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ABSTRACT

Techniques for integrating English-as-a-Second-Language (ESL) instruction and mathematics instruction, particularly the language of mathematics, are discussed. Focus here is on mathematics instruction for limited-English-speaking students in the mainstream classroom. First, examples of the academic language of mathematics that students must develop are outlined. Some instructional techniques that incorporate students' own language and background in the construction of mathematics concepts and formal mathematics register are then offered. These include: techniques to make mathematics lessons more comprehensible and ways to promote interaction and provide a classroom environment conducive to language and literacy acquisition; methods for maintaining high expectations for language minority students and helping them develop higher-order cognitive and metacognitive skills and strategies; and assessment of language development alongside conceptual knowledge. A concluding discussion looks at the roles of content-area and ESL teachers in the overall education of language minority students and the need for collaborative efforts to ensure effective language and knowledge development throughout the school day. Contains a 13-item list of sources of information and materials, and a 36-item bibliography. (MSE)

From 1 to Z: Integrating Math and Language Learning

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Do children need much language proficiency for mathematics and do mathematics teachers have any responsibility for student language development? It is generally believed that language minority students need less language proficiency to be mainstreamed into mathematics classes. But mathematics has a unique register that students must ultimately learn, particularly with the trend toward more communication in mathematics classes today, and students may need more help to become proficient in the academic language and mathematics register of mathematics classes. I believe that both language teachers and mathematics teachers have a responsibility to help students develop mathematical English. Whatever the methods, either bilingual or ESL ones, used to prepare them to be mainstreamed into mathematics classes, students may need continued support to develop academic language skills in mathematics and other content-area classes. In some programs, students are mainstreamed into mathematics classes earlier than other classes. Some ESL and bilingual programs may not adequately prepare students with the language and communication skills to meet the challenge of mathematics classes (Brenner, 1994). It is important that both language teachers and mathematics teachers give students the support to continue developing their academic language skills after they have been mainstreamed into mathematics classes.

ESL teachers, incorporating content-based ESL techniques into their classes, may include mathematics content with language instruction in their classes. Such integration of content and language, with emphasis on learning mathematics language, can help provide language minority students with support they may need to understand and communicate in mathematics classes, particularly when such language classes are concurrent with mathematics classes. However, mathematics and language learning can be integrated not only within language classes but within mathematics classes as well. Rather than existing in isolation, mathematical concepts and the linguistic context that surrounds them are intertwined, so it is unrealistic to think that they can be taught in isolation (Zepp, 1989). Some teacher may think that language minority students may not need much language knowledge and skills to function in mathematics classes which may focus on mathematical symbols and their manipulation. Though much mathematics instruction focuses on representing ideas with symbols and manipulating those symbols, students still understand mathematics by linking those forms with meaning (Kessler, 1987). Focus on symbol manipulation may make some language minority students with less developed English proficiency feel comfortable, but it is not helpful in the long run as more is required than symbol manipulation, particularly at higher levels (Clarkson and Thomas, 1993). This is particularly true in current mathematics instruction. Current trends and standards place increasing emphasis on communication in mathematics instruction and assessment (Brenner, 1994). Even with students that have limited English proficiency, overreliance on symbol manipulation and lack of sufficient attention to mathematics language development deprives language minority students of the chances to develop

the skills they need to succeed in instructional and assessment tasks. Students with weak language skills need more, not less, experience with mathematics language (MacGregor, 1993). Those students who seem to be proficient in the kinds of language needed for social interaction may not be proficient in the types of academic language needed for success in mathematics classes. Rather than take it for granted that students have the abilities to use language for learning and ignore students' language development needs, mathematics teachers need to develop an appreciation of the language demands of learning mathematics, an understanding of the language learning process, and the abilities to diagnose when it is necessary to work on language (MacGregor, 1993). They should also develop sensitivities to other cultures and an understanding of different cultural expectations for classroom discourse (Miller, 1993).

With a focus on student development of mathematics language and content in mainstream classes, this paper discusses major aspects of mathematics education of language minority students that educators may need to be aware of. The paper first provides examples of the aspects of the academic language of mathematics that students need to develop and discusses instructional approaches that incorporate students own language and background knowledge in the construction of mathematics concepts and formal mathematics register. There is an overview of techniques to make mathematics lessons and text comprehensible as well as ways to promote more interaction and provide a classroom environment conducive to classroom language and literacy acquisition. There is a discussion on maintaining high expectations for language minority students and helping students develop higher order problem-solving and metacognitive skills and strategies. Assessing language development together with conceptual knowledge acquisition is covered. The paper concludes with a discussion of the roles that content-area teachers and ESL teachers have in the overall education of language minority students and the need for collaborative efforts to ensure effective language and knowledge development throughout the school day.

This paper is not intended as the comprehensive source of information about integrating language and mathematics, but as only a starter and introduction to other sources that cover individual aspects in much more breadth and depth. Some issues are not discussed at all, such as the effect of cultural background upon mathematics understanding and learning, culturally-influenced interaction and learning styles, the effects of parents' and teachers' attitudes toward mathematics upon students' attitudes and performance, integrating multiculturalism into mathematics, using literature to introduce and discuss mathematical concepts, and other issues. Their omission is a reflection of the lack of time of the author and not a reflection of any degree of importance, and teachers interested in integrating mathematics and language learning need to look into these issues as well.

Mathematics Register

To function effectively in mathematics classes, students need to develop knowledge of mathematics as a language. Teachers should take an instructional approach that provides opportunities for students to develop mathematical language and listening, speaking, reading and writing skills as they acquire mathematical skills (Spanos, Rhodes, Dale, & Crandall, 1988). Such language includes specific vocabulary, syntax, and other features of the mathematics register that represent mathematical concepts as well as the language that is used to teach mathematics. The following information on the register or language of mathematics is a compilation of information from Spanos et al. (1988), Kessler (1989), MacGregor (1993), Allen (1990), and Zepp (1989). The information is only a sample of some of the features and is by no means complete. Teachers should get more information from not only these sources listed above, but also from investigative work, observation, and reflection they can do themselves to determine the language demands of their specific classes.

Semantics

Vocabulary

Words that are specific to mathematics

divisor, denominator, quotient, coefficient

Everyday words that take on new meaning

equal, rational, irrational, column, table

Words that indicate the same mathematics operation

add, plus, combine, and, sum, increased by

Collocations of words to express mathematical notions

Word-symbol correspondence

Referential

For example, recognizing that "a number" and "the number" refer to the same quantity in the sentence:

"Five times a number is two more than ten times the number."

Syntax

Comparatives

greater than/less than, *n* times as much, as...as, -er than

Prepositions: by - multiplied by, increased by; divided by, divided into

Prepositions and word order:

take one third of

decrease by one third

decrease to one third

one third less than

less than one third

Lack of one-to-one correspondence between mathematical symbols and the words they represent and wrong word order

eight divided by 2

The number *a* is five less than the number *b* - $a = b - 5$ (not $a = 5 - b$)

Student production of "divide the number of sides by 360" instead of "divide 360 by the number of sides"

Numbers used as nouns (rather than adjectives)

Logical connectors

Texts include signals for addition or similarity, contradiction, cause/effect or reason/result, chronological or logical sequence.

Research indicates that logical connectors are critical to the development of mathematical reasoning in English as a Second Language (Dawe, 1983).

Syntactic variation on a single semantic notion

How many were there in all? How many were there then? How many more were there? How many were left? How many fewer were there?

Mathematics discourse

Cohesion and coherence

Discourse competence involves functional use of language

Heuristic (tell me why): exploring the world around and inside one

Imaginative (let's pretend): creating a world of one's own

Informative (I've got something to tell you): communicating new information

Representative function (telling how things are)

Directive function (requesting others to do things

Other discourse features such as classifying, evaluating, initiating problems and questions, raising conjectures, convincing others of validity of their viewpoint, using mathematical forms of argumentation

Discourse of written texts:

conceptually packed and have high density

require up-and-down as well as left-to-right eye movements

require a reading rate adjustment because they must be read more slowly than natural language texts, may require multiple readings

use numerous symbolic devices such as charts and graphs

contain a great deal of technical language with precise meaning

lack of redundancy or paraphrasing (particularly in word problems)

Building on students' knowledge of content and language

As can be seen from the descriptions of some aspects of mathematics language above, students need more than the knowledge and skills of language that they may use everyday in social interaction. Teachers need to help students' develop such complex vocabulary, structures, and discourse to be able to communicate effectively in the formal register of mathematics. Developing mathematics language together with new, often abstract mathematical concepts needs to be emphasized in mathematics classes. However, an emphasis on mathematical language and content knowledge development does not necessarily devalue students' own language and knowledge in the construction of mathematical concepts. Teachers can help make mathematics learning more meaningful and effective by encouraging

students to use their own background knowledge and language in the construction of new knowledge of mathematics concepts and language:

> The language of mathematics must be meaningful to mathematics learners if they are to be able to communicate mathematically and solve mathematics problems. Meaning can be enhanced by classrooms which regularly and explicitly emphasize relationships between students' informal, intuitive, or physical notions of concepts and related technical terminology and symbols. Teachers need to acknowledge...the value of students' own interpretations of language and ideas. The value of students' own language in explaining their own thinking must also be acknowledged and encouraged. (Frid, 1993, p. 38)

Teachers can recognize and encourage students' use of their linguistic and cultural backgrounds in constructing new knowledge by making meaningful problems based on real situations from students' lives (Brenner, 1994) and allowing at times students' use of their own everyday language in the efforts to learn new concepts and language. Teachers need to find ways to link new knowledge with students' knowledge and experience and to link students informal representations of concepts to formal one in the mathematics register. Through their own experiences and language, students can try to make sense of concepts before learning the vocabulary and language of the concepts, using this understanding as the basis for further learning (Gallard & Tippins, 1994). Students' use of their own everyday language can be for the purpose of making links between familiar language and mathematics language. Such everyday language includes not only common second language vocabulary and structures that students may have acquired in social interaction but also use of their native language when it is feasible, such as in mainstream mathematics classes with significant numbers of students from one particular language background and a teacher or aide who knows some of the language, or among students from the same language background grouped together. The important thing to remember is that the use of everyday and native language is still a means to the learning and acquisition of mathematics language. Though teachers can use common language in discussing new concepts and can encourage students to use their own language when defining and discussing new vocabulary and concepts, at some point both teachers and students have to use appropriate mathematics vocabulary and language in their communications about mathematics (Malone and Miller, 1993).

In such constructivist approaches to mathematics learning, teachers can act as mediators between interpretations constructed by the class and general mathematics conventions (Frid, 1993). Biddulph & Osbourne, (1984; cited in Begg, 1993), suggest a constructivist approach, the interactive teaching approach, in which teachers interact with students to challenge, modify, and extend their ideas. Teachers provide experiences through which students can explore, raise questions, and suggest explanations and then carry out whole class or group activities to help students evaluate the experience, investigate their questions and

suggestions, and draw conclusions (Begg, 1993, p. 284). In another constructivist approach, the problem-centered approach (Wheatley, 1991), teachers select problematical tasks and help students plan and carry out investigations in collaborative small group-work, after which students present and discuss solutions in the whole class (Begg, 1993, p. 284). In such approaches, it is important for teachers to use before-learning, during-learning, and after-learning questions to find out students' prior knowledge and experiences, monitor students' progress, provide feedback, and assess what students have learned (Begg, 1993, pp. 285-286). With a focus on both language and content, either through questioning or activities such as semantic mapping, such assessment can provide indications on what language and conceptual knowledge students have or are acquiring throughout the lesson so that they can help students make the transition from their own knowledge and language to more complex conceptual knowledge and formal mathematics language.

Enhancing comprehensibility and providing input

Making language comprehensible is one of the main ways to not only help students understand and learn mathematics concepts but also to facilitate their acquisition of academic language as well. According to Krashen (1982), language is acquired through receiving comprehensible input. Comprehensible input is based on prior knowledge, extralinguistic clues, context, and linguistic competence (Krashen, 1982). One way of ensuring that lessons are comprehensible to language minority students is adapting the language of the lessons to students' needs or proficiency levels and checking frequently for understanding. Such adaptation can include using a slower rate of speech, clear enunciation, controlled vocabulary, controlled sentence length, controlled syntax, use of cognates, limited use of idiomatic expressions, definition of words with double meanings, providing synonyms or other descriptive clues, use of longer and natural pauses, repeat and review, use of fewer pronouns, stressing high frequency words, and, if lecture style is necessary, presenting information in simplified, shorter lecture form. Checking frequently for understanding of concepts can include checking for comprehension, eliciting requests for clarification, repeating information, paraphrasing statements of information, expanding statements of information, posing a variety of questions, posing questions at different levels, and facilitating teacher-to-student and student-to-student interaction. Other things to consider when presenting information to students include announcing the lesson's objectives and activities, writing legibly, developing and maintaining routines, listing and reviewing instructions step-by-step, and providing frequent summations of the salient points of the lesson (Short, 1991, p. 4). Using lesson markers that signal and let students know where they are at in a lesson is also useful (Wong Fillmore, 1985; cited in Faltis, 1993).

Many mathematics activities and lessons may be what Cummins (1992) describes as context-reduced and cognitively demanding. Providing contextual support is the primary way to help make the language and content of lessons and texts comprehensible. Contextual support includes visuals such as pictures, overheads, graphics, diagrams, manipulatives, props, real objects and materials, illustrations, body and facial gestures and expressions, etc. There is definitely some value to using physical objects and manipulatives to provide experiences that combine mathematics and language. According to Reilly (1988; cited in Allen, 1990, p.9):

> Lessons that teach new concepts in mathematics should use graphics, manipulatives, and other hands-on, concrete materials that clarify and reinforce meanings in mathematics communicated through language.

Charbonneau and John-Steiner argue that, among other things, teachers need to utilize the strengths of "learning through the visual, observational mode, and the tactile, manipulative mode" (p. 99). Cantoni-Harvey describes how, as children learn abstract mathematical structures from concrete manipulative situations, the teacher at the elementary level provides "appropriate materials, introduces new vocabulary, and manages the flow of physical and verbal responses." (1987, 131-132; cited in Allen, 1990, p. 11). The utilization of manipulatives is not limited to whole-class activities. DeAvila (1983; cited in Kessler, 1987) has demonstrated cooperative learning activities that incorporate manipulative materials, graphics and other aids to make the activities meaningful and that facilitate student acquisition of mathematics and language. In whatever format, activities that use manipulatives and other materials can provide contextual support to learning mathematics and language.

Providing opportunities for language and content acquisition through interaction

Students need opportunities to use mathematical language in meaningful situations through interaction with teachers, peers, and others. A variety of class, group, and cooperative learning activities can be employed to ensure that LEP students have opportunities to negotiate for meaning, gain comprehensible input, and produce language in meaningful contexts. Whole class activities do not have to be limited to recitation style lectures with teacher-student interaction limited to asking and answering display questions. Teacher can incorporate discourse and interaction styles that provide more of a natural language context of communication. The constructivist approaches discussed above involve more interaction among students and between teachers and students than recitation style lectures. Another way to incorporate more of a natural language environment in student-teacher interaction and communication is through the use of formats such as instructional conversations (Rueda, Goldenberg, & Gallimore, 1992). Instructional conversations can be characterized by provision of challenging but non-threatening atmosphere, teacher responsiveness to student contributions, promotion of discussion and

connected discourse, general participation among students in which they influence the selection of turns, focus on a theme, activation and use of students' background knowledge, promotion of more complex language and expression through a variety of elicitation techniques, and promotion of students' supporting their positions with text, pictures, reasoning, etc. (pp. 5 and 6). According to Bell (1993), the instructional conversational model has been found to be effective with students who are learning mathematics in a second language. Brenner (1994, p. 240) also advocates teachers' "pursuing ideas raised by students and expanding upon them in formal mathematical terms" as a way of facilitating students' mathematical language development.

Increasing interaction through the use of groupwork and cooperative learning activities also contributes to language development. In general, groupwork and cooperative activities are considered to be one of the major ways we can help students develop academic language skills in the classroom. Such activities provide opportunities to for students to use language that is related to the task at hand, expose learners to increased amounts of complex language input, and provide more opportunities for the learners to refine their communication skills through natural second language practice and negotiation of meaning through talk (McGroarty, 1992). Cooperative activities provide content support for linguistically diverse students and can help maximize the rate at which secondary students acquire the English language, content area knowledge, and interpersonal skills needed for success in school (Holt, Chips, & Wallace, 1991).

In mathematics classes in particular, cooperative and group activities can contribute to students' development of mathematical language and problem-solving strategies. By articulating their strategies and reasoning within a group, particularly during problem-solving, students can provide insights into their processes of thinking and schema development to other students (and to teachers) and foster metacognition (McTighe and Clemson, 1991, cited in Cooper et al., 1993; Bickmore-Brand, 1993). Talking through problems helps students "to gradually become comfortable listening to and using mathematics language" (Spanos et al., 1988, p. 236). Groupwork provides opportunities for children to "expose their misconceptions and begin to resolve them through discussion" (Gooding and Stacey, 1993).

However, just putting students together and having them work together on tasks may not always lead to increased learning. Teachers may have to deal with a number of problems that may arise in group work that can limit interaction, content and language learning. Without appropriate structuring, debilitating patterns of behavior such as nonproductive conflict, helplessness, rebelling against the task, etc., may occur (Johnson and Johnson, 1991; cited in Cooper et al., 1993). Communication may be limited, one-way, and destructive (Cooper et al., 1993). Status problems of dominance and nonparticipation may occur because of different status orderings within groups (that reflect academic, peer, and social status) and different expectations for competence from high and low status peers,

leading to situations in which low status students have limited access to the task and fewer opportunities to talk and contribute (Cohen, 1994) . Also, weak students may deal with language problems by avoiding verbal interaction that would foster language development by having other students show them instead of explain (Marland, 1977; cited in MacGregor, 1993). Though teachers may attempt to promote more of a student-centered class environment that provides opportunities for peer interaction, these and other problems may limit the type of communication, negotiation of meaning, and opportunities to comprehend and use language in meaningful contexts that can be conducive to academic language development.

Teachers may have to spend a good deal of time and effort in designing, structuring, and monitoring group work to make sure that such activities promote content and language learning. Care should be taken in designing cooperative and group activities to ensure that all students can and will participate. Second language students need a task at which they can contribute to the group and feel a sense of accomplishment (Short, 1990). It is important to find tasks, even nonverbal ones, that students at varying degrees of language proficiency can accomplish in cooperative groups. To deal with problems of status, Cohen (1994) recommends assigning tasks that require a range of intellectual abilities while convincing students that no one student will have all the necessary abilities but that everyone will have some of the necessary abilities. She also suggests another method of assigning competence to low status students by observing and evaluating students' contributions to the group and making the evaluation of the contribution known to other students. Training students for cooperation and teaching students how to work together is also recommended. To ensure successful social interaction within groups that emphasize assigned roles and two-way interaction with questions, suggestions, and raising alternatives, teachers may need to introduce, explain, demonstrate and clarify the social-interaction roles within groups that are expected of students (Cooper et al., 1993). Teachers may also need to help students develop the abilities to verbalize and communicate their own problem-solving process to other students.

These and other ways of promoting peer interaction and discussion in efforts to facilitate students' development of language and reasoning abilities should be further explored by teachers and researchers. In classes that follow the NCTM standards, children will need these skills, as the standards give emphasis on ways that teachers can enhance students oral expression of ideas in peer discussion (Brenner, 1993).

Developing Literacy Skills in Mathematics

As well as the language of mathematics, students may need to further develop reading and writing skills. In mathematics classes, the textbook is the most common resource used (Travers, 1988). Reading mathematics text is considerably different and more difficult than narrative text or other kinds of expository text, as can be seen from the description of discourse

features of written mathematics text above. Students cannot just rely on everyday reading skills such as skimming, sampling the text, and using knowledge of the world to support comprehension, as they have to use much care and precision in interpreting text (MacGregor, 1993). In reading word problems, students have to use the difficult literacy skill of selecting and reorganizing information (MacGregor, 1993). Due to sociocultural aspects of many word problems (Kessler, 1987), insufficient language proficiency, and problems with structure and word order, it may be difficult for students to construct a representation of written word problems upon which to apply rules and solve.

Since students may not have adequate skills to read and comprehend mathematics text, instruction in reading mathematics text is an important part of the mathematics class, particularly if the class is textbook-oriented. There are a number of programs designed to improve text comprehension in mathematics classrooms (see Blanton, 1991, for discussion of a number of these). For example, Greabell and Anderson (1992) discuss how to apply Directed Reading Activity strategies to mathematics activities. Whatever the method, the important thing is that mathematics teachers take some responsibility for student reading development:

> Instruction in mathematics should emphasize reading carefully and analytically in order to understand meanings, thinking about what is being read, and then translating and formulating English symbols into the special symbols of mathematics. (Aiken, 1977; cited in Blanton, 1991)

Writing in mathematics can help students understand and clarify mathematical concepts, provide an interesting setting for children to construct mathematical meaning, and help children develop mathematical language (Waters and Montgomery, 1993). Kessler (1987, p. 6) found that the scores of children who participate extensively in writing activities go up in both language and mathematics. Writing mathematics reports, investigations, and other forms of writing in mathematics is also different from other types of writing for other subjects such as writing essays in English or reports of experiments in science, and teachers just can't assume that students can simply use their skills in essay or report writing (Stephens, 1993). Writing in mathematics usually consists of routine writing and recording activities, such as writing to explain, daily diaries, journals, etc., and explorative writing, such as mathematics logs, written proposals, report writing, resumes or portfolios (see Waters and Montgomery, 1993 for descriptions of these activities). Children can write their own word problems (Kessler, 1987). Writing prompts that elicit written responses to specific mathematics questions or problems is a constructive writing-to-learn activity that gives students practice in communicating their knowledge of mathematics, helps students clarify concepts, and assists teachers with informal assessment of students' understanding of mathematical language and concepts (Miller, 1992).

Maintaining high expectations: Helping students develop higher order skills and strategies

One of the goals in the NCTM standards is that students develop confidence in their ability to do mathematics. One of the ways that teachers can help language minority students gain confidence is providing opportunities for students to experience success in activities that are comprehensible (Dale & Cuevas, 1987, p. 31). However, teaching for success does not mean simplifying or watering down the mathematics curriculum to make it easier for students to succeed, but supplementing, finding ways of presenting material, and designing activities to make content more comprehensible and to help students in making links between their knowledge and language and that of mathematics classes, as discussed above. One problem with some teachers of language minority students is focusing on lower level skills while students are developing language proficiency. Sometimes higher level mathematics curriculum is denied to language minority students (Oakes, 1990; cited in Brenner, 1994). Some teachers may not encourage minority students to take difficult classes or reduce challenges set for them out of fear that they may fail (Cocking & Chipman, 1993, p. 32). Some studies and observations of classrooms with language minority students have indicated emphases on low level learning activities (even with higher level students) and stress on more learning of facts that more complex types of information processing (Simmons, 1985, cited in Cocking & Chipman, 1988; DeAvila, 1988, p. 104).

Particularly in classes that follow NCTM standards that emphasize development of more advanced processes such as problem solving, reasoning, estimation, and communication (Brenner, 1994), language minority students need access to higher level mathematics curriculum and development of higher level cognitive and metacognitive skills and strategies. Teachers need to incorporate instruction that will help language minority students develop such processes and strategies. As mentioned above, think-aloud activities by teachers and peers as they collaboratively work on mathematics problems can model the reasoning that is used to work on the problems and foster metacognition. To encourage students to verbalize their thoughts while solving problems in groups, teachers may need to teach the language (words, phrases, structure, etc.) of thinking aloud (Reeves, 1986; cited in Brand, 1993). Teachers can model problem-solving and other thinking strategies by using an overhead projector or the blackboard while working on tables, graphs, or formulas (Cooper et al., 1993). Modeling language and strategies and gradually allowing students more control and contributions as they develop competence can be effective in helping students develop communication and problem-solving skills. In one study in which teachers modeled exploratory language and problem-solving strategies, students "gave more elaborate answers and used more mathematical language during discussion with their peers" (Brenner, 1994). It is critically important for students to learn to think mathematically:

> In particular, the ability to think mathematically appears to be the crucial element in mathematics achievement. It may be at this cognitive and metacognitive level that language and mathematics are most intricately related. (Dale & Cuevas, 1987, p. 25.)

It is important that students develop the metacognitive abilities to monitor their thinking. This is particularly important in approaches that focus on students' construction of knowledge, as students need to monitor and evaluate their knowledge representations with concepts they are learning in textbooks and lessons. In class, group, and individual work students can develop the skills to become responsible for determining whether the knowledge they have constructed is viable and coherent with the conventions of mathematics textbooks and other resources (Frid, 1993, p. 38). Also, since there is increasingly more emphasis in some mathematics curriculum standards on students' development of abilities to communicate how they arrived at solutions and reflect on their processes, oral and written communication skills and strategies should be stressed. In short, the ability of language minority students to think in mathematics, be aware of and express their thoughts, and regulate their thinking are all important goals that should be addressed in mathematics classes.

Assessment: Assessing language and content learning

Current trends in mathematics assessment seem to be leaning more towards tasks that involve more linguistic responses, such as open-ended tasks, journal writing, and tasks that call for student reflection and articulation of their thoughts, than previous ones that demanded little language facility of students (Clarke, 1993). Such trends make it even more imperative that language minority students develop the language proficiency and skills to articulate and demonstrate their knowledge verbally and in written form. Mathematics teachers need to monitor and assess their students' academic language development as well as their learning of conceptual knowledge.

Language and content assessment can be integrated and done in a number of ways. To assess what students know about a concept and what vocabulary and language they use to express it, activities such as semantic mapping or semantic feature analysis can be used to not only activate students' conceptual and language knowledge but also for informal assessment of such. Ongoing assessment during instruction and activities can be provided information about students' conceptual and language development and be used to revise instruction (Buchanan & Helman, 1993). Some of the things Buchanan & Helman (1993) suggest are checklists that can be used to assess not only content but also students' listening abilities or use of appropriate vocabulary or anecdotal records through observation which can give information about students' development of conceptual knowledge as well as their oral abilities, such as how well students can respond to teacher questions or explain their reasoning to students or teachers (pp. 7-8). Rating scales and rubrics can be used to assess students' understanding and

performance on problems as well as students' language usage, and portfolios, including homework, logs, writing samples, etc., can assess students' math knowledge and written language abilities (Buchanan & Helman, 1993). These and other forms of assessment can provide useful alternatives and supplements to tests and worksheets. They can give teachers more insight into their students' progress in terms of language development as well as learning of content knowledge and mathematics skills.

Expanding the roles of content area and ESL teachers/specialists: Collaboration is the key

This paper has discussed how mathematics teachers can and should take responsibility for the continued language and literacy development of language minority students. This doesn't mean shifting the responsibilities away from language teachers or even reading specialists. Teachers should collaborate to provide opportunities for language and literacy development throughout the school day.

> ...instruction, support, and guidance for the ESL learner in the development of second language and literacy skills should not be divided up into areas of responsibilities to be taken care of by teachers and specialists in classes that are isolated from each other, but can and should be a collaborative effort among content area teachers, ESL teachers/language development specialists, and reading teachers/specialists. While none of them alone may have the expertise to deal with the variety of potential language, knowledge, and literacy problems that ESL students may face, collectively they can combine their knowledge and expertise to more effectively implement the kinds of assessment, analysis, planning, and creation of resources needed to help ESL students succeed in reading and learning content area text. While none alone may have the time to even consider taking on such tasks, splitting up the tasks and concentrating on those they are most capable of doing will help make the time problem more manageable. Developing the awareness that content, language, and reading development can be integrated, and showing how teachers can collaborate to make such integration a reality may help change attitudes against collective responsibilities for the total education of ESL students (Kang, 1994).

Teachers can collaborate in the education of second language students in several ways. In the area of coordination of curriculum and instruction, teachers and specialists can collaborate to organize and implement curriculum in a way that each teacher's efforts builds upon, supports, and reinforces the efforts of others. Instead of organizing the structure and language that is being taught in ESL classes according to some arbitrary structure, the structure taught in the ESL class can be sequenced to reflect the needs of the students in their content area classes. Upon analysis of the objectives and the language abilities students need to succeed in mathematics classes, ESL teachers can use mathematics content to form the basis of language activities (Dale & Cuevas,

1987). Activities such as mathematics problem-solving activities can be incorporated into the ESL class to encourage interaction and academic language use in group work. Both ESL and content area teachers can become more sensitive to the demands that specific registers of different content areas place upon students and find ways to integrate mathematics and language learning in content-area and ESL classes.

In terms of collaborative efforts to create resources, teachers can share the time, expertise, and responsibilities for joint creation of resources to help ESL student in mainstream classes. ESL teachers/ specialists and content area teachers can work together as resource personnel to provide, adapt, and supplement material that can make subject matter more comprehensible as well as more conducive to language and reading development. Mathematics teachers can help design and develop math-based materials to be used in ESL classes.

Also, training, consultation, and research can be a collaborative effort. Channels and forums for consultation can be set up so that regular content area teachers can get advice on specific students, problems, materials, etc., from reading and language teachers. Inservice training sessions can be set up so that content area teachers can receive training on how to integrate content, reading, and language development. Language and content area teachers can collaborate to conduct action research to find out what works with specific classes and students.

The education of language minority students should be the responsibility of all teachers. Collaboration among teachers can provide an environment in which second language and literacy development can exist across the curriculum and school day, leading to increased opportunities for academic success among ESL students.

Sources for Information and Materials

The Center for Applied Linguistics, 1118 22nd Street N. W., Washington, D.C., 20037 is the place for teacher resources, guides, handbooks, etc. This address is the same for the Center for Language Education and Research (CLEAR), National Clearinghouse for Bilingual Education, and the National Center for Research on Cultural Diversity and Second Language Learning, all of which publish guides and reports concerning the education of language minority students.

Some good articles and books on mathematics and language instruction:

Brenner, M. E. (1994). A communication framework for mathematics: Exemplary instruction for culturally and linguistically diverse students. In B. McLeod (Ed.), Language and learning: Educating linguistically diverse students (pp. 233-267). Albany, NY: State University of New York Press (A very good presentation of innovative instructional practices and models discussed in terms of NCTM Standards and reforms and constructivist approaches to education.)

Buchanan, K. & Helman, M. (1993). Reforming mathematics instruction for ESL literacy students. Washington, DC: National Clearinghouse for Bilingual Education. (This guide discusses the NCTM Standards in terms of language minority students, has a good section on assessment, and contains some sample lesson plans.)

Cocking, R. R., & Mestre, J. P. (1988). Linguistic and cultural influences on learning mathematics. Hillsdale, NJ: Lawrence Erlbaum Associates. (particularly one chapter that provides a detailed description of mathematics register: Spanos, G., Rhodes, N. C., Dale, T. C., & Crandall, J. Linguistic features of mathematical problem solving: Insights and applications.)

Dale, T. C., & Cuevas, G. J. (1987). Integrating language and mathematics learning. In J. Crandall (Ed.), ESL through content-area instruction. Englewood Cliffs, NJ: Prentice Hall Regents. (Authors have been doing research on and designing programs for language minority students and mathematics for a long time. This article is a fairly thorough discussion of integrating language learning and mathematics, with a look at different programs and practices and containing many examples for both content and ESL teachers.)

Stephens, M., Waywood, A., Clarke, D., & Izard, J. (1993). Communicating mathematics: Perspectives from classroom practice and current research. Hawthorn, Australia: Australian Council for Educational Research Ltd. (Many good articles that discuss, among other things: groupwork, interaction, and communication in mathematics; good classroom practices; writing; assessment; influence of culture and linguistic background.)

Three excellent general books on teaching language minority students that are written for content area teachers are:

Faltis, Christian J. (1993) Joinfostering: Adapting teaching strategies for the multilingual classroom. New York: MacMillan Publishing Company.

P. A. Richard-Amato & M. A. Snow. (1992). The multicultural classroom: Readings for content-area teachers. White Plains, NY: Longman.

Peitzmann, Faye & Gadda, George. (1991). With different eyes: Insights into teaching language minority students across the disciplines. Los Angeles: California Academic Partnership Program. (available from: UCLA Center for Academic Interinstitutional Programs, Graduate School of Education, Gayley Center, Suite 304, Los Angeles, California, 90024-1372.

A good handbook for integrating language and content instruction is:

Short, Deborah J. (1991). Integrating language and content instruction: Strategies and techniques. Washington, DC: National Clearinghouse for Bilingual Education. (in ERIC: ED 338 111). It includes sample lesson plans.

Another good, brief handbook is:

Hamayan, E. V., & Perlman, R. (1990). Helping language minority students after they exit from bilingual/ESL programs: A handbook for educators. Washington, D.C.: National Clearinghouse for Bilingual Education.

A good handbook for cooperative learning in the secondary school (written with language minority students in mind) is:

Holt, Daniel D., Chips, Barbara, & Wallace, Diane. (1992). Cooperative learning in the secondary school: Maximizing language acquisition, academic achievement, and social development. Washington, DC: National Clearinghouse for Bilingual Education.

Another excellent article on teaching language minority students is:

Fillmore, L. W. (1989). Teaching English through content: Instructional reform in programs for language minority students. In J. Esling (Ed.), Multicultural education and policy: ESL in the 1990s (pp. 125-143). Toronto, OISE Press.

Bibliography

Allen, B. (1990). Content-based second language instruction: Teaching English through mathematics in grades 1-4. EDRS Document No. ED 328 085.

Bell, G. (1993). Mathematics X language = language X mathematics. In M. Stephens, A. Waywood, D. Clarke, & J. Izard (Eds.), Communicating mathematics: Perspectives from classroom practice and current research (pp. 140-150). Hawthorn, Australia: Australian Council for Educational Research Ltd.

Bickmore-Brand, J. (1993). Applying language-learning principles to mathematics teaching. In M. Stephens, A. Waywood, D. Clarke, & J. Izard (Eds.), Communicating mathematics: Perspectives from classroom practice and current research (pp. 79-90). Hawthorn, Australia: Australian Council for Educational Research Ltd.

Blanton, M. (1991). Teaching reading in the math classroom. The Clearing House, 64(3), 162-164.

Brenner, M. E. (1994). A communication framework for mathematics: Exemplary instruction for culturally and linguistically diverse students. In B. McLeod (Ed.), Language and learning: Educating linguistically diverse students (pp. 233-267). Albany, NY: State University of New York Press.

Buchanan, K. & Helman, M. (1993). Reforming mathematics instruction for ESL literacy students. Washington, DC: National Clearinghouse for Bilingual Education.

Charbonneau, M. P., & John-Steiner, V. (1988). Patterns of experience and the language of mathematics. In R. R. Cocking & J. P. Mestre (Eds.), Linguistic and cultural influences on learning mathematics (pp. 91-100). Hillsdale, NJ: Lawrence Erlbaum Associates.

Clarkson, P., & Thomas, J. (1993). Communicating mathematics bilingually. In M. Stephens, A. Waywood, D. Clarke, & J. Izard (Eds.), Communicating mathematics: Perspectives from classroom practice and current research (pp. 263-273). Hawthorn, Australia: Australian Council for Educational Research Ltd.

Cooper, T., Atweh, B., Baturo, A., & Smith, R. (1993). Higher intellectual functioning, empowerment and oral communication in mathematics instruction. In M. Stephens, A. Waywood, D. Clarke, & J. Izard (Eds.), Communicating mathematics: Perspectives from classroom practice and current research (pp. 1-25). Hawthorn, Australia: Australian Council for Educational Research Ltd.

Cummins, J. (1992). Language proficiency, bilingualism, and academic achievement. In P. A. Richard-Amato & M. A. Snow (Eds.), The multicultural classroom: Readings for content-area teachers (pp. 16-26). White Plains, NY: Longman.

Dale, T. C., & Cuevas, G. J. (1987). Integrating language and mathematics learning. In J. Crandall (Ed.), ESL through content-area instruction. Englewood Cliffs, NJ: Prentice Hall Regents.

Dawe, L. (1983). Bilingualism and mathematical reasoning in English as a second language. Educational Studies in Mathematics, 14, 325-353.

Faltis, Christian J. (1993) Joinfostering: Adapting teaching strategies for the multilingual classroom. New York: MacMillan Publishing Company.

Fillmore, L. W. (1989). Teaching English through content: Instructional reform in programs for language minority students. In J. Esling (Ed.), Multicultural education and policy: ESL in the 1990s (pp. 125-143). Toronto, OISE Press.

Frid, S. (1993). Communicating mathematics: A social sharing of language and decisions pertaining to truth and validity. In M. Stephens, A. Waywood, D. Clarke, & J. Izard (Eds.), Communicating mathematics: Perspectives from classroom practice and current research (pp. 26-40). Hawthorn, Australia: Australian Council for Educational Research Ltd.

Gallard, A. J., & Tippins, D. J. (1994). Language diversity and science learning: The need for a critical system of meaning. In B. McLeod (Ed.), Language and learning: Educating linguistically diverse students (pp. 269-288). Albany, NY: State University of New York Press.

Gooding, A., & Stacey, K. (1993). How children help each other learn in groups. In M. Stephens, A. Waywood, D. Clarke, & J. Izard (Eds.), Communicating mathematics: Perspectives from classroom practice and current research (pp. 41-50). Hawthorn, Australia: Australian Council for Educational Research Ltd.

Greabell, L. C., & Anderson, N. A. (1992). Applying strategies from the Directed Reading Activity to a Directed Mathematics Activity. School Science and Mathematics, 92(3), 142-144.

Hamayan, E. V., & Perlman, R. (1990). Helping language minority students after they exit from bilingual/ESL programs: A handbook for educators. Washington, D.C.: National Clearinghouse for Bilingual Education.

Holt, Daniel D., Chips, Barbara, & Wallace, Diane. (1992). Cooperative learning in the secondary school: Maximizing language acquisition, academic achievement, and social development. Washington, DC: National Clearinghouse for Bilingual Education.

Kang, H. W. (1994). Helping Second Language Readers Learn from Content Area Text through Collaboration and Support. Journal of Reading, 37(8), 646-652.

Kessler, C. (1986). Mathematics and language intersections for Hispanic bilingual students. Paper presented at the Annual Meeting of the National Council of Teachers of Mathematics, Washington, D. C., April 1-8. EDRS Document No. ED 267 995.

Kessler, C. (1987). Linking mathematics and second language teaching. Paper presented at the Annual Meeting of the Teachers of English to Speakers of Other Languages, Miami Beach, FL., April 21-27. EDRS Document No. ED 289 357.

Krashen, S. D. (1982). Principles and practice in second language acquisition. New York: Pergamon Press.

MacGregor, M. (1993). Interaction of language competence and mathematics learning. In M. Stephens, A. Waywood, D. Clarke, & J. Izard (Eds.), Communicating mathematics: Perspectives from classroom practice and current research (pp. 51-59). Hawthorn, Australia: Australian Council for Educational Research Ltd.

Malone, J., & Miller, D. (1993). Communicating mathematical terms in writing: Some influential variables. In M. Stephens, A. Waywood, D. Clarke, & J. Izard (Eds.), Communicating mathematics: Perspectives from classroom practice and current research (pp. 177-190). Hawthorn, Australia: Australian Council for Educational Research Ltd.

McGroarty, M. (1992). Cooperative learning: The benefits for content-area teaching. In P. A. Richard-Amato & M. A. Snow (Eds.), The multicultural classroom: Readings for content-area teachers (pp. 58-69). White Plains, NY: Longman.

Miller, L. D. (1992). Begin mathematics class with writing. The Mathematics Teacher, 85(5), 354-355.

Miller, L. D. (1993). Making the connection with language. Arithmetic Teacher, February 1993, 311-316.

Rueda, R., Goldenberg, C., & Gallimore, R. (1992). Rating instructional conversations: A guide. Washington, D.C.: National Center for Research on Cultural Diversity and Second Language Learning.

Short, Deborah J. (1991). Integrating language and content instruction: Strategies and techniques. Washington, DC: National Clearinghouse for Bilingual Education.

Spanos, G., Rhodes, N. C., Dale, T. C., & Crandall, J. (1988). Linguistic features of mathematical problem solving: Insights and applications. In R. R. Cocking & J. P. Mestre (Eds.), Linguistic and cultural influences on learning mathematics (pp. 221-240). Hillsdale, NJ: Lawrence Erlbaum Associates.

Stephens, M. (1993). Valuing and fostering mathematical writing in the Victorian Certificate of Education. In M. Stephens, A. Waywood, D. Clarke, & J. Izard (Eds.), Communicating mathematics: Perspectives from classroom practice and current research (pp. 164-176). Hawthorn, Australia: Australian Council for Educational Research Ltd.

Travers, K. J. (1988). Opportunity to learn mathematics in eighth-grade classrooms in the United States: Some findings from the second international mathematics study. In R. R. Cocking & J. P. Mestre (Eds.), Linguistic and cultural influences on learning mathematics (pp. 187-199). Hillsdale, NJ: Lawrence Erlbaum Associates.

Waters, M., & Montgomery, P. (1993). Writing in mathematics: Multiplying the learning. In M. Stephens, A. Waywood, D. Clarke, & J. Izard (Eds.), Communicating mathematics: Perspectives from classroom practice and current research (pp. 191-208). Hawthorn, Australia: Australian Council for Educational Research Ltd.

Zepp, Raymond. (1989). Language and mathematics education. Hong Kong: UEA Press Ltd. EDRS Document No. ED 312 143.