strategies. They are:

STANDARD 1: WORTHWHILE MATHEMATICAL TASKS

The teacher of mathematics should pose tasks that are based on:

- sound and significant mathematics;
- knowledge of students' understandings, interests, and experiences;
- knowledge of the range of ways that diverse students learn mathematics;

and that

- engage students' interests;
- develop students' mathematical understandings and skills;
- stimulate students to make connections and develop a coherent framework for mathematical ideas;
- call for problem formulation, problem solving, and mathematical reasoning;

- promote communication about mathematics;
- represent mathematics as an ongoing human activity;
- display sensitivity to, and draw on, students' diverse background experiences and dispositions;
- promote the development of all students' dispositions to do mathematics.

STANDARD 2: THE TEACHER'S ROLE IN DISCOURSE

The teacher of mathematics should orchestrate discourse by:

- posing questions and tasks that elicit, engage, and challenge each student's thinking ability;
- listening carefully to students' ideas;
- asking students to clarify and justify their ideas orally and in writing;
- deciding what to pursue in depth from among the ideas that students bring up during a discussion;
- deciding when and how to attach mathematical notation and language to students' ideas;
- deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;
- monitoring students' participation in discussions and deciding when and how to encourage each student to participate.

STANDARD 3: STUDENTS' ROLE IN DISCOURSE

The teacher of mathematics should promote classroom discourse in which students:

- listen to, respond to, and question the teacher and one another;
- use a variety of tools to reason, make connections, solve problems, and communicate;
- initiate problems and questions;
- make conjectures and present solutions;
- explore examples and counterexamples to investigate a conjecture;
- try to convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers;
- rely on mathematical evidence and argument to determine validity.

STANDARD 4: TOOLS FOR ENHANCING DISCOURSE

The teacher of mathematics in order to enhance discourse, should encourage and accept the use of:

- computers, calculators, and other technology;
- concrete materials used as models;
- pictures, diagrams, tables, and graphs;
- invented and conventional terms and symbols;
- metaphors, analogies, and stories;
- written hypotheses, explanations, and arguments;
- oral presentations and dramatizations.

STANDARD 5: LEARNING ENVIRONMENT

The teacher of mathematics should create a learning environment that fosters the development of each student's mathematical power by:

- providing and structuring the time necessary to explore sound mathematics and grapple with significant ideas and problems;

- using the physical space and materials in ways that facilitate students' learning of mathematics;
- providing a context that encourages the development of mathematical skill and proficiency;
- respecting and valuing students' ideas, ways of thinking, and mathematical dispositions;

and by consistently expecting and encouraging students to:

- work independently or collaboratively to make sense of mathematics;
- take intellectual risks by raising questions and formulating conjectures;
- display a sense of mathematical competence by validating and supporting ideas with mathematical argument.

STANDARD 6: ANALYSIS OF TEACHING AND LEARNING

The teacher of mathematics should engage in ongoing analysis of teaching and learning by:

- observing, listening to, and gathering other information about students to assess what they are learning;
- examining effects of the tasks, discourse, and learning environment on students' mathematical knowledge, skills, and dispositions;

in order to:

- ensure that every student is learning sound and significant mathematics and is developing a positive disposition toward mathematics;
- challenge and extend students' ideas;
- adapt or change activities while teaching;
- make plans both short- and long-range;
- describe and comment on each student's learning to parents and administrators, as well as to the students themselves.

The movement toward this vision of instruction for mathematical empowerment of all students is strongly dependent upon the environment of the classroom, an environment governed in a large part by the decision-making role of the classroom teacher. The NCTM teaching standards identify five major components necessary in the instructional environment for the mathematics classroom and tie these components directly to teachers asking, and encouraging students to ask, appropriate and stimulating questions. The five major instructional components and suggestions for questions are (NCTM, 1991, pp. 3-4):

- **Helping students work together to make sense of mathematics**
  "What do others think about what Janine said?"
  "Do you agree? Disagree?"
  "Does anyone have the same answer but a different way to explain it?"
  "Would you ask the rest of the class that question?"
  "Do you understand what they are saying?"
  "Can you convince the rest of us that that makes sense?"

- **Helping students to rely more on themselves to determine whether something is mathematically correct**
  "Why do you think that?"
  "Why is that true?"

"How did you reach that conclusion?"
"Does that make sense?"
"Can you make a model to show that?"

- **Helping students learn to reason mathematically**
  "Does that always work?"
  "Is that true for all cases?"
  "Can you think of a counterexample?"
  "How could you prove that?"
  "What assumptions are you making?"

- **Helping students learn to conjecture, invent, and solve problems**
  "What would happen if . . .." What if not?"
  "Do you see a pattern?"
  "What are some possibilities here?"
  "Can you predict the next one? What about the last one?"
  "How did you think about the problem?
  "What decision do you think he should make?"
  "What is alike and what is different about your method of solution and hers?"

- **Helping students to connect mathematics, its ideas, and its applications**
  "How does this relate to . . .?"
  "What ideas that we have learned before were useful in solving this problem?"
  "Have we ever solved a problem like this one before?"
  "What uses of mathematics did you find in the newspaper last night?"
  "Can you give me an example of . . .?"

Instructional Strategies

The following diagrams are examples of one teacher's planning efforts to connect measurement and geometry concepts to life, earth, and physical science units:

Measurement

Weather
Measure, record
- temperature °C °F
- volume of rainfall
- feet of snow

Dental Health
- Measure time spent brushing teeth
- Measure time spent in dentist office

Human Body
Estimate and measure body parts in inches. Draw a picture of yourself to show results. Estimate the weight of classmates. Organize/sequence from least to most weight. Weigh on scales, collect data, check predictions.

Food Groups
Use foods to help measure capacity of various containers. Weigh and compare fresh fruit vs. dried fruit.

Simple Machines
Use small cars (wheels) and ramps (inclined planes). Race cars off ramps, measure distance in inches, feet. Does incline affect distance?

Ecology
Buy an acre of rainforest. Measure and mark off an acre of land.

Plants
Students keep a record of plant growth, using inches, cm. Meaure weight of various seeds. Record volume of water used to feed plants.

Insects
Students measure wings or length of various insects. Measure length of ant food trails—distance to mound.

Dinosaurs
Use feet to measure length of dinosaurs. Compare capacity; e.g., 1 dinosaure = 1 bus. How do their masses compare?

Animals
Bring in stuffed animals to create a classroom zoo. Sequence animals by height, weight, longest tail, arms, legs, wings, etc. Use string and inches to verify and graph results.

Earth science
Predict the weight sequence of a set of rocks. Use scales to verify the sequence. Weigh different soil types. Do a cup of sand and a cup of top soil weigh the same?

Geometry

Weather
Critical Thinking—Write a paragraph. What if . . . hail were shaped like cones, cubes, pyramids, etc. Create unique cloud shapes using precut construction paper shapes.

Dental Health
Discuss various teeth and their jobs. what shape (3-D) do these teeth resemble? Critical thinking: What if our teeth were all shaped like cones, spheres? Etc.

Animals
Use measurement info on animals to graph results. Use pattern blocks to create unique animals.

Space
Use three-dimensional shapes to make a rocket ship. Use construction paper to create a space-shape monster. Write data identifying the monster's various shape/body parts. Identify planets as spheres.

Human Body
Bilateral symmetry—Art project. Draw one side of body; fold, trace other side. Plot measurements of student height or weight on line graphs.

Simple Machines
Explore angles of inclined planes for racing activities. Does incline affect the distance the car travels? Rotational symmetry of various wheels and gears create efficiency of machines. Use toy wheels, gears, etc. to create a unique machine. What job does it perform?

Insects
Identify symmetry in a variety of insects. Use art to help with this concept; i.e., making butterflies with paint on one side of paper, folding to create mirror image. Look at rotational symmetry in bee hives and at the shape of cells in hives.

Food Groups
Bring in a variety of foods. Have students identify foods that closely match geometric solids: cube, cylinder, cone, pyramid, and sphere.

Plants
Identify the shapes found in real plants; e.g., leaves, petals, stems. Use pattern blocks to create a unique plant. Identify shapes. Plot growth of plants to create a line graph.

Appropriate questioning techniques and meaningful problem-solving situations are two major strategies for effective mathematics instruction.

Uses of Technology and Manipulatives

Calculators and computers are tapped for important roles in mathematics at all levels and across topics. Changes in technology and the broadening of the areas in which mathematics is applied have resulted in growth and changes in the discipline of mathematics itself. The new technology has altered the very nature of the problems important to mathematics and the methods mathematicians use to investigate them.

The NCTM *Curriculum Standards* (1989) call for the following regarding technology in the classroom:

- appropriate calculators for all students at all times
- a computer for every classroom for demonstration
- access to a computer for individual and group work
- students learning to use the computer as a tool for processing information and performing calculations to solve problems

Calculators and computers offer teachers and students an important learning aid. Their potential is great and as yet untapped both in developing concepts and in developing positive attitudes and persistence in problem solving.

Computers can be utilized in a variety of ways in the mathematics classroom, and the appropriateness of a particular approach depends on the goals. Three qualitatively different methods suggested by R. Taylor in *The Computer in the School: Tutor, Tool, Tutee* are:

- as a sophisticated teaching machine
- to be programmed (or taught) by the student
- as a mode for applications in research and development through software that displays graphs, manipulates symbols, analyzes data, and performs mathematical procedures. Applications such as spreadsheets, word processing, data bases, and communication packages have the appeal of matching the classroom's use of technology with that of society's.

Calculator use is not for the purpose of replacing paper-and-pencil computations but to reinforce them. According to N. Kober in *Ed Talk: What We Know About Mathematics Teaching and Learning*, calculator use is apt to sustain independent thought, not replace it. For example, students can be challenged to invent calculator algorithms to replace procedures taught in textbooks. The students explain why their procedures work and debate the advantages and disadvantages of their procedures over others. Calculators are programmable, produce graphics, and work in fractional and algebraic notation. Teachers need to be innovative; they need to experiment and share ideas.

Furthermore, manipulatives offer an excellent way to enable students to connect between mathematical ideas. Learning is enhanced when students are exposed to a concept in a variety of manipulative contexts. As an example, fractions represented with pattern blocks, fraction bars, fraction circles, and Cuisenaire rods help students understand the concept of fraction independent of the physical representation. In addition to using manipulatives for new concepts, activities should be oriented to help students connect between concrete, pictorial, and abstract representations of ideas.

However, the use of manipulatives should not become an end in itself. Learning the motions of modeling addition and subtraction with Cuisenaire rods does not guarantee understanding of the mathematical relationship between these inverse operations. It is important not only that the concrete manipulation of materials closely matches the mathematical concept being developed but that the actions are accompanied with appropriate questioning by the teacher and reflection by the student.

In the instructional uses of both technology and manipulatives, the goal is to enhance mathematical thinking. Again, the teacher's role as questioner and decision maker influences the effectiveness of the incorporation of these tools.

Essential Elements of Instruction

Essential Elements of Grade 2 Mathematics with Sample Learning Objectives and Sample Clarifying Activities

The State Board of Education in 1989 revised the essential elements of instruction for mathematics, Grades 1-8. These revised essential elements follow closely the recommendations made by the National Council of Teachers of Mathematics in its nationally recognized *Curriculum and Evaluation Standards for School Mathematics*. According to the Texas Education Agency (1989), "The mathematics curriculum review committee and the Agency [TEA] have tried to be sensitive to a balance between changes expected of teachers and improvements necessary to help students learn mathematics more effectively." Some of these major changes include:

- narrowing the spiral of the curriculum—beginning some topics later and finishing some topics sooner in the curriculum to eliminate some of the redundance
- revising the role of review in the curriculum so that the majority of each grade level is new material and so that review is placed in relevant contexts
- emphasizing the development of problem-solving skills in relevant and interesting situations
- incorporating calculators and computers throughout all grades as problem-solving tools
- adding an essential element on patterns, relations, and functions
- separating the teaching of operations and computation so that all students learn the meaning of the operations
- strengthening the areas of probability, statistics, and geometry
- emphasizing the importance of communication in mathematics

- building on a sound foundation of concepts rather than on rote procedures
- putting mathematics into meaningful contexts

The following essential elements and descriptors for second grade mathematics are annotated with sample learning objectives and sample clarifying activities (except for EE1: Problem-Solving). The learning objectives give a brief look at the developmental components of the concept(s) in the descriptor. The sample clarifying activities are addressed to the teacher and provide a glimpse of what student engagement with this concept might look like in the classroom.

Each set of sample learning objectives and sample clarifying activities is meant to be viewed as an integrated whole (not necessarily matched one-to-one) to clarify the descriptors and to identify connections among them, as well as connections to meaningful problem situations. The Problem-Solving strand therefore is annotated only with sample learning objectives and is connected to the other strands through the language and situations used in their sample clarifying activities.

Many of the activities involve the use of manipulatives and common materials such as hundreds charts or grid paper. A list of these manipulatives can be found in the References and Resources. Also in the References and Resources are lists of the children's trade books, teacher resource books, and software cited in the activities as examples of instructional materials.

The revised essential elements, sample learning objectives, and sample clarifying activities for Grade 2 are:

(1) **Problem Solving.** Experience in solving problems designed to systematically develop students' problem-solving abilities through a variety of strategies and approaches. The student shall be provided opportunities to engage in the following types of activities:

(A) **develop an organized approach to solving application and nonroutine problems appropriate for grade two;**

Sample Learning Objectives

Involving patterns, relations, and functions; number and numeration concepts; operations and computation; measurement; geometry; probability, statistics, and graphing

(B) **analyze problems by identifying relationships, discriminating relevant from irrelevant information, sequencing, observing patterns, prioritizing, and questioning;**

Sample Learning Objectives

Make inferences and predictions

(C) communicate an understanding of a problem by describing and discussing the problem and recording the relevant information;

Sample Learning Objectives

Demonstrate creative thinking through fluency, flexibility, elaboration, creation of new ideas, and spontaneity

Estimate outcomes including appropriate units for outcomes

(D) select appropriate strategies from a variety of approaches;

Such as: acting it out; making a model; drawing a picture; guessing and checking; making a diagram, chart, or graph; finding a pattern; using a simpler problem; working backwards; etc.

(E) select appropriate materials and methods for solutions; and

Sample Learning Objectives

Such as: concrete manipulatives, mental computation, paper and pencil (pictorial and/or symbolic), calculator, or computer

Reflect on the problem-solving process and solution of a problem by evaluating outcomes for reasonableness (including appropriateness of units), make revisions as needed, describe and discuss the process and solution, and make a decision based on the solution

(F) generate and extend problems.

(2) Patterns, Relations, and Functions. Use of models and patterns to develop the concepts of relations and functions. The student shall be provided opportunities to:

(A) identify and extend patterns of objects and symbols;

Sample Learning Objectives

Identify, reproduce, and extend rhythmic patterns.

Sample Clarifying Activities

Have students use a variety of instruments to play a musical pattern for their partners. The partner can record the pattern with pictures, words, or symbols and play it back.

Identify, reproduce, and extend patterns made up of concrete objects.

Have students identify and describe patterns in nature; e.g., from plant posters or pictures of crystals. Students can extend the patterns to predict what a larger plant or crystal would look like.

Identify and extend patterns of pictures, geometric shapes, or symbols.

Give students a scrap of patterned wallpaper to paste in the middle of a blank sheet of paper. Students can extend the wallpaper pattern to fill the paper.

Create and describe verbally patterns using concrete objects, pictures, or symbols.

Have students create patterns using toothpicks, popsicle sticks, pictures, numbers, or letters. Students can describe their patterns and look for similarities and differences between the patterns.

(B) compare and order whole numbers;

Sample Learning Objectives	*Sample Clarifying Activities*
Create number patterns involving one more, two more, three more, four more, five more, and ten more.	On a hundreds chart, have students color in the squares that are one more than the previous square colored. Have them use the same procedure for two more, three more, four more, five more, and ten more, using a different hundreds chart each time. Students can repeat the activity putting all the patterns on the same hundreds chart to see where the patterns overlap. Make a class chart of summary statements about the patterns.
Order sets of numbers from greatest to least and from least to greatest.	Have students cut out numbers from newspapers or magazines and order them from greatest to least and least to greatest, using a number line or hundreds chart. Students can discuss other possible strategies for ordering the numbers (e.g., how many in certain place value positions).

(C) construct a set of ordered pairs by pairing members of two sets of concrete objects according to a given rule;

Sample Learning Objectives

Collect two related sets of information from a given physical situation, a given rule.

Organize the data into a two-column chart showing each element of one set paired with a related element from the other set.

Translate the information in the two-column chart into ordered pairs.

Sample Clarifying Activity

Have students use a balance scale to measure the mass of pairs of scissors in terms of small washers. Students can record the data in a chart.

pairs of scissors	number of washers
0	0
1	12
2	24
.	.
.	.
.	.

The information on the chart can also be written as ordered pairs: (0,0), (1,12), (2, 24), (3,36) . . . ..

(D) develop basic fact strategies; and

Sample Learning Objectives

Separate a given set of objects into two groups in as many ways as possible.

Sample Clarifying Activities

Give pairs of students a set of 12 two-color counters. Students can use the two colors on the counters to display all of the possibilities for separating 12 objects into 2 groups. (Students should include a group of 12 and a group of 0 as possibilities.)

Find patterns in the addition facts table.

Use the patterns in the addition facts table to generate addition and subtraction strategies.

Have partners complete blank addition facts tables and list patterns found. Each student can then generate an addition or subtraction strategy based on one of the patterns and present it to the class.

(E) demonstrate the relationship between the actions of joining equivalent sets and separating a set into equivalent sets.

Sample Clarifying Activity

Have students work in groups of 4 and give each student 3 interlocking cubes. Students place all the cubes in the center of the table and count them. Ask, "If we separate this set of cubes evenly among the members of your group, how many would each of you get?" Students can verify their prediction. Follow up with questions like "If you had started with 4 cubes each and put them in the center, how many would there be? If you took that amount and divided it evenly again, how many would you each get? What if you separated that amount among only 2 people?" Etc.

(3) **Number and Numerations Concepts.** Concepts and skills associated with the understanding of numbers and the place value system. The student shall be provided opportunities to:

(A) write a number sentence to compare numbers, including different names for the same number;

Sample Learning Objectives	*Sample Clarifying Activities*
Rename numbers using various number sentences (different names for the same number; i.e., 2 + 3 = 4 + 1).	During opening exercises, calendar time, or some other convenient time, have students brainstorm as many names for the number that is the day's date as they can think of. Record their ideas on a chart, one for each number, and post them in the room. Have students look for patterns in the charts.
Compare numbers and identify which is less and which is greater.	Have students find numbers in newspapers and discuss strategies for determining which is greater or less.
Read and write sentences using symbols < and >.	Have students use magnetic "greater than" or "less than" symbols to compare numbers on the chalkboard or magnetic board and share their reasoning with the class.

(B) use concrete models of hundreds, tens, and ones to develop the concept of place value;

Sample Learning Objectives

Use concrete models to rename tens and ones.

Use concrete models to rename hundreds and tens.

Use concrete models to rename thousands and hundreds.

Identify patterns in place value.

Sample Clarifying Activities

Make a large place-value floor chart from a shower curtain or rug. Put out a pile of single cubes, popsicle sticks, or bottle caps, and have students group them in cups or sacks to show the number of 1000s, 100s, 10s, and 1s. Students can then read the amount from the chart.

Have students fill in a hundreds chart and work with partners to investigate what patterns can be found. Students can use crayons or markers to reveal patterns. (Example: The same number in both the ones and tens places.)

(C) use models of hundreds, tens, and ones to conceptualize, read, and write numbers;

Sample Learning Objectives

Tell, read, and write stories using various numbers and model the numbers used.

Use concrete models, pictorial models, and number lines to explore various relationships of numbers using words such as *about, near, closer to, between, a little less than,* etc.

Devise plans to recognize the relative values of numbers.

Sample Clarifying Activities

Have students work in groups of four. Each member writes a story problem incorporating two-, three-, or four-digit numbers. As one student shares the story, the other three students can model the number, each using one of these three means—concrete models, pictorial models, and a calculator. Rotate types of models as each participant shares a story.

Using three overhead spinners, create four 3-digit numbers. Have students record the numbers in ascending or descending order and discuss the strategies they used to determine the relationships of the numbers.

(D) determine whether a number is even or odd by pairing objects in a set;

Sample Clarifying Activity

Have students draw a number from a bag and predict whether the number is even or odd. Have that number of students pair up for a dance and conclude whether the number is even or odd. Students can look for patterns in the even and odd numbers. *Madeline* by Ludwig Bemelmans can be used as a motivating story.

(E) **use concrete models for fractions to investigate different physical representations for the same fractional parts of whole objects or sets of objects;**

Sample Learning Objectives

Recognize fractions using models of whole objects.

Recognize fractions using models of sets of objects.

Communicate the meanings of the numerator and denominator of a fraction.

Read and write fractions for halves, thirds, fourths, and sixths.

Sample Clarifying Activities

Have students fold or cut paper shapes (rectangle, square, triangle, circle) to show fractional parts (1/2, 1/3, 1/4, 1/6). Students can name and describe each fraction to connect it to its symbol.

Have students spill a predetermined number of two-colored counters onto their desktops and identify the fractional part that is red and the fractional part that is yellow. Students can name and describe each fraction to connect it to its symbol.

(F) **demonstrate and write the value of various collections of coins; and**

Sample Learning Objectives

Count money using pennies, nickels, dimes, quarters, or concrete models through 99 cents.

Use the cent symbol to write the value of a set of coins through 99 cents.

Demonstrate various collections of coins that equal a given value through 99 cents.

Sample Clarifying Activities

Give each pair of students a set of coins and a hundreds chart. One partner selects each coin, names its value, and hands it to the other partner. That student places each coin in the appropriate space on the hundreds chart to indicate the running total value.

1	2	3	4	5	6	7	8	9	D
11	12	13	14	N	P	P	P	19	20

Have students work in small groups with real coins or models of coins. Each member shows a different way to make a given value. Students can record their combinations of coins, share their reasoning, and determine if there are still other ways to show the given value.

(G) **use ordinals.**

Sample Learning Objectives

Use ordinal numbers to identify position.

Sample Clarifying Activities

As students line up for an activity (lunch, physical education, etc.) have them orally identify their positions.

Read and write ordinal numbers.	Have students work together as a class to make a peanut butter and jelly sandwich. Have students use ordinal numbers to communicate in writing the steps required to make the sandwich. Students may choose other home tasks such as washing dishes to describe using ordinal numbers.

(4) **Operations and Computation.** Use of manipulatives to develop the concepts of basic operations on numbers and to apply these concepts to the computational algorithms. The student shall be provided opportunities to:

(A) make generalizations about ordering and grouping, and identify patterns in addition and subtraction;

Sample Learning Objectives	*Sample Clarifying Activities*
Make generalizations about the ordering (commutative) property and grouping (associative) property of addends in addition.	Have students use two different colors of interlocking cubes to explore addition facts to 10 and record the different combinations of addends found. Students can reverse the order of the colors to illustrate the commutative property. Students can use the same manipulati\ 's to investigate the associative property and share strategies for adding three and four addends.
Investigate number patterns involving certain quantities more/less (including 0, the identity element for addition).	Have students use counters to model these addition facts: $0 + 2 = 2$ $1 + 2 = 3$ $2 + 2 = 4$ $3 + 2 = 5$ . . . then repeat using -2: $2 - 2 = 0$ $3 - 2 = 1$ $4 - 2 = 2$ $5 - 2 = 3$ . . .
Use the patterns created and the inverse relationship between addition and subtraction as strategies for learning addition and subtraction facts.	Have students compare the number sentences from the previous activity to determine and explain patterns. Extend the activity by increasing the values of the addends to two- and three-digit numbers. Have students look for similarities and differences in the patterns.

(B) demonstrate an understanding of multiplication and division and their inverse relationship by joining equivalent sets of objects and separating a set into equivalent sets;

Sample Learning Objectives	*Sample Clarifying Activities*
Use manipulatives to demonstrate the meaning of multiplication by joining equivalent sets of objects and relating multiplication to repeated addition.	Have students draw three trees and place two red counters on each tree. Ask, "How many apples do you see?" Students can record their observation using repeated addition (2 + 2 + 2 = 6) and multiplication (3 groups of 2; 3 x 2 = 6). Students can repeat the activity using different amounts of apples on each tree.
Use manipulatives to relate multiplication to an array.	Have students place counters on inch grid paper to show arrays such as 3 x 2, 1 x 7, 6 x 3. Students can record the array using multiplication sentences: 3 x 2 = 6.
Use manipulatives to demonstrate the grouping concept of division by separating a set of objects into equivalent sets and relating division to repeated subtraction.	Have students draw one big tree and place all 6 red counters on the tree. Ask questions such as "If you ate 2 apples each day, how many days would you get to eat apples from this tree?" Students can group the apples and record their results with repeated subtraction (6 - 2 = 4, 4 - 2 = 2, 2 - 2 = 0 means 3 days) or with a division sentence (6 ÷ 2 = 3). Students can then make up their own questions.
Use manipulatives to demonstrate the sharing concept of division by separating a set of objects into a specific number of sets.	Have each student put 12 apples on the tree. Model questions such as "If 3 people shared the apples, how many would each one get?" Students can record their results with a division sentence (12 ÷ 3 = 4) and then ask questions about 4 friends or 2 friends or 5 friends.
Use manipulatives to show the inverse relationship between multiplication and division.	Have each pair of students use beans to show two sets of four and record the multiplication equation. The partners then can reverse the process by dividing the eight beans into two equal sets and recording the division equation. Students can repeat the activity using different quantities of beans.

(C) select the correct operation and solve real-life problems involving addition and subtraction using a calculator when appropriate;

Sample Learning Objectives

Create and solve addition and subtraction problems, using a calculator when necessary.

Select the correct operation, addition or subtraction, for given word problems.

Solve problems by writing number sentences involving addition and subtraction facts.

Sample Clarifying Activity

Have students work in pairs to illustrate a fishing scene. Each partner writes an addition and a subtraction story problem about the picture. Partners can solve each others' problems using manipulatives and record their results on paper or on calculators.

(D) use the inverse relationship between addition and subtraction to complete number facts;

Sample Clarifying Activity

Give each pair of students a picture of a story setting with 2 distinct sets of objects in it, for example a beach with 4 sand dollars and 6 conch shells. One partner writes an addition sentence about the picture, and the other partner writes a subtraction sentence. Then they switch places; the one who wrote an addition sentence writes a different subtraction sentence and vice versa, so that the 4 *fact family* equations are generated.

(E) use physical models to solve addition and subtraction problems whether or not regrouping is necessary;

Sample Learning Objectives

Use manipulatives and mathematical arguments to devise strategies to solve two-digit and three-digit addition problems.

Use manipulatives and mathematical arguments to solve two- and three-digit subtraction problems.

Sample Clarifying Activity

Give students computational situations that are not basic facts like 14 + 18 or 23 - 9; encourage them to devise a strategy to do the computations. Have students share their various strategies with the class for discussion and agreement or disagreement.

(F) use a multiplication or division number sentence to describe modeled situations;

Sample Clarifying Activity

Have students create a scene in the classroom that has the same number of students in each group or table. Have students determine the number of groups and the number of children in each group. Students can write multiplication and division number sentences to describe the situation. Have them generate stories and questions to go with each number sentence.

(G) recall basic addition and subtraction facts and complete addition number sentences with a missing addend;

Sample Clarifying Activity

After showing students *Anno's Counting House* by Mitsumasa Anno, have them write a number sentence with a missing addend to describe one of the situations; e.g., 4 (children in the old house)+ _____ (children in the old house) = 10 (total children). Students can use two-color counters or knowledge of basic facts to solve the equations.

(H) add two, three, or four single digit addends; and

Sample Clarifying Activity

Have students roll a die twice, then add and record the number sentence. Students can roll three times for 3 addends and four times for 4 addends. A decahedral die (from a polyhedral dice set) will generate numbers 0—9.

(I) illustrate the connection between concrete models and the addition algorithm, and use the algorithm with two- and three-digit numbers.

Sample Clarifying Activity

Have pairs of students use three spinners (one each for the hundreds, tens, and ones places) to spin a three-digit number. Have them show that number on an abacus, with place value blocks, and with numerals on a place value chart. Students then spin a second number and show the sum of the two using the three models.

(5) Measurement. Concepts and skills using metric and customary units through the use of concrete models. The student shall be provided opportunities to:

(A) estimate and measure the length, width, and height of objects;

Sample Learning Objectives	*Sample Clarifying Activities*
Use nonstandard units to estimate and measure the length, width, and height of objects.	Have students trace and cut out the shapes of their feet. Students can estimate and measure length, width, and height of objects in the classroom (such as tables, bookcases, chairs) using footprints. Other nonstandard units may include string, straws, pencils, crayons, paper clips.
Identify concrete models that approximate one centimeter, one decimeter, and one meter.	Have students identify and label items in the classroom that approximate one centimeter, one decimeter, or one meter (school box, pencils, crayons, tables, paper clips).
Identify concrete models that approximate one inch, one foot, and one yard.	Have students collect items from home such as spoons, paper towel rolls, ribbon, straws, and shoelaces and display the items according to their approximate lengths.
Estimate lengths and use a measuring instrument to measure objects using inches, feet, and yards.	Have students work with partners to estimate and measure lengths of body parts (arms, fingernails, fingers, hands, feet, legs) using inches, feet, and yards. Students can compare their estimates with actual measurements.
Estimate lengths and use a measuring instrument to measure objects using centimeters, decimeters, and meters.	Have each student make an outline chart of his or her partner's body on paper. Students can estimate and measure arms, legs, and fingers and record estimates on the chart in red and actual measurements in blue, using appropriate metric units.

(B) measure the weight of an object using nonstandard units;

Sample Clarifying Activity

Have students use a balance scale and wooden cubes, nails, washers, or bottle caps to weigh selected items from their school supplies (glue, crayons, pencils, markers). Students can record their measurements to compare with the measurements of others in the class.

(C) identify concrete models that approximate weight/mass units for pound and kilogram;

Sample Clarifying Activity

Make three hoops of yarn circles labeled "less than a pound," "more than a pound," and "about 1 pound." Have students estimate weights of selected items and place them in the corresponding circles. Students can then use a balance scale to test their predictions.

(D) estimate and measure the weight/mass of an object;

Sample Learning Objectives	*Sample Clarifying Activities*
Estimate weights and use balance scales to measure objects in kilograms and pounds.	Have students go on a scavenger hunt to find objects that weigh about 1 kilogram, 2 kilograms, 3 kilograms, etc. Students can use a balance scale to test their choices.
Estimate weights and use spring scales to measure objects in kilograms and pounds.	Have students predict weights of and measure food items (canned goods, bread, grapefruit, bags of beans, rice) using a spring scale, as in a grocery store setting.

(E) estimate concepts of time;

Sample Learning Objectives	*Sample Clarifying Activities*
Use an understanding of 1 minute to estimate what can be done in 2 or 5 minutes.	Set a timer for 1 minute. Have students write their names, draw circles, or do a similar activity for 1 minute. Students can then predict how many times the activity could be done in 2 or 5 minutes.
Estimate what can be done in one hour (60 minutes).	Have students brainstorm and list things that can be done in 60 minutes. In a follow-up discussion, students can justify their choices.

(F) tell time on traditional clocks; and

Sample Learning Objectives	*Sample Clarifying Activities*
Tell time to the nearest hour on a traditional clock.	Have students construct a double clock with a traditional face on one side and a digital readout on the other side. As students read or listen to a story involving time, they can place the hands in position and show the correct time on the digital side.

Identify the minutes past the hour on a traditional clock.

Relate the position of the numbers on the clock face to fraction parts of a circle.

Have students divide the face of a clock in half and color the first 30 minutes of the hour red and the second half blue. Students can discuss what numbers are in the first and second halves and how many minutes are in each half. Have students predict the numbers and how many minutes in each fourth of an hour and use the clock face to test their predictions.

(G) solve application and nonroutine problems involving length, weight, and time.

Sample Clarifying Activity

Collect a variety of spo· . balls. Have students predict the distance eacl ·an be kicked. Students can then kick the balls, measure, and graph the data. Have students look for patterns in the results.

(6) Geometry. Properties and relationships of geometric shapes and their applications. The student shall be provided opportunities to:

(A) identify, describe, and compare two-dimensional shapes and three-dimensional figures;

Sample Learning Objectives

Explore and compare three-dimensional shapes introducing appropriate math vocabulary.

Identify cone, cylinder, sphere, cube, and rectangular prism.

Identify faces, edges, and corners on three-dimensional figures.

Identify flat and curved surfaces on three-dimensional figures.

Sample Clarifying Activities

Using a pillowcase full of three-dimensional real-world objects, have students feel (without looking) and describe the figure they picked. Have a cone, cylinder, sphere, cube, and rectangular prism displayed and, as the student describes the object in the pillowcase, the other class members choose and name the three-dimensional figure.

Have the above three-dimensional objects placed on a floor chart (one shape at the top of each column). Each row will be identified by these questions:

How many flat faces?
How many edges?
How many corners?

Lead a whole-group discussion to answer the questions for each shape and justify each response.

Explore and compare two-dimensional shapes introducing appropriate math vocabulary. Identify sides and corners on two-dimensional shapes.	Have students work in pairs and select 10 attribute blocks. One person creates a picture (hidden from partner) and describes the picture while the partner tries to reproduce the scene. Extend the activity by having students choose different numbers of blocks, or one partner can create a scene, trace around the blocks, and give the outline to the other partner to try to fill in the picture with the blocks while verbally identifying the two-dimensional shapes.
Name the two-dimensional shapes that make a three-dimensional figure.	Have students work in groups of four with a piece of newsprint folded twice to make four sections. Each group is given four different three-dimensional figures. Each group member presses the faces and bases of their 3-D figure in chocolate pudding, then stamps them on the newsprint. (A cube would have six squares pressed on the newsprint.) Have students hide the figures and rotate the stamped pictures from group to group, having each group identify which figure made each set of impressions.
Recognize and identify solid and plane figures in real-world objects such as boxes, cans, and balls.	Make wall charts labeled "cylinder," "sphere," "rectangular prism," etc. Have students cut out or draw pictures of real-world objects and paste them on the appropriate charts. Students can discuss why some charts seem to have more examples than others.

(B) investigate congruence and symmetry using models, drawings, and computer graphics;

Sample Learning Objectives	*Sample Clarifying Activities*
Explore the concept of congruence and identify congruent figures.	Have students use pattern blocks to create an animal. Students can identify any edges or sides that are congruent and justify their choices.
Identify symmetrical figures and their lines of symmetry.	Have students trace around their animals, cut them out, and fold them to see if any are symmetrical. Ask questions like, "Does it have more than one line of symmetry? If the animai is not symmetrical, can you make one that is?"
Investigate congruence and symmetry using computer graphics.	Have students use software such as Ventura's "Hands-On Math"or "Geodraw" to draw congruent and symmetrical shapes.

(C) investigate perimeter using concrete models; and

Sample Learning Objectives	*Sample Clarifying Activities*
Explore the concept of perimeter by identifying the distance around an object.	Have students use a 1-decimeter piece of string to find an object in the room whose perimeter is about 1 decimeter. Students can wrap the string around each object, identifying it's perimeter.
Express the distance around an object in non-standard and standard units of length.	Have students use paperclips, fingers, and crayons or rulers and tape measures to measure the perimeters of various classroom objects. Have students discuss various strategies for finding perimeters such as measuring each side and adding the measurements.

(D) solve application and nonroutine problems involving geometry.

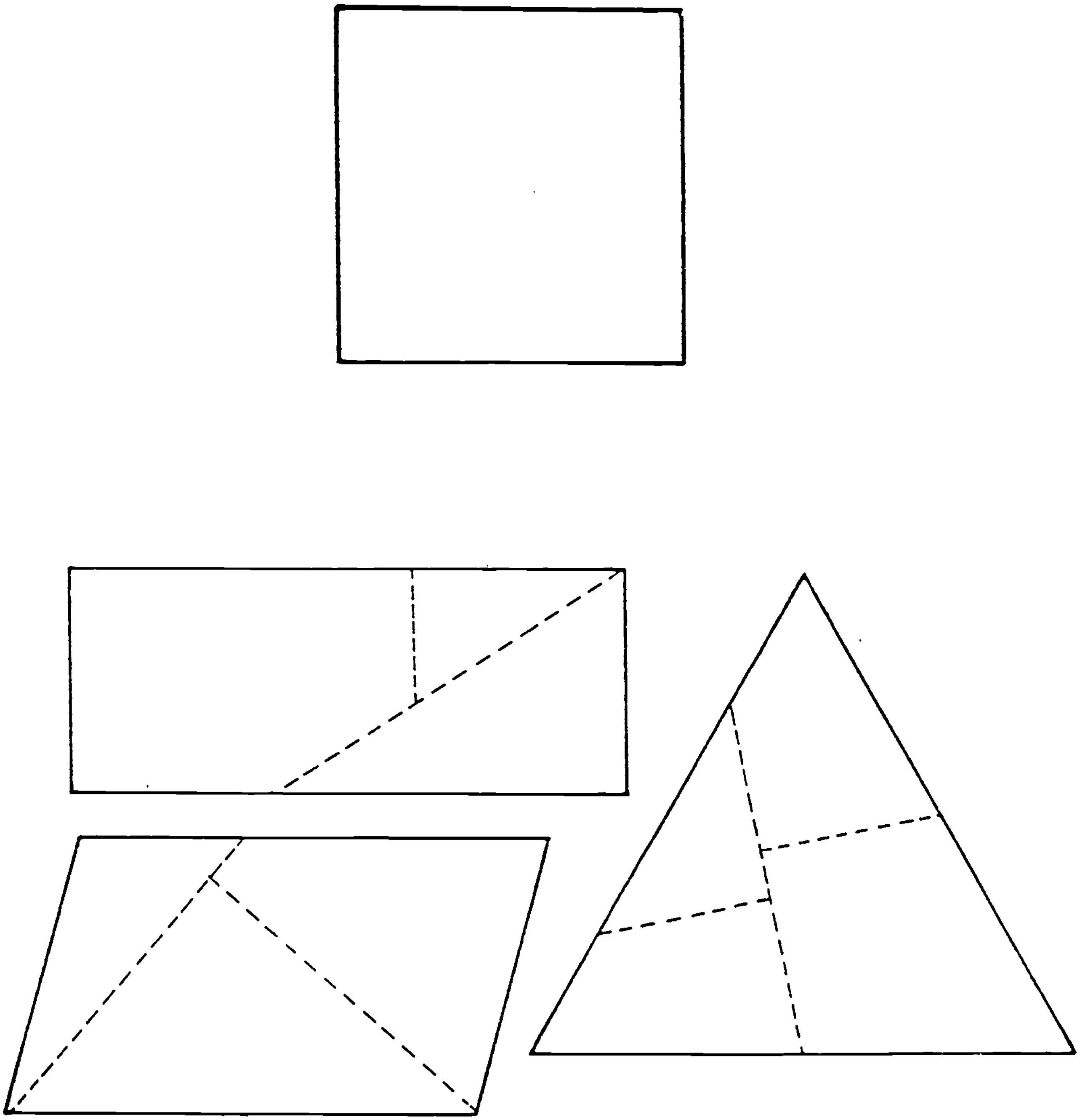
Sample Clarifying Activity

Give each group a copy of the shapes on the worksheet. Have students cut out the rectangle, triangle, and parallelogram, keeping the pieces from each shape separate. Have students reconstruct the original shapes. (Outlines of each shape could be drawn to facilitate the activity.) Students can then use the pieces from each of the three shapes to construct the square.

(7) Probability, Statistics, and Graphing. Use of probability and statistics to collect and interpret data. The student shall be provided opportunities to:

(A) collect and organize data;

Sample Learning Objectives	*Sample Clarifying Activities*
Identify categories around which to organize collected data.	Have each student place one shoe into a pile, discuss the similarities and differences among the shoes, and explore ways of grouping the shoes. Students can determine the number of columns needed to display the organized shoes.
Use a chart to organize collected data.	Have students develop a large floor chart from a shower curtain, dividing it into the pre-determined number of columns. Students can arrange the shoes on the chart and make summary statements about the data.



(B) make pictographs and bar-type graphs;

Sample Learning Objectives	*Sample Clarifying Activities*
Place data from a chart into a pictograph.	Have students draw and cut out pictures of shoes and replace the real shoes on the shower curtain chart with pictures of shoes. Students can explore the concept of having one shoe picture mean two or more real shoes.
Place data from a chart into a bar-type graph.	Have students replace each shoe on the chart with an interlocking cube, one color for each category. Students can then link the cubes of the same color into bars and stand them next to each other on a labeled card to make a three-dimensional bar graph.

(C) describe data displayed on a bar graph;

Sample Clarifying Activity

Have students record as many facts as they can about the shoe data and share their favorite observations with the rest of the class.

(D) draw conclusions and make predictions based on graphed data;

Sample Learning Objectives	*Sample Clarifying Activities*
Make summary statements based on graphed data.	Have students identify questions about the class's shoes that can be answered from the data on the graph and questions that cannot be answered using the graph.
Make predictions based on graphed data.	Have students predict how the graph of shoes might change using a different population (different age group, occupations, sports players, etc.).

(E) solve application and non-routine problems for situations involving graphs;

Sample Clarifying Activity

Have students pretend they own a children's shoe store and use the bar graph to determine the amount of each kind of shoe to stock in their store. Students should justify their choices.

(F) display all arrangements of a given set of objects and identify the pattern; and

Sample Learning Objectives	*Sample Clarifying Activities*
Use concrete objects, pictures, or symbols to depict all arrangements.	Have students cut patterns of cones and scoops of ice cream to depict all double dip combinations that could be made from 3, 4, or 5 different flavors.
Organize the arrangements and identify patterns.	Have students arrange scoops of ice cream in some organized way, like putting all of the ones with vanilla on top together in a group.
Use patterns to determine the possible number of arrangements.	Have students use their organized arrangement to predict the possible number of variations of double dips of ice cream. Students can analyze the patterns to find pairs of scoops they may have left out.

(G) explore the likelihood of an event occurring.

Sample Learning Objectives	*Sample Clarifying Activity*
Collect data about an event. Organize the data to look for patterns. From the organized data, make conclusions about the likelihood of an event.	Give each pair of students a bag containing six red beads and three black beads. Have students explore the likelihood of drawing a black bead from the bag by drawing one bead, recording the color, putting it back and repeating this several times. Students can organize their data and compare their data with that of other groups to make conclusions.

Texas Assessment of Academic Skills

Focus

The Texas Education Agency implemented the Texas Assessment of Academic Skills (TAAS) testing program in 1990. The program is in effect for the 1990-1995 period. The purpose of the assessment program is to provide Texas schools with an accurate measure of student achievement. The scope of content of the TAAS includes more of the instructional targets delineated in the essential elements than previous state assessments. Every section of the TAAS test contains a certain number of broad objectives. These objectives remain constant from grade to grade because they represent the core concepts that form the basis for a sound instructional progression from Grade 1 through Grade 12. What will differ from grade to grade are the instructional targets—or essential elements that comprise each objective. A portion of this extended set of instructional targets is selected for assessment annually, but not every target is tested every year.

The broadened scope of the TAAS assessment program allows for a different focus, one that addresses the academic requirements of the 1990s. Skill areas that demand little more than rote memorization are de-emphasized, while areas that improve a student's ability to think independently, read critically, write clearly, and solve problems logically receive increased emphasis. This emphasis is in keeping with current national trends in education, which stress the importance and necessity of teaching students higher order thinking skills.

Domains, Objectives, and Targets

The TAAS features three domains—concepts, operations, and problem solving. Each domain contains objectives that are derived from the essential elements. For every objective, there are instructional targets that describe the kinds of mathematical experiences that will reflect that objective. Each instructional target was taken for the most part directly from the essential elements as delineated in the *State Board of Education Rules for Curriculum*. Each target is defined in behavioral terms appropriate for pencil-and-paper testing.

Although TAAS testing begins at Grade 3, it is important for Grade 2 teachers to be aware of the Grade 3 targets for TAAS. Grade 2 teachers are responsible for developing foundations of understanding which will be the base on which the rest of the mathematics curriculum stands. Therefore, the TAAS objectives and targets for Grade 3 are listed below.

DOMAIN: Concepts

Objective 1: The student will demonstrate an understanding of number concepts.

(a) Compare and order whole numbers
(b) Use whole number place value
(c) Use odds, evens, and skip counting
(d) Recognize and compare fractions using pictorial models
(e) Translate whole numbers (name to numeral/numeral to name)
(f) Recognize decimal place value (tenths and hundredths; using models)

Objective 2: The student will demonstrate an understanding of mathematical relations, functions, and other algebraic concepts.

(a) Use whole number properties and inverse operations
(b) Determine missing elements in patterns
(c) Use number line representations for whole numbers
(d) Classify objects

Objective 3: The student will demonstrate an understanding of geometric properties and relationships.

(a) Recognize two- and three-dimensional figures
(b) Describe and compare two- and three-dimensional shapes
(c) Identify informal representations of congruence and symmetry

Objective 4: The students will demonstrate an understanding of measurement concepts using metric and customary units.

(a) Use measurement units of time, length, temperature, and weight/mass
(b) Find perimeter

Objective 5: The student will demonstrate an understanding of probability and statistics.

(a) Interpret and use charts, tables, bar graphs, and pictographs

DOMAIN: Operations

Objective 6: The student will use the operation of addition to solve problems.

(a) Add whole numbers
(b) Add money using models

Objective 7: The student will use the operation of subtraction to solve problems.

(a) Subtract whole numbers
(b) Subtract money using models

Objective 8: The student will use the operation of multiplication to solve problems.

(a) Use modeling and patterns of whole number multiplication concepts to generate basic facts

Objective 9: The student will use the operation of division to solve problems.

(a) Recognize modeling of division

DOMAIN: Problem Solving

Objective 10: The student will estimate solutions to a problem situation.

(a) Estimate with whole numbers

Objective 11: The student will determine solution strategies and will analyze or solve problems.

(a) Select strategies or solve problems using addition and subtraction with whole numbers

Objective 12: The student will express or solve problems using mathematical representation.

(a) Identify solution sentences
(b) Recognize models for problems
(c) Interpret charts, tables, pictographs, and bar graphs and use the information derived to solve problems
(d) Identify or solve story problems depicting the joining (addition/multiplication) and separating (subtraction/division) processes

Objective 13: The student will evaluate the reasonableness of a solution to a problem situation.

(a) Evaluate reasonableness

Sample Lessons for Teaching Grade 2 Mathematics

The following sample lessons represent the kind of mathematical experiences recommended for students in second grade mathematics classes. These expanded sample activities include ideas for motivational introductions, exploratory questions to ask during activities, summary questions for reflection after exploring the concept, and ideas for extension and assessment. They are included as examples of significant, mathematical tasks that address the state's curriculum requirements in light of the national recommendations. Note that each sample activity involves essential element descriptors, as well as objectives from the Grade 3 Texas Assessment of Academic Skills (TAAS). Several different manipulatives are included in these activities. It is important that students use these manipulatives as they work through the activities. Manipulatives and concrete objects enable elementary school students to better understand the mathematical problems and concepts they so often struggle to learn. Students' experiences with manipulatives are recommended in the essential elements, TAAS's instructional targets, and NCTM's *Curriculum and Evaluation Standards for School Mathematics*.

Most activities in this section recommend that students work together in pairs or small groups. Working together in cooperative groups promotes communication and mathematical confidence and enhances students' problem-solving abilities. At the end of each activity is space to add your own notes and ideas.

EE: 2A *Related EEs: 1A, 1B, 1C, 6A, 6B, 6D* *Grade 3 TAAS Objectives: 2, 3, 11*

Objective The student will extend a geometric pattern in all directions and discuss the strategies used to extend the pattern.

Activity Wallpaper Etcetera

Materials Wallpaper samples, felt-tipped markers, manilla paper, colored pencils, glue, and crayons

Resources *Cooperative Learning, Math Success* by Lynn Molyneux

Procedure

Introduction:

1. Have each student glue a small sample of wallpaper in the middle of a piece of manilla paper and brainstorm what it would look like on a wallpapered wall.
2. Let the students discuss with their partners what they need to do to extend the patterns.
3. As they work with their partners, students can extend the patterns in all directions using the crayons, felt-tipped markers, or colored pencils.

Exploration:

- What patterns do you see in the sample?
- Are shapes turned, flipped, or slid in the pattern?
- How could you decide what comes next in each direction?

Extension:

Have each student design his or her own wallpaper scrap for a partner to extend.

Summary:

- What patterns did you find?
- How did you decide what came next in the pattern?
- How is this part of the pattern related to this other part? Is it the same shape and size (congruent)? Is it turned? Flipped? Slid?

Assessment

Questions:

(See summary questions.)

Observations:

- Did students complete the design?
- Did students use turning, sliding, or flipping to complete the design?
- Did students use geometric vocabulary to describe the design?

Tasks:

Use new geometric vocabulary to describe a given wallpaper sample.

EEs: 2A, 2C, 2D *Related EEs: 1A, 1B, 1C, 1D, 4B, 4C, 4F* *Grade 3 TAAS Objectives: 2, 6, 8, 11, 12*

Objective The student will generate numerical sequences.

Activity Number Sequences

Materials Overhead projector, transparencies

Resources *Cooperative Learning, Math and Success* by Lynn Molyneux.

Procedure

Introduction:

1. Have students work in cooperative groups.
2. Begin by presenting a familiar numerical sequence:
   "Let's start with 1 bike. How many tires does Tomé need? How many tires does Tomé need with 2 bikes? If we write the number of bikes in a row such as 1,2,3,4,5, . . . [Demonstrate on the overhead.], we can write the number of tires under it.
   number of bikes 1, 2 , 3 , 4 , 5 , . . .
   number of tires 2 , 4 , 6 , 8, 10, . . .
   Sometimes we want to express this in what mathematicians call a set of ordered pairs. This problem can be written as { (1,2), (2,4), (3,6), (4,8), (5,10), . . .}. For 6 bikes, how many tires will Tomé need to buy? The ordered pairs show (number of bikes, number of tires)."
3. Allow the groups time to discuss and explain their ideas. Each group should make sure everyone in the group understands the information.
4. "If Tomé wanted to buy 20 bikes, can we find out how many tires he needs to buy without completing the chart?" Allow the groups time to discuss and explain their ideas.
5. "Can someone find a rule that fits our pattern?" (n x 2 where n = number of bikes)
6. Read Situation 2 aloud: A movie ticket costs $[illegible].00. Show information about what it would cost for certain numbers of people to go to the movies. Have each group generate a table and a set of ordered pairs to share with the class.
7. Have each group consider, "What rule fits our pattern? How much would it cost for 40 people to go to the movies?"
8. Continue with other related sequences:
   - number of sides in a shape vs. number of angles
   - number of feet vs. number of inches in a measurement
   - number of people vs. number of cars needed for a field trip
9. Allow the groups time to share what they have learned.

Exploration:

- How do the chart and the ordered pairs help you find a pattern?
- Do the numbers in the sequence increase? Is it an addition pattern? Is it a multiplication pattern?
- Do the numbers in the sequence decrease?
- Is it a subtraction pattern?
- Is it a division pattern?
- How are the numbers in the new sequence related to the 1, 2, 3, 4, 5, . . . sequence?

Extension:

- Have the students develop their own number sequences for their partners to solve.
- Have students design a situation to fit a given number sequence.

Summary:
- How does the chart help you to visualize the pattern?
- In what ways do the ordered pairs describe the pattern?
- How does the rule help improve the predictions you make?

Assessment

Questions:
- How can you describe the relationship between each number in the new sequence and the numbers in the 1, 2, 3, 4, 5, . . . sequence?
- How can you show the relationship between the two sequences in a set of ordered pairs?
- What was the most important thing you learned?

Observations:
- Are the students using the chart correctly?
- Are the students able to express their observations in mathematical terms?
- Are the students interpreting the ordered pairs correctly?

Tasks:
- In your journal, generate a number sequence to fit a situation and explain its pattern.
- Given a set of ordered pairs describing a numerical sequence, describe the rule that is represented by the ordered pairs.

EE: 3B *Related EEs: 1A, 1E, 1F, 4A, 4I* *Grade 3 TAAS Objectives: 1, 6, 7, 10, 11, 12, 13*

Objective The student will use number sense and base ten materials to explore different ways to represent given numbers.

Activity Exploring Place Value Materials

Materials Base ten materials

Procedure

Introduction:

1. Have students work in cooperative groups to initially explore base ten materials with this question in mind: "If we are going to work with these materials, what do we need to know to use them to communicate with each other about numbers?"
2. Have students discuss the relationships between the different pieces. Students could even make their own sets of units, longs, and flats out of centimeter graph paper, based on their investigations of teacher-made or manufactured materials.
3. Give students a number to model with the materials: "Show 27." Encourage creativity and diversity; e.g., 27 units, equals 1 long and 17 units or 3 longs with 3 units placed on top of them so that 27 worth of the longs is showing.
4. Circulate and observe each group's work to form questions for the summary discussion.

Exploration:

- How does your representation show 27?
- What does the white cube (or square, or smallest piece) represent? The orange (or long)?
- Why did you choose this representation?
- Is there another way you could show 27?
- Show 27 with the greatest number of pieces.
- Show 27 with the least number of pieces.

Extension:
How many different ways could you model this one number? Illustrate each way using graph paper, manila paper, and markers.

Summary:

- Who thinks they showed 27 with the most number of pieces? Explain how it represents 27.
- Who thinks they showed 27 with the least number of pieces? Explain how yours represents 27.
- What are some other ways you showed 27? Explain how they work.
- What values did you use for the pieces? What would happen if you changed the values for the pieces? Can anyone give an example?
- Make a class graph showing all the ways used to make 27. Encourage new ones with questions like: "Could we use the flat to show 27? Could we use subtraction?"
- What are the advantages of each of these representations? What are the disadvantages? Have students discuss pros and cons of each representation.

Assessment

Questions:
(See summary questions.)

Observations:

- Were students using only units, only longs, or combinations of both?
- Were students using the concept of proportional value for the different sized pieces?

Tasks:

- Give students graph paper, manilla paper, and a new number. Have students use the graph paper to cut representations of units, longs, and flats to show many different representations of their number. Have them glue their representations on the manilla paper and write an accompanying paragraph in their journals.
- Have each group brainstorm a written list of summary statements for the activity. Compile them into a class list.

Objective The student will use concepts of base ten place value to determine the value of eggs found in an egg hunt.

Activity Don't Count Your Chips Before They Hatch

Materials Paper eggs (red, white, blue); place value mats; base ten materials; plastic eggs (40); red,white, blue poker chips (two sets of 100); baskets (one for each group); prizes; calculators

Procedure

Introduction:

1. Set aside the following materials for each group:
   - place value eggs made of construction paper: large blue eggs marked with 100, 200,. . . 900; medium red eggs marked with 10, 20, . . . 90; and small white eggs marked with 1, 2, . . . 9
   - set of base ten materials and place value mats and one calculator
   - plates or paper on which to write 3-digit numbers
2. In advance, place 2-3 chips in each plastic egg. Do not use more than 10 blue chips in all 40 eggs. Hide the plastic eggs.
3. Tell the students they are going to have an egg hunt. They will get prizes if they can determine the value of the eggs they find.
4. With students in groups of three, determine group roles (egg maker, base ten materials manipulator, recorder). One student shows a 3-digit number using paper eggs, one builds the number with concrete manipulatives, and one records the number in two ways. Have students rotate the roles so that all will have a chance to do each job.
5. Write a 3-digit number on the chalkboard (such as 567). Ask, "What does each digit in 567 represent? What are two ways you can write 567? What happens when there are no tens? Show 305 with your eggs. What happens when there are no ones? Show the number 350 with your eggs."
6. After this discussion, tell students that they get to hunt for eggs with place value chips inside: white = 1, red = 10, blue = 100. (Note that the chips are in the same colors as the corresponding paper eggs).
7. The students in the groups of three now have roles of basket carrier, exchanger, and recorder. When the students open the eggs back in the classroom, they are to arrange the chips on the place value mat. As a group, they are to determine the total points contained in the egg. If exchanges need to be made, the exchanger takes the appropriate chips to the bank (the teacher or an assigned student).
8. Chart the class results on the chalkboard and give prizes (stickers or certificates for the most eggs collected, highest number of points, most cooperative, correct totals, etc.)

Exploration:

- How is your group deciding to share the work?
- What is the value of a blue chip? Red? White?
- What exchanges did your group have to make?
- How did you know when to exchange chips?
- How did you arrive at your total?
- What happened to your total when you found a blue chip in an egg? A white chip? A red chip?
- What patterns did you notice in the ways the total changed each time?
- What other strategies could you have used?

- Can you use the calculator in any of your strategies? If so, how?

Extension:
- Play Beanbag Toss. Mark off a game board on the floor with tape and label the areas with 1, 10, or 100 points. Toss the beanbags onto the gameboard. Use a slate or paper to record the total points.
- Make an abacus using egg carton lids, plastic straws and beads or interlocking cubes (red, white, and blue to continue the colors in the egg hunt). Explore 10 more, 10 less, 100 more, 100 less.

Summary:
- How are the chips in the eggs different from using the base ten materials?
- What are the advantages of using different colors rather than different sizes to represent each place value?
- What are the disadvantages?
- What happened when you had 10 or more white chips?
- What happened when you had 10 or more red chips?
- What do 10 red chips represent?
- What do you think would happen if you had 10 blue chips?
- Would you have been able to have a egg hunt with base ten materials?
- When you were hunting the eggs, were you able to tell if an egg contained ones, tens, or hundreds? Why or why not?
- Could you tell how much the egg was worth by guessing how many chips were in it? Why or why not?
- If you had a choice to use these chips or other concrete place value models as money, which would you choose? Why?
- What else could you use to represent ones, tens, and hundreds?
- Did you use the calculator in any of your strategies? If so, how?

Assessment

Questions:
(See summary questions.)

Observations:
- Were students using place-value strategies for determining the value of each egg?
- Were students using place-value strategies for determining the totals of their eggs?

Tasks:
- Represent a given 3-digit number with some kind of physical material.
- Design an odometer device. Represent situations such as, "I had 585 miles on my car yesterday. Today I drove 36 more miles. What should the odometer read today?"
- Write a journal entry comparing the use of base ten materials with the use of colored chips to represent numbers.

EEs: 4C (4F) *Related EEs: 1A, 1C, 1F, 4D, 4G, 4H* *Grade 3 TAAS Objectives: 6, 7, (8, 9), 11, 12*

Objective The student will write, select the correct operation, and solve real-life problems with a Texas theme involving addition and subtraction (or multiplication and division).

Activity Texas Problem Solving

Materials Texas recording sheet, assortment of manipulatives, calculators

Procedure

Introduction:

1. Have students choose three numbers to use to make up one addition and one related subtraction story problem. These numbers could be generated through exploration with calculators, then verified with manipulatives or reasoning. (Appropriate numbers could be used to generate multiplication and division story problems.)
2. Have students write their story problems on the Texas recording sheet and record the number sentences for the solutions on the back.
3. Have students trade their recording sheets and work one another's problems, using manipulatives if necessary.

Exploration:

- How can you generate a number sentence to use?
- How do you know what kind of story to make to go along with that number sentence?
- What patterns do you see in the number sentences for your addition and subtraction stories? Multiplication and division stories?
- How might these patterns be useful in solving these problems?
- What makes a story problem interesting to you?
- What manipulatives might you use to solve this problem?

Extension:

Choose your own topic to write story problems about. Make a book of problems.

Summary:

- How did you select the number sentences to use for your stories?
- How did you decide what situations to use in your stories?
- Did writing your stories help you solve anyone else's problems? How did it help?
- What relationships did you see between addition and subtraction? Multiplication and division?
- How did you use the calculator?
- How did you decide what number sentence to use to solve someone else's story?
- How did you decide if your solution was reasonable?

Assessment

Questions:

(See summary questions.)

Observations:

- Were students connecting the number sentences to appropriate physical situations?
- Were students writing appropriate questions for their number sentences?
- Were students recognizing the relationship between addition and subtraction? Multiplication and division?
- Were students using their understanding of the operations to solve other student's story problems?
- Were students using their understanding of number and operations to judge whether their solutions were reasonable or not?

Tasks:

- Given a number sentence, write an appropriate story problem.
- In your journal, make a list of situations (questions) that could possibly go with an addition number sentence, subtraction number sentence, etc.

EEs: 4E, 4I | *Related EEs: 1A, 1D, 1E, 2B, 3B* | *Grade 3 TAAS Objectives: 1, 6, 7, 11, 13*

Objective The student will use the concept of place value to make trading decisions in a game.

Activity The Trading Game

Materials Laminated blank place value boards or blank paper; colored tiles or chips; dice

Procedure

Introduction:

1. Have students divide the mat or board into four sections.
2. Let the class decide which color will go in each section.
3. The object of the game is to get 1 chip of the last color (on the left).
4. Make a rule that 5 of the first color (on the right) equals 1 of the next color (to the left).
5. Have students decide in their groups, as they play the game, how to get to the next colors on the board.

Exploration:

- How did you decide to make the next color? Why?
- How do you know when to make a trade for another color?
- Some of you aren't putting on the number of colored tiles the dice say. For example, I saw you put on only one tile when you rolled a 5. What are you doing instead? Why?
- What kinds of strategies are you using for making faster trades?
- Can you tell what number you need to roll to make the next trade?
- Can you tell by looking at your tiles on your board how far you are from winning?

Extension:

- Change the trading rule from 5 to 3 or 7 to 10. Predict how these changes will affect the game.
- Use polyhedral dice other than the cube. Predict how the game will change.
- Figure out the total number of tiles it takes to win a game with a 5-trade rule, a 3-trade rule, a 10-trade rule.
- Start with one tile on the far left space and make a rule that whatever is rolled on the dice must be taken away from the place on the right until only one chip in the far right space remains. Figure out how to do this.

Summary:

- What shortcuts or strategies did you use to put tiles on the board?
- How did you use mental arithmetic?
- What were the advantages of using mental arithmetic and trading strategies?
- How would the game change if the rules changed? (For example, trade for 6 or 2 instead of 5.)

Assessment

Questions:

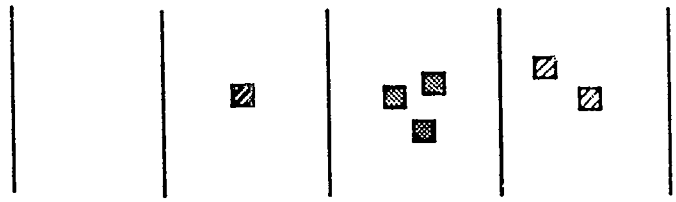
- How would you change your strategy for the next game? Why?
- What if we use a "trade for 10" rule? How would the game change? How would your strategies/shortcuts change?
- (See also summary questions.)

Observations:

- Were students trading accurately?
- Were students developing shortcuts?
- Were students discussing what would happen if they got certain kinds of rolls?

Tasks:

- Have students write a summary statement of what they learned from the game.
- Give representations of boards at different stages of the game and have students analyze what has happened and what will happen. Example: Given a board that looks like this and a trading rule of 6:



What happens if you roll a 6? A 4? What will you need to do to win this game?

Objective The student will use appropriate measurement tools and units to measure lengths and masses of real-life objects.

Activity Measuring Real Stuff

Materials A collection of non-standard measurement units (cubes, cups, bottles, toothpicks, paper clips, interlocking cubes, links, balances); a variety of objects to be measured (school supplies, desks, books, shelves, colored tiles, blocks); paper and pencil; simple balance beam

Resources *Amelia Bedelia* by Peggy Parish

Procedure

Introduction:

1. Read *Amelia Bedelia* and have students discuss some of the ways Amelia tried to measure things (with heights of cups rather than what they contained). Lead into a discussion of the importance of choosing appropriate measuring instruments and units.
2. Provide students with real-life objects to be measured.
3. Have students choose two different units of measure to use to measure an object and record the two measurements. Discussion may be needed here on the importance of a measurement containing both a number and a unit (e.g., 8 paper clips).
4. Have students measure several objects, organize their data, and look for patterns or possible conjectures about the measurements and the units.

Exploration:

- Why did you choose the unit of measure that you used?
- What could you use the paper clips to measure?
- What other units could be used to measure the same thing as the paper clips?
- What kinds of measurements could you make of the scissors?
- Why did you need more (paper clips) than (toothpicks) to measure that object?
- Which of your measurements was the heaviest? The lightest? How do you know?
- What do the numbers in the measurements tell you?
- Why are the unit words in the measurement important?

Extension:

Use standard units of measure (centimeters, grams, ounces). Compare your results to the results of other groups. What are the advantages of these standard units?

Summary:

- What were some of the characteristics that you chose to measure on the objects?
- Why did you choose the unit of measure that you used?
- About how many tiles did the notebook weigh? About how many blocks did the notebook weigh? Why did you use more tiles than blocks to measure the book?
- How did you decide if your measurements were reasonable or not? How did you use estimation?
- If a book was 25 paper clips long, about how many pens long do you think it would be? Why?
- What conjectures could you make about the result of using different measurement units to measure the same object?

Assessment

Questions:
(See summary questions.)

Observations:
- Did students choose appropriate measurement units?
- Did students use the measurement tools appropriately?
- Did students recognize the properties of measuring (units must be connected but not overlapping; units must have the same characteristic that is being measured).
- Did students display an understanding that the smaller the unit, the more it takes of them to make the same measurement?

Tasks:
- Given some nonstandard units, estimate measurements of objects. Explain why you think your estimate is reasonable.
- In your journal, explain what could be happening in situations like, "I measured the desk and it is 15 pencils long, but my partner says it is 20 pencils long. Which one of us is right?" Make up a situation where two people could measure the same object correctly but get what sound like different measurements.

EEs: 5B, 5C, 5D, 5G　　*Related EEs: 1A, 1B, 1C, 1D, 1E, 4C, 4D*　　*Grade 3 TAAS Objectives: 4, 6, 7, 10, 11, 12, 13*

Objective The student will use mathematics to compare mass and volume of popped and unpopped corn.

Activity What's Poppin'?

Materials Popcorn (unpopped), popcorn popper (preferably one not needing oil), a variety of measurement tools for mass and volume, chart paper and materials for recording

Resources *Popcorn* by Tomie de Paola

Procedure

Introduction:

1. Motivate an interest in the change in popcorn kernels when they are popped. For example:
   - Show the class a container of popped popcorn (made previously from 2 oz. of kernels) and 2 oz. of unpopped kernels. Let the class hypothesize which would give them more to eat. Encourage reasoning to justify their choices.
   - Present the class with the problem of planning a party with popcorn for refreshments. How many ounces of popcorn kernels would we need to pop for everyone to have 400mL of popped popcorn?
   - Present the class with the problem of making the decision of what kind of popcorn to buy for party refreshments. (One of the criteria for deciding would be which popcorn provides the most to eat.) How can we determine which one is best?
   - Read *Popcorn* by Tomie de Paola.
2. Work with the whole class to develop a plan for comparing the popcorn before and after it is popped. Important points of discussion would include characteristics that you would want to measure, tools and units to use to measure the popcorn kernels before popping compared to tools and units to use to measure the popcorn after popping, and methods for recording data.
3. Have students make predictions about the measurements they have decided to make and graph the predictions.
4. Have each group measure out 2 oz. of kernels to pop (or have popped by the teacher). Make sure each group makes the agreed upon measurements before popping the kernels.
5. Have each group make the agreed upon measurements of the popped popcorn and record its data.
6. Combine the groups' data into a class set of data. For example:

Mass of kernels	Mass of popped corn	Volume of kernels	Volume of popped corn

7. Have students construct questions, statements, and story problems from the data.

Exploration:

- Do you need to count the kernels? Why or why not?
- Why did you choose this type of measurement device over another?
- How will you record your information? Why?
- What things do you need to make sure to keep the same, if anything? Why?

- What did you decide to do with the unpopped kernels? Why?
- How did the mass of the popcorn change after being popped?
- How did the volume of the popcorn change after being popped?
- Do you see a pattern?
- How could you explain the changes you see in your data?
- How do you think your data compares with another group's?
- What do you think would happen if you measured and popped another 2 oz. of popcorn?

Extension:
- Use other types of food to investigate changes. For example, compare peanuts (or other seeds) before and after shelling, compare grapes (or other fruits) before and after drying, compare popsicles before and after melting, compare beans before and after soaking.
- Discuss how the data collection should change if salt, butter, or oil is used when popping the corn. Use the new plan to collect a new set of data and compare it to the old.
- From the data, determine how many ounces of popcorn would need to be popped for each person in the class to have 1 cup.
- Predict what would happen if 2 oz. of popcorn was popped for only half the amount of time. Would the volume be half as much?

Summary:
- What techniques did your group use to measure? Which ones do you think worked the best?
- What changes did you notice in the mass after popping?
- What changes did you notice in the volume after popping?
- What patterns did you notice in the changes in mass? Volume?
- If it takes up so much more space, why didn't it weigh more?
- How can we use the data to answer our question (e.g., What volume of kernels is needed to provide each person in the class with a certain amount for a party?).
- How was your groun's techniques for measuring and recording data different from, or similar to, the techniques of another group?
- If we repeated the experiment, would the results be the same? Why or why not?
- Why did we all start with 2 oz. of kernels?
- Did the number of kernels you started with match the number of pieces of popcorn after popping? Why or why not?
- How did we use mathematics in this activity?
- How could we use the information we gained in this activity?

Assessment

Questions:
- What are some questions you could answer using the data from the table?
- What are some mathematical stories you could write using the data from the table?
- (See also summary questions.)

Observations:
- Did student discussion indicate an understanding of the problem?
- Did student discussion indicate an understanding of the characteristics to be measured?
- Did students develop appropriate strategies for measuring?
- Did students develop appropriate strategies for recording data?
- Were students working collaboratively within the group?

Tasks:
- Write, or dictate, a story about how this information might be helpful to you in the future.
- Write about, dictate, or draw a picture of a situation using the data in the table.

Objective The student will describe and compare attributes of three-dimensional geometric figures.

Activity Roll or Slide?

Materials Geometric objects (both commercial and real-world: boxes, cans, balls, ice cream cones); observation-recording chart; ramp (a board with one end on a stack of books)

Procedure

Introduction:

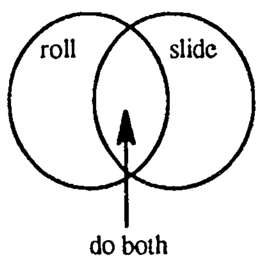
1. Display a collection of solids.
2. Select one object at a time and have students identify other, similarly shaped objects in the room.
3. Allow time for students to handle the solids and observe the attributes of each one.
4. Create a ramp on a table or on the floor to test each solid's ability to roll or slide.
5. Before releasing the object down the ramp, have students predict whether each one will roll, slide, or be able to do both.
6. Release the solid and have students observe the results. There may be some discussion about the need for more than one trial.
7. Have students complete the chart with predictions, then use the objects and the ramp to test their predictions and correct their charts, if necessary.

Exploration:

- What is this shape's name?
- How many of its attributes can you name?
- Can you find other objects with the same attributes?
- How are these two objects alike? How are they different?
- Why did you predict this object would roll? Slide? Both?
- What decisions do you have to make in testing the objects?

Extension:

- Instead of using the recording chart, make a graph on the floor or on a desktop using the actual objects.
- Use yarn to make a large Venn diagram on the floor or on the desktop. Label the circles and the intersection an place the actual objects inside the appropriate sections.



Summary:

- What are some things you discovered?
- Were your predictions correct at first? Did they improve? Why?
- How is the cube different from the sphere?
- Are they alike in any ways?

- How is the ice cream cone different from the can?
- Did more objects roll or slide?
- What pattern did you see in the objects that did both?
- Did some objects have more than one way of being tested? Which shapes had the most ways? Which shapes had the least?

Assessment

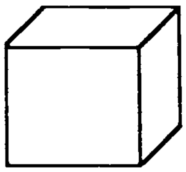
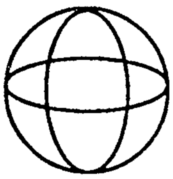
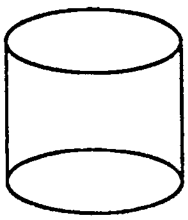
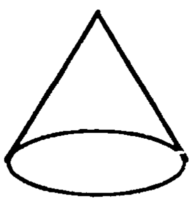
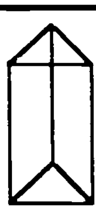
Questions:
(See summary questions.)

Observations:
- Can students name and describe the geometric solids?
- Did they use accurate vocabulary to describe similarities and differences in the shapes?
- Did they test their objects on the ramp from every possible position?
- Did the students' predictions become more accurate as the activity continued?
- Were students able to explain their reasoning?

Tasks:
- Design an object that would only roll.
- Design an object that would only slide.
- Design an object that can do both.
- Write three summary statements in your mathematics journal about shapes that roll, slide, or do both.

Real-World Solids

Name: ______________________________

Object	Roll?	Slide?	Both?
 cube			
 sphere			
 cylinder			
 cone			
 rectangular prism			

EEs: 7A, 7B, 7C, 7D | *Related EEs: 1A, 1B, 1C, 1D, 1E, 3E, 4G* | *Grade 3 TAAS Objectives: 1, 5, 6, 10, 11, 12, 13*

Objective The student will use sampling and knowlege of fractions to draw conclusions about the results of spinning spinners and making sums.

Activity Spinning Sums

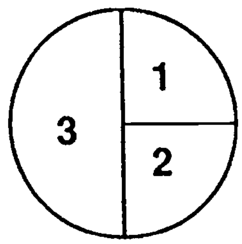
Materials Copies of spinners, large paper clips, markers, one-inch or two-cm grid paper

Resources *A Collection of Math Lessons from Grades 1 through 3* by Marilyn Burns and Bonnie Tank

Procedure

Introduction:

1. Have each student mark one of the spinners like this:



2. Ask students what results they think they would get after spinning the spinner many times, and why.
3. Have each student spin the spinner and record the results until one of the numbers has occurred 10 times.
4. Have students work with partners to translate their results onto a bar graph on the grid paper (one square colored per number spun), cut the bars apart, and glue them onto a class bar graph to show the class's results.
5. Discuss the results represented on the class bar graph.
6. Discuss the following situation: Suppose you and your partner each spin your spinner at the same time and you record the sum of the two numbers that are spun. What results would you expect to get after doing this several times? Why?
7. Have students perform the experiment (spinning two spinners and recording the sum of the two numbers spun), record their results on a bar graph, cut apart the bars, and glue them onto a class bar graph.
8. Discuss the class's results.

Exploration:

For spinning a single spinner:

- What number do you think will occur most often? Why?
- How do you think the arrangement of the space on the spinner might affect the outcomes?
- What is important about the way you spin the spinner?
- How can we record our data? What other ways can we think of? Which of these ways do you think would work the best? What does each method of recording show us that the others do not?
- Do you see a pattern in the data? Do your data match your predictions?
- Why do you think the data turned out the way it did?
- Do the class's data look the same as your data? Why or why not?

For spinning two spinners to make a sum:
- How are you going to record your data?
- How many different sums are possible?
- Do you expect any sum to occur more often than any other? Why or why not?
- How do you think the arrangement of the space on the spinner might affect these outcomes? Why?
- Do you see a pattern in the data? Do your data match your predictions?
- Why do you think the data turned out the way it did?
- Do the class's data look the same as your data? Why or why not?
- If you spun the spinners several times for another set of sums, would you get the same results? Why or why not?

Extension:
- Compare your results with another group's.
- Try the experiments with a different kind of spinner (e.g., putting the numbers in different places, making different-sized sections, putting different numbers on the spinners).
- Repeat your trials to see if you get the same results.
- Design a spinner that would give whole number sums from 5 through 8 with 8 occurring most often.

Summary:
- What conclusions can you make from the combined data of the whole class?
- Did the class data support your prediction? If not, how was it different?
- Why do you think ____ occurred so often?
- Why did some numbers occur so many more times than others?
- Did any outcomes occur exactly the same number of times? Would you expect any of them to? Why or why not?
- If you repeated the experiment, what would you expect to be the same? What different?
- What questions can't be answered from your data?
- What did you learn from this activity?
- What are you still not sure about in this activity?
- In what other situation(s) might you need to use this same kind of thinking?
- How did you use mathematics to help you make your conclusions?

Assessment

Questions:
- What if the two small sections of the spinner were labeled 2 and 3 and the large section was 1? How would you expect the results to change and why?
- What conditions are important when making the spins? Why?
- (See also summary questions.)

Observations:
- Were students actively participating?
- Were partners working together to solve the problem?
- Were students designing an appropriate way to record the outcomes?
- Were students discussing alternative approaches to recording the outcomes?
- Were students using appropriate thinking strategies for solving the problem, or were they making random decisions?
- Were students using their data to support conclusions they were making?
- Were students using knowledge about number to make their conclusions?

Tasks:

- Given a spinner, predict the possible outcomes and a reasonable pattern to expect in the results.
- Given a set of results (perhaps some graphed data), choose or design a spinner that would be likely to produce these results.

Objective The student will display all possible arrangements of a given set of objects and find a pattern in the results.

Activity Double Dipping

Materials 60 construction circles per group, 12 each of 5 different colors (to represent the ice cream scoops); 30 paper cones per group; a large piece of butcher paper per group (to glue the different kinds of cones for the Ice Cream Shop poster); glue

Procedure

Introduction:

1. Tell the class to imagine they are working in an ice cream shop and must make a poster showing all the different kinds of double-scoop cones that the shop can make from 5 flavors.
2. To begin, give the whole class a similar but less complex problem. "Suppose your shop had only two flavors, how many different cones could you make?" Have the students arrange the cones and ice cream on their tables as you record their responses on the chalkboard or the overhead. Discuss the meaning of "different cones." Listen for questions such as "Is chocolate-vanilla the same as vanilla-chocolate?" "Can we use chocolate-chocolate as one type of cone?" Let each group make their own decisions about what is "different." Note that groups counting vanilla-chocolate as the same as chocolate-vanilla will have results different from groups that count them as different arrangements.
3. When the class understands what has been presented so far, discuss the problem to be solved. "Our shop did so well with 2 flavors, we are ready to expand to 5 flavors. In your groups, you are to find all the different kinds of double-scoop cones your shop can make with 5 flavors. Each group is to use the circles and cones to make a poster showing all the different kinds of cones that can be made."

Exploration:

While the groups are making their cones, you can ask the following questions:

- How do you know you have found all the kinds of cones?
- Do you see any that are the same?
- What strategies are you using?
- Are you satisfied with your strategy? Why?
- Does everyone in your group understand your strategy? If I asked you to, could you explain your strategy to someone from another group?

Extension:

- "Now that you have found all the kinds of double-scoop cones, can you find how many cones you can make if you had the same number of flavors (5) but 2 different kinds of cones? Use paper and markers or crayons to sketch your answer."
- Suppose you own a pizza shop and have 5 different toppings available. How many different 2-topping pizzas could you make?
- Make an organized list of all the different pizzas you could make. How will you know when you have found them all?
- Suppose you decided to sell 3-topping pizzas? How many different pizzas could you make? Make an organized list and be prepared to describe how you knew you had found them all.
- How is this activity like the ice cream shop activity? How is it different?

Summary:
The end result will vary depending on the rules that the group used. If chocolate-vanilla is considered to be the same as vanilla-chocolate, then there are 15 possible different cones (counting chocolate-chocolate, etc.) If, however, the groups decided that chocolate-vanilla is different from vanilla-chocolate, there are 25 possible arrangements. The following questions should be used to discuss and summarize the activity:

- How did you organize as a group?
- How did you decide when you had found all the cones? Did you think your group found all of them?
- What plan did you use to arrange your cones on the poster? Did you use a certain order?
- Did you see or use any patterns? (How many cones have chocolate on top? Vanilla on top?)
- Can you explain and write a description of your pattern?

Assessment

Questions:

- What questions could be answered from the information on your poster? What questions could not be answered by the information on your poster?
- (See also summary questions.)

Observations:

- Were students using organized searches for the different cones?
- Did students use appropriate methods for organizing their results on their poster displays?
- Were students able to explain their answers?
- Were students working together in their groups?

Tasks:

- Write a short report to go with your poster, describing the problem and your solution.
- In your mathematics journal, write a description of a pattern you observed in the double-dip ice cream cones.

Evaluation

Philosophy

NCTM's *Professional Standards for Teaching Mathematics* and *Curriculum and Evaluation Standards for School Mathematics* (the *Standards*) emphasize the connection between assessment of students and analysis of instruction. In other words, mathematics teachers should monitor students' learning (both formatively and summatively) in order to assess and adjust teaching. Teachers must observe and listen in order to tailor teaching strategies. Information about what students are understanding should be used to revise and adapt short- and long-range plans, and students' understandings should guide teachers in shaping the learning environment. Also, teachers are responsible for describing students' learning to administrators, parents, and students themselves.

Students' mathematical power depends on various understandings, skills, and dispositions. The development of students' abilities to reason mathematically—to conjecture, justify, and revise based on evidence and to analyze and solve problems—must be assessed. A student's disposition toward mathematics (confidence, interest, perseverance, etc.) is also a key dimension that teachers should monitor.

The importance of using assessment to improve instruction is crucial. Information should be gathered from multiple sources using numerous assessment techniques and modes that are aligned with the curriculum. Assessment techniques must reflect the diversity of instructional methods implied in the *Standards* and the various ways students learn and process information. Instructional decisions should be based on this convergence of information from different sources.

In summary, the following aspects of students assessment and program evaluation should receive increased and decreased attention (NCTM 1989):

Increased Attention

- Assessing what students know and how they think about mathematics
- Having assessment be an integral part of teaching
- Focusing on a broad range of mathematical tasks and taking a holistic view of mathematics
- Developing problem situations that require the applications of a number of mathematical ideas
- Using multiple assessment techniques, including written, oral, and demonstration formats
- Using calculators, computers, and manipulatives in assessment
- Evaluating the program by systematically collecting information on outcomes, curriculum, and instruction
- Using standardized achievement tests as only one of many indicators of program outcomes

Decreased Attention

- Assessing what students do not know
- Having assessment be simply counting correct answers on tests for the sole purpose of assigning grades
- Focusing on a large number of specific and isolated skills organized by a content-behavior matrix
- Using exercises or word problems requiring only one or two skills
- Using only written tests
- Excluding calculators, computers, and manipulatives from the assessment process
- Evaluating the program only on the basis of test scores
- Using standardized achievement tests as the only indicator of program outcomes

Types of Evaluation

While paper and pencil tests are one useful medium for judging aspects of students' mathematical knowledge, teachers need information gathered in a variety of ways and using a range of sources. Observing, interviewing, and closely watching and listening to students are all important means of assessment. While monitoring students, teachers can evaluate the learning environment, tasks, and discourse that have been taking place. Using a variety of strategies, teachers should assess students' capacities and inclinations to analyze situations, frame and solve problems, and make sense of concepts and procedures. Such information should be used to assess how students are doing, as well as how well the tasks, discourse, and environment are fostering students' mathematical power and then to adapt instruction in response.

Assessment instruments and techniques should be properly aligned with the curriculum to enable educators to draw conclusions about instructional needs, progress in achieving the goals of the curriculum, and the effectiveness of a mathematics program. That is, the content, processes, and skills assessed must reflect the goals, objectives, and breadth of topics specified in the curriculum. The particular emphases of the assessment should reflect the emphases of instruction. For example, primary children, whose understanding of fractions is closely tied to the use of physical materials, should be encouraged to use such materials to demonstrate their conceptual knowledge. Assessment items need to be structured around the central ideas of the curriculum and need to provide opportunities for students to demonstrate their understanding of the connections among major concepts. In addition, assessment must reflect the relative emphasis placed on technology during instruction; to the extent that calculators and computers have been important during instruction, they should also be available during assessment.

Assessment techniques suggested in the *Standards* include multiple-choice, short-answer, discussion, and open-ended questions; interviews; homework; projects; journals; essays; portfolios; presentations; and dramatizations. Assessment can occur during and after whole-group explorations, during whole-group discussions, in sharing sessions, during individual conferences, during small-group conferences, while students are working on projects, after completion of projects, when students are engaged in self-evaluation tasks, and continually while students are explaining, justifying, debating, and questioning ideas and concepts.

Using Portfolios. Student portfolios are becoming more prevalent as a means of keeping a record of student progress in mathematics. Teachers have always kept folders of students' work, but portfolios should have more focus and be more important for assessment. An assessment portfolio is a planned selection of a student's work collected throughout the school year. Teachers as well as students should be allowed to choose the items to be included in portfolios, since it gives a good indication of what is valued in the work the students do throughout the school year. A portfolio might include samples of student-produced written descriptions of the results of practical or mathematical investigations; pictures and dictated reports from younger students; extended analyses of problem situations and investigations; descriptions and diagrams of problem-solving processes; statistical and graphic representations; responses to open-ended questions or homework problems; group reports and photographs of student projects; copies of awards or prizes; video, audio, and computer-generated examples of student work; and other material based on project ideas developed with colleagues.

Using Writing. Communication in mathematics has become important as we move into an era of a thinking curriculum. Journals, logs, problem-solving notebooks, explanations, justifications, and reflections are ways to include writing in the mathematics curriculum. Students should be urged to discuss ideas with each other, and to ask questions, to diagram and graph problem situations for clarity. Writing in mathematics classes, once rare, is now vital. In particular, mathematics journals can include the following:

- vocabulary definitions written in the student's own words along with explanations of how the terms are used in mathematics

- rules or procedures written as if explained to a friend in a letter or to another student who was absent during the instruction

- free writing, including what students think they will learn in an experience, descriptions of accomplishments, how students can use what they've learned, what isn't fully understood or is causing difficulty, examples in the real world related to the mathematics learned, a discovery made or additional ideas and conjectures related to the topic, and what else students might want to learn about.

These writing experiences are also important mathematics learning experiences in that they:

- help students become more active in their own learning

- help students internalize what they are learning to make it more meaningful

- allow students to express their feelings and attitudes toward mathematics

- give students a source they can use for studying

- allow students to reflect upon and clarify their own thinking

- give students the opportunity to share with each other what they are learning, also allowing them to learn from one another

- allow students to go beyond what they are learning in class and to make conjectures and connections

- give students the opportunity to think of mathematics as existing outside the classroom

- give students the opportunity to communicate with the teacher in an informal setting

- give the teacher an idea of how students are thinking

- allow the teacher to informally assess student learning (whether it be pre- or post-assessment)

Using Teacher Observations. Teacher observations can be broken down into two levels: formal and informal. Formal observations include checklists, comment cards, and summaries. Informal observations include mental notes. Students should be observed both individually and as they work in groups. When using observations, a teacher should look for students' learning styles, students' ideas, communication techniques, cooperation strategies, and use of manipulatives. Some possible questions that can guide observations of students doing mathematics are:

- Does the student consistently work alone or with others?

- Does the student try to explain organizational and mathematical ideas?

- Does the student synthesize and summarize his or her own or a group's thinking?

- Does the student work with the group to agree on a plan or structure for tackling the task?

- Individually or within the group, does the student choose and use appropriate manipulatives?

Using Questioning. Asking the right question is an art to be cultivated by all educators. Low-level quizzes that ask for recall or simple computations are over used and over done. Using good, high-level open-ended questions that give students a chance to think are one of the goals of mathematics assessment. These questions might be used as teaching or leading questions as well as for assessment purposes. Both questions and responses may be oral, written, or demonstrated by actions taken. When using oral questions, the teacher can prepare a list of possible questions ahead of time. (For examples, see the sample activities in the previous section.) The teacher should allow for plenty of wait time. The teacher may keep a written record of observations during the questioning time to use for formal assessment. Questioning for assessment should occur in several places during instruction:

- during introductory activities to assess students' prior knowledge and experience
- during exploration to focus students' attention on important concepts and connections
- after instruction, in order for students to summarize results, reflect on their experience, and clarify their thoughts

Using Student Presentations. Student presentations can take many forms, including oral explanations, oral presentations, and projects. One of the best ways to assure the connection between instruction and assessment is to embed assessment into instruction. When students become involved in projects or investigations, assessment becomes natural and invisible. Student presentations may be related to connections within mathematics and connections outside mathematics. When evaluating student presentations, the teacher should look for whether the student can identify and define the problem; make a plan; collect needed information; organize the information and look for patterns; discuss, review, revise, and explain results; and produce a quality product or report.

Using Performance Assessment. Performance assessment involves giving a group of students, or an individual student, a mathematical task that may take from half an hour to several days to complete or solve. The object of this form of assessment is to look at how students are working, as well as at the completed tasks or products. Performance assessment requires the teacher to look at how students solve a problem. Performance activities may be videotaped, tape recorded, or recorded in writing. The task might be from any mathematical content area and might include some connections such as with science, social studies, language arts, or fine arts. Performance assessment is an excellent place for students to use manipulatives to demonstrate understanding of mathematics content. Information from performance assessment can be recorded using rubrics that assign point values to important aspects of the problem-solving process. For example, the following assessment criteria could be used during observation or based on written work to judge a student's involvement in problem solving:

1. Understanding the Problem:

0 points	Does not understand the problem
1 point	Misunderstands part of the problem
2 points	Completely understands the problem

2. Choosing and Implementing a Solution Strategy

0 points	Makes no attempt or uses a totally inappropriate strategy
1 point	Chooses a partly correct strategy based on interpreting part of the problem incorrectly
2 points	Chooses a strategy that could lead to a correct solution if used without error

3. Getting the Answer

0 points	Gets no answer or a wrong answer based on an inappropriate solution strategy
1 point	Makes a copying error or computational error; gets partial answer for a problem with multiple answers; or labels answer incorrectly
2 points	Gets correct solution

Some excellent resources on assessment, in addition to the NCTM *Curriculum and Evaluation Standards*, include *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions* (1991) and *Assessment alternatives in Mathematics* (Stenmark, 1989).

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Wahl, J. and Wahl, S. (1976). *I can count the petals of a flower*. Reston, VA: National Council of Teachers of Mathematics.

Children's Trade Books

Numerous children's books have the potential for motivating rich mathematics activities. This short list gives the bibliographic information of the books mentioned in the activities in this series of curriculum guides for elementary mathematics (Grades 1 - 5).

Anno, M. (1982). *Anno's counting house.* New York: Philomel Books.

Anno, M. (1983). *Anno's mysterious multiplying jar.* New York: Philomel Books.

Aardema, V. (1976). *Why mosquitoes buzz in people's ears.* New York: Dial Books for Young Readers.

Bemelmans, L. (1960). *Madeline.* New York: Viking Children's Books.

Bishop, C. H. (1938). *Five Chinese brothers.* New York: Coward.

Brier, C. (1983). *The shoemaker and the elves.* New York: Lothrop, Lee & Shepard.

Carle, E. (1977). *The grouchy ladybug.* New York: Scholastic, Inc.

Carle, E. (1989). *The very hungry caterpillar.* New York: Putman.

Dahl, R. (1964). *Charlie and the chocolate factory.* New York: Alfred Knopf.

de Paola, T. (1978). *The popcorn book.* New York: Holiday House.

Ehlert, L. (1990). *Fish eyes: A book you can count on.* San Diego: Harcourt, Brace, Jovanovich.

Faucher, E. (1989). *Honey, I shrunk the kids.* New York: Scholastic, Inc.

Flournoy, V. (1985). *The patchwork quilt.* New York: Dial Books for Young Readers.

Freeman, D. (1968). *Corduroy.* New York: Viking Press.

Giganti, Jr., P. (1992). *Each orange had 8 slices.* New York: The Trumpet Club.

Haley, G. E. (1971). *Noah's ark.* New York: Atheneum.

Hogrigan, N. (1966). *Always room for one more.* New York: Holt.

Hulme, J. N. (1991). *Sea squares.* New York: Hyperion Books.

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Silverstein, S. (1981). *A light in the attic.* New York: Harper and Row.

Silverstein, S. (1974). *Where the sidewalk ends.* New York: Harper and Row.

Singer, M. (1991). *Nine o'clock lullaby.* New York: HarperTrophy.

Steptoe, J. (1980). *Daddy is a monster . . . sometimes.* New York: HarperTrophy.

Ward, C. (1988). *Cookie's week.* New York: Putnam.

West, C. (1987). *Ten little crocodiles.* New York: Barron's.

Yolen, J. (1987). *Owl moon.* New York: Philomel Books.

Software

The following list contains bibliographic information for the software packages mentioned in this series of curriculum guides for elementary mathematics (Grades 1 - 5). Other appropriate software may be obtained from these and other companies.

Blockers and Finders from WINGS for learning/Sunburst Communications, 1600 Green Hills Road, P.O. Box 660002, Scotts Valley, CA 95067-0002.

Geometric preSupposer from WINGS for learning/Sunburst Communications, 1600 Green Hills Road, P.O. Box 660002, Scotts Valley, CA 95067-0002.

Hands-On Math: Volumes 1, 2, and 3 from Ventura Educational Systems, 3440 Brokenhill Street, Newbury Park, CA 91320.

Suggested Manipulatives

The following is a list of the manipulative materials used in the activities in this series of curriculum guides for elementary mathematics (Grades 1 - 5):

Calculators
Base ten blocks
Coins and bills (play or real money)
Interlocking cubes
Colored tiles
Pattern blocks
Cuisenaire rods

Graphing floor mat
Polyhedral dice (including the regular cube)
Colored chips
Two-color counters

Attribute blocks
Geoblocks
Geoboards
Tangrams
Plastic mirrors
Wooden or plastic models of geometric solids

Balance scales and masses (customary and metric)
Spring scales
Tape measures (customary and metric)
Rulers (customary and metric)
Meter sticks and yardsticks
Trundle wheels
Graduated cylinders
Measuring cups and spoons
Stopwatches

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(1) acceptance policies on student transfers from other school districts;

(2) operation of school bus routes or runs on a nonsegregated basis;

(3) nondiscrimination in extracurricular activities and the use of school facilities;

(4) nondiscriminatory practices in the hiring, assigning, promoting, paying, demoting, reassigning, or dismissing of faculty and staff members who work with children;

(5) enrollment and assignment of students without discrimination on the basis of race, color, or national origin;

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