

DOCUMENT RESUME

ED 384 493

SE 055 269

TITLE Guidelines for Teaching Grade 1 Mathematics.
 INSTITUTION Texas Education Agency, Austin. Div. of Curriculum Development.
 PUB DATE [94]
 NOTE 78p.; For related guides, see SE 055 270-276.
 AVAILABLE FROM Texas Education Agency, 1701 North Congress Avenue, Austin, TX 78701-1494 (Publication No. CU4 211 12: \$4).
 PUB TYPE Guides - Classroom Use - Teaching Guides (For Teacher) (052)
 EDRS PRICE MF01/PC04 Plus Postage.
 DESCRIPTORS *Behavioral Objectives; Courseware; Grade 1; Interdisciplinary Approach; Lesson Plans; Manipulative Materials; *Mathematics Curriculum; *Mathematics Instruction; Primary Education; State Curriculum Guides; *Student Educational Objectives; *Student Evaluation; *Teaching Methods; Technology Connections (Mathematics); *NCTM Professional Teaching Standards; Texas Education Agency
 IDENTIFIERS

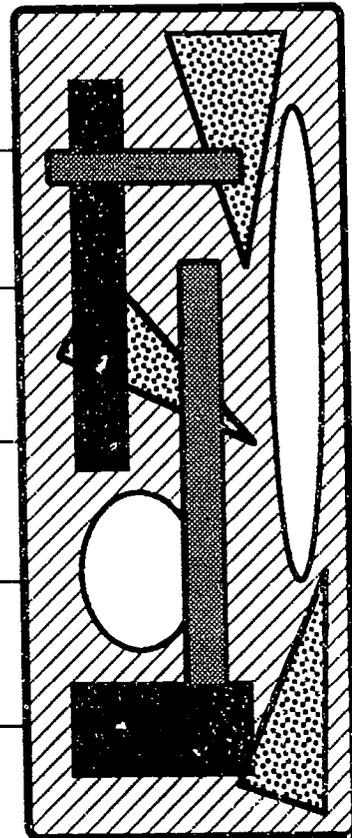
ABSTRACT

This document Agency is designed to assist teachers and other school personnel in the planning and teaching of the first grade mathematics course. Contents include: (1) Overview of Grade 1 Mathematics (mission statement, purpose and philosophy, goals, National Council of Teachers of Mathematics' Professional Standards for Teaching Mathematics, instructional strategies, and uses of technology and manipulatives); (2) Essential Elements of Instruction with sample learning objectives and sample clarifying activities; (3) Texas Assessment of Academic Skills (TAAS) (focus, domains, objectives, and targets); (4) Sample Lessons for Teaching Grade 1 Mathematics; and (5) Evaluation (philosophy and types of evaluation). TAAS features three domains: concepts, operations, and problem solving. The Essential Elements are: problem solving; patterns, relations, and functions; number and numeration concepts; operations and computation; measurement; geometry; and probability, statistics, and graphing. Suggested resources include children's trade books, software, and suggested manipulatives. Contains 22 references.
 (MKR)

 * Reproductions supplied by EDRS are the best that can be made *
 * from the original document. *

ED 384 493

GUIDELINES FOR TEACHING GRADE 1 MATHEMATICS



Texas Education Agency
Austin, Texas

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION
CENTER (ERIC)

- This document has been reproduced as received from the person or organization originating it.
- Minor changes have been made to improve reproduction quality.
- Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

PERMISSION TO REPRODUCE THIS
MATERIAL HAS BEEN GRANTED BY

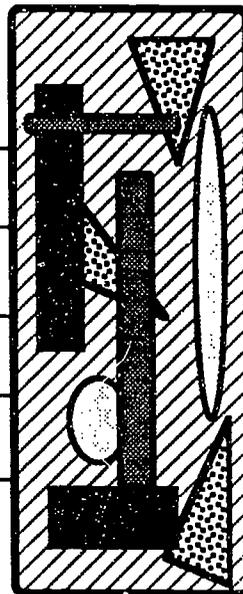
L. Kemp

TO THE EDUCATIONAL RESOURCES
INFORMATION CENTER (ERIC)

2
BEST COPY AVAILABLE

255 269

GUIDELINES
FOR
TEACHING
GRADE **1**
MATHEMATICS



This publication is not copyrighted; any or all sections may be duplicated.
After an initial free distribution to authorized institutions, additional copies may
be purchased for \$4.00 from the Publications Distributions Office.

Texas Education Agency
1701 North Congress Avenue
Austin, Texas 78701

FOREWORD

Guidelines for Teaching Grade 1 Mathematics is designed to help teachers and other school district personnel plan and teach first grade mathematics. The publication presents the philosophy and intent of the course and discusses the required essential elements, TAAS instructional targets, instructional strategies, and the use of technology and manipulatives. Also included are sample objectives and activities to illustrate how the essential elements for first grade mathematics can be taught. School district personnel may want to use these suggestions to develop their own curriculum documents for the course.

We hope these guidelines will be useful in planning and teaching mathematics in Grade 1 and in equipping the mathematics classroom.

Lionel R. Meno
Commissioner of Education

STATE BOARD OF EDUCATION

(State Board for Vocational Education)

CAROLYN HONEA CRAWFORD, Beaumont
Chairman of the State Board of Education
District 7

WILL D. DAVIS, Austin
Vice Chairman of the State Board of Education
District 10

MARY HELEN BERLANGA, Corpus Christi
Secretary of the State Board of Education
District 2

Board Members

ALMA A. ALLEN, Houston
District 4

GERALDINE MILLER, Dallas
District 12

JACK CHRISTIE, Houston
District 6

RENE NUÑEZ, El Paso
District 1

ELEANOR N. CONRAD, Dallas
District 15

ROBERT H. OFFUTT, San Antonio
District 5

MONTE HASIE, Lubbock
District 15

DIANE PATRICK, Arlington
District 11

WILLIAM L. HUDSON, Wichita Falls
District 14

MARY KNOTTS PERKINS, Lufkin
District 8

PATSY JOHNSON, Sulphur Springs
District 9

ESTEBAN SOSA, San Antonio
District 3

LIONEL R. MENO
Commissioner of Education
(Executive Officer of the State Board of Education)

**Committees
of the
State Board of Education**

PERSONNEL

RENE NUÑEZ, Chairman
ALMA A. ALLEN
JACK CHRISTIE
ELEANOR N. CONRAD
DIANE PATRICK

STUDENTS

GERALDINE MILLER, Chairman
MARY HELEN BERLANGA
PATSY JOHNSON
ROBERT H. OFFUTT
MARY KNOTTS PERKINS

SCHOOL FINANCE

WILL D. DAVIS, Co-chairman
WILLIAM L. HUDSON, Co-chairman
CAROLYN HONEA CRAWFORD
MONTE HASIE
ESTEBAN SOSA

LONG-RANGE PLANNING

MARY KNOTTS PERKINS, Chairman
ALMA A. ALLEN
JACK CHRISTIE
ELEANOR N. CONRAD
CAROLYN HONEA CRAWFORD
WILL D. DAVIS
PATSY JOHNSON
DIANE PATRICK

PERMANENT SCHOOL FUND

ESTEBAN SOSA, Chairman
MARY HELEN BERLANGA
MONTE HASIE
WILLIAM L. HUDSON
GERALDINE MILLER
RENE NUÑEZ
ROBERT H. OFFUTT

TEXAS EDUCATION AGENCY

This publication was developed by Agency staff members in the Office of the Deputy Commissioner for Curriculum, Assessment, and Professional Development.

LINDA CIMUSZ
Deputy Commissioner for Curriculum, Assessment, and Professional Development

MARVIN VESELKA
Associate Commissioner for Curriculum, Assessment, and Textbooks

ANN SMISKO
Director, Curriculum Development and Textbooks

Mathematics Section

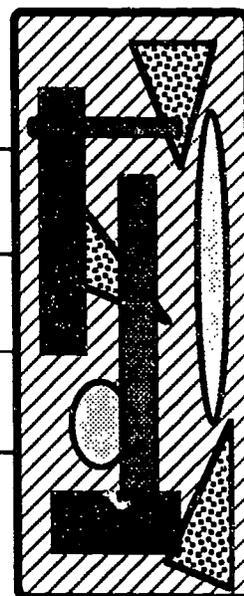
BILL HOPKINS
Director of Mathematics

BARBARA MONTALTO
Assistant Director of Mathematics

CONTENTS

Overview of Grade 1 Mathematics	1
Mission Statement	1
Purpose and Philosophy	1
Goals	2
National Council of Teachers of Mathematics: Professional Standards for Teaching Mathematics	4
Instructional Strategies	7
Uses of Technology and Manipulatives	10
Essential Elements of Instruction	12
Essential Elements of Grade 1 Mathematics with Sample Learning Objectives and Sample Clarifying Activities	12
Texas Assessment of Academic Skills	32
Focus	32
Domains, Objectives, and Targets	32
Sample Lessons for Teaching Grade 1 Mathematics	35
Evaluation	56
Philosophy	56
Types of Evaluation	58
References and Resources	62
Acknowledgments	68

Overview of Grade 1 Mathematics



Mission Statement

Guidelines for Teaching Grade 1 Mathematics is one in a series of eight documents for the first through the eighth grades designed to assist teachers and other school personnel in the planning and teaching of elementary mathematics. The discussions of philosophy, goals, instructional strategies, uses of technology and manipulatives, and aspects of evaluation are provided as starting points for districts to begin the process of developing their own curriculum documents. The essential elements of instruction for each grade level are supported with sample learning objectives, sample clarifying activities, and complete sample lessons. These guidelines should prove useful to district personnel in: (1) planning curriculum, (2) planning instruction, and (3) equipping classrooms for mathematics teaching and learning.

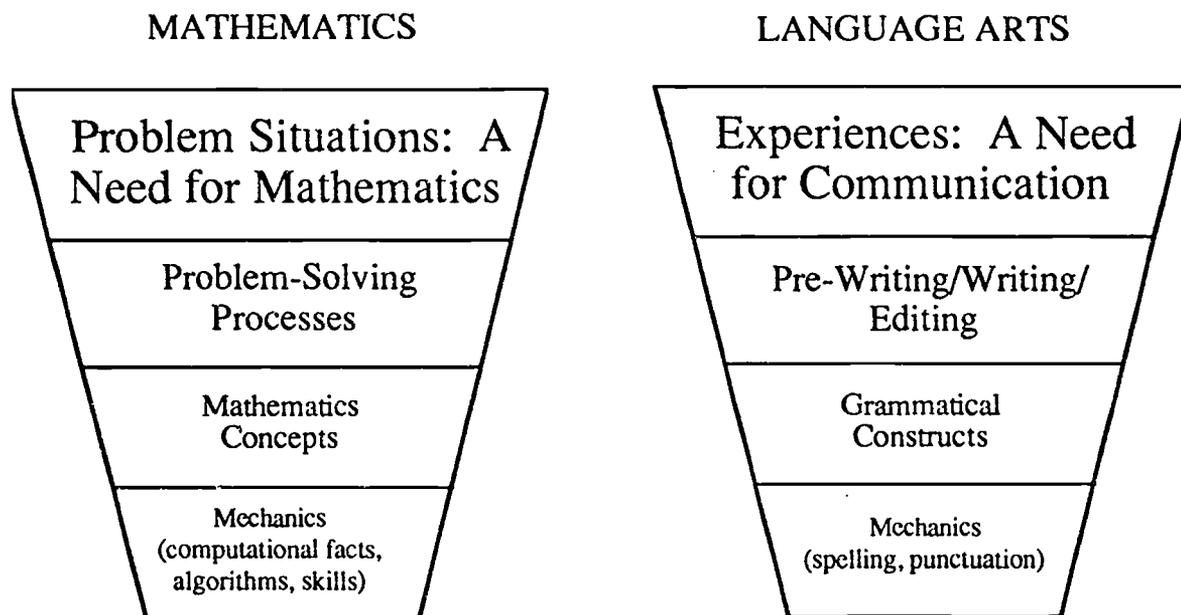
Purpose and Philosophy

Mathematics is useful, exciting, and creative and can be enjoyed by all elementary school students. Problem-solving skills and logical reasoning are developed while students explore and make sense of their world through rich, worthwhile mathematical experiences. Unfortunately, mathematics has been viewed by many students as boring, irrelevant, and routine and as externally dictated by a rigid system of rules governed by standards of speed, accuracy, and memory. In the past, computational facility has been emphasized instead of a broad, integrated view of mathematics. While computational skills are important, learner characteristics and the vitality of mathematics itself cannot be overlooked. Mathematics in the elementary grades should be broad-based and concept driven and should reflect relevant mathematics and connections between mathematics concepts and between these concepts and other disciplines.

Children enter elementary school with a natural curiosity and enthusiasm for learning. Mathematics experiences at the elementary level should tap into these characteristics for children to begin developing mathematical power—the ability to think and communicate, drawing on mathematical ideas and using mathematical tools and techniques. The attitudes students form in

elementary school toward mathematics will determine the choices they make of future mathematics coursework and consequently the availability or loss of educational and career opportunities.

The elementary school mathematics curriculum should emphasize the processes of problem solving, reasoning, communication, and making connections within the contexts of investigating geometry, measurement, probability, statistics, graphing, patterns, and functions, as well as with number, numeration, and operation concepts. Problem solving should be the focus of instruction with skills and concepts being introduced, developed, and applied through meaningful problem situations. Mathematics instruction needs to begin with meaning and purpose in much the same way as elementary teachers present language arts instruction, as reflected in the following graphic illustration:



All students need rich and relevant problem-solving experiences with appropriate teacher guidance and questioning. Such experiences will empower students to build meaning for the mathematics they encounter today and to strengthen reasoning skills needed for the mathematics of tomorrow.

Goals

According to *Curriculum and Evaluation Standards for School Mathematics* developed by the National Council of Teachers of Mathematics (NCTM), the five overall curriculum goals for students are:

- learning to value mathematics
- becoming confident in their ability
- becoming mathematical problem solvers
- learning to communicate mathematically
- learning to reason mathematically

Moreover, the educational system of today demands new societal goals for education:

- mathematically literate workers
- lifelong learning
- opportunity for all
- an informed electorate

Specifically, teaching the mathematics curriculum to elementary school students must be related to the characteristics of the learners and their needs today and in the future.

Everybody Counts (National Research Council, 1989) posits that "self-confidence built on success is the most important objective of the mathematics curriculum" (p. 45). Individuals must be able to use mathematics in their later lives—as employees, parents, and citizens. Ability and disposition to do so often depends on attitudes toward mathematics developed in school.

Through the use of worthwhile mathematical activities investigated in cooperative, group environments, teachers of elementary mathematics can empower their students with strong mathematical understanding and disposition.

National Council of Teachers of Mathematics: Professional Standards for Teaching Mathematics

The *Professional Standards for Teaching Mathematics (NCTM, 1991)* are based on four assumptions about the practice of teaching. These assumptions are abbreviated versions of the more extensive ones found in the original document (NCTM, 1991, pages 21-22).

- (1) The goal of teaching mathematics is to help all students develop mathematical power. Teachers must help every student develop conceptual and procedural understandings of number, operations, geometry, measurement, statistics, probability, functions, and algebra and the connections among ideas. They must engage all students in formulating and solving a wide variety of problems, making conjectures and constructing arguments, validating solutions, and evaluating the reasonableness of mathematical claims.
- (2) What students learn is fundamentally connected with how they learn it. Students' opportunities to learn mathematics are a function of the setting and the kinds of tasks and discourse in which they participate.
- (3) All students can learn to think mathematically. The goals such as learning to make conjectures, to argue about mathematics using mathematical evidence, to formulate and solve problems, and to make sense of mathematical ideas are not just for some group thought to be "bright" or "mathematically able."
- (4) Teaching is a complex practice and hence not reducible to recipes or prescriptions. First of all, teaching mathematics draws on knowledge from several domains: knowledge of mathematics, of diverse learners, of how students learn mathematics, of the contexts of the classroom, school, and society. Good teaching depends on a host of considerations and understandings. Good teaching demands that teachers reason about pedagogy in professionally defensible ways within particular contexts of their own work.

The *Professional Standards for Teaching Mathematics* identifies a particular set of instructional standards for the effective teaching of mathematics. The standards describe the nature of the tasks, patterns of communication, the learning environment, and the analysis of instruction. More specifically, five of these standards focus on instructional strategies. They are:

STANDARD 1: WORTHWHILE MATHEMATICAL TASKS

The teacher of mathematics should pose tasks that are based on:

- sound and significant mathematics;
- knowledge of students' understandings, interests, and experiences;
- knowledge of the range of ways that diverse students learn mathematics;

and that

- engage students' interests;
- develop students' mathematical understandings and skills;
- stimulate students to make connections and develop a coherent framework for mathematical ideas;
- call for problem formulation, problem solving, and mathematical reasoning;

- promote communication about mathematics;
- represent mathematics as an ongoing human activity;
- display sensitivity to, and draw on, students' diverse background experiences and dispositions;
- promote the development of all students' dispositions to do mathematics.

STANDARD 2: THE TEACHER'S ROLE IN DISCOURSE

The teacher of mathematics should orchestrate discourse by:

- posing questions and tasks that elicit, engage, and challenge each student's thinking ability;
- listening carefully to students' ideas;
- asking students to clarify and justify their ideas orally and in writing;
- deciding what to pursue in depth from among the ideas that students bring up during a discussion;
- deciding when and how to attach mathematical notation and language to students' ideas;
- deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;
- monitoring students' participation in discussions and deciding when and how to encourage each student to participate.

STANDARD 3: STUDENTS' ROLE IN DISCOURSE

The teacher of mathematics should promote classroom discourse in which students:

- listen to, respond to, and question the teacher and one another;
- use a variety of tools to reason, make connections, solve problems, and communicate;
- initiate problems and questions;
- make conjectures and present solutions;
- explore examples and counterexamples to investigate a conjecture;
- try to convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers;
- rely on mathematical evidence and argument to determine validity.

STANDARD 4: TOOLS FOR ENHANCING DISCOURSE

The teacher of mathematics in order to enhance discourse, should encourage and accept the use of:

- computers, calculators, and other technology;
- concrete materials used as models;
- pictures, diagrams, tables, and graphs;
- invented and conventional terms and symbols;
- metaphors, analogies, and stories;
- written hypotheses, explanations, and arguments;
- oral presentations and dramatizations.

STANDARD 5: LEARNING ENVIRONMENT

The teacher of mathematics should create a learning environment that fosters the development of each student's mathematical power by:

- providing and structuring the time necessary to explore sound mathematics and grapple with significant ideas and problems;

- using the physical space and materials in ways that facilitate students' learning of mathematics;
- providing a context that encourages the development of mathematical skill and proficiency;
- respecting and valuing students' ideas, ways of thinking, and mathematical dispositions;

and by consistently expecting and encouraging students to:

- work independently or collaboratively to make sense of mathematics;
- take intellectual risks by raising questions and formulating conjectures;
- display a sense of mathematical competence by validating and supporting ideas with mathematical argument.

STANDARD 6: ANALYSIS OF TEACHING AND LEARNING

The teacher of mathematics should engage in ongoing analysis of teaching and learning by:

- observing, listening to, and gathering other information about students to assess what they are learning;
- examining effects of the tasks, discourse, and learning environment on students' mathematical knowledge, skills, and dispositions;

in order to:

- ensure that every student is learning sound and significant mathematics and is developing a positive disposition toward mathematics;
- challenge and extend students' ideas;
- adapt or change activities while teaching;
- make plans both short- and long-range;
- describe and comment on each student's learning to parents and administrators, as well as to the students themselves.

The movement toward this vision of instruction for mathematical empowerment of all students is strongly dependent upon the environment of the classroom, an environment governed in a large part by the decision-making role of the classroom teacher. The NCTM teaching standards identify five major components necessary in the instructional environment for the mathematics classroom and tie these components directly to teachers asking, and encouraging students to ask, appropriate and stimulating questions. The five major instructional components and suggestions for questions are (NCTM, 1991, pp. 3-4):

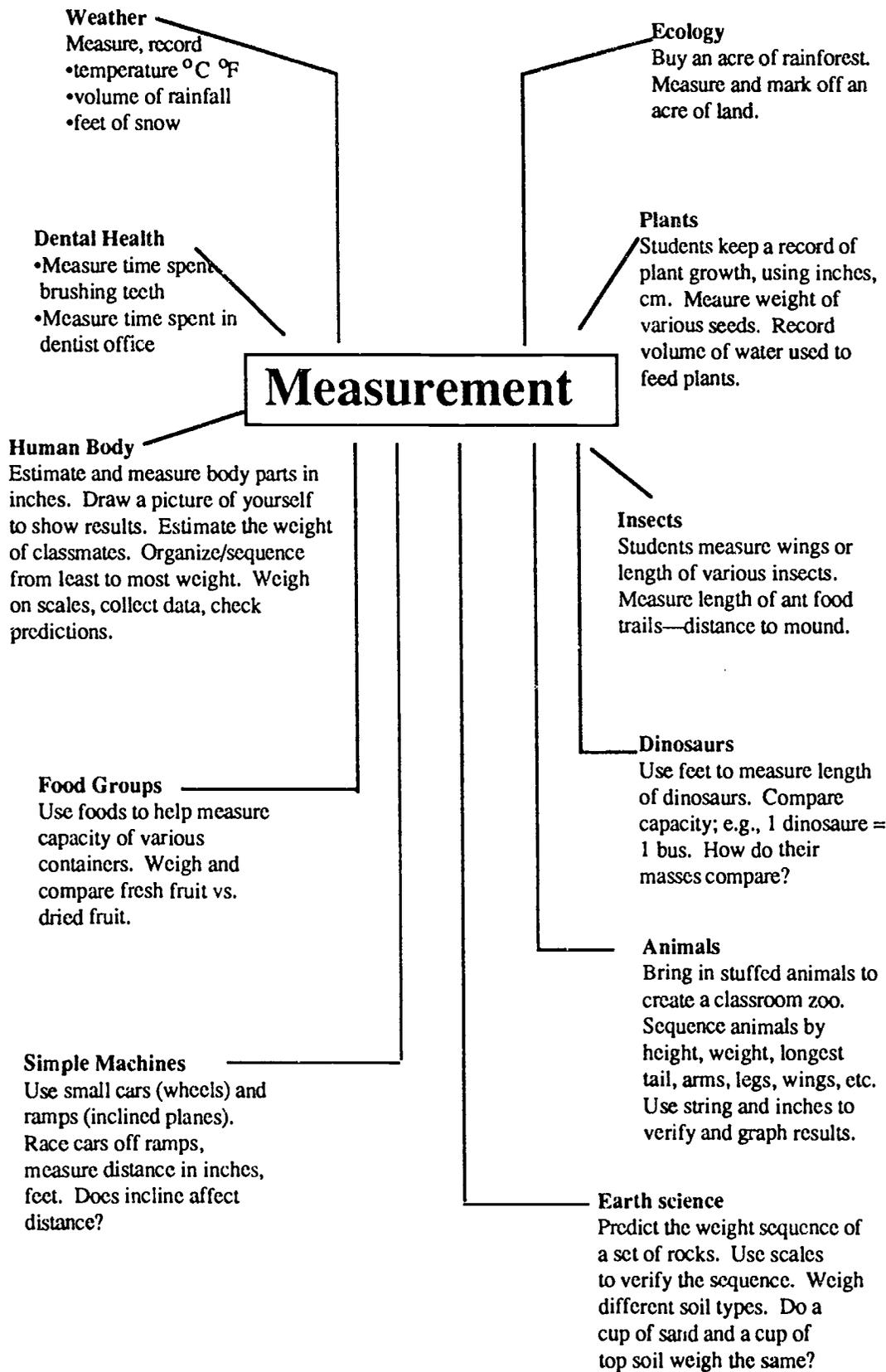
- **Helping students work together to make sense of mathematics**
 - "What do others think about what Janine said?"
 - "Do you agree? Disagree?"
 - "Does anyone have the same answer but a different way to explain it?"
 - "Would you ask the rest of the class that question?"
 - "Do you understand what they are saying?"
 - "Can you convince the rest of us that that makes sense?"
- **Helping students to rely more on themselves to determine whether something is mathematically correct**
 - "Why do you think that?"
 - "Why is that true?"

"How did you reach that conclusion?"
"Does that make sense?"
"Can you make a model to show that?"

- **Helping students learn to reason mathematically**
 - "Does that always work?"
 - "Is that true for all cases?"
 - "Can you think of a counterexample?"
 - "How could you prove that?"
 - "What assumptions are you making?"
- **Helping students learn to conjecture, invent, and solve problems**
 - "What would happen if . . ." What if not?"
 - "Do you see a pattern?"
 - "What are some possibilities here?"
 - "Can you predict the next one? What about the last one?"
 - "How did you think about the problem?"
 - "What decision do you think he should make?"
 - "What is alike and what is different about your method of solution and hers?"
- **Helping students to connect mathematics, its ideas, and its applications**
 - "How does this relate to . . .?"
 - "What ideas that we have learned before were useful in solving this problem?"
 - "Have we ever solved a problem like this one before?"
 - "What uses of mathematics did you find in the newspaper last night?"
 - "Can you give me an example of . . .?"

Instructional Strategies

The following diagrams are examples of one teacher's planning efforts to connect measurement and geometry concepts to life, earth, and physical science units:



Weather

Critical Thinking—Write a paragraph. What if . . . hail were shaped like cones, cubes, pyramids, etc. Create unique cloud shapes using precut construction paper shapes.

Animals

Use measurement info on animals to graph results. Use pattern blocks to create unique animals.

Human Body

Bilateral symmetry—Art project. Draw one side of body; fold, trace other side. Plot measurements of student height or weight on line graphs.

Simple Machines

Explore angles of inclined planes for racing activities. Does incline affect the distance the car travels? Rotational symmetry of various wheels and gears; create efficiency of machines. Use toy wheels, gears, etc. to create a unique machine. What job does it perform?

Food Groups

Bring in a variety of foods. Have students identify foods that closely match geometric solids: cube, cylinder, cone, pyramid, and sphere.

Dental Health

Discuss various teeth and their jobs. What shape (3-D) do these teeth resemble? Critical thinking: What if our teeth were all shaped like cones, spheres? Etc.

Space

Use three-dimensional shapes to make a rocket ship. Use construction paper to create a space-shape monster. Write data identifying the monster's various shape/body parts. Identify planets as spheres.

Insects

Identify symmetry in a variety of insects. Use art to help with this concept; i.e., making butterflies with paint on one side of paper, folding to create mirror image. Look at rotational symmetry in bee hives and at the shape of cells in hives.

Plants

Identify the shapes found in real plants; e.g., leaves, petals, stems. Use pattern blocks to create a unique plant. Identify shapes. Plot growth of plants to create a line graph.

Geometry

Appropriate questioning techniques and meaningful problem-solving situations are two major strategies for effective mathematics instruction.

Uses of Technology and Manipulatives

Calculators and computers are tapped for important roles in mathematics at all levels and across topics. Changes in technology and the broadening of the areas in which mathematics is applied have resulted in growth and changes in the discipline of mathematics itself. The new technology has altered the very nature of the problems important to mathematics and the methods mathematicians use to investigate them.

The NCTM *Curriculum Standards* (1989) call for the following regarding technology in the classroom:

- appropriate calculators for all students at all times
- a computer for every classroom for demonstration
- access to a computer for individual and group work
- students learning to use the computer as a tool for processing information and performing calculations to solve problems

Calculators and computers offer teachers and students an important learning aid. Their potential is great and as yet untapped both in developing concepts and in developing positive attitudes and persistence in problem solving.

Computers can be utilized in a variety of ways in the mathematics classroom, and the appropriateness of a particular approach depends on the goals. Three qualitatively different methods suggested by R. Taylor in *The Computer in the School: Tutor, Tool, Tutee* are:

- as a sophisticated teaching machine
- to be programmed (or taught) by the student
- as a mode for applications in research and development through software that displays graphs, manipulates symbols, analyzes data, and performs mathematical procedures. Applications such as spreadsheets, word processing, data bases, and communication packages have the appeal of matching the classroom's use of technology with that of society's.

Calculator use is not for the purpose of replacing paper-and-pencil computations but to reinforce them. According to N. Kober in *Ed Talk: What We Know About Mathematics Teaching and Learning*, calculator use is apt to sustain independent thought, not replace it. For example, students can be challenged to invent calculator algorithms to replace procedures taught in textbooks. The students explain why their procedures work and debate the advantages and disadvantages of their procedures over others. Calculators are programmable, produce graphics, and work in fractional and algebraic notation. Teachers need to be innovative; they need to experiment and share ideas.

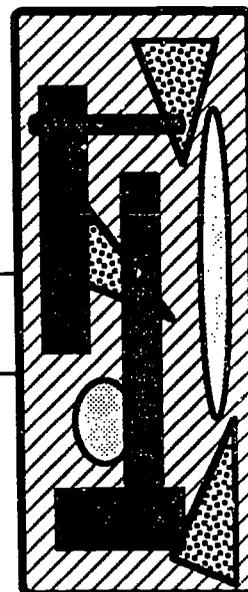
Furthermore, manipulatives offer an excellent way to enable students to connect between mathematical ideas. Learning is enhanced when students are exposed to a concept in a variety of manipulative contexts. As an example, fractions represented with pattern blocks, fraction bars, fraction circles, and Cuisenaire rods help students understand the concept of fraction independent of the physical representation. In addition to using manipulatives for new concepts, activities

should be oriented to help students connect between concrete, pictorial, and abstract representations of ideas.

However, the use of manipulatives should not become an end in itself. Learning the motions of modeling addition and subtraction with Cuisenaire rods does not guarantee understanding of the mathematical relationship between these inverse operations. It is important not only that the concrete manipulation of materials closely matches the mathematical concept being developed but that the actions are accompanied with appropriate questioning by the teacher and reflection by the student.

In the instructional uses of both technology and manipulatives, the goal is to enhance mathematical thinking. Again, the teacher's role as questioner and decision maker influences the effectiveness of the incorporation of these tools.

Essential Elements of Instruction



Essential Elements of Grade 1 Mathematics with Sample Learning Objectives and Sample Clarifying Activities

The State Board of Education in 1989 revised the essential elements of instruction for mathematics, Grades 1-8. These revised essential elements follow closely the recommendations made by the National Council of Teachers of Mathematics in its nationally recognized *Curriculum and Evaluation Standards for School Mathematics*. According to the Texas Education Agency (1989), "The mathematics curriculum review committee and the Agency [TEA] have tried to be sensitive to a balance between changes expected of teachers and improvements necessary to help students learn mathematics more effectively." Some of these major changes include:

- narrowing the spiral of the curriculum—beginning some topics later and finishing some topics sooner in the curriculum to eliminate some of the redundancy
- revising the role of review in the curriculum so that the majority of each grade level is new material and so that review is placed in relevant contexts
- emphasizing the development of problem-solving skills in relevant and interesting situations
- incorporating calculators and computers throughout all grades as problem-solving tools
- adding an essential element on patterns, relations, and functions
- separating the teaching of operations and computation so that all students learn the meaning of the operations
- strengthening the areas of probability, statistics, and geometry
- emphasizing the importance of communication in mathematics

- building on a sound foundation of concepts rather than on rote procedures
- putting mathematics into meaningful contexts

The following essential elements and descriptors for first grade mathematics are annotated with sample learning objectives and sample clarifying activities (except for EE1: Problem-Solving). The learning objectives give a brief look at the developmental components of the concept(s) in the descriptor. The sample clarifying activities are addressed to the teacher and provide a glimpse of what student engagement with this concept might look like in the classroom.

Each set of sample learning objectives and sample clarifying activities is meant to be viewed as an integrated whole (not necessarily matched one-to-one) to clarify the descriptors and to identify connections among them, as well as connections to meaningful problem situations. The Problem-Solving strand therefore is annotated only with sample learning objectives and is connected to the other strands through the language and situations used in their sample clarifying activities.

Many of the activities involve the use of manipulatives and common materials such as hundreds charts or grid paper. A list of these manipulatives can be found in the References and Resources. Also in the References and Resources are lists of the children's trade books, teacher resource books, and software cited in the activities as examples of instructional materials.

The essential elements, sample learning objectives, and sample clarifying activities for Grade 1 are:

- (1) **Problem Solving.** Experience in solving problems designed to systematically develop students' problem-solving abilities through a variety of strategies and approaches. The student shall be provided opportunities to engage in the following types of activities:
 - (A) **develop an organized approach to solving application and nonroutine problems appropriate for Grade 1;**

Sample Learning Objectives

Involving patterns, relations, and functions;
number and numeration concepts; operations
and computation; measurement; geometry;
probability, statistics, and graphing

- (B) **analyze problems by identifying relationships, discriminating relevant from irrelevant information, sequencing, observing patterns, prioritizing, and questioning;**

Sample Learning Objective

Make inferences and predictions

- (C) **communicate an understanding of a problem by describing and discussing the problem and recording the relevant information;**

Sample Learning Objectives

Demonstrate creative thinking through fluency,
flexibility, elaboration, creation of new ideas,
and spontaneity

Estimate outcomes including appropriate units for outcomes

(D) select appropriate strategies from a variety of approaches;

Sample Learning Objectives

Such as: acting it out; making a model; drawing a picture; guessing and checking; making a diagram, chart, or graph; finding a pattern; using a simpler problem; working backwards; etc.

(E) select appropriate materials and methods for solutions; and

Sample Learning Objectives

Such as: concrete manipulatives, mental computation, paper and pencil (pictorial and/or symbolic), calculator, or computer

Reflect on the problem-solving process and solution of a problem by evaluating outcomes for reasonableness (including appropriateness of units), make revisions as needed, describe and discuss the process and solution, and make a decision based on the solution

(F) generate and extend problems.

(2) Patterns, Relations, and Functions. Use of models and patterns to develop the concepts of relations and functions. The student shall be provided opportunities to:

(A) identify and describe patterns in real-life situations;

Sample Learning Objectives

Locate and describe patterns on common objects or in sets of objects.

Identify patterns used in familiar literature (songs, poems, and fairy tales).

Sample Clarifying Activities

Have students locate and describe patterns in everyday objects such as wrapping paper, floor tiles, fabrics, clothing, and wallpaper.

Have students identify the pattern of character responses in familiar stories such as "The Little Red Hen" and "The Three Little Pigs" or popular music like a rap song. When reading or hearing a new story with a pattern, students can make predictions about what comes next and then read on to check their predictions.

(B) identify and extend patterns made up of sets of concrete objects, symbols, or shapes;

Sample Learning Objectives

Identify, reproduce, and extend rhythmic patterns.

Identify and extend patterns made up of sets of concrete objects.

Identify and extend patterns made up of symbols or shapes.

Sample Clarifying Activities

Have students identify, repeat, and continue a sound pattern such as clap hands, slap knees. One student can be selected to extend the pattern (hands, knees, shoulders) for others to copy.

Have students use common objects such as keys, buttons, or various pasta shapes to create a simple pattern for a partner to copy and extend.

As a new picture is placed on the calendar for each day, have students identify the pattern (hearts of red, pink, and white, for example, in February). When the pattern has been identified after a number of days, students can predict what the picture will be for a day later in the month.

(C) order objects according to a given attribute; and

Sample Learning Objectives

Sort items according to a given attribute.

Identify the sorting rule for a group of objects.

Sort differently a group of objects according to a new rule.

Order sorted items according to size, weight, or other attribute.

Sample Clarifying Activities

After they have heard or read the story "A Lost Button" from the book *Frog and Toad Are Friends* by Arnold Lobel, students can sort a group of buttons to determine if they have a button that could be the missing one.

Have students secretly sort some common objects into two groups such as school items and home items. Then have them identify how the objects were grouped by naming other objects that could be put into each group.

Have students work together to find a new way to sort the set of objects in the previous activity. For example, the two new groups could be things made of plastic and things made with paper.

Have students order a group of leaves from smallest to largest. Students can then discuss the leaves' ability to absorb sunlight and water related to their different sizes.

(D) demonstrate the relationship between the actions of joining and separating sets.

Sample Learning Objectives

Use sets of objects to demonstrate the action of joining and separating.

Connect the actions of joining and separating sets to related stories.

Sample Clarifying Activities

Identify two groups of students such as girls with long sleeves and girls who are not wearing long sleeves. Students can take a photograph or draw a picture to show that the girls in one group can be joined with the girls in the other group to show all of the girls in the class. In a similar manner, students can show that the number of girls with long sleeves can be separated from all of the girls to show the girls who are not wearing long sleeves.

Have students paint and cut out 10 fish. While listening to the book *Fish Eyes* by Lois Ehlert, students can use their fish to model the joining actions in the book. In a different activity, students can model the actions of separating sets while listening to the book *Ten Little Crocodiles* by Colin West.

(3) Number and Numeration Concepts. Concepts and skills associated with the understanding of numbers and the place value system. The students shall be provided opportunities to:

(A) compare sets by size and quantity;

Sample Learning Objectives

Match objects to determine and create equivalent sets.

Use estimation and matching to identify or create a set with more/less than another set.

Sample Clarifying Activities

Give each pair of students two small cups with a supply of pebbles, cubes, pasta, seeds, beads, or beans. Have one student place a number of items in one cup. The other student must put a set with the same number of objects in the other cup and explain the strategy used to make the equivalent set.

Have students work with partners, each with a small cup and a bunch of small objects. Each partner puts a set of small objects in a cup, without the other partner looking. They then show their cups, estimate the number of objects in each others' cups without counting, and choose which cup has more (or less). Together, the partners decide on a strategy for comparing the sets to test their estimates.

Recognize one more and one less.

After they have listened to the book *Always Room for One More* by Nonny Hogrigan or the poem "Eight Balloons" from *A Light in the Attic* by Shel Silverstein, have students identify if one more or one less occurs in each situation.

Identify the greatest/least set.

Show students several different containers and a box of lima beans or cubes. Have students estimate which container holds the greatest/least number of lima beans or cubes. Have students justify their estimations verbally and develop strategies to explore the containers' capacities and test their predictions.

(B) compare numbers verbally;

Sample Learning Objectives

Compare numbers using the words *greater than*, *less than*, and *equal to*.

Sample Clarifying Activities

Give each pair of students a set of numeral cards and three word cards saying "is greater than," "is less than," "is equal to." One partner constructs a number sentence with the cards; the other partner reads it, states whether it is true or false, and uses manipulatives or other strategies to justify the response.

Order numbers using the words *before*, *after*, and *between*.

Have students guess a mystery number by asking only yes/no questions about what it comes before, after, or between. Sample question: "Is it a number between 20 and 30?" Results of answers can be displayed on a number line. For example, if the answer to the above question is "no," students will cross out all numbers between 20 and 30, recognizing that 20 and 30 are still possibilities for the mystery number.

(C) explore the concept of place value;

Sample Learning Objectives

Use concrete models of tens and ones to conceptualize and name numbers through 100.

Sample Clarifying Activities

Have students work with partners to use longs and units from base ten materials to model a given whole number. One partner can select the pieces while the other partner uses the constant addition function to count their values (10, 20, 30, 31, 32, 33, . . .).

Investigate place-value patterns in numerals.

Have students fill in the numerals on a hundreds chart and find as many patterns as they can to share with the rest of the class. On a large chart, make a class list of the patterns found.

(D) read and write numerals through the highest number conceptualized;

Sample Learning Objectives

Associate two-digit numerals with models of numbers.

Read and write numerals through 100.

Sample Clarifying Activity

Write two-digit numbers from 10-99 on strips of paper and place them in a paper bag. Each group of three students draws a number from the bag. One student in the group displays the number using concrete models such as base ten blocks. One student locates the number on the hundreds chart or number line. The third student listens to any questions or discussion of the other two and clarifies, if necessary.

(E) read number words;

Sample Learning Objective

Associate number words with symbols and sets (zero through ten).

Sample Clarifying Activities

Give each group of students a set of index cards each with a number word (zero through ten) or a numeral (0 through 10). Students shuffle the cards and randomly place the cards on the table face down. Students take turns trying to find matching words and numerals by turning over two cards at a time. The student with the most pairs of matching cards is the winner.

Have students cut out number word phrases (e.g., two dogs were seen, ten people came) from newspapers, magazines, or old readers; illustrate them with the appropriate number of objects; and post them on the bulletin board.

(F) orally count objects by ones, twos, fives, and tens through the highest number conceptualized;

Sample Learning Objectives

Orally count various types of objects by ones.

Orally count various types of objects by twos.

Orally count various types of objects by fives.

Orally count various types of objects by tens.

Sample Clarifying Activity

Have students work with partners. One student counts a set of small objects (beans, cubes, chips, pasta) by ones, then twos, then fives, then tens. The other student uses the constant function on a calculator to record the quantities and to connect them to the symbols and the words.

(G) use ordinal numbers;

Sample Learning Objectives

Orally list ordinal numbers (first through tenth).

Identify ordinal positions (first through tenth).

Sample Clarifying Activities

Have students use ordinal numbers to describe the situations in *Why Mosquitoes Buzz in People's Ears* by Verna Aardema, or *Five Chinese Brothers* by Claire Hatcher Bishop, or "When Tillie Ate the Chili" from *The New Kid on the Block* by Jack Prelutsky.

Have students work together as a class to make a peanut butter and jelly sandwich. Have students use ordinal numbers to communicate in writing the steps required to make the sandwich. Students may choose other home tasks, such as washing dishes to describe using ordinal numbers.

(H) orally identify fractional parts of whole objects or sets of objects;

Sample Learning Objectives

Identify equal parts of whole objects.

Identify halves, thirds, and fourths, using sets of objects.

Recognize that two halves, three thirds, and four fourths make a whole, using sets of objects.

Sample Clarifying Activities

Provide students with a candy bar that comes divided in sections. Have students identify the equal parts of the candy bar.

Give each pair of students a set of two-color counters or lima beans painted on one side. Have one partner spill a number of counters or beans from a cup onto the table. The other partner will name the fractional part of each set that is each color. For example, two of the three beans or $\frac{2}{3}$ are red.

(I) identify coins; and

Sample Learning Objectives

Identify a penny and its critical attributes.

Identify a nickel and its critical attributes.

Identify a dime and its critical attributes.

Identify a quarter and its critical attributes.

Sample Clarifying Activities

Give each student real coins or concrete models of pennies, nickels, dimes, and quarters; crayons; and paper. Have students arrange the coins in a pattern, lay the paper over the coins, and create crayon rubbings. Students can trade their pictures and identify the coins in the different rubbings.

Have students sort and resort a group of mixed coins according to various attributes; e.g., coins that have buildings on them versus coins that don't.

(J) describe the value of coins, using real coins or concrete models of coins.

Sample Learning Objectives

Associate a penny with its value.

Associate a nickel with its value in terms of pennies, using real coins or concrete models of coins.

Associate a dime with its value in terms of pennies and nickels, using real coins or concrete models of coins.

Associate a quarter with its value in terms of pennies, nickels, and dimes, using real coins or concrete models of coins.

Count sets of coins.

Sample Clarifying Activities

Have students draw or cut out pictures from several types of stores, such as candy stores, toy stores, record stores, and pet stores in a shopping mall. Have students label each drawing with the cost of an item from that store. In pairs, students can sort their pictures according to the types of stores. They then can use real or concrete models of coins to purchase an item, saying the value of the coins and counting the amount.

Give each pair of students a set of coins and a hundreds chart. One partner selects each coin, names its value, and hands it to the other partner. That student places each coin in the appropriate space on the hundreds chart to indicate the running total value.

(4) Operations and Computation. Use of manipulatives to develop the concepts of basic operations on numbers and to apply these concepts to the computational algorithms. The student shall be provided opportunities to:

(A) demonstrate an understanding of addition and subtraction and their inverse relationship by joining, separating, and comparing sets of objects;

Sample Learning Objectives

Use manipulatives to show the meaning of addition by telling how many there are to begin with, how many are joining, and how many there are in all.

Use manipulatives to show the meaning of subtraction by telling how many there are to begin with, how many are going away or being separated, and how many are left.

Sample Clarifying Activities

Have each student create a picture mat for a concept currently being taught in the classroom, such as the beach when the class is studying the ocean environment. Students can use concrete objects such as shells to act out and solve an oral word problem involving addition created by the teacher or by the students.

Give each student a number of small edible treats such as goldfish crackers. Students demonstrate a model for subtraction by eating the appropriate number of treats to determine how many are left.

Use manipulatives to show the meaning of subtraction by comparing two sets of objects and telling how many greater or fewer one set is than the other.

Select a topic where the students will have exactly one of two characteristics such as boy/girl, lunch from home/cafeteria, or buttons on shirt/no buttons. Have each student select one of two colors of linking cubes according to the appropriate characteristics. Students can stack the cubes according to color, compare the quantities, and state which characteristic occurs more or less often.

Use manipulatives to show the inverse relationship between addition and subtraction.

Have each student create a picture mat of a bunk bed (one bed on top of the other). Students then place eight counters on the mat, some on the top bunk and some on the bottom, and describe the situations they have created. "There are 5 cubes on the top and 3 cubes on the bottom, so that makes 8 in all. If there are 8 cubes in all and 3 are on the bottom, then 5 cubes must be on the top."

(B) explore multiplication and division and their inverse relationship by joining equivalent sets of objects and separating a set of objects into equivalent sets;

Sample Learning Objectives

Sample Clarifying Activities

Use manipulatives to explore the meaning of multiplication by joining equivalent sets of objects.

Have the class brainstorm a list of things that come in threes (stop lights, tricycle wheels). Students can work in groups of 3 to use linking cubes to make models for problems based on this list. For example, if there are four tricycles outside, how many wheels would there be? Or read *Each Orange Had 8 Slices* by Paul Giganti, Jr.

Use manipulatives to explore the grouping concept of division by separating a set of objects into equivalent sets.

Have students create a story mat that pictures five or six containers such as boxes, cartons, or wagons. Specify the number of counters that can fit in each container. Name a quantity and ask students to determine how many containers are needed to hold that quantity. For example, if there are 16 children and 4 children will fit in each car, how many cars will be needed to take the children on a trip?

Use manipulatives to explore the sharing concept of division by separating a set of objects into a specified number of sets.

Read the book *The Doorbell Rang* by Pat Hutchins and have students model the process of division by separating given amounts of cookies into a specified number of plates. For example, if you had 15 cookies and 3 people, how would they be shared? What if there were 4 people?

Use manipulatives to explore the inverse relationship between multiplication and division.

Have students work in partners with a set of objects. One partner selects a number of objects (say 10), arranges them in equal groups (e.g., five groups of two objects), and makes a statement such as "Ten objects split into 5 groups means 2 in each group." The other partner rejoins the group into one set and makes a statement such as "Five groups of 2 objects each means 10 objects altogether."

(C) demonstrate properties of addition (commutative, associative, identity) with concrete models;

Sample Learning Objectives

Use manipulatives to demonstrate the commutative property of addition.

Sample Clarifying Activities

Have students make two color trains with interlocking cubes such as 4 reds and 3 blues. After making an oral statement or writing a number sentence to describe the train ($4 + 3 = 7$), they then switch the order of the colors (or flip the train over) and write or state the new number sentence ($3 + 4 = 7$) for this train.

Use manipulatives to demonstrate the associative property of addition.

Gather data on the number of students wearing long pants (perhaps 8), short pants (5), and skirts (6). Students can first group those with pants ($8 + 5 = 13$) to write a number sentence about the whole class ($13 + 6 = 19$). The students who don't have long pants can then be grouped ($5 + 6 = 11$) to write a number sentence about the whole class ($8 + 11 = 19$). Have students compare these sentences to each other and to the combined sentence for the whole class, $8 + 5 + 6 = 19$.

Use manipulatives to demonstrate the identity element of addition.

Have students place a number of counters in one hand but no counters in the other hand. What is special about adding 0? Students can write number sentences using zero as an addend and describe the patterns they see.

(D) use an addition or subtraction number sentence to describe the joining, separating, or comparing of sets; and

Sample Learning Objectives

Join two sets of concrete objects and record an addition sentence to describe the operation.

Sample Clarifying Activities

Have students sort lunches brought from home. An addition sentence can be written to describe the lunches. For example, if there are 7 lunches with apples and 9 lunches without, then there are $7 + 9 = 16$ total lunches.

Separate a set of concrete objects into two sets and record a subtraction sentence to describe the operation.

Have students gather a known number of crayons and separate the broken crayons from the unbroken ones. Students can count one of the groups and write a subtraction sentence to describe how many are in the other group. For example, out of 24 crayons, 10 are broken and $24 - 10 = 14$ are unbroken.

Compare two sets of concrete objects and record a subtraction sentence to describe the operation.

Have students work in pairs. Each student takes out all of the pencils or markers in his or her desk. The students compare the number of pencils to determine who has more. The partners write a subtraction sentence to describe the situation. "I have 8 markers. You have 6 markers. $8 - 6 = 2$. I have 2 more markers than you do."

(E) complete (write) addition and subtraction sentences.

Sample Learning Objectives

Sample Clarifying Activities

Solve problems involving addition facts using number sentences and using calculators.

Have students write addition number sentences to describe the paired pictures in the book *Annie's One to Ten* by Annie Owen. The students can use calculators to explore the different possible combinations of addends to make the sum of 10.

Solve problems involving subtraction facts using number sentences and using calculators.

Have students write subtraction number sentences to describe the hidden people behind the windows in the book *Anno's Counting House* by Mitsumasa Anno. Calculators can be used to allow students to explore the situations pictured in the book.

(5) Measurement. Concepts and skills using metric and customary units. The student shall be provided opportunities to:

(A) measure objects using different nonstandard units;

Sample Learning Objectives

Sample Clarifying Activities

Estimate and then measure the length, width, and/or height of objects, using a nonstandard unit, and compare the results.

Have students record estimates for how many paper clips long they think are each of several school objects, such as a pencil, scissors, or bottle of glue. Students can then use paper clips to measure each object and record the measurements. Have students compare their estimates with their measurements and discuss any trends.

Compare measures of length, width, and/or height of the same objects, using two or more nonstandard units.

Have students use popsicle sticks to measure the heights of several objects and record the measurements. Students can then use drinking straws as the unit of measure. Have students compare the two measurements and discuss the importance of both the quantity and the unit in a measurement; e.g., the desk is 8 popsicle sticks or 6 drinking straws long.

Order a group of objects from shortest to longest.

Have students work in pairs to order a group of objects, such as ribbons or pencils from shortest to longest.

(B) identify concrete models that approximate units for centimeter, decimeter, inch, and foot;

Sample Learning Objectives

Sample Clarifying Activities

Identify concrete models that approximate one centimeter.

Give students a model of 1 centimeter (e.g., the side of a base ten units cube) and have them go on a scavenger hunt for objects in the room or outside that are about 1 centimeter long. List these on a classroom chart.

Identify concrete models that approximate one decimeter.

Give students a model of 1 decimeter (e.g., a tens piece from the base ten materials) and have them go on a scavenger hunt for objects in the classroom or outside that are about 1 decimeter long. List them on a classroom chart.

Identify concrete models that approximate one inch.

Give students a model of 1 inch (e.g., the edge of any pattern block, except for the long side of the red trapezoid) and have them go on a scavenger hunt for objects in the classroom or outside that are shorter than 1 inch, longer than 1 inch, or about 1 inch in length. List the items on 3 classroom charts.

Identify concrete models that approximate one foot.

Give students a model of 1 foot (e.g., a 12-square strip cut from inch grid paper) and have them go on a scavenger hunt for objects in the classroom or outside that are about 1 foot in length. Have students discuss their strategies for finding the objects.

(C) use concrete unit models for inches, feet, centimeters, and decimeters;

Sample Learning Objectives

Use concrete models for 1 centimeter and 1 decimeter to measure objects.

Use concrete models for 1 inch and 1 foot to measure objects.

Estimate lengths and use the measuring instrument to measure objects in centimeters.

Estimate lengths and use the measuring instrument to measure objects in inches.

Sample Clarifying Activities

Have students use base ten blocks (unit cube for cm, 10-rod for dm), Cuisenaire Rods (white for cm, orange for dm), or centimeter grid paper cut into 10cm strips and 1cm squares to measure objects. Have students discuss their strategies for deciding about partial units.

Have students use edges of pattern blocks, except for the long side of the red trapezoid, or inch grid paper cut into 1-inch squares and 12-inch strips to measure objects. Have students discuss their strategies for deciding about partial units.

Have students estimate the lengths of several objects and then use a centimeter ruler to determine the actual lengths in centimeters.

Have students estimate the lengths of several objects and then use an inch ruler to determine the actual lengths in inches.

(D) compare two objects according to weight;

Sample Learning Objectives

Estimate and compare the weight (mass) of two objects using a balance scale.

Estimate and compare the weight (mass) of two objects using nonstandard units.

Sample Clarifying Activities

Have students use a balance scale to determine which of two objects is heavier or lighter.

Have students use marbles (or a similar material) to weigh objects on the balance scale and record the weight in terms of how many marbles are as heavy as the object. After weighing a second object, students can compare the weights of the two objects in marbles to determine which is heavier and which is lighter.

(E) explore the concept of time; and

Sample Learning Objectives

Explore the concept of time in terms of what can be done in one minute.

Explore hours and half-hours and connect to fraction concepts using traditional clocks.

(F) read a calendar.

Sample Learning Objectives

Read a calendar in terms of months.

Read a calendar in terms of weeks.

Read a calendar in terms of days/dates.

Sample Clarifying Activities

Ask students to predict how many times they can clap their hands in one minute. After predictions are made, students can count the claps they make for one minute. Ask students why there were differences in the number of times people could clap. Have students make estimates and trials for other activities to see what can be done in one minute. (How many hops? How far can you run?) Ask students to brainstorm lists of things that take about one minute, more than one minute, or less than one minute.

Have students make a paper plate clock face. As they listen to a book such as *Nine O'Clock Lullaby* by Marilyn Singer, students can show a time on the hour or half-hour on their clocks.

Sample Clarifying Activities

Have students read or listen to the book *Chicken Soup With Rice* by Maurice Sendak. Students can create a picture for each month of the year to create their own calendars.

Have students create their own book about "a week in the life" in the pattern of books they have heard such as Eric Carle's *The Very Hungry Caterpillar* or Cindy Ward's *Cookie's Week*. Each of the seven days of the week should have some event described and pictured.

Use the class calendar to highlight happenings and regularly scheduled activities such as computer lab days and library visits. Students can use the calendar to count down to events like field trips and special units of learning. Students can play a game where one student names a date and another tells on which day it falls.

(6) **Geometry.** Properties and relationships of geometric shapes and their applications. The student shall be provided opportunities to:

(A) **use physical materials to construct simple geometric shapes and combine shapes to form new shapes;**

Sample Learning Objectives

Use physical materials to construct three-dimensional shapes.

Use physical materials to construct models of two-dimensional shapes.

Explore and manipulate geometric shapes to form a new shape.

Make a record of an exploration with shapes.

Sample Clarifying Activities

Have students use six congruent squares and tape to construct a cube.

Have students use toothpicks (for sides) and marshmallows or gumdrops (for corners) to construct models of two-dimensional geometric shapes. Other materials such as straws, string, and cubes can also be used. Have students discuss the characteristics of each.

Have students use triangles from the pattern blocks or tangrams to make a square. Or students can cut a rectangle into two pieces and discuss the different shapes they can make.

After they have created a new shape, design, or picture with pattern blocks, tangrams, or other geometric shapes, have students paste construction paper pieces of the same size and color to make a record of their work. Students can use the pictures to investigate number ideas, patterns, measurement estimates, etc.

(B) **identify two-dimensional shapes;**

Sample Learning Objectives

Identify objects that are models for squares, triangles, rectangles, and circles.

Describe the attributes of a square, triangle, rectangle, and circle.

Sample Clarifying Activities

Have students brainstorm a list of objects that model each of the shapes. The list can be added to over a period of time as students discover new objects for each shape. Each of the students can contribute a page in a class book for each shape.

Have students close their eyes and feel attribute blocks or shapes cut from cardboard to determine the number of sides and corners in a square, triangle, rectangle, and circle.

Have students complete a chart to identify the number of sides and corners for each shape. Students can look for patterns in the chart.

(C) compare three-dimensional objects describing similarities and differences using appropriate mathematical language; and

Sample Learning Objectives

Identify two-dimensional shapes on three-dimensional objects.

Compare three-dimensional objects by identifying similarities and differences.

Sample Clarifying Activities

Have students trace or stamp the outside faces of a variety of common objects. When tracing the faces of a cube, for example, six congruent squares would be drawn or stamped. Students can compare sets of faces from different objects and group them; e.g., all the objects that have two circular faces.

Have students take milk cartons or cereal boxes and cut them apart to identify the two-dimensional shapes found on the object.

Have students sort a group of objects according to given attributes. For example, tell them "Find all of the objects which have at least one face which is round like a circle" or "Find all of the objects that have at least one face that is shaped like a triangle."

Students can choose two 3-dimensional objects and tell how they are alike and different.

(D) locate the interior and exterior of plane figures.

Sample Learning Objectives

Identify points inside, outside, and on figures.

Identify open and closed figures.

Sample Clarifying Activities

Have students use one rubber band on a geoboard to construct a closed figure. Give them three different colored dots to mark places inside, outside, and on their figures. Students can then record this exploration on dot paper with pencils and crayons.

Have students use a drawing software program to draw a figure on the computer. When they try to fill in this figure, they will soon discover if the figure they drew was open or closed. If a computer program is not available, the students can engage in a similar activity by closing their eyes and drawing shapes with a pencil on a piece of paper. Closed figures can be colored in by students, but open figures must be left uncolored.

(7) **Probability, Statistics, and Graphing.** Use of probability and statistics to collect and interpret data. The student shall be provided opportunities to:

(A) **collect data;**

Sample Learning Objectives

Collect and sort concrete objects into various categories and describe how the objects were sorted.

Collect and sort pictures of objects in various categories and describe how the pictures were sorted.

Sample Clarifying Activities

Have students sort fairy tale, sports, and biography books from the school library and describe how they were sorted.

Have students bring from home individual pictures of their family members. Have students explore and describe various ways to sort the pictures.

(B) **make pictographs and bar-type graphs using objects, pictures of objects, or colored cells;**

Sample Learning Objectives

Arrange concrete objects to create a real graph.

Arrange pictures of objects to create a pictograph, with each picture representing a single unit.

Use colored squares to create a bar-type graph, with each cell representing a single unit.

Sample Clarifying Activities

Have students arrange on a floor graphing mat concrete objects such as wooden shapes that roll, stack, or slide; their favorite cookies, cereals, or toothpastes; their favorite M&M or Skittle flavored candy; citrus fruit; types of milk they drink; or types of shoelaces each student is wearing. Students can then discuss what questions can and can't be answered with information from the graph.

Have students represent the concrete floor graph with pictures on a wall chart, matching the pictures to the concrete data.

After a discussion of differences between the real graph and the picture graph, have students place colored squares of construction paper on grid paper in a bar-type format to represent the data from the concrete and picture graphs. Students can then compare the three types of graphs.

(C) draw conclusions and make informal predictions based on experiences or graphed data;

Sample Learning Objectives

Make summary statements based on graphed data.

Make informal predictions based on graphed data.

Use graphed data to make decisions or to solve problems.

Sample Clarifying Activities

Have students make 20 spins with a spinner colored $\frac{1}{2}$ red, $\frac{1}{4}$ blue, and $\frac{1}{4}$ yellow and record the results in a bar graph. Students can then make true statements about the results. For example, "There are more red spins than yellow spins. There are only 2 more blue spins than yellow spins. There were no green spins."

From the results of the data from the 20 spins with the spinner above, have students predict results for a spinner colored $\frac{1}{2}$ yellow, $\frac{1}{4}$ red, and $\frac{1}{4}$ blue. Students can then spin to test their predictions.

After students have collected and graphed information such as the months of their birthdays, have them brainstorm ways that information could be used. For example, the teacher could know how many pencils needed for presents each month.

(D) identify events that are sure to happen, are sure not to happen, and those which we cannot be sure about; and

Sample Learning Objectives

Analyze events to determine what is certain, uncertain, and impossible.

Analyze events to determine what is likely, unlikely.

Sample Clarifying Activities

Give each student three index cards, one marked "certain," one "uncertain," and one "impossible." Present a series of situations (the sun will shine somewhere in the world tomorrow) or read a story such as *Daddy is a Monster . . . Sometimes* by John Steptoe and have students respond to information by holding up one of the cards. Students who disagree can discuss their reasoning for choosing "certain" or "uncertain" or "impossible."

Have students listen to or read stories such as *The Queen of Eene* by Jack Prelutsky or poems in *Where the Sidewalk Ends* by Shel Silverstein. Students can select events from these stories or poems, post them on a classroom chart under the headings "Likely" or "Unlikely" and discuss their reasoning for their choice of heading.

(E) build patterns which display various arrangements of a given set of objects.

Sample Learning Objectives

Use concrete objects or pictures to depict possible arrangements.

Use symbols to represent possible arrangements.

Look for patterns in the representations of possible arrangements.

Use patterns to predict or justify the possible number or arrangements.

Sample Clarifying Activities

Have students predict the possible different clothing combinations for 3 different colors of sweat shirts and 3 different colors of sweat pants. After making and discussing their predictions, students can work in small groups to cut out shirts and pants from 3 colors of construction paper and explore the possible outfits. The groups can use pictures or words to record the possible outfits.

For a class assignment, you want the students to work together in groups of 3. How many different combinations of boys and girls can be made for a group of 3? Have students use a "B" for boy and a "G" for girl to write down all of the different possible groups. (GGG, GGB, GBB, BBB).

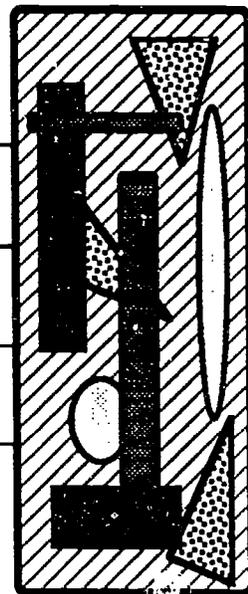
Have students choose two sheets of construction paper from 4 available colors to make a picture. One sheet must be the background and cannot be torn. Shapes or designs may be torn from the other sheet and pasted onto the background. When all pictures are completed, ask the students to look at them and to find a way to sort them. They might be sorted first by background color (all of the blues, then all of the reds, and so on) or by the color that was pasted. After students have had an opportunity to sort the pictures, have them think about questions like "How many different kinds of pictures with red background could there be? How many different kinds of pictures with red pasted shapes could there be? Why? Are there any possible pictures missing? What are they? How did you figure that out?"

Texas

Assessment of

Academic

Skills



Focus

The Texas Education Agency implemented the Texas Assessment of Academic Skills (TAAS) testing program in 1990. The program is in effect for the 1990-1995 period. The purpose of the assessment program is to provide Texas schools with an accurate measure of student achievement. The scope of content of the TAAS includes more of the instructional targets delineated in the essential elements than previous state assessments. Every section of the TAAS test contains a certain number of broad objectives. These objectives remain constant from grade to grade because they represent the core concepts that form the basis for a sound instructional progression from Grade 1 through Grade 12. What will differ from grade to grade are the instructional targets—or essential elements that comprise each objective. A portion of this extended set of instructional targets is selected for assessment annually, but not every target is tested every year.

The broadened scope of the TAAS assessment program allows for a different focus, one that addresses the academic requirements of the 1990s. Skill areas that demand little more than rote memorization are de-emphasized, while areas that improve a student's ability to think independently, read critically, write clearly, and solve problems logically receive increased emphasis. This emphasis is in keeping with current national trends in education, which stress the importance and necessity of teaching students higher order thinking skills.

Domains, Objectives, and Targets

The TAAS features three domains—concepts, operations, and problem solving. Each domain contains objectives that are derived from the essential elements. For every objective, there are instructional targets that describe the kinds of mathematical experiences that will reflect that objective. Each instructional target was taken for the most part directly from the essential elements as delineated in the *State Board of Education Rules for Curriculum*. Each target is defined in behavioral terms appropriate for pencil-and-paper testing.

Although TAAS testing begins at Grade 3, it is important for Grade 1 teachers to be aware of the Grade 3 targets for TAAS. Grade 1 teachers are responsible for developing foundations of

understanding which will be the base on which the rest of the mathematics curriculum stands. Therefore, the TAAS objectives and targets for Grade 3 are listed below.

DOMAIN: Concepts

Objective 1: The student will demonstrate an understanding of number concepts.

- (a) Compare and order whole numbers
- (b) Use whole number place value
- (c) Use odds, evens, and skip counting
- (d) Recognize and compare fractions using pictorial models
- (e) Translate whole numbers (name to numeral/numeral to name)
- (f) Recognize decimal place value (tenths and hundredths; using models)

Objective 2: The student will demonstrate an understanding of mathematical relations, functions, and other algebraic concepts.

- (a) Use whole number properties and inverse operations
- (b) Determine missing elements in patterns
- (c) Use number line representations for whole numbers
- (d) Classify objects

Objective 3: The student will demonstrate an understanding of geometric properties and relationships.

- (a) Recognize two- and three-dimensional figures
- (b) Describe and compare two- and three-dimensional shapes
- (c) Identify informal representations of congruence and symmetry

Objective 4: The students will demonstrate an understanding of measurement concepts using metric and customary units.

- (a) Use measurement units of time, length, temperature, and weight/mass
- (b) Find perimeter

Objective 5: The student will demonstrate an understanding of probability and statistics.

- (a) Interpret and use charts, tables, bar graphs, and pictographs

DOMAIN: Operations

Objective 6: The student will use the operation of addition to solve problems.

- (a) Add whole numbers
- (b) Add money using models

Objective 7: The student will use the operation of subtraction to solve problems.

- (a) Subtract whole numbers
- (b) Subtract money using models

Objective 8: The student will use the operation of multiplication to solve problems.

- (a) Use modeling and patterns of whole number multiplication concepts to generate basic facts

Objective 9: The student will use the operation of division to solve problems.

- (a) Recognize modeling of division

DOMAIN: Problem Solving

Objective 10: The student will estimate solutions to a problem situation.

- (a) Estimate with whole numbers

Objective 11: The student will determine solution strategies and will analyze or solve problems.

- (a) Select strategies or solve problems using addition and subtraction with whole numbers

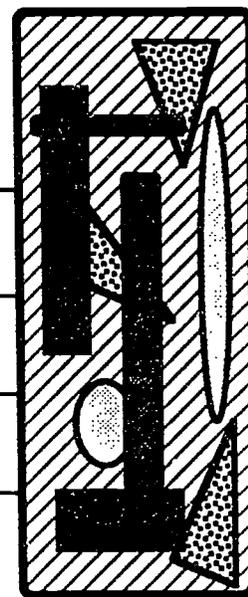
Objective 12: The student will express or solve problems using mathematical representation.

- (a) Identify solution sentences
- (b) Recognize models for problems
- (c) Interpret charts, tables, pictographs, and bar graphs and use the information derived to solve problems
- (d) Identify or solve story problems depicting the joining (addition/multiplication) and separating (subtraction/division) processes

Objective 13: The student will evaluate the reasonableness of a solution to a problem situation.

- (a) Evaluate reasonableness

Sample Lessons for Teaching Grade 1 Mathematics



The following sample lessons represent the kind of mathematical experiences recommended for students in first grade mathematics classes. These expanded sample activities include ideas for motivational introductions, exploratory questions to ask during activities, summary questions for reflection after exploring the concept, and ideas for extension and assessment. They are included as examples of significant, mathematical tasks that address the state's curriculum requirements in light of the national recommendations. Note that each sample activity involves essential element descriptors, as well as objectives from the Grade 3 Texas Assessment of Academic Skills (TAAS). Several different manipulatives are included in these activities. It is important that students use these manipulatives as they work through the activities. Manipulatives and concrete objects enable elementary school students to better understand the mathematical problems and concepts they so often struggle to learn. Students' experiences with manipulatives are recommended in the essential elements, TAAS's instructional targets, and NCTM's *Curriculum and Evaluation Standards for School Mathematics*.

Most activities in this section recommend that students work together in pairs or small groups. Working together in cooperative groups promotes communication and mathematical confidence and enhances students' problem-solving abilities. At the end of each activity is space to add your own notes and ideas.

Objective The student will use geometric shapes to generate patterns.

Activity Geo-Wrap Vests

Materials Various colors of construction paper, foam cut-out geometric shapes (i.e., squares, circles, triangles, rectangles), tempera paint, butcher paper or large brown grocery bags to make vest cut-outs

Resources *Aida* by Leontyne Price, encyclopedias or other reference books

Procedure

Introduction:

1. Read *Aida* to class and show students pictures of the African garments.
2. Show other pictures of African garments.
3. Have students identify patterns in the garments.
4. Explain to students that Ashanti Kente Cloth originated in West Africa and the Kitanga Cloth is from East Africa. Explain that, in the United States, African-Americans are wearing the Kente Cloth as scarves (head wraps), dashikis (tunics) and other garments. (If you can obtain a piece of Kente Cloth, have students discuss the patterns they see in the cloth.)
5. Show students how to use tempera paint and foam cut-out geometric shapes to make imprints onto butcher paper or grocery bags cut into the shape of a vest.
6. Have students make geo-wrap vests, similar to dashikis, with a pattern, so they can have a fashion show and model for the other classes.

Exploration:

- What are some other patterns you have learned about or used?
- What things can you think of that do not have patterns?
- What is the difference between looking at a picture with a pattern and looking at one without a pattern?
- What geometric shapes are you planning to use to create your pattern on your vest? Why?
- How many times do you think you will be able to repeat your pattern on your vest? How did you arrive at your estimate?

Extension:

Students can use computer software (e.g., *Print Shop*) to generate invitations with patterned borders to match or coordinate with their vests.

Summary:

- How would you describe your pattern?
- Why did you use the shapes you used? How did you decide on your pattern?
- How is your pattern different from others in your class?
- If you had to sort the vests the class made into different groups, what groups would you make? What characteristics would you use for sorting?

Assessment

Questions:

(See summary questions.)

Observations:

- Are students using appropriate geometric vocabulary when they discuss their patterns?
- Are students able to identify characteristics of patterns?

Tasks:

- Write in your mathematics journal a description of the pattern on your vest.
- Write an invitation to another class to come view the geo-wrap vests. Describe the activity you did and what you learned from it in a way that would make the class interested in coming. Decorate the invitation with an example of your pattern around the border.

Objective The student will explore attributes of geoblocks.

Activity Classifying Geoblocks

Materials Set of geoblocks, grid paper, balance (available if students request it or on supply table to stimulate curiosity)

Procedure

Introduction:

1. Have class brainstorm traits of a select group of students. Emphasize how students may be a member of more than one set.
2. In a whole-class setting, let students explore the geoblocks and brainstorm a short, beginning list of attributes.
3. Have students continue brainstorming attributes in small groups.
4. Have students in small groups classify their set of blocks according to at least three attributes and display their different classifications. (The blocks could be originally distributed in prearranged groups in order for students to focus on particular attributes, or types of blocks could be evenly divided among groups so each group has the same set, or each small group of students could randomly draw a set of blocks to investigate.)

Exploration:

- How are these two blocks alike? How are they different?
- How many different attributes can you find?
- Which category would this block go in? Why?
- Which block would not go in that category? Why not?
- What words do you know that can help you describe these blocks?
- Explain your category and show the teacher some examples.

Extension:

- Order blocks by weight. (Predict first.)
- Use combinations of blocks to make new shapes.
- What combinations of blocks can be used to make a rectangular prism?
- Make a Venn diagram showing relationships between the blocks.
- Make jackets for several blocks by tracing their faces. (If cut from paper, jackets should fold up to represent the original block.) Make a set of task cards for other groups. Can jackets be matched to corresponding blocks?

Summary:

- Discuss the vocabulary used to describe the attributes.
- Explain one of your categories (different from what any other group has explained).
- What criteria did you use to place a block in that group?
- Which block(s) fit the most of your categories?
- Did any block fit in all of your categories?
- Does any block fit in every category thought of by the class? Why or why not?
- Is there a category that would contain exactly one block? Describe it.
- Can you find any real-life examples (in the classroom, at home, etc.) of each kind of block?

Assessment:

Questions:

- Can you show me a block that has _____ (an attribute or combination of attributes)?
- (See summary questions.)

Observations:

- Were students using appropriate geometric vocabulary?
- Could students justify their classifications?
- Were students testing each block for a particular category?

Tasks:

- Categorize objects in your desks like you did the blocks.
- Write a journal entry (or contribute to a class experience chart) explaining your classifications.
- Write a journal entry explaining new vocabulary you learned.
- Given a block, list as many attributes of it as you can.
- Write a question about the blocks for another group to answer.
- Write an answer about the blocks and have another group come up with the question. Think of three possible questions of your own.

Objective Students will explore various ways to represent and verify halves, thirds and fourths.

Activity To Be (Half) or Not To Be (Half)?

Materials Interlocking cubes, variety of measurement tools (e.g., balance scales, tape measures, grid paper)

Procedure

Introduction:

1. Have a large bag or box filled with interlocking cubes. Discuss with the students that you would like for them to use half of the cubes in the activity and you will share the other half with the class next door so they can do the activity, too. How can the set of cubes be divided into halves?
2. Ask for students' ideas on how to verify that the set of cubes will have been divided into two halves. These ideas can be recorded on a chart or on the board for future reference, if desirable.
3. After brainstorming some ideas for halves, ask students what would happen if there were three classes that wanted to use the cubes at the same time? What would need to be done? What could be done to verify that the group of cubes was divided into thirds?
4. Discuss how some problems can be investigated more easily by looking at a simpler problem that is much like the bigger problem, and give each group of students a bag of cubes (50 or so, or let each student grab two handfuls of cubes to combine at his or her table).
5. Have each group of students divide its set of cubes into halves and find as many ways as possible to verify that the set has been divided into halves.
6. After investigating halves, each group of students should explore ways to verify thirds and fourths.
7. Each group should write about or draw a picture of the ways they have used to verify their fractions in order to compare and contrast their ways with those of other groups.

Exploration:

- What is it important for you to do when you make halves? Thirds? Fourth?
- How do you know when you have fair shares, or equal parts, (or whatever language the students begin using at this time)?
- What did you do if you had left-overs?
- What other ways can you compare the parts, without counting the cubes in them?
- How could you use the balance scale to compare the parts?
- How could you use the grid paper to compare the parts?
- How could you use shapes or designs to compare the parts?
- Which of the ways you used to verify halves worked also for thirds? Which of the ways, if any, did not work?
- Which of the ways you used to verify halves worked also for fourths? Which did not work?
- If you divided your set of cubes first into halves, could you divide it into thirds or fourths? Why or why not?
- If you divided your set of cubes first into fourths, could you make thirds or halves? Why or why not?
- How did your group decide to record your methods?

Extension:

- Use a variety of materials with which to represent fractional parts; e.g., candy bars, bags of candies, fruit.
- Bring something from home that shows halves, thirds, fourths.
- Make a symmetrical picture, and relate it to halves. Make a picture that relates to thirds, to fourths. Can you make a picture that shows all three of these fractions at the same time?
- Take a nature walk and collect various types of wildflowers that can be used to show halves, thirds, or fourths. (See *I Can Count the Petals of a Flower* from NCTM.)

Summary:

- What techniques did your group use to verify halves (thirds and fourths)? Which ones do you think worked the best?
- What things were important in order to verify halves? Thirds? Fourth?
- Were you able to show halves for every set of cubes? Thirds? Fourth? Why or why not?
- What did your group decide to do when there were leftovers? Why?
- How were your techniques for halves different from your techniques for thirds? For fourths?
- Did you notice any patterns or connections between halves and thirds? Between halves and fourths? Between thirds and fourths?
- How is your group's technique different from, or similar to, the technique of another group?
- In addition to cubes, what could you use to show halves? Thirds? Fourth?
- What are some circumstances in which you might use your knowledge of halves? Thirds? Fourth?
- How could you relate the ideas of halves, thirds, and fourths to everyday life, to things around our classroom and homes?

Assessment

Questions:

- How do you know when something has been divided into halves? Into thirds? Into fourths?
- How can you use (some particular measurement instrument) to verify halves? Thirds? Fourth?
- How did your group organize itself?
- How could your group improve in its working together?
- (See also summary questions.)

Observations:

- Did student discussion indicate an understanding of the problem?
- Did student discussion indicate an understanding of the connections between the fraction names and their representations?
- Did students develop appropriate strategies for verifying fair shares?
- Were students working individually within the group or collaborating?

Tasks:

- Write, or dictate, a story about how this information might be helpful to you in the future. (For example, your group had a lump of clay, and each person used some to make a different animal. How can you prove to the teacher that the clay was shared evenly?)
- Write about, dictate a description of, or draw a picture of one of the techniques your group used to verify halves, thirds, or fourths. Tell what you think are the advantages and disadvantages of this particular technique.

EEs: 3I, 3J

Related EEs: 1A, 1B, 1D, 1E,
1F, 4D, 4E

Grade 3 TAAS Objectives: 1, 6, 7, 10,
11, 12, 13

Objective The student will be able to identify and describe the value of coins, using real coins or concrete models of coins.

Activity Math in the Mall

Materials Store signs on construction paper: Toy Store, Jewelry Store, Department Store, Card Shop; old greeting cards; toys or pictures of toys; pictures of jewelry or old jewelry; pictures of department store items; price tags from 1¢ to 99¢; real or concrete models of pennies, nickels, dimes, quarters; calculators.

Procedure

Introduction:

1. Generate class discussion on the following questions:
 - Did people in the United States always have coins to buy things?
 - What objects did Native Americans use instead of money to buy items (bartering)?
 - Have you heard your parents talking about being on a budget?
 - Why do you think people have budgets?
 - How can a budget help when you go shopping?
2. Tell students that they are going to act out going to the mall to purchase items from several stores. Have them bring items or pictures of items for the stores to school.
3. Students will work with partners: one will be the customer and one will be the store clerk.
4. Distribute coins to each student before the students begin shopping.
5. The budget that each person will be on will be the amount of money received from the teacher. Students can make purchases from only two different stores.
6. The customer will select an item to purchase, select the correct amount of money from their collection of coins, and give it to the store clerk who will check the amount. If a student wants to purchase two items from the same store, he or she may use the calculator to find the total cost.
7. Partners should take turns being the customer and the store clerk.
8. Each student can take a mathematics journal to the store and write down the items and the coins used to pay for each item. Calculators can be used to record totals of the two items.

Exploration:

- How many of each coin do you have?
- What are the values of each coin?
- What strategy did you use to count your coins? Can you think of another way to count them?
- How did you decide what items to buy?
- How did you determine what coins were needed to buy each item?
- Were there more combinations of coins you could have used?
- How did you determine if you had enough money for the two items you wanted?
- Can you think of another way to find the total of what you spent?
- How can the calculator be useful in your decision making?

Extension:

Have students investigate the number of tens and the number of fives in the amount of their purchases, using the calculator.

Summary:

- What kinds of coins did you use the most? Why?
- What strategies did you use for counting your coins? Why did you use that strategy?
- How does your strategy compare to someone else's?
- What kinds of discussions did you and your partner have about the coins when you were purchasing items? How did you use mathematics in these discussions?
- What did you learn in this activity? How do you think you will use this information in other situations?

Assessment

Questions:

(See summary questions.)

Observations:

- Did students recognize the coins and their corresponding values?
- Were students using appropriate techniques for counting combinations of coins?
- Did students use a variety of ways to count collections of coins?

Tasks:

- Write in your mathematics journal a description of how you chose the two items you purchased.
- Make a chart showing how many pennies, nickels, dimes, and quarters were used to purchase each item.
- Make a chart showing other coin combinations that could be used to purchase each item.

- Objective** The student will explore numbers that are related to square arrays by joining equivalent sets of objects.
- Activity** The Squares Dance
- Materials** A variety of sets of objects, such as colored tiles, interlocking cubes, base ten blocks, counters, pattern blocks; calculators; large sheets of paper
- Resources** *Sea Squares* by J. N. Hulme. *Bunches and Bunches of Bunnies* by L. Mathews

Procedure*Introduction:*

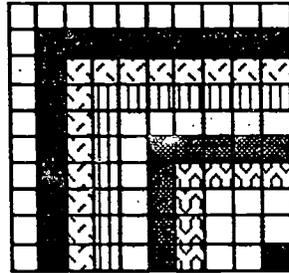
1. The book *Sea Squares* provides the focus for this lesson. The book is called *Sea Squares* because it deals with square numbers. During the first reading of the book, allow the children to enjoy the beautiful illustrations. A discussion of the book should follow the reading.
2. After the initial discussion, tell the students that you have noticed a pattern in the telling of the story. Ask them to look for a pattern as you read the book a second time. "What did you notice? What kinds of patterns did you see? Did you see any number patterns?" (Students may recognize that the number of animals increases on each subsequent page. They may also have noticed that the number of objects the animal has matches the number of animals. For example, 8 octopuses have 8 legs each. An observant child may have noticed that the border on the right side hints at what animal and how many comes next.)
3. Use square tiles to demonstrate how a square is made with the same number of tiles in every row and every column (making the same number of tiles along each edge). Show students how a square can be made with 1 tile (1 along each edge), 4 tiles (2 along each edge), and 9 tiles (3 along each edge).
4. Experiment with the class on making a square with 2 or 3 tiles, and discuss what happens.
5. Have students work in small groups to find out what other numbers can be used to make squares.
6. Students in each group must decide on a material to help them solve the problem. Brainstorm with the groups some possible methods for solving the problem and record these ideas on the blackboard. (Students' ideas may include drawing a picture, using interlocking cubes or other materials, looking for a pattern, or making a model.)
7. Each group will be given one large sheet of paper to record its findings and how group members solved the problem.

Exploration:

- How do you know that the shape you are making is a square?
- We talked about ways to make squares with edges of 1 or 2 or 3. How many might you put on the edge of the next square?
- I see you have tried to make a square with 12 cubes. Will that many work? How do you know?
- Have you figured out any strategies for deciding whether a certain number of tiles will work or not? What made you think of those strategies?
- How are you going to remember which numbers worked? Can you write that on your paper somehow?
- Do you see any patterns in the numbers you have found so far?

Extension:

- Use counters to make up and solve oral word problems. "There are 2 children. Each child has 2 mittens. How many mittens are there?" "There are 3 tricycles. Each tricycle has 3 wheels. How many wheels are there?"
- Find the location of the square numbers on a hundreds chart. Is there any pattern?
- Write on the blackboard the numbers through 100 which make squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100. Use a different color of cube to add on to make each new square. How many of each color is needed to build each new square?



1, 3, 5, 7, 9, 11, 13, 15, 17, 19. (Students may notice a pattern of odd numbers.)

- As a take-home activity, look for and draw other places where you find patterns of squares (floor tiles, bathroom tiles, checkerboards, wallpaper, wrapping paper, and so on).

Summary:

- What numbers made squares?
- What numbers did not make squares?
- What strategies did you use to decide if a number could make a square or not?
- Did you have to try every number, or did you figure out a way to be able to tell without actually making a square?

Read to the students the book *Bunches and Bunches of Bunnies* and have them compare it to the book *Sea Squares*. (The pattern of numbers is the same.)

- How is this book like *Sea Squares*?
- In this book, 2 groups of 2 make 4 and 3 groups of 3 make 9. How is that like making the squares with tiles?
- How would you describe your other squares in this language: "___ groups of ___"?
- Show with the tiles in your squares the equal groups.

Assessment

Questions:

(See summary questions.)

Observations:

- Were students able to communicate with one another about making the squares?
- Did their discussion show an understanding of the problem?
- Were students making connections between the square arrays and the equal groups?
- Were students working individually within the group or collaborating?

Tasks:

- Working in pairs, create a page for a squares book. For example, make a page showing many representations of 7 groups of 7.
- In your mathematics journal (or in a classroom discussion for an experience chart), describe what you learned about square numbers.

Objective The student will develop a sense of measurement as a description of a particular characteristic of an object in order to communicate information about that object. In addition, the student will generate physical references for standard measurement units by comparing the properties and characteristics of objects in the local environment to given measures.

Activity Measurement Scavenger Hunt

Materials For use with both English and metric units, a variety of measuring devices such as balance scales and masses, tape measures, trundle wheels, graduated cylinders, rulers, spring scales, measuring cups and spoons, stopwatches; a list of measurements to find objects to fit; large sheets of newsprint, each labeled with a particular measurement, on which groups can post names or pictures of the objects they find to fit each measurement

Procedure

Introduction:

1. Post on the bulletin board, blackboard, or overhead transparency a list of measurements; e.g., 1 foot, 1 inch, 1 meter around, 1 decimeter, 1 minute, 1 centimeter, 100 grams.
2. Have students discuss what they know about these measurements; for example, what kinds of things each might measure, what the characteristic is that is being measured, or what the unit of measure itself might look like.
3. Challenge each group of four to find something in the classroom to fit each given measurement. Label a sheet of newsprint with each measurement and have each group post their objects on the appropriate charts (by writing names or drawing pictures). Encourage each group to find examples that are different from the examples of the other groups.
4. Discuss the items found by each group and the strategies they used in finding and testing items.

Exploration:

- What tools can you use to do each measurement?
- Can you use some tools more often than others?
- Would any tool fit every kind of measurement?
- Does your measurement always have to be exact?
- How do you know when you are close enough?
- How do you know that the object fits the measurement?
- Why did you use this tool and not that one?
- Is there another tool you could have used to make this measurement?
- If you use another tool (for example, the ruler instead of the tape measure) will you get the same result? Why or why not?
- How can you use the information on the charts to get more ideas about what to investigate?

Extension:

- Identify a measurement not on the list and make a chart of items for it.
- Group the measurements that use the same kinds of tools.
- Select the list of objects from one of the charts and discuss ways in which all the objects in the list are alike. Are there any characteristics they all share?

Summary:

- What did you need to know to do these measurements?
- Did you need any new information to make the measurements?
- Why did you use the measurement tool you used for _____?
- Were some tools more effective than others for measuring? Why? How did you determine whether they were more effective or not?
- Which tool did you like the best? Why?
- Which tool did you like the least? Why?
- How did your group decide which objects to try?
- Did you have to try many things to find a measure? Did what you tried help you in your next guess? Why or why not?
- Which measurement was easiest for your group to find? Why do you think that was so?
- Which measurement was hardest for your group to find? Why do you think that was so?
- Was there any measurement you didn't find? What objects did you try in looking for it?
- Did the items found by other groups and posted on the charts help your group? How?
- What measurements surprised you? Why?
- How are all the things on this list alike? How are they different?
- What does *measurement* mean?
- What did you learn from this activity that you didn't know before?
- How could you use what you have learned in another situation, like at home or while shopping?

Assessment

Questions:

- If you had to measure (object), which tool would you use? Why?
- (See also summary questions.)

Observations:

- Were students actively participating?
- Were students making measurements appropriately?
- Were students making appropriate suggestions for objects to measure?
- Were students using appropriate instruments for each type of unit?
- Were students using knowledge about measurement units?

Tasks:

- Make a class definition for measurement (to write in your journal or display on a language chart).
- Keep a scrapbook of measurements that you see in the newspaper, in magazine articles, or advertisements. Match them with the kinds of measuring tools that would be used.

Objective The student will demonstrate an understanding of the way a balance scale works.

Activity Making a Balance Scale

Materials 12-inch rulers (1 per group), paper cups (2 per group), tape (or sticky tack), three pencils taped together in a stack with two on the bottom and one on the top (1 set per group) to act as a fulcrum, small objects to put in the cups or tongue depressors (1 per student), small plastic medicine cups (2 per student), pencils or pens to use as fulcrums, small objects to put in the cups

Procedure

Introduction:

1. Begin the lesson by discussing the actions of a seesaw. For students who have not seen a seesaw before, conduct a class discussion about what a seesaw does or visit a playground.
2. Demonstrate the action of a seesaw with a 12-inch ruler, on which plastic drinking cups have been attached to the ends, balanced on a fulcrum (a stack of pencils taped together) (also an example of a first-class lever). Stress that the fulcrum needs to be in the middle for the balance to work correctly. Hold up two objects and allow the students to predict what will happen when you put one object in each cup. Have students justify their predictions.
3. Place the objects in the cups and allow the students to discuss their observations.
4. Have students work in small groups to make their own balance scale and test various objects.

Exploration:

- What do you think will happen when these objects are placed in the cups?
- Why do you think this will happen?
- What do you think is happening here?
- When you look at the objects before they are placed in the cups, is there something that helps you predict which side will go down?
- Do you notice a pattern?
- Have you found any objects that remain balanced in the cups? What does that mean?
- What could you put in the cups that would keep the balance balanced?
- What if you put the same number of pennies in each cup? Try it.
- What if you put the same number of beans in each cup? Try it.

Extension:

- Use your balance to find things that are heavier (lighter) than some given object. Make a poster of all those things.
- Write some sentences comparing your balance scale to the balance scale in the classroom, how they are alike and how they are different.
- Use your balance scale to investigate number sentences like $6 + \underline{\quad} = 8$.

Summary:

- How does your balance work?
- What information does it give you?
- How did you make your predictions about what would happen with each set of objects?
- How might you use this tool, or one like it, in the real world?
- This is a store-bought balance scale that works the same way your balance does. How do you think you would use it?

Assessment

Questions:

- What would you use a balance scale to measure?
- (See also summary questions.)

Observations:

- Were students actively participating?
- Were students using the balance appropriately?
- Were students interpreting the actions of the balance appropriately?
- Were students making connections between their balance and the commercial balance scale?

Tasks:

- Make a class description of how to use a balance scale (to write in your journal or display on a language chart).
- Demonstrate how to compare the mass/weight of two objects, using a balance scale. Describe the information the balance scale can give you about the objects.

Objective The student will explore and manipulate geometric shapes to form new shapes.

Activity The Shape Machine

Materials Pattern blocks, attribute blocks, or other materials with a variety of shapes; construction paper cut-outs of squares, circles, triangles, and rectangles; glue; calculators; recording sheet (optional)

Resources *The House That Makes Shapes* by J. Potts

Procedure

Introduction:

1. Show the cover of the book *The House That Makes Shapes* with the title covered. Ask the students to make predictions of what they think the book will be about. Ask them to observe the house. Does it provide any clues?
2. Read and discuss *The House That Makes Shapes*. Ask questions such as "How did the shapes appear? Why did the man make things with the shapes? What did the man do first?"
3. Have students examine the house the man made. Are there any patterns in the way he used the shapes? Students may find groups of three in the windows, columns, and stars.
4. Have students create their own constructions made of shapes (pattern blocks, attribute blocks, etc.). After students have had some time to explore with the blocks, allow them to walk around to see what other students have created. Someone else's idea may spark an adaptation or an original thought.
5. Following the period of exploration, put the blocks away and give each student a small piece of paper on which to draw a plan for a construction using only squares, circles, triangles, and rectangles.
6. Each student must determine the total number of each shape needed to make his or her construction. In each group of 3 or 4 students, the leader uses a calculator to determine the total number of each shape needed by the group for its constructions. The materials person in the group is then sent to obtain the appropriate number of each shape for the group. The information could be arranged in a chart such as:

The number of each shape we needed to build			
squares	circles	triangles	rectangles
27	13	10	32

- or the information could be shown in a bar graph (see the page following this activity).
7. Each member of the group takes the number of shapes needed to make his or her construction according to the original plan. The students then paste their shapes to make the construction as on the plan. If there is a problem during the activity, the groups must determine how the problem occurred (miscount from the plan, incorrect total for the group, change of plan, etc.) before they can obtain more shapes.

Exploration:

- What shapes are you using in your plan? Why?
- How many of each shape are you using?
- What shape does your group need the most of? The least? Is that the same as for your plan?
- How did your leader determine how many of each shape was needed?
- How did the materials person make sure he or she got that many of that shape?
- What shapes did you like to use the best? The least? Why?
- How do you think the plans affected which shapes were used?

Extension:

- Make a bar graph of the shapes your group used. Compare it to other groups' shapes.
- Examine things at home. What kinds of shapes do you see? Which shape do you see most often? Least often? Why?

Summary:

- What shapes did you use the most? The least? Why?
- What shapes did your group use the most? The least?
- Were your number of shapes the same or different than other groups?
- What kinds of plans used circles most?
- What kinds of plans used rectangles most?
- What kind of plan could you make that would need a lot of triangles?
- How did your group organize your information about the shapes you needed?
- What problems did you encounter with your numbers of shapes? How did you solve them?

Assessment

Questions:

(See summary questions.)

Observations:

- Were students actively participating?
- Were students organizing information appropriately?
- Were students making appropriate suggestions for solving problems they encountered?
- Were students using appropriate geometric vocabulary in their discussions?

Tasks:

- Make a class chart and bar graph of the numbers of shapes that were used. Compare it to the information from another class that did the same activity.
- Keep an organized scrapbook of magazine pictures that represent different geometric shapes.

Objective The student will collect and organize information to determine whether more of the surface area of the earth is covered by land or by water.

Activity More Land or Water?

Materials Inflatable globe (one per pair of students); small sticky dots (one per child); paper/pencil for recording data; newsprint/markers for recording combined data; interlocking cubes and balance scales or graduated cylinders, spoons, sand, and water (for comparing data)

Resources *Alexander and the Terrible, Horrible, No Good, Very Bad Day* by J. Viorst

Procedure

Introduction:

1. Read a story like *Alexander and the Terrible, Horrible, No Good, Very Bad Day* by Judith Viorst that involves cross-continent traveling. (Remember, he decides he'll just go to Australia!)
2. Locate on the globe the places named in the story and discuss what kind of route would need to be used to get there—travel across land vs. travel across water.
3. Ask students whether they think there is more land or more water covering the surface of the earth.
4. Ask them to explain their predictions.
5. Brainstorm with students ways they could find out which covers more of the earth, land or water. Suggestions might include finding a puzzle of the globe, putting the water pieces in one pile and the land pieces in another and comparing the two piles.
6. Demonstrate for the students one way to collect random samples of data by tossing the globe lightly, catching it, and looking to see whether your index finger is touching land or water. Discuss ways to make sure that the catch is as random as possible (e.g., toss or roll the globe between two people).
7. Have each student place a sticky dot on the nail of the right index finger to mark it as the one to look at after catching the globe.
8. Have each group identify a tosser, a catcher, a watcher, and a recorder. The tosser should toss (or roll) the globe to the catcher five times while the watcher and recorder collect and record the data. The jobs should then be rotated until every person in the group has performed every job and the group has 20 pieces of data recorded.
9. Have each group suggest ways to organize its own data to make comparisons and to come to a conclusion. Organization and comparison of the data could be done by representing the data with interlocking cubes (maybe blue for landing on water and green for land) and comparing the two sets by making groups of tens, making a bar graph, or weighing them on a balance scale. The data could be represented also by placing a teaspoonful of sand in a graduated cylinder for each land tally and a teaspoonful of water in another graduated cylinder for each water tally; then comparisons of the volumes could be made.

Exploration:

- How can you record where your finger lands on the globe each time?
- How are you going to record your result if your finger lands on both water and land at the same time?
- What are some different ways of recording the tosses?
- Do you think rolling the ball gives you different results than tossing it?

- How can you decide whether your finger landed more times on land or water?
- What kind of techniques can we use to compare the two sets of data?
- What conclusions can you make from your data?
- Do you think your data is just like any other group's data? Why or why not?
- Do you think if you did this experiment again that you would get exactly the same set of data? How do you think it might be the same? How do you think it might be different?

Extension:

- Toss the globe five more times each to see if it changes your conclusion.
- Use a piece of centimeter graph paper to estimate the surface area of the globe covered by water and the surface area covered by land.

Summary:

- How did your group decide to represent your data?
- What techniques did your group use to compare the land to the water? Which ones do you think worked the best?
- What conclusions did you make from your data? Why?
- How is your group's data different from the data of another group?
- How are the data from all the groups alike?
- What conclusions can you make from the combined data of the whole class? How are these conclusions alike or different from those made from your group's data?
- Why do you think the combined results did/didn't match your group's results?
- What if the experiment was done with adult hands? with a baby's hands? Would the results change? Why or why not?
- What if the globe were larger or smaller? Would the results change? Why or why not?
- How would the results be affected if we tossed the globe fewer times? more times?
- Would you need to travel in a car or a boat more often from America to Europe?
- If you were a world traveler, would you rather have a car or a boat? Why?

Assessment

Questions:

- What questions could the graph (or other data representations) answer?
- What questions could not be answered from the data?
- How are the ways we presented the data alike?
- How are they different?
- If you went around the world, would you spend more time in a boat or in a car? Why?
- From the shuttle, do astronauts see land or water more often?
- How did your group organize itself?
- How could your group improve in its working together?
- (See also summary questions.)

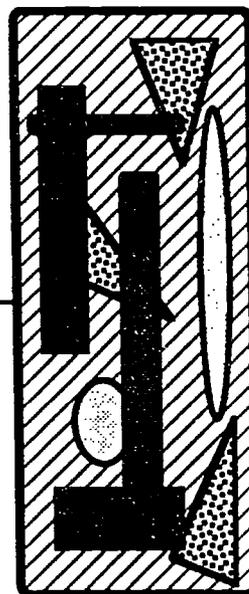
Observations:

- Did student discussion indicate an understanding of the problem?
- Did student discussion indicate an understanding of the connections between the data and the problem to be solved?
- Did students develop appropriate strategies for solving the problem or were they randomly guessing?
- Were students working individually within the group or collaborating?

Tasks:

- Write, or dictate, a story about how this information might affect your future. (For example, what implications does it have for population growth? For food resources? What happens when the land is filled with people?)
- Suppose that after watching a scary movie your little brother or sister is really afraid of a meteor coming down and crashing on his or her head. How could you use this information to help the child be less scared?

Evaluation



Philosophy

NCTM's *Professional Standards for Teaching Mathematics* and *Curriculum and Evaluation Standards for School Mathematics* (the *Standards*) emphasize the connection between assessment of students and analysis of instruction. In other words, mathematics teachers should monitor students' learning (both formatively and summatively) in order to assess and adjust teaching. Teachers must observe and listen in order to tailor teaching strategies. Information about what students are understanding should be used to revise and adapt short- and long-range plans, and students' understandings should guide teachers in shaping the learning environment. Also, teachers are responsible for describing students' learning to administrators, parents, and students themselves.

Students' mathematical power depends on various understandings, skills, and dispositions. The development of students' abilities to reason mathematically—to conjecture, justify, and revise based on evidence and to analyze and solve problems—must be assessed. A student's disposition toward mathematics (confidence, interest, perseverance, etc.) is also a key dimension that teachers should monitor.

The importance of using assessment to improve instruction is crucial. Information should be gathered from multiple sources using numerous assessment techniques and modes that are aligned with the curriculum. Assessment techniques must reflect the diversity of instructional methods implied in the *Standards* and the various ways students learn and process information. Instructional decisions should be based on this convergence of information from different sources.

In summary, the following aspects of students assessment and program evaluation should receive increased and decreased attention (NCTM, 1989):

Increased Attention

- Assessing what students know and how they think about mathematics
- Having assessment be an integral part of teaching
- Focusing on a broad range of mathematical tasks and taking a holistic view of mathematics
- Developing problem situations that require the applications of a number of mathematical ideas
- Using multiple assessment techniques, including written, oral, and demonstration formats
- Using calculators, computers, and manipulatives in assessment
- Evaluating the program by systematically collecting information on outcomes, curriculum, and instruction
- Using standardized achievement tests as only one of many indicators of program outcomes

Decreased Attention

- Assessing what students do not know
- Having assessment be simply counting correct answers on tests for the sole purpose of assigning grades
- Focusing on a large number of specific and isolated skills organized by a content-behavior matrix
- Using exercises or word problems requiring only one or two skills
- Using only written tests
- Excluding calculators, computers, and manipulatives from the assessment process
- Evaluating the program only on the basis of test scores
- Using standardized achievement tests as the only indicator of program outcomes

Types of Evaluation

While paper and pencil tests are one useful medium for judging aspects of students' mathematical knowledge, teachers need information gathered in a variety of ways and using a range of sources. Observing, interviewing, and closely watching and listening to students are all important means of assessment. While monitoring students, teachers can evaluate the learning environment, tasks, and discourse that have been taking place. Using a variety of strategies, teachers should assess students' capacities and inclinations to analyze situations, frame and solve problems, and make sense of concepts and procedures. Such information should be used to assess how students are doing, as well as how well the tasks, discourse, and environment are fostering students' mathematical power and then to adapt instruction in response.

Assessment instruments and techniques should be properly aligned with the curriculum to enable educators to draw conclusions about instructional needs, progress in achieving the goals of the curriculum, and the effectiveness of a mathematics program. That is, the content, processes, and skills assessed must reflect the goals, objectives, and breadth of topics specified in the curriculum. The particular emphases of the assessment should reflect the emphases of instruction. For example, primary children, whose understanding of fractions is closely tied to the use of physical materials, should be encouraged to use such materials to demonstrate their conceptual knowledge. Assessment items need to be structured around the central ideas of the curriculum and need to provide opportunities for students to demonstrate their understanding of the connections among major concepts. In addition, assessment must reflect the relative emphasis placed on technology during instruction; to the extent that calculators and computers have been important during instruction, they should also be available during assessment.

Assessment techniques suggested in the *Standards* include multiple-choice, short-answer, discussion, and open-ended questions; interviews; homework; projects; journals; essays; portfolios; presentations; and dramatizations. Assessment can occur during and after whole-group explorations, during whole-group discussions, in sharing sessions, during individual conferences, during small-group conferences, while students are working on projects, after completion of projects, when students are engaged in self-evaluation tasks, and continually while students are explaining, justifying, debating, and questioning ideas and concepts.

Using Portfolios. Student portfolios are becoming more prevalent as a means of keeping a record of student progress in mathematics. Teachers have always kept folders of students' work, but portfolios should have more focus and be more important for assessment. An assessment portfolio is a planned selection of a student's work collected throughout the school year. Teachers as well as students should be allowed to choose the items to be included in portfolios, since it gives a good indication of what is valued in the work the students do throughout the school year. A portfolio might include samples of student-produced written descriptions of the results of practical or mathematical investigations; pictures and dictated reports from younger students; extended analyses of problem situations and investigations; descriptions and diagrams of problem-solving processes; statistical and graphic representations; responses to open-ended questions or homework problems; group reports and photographs of student projects; copies of awards or prizes; video, audio, and computer-generated examples of student work; and other material based on project ideas developed with colleagues.

Using Writing. Communication in mathematics has become important as we move into an era of a thinking curriculum. Journals, logs, problem-solving notebooks, explanations, justifications, and reflections are ways to include writing in the mathematics curriculum. Students should be urged to discuss ideas with each other, and to ask questions, to diagram and graph problem situations for clarity. Writing in mathematics classes, once rare, is now vital. In particular, mathematics journals can include the following:

- vocabulary definitions written in the student's own words along with explanations of how the terms are used in mathematics
- rules or procedures written as if explained to a friend in a letter or to another student who was absent during the instruction
- free writing, including what students think they will learn in an experience, descriptions of accomplishments, how students can use what they've learned, what isn't fully understood or is causing difficulty, examples in the real world related to the mathematics learned, a discovery made or additional ideas and conjectures related to the topic, and what else students might want to learn about.

These writing experiences are also important mathematics learning experiences in that they:

- help students become more active in their own learning
- help students internalize what they are learning to make it more meaningful
- allow students to express their feelings and attitudes toward mathematics
- give students a source they can use for studying
- allow students to reflect upon and clarify their own thinking
- give students the opportunity to share with each other what they are learning, also allowing them to learn from one another
- allow students to go beyond what they are learning in class and to make conjectures and connections
- give students the opportunity to think of mathematics as existing outside the classroom
- give students the opportunity to communicate with the teacher in an informal setting
- give the teacher an idea of how students are thinking
- allow the teacher to informally assess student learning (whether it be pre- or post-assessment)

Using Teacher Observations. Teacher observations can be broken down into two levels: formal and informal. Formal observations include checklists, comment cards, and summaries. Informal observations include mental notes. Students should be observed both individually and as they work in groups. When using observations, a teacher should look for students' learning styles, students' ideas, communication techniques, cooperation strategies, and use of manipulatives. Some possible questions that can guide observations of students doing mathematics are:

- Does the student consistently work alone or with others?
- Does the student try to explain organizational and mathematical ideas?
- Does the student synthesize and summarize his or her own or a group's thinking?
- Does the student work with the group to agree on a plan or structure for tackling the task?

- Individually or within the group, does the student choose and use appropriate manipulatives?

Using Questioning. Asking the right question is an art to be cultivated by all educators. Low-level quizzes that ask for recall or simple computations are over used and over done. Using good, high-level open-ended questions that give students a chance to think are one of the goals of mathematics assessment. These questions might be used as teaching or leading questions as well as for assessment purposes. Both questions and responses may be oral, written, or demonstrated by actions taken. When using oral questions, the teacher can prepare a list of possible questions ahead of time. (For examples, see the sample activities in the previous section.) The teacher should allow for plenty of wait time. The teacher may keep a written record of observations during the questioning time to use for formal assessment. Questioning for assessment should occur in several places during instruction:

- during introductory activities to assess students' prior knowledge and experience
- during exploration to focus students' attention on important concepts and connections
- after instruction, in order for students to summarize results, reflect on their experience, and clarify their thoughts

Using Student Presentations. Student presentations can take many forms, including oral explanations, oral presentations, and projects. One of the best ways to assure the connection between instruction and assessment is to embed assessment into instruction. When students become involved in projects or investigations, assessment becomes natural and invisible. Student presentations may be related to connections within mathematics and connections outside mathematics. When evaluating student presentations, the teacher should look for whether the student can identify and define the problem; make a plan; collect needed information; organize the information and look for patterns; discuss, review, revise, and explain results; and produce a quality product or report.

Using Performance Assessment. Performance assessment involves giving a group of students, or an individual student, a mathematical task that may take from half an hour to several days to complete or solve. The object of this form of assessment is to look at how students are working, as well as at the completed tasks or products. Performance assessment requires the teacher to look at how students solve a problem. Performance activities may be videotaped, tape recorded, or recorded in writing. The task might be from any mathematical content area and might include some connections such as with science, social studies, language arts, or fine arts. Performance assessment is an excellent place for students to use manipulatives to demonstrate understanding of mathematics content. Information from performance assessment can be recorded using rubrics that assign point values to important aspects of the problem-solving process. For example, the following assessment criteria could be used during observation or based on written work to judge a student's involvement in problem solving:

1. Understanding the Problem:

0 points	Does not understand the problem
1 point	Misunderstands part of the problem
2 points	Completely understands the problem

2. Choosing and Implementing a Solution Strategy

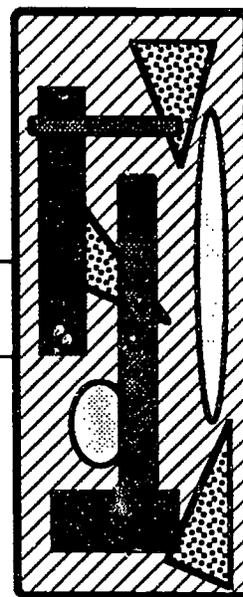
- 0 points Makes no attempt or uses a totally inappropriate strategy
- 1 point Chooses a partly correct strategy based on interpreting part of the problem incorrectly
- 2 points Chooses a strategy that could lead to a correct solution if used without error

3. Getting the Answer

- 0 points Gets no answer or a wrong answer based on an inappropriate solution strategy
- 1 point Makes a copying error or computational error; gets partial answer for a problem with multiple answers; or labels answer incorrectly
- 2 points Gets correct solution

Some excellent resources on assessment, in addition to the NCTM *Curriculum and Evaluation Standards*, include *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions* (1991) and *Assessment alternatives in Mathematics* (Stenmark, 1989).

References and Resources



- Baker, A. and Baker, J. (1991). *Maths in the mind: A process approach to mental strategies*. Portsmouth, NH: Heinemann.
- Burns, M. (1987). *A collection of math lessons from grades 3 through 6*. Sausalito, CA: Math Solution Publications.
- Burns, M. (1992). *About teaching mathematics*. Sausalito, CA: Math Solution Publications.
- Burns, M. and Tank, B. (1988). *A collection of math lessons from grades 1 through 3*. Sausalito, CA: Math Solution Publications.
- Fun with foods*. (1987). Fresno, CA: AIMS Education Foundation.
- Geddes, D. (1992). *Geometry in the middle grades: Addenda Series, Grades 5-8*. Reston, VA: National Council of Teachers of Mathematics.
- Math + Science = Solution*. (1987). Fresno, CA: AIMS Education Foundation.
- Mathematical Sciences Education Board. (1989). *Everybody counts: A report to the nation on the future of mathematics education*. Washington, DC: National Research Council.
- Stenmark, J. K. (Ed.) (1991). *Mathematics assessment: Myths, models, good questions, and practical suggestions*. Reston, Va: National Council of Teachers of Mathematics.
- Middle Grades Mathematics Project. (1986). *Factors and multiples*. Menlo Park, CA: Addison-Wesley.
- Middle Grades Mathematics Project. (1986). *Mouse and elephant: Measuring growth*. Menlo Park, CA: Addison-Wesley.
- Middle Grades Mathematics Project. (1986). *Spatial visualization*. Menlo Park, CA: Addison-Wesley.

- Molyneux, L. (1992). *Cooperative learning, math and success*. Canandaigua, NY: Trellis Books.
- National Council of Teachers of Mathematics. (various years). *Arithmetic Teacher* (a journal of philosophy and activities for teachers of elementary mathematics).
- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- Parker, T. (1984). *In one day*. Boston, MA: Houghton Mifflin.
- Pollard, J. (1985). *Building toothpick bridges*. Palo Alto, CA: Dale Seymour Publications.
- Popping with power*. (1987). Fresno, CA: AIMS Education Foundation.
- Stenmark, J. K. (1989). *Assessment alternatives in mathematics*. Berkeley, CA: Lawrence Hall of Science, University of California.
- Stenmark, J. K., Thompson, V., and Cossey, R. (1986). *Family math*. Berkeley, CA: Lawrence Hall of Science, University of California.
- Wahl, J. and Wahl, S. (1976). *I can count the petals of a flower*. Reston, VA: National Council of Teachers of Mathematics.

Children's Trade Books

Numerous children's books have the potential for motivating rich mathematics activities. This short list gives the bibliographic information of the books mentioned in the activities in this series of curriculum guides for elementary mathematics (Grades 1 - 5).

- Anno, M. (1982). *Anno's counting house*. New York: Philomel Books.
- Anno, M. (1983). *Anno's mysterious multiplying jar*. New York: Philomel Books.
- Aardema, V. (1976). *Why mosquitoes buzz in people's ears*. New York: Dial Books for Young Readers.
- Bemelmans, L. (1960). *Madeline*. New York: Viking Children's Books.
- Bishop, C. H. (1938). *Five Chinese brothers*. New York: Coward.
- Brier, C. (1983). *The shoemaker and the elves*. New York: Lothrop, Lee & Shepard.
- Carle, E. (1977). *The grouchy ladybug*. New York: Scholastic, Inc.
- Carle, E. (1989). *The very hungry caterpillar*. New York: Putman.
- Dahl, R. (1964). *Charlie and the chocolate factory*. New York: Alfred Knopf.
- de Paola, T. (1978). *The popcorn book*. New York: Holiday House.
- Ehlert, L. (1990). *Fish eyes: A book you can count on*. San Diego: Harcourt, Brace, Jovanovich.
- Faucher, E. (1989). *Honey, I shrunk the kids*. New York: Scholastic, Inc.
- Flournoy, V. (1985). *The patchwork quilt*. New York: Dial Books for Young Readers.
- Freeman, D. (1968). *Corduroy*. New York: Viking Press.
- Giganti, Jr., P. (1992). *Each orange had 8 slices*. New York: The Trumpet Club.
- Haley, G. E. (1971). *Noah's ark*. New York: Atheneum.
- Hogryan, N. (1966). *Always room for one more*. New York: Holt.
- Hulme, J. N. (1991). *Sea squares*. New York: Hyperion Books.
- Hutchins, P. (1987). *The doorbell rang*. New York: Scholastic, Inc.
- Lobel, A. (1983). *Frog and toad are friends*. New York: Harper.
- Mathews, L. (1978). *Bunches and bunches of bunnies*. New York: Scholastic, Inc.
- McMillan, B. (1991). *Eating fractions*. New York: Scholastic, Inc.

- Owen, A. (1988). *Annie's one to ten*. New York: Alfred Knopf.
- Parish, P. (1963). *Amelia Bedelia*. New York: Harper and Row.
- Peterson, J. (1967). *The Littles*. New York: Scholastic, Inc.
- Potts, J. (1992). *The house that makes shapes*. New York: Hyperion.
- Prelutsky, J. (1984). *New kid on the block*. New York: Greenwillow.
- Prelutsky, J. (1978). *The Queen of Eene*. New York: Greenwillow.
- Price, L. (1990). *Aida*. New York: Harcourt, Brace, Javonovich.
- Sendak, M. (1962). *Chicken soup with rice*. New York: Harper and Row.
- Silverstein, S. (1981). *A light in the attic*. New York: Harper and Row.
- Silverstein, S. (1974). *Where the sidewalk ends*. New York: Harper and Row.
- Singer, M. (1991). *Nine o'clock lullaby*. New York: HarperTrophy.
- Step toe, J. (1980). *Daddy is a monster . . . sometimes*. New York: HarperTrophy.
- Ward, C. (1988). *Cookie's week*. New York: Putnam.
- West, C. (1987). *Ten little crocodiles*. New York: Barron's.
- Yolen, J. (1987). *Owl moon*. New York: Philomel Books.

Software

The following list contains bibliographic information for the software packages mentioned in this series of curriculum guides for elementary mathematics (Grades 1 - 5). Other appropriate software may be obtained from these and other companies.

Blockers and Finders from WINGS for learning/Sunburst Communications, 1600 Green Hills Road, P.O. Box 660002, Scotts Valley, CA 95067-0002.

Geometric preSupposer from WINGS for learning/Sunburst Communications, 1600 Green Hills Road, P.O. Box 660002, Scotts Valley, CA 95067-0002.

Hands-On Math: Volumes 1, 2, and 3 from Ventura Educational Systems, 3440 Brokenhill Street, Newbury Park, CA 91320.

Suggested Manipulatives

The following is a list of the manipulative materials used in the activities in this series of curriculum guides for elementary mathematics (Grades 1 - 5):

Calculators
Base ten blocks
Coins and bills (play or real money)
Interlocking cubes
Colored tiles
Pattern blocks
Cuisenaire rods

Graphing floor mat
Polyhedral dice (including the regular cube)
Colored chips
Two-color counters

Attribute blocks
Geoblocks
Geoboards
Tangrams
Plastic mirrors
Wooden or plastic models of geometric solids

Balance scales and masses (customary and metric)
Spring scales
Tape measures (customary and metric)
Rulers (customary and metric)
Meter sticks and yardsticks
Trundle wheels
Graduated cylinders
Measuring cups and spoons
Stopwatches

ACKNOWLEDGMENTS

The following mathematics educators assisted in the development of this series of curriculum guides for mathematics, Grades 1 - 5.

Jane F. Schielack, Ph.D.

Texas A&M University
Project coordinator

Charles E. Lamb, Ph.D.

Texas A&M University
Consulting editor

Jennie Bennett—Project Writer

University of Houston

Wayne Gable—Project Writer

Austin ISD

Judy A. Abbott

University of Texas at Austin

Provided assistance in organization and preparation of materials.

Advisory Committee for Grades 1 - 2

Jennie Bennett, Fort Bend ISD

Merry Brown, Brazosport ISD

Kathleen Buchert, Region III ESC

Pat Echols, Klein ISD

Cathy Rahlfs, Humble ISD

Patti Wooten, Aldine ISD

Development of materials for this publication was funded with Title II, Dwight D. Eisenhower Federal Funds (P.L. 100-297, as amended) administered by the Texas Staff Development Project, Bonnie McNemar project director.

COMPLIANCE STATEMENT

TITLE VI, CIVIL RIGHTS ACT OF 1964; THE MODIFIED COURT ORDER, CIVIL ACTION 5281, FEDERAL DISTRICT COURT, EASTERN DISTRICT OF TEXAS, TYLER DIVISION

Reviews of local education agencies pertaining to compliance with Title VI Civil Rights Act of 1964 and with specific requirements of the Modified Court Order, Civil Action No. 5281, Federal District Court, Eastern District of Texas, Tyler Division are conducted periodically by staff representatives of the Texas Education Agency. These reviews cover at least the following policies and practices:

- (1) acceptance policies on student transfers from other school districts;
- (2) operation of school bus routes or runs on a nonsegregated basis;
- (3) nondiscrimination in extracurricular activities and the use of school facilities;
- (4) nondiscriminatory practices in the hiring, assigning, promoting, paying, demoting, reassigning, or dismissing of faculty and staff members who work with children;
- (5) enrollment and assignment of students without discrimination on the basis of race, color, or national origin;
- (6) nondiscriminatory practices relating to the use of a student's first language; and
- (7) evidence of published procedures for hearing complaints and grievances.

In addition to conducting reviews, the Texas Education Agency staff representatives check complaints of discrimination made by a citizen or citizens residing in a school district where it is alleged discriminatory practices have occurred or are occurring.

Where a violation of Title VI of the Civil Rights Act is found, the findings are reported to the Office for Civil Rights, U.S. Department of Education.

If there is a direct violation of the Court Order in Civil Action No. 5281 that cannot be cleared through negotiation, the sanctions required by the Court Order are applied.

TITLE VII, CIVIL RIGHTS ACT OF 1964 AS AMENDED BY THE EQUAL EMPLOYMENT OPPORTUNITY ACT OF 1972; EXECUTIVE ORDERS 11246 AND 11375; EQUAL PAY ACT OF 1964; TITLE IX, EDUCATION AMENDMENTS; REHABILITATION ACT OF 1973 AS AMENDED; 1974 AMENDMENTS TO THE WAGE-HOUR LAW EXPANDING THE AGE DISCRIMINATION IN EMPLOYMENT ACT OF 1967; VIETNAM ERA VETERANS READJUSTMENT ASSISTANCE ACT OF 1972 AS AMENDED; IMMIGRATION REFORM AND CONTROL ACT OF 1986; AMERICANS WITH DISABILITIES ACT OF 1990; AND THE CIVIL RIGHTS ACT OF 1991.

The Texas Education Agency shall comply fully with the nondiscrimination provisions of all federal and state laws, rules, and regulations by assuring that no person shall be excluded from consideration for recruitment, selection, appointment, training, promotion, retention, or any other personnel action, or be denied any benefits or participation in any educational programs or activities which it operates on the grounds of race, religion, color, national origin, sex, disability, age, or veteran status (except where age, sex, or disability constitutes a bona fide occupational qualification necessary to proper and efficient administration). The Texas Education Agency is an Equal Employment Opportunity/Affirmative Action employer.



Texas Education Agency
1701 North Congress Avenue
Austin, Texas 78701-1494

CU4 211 12