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ABSTRACT

This paper, in a fashion easy to follow, illustrates the interesting relationship between structural equation modeling and canonical correlation analysis. Although computationally somewhat inconvenient, representing canonical correlation as a structural equation model may provide some information which is not available from conventional canonical correlation analysis. Hierarchically, with regard to the degree of generality of the techniques, it is suggested that structural equation modeling stands to be a more general approach. For researchers interested in these techniques, understanding the interrelationship among them is meaningful, since our judicious choice of these analytic techniques for our research depends on such understanding. Three tables and two figures present the data. (Contains 25 references.) (Author)

Structural Equation Modeling and Cancnical Correlation Analysis: What Do They Have in Common?

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Abstract

The paper, in a fashion easy to follow, illustrates the interesting relationship between structural equation modeling and canonical correlation analysis. Although computationally somewhat inconvenient, representing canonical correlation analysis as a structural equation model may provide some information which is not available from conventional canonical correlation analysis. Hierarchically, with regard to the degree of generality of the techniques, it is suggested that structural equation modeling stands to be a more general approach. For researchers interested in these techniques, understanding the interrelationship among them is meaningful, since our judicious choice of these analytic techniques for our research depends on such understanding.

INTRODUCTION

The utilization of multivariate methods has been widely recognized as to be important in social and behavioral science research. The importance mainly stems from two considerations: 1) we intend to honor the complex social reality in which we operate and which we eventually want to generalize to; 2) we intend to avoid inflating experiment-wise error rate in our statistical analysis (Fish, 1988; Johnson & Wichern, 1988; SAS/STAT User's Guide, Version 6, Vol. 4, 1989; Stevens, 1986). Among the multivariate statistical techniques, canonical correlation analysis has occupied an important strategic position. It has often been conceptualized as a unified approach to many parametric statistical testing procedures, univariate and multivariate alike (Baggaley, 1981; Dunteman, 1984; Fornell, 1978; Knapp, 1978; Kshirsagar, 1972; SAS/STAT User's Guide, Version 6, Vol. 4, 1989; Thompson, 1984, 1991).

Many popular parametric statistical techniques share one thing in common: they are designed to analyze linear relationships among variables. It has been shown that many of the seemingly different techniques are really not that different from one another after all (Fan, 1992; Knapp, 1978; Thompson, 1991). More specifically, "most of the practical problems arising in statistics can be translated, in some form or the other, as the problem of measurement of association between two vector variates **x** and **y**" (Kshirsagar, 1972, p. 281). Canonical correlation analysis, which summarizes the relationships between two groups (vectors) of variables, naturally brings order to the superficial chaos of a myriad of analytic techniques. Theoretically or empirically, the interesting relationship between canonical correlation analysis and many other statistical testing procedures ranging from simple correlation, t-test, to MANOVA, discriminant analysis has been shown many times (Knapp, 1971; Kshirsagar, 1972; Tatsuoka, 1989; Thompson, 1991).

Structural equation modeling (SEM) has been heralded as a unified model which joins methods from econometrics, psychometrics, sociometrics, and multivariate statistics (Bentler, 1994). The generality and wide applicability of structural equation modeling approach has been amply demonstrated (Jöreskog & Sörbom, 1989; Bentler, 1992). The statistical relationship among structural equation modeling and canonical correlation analysis, however, is less obvious. Consequently such relationship is less known by researchers, despite the insightful discussion on this topic provided by Bagozzi, Fornell and Larcker (1981). The purpose of this paper is to use concrete analysis examples to show the relationship among structural equation modeling and canonical correlation analysis. Also, this paper attempts to raise some questions about whether the structural equation modeling approach to canonical analysis will allow us to accomplish more than conventional canonical correlation approach. For researchers who are interested in these sophisticated analytical tools, to understand the underlying relationship among these seemingly different methods will contribute to their judicious choice among these techniques as they encounter different research situations.

SEM AND CANONICAL CORRELATION ANALYSIS

Canonical Correlation Analysis

Pioneered by Hotelling (1935), canonical correlation analysis seeks to identify and measure the association between two sets of variables. Canonical correlation analysis can be understood as the bivariate correlation of two synthetic variables which are the linear combinations of the two sets of original variables (Johnson & Wichern, 1988; Thompson, 1984, 1991).

The two sets of original variables are linearly combined to produce pairs of synthetic variables which have maximum correlation, with the restriction that each member of each subsequent set of such synthetic variables is orthogonal to all members of all other sets. The maximum number of such pairs of synthetic variables which can be produced equals the number of variables in the smaller set of the two. In this sense, the synthetic variables in canonical correlation analysis, which are the linear combinations of the original variables, are similar to the synthetic variables produced in some other multivariate analysis techniques such as principal component analysis, discriminant analysis, etc. The difference is that, in different statistical analysis, the original variables are linearly combined to satisfy different criteria. For example, in discriminant analysis, the original variables are linearly combined to produce synthetic variables which maximizes the ratio of between-group variance to within-group variance, so that different groups can be maximally differentiated on the synthetic variables. In principal component analysis, the synthetic variables (principal components) are constructed so that the variance on the synthetic variables are maximized, and the maximum amount of variance in the original variables can be accounted for by the smallest number of these principal components.

As can be expected for a multivariate statistical method, in canonical correlation analysis, eigenstructures of certain matrices play a crucial role in deriving the linear coefficients needed to produce the synthetic variables and in deriving the canonical correlation coefficients for different canonical functions. Assume two sets of variables **X** and **Y**, with **X** set is the smaller of the two. When combined, the two sets of variables have the following partitioned correlation matrix:

$$R = \begin{bmatrix} R_{xx} & R_{xy} \\ R_{yx} & R_{yy} \end{bmatrix}$$

The derivation of the linear coefficients for combining original variables into canonical variates is based on the following two matrices **A** and **B**, which have the same eigenvalues λ_i, but have different eigenvectors $\mathbf{a}_i$ and $\mathbf{b}_i$ associated with the eigenvalues λ_i.

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$$A = R_{xx}^{-1} R_{xy} R_{yy}^{-1} R_{yx}$$
$$B = R_{yy}^{-1} R_{yx} R_{xx}^{-1} R_{xy}$$

The elements in the eigenvectors a_i and b_i turn out to be the linear coefficients needed for the two original sets of variables X and Y respectively. These coefficients are often referred to as canonical function coefficients (Thompson, 1984) or canonical weights (Pedhazur, 1982). Once the linear coefficients are obtained, we use them to derive a pair of synthetic variables (canonical variates):

$$X_i^* = a_i' X = a_1x_1 + a_2x_2 + \ldots + a_ix_i$$
$$Y_i^* = b_i' Y = b_1y_1 + b_2y_2 + \ldots + b_jy_j$$

The correlation between X^*_i and Y^*_i is maximized, subject to the restriction that each subsequent canonical function is orthogonal to all previous canonical functions. The square root of the associated eigenvalue λ_i is the correlation between X* and Y* ($R_{X^*Y^*} = \lambda_i^{§}$). Once the first eigenvectors and associated eigenvalue are determined, a second pair of canonical variates can be derived based on the next eigenvector-eigenvalue pair.

Canonical structural coefficients, i.e., the correlation coefficients between the original variables (X and Y) with their respective canonical variates, can be obtained once canonical function coefficients are known by premultiplying the function coefficient vectors with the associated correlation matrix of the original variables:

$$r_{x*x} = R_{xx} a_i$$
$$r_{y*y} = R_{yy} b_j$$

For every observed variable, it is not only correlated with its own canonical variate of its own set, but it may also be correlated with the canonical variate of the other variable set. This correlation is termed "cross-loading" (Bagozzi, Fornell, & Larcker, 1981) or "index coefficients" (Johnson & Wichern, 1988; Thompson, 1984). The relationship between index coefficients and function coefficients are as follows:

$$r_{y*x} = R_{xy} b_j$$
$$r_{x*y} = R_{yx} a_i$$

Furthermore, the index coefficients are related to canonical correlation coefficient (r_c) as follows:

$$r_c^2 = \lambda_i = r'_{x*y} R_{yy}^{-1} r_{x*y}$$
$$= r'_{y*x} R_{xx}^{-1} r_{y*x}$$

Structural Equation Modeling

Structural equation models have been discussed in literature extensively (J-reskog & S-rbom, 1989; Bentler & Weeks, 1980). Different theoretical models have been proposed which essentially accomplish the same goals (McDonald, 1978, 1980; McArdle, 1980; Bentler & Weeks, 1980; J-reskog & S-rbom, 1988). Among these models, EQS (Bentler, 1992; Bentler & Weeks, 1980) and LISREL (J-reskog & S-rbom, 1988) models have become widely known among researchers in different substantive research areas.

EQS model represents the linear relationship among variables (either observed or latent) as:

$$\eta = \beta\eta + \gamma\xi$$

Where, η correspond to endogenous variables (either observed or latent), while ξ correspond to exogenous variable (also either observed or latent). γ describes the relationship from exogenous to endogenous variables, and β describes the relationship between endogenous to other endogenous variables.

Similar to EQS model but with somewhat more elaboration, LISREL model specifically distinguishes between relationships between observed and latent variables (two measurement models for exogenous and endogenous latent variables respectively) and those between latent variables themselves (structural model):

$$\begin{aligned} \eta &= B\eta + \Gamma\xi + \zeta \\ y &= \Lambda_y\eta + \epsilon \\ x &= \Lambda_x\xi + \delta \end{aligned}$$

Structural Model and Canonical Analysis

SEM Representation of Canonical Correlation Analysis

To represent canonical correlation analysis using structural equation modeling turns out to be not so straight forward. Following the demonstrations by Bagozzi et al. (1981), the canonical variate pair (X* and Y*) can be reduced to a model with a single latent variable (either X* or Y*), rather than two latent variable model (both X* and Y*). Based on this approach, the canonical correlation analysis can be represented as a Multiple Indicators/Multiple Causes (MIMIC) model. The distinguishing feature of MIMIC model is that the latent variable has both causal indicators and effect indicators (MacCallum & Browne, 1993). The application and implementation of MIMIC model in research has been discussed elsewhere (Goldberger, 1972; Jöreskog & Sörbom, 1988). The MIMIC model representing the first canonical variate X^*_1 is presented in Figure 1.

Insert Figure 1 about here

In the model of Figure 1, the Γ matrix (the effects of causal indicators on the latent variable η) contains function coefficients for the first canonical function for **X** variables. The Λ_y matrix (the effects of the latent variable η on its effect indicators) contains the index coefficients of **Y** set variables on the first canonical variate X^*_1. As discussed before, this SEM representation of canonical correlation analysis only deals one canonical variate (X_1^* or Y_1^*) at a time, and in this case, the latent variable η is actually the first canonical variate X^*_1 for **X** variables.

Since the latent variable η in Figure 1 represents the first canonical variate X^*_1, based on the concept of canonical correlation that the canonical variate is a linear combination of observed variables without error, the disturbance term (ζ) for the latent variable η should be fixed to be zero. Also, since canonical correlation analysis is symmetrical, canonical variate

Y^*_1 for Y variable set can easily be represented by reversing the direction of the MIMIC model, i.e., by changing the causal indicators into effect indicators, and the effect indicators into causal indicators.

For the first canonical function, once canonical function coefficients and index coefficients are obtained from SEM model, canonical correlation coefficient and canonical structure coefficients can be derived based on the formulas presented before. After the first canonical function depicted in Figure 1 is solved, the second canonical function can be built over the first by imposing another latent variable on the MIMIC model, and constraining all the coefficients of the original model to be equal to the values already obtained for the model. Since canonical functions are orthogonal to each other, the new latent variable is specified to have or receive no effect from the first latent variable. The first two canonical functions are depicted in Figure 2. In the same vein, more latent canonical variates can be sequentially added to the MIMIC model to accommodate more canonical functions as data permit.

Insert Figure 2 about here

From the discussion above, it appears somewhat cumbersome to represent canonical correlation analysis using SEM approach, since several related models have to be analyzed and some additional calculation is also necessary to obtain all the results typically obtained in a canonical correlation study.

These inconveniences aside, SEM approach does seem to be able to provide some additional information which canonical correlation analysis does not have. Two such potential advantages are 1) significance testing for canonical function coefficients; 2) significance testing for individual canonical functions.

Significance Testing for Coefficients

For some multivariate methods such as discriminant analysis, factor analysis, etc., theoretical sampling distributions are either not available or extremely complex for different types of coefficients, thus it is difficult or impossible to conduct statistical significance testing for these coefficients. The same is true for canonical correlation analysis.

The lack of sampling distributions for many coefficients in some multivariate methods contributes to the subjectivity of certain some decision process. For example, in canonical correlation analysis, what value of function coefficients should we use as the criterion to judge the importance of contribution of observed variables to the canonical function? Since significance testing is not available in canonical analysis for the coefficients, we have one less tool for the purpose. SEM approach to canonical analysis seems to be capable of providing us with this information. In SEM approach depicted in Figure 1, since Γ and Λ_y contain canonical function and index coefficients respectively, and standard errors for these coefficients are also available from SEM, critical *t* ratios can be calculated to give us some sense about how much sampling error we can expect for these coefficients.

Testing for Individual Canonical Functions

Mathematically, the likelihood ratio test in canonical analysis compares the sample restricted generalized variance under the null hypothesis with the unrestricted generalized variance as follows:

$$\frac{\begin{vmatrix} R_{xx} & 0 \\ 0 & R_{yy} \end{vmatrix}}{\begin{vmatrix} R_{xx} & R_{xy} \\ R_{yx} & R_{yy} \end{vmatrix}} = \frac{|R_{xx}|\ |R_{yy}|}{\begin{vmatrix} R_{xx} & R_{xy} \\ R_{yx} & R_{yy} \end{vmatrix}}$$

This test indicates that the necessary and sufficient condition for canonical correlations to be zero is $\mathbf{R}_{xy}$ ($\mathbf{R}_{yx}$) being zero.

Due to the complexity of distribution theory for sample canonical correlation coefficients (Johnson & Wichern, 1988; Kirsagar, 1972), the likelihood ratio test in canonical analysis is constructed not for testing individual canonical functions, but rather, for sequentially testing several canonical functions as a group. For example, in a situation where three canonical functions are derived, three likelihood ratio tests will be conducted. The first tests the H_0 that all canonical functions are zeros, the second tests the H_0 that the second and the third canonical functions are zeros, and the third tests the H_0 that the last canonical function is zero. If the first two tests are statistically significant and the third is not, it will be concluded that the first two canonical functions are statistically significant while the third is not. Such conclusion sounds as if we had conducted significance tests for individual canonical functions when, in reality, we have not. Strictly speaking, only the last test for the third canonical function is a true test for individual function, and all previous tests are not. Theoretically, in the situation described above, it is possible that the second canonical function by itself does not account for a statistically significant portion of data covariance, but only together with the third canonical function, they jointly account for a statistically significant portion of data covariance. This possibility will not be known under the conventional likelihood ratio test in canonical analysis.

If structural equation model can be formulated to represent canonical analysis, one interesting question will be whether statistical significance test can be conducted for individual canonical function, instead of the sequential testing process described above. In other words, can we independently test for the null hypothesis that the first canonical correlation itself is zero, or the second, and the third, instead of testing for all three, last two, and last one? Conceptually, this seems to be possible in SEM using the approach of nested models and then test for the statistical significance of differential χ^2 between two nested models, one with constraints while the other without.

Within the framework of LISREL model, the implied correlation matrix for standardized variables for our MIMIC model is as follows:

$$R = \begin{bmatrix} R_{xx} & R_{xy} \\ R_{yx} & R_{yy} \end{bmatrix} = \begin{bmatrix} \Lambda_x \phi \Lambda_x' & \Lambda_x \phi \Gamma' \Lambda_y' \\ \Lambda_y \Gamma \phi \Lambda_x' & \Lambda_y (\Gamma \phi \Gamma') \Lambda_y' + \theta_\epsilon \end{bmatrix}$$

It is easy to see that once Λ_y is constrained to be zero, $\mathbf{R}_{xy}$ ($\mathbf{R}_{yx}$) will be zero so that the restricted condition for the likelihood ratio test in conventional canonical analysis will be satisfied. This suggests that constraining the Λ_y to be zero conceptually represents a restricted model of zero canonical correlation. That zero Λ_y means zero canonical correlation is also shown through the relationship between canonical index coefficients ($\mathbf{r}_{x^* y} = \Lambda_y$) and canonical correlation coefficient. (r_c):

$$\begin{aligned} r_c^2 &= r_{x* y}' R_{yy}^{-1} r_{x* y} \\ &= r_{y* x}' R_{xx}^{-1} r_{y* x} \end{aligned}$$

The flexibility of testing for nested models approach in SEM can easily accommodate testing for such unrestricted vs restricted models, though in this case of MIMIC model, the restricted model seems to be a somewhat strange model with only causal indicators and no effect indicators. It is not clear what methodological ramifications such a restricted model may have on SEM estimation process.

An Example

An example in SAS STAT User's Guide (SAS Institute, 1988) is used as an illustration for the relationship between canonical correlation analysis and SEM. The data contain three physical variables (measurements of Weight, Waist and Pulse) and three exercise variables (numbers of Chins, Situps and Jumps completed). The correlation matrix for the three variables is presented in Table 1, and the sample size was altered in order to yield statistically significant results for the canonical functions derived from the data (original sample size for the data matrix is 20).

Table 2 presents the results of canonical function coefficients from both SEM and canonical analysis approach. The values for the coefficients from the two approaches are essentially the same, thus not repeated in the table. The critical *t* ratios for the function coefficients based on SEM approach (*t* ratios are for unstandardized coefficients) are also presented in the table together with the coefficients. As discussed previously, conventional canonical analysis does not provide estimate for standard errors for the coefficients, thus making it difficult to make statistical assessment for the relative importance of the function coefficients, i.e., the relative contribution of observed variables to the canonical variate. In SEM approach, such information becomes available, assuming that the data satisfy the SEM theoretical assumptions in order for such estimates to be valid.

It is observed that statistical importance of the coefficients may not be accurately assessed by its absolute value, since such values should be assessed in relation to its standard error. For example, the variable Chins has function coefficient -.35 on the first canonical function, and -.37 on the second canonical function. Without knowledge of the standard errors, as in conventional canonical analysis, it may be concluded that the variable Chins is about equally important for the first and second canonical functions. But when judged in relation to the standard errors, the interpretation will change dramatically. This variable is much more important for the first canonical function (critical ratio 4.59) than for the second canonical function (critical ratio .88, and statistically not different from zero). This example shows some advantage of using SEM to represent conventional canonical correlation analysis. Such information about standard errors for the coefficients is meaningful and important in the statistical sense.

Table 3 presents the results of SEM nested-model approach for testing each of the three individual canonical functions as discussed above, together with the results from the sequential testing in canonical correlation analysis. It is seen that for this dataset, the results from the two approaches are very similar. It is unclear whether this (the same results based on the two approaches) will hold for different data sets. Theoretically, the sequential testing approach in canonical analysis is somewhat different from testing individual canonical functions, since more than one canonical functions are being tested at a same time except for the last canonical function. Assuming that the nested model approach in SEM for testing canonical correlation as discussed above is correct, there seems to be some slight conceptual difference between this and the sequential testing approach. It is unclear why no obvious difference between the two approaches occurs.

The solution to the model presented in Figure 1 does not directly provide canonical correlation coefficient or canonical structural coefficients. By using formulas presented previously, these can be obtained with some calculation. Although this may be burdensome computationally, it does not negate the fact that canonical correlation analysis can be conceptually and statistically represented as a structural equation model. Such a relationship is meaningful in the sense that it further demonstrates the generality and versatility of structural equation modeling approach to data analysis.

At this time, several questions remain with regard to the relationship between canonical analysis and structural equation modeling. First, is there a better model representation for canonical analysis other than what was presented in Bagozzi et al. (1981)? Can we represent canonical analysis using a different model so that we may directly obtain those statistics, especially canonical correlation coefficient, which we can not directly obtain from the current model? Several models have been tried, but this current model seems to be the only one which works.

Another question is related to the nested model testing approach for testing individual canonical functions. As discussed previously, it is not certain the approach tried in this paper is a sound approach statistically, since the restricted model seems to be a strange model, although the test based on the differential χ^2 seem to provide reasonable results. Are there better ways to construct statistical test for individual canonical function? Also on this point, the conventional sequential testing approach and the nested model testing approach seem to provide very similar results. Assuming that the nested model testing approach is correct, is this a coincidence due to data, or they are essentially the same? These and some other minor questions aside, the relationship between conventional canonical analysis and structural equation modeling is unmistakable. Researchers interested in these methods may benefit from the understanding of such underlying relationship. The statistical relationship between the approaches may also provide us with some new insight for the conventional canonical correlation analysis.

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Table 1

Correlation Data Matrix Used for Presentation (N = 213)

Weight	1.000					
Waist	0.870	1.000				
Pulse	-0.366	-0.353	1.000			
Chins	-0.390	-0.552	0.151	1.000		
Situps	-0.493	-0.646	0.225	0.696	1.000	
Jumps	-0.226	-0.192	0.035	0.496	0.669	1.000

Table 2

Testing for Canonical Functions: SEM Nested Model Approach and Sequential Testing Approach in Canonical Analysis

SEM		Sequential Testing
H_0: 1st $r_c=0$	$\chi^2_{diff(df=3)} = 212.01$	H_0: All 3 $r_c=0$
	$p < .0001$	$p < .0001$
H_0: 2nd $r_c=0$	$\chi^2_{diff(df=3)} = 8.62$	H_0: Both 2nd & 3rd $r_c=0$
	$p < .05$	$p < .05$
H_0: 3rd $r_c=0$	$\chi^2_{diff(df=3)} = 1.12$	H_0: The 3rd $r_c=0$
	$p > .10$	$p > .10$

Table 3

Standardized Canonical Function Coefficients and Critical Ratios from Canonical Analysis and SEM Estimation

Variable Group	Variable Names	Function I	Function II	Function III
Physical	Weight	-.77 (7.23)[a]	-1.88 (11.44)	-.20 (.11*)
	Waist	1.57 (12.91)	1.17 (4.07)	.51 (.35*)
	Pulse	-.06 (1.04*)	-.23 (.64*)	1.05 (1.04*)
Exercise	Chins	-.35 (4.59)	-.37 (.88*)	-1.30 (1.59*)
	Situps	-1.05 (11.25)	.12 (.21*)	1.24 (2.07)
	Jumps	.72 (9.60)	1.06 (5.53)	-.41 (.37*)
Canonical r		r_{c1} = .795	r_{c2} = .199	r_{c3} = .072
SEM Fit		$\chi^2_{df=3}$= 9.73	$\chi^2_{df=3}$= 1.12	$\chi^2_{df=3}$= .000
		$p < .001$	$p > .10$	

a critical ratios estimated from SEM in parenthesis

* statistically not different from zero

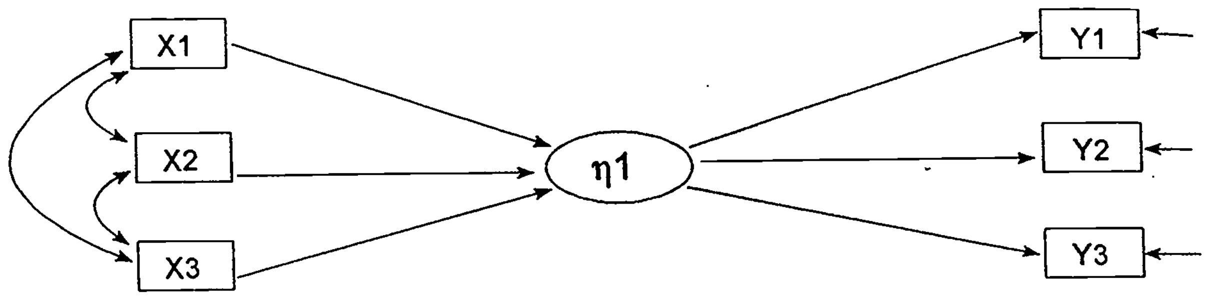

Figure 1: Structural Equation Model Representation of the First Canonical Function

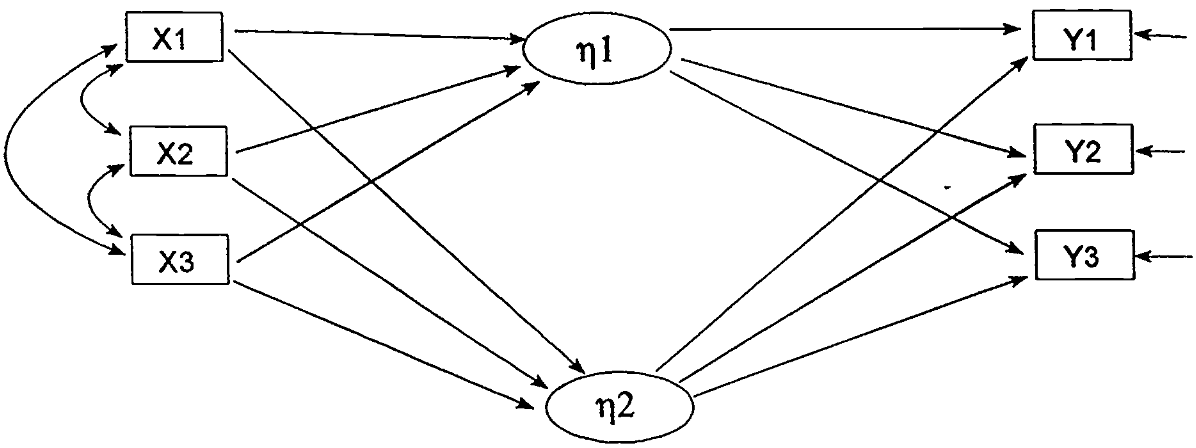

Figure 2 Structural Equation Model Representation of the First Two Canonical Functions