DOCUMENT RESUME

ED 379 153 SE 055 728

AUTHOR Owens, Douglas T.; Menon, Ramakrishnan
TITLE Understanding Children's Development of Rational Number Concepts: Final Research Report.
INSTITUTION British Columbia Univ., Vancouver. Dept. of Mathematics and Science Education.
SPONS AGENCY Social Sciences and Humanities Research Council of Canada, Ottawa (Ontario).
PUB DATE [91]
CONTRACT 410-90-1369
NOTE 270p.
PUB TYPE Reports - Research/Technical (143)

EDRS PRICE MF01/PC11 Plus Postage.
DESCRIPTORS *Decimal Fractions; Educational Change; *Elementary School Students; Foreign Countries; Fractions; Grade 4; Grade 6; Intermediate Grades; Interviews; *Manipulative Materials; Mathematics Education; *Mathematics Instruction; *Number Concepts; Observation; *Rational Numbers; Student Journals; Teaching Methods
IDENTIFIERS British Columbia; Teacher Researcher Cooperation

ABSTRACT

This document reports on a small-group teaching experiment whose goal was to understand how fourth- and sixth-grade children develop concepts of common and decimal fractions. Both the Grade 6 and the Grade 4/5 children were taught common and decimal fractions through discussion and by using manipulatives, beginning with basic concepts of common fractions. Results showed: (1) common fractions were initially interpreted as parts of a whole, using region models; (2) Grade 6 students seemed to find manipulatives helpful, but Grade 4 students used manipulatives less and seemed more interested in completing as many questions as they could without recourse to manipulatives; (3) students were able to relate common and decimal fractions; (4) most students could develop fraction and decimal concepts, learn the associated operations meaningfully, and perform satisfactorily on end-of-unit tests; and (5) teachers felt they had learned much from their involvement in the project, including: the need for clarification of roles of researcher and teacher, importance of student interviews, usefulness of journals and student-constructed questions, and judicious use of worksheets. Appendices contain reports of lessons in Grade 4 and Grade 6, project evaluation by teachers and researchers, pre- and post-test questions, interview questions and transcripts, and worksheets. Contains 14 references. (MKR)

UNDERSTANDING CHILDREN'S DEVELOPMENT

OF RATIONAL NUMBER CONCEPTS

FINAL RESEARCH REPORT

To

Social Sciences and Humanities Research Council of Canada
Grant Number 410 - 90 - 1369

DOUGLAS T. OWENS
Principal Investigator

RAMAKRISHNAN MENON
Research Assistant

UNIVERSITY OF BRITISH COLUMBIA
Department of Mathematics and Science Education
2125 Main Mall
Vancouver, B. C.
V6T 1Z4

TABLE OF CONTENTS

1990-91 Research Report

Overview and Main Findings

Background and Rationale

The 1990-91 project fundedby SSHRCC (Grant # 410-90-1369) is an extension of the 1988-89 funded project (SSHRCC Grant #410-88-0678) and it builds upon the 1986-87 study (SSHRCC Grant #410-86-0632). The 1988-89 project was a small-group teaching experiment with the aim of understanding how children develop concepts of common and decimal fractions. The new project for 1990-91 was to revise this instructional sequence and add additional instrucional activities for 1) addition and subtraction for decimals at grades 4 and 6, and 2) multiplication of decimals at grade 6. These areas of operations with decimal fractions build upon the 1986-87 study. The extension is in the facet of conducting the investigation under classroom conditions to examine children's concept development under those conditions.

Objectives

It is important to understand the kinds of concepts students develop while taking part in learning experiences. The purpose of this project is to study the nature of children's understanding of common and decimal fraction concepts and selected decimal fraction operations under carefully monitored classroom conditions. The objective of this project is to build models, that is, plausible explanations (Cobb and Steffe, 1983) of children's mental activity in their construction of the concepts of common and decimal fractions as they encountered carefully sequenced instructional materials under classroom conditions. Inherent in accomplishing these goals is the need to interpret the cognitive processes of children as they construct or give meaning to common and decimal fraction concepts.

Scholarly Significance

Skemp (1976) contrasted instrumental and relational mathematics. "The kind of learning which leads to instrumental mathematics consists of the learning of an increasing number of fixed plans, by which students can find their way from particular starting points . . . to required finishing points . . . In contrast, learning relational mathematics consists of building up a conceptual structure (schema) from which its possessor can . . . produce an unlimited number of plans . . . " (p. 25). Instrumental understanding, according to Skemp, could be called "rules without reasons." Relational understanding is what some in mathematics education would call "understanding."

Hiebert and Lefevre (1986) described conceptual knowledge as knowledge that is rich in relationships. They defined procedural knowledge as (1) the symbol representation system of mathematics (also called form) or (2) rules, algorithms, procedures, or "step-by-step instructions that prescribe how to solve mathematical tasks" (p. 6). Hiebert and Wearne (1986) noted three places or sites where links between conceptual and procedural knowledge are especially important in solving a problem. Site 1 is the point at which symbols are interpreted. These may be symbols for numbers or for operations. Site 2 is where the symbols are manipulated by a set of procedures. Site 3 is when an answer to the problem is formulated after the symbol manipulation. It is here that connections between the conceptual knowledge and procedures are especially important, for example in determining the reasonableness of the answer. Hiebert and Wearne (1986) have data with students dealing with decimals to suggest that connections at all three sites are lacking. Especially crucial are sites 1 and 3, the conceptual zones. "To observe students work with decimal fractions is to observe students struggling with written symbols they do not understand" (p. 219).

"What seemed to be missing was a link between their conceptual knowledge and a notion that written answers should be reasonable" (p. 220).

Owens (1985) found that students can compute in decimal multiplication. However, when students were asked to estimate the product and place the decimal point in a question like, 3.25 x 6.52 = 2119, where the ending zeroes have been dropped, the students persisted in counting the decimal places. In Resnick's terms, these students knew the syntactic rules, but their use of the rules did not "reflect the underlying semantics or meaning" (Resnick, 1982, p. 137).

These examples have been cited to indicate the acknowledgement from the literature that individuals can "do" mathematics that they do not understand. This area of rational numbers is one in which this is often the case. The concepts under consideration in this study are central to the understanding of the procedures with rational numbers. The proposal was to study the nature of children's understanding of the concepts and the nature of acquiring the concepts.

Theoretical Approach

Herscovics and Bergeron (1984) contrast a constructivist versus a formal approach to teaching mathematics. The constructivist approach assumes that the way a person learns mathematics is to construct knowledge based on previous knowledge. This is not to say that the child re-invents mathematics spontaneously, but does so under the influence of instruction. The constructivist approach focuses on what the learner knows and on what stimulation is appropriate to aid the child in constructing new understandings. By contrast a formal approach would see knowledge transmitted from the knower to the learner.

The principal investigator claims a constructivist approach to learning mathematics as an underlying model. He ascertained the understandings of the students about the concepts under consideration, and provided activities which are purported to embody the concepts. Then the constructions which the children made were observed in a series of interviews. Interpretations of their thinking were made in light of the literature on understanding and on conceptual and procedural knowledge.

Kilpatrick (1981a) calls for

> more of our attention to the study of mathematical learning and thinking as they occur in school. We must take full account in our research of the multiple contexts in which both learning and thinking occur. Each is embedded in interacting systems of the pupil's cognitions, the subject matter and the social setting. We have tended to concentrate on at most one of these systems, and we have neglected interactions within the system, not to mention interactions between systems. (p. 371)

In pointing out the importance of constructs and how teachers may borrow them to guide practice, Kilpatrick (1981b) called for teachers to be more involved in the design and conduct of research.

> This view suggests why teachers should be active researchers, why they should develop a research attitude. Teachers should not stop at being borrowers; they should become collaborators. . . . Research in our field is disciplined inquiry directed at mathematics teaching and learning. It is stepping out of the stream of daily classroom experience and stopping to reflect on it. It is becoming conscious of the constructs we are using and then trying other constructs on for size.

The study took place in classrooms with the collaboration of teachers. The instructional phase was much like the teaching experiment described by Bell, Swan and Taylor (1981). The data collection (particularly the interviews and video taping) drew heavily on the methodology of the small-group teaching experiment.

Post, et al. (1985) described an 18-week teaching experiment with six children at each of two sites. Each child was interviewed individually 11 times at eight-day intervals. A participant observer was present and took notes during the instruction.

Steffe (1983) indicates three aspects of a constructivist teaching experiment: Teaching episodes, individual interviews, and modelling. The teaching episodes involve a teacher, a child and an adult witness. The role of the witness is to interpret the communication between the two participants. The teaching episodes are routinely video taped for three reasons: (1) planning the next teaching episode; (2) forming a revised hypothesis of the student's knowledge and test hypotheses; and (3) making public the teaching episodes together with the intentions and interpretations of the teacher.

The second aspect of the teaching experiment is individual interviews. The purpose of the interviews is to explain how the child interprets and solves the tasks. This allows for the third and most important aspect of the teaching experiment which is modelling. A model "is no more than a plausible explanation of children's constructive activities. One can never claim a correspondence between the model and children's inaccessible mathematical realities. Although a model can be viable, it can never be verified" (Cobb and Steffe, 1983 p. 92-93). The models must be specific enough to account for a particular instructional setting, but general enough to account for the mathematical progress of other children.

These have been cited to indicate some precedents to the methodology proposed below. In addition, the instructional position of the proposed research has been put into perspective, and several processes defined which will be used in the procedures described later.

Social Relevance

In schools, especially with the mathematics curriculum in British Columbia, decimal fractions are being introduced earlier and complex computation with common fractions is being delayed until grade eight. This study will indicate at two grade levels, the quality of conceptual understanding that can be anticipated when students encounter concepts of common and decimal fractions, and decimal operations. Typically one notational system is taught and then the other, but in this study they are integrated. The investigator conjectures that once children have understood the concepts, that computation algorithms will be more meaningful, hence easier. With meaningful concepts, children will be better equipped to monitor their computational procedures and outcomes.

Purpose of Study

Generally, common fractions and decimal fractions are taught separately, one after the other. Even though there is occasional reference to common fractions when teaching decimal fractions, there is seldom any sustained effort to *integrate* the two. This project (Grant # 410-90-1369) focussed on building plausible explanations of how children construct common and decimal fraction concepts resulting from carefully sequenced instructional materials under classroom conditions. Specifically, our focus questions were:

1. How do children interpret common and decimal fraction concepts?
2. Do manipulatives help children construct common and decimal fraction concepts?
3. Can students relate common and decimal fractions?
4. Will conceptual understanding of common and decimal fractions help students learn the associated operations more meaningfully?
5. How well do students exposed to conceptual development of common and decimal fractions perform on end of unit tests?
6. What do teachers as collaborative researchers learn from their involvement in the project?

Participants

The project involved a grade 6 teacher, A, with 22 pupils and a Grade 4/5 teacher, B, with 26 pupils (with only the 16 Grade 4 included in the study) from two different schools in Richmond, British Columbia. A, though an experienced teacher, had very little experience teaching mathematics, while B had considerable experience teaching mathematics. The researcher, from the University of British Columbia, has had extensive experience in classroom-based research on common and decimal fractions, and the student assistant, a Doctoral student, had also studied students' common and decimal fraction concepts in an Asian country.

Overview

Both the Grade 6 and the Grade 4/5 children were taught common and decimal fractions through discussion and by using manipulatives, beginning with basic concepts of common fractions. By the end of the study, the Grade 6 children could work word problems involving decimal multiplication by referring to common fraction concepts. These children also performed well on a final unit test on common and decimal fractions. More than a month after the study, they could solve word problems involving multiplication of decimal fractions. Also, by the end of the study, the Grade 4 children could work word problems involving decimal addition and subtraction by referring to common fraction concepts. These children, too, did well in a final unit test on common and decimal fractions.

Research Method

The grade 6 study was for the period January 23, 1991 to March 27, 1991 and the grade 4/5 study lasted from April 10, 1991 to June 14, 1991. There were 26 and 28 instructional sessions.respectively for the grade 4/5 and grade 6 classes.

The grade 4 instructional sequence included experiences to provide children opportunities to understand common and decimal fraction concepts. The focus was on concept referents which applied equally well to common and decimal fractions, and the interrelationships between the two systems. The concepts sequence was followed by activities in addition and subtraction algorithms and applications for decimal fractions. The grade 6 instructional sequence included the above with the addition of algorithms and applications for decimal multiplication. Written tests were given periodically, at the end of each topic, to monitor the progress of the class, and especially the targeted interviewees. Due to the time of year the teachers preferred to teach the topics, data collection in grade 6 preceded that in grade 4 .

The interview tasks were similar to tasks encountered during instruction and on written tests. Students' explanations were used as the data to generate hypotheses of the children's thinking about the concepts. Inferences were made about the character or quality of students' mental constructions of the concepts. That is, models of the children's cognitive structures were built by the investigator. Video and field notes were used to explain why students might have made these constructions.

Procedure

A pretest on common and decimal fractions was given. Based on test scores and participant teachers' recommendations, 6 students (3 boys and 3 girls) covering a range of mathematical ability at each grade level were identified for a series of interviews. The interviewees were chosen using the following criteria: 1) 2 low, 2 middle, and 2 high achievers in fraction concepts and general mathematical ability; 2) both sexes represented at each level. Exceptional students were excluded. The interviews were held at 5- to 8-instructional-day intervals. There were 5 interviews involving six grade 6 students and 4 interviews involving the six grade 4/5 students, giving a total of 54 videotaped interviews.

The research team met for 3 days or 6 half days to define roles of team members and to plan the instructional sequence. Both teachers and the graduate student were cosidered research assistants. Together, an overall instructional sequence was planned. Although the instructional sequences were prepared in advance, modifications were made when and where appropriate. The principal investigator and student assistant designed and prepared worksheets and manipulatives to be used in class. They also recorded class lessons, discussions and individual student-researcher interviews. Some worksheets from a previous study (SSHRCC # 410-88-0678) were also used. Worksheets were to be used as starting points for discussion rather than as practice exercises that had to be completed.

The grade 4 instructional sequence included experiences to help children understand and integrate common and decimal fraction concepts. After the concept development phase, word problems were used to initiate exploration of addition and subtraction algorithms for decimal fractions. The grade 6 instructional sequence was similar, but in addition, had word problems and estimation tasks leading to the multiplication algorithm for decimal fractions. Data collection in Grade 6 preceded that in Grade 4.

Interview tasks were similar to those met in class and on written tests. Student explanations on the interview tasks were used to infer plausible constructions (models) of their thinking about common and decimal fractions.

After data were collected, the project team met to evaluate the project. During this phase, video tapes of classroom lessons and student interviews were viewed by the teachers, followed by audio taped team discussions and written comments from the teachers. Subsequently, the student assistant synthesized these discussions and written comments as a draft report of the project evaluation and the other team members commented on this draft.

Results and discussion

For ease of reading, the study questions are repeated here, and a question-answer format used to present salient aspects of the study. (Q1 indicates question 1 and A1 indicates answer to question 1).

Q1. How do children interpret common and decimal fraction concepts?

A1. Common fractions were initially interpreted as part of a whole, using region models, with parts having to be of equal size. For example, 3/4 was usually represented by shading 3 out of 4 equal parts from a region (rectangular or circular) model. Only one interviewee represented 3/4 using a discrete (set) model, colouring 3 out of 4 circles. Another interviewee (Grade 6) liked to use number lines to explain fraction concepts involving comparison or ordering, although she used region models to represent initial fraction concepts. But irrespective of whether the region or the set model was used to represent fractions, all the students used diagrams of one sort or another. For example, in the interviews, some students said they visualised pictures even if they did not use actual drawings.

While most students had no difficulty in recognizing fractions in set models, some did have difficulty when the elements of the set were of different sizes. For example, when the Grade 6 interviewees were shown 8 balls of different sizes, one of them said he could not find a fraction of the balls because the sizes of the balls were not equal. But when he was asked to find 1/2 of his class of 24, he gave an answer of 12. When the interviewer reminded him that the students of his class were not of equal size, he replied that it would be easier to find 1/8 of the balls if they were of equal sizes but for the kids, it did not matter. In other words, students felt that size, rather than numerosity had to be taken into account, in this discrete case of balls, but not for the case of everyday experiences such as fraction of students in a class because, for example, they could represent the number of girls as a fraction of the number of students in the class. It looks as if students experiential knowledge plays a part in contextualizing their understanding of fraction concepts. The students would have had experiences with parts of a class, but not with different-sized balls in one place. For example, a Grade 4 interviewee discarded his initial answer of 3 girls, when trying to solve a problem (involving 27 students where 1/3 were girls), saying that there could not be just 3 girls in a class. In his experience, all the classes that he knew about had more than three girls in each class.

Other than pictures and experience, students used manipulatives to help interpret fraction concepts. But there was a difference in how the manipulatives were used by the grade 4 and grade 6 students. While most Grade 6 students used manipulatives to model and develop fraction concepts, most of the Grade 4 students used manipulatives more to check their answers preferring pictures and diagrams to develop concepts. One reason for this difference could be that the Grade 4 teacher used more diagrams in her explanations, compared to the Grade 6 teacher who relied more on the manipulatives.

In terms of the type of everyday models used for fraction concepts, both groups of students favoured models of chocolate bars and pizzas to represent and explain fraction concepts, when not using manipulatives. For example, they would talk about a fraction being bigger or smaller by relating it to the number of people sharing a chocolate bar or pizza: more people implied smaller pieces/shares and hence smaller fractions. Once again, students seemed to make fraction concepts more meaningful for themselves by relating fractions to their everyday experiences.

Word problems involving fraction concepts and operations seemed to lend themselves to more meaningful exploration of the underlying principles as opposed to questions presented in symbolic (number) form. For example, the problem "A bamboo plant grows 3 tenths of a meter each day. How many tenths of a meter would it grow in 7 days?" was solved by the Grade 6 interviewees either by multiplication or by repeated addition. They also had no difficulty writing the answer in both common and decimal fraction notation. However, 0.4 x 0.2 was not always answered as 0.08, possibly because of the strong perceptual cues and relationship with addition of decimal fractions and also because the symbols did not lend themselves readily to real-life experience.

Q2. Do manipulatives help children construct common and decimal fraction concepts?

A2. The Grade 6 students seemed to find manipulatives helpful in two ways: to develop conceptual understanding of common and decimal fractions; and to use the manipulatives as a tool for communicating their understanding, by using them to explain a concept, or the solution of a problem, either to their peers or to their teacher.

The Grade 4 students used manipulatives less and seemed more interested in completing as many questions as they could (in class) without recourse to manipulatives. Some even said that they thought using manipulatives was a waste of time because they "already know how to get the answer". Even so, when urged to use manipulatives for questions which they had difficulty with, they found manipulatives did help their understanding.

The interviews showed that students did find manipulatives helpful in constructing common and decimal fraction concepts. It looked as if Grade 6 students used manipulatives as an integral aspect of learning fraction and decimal concepts, while the Grade 4 students used them more peripherally, either as a check for already obtained answers, or to help them overcome some difficulties. The Grade 4 teacher felt that one reason for their difficulty in getting out of the "have to complete these questions" attitude was their previous experience in being allowed to work on a task of their choosing once they had completed their set exercises/worksheets. In other words, they seemed to view the worksheets as practice exercises rather than as exploratory exercises to initiate meaningful discussion.

Q3. Can students relate common and decimal fractions?

A3. There is ample evidence that students can relate common and decimal fractions. Because the emphasis throughout had been on decimal fractions being just another way of representing common fractions, students did not view the former as a completely new topic. For example, though 0.5 was the written form, students read it as 5 tenths. Students had plenty of practice in renaming a given fraction in a number of different, but equivalent forms, such as 1/2 = 10/20 = 5/10 = 0.5 = 0.50 = 50/100. During the interviews, students demonstrated facility in moving from common fraction to decimal fraction representation and vice versa. For example, using a metre rule, they noticed the equivalence of 1/2 to the 50 cm mark or 50/100, and 1/4 to the 25 cm mark or 25/100. As another example, most Grade 6 interviwees could write one eighth and 3 eighths as decimal fractions, especially after they had shaded in a square divided into 1000 equal parts. They could also correctly order a series of fractions like 1/9, 0.1 and 1/12, as well as locate fractions on a number line giving both the decimal and common fraction equivalents for the points located, or even for the points previously marked. For example, they could use 0.6 that was already marked on a number line, to locate 20 hundredths, reading and writing it as 20/100, 2/10, 0.20 or as 0.2, with hardly any difficulty.

Q4. Will conceptual understanding of common and decimal fractions help students learn the associated operations more meaningfully?

A4. We found that given more time than is usually allotted to the study of these topics, as well as suitable manipulatives, class and group discussions, most students could develop fraction and decimal concepts and learn the associated operations more meaningfully.

For example, students used the fraction strips to identify and order fractions. Even when the units were changed, for instance when a strip originally representing 2 fifths was to be considered as representing 1 fifth, students could demonstrate what the (new) whole would look like. Students could also demonstrate why, when the denominator increases (while the numerator remains constant), a smaller fraction is obtained, by talking about smaller shares when dividing something equally among more people. Also, when word problems about decimal fraction addition, subtraction and multiplication were given, students could justify why it made sense to place the decimal point as they did. So, when it came to doing sums with just numbers and no words, they had no difficulty placing the decimal point. All these inferences were borne out during the interviews, too.

Q5. How well do students exposed to conceptual development of common and decimal fractions perform on end of unit tests?

A5. For the Grade 6, the pretest mean score was 39% and the posttest mean score was 61%. The corresponding scores for the Grade 4 were 26% and 66%. Also both teachers were satisfied with their students' performance in the end of unit tests, with many students scoring more than 85%.

Q6. What do teachers as collaborative researchers learn from their involvement in the project?

A6. The teachers felt they had learned quite a lot from their involvement in the project. Below is a summary of what they felt they had learned:

a) The need for clarification of roles. Both had initially believed they were helping the researcher with his agenda, and in doing so, they might learn some useful and innovative approaches to the teaching and learning of common and decimal fractions. While teacher A had initially been very dependent on the researcher, and had insisted on numerous planning meetings as she had no confidence in teaching these topics, she slowly started taking more control of what was to be taught and how it was to be taught. Teacher B felt her role was not clearly defined, and felt that she should not "interfere" for fear of causing an adulteration or bias in the study. So, while she was very supportive of the project, she did not try to modify it to suit her perception of teaching-learning in her class.

b) The importance of student interviews. Both commented that they found the interviews an extremely valuable technique to assess children's mathematical understanding. They suggested that viewing the interview tapes should be done as early as posible, so that feedback from the interviews could help teachers understand individual differences and help plan future lessons. They also felt they had to make interviews an aspect of their teaching repertoire, even though they might be time-consuming. They suggested they could interview a student or two every day, while the others were doing seat work.

c) The usefulness of journals. While both agreed student mathematics journals were beneficial, especially in terms of the feedback obtained from the students, they felt they needed more resources on how to use these optimally. They also felt as research participants, they should keep journals not only to help remember what had transpired in lessons, but also to serve as a means of communication between the teachers and the researcher.

d) The usefulness of student-constructed questions. The teachers felt student-constructed questions motivated students and also provided opportunities for deeper understanding and more challenging questions. Moreover, they were a valuable source of peer and social interaction because the questions were generated through group discussion, and each member of the group had to understand and explain the solution to the problem generated. One drawback was the lack of focus on specific mathematical topics. For example, though there were many good questions, some of them were only cursorily concerned with fractions, the topic under discussion at that time.

e) The use of worksheets. The worksheets were originally meant to be used as starting points for discussion. While the worksheets were utilised as discussion points in Grade 6, the Grade 4 students tended to use them as practise exercises. According to teacher B, two reasons for the Grade 4 students' use of worksheets were (a) their prior experience in using worksheets as something to be completed in a set amount of time before going on to their own choice of activities, and (b) their resemblance to textbook exercises. She suggested the use of larger fonts, more illustrations, and fewer questions per page, so as to be able to concentrate on one question at a time, in order to enhance discussion.

f) Enhanced confidence. Although the teachers agreed that how much they got out of a project is a function of how actively their pursued their own agenda, and that learning from the project was emergent and resulted from reflection, their experience in the project had given them greater confidence not only to participate in similar projects, but also to share their experiences with their colleagues possibly in workshop sessions.

Papers and presentations arising out of this project

Two papers, arising directly out of this project, are presently being prepared for submission to jounals. They are entitled "Understanding multiplication of decimal fractions" and "Students' number line representations of common and decimal fractions."

Three presentations have been given by Douglas T. Owens so far using data from this project:

Owens, D.T. "Meaningful integration of common and decimal fractions." Presentation at the Annual Conference of the Mathematics Council, Alberta Teacher's Association, October 31-Novermber 2, 1991, Edmonton.

Owens, D.T. "Integrating common and decimal frations in grade six (11 years old)." Short Presentation at the 7th International Congress on Mathematical Education, August 17-23, 1992, Quebec.

Owens, D.T. "The fraction connection: Integrating decimal and common fractions." Session presented at the British Columbia Association of Mathematics Teachers Provincial Coonference, October 16, 1992, Pitt Meadows.

A further two papers have been accepted for the forthcoming 1993 NCTM Research Presession and AERA Annual Meeting.

REFERENCES

Bell, A., Swan, M. & Taylor, G. (1981). Choice of operation in verbal problems with decimal numbers. Educational Studies in Mathematics, 12, 399-420.

Cobb, P. & Steffe, L.P. (1983). The constructivist researcher as teacher and model builder. Journal for Research in Mathematics Education, 14, 83-94.

Herscovics, N. & Bergeron, J.C. (1984). A constructivist vs. a formalist approach in the teaching of mathematics. In B. Southwell et al. (Eds.), Proceedings of the Eighth International Conference for the Psychology of Mathematics Education, (pp. 190-197), Sydney, Australia.

Hiebert, J. & Lefevre P. (1986). Conceptual and procedural knowledge in mathematics: an introductory analysis. In J. Hiebert (Ed.), Conceptual and Procedural Knowledge: The Case of Mathematics., (pp1-27), Hillsdale, N.J.: Lawrence Erlbaum Associates.

Hiebert, J. & Wearne, D. (1986) Procedures over concepts: The acquisition of decimal number knowledge. In J. Hiebert (Ed.), Conceptual and Procedural Knowledge: The Case of Mathematics, (pp.199-223), Hillsdale, N.J.: Lawrence Erlbaum Associates.

Kilpatrick J. (1981a). Research on mathematical learning and thinking in the United States. Recherches en Didactique des Mathématiques, 2, 3, 363-379.

Kilpatrick J. (1981b). The reasonable ineffectiveness of research in mathematics education. For the Learning of Mathematics, 2, 2, 22-29.

Owens, D. (1985). Decimal concepts and operations. What do students think. Proceedings of the Seventh Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, (pp. 218-223), 2-5 October, Columbus,OH.

Owens D.T. (1986) Understanding decimal concepts and operations. Paper presented at the 64th Annual Meeting of the National Council of Teachers of Mathematics, 2-5 April, Washington, D.C.

Owens, D.T. & Haggerty S.M. (1987). Decimal concepts and operations: A teaching experiment. A research report prepared for Social Sciences and Humanities Research Council of Canada, Grant# 410-86-0632.

Post, T.R., Wachsmuth, I., Lesh, R., & Behr, M.J.. (1985). Order and equivalence of rational numbers: a cognitive analysis. Journal for Research in Mathematics Education, 16, 18-36.

Resnick, L.B. (1982). Syntax and semantics in learning to subtract. In T.P. Carpenter, J.M. Moser, and T.A. Romberg (Eds.), Addition and subtraction: A cognitive perspective, (pp. 136-155), Hillsdale, N.J.: Lawrence Erlbaum Associates.

Skemp, R.R. (1976). Relational understanding and instrumental understanding. Mathematics Teaching, 77, 20-26.

Steffe, L.P. (1983). The teaching experiment methodology in a constructivist research program. In M. Zweng (Ed.), Proceedings of the Fourth International Congress of Mathematical Education, (pp. 469-471), Boston: Birkhauser et al.

APPENDIX A

Report of Lessons in Grade 6, Brighouse Elementary, Richmond

Lesson 1 (Wednesday, January 23, 1991) - Common fraction concepts (regions) and symbols (including mixed numbers).

Worksheet 1 was distributed. Most students had little difficulty with identifying fractions represented by shaded regions. The question on mixed numbers, Question 3, where students had to identify shaded regions larger than a unit and give the appropriate mixed number as well as the common fraction symbol, proved quite difficult. The fraction strips seemed to help.

Lesson 2 (Friday, Jan. 25, 1991) - Units, equal parts mixed numbers and comparing fractions.

The fraction strips showing eighths, fourths and halves were discussed. Students were asked to show the appropriate fraction strip when the fraction name was mentioned orally and also when the fraction symbol was written on the board. Then students were given a page on which several regions with shaded parts were shown. They were to id ntify the shaded parts representing a half, a fourth and a third. The emphasis was on the number of parts and the equality of parts. Different names for a whole unit (4 fourths, 8 eighths etc.) were also discussed. Students also had to use the fraction strips to show various fractions grater than one, such as 10 eighths, 2 and 3 eighths etc. Question 3 (Worksheet 1) on mixed numbers was reviewed. Various fractions were compared (e.g. "which is bigger, 1/4 or 1/8? Why?"), both by using the fraction strips and also without using them, for example, by thinking of sharing chocolate bars equally.

Lesson 3 (Monday, Jan. 28, 1991) - Fraction concepts (equal parts, total number of parts, number of parts considered; same area but different shapes; different units; drawing the unit, given a part; mixed numbers to common fractions and vice versa).

Reviewed three important factors affecting fractions (equal parts, total number of parts, number of parts considered). Worksheet 2 was distributed. Writing of fraction symbols, given fraction names in words, including fractions larger than one. Exercises on identifying fractions represented by different-shaped regions. Some actually worked out the a[illegible]ncerned, while others tried to transform one shape to another. Also discussion on whether it makes sense to talk about, say, a "bigger" half (e.g. different units). Drawing the unit, given a part. Changing mixed numbers to common fractions and vice versa. One of the students used an algorithm to change mixed numbers to common fractions, but was unableto explain why.

Lesson 4 (Wednesday, Jan. 30, 1991) - Comparing and ordering fractions.

Revised Question 7 (comparing fractions less than one), Worksheet 1. Discussed which of a pair of fractions (including fractions larger than one) was larger (using and without using fraction strips). Questions 5 and 6 (comparing two fractions and ordering three fractions), Worksheet 2, were discussed. Students used fraction strips and pictures (rectangles and circles) to explain their thinking. Question 1 (ordering three fractions), Worksheet 3, was attempted.

Lesson 5 (Friday, February 1, 1991) - Student-constructed questions; number lines.

Each group of students were to prepare 3 questions on fractions, together with the answers. In the meantime, the teacher worked with 7 students who needed help, either because they could not understand the previous lessons or because they had missed a few classes. A discussion as to what was easy or difficult about making questions while working in groups was then held. Fraction strips and "longs" (a 10 cm x 1 cm rectangular strip divided into 10 cm squares) were then used to draw and mark number lines to show fractions. At least three students had difficulty marking and using the number lines. Page 148, on marking off unmarked number lines, was completed.

Lesson 6 (Monday, Feb. 4, 1991) - Continue number lines.

Basic ideas on fractions (equal parts, total number of parts and number of parts considered) were reviewed. Worksheet 4, consisting of further exercises on number lines, was distributed. Some students had difficulty locating 2/5 etc. on a number line marked in tenths. Page 145 (Worksheet 3), on locating pairs of fractions on number lines, was attempted. The first three were quite easy, but those with different denominators posed more difficulty. Review questions were given.

Lesson 7 (Wednesday, Feb. 6, 1991) - Review of number line; second interview.

The IMPACT sheets for getting feedback on lessons were distributed to the class. Difficulties with the number line surfaced during the interview. So the teacher followed up with discussion and remediation activities. There was no audiotape for this lesson.

Lesson 8 (Friday, Feb. 8, 1991) - Decimal fractions, relating decimal fractions with common fractions, number lines and "flats".

Most students could show decimal and related common fractions less than one on the number lines, but had a little difficulty with fractions larger than one (Question 1, Worksheet 5). Exercises on page 156 (using diagrams of flats a 10 cm x 10 cm square partitioned into 100 cm squares as well as diagrams of 10 longs joined to form a square) were discussed, using the OHP. Page 157 had similar exercises, together with writing common fractions and associated decimal fractions. Writing common fractions as decimal fractions and vice versa (even fractions larger than one) seemed to be quite easy. Most students worked ahead, to page 158, Worksheet 6, doing exercises on equivalent fractions (both common and decimal fractions, based on diagrams of flats). One student commented that the Math lesson was more like a language lesson as there was a lot of discussion and very little of just "doing" sums, just to get the correct answer. Students seemed to have integrated decimal fractions with common fractions. Possibly the revision of renaming/regrouping whole number on Tuesday and Thursday when the project lessons were not on helped in this integration.

<u>Lesson 9</u> (Monday, Feb. 11, 1991) - Feedback on worksheets; comparing and renaming common and decimal fractions.

Dr. Owens gave directions on how to complete the feedback form/questionnaire on worksheets used in the project so far, after which students completed the form. Then they tried the exercises on comparing and renaming fractions (both common and decimal fractions), pages 160 to 164, Worksheet 6.

<u>Lesson 10</u> (Wednesday, Feb. 13, 1991) - Problems relating fractions to real-life situations; renaming on number line.

Discussion of review paper. Students worked in groups to come up with real-life situations involving fractions (commom and decimal). The teacher presented a problem about the number (in fractions) of immigrants to Canada according to country of origin. This generated many diferent approaches to its solution. The exercise on locating and renaming fractions on the number line (page 165, Worksheet 6) was tried. Exercises (pages 166 and 167, Worksheet 6) on locating another fraction based on a marked fraction on the number line (e.g. mark 0.60, given the location of 3/10), as well as renaming the sum of some common fraction as a decimal fraction (e.g. 2 + 12/100 = 3.2) were also done.

Leson 11 (Friday, Feb. 15, !991) - Renaming, comparing and ordering common and decimal fractions; student-constructed problem.

Exercises on comparing and ordering common and decimal fractions, such as "Arrange from largest to smallest the group of fractions 3/6, 0.3, 3/25, 0.03, 3/2," were given (pages 168 to 171), from Worksheet 6. One group's question " If you had 50 m of metal and it takes 2/6 of it to make a car, how many cars could you make?" was attempted by all the others and discussed.

Lesson 12 (Monday, Feb. 18, 1991) - Ordering common and decimal fractions.

Group discussion on Question 1, Worksheet 7 (ordering 3/6. 0.3, 3/25, 0.03, 3/2). One group explained that 3/6 > 0.3 because "dividing by 6 gives bigger parts than dividing by 10". They expanded on this by relating this to sharing a chocolate bar among 6 and among 10 people. Another solution was to try and make the denominators the same. One group devised the following problem: How many lamps do you need to light up the room, when one lamp can light 0.2 of the room and the whole room is 41 square metres?

Lesson 13 (Wednesday, Feb. 20, 1991) - Word problem; ordering, using the idea of how close the fraction is to one.

The problem (see lesson 12) devised by a group was discussed. Quite a few students did not realise that the given area of the room was actually extraneous information (and according to the group who made up this problem, the extraneous information was puposely put in to "throw them off"!). Some used the longs to cover up the flat to arrive at the answer to this problem, while others used a number line. When asked what sort of problem it was, many could relate it to the coat problem they had done some time ago (e.g. how many coats from a 10 m length of material if one coat needs 2 1/2 m?). Next, the students tried to order the fractions: 0.8, 98/100, 6/8.

Students had different ways of doing this. However, some still had difficulties. Further exercises on ordering were done. The idea of how many pieces are needed to make up one (the whole unit), that is, the amount missing semed to be okay, but using the "missing" part to decide on which fraction were bigger/smaller seemed quite difficult. At the end of the lesson, the conclusion was that there were two ways of deciding which fraction was bigger: by checking with whole numbers and by finding out how much was missing to make the whole.

<u>Lesson 14</u> (Monday, Feb. 25, 1991) - Identifying and ordering fractions close to 0, 1/2, and 1; word pıoblem.

Students identified and ordered fractions close to 0, 1/2 and 1 (Worksheet 7). Plenty of discussion was generated with the fractions 6/10 and 3/5. Students explored the strips representing 25/100 and 20/100. The solution to the doghouse problem posed by one group was discussed. The problem: A carpenter wants to build some doghouses. He has 100 sq. ft. of lumber. One doghouse needs 4.5 sq. ft. of wood. How many doghouses can he make? There were various approaches to the problem (e.g. guess and check, by multiplying, by using flats and cubes).

<u>Leson 15</u> (Wednesday, Feb. 27, 1991) - Word problem; tenths, fifths and hundredths, using flats and longs as the unit.

The following problem, constructed by a group of students in the class, was discussed: A carpenter is building a house. The area of the house is 140.5 sq. m. There are 7 rooms. The area of each of the three bedrooms is 18 sq. m. What is the length and width of each oom? The kitchen, living room and dining room are equal in area but are bigger than the bedrooms. The area of the 7th room is 4 sq. m. What is the area of the kitchen? Most of the students used a guess and check method by trying out various products (e.g. 27.5 x 3 = 82.5). The group that designed the problem were aware that there were a number of possibilities for the dimensions of the room, but they disregarded some which seemed unrealistic. They also said that they learned more by preparing their own questions than by just answering questions. Then students showed the equivalence of various fractions (e.g. 20/100 = 1/5 = 200/1000 = 0.2 = 0.20) and did exercises on pages 181 and 182 of Worksheet 8 (shading flats and longs, given the common or decimal fraction name and also writing out the related common/decimal fraction equivalences). Most students found the exercises on these pages easy. Fraction strips were also used to do the number line exercises on page 183. Exercises on identifying fifths and ordering fractions (page 184) were also done.

Lesson 16 (Friday, March 1, 1991) - Learning log; equivalent fractions (common and decimal) and ordering fractions, using flats, longs, number lines and diagrams.

Students wrote in their learning log about their feelings about student-constructed questions and working in groups. One student presented a word problem on buying and sharing pizzas (also involved GST). Previous work on equivalent fractions (both common amd decimal), and ordering fractions, pages 183 and 184, were discussed and reviewed. Questions on fourths of a flat and long (including shading and writing equivalent common and decimal fraction names), page 185 (Worksheet 8) were done. Identifying and renaming common and decimal fractions, including the use of number lines, pages 187 to 188, were also attempted.

Lesson 17 (Monday, March 4, 1991) - Word problem; fourths and eighths (using flats and decimal fraction equivalents, including hundredths and thosandths).

The pizza problem presented during the previous lesson was discussed and solved quite easily. Exercises done during the last lesson were checked. The students then tried to partition the flat into fourths and eighths. Some students noticed that multiplying by 10 was involved when renaming decimal fractions (e.g. 3/10 = 30/100

= 300/1000). THe fourths were done quite easily, but the eighths, involving thousandths were a bit difficult for the students.

Lesson 18 (Wednesday, March 6, 1991) - Thousandths; ordering common and decimal fractions.

Shading the flats (partitioned into thousand parts) to represent fourths, eighths and thousandths (page 194). Use of metre stick to relate cm to common and decimal fractions. One of the students showed the Picard method she had learned in France on renaming fractions to other equivalent fractions. Exercises on ordering (e.g. find the largest and smallest of 3/4, 1/3, 0.3), page 193, Worksheet 8. Discussion and exercises on numbers between given decimal fractions (e.g find a decimal fraction between 2.74 and 2.73), page 197, Worksheet 8. Finally, the students wrote in their learning log about what they used to think about learning fractions and what they think about it now.

Lesson 19 (Monday, March 11, 1991) - Student feelings about posttest performance; word problem; idea of 1/3 as a decimal fraction; addition and subtraction of decimal fractions.

Students were asked about how they felt they had done in the posttest as compared to the pretest. All felt they had improved (Actually, one student did worse on the posttest: he said he had guessed for the pretest, but did not guess for the posttest!). From the teacher's analysis, basic concepts of common and decimal fractions seem to be weil-understood. The most difficult topics seemed to be finding fractions in between given fractions and operations on fractions. A group of students presented their problem on a housing developer who wanted to make a certain amount of profit. Dr. Owens then discussed the idea of 1/3 as a decimal fraction, using the 30/100 strip from a flat and relating it with the flat and cubes. Many students seemed to grasp the idea of a recurring decimal. Then addition and subtraction of decimal fractions in the context of a word problem (the Lewis family trip) was discussed.

Lesson 20 (Wednesday, March 13, 1991) - Addition and subtraction of decimal fractions in the context of word problems.

Various real-life situations involving addition and subtraction of decimal fractions via word problems (the Lewis family trip, fabric needed for a costume party, downhill skiing in the World Cup race, World track and field championship, amount of precipitation etc.) were discussed. The addition and subtraction worksheet (consisting of word problems)was distributed. The students worked on it and checked their answers with each other (this was done according to "once you have finished, pair up with someone else who has also finished and discuss your solutions. Then find two more people who have finished and discuss your solutions in groups of four"). The rule of "lining up" the digits according to their place value arose naturally out of the discussion. Then students were asked to prepare problems of their own. They did this, mostly working in pairs (some worked in groups, while others preferred to work individually).

Lesson 21 (Friday, March 15, 1991) - Addition and subtraction of common fractions.

Addition and subtraction of common fractions was introduced using word problems. Most students renamed the fractions so as to be able to deal with like denominators. Exercises from the text book were also attempted (page 321, # 20; page 322, # 31 - 34). Because renaming and equivalent fractions seemed to be pretty well-understood, there was not much problem dealing with these two operations with common fractions.

Lesson 22 (Monday, March 18, 1991) - Decimal fraction multiplication.

The worksheet containing word problems in decimal fraction multiplication was distributed. Students were to work in groups on the worksheet problems. Questions 1, 2, 3, 5 and 6 were discussed, while Question 4 was postponed to another day, as it was considered too difficult at this stage. The students who knew the multiplication algorithm were advised not to tell their friends about it. So there were many interesting approaches to the multiplication problems, including estimating, lining up the digits as in addition etc.

Lesson 23 (Wednesday, March 20, 1991) - Exploring some rules for multiplying decimal fractions.

Students first were reminded about the problems they had done in the previous lesson (Lesson 22) and then were set towork on a worksheet where they were to choose an answer from three given alternatives and give reasons for their choice (e.g. 39 x 1.2 = 046.80, 46.8 or 39.78). Some children suggested that 39 was near 40 and 1.2 was near 1 and so 39 x 1.2 would be close to 40 x 1 = 40. Therefore the answer should be 39.78; others (a very small minority!) disagreed and thought the answer should be 46.8. Similarly, a lot of discussion was generated for the question 3.9 x 0.12, where the choices were 0.108, 4.680 and 0.468. Interestingly enough, some of the "weaker" students chose 0.468 and refused to be influenced by the "brighter" ones who chose the other alternatives. One of the "weaker" ones even explained her choice of 0.468 by converting 0.12 to 12/100 and approximating it to 10/100, saying that the answer should be close to 4 x 10/100 = 40/100 = 0.40. Pretty sophisticated thinking! Students seemed to enjoy testing their rules and it was gratifying to find that not only did they forget about the time (so involved were they), but they did not ask to be just given the rule!

Lesson 24 (Friday, March 22, 1991) - Learning log; testing rules for multiplying decimal fractions.

The students wrote in their learning log about how they felt when tryimg to find a rule for multiplying decimal fraction. They were also asked to comment on why they did not feel the lessons seemed to take as long as before. Students continued testing their rules about multiplying decimal fractions. This time they used the calculators to check their answers and try to work out a rule based on the answers shown by the calculators. They were also asked to compare with mixed numbers(e.g. 3.2 x 2.9 = 3 2/10 x 2 9/10) and estimate the answers by rounding. Students also worked out some more exercises from the worksheet and tried to explain how they got their answers.

Lesson 25 (Monday, March 25, 1991) - Testing rules for decimal fraction multiplication.

Students multiplied decimal fractions greater than one (e.g. 4.9 x 2.3) as well as those less than one (e.g. 0.55 x 0.36) and tried to "stabilise" their rules. One of the sttudents pointed out that multiplying should result in a larger number. The teacher asked the students to use the flats (or pictures of flats) to show 0.4 x 0.3 as well as a few similar numbers to find out whether multiplication always resulted in a larger number. Then students tried to multiply a whole number with a decimal fraction less than one (e.g. 0.9 x 29). Most children seem confident of the multiplication algorithm now. The difficulty earlier was in estimating the product when one factor was less than one.

Lessson 26 (Wednesday, March 27,1991) - Decimal fraction multiplication word problems.

Review of multiplying decimal fractions with whole numbers and other decimal fractions. More word problems involving decimal multiplication were attempted and discussed. Use of the word "of" to mean multiplication, especially in situations involving a part of something. The students were asked to summarise what they had learned so far, by writing down the "big" ideas, in their learning logs. This was the last day of the project lessons.

Report of Lessons in Grades 4 and 5 (Division 6), Blundell Elementary, Richmond

Lesson 1 (Wednesday, April 10, 1991) - To get an idea of the children's fraction concepts.

Discussion based on 5 questions handed out to children, to find out their initial concepts of common fractions. Q1 asked about whether it makes sense to talk about a "bigger half" when an apple is cut into two. Q2 required the children to come up with different ways of dividing a square into 2 halves. Q3 was to find out whether the whole unit could be constructed, given a fractional part. Q4 probed students' ideas of fractions when comparing across different units. Q5 tried to indicate whether experience in whole number ordering influenced ordering of fractional numbers. Most difficulties were with Q4, where most students thought that 1/4 > 1/3 because 4 > 3. However, about six of them explained why 1/4 < 1/3, by drawing diagrams and talking about the the size of the fractional pieces.

Lesson 2 (Friday, April 12, 1991) - Paper-folding to demonstrate some common fractions.

Paper-folding (actually "strip-folding")to illustrate 1/2, 1/4, 1/8, 1/3, 1/6 (words, rather than fraction symbols were used). Some children had difficulty in differentiating "eights" and "eighths". Worksheet 1, page 1 (different-shaped parts of regions). Most students were not clear about what was expected of them in the worksheet. Since it was given immediately after the paper-folding activity involving congruent parts of a region, they were put off when presented with different-shaped parts of a region. This was especially so as they thought the worksheet was a "practice" exercise on what they had just done. The worksheet was not used as a starting point for discussion. The teacher felt that before Worksheet 1 is to be completed, more preparatory lessons had to be given.

Lesson 3 (Monday, April 15, 1991) - Emphasizing equal parts of a region to identify fractions.

Review of Lesson 1 discussion questions, using diagrams on chalkboard and fraction chart prepared by teacher. Worksheet on recognizing equal parts (taken from workbook by teacher) was distributed. Most children could do all the questions on this (teacher-selected) worksheet. The lesson was of shorter duration today (30 minutes compared to one hour)because of class photograph being taken.

Lesson 4 (Wednesday, April 17, 1991) - Identifying fractions, given shaded regions; identifying the larger of a pair of fractions.

Went over a few exercises on the chalkboard (shading regions, given fractional names). Returned Worksheet 1 for students to reconsider their answers. Discussion of whether a pair of parallel chords trisecting an imaginary diameter could give rise to "thirds". The fractions 1/2 and 1/3, and 1/4 and 1/3 were compared as to which were bigger. Again, fraction symbols were not used.

Lesson 5 (Friday, April 19, 1991) - Using fraction strips to demonstrate some common fractions.

Used fraction strips for the first time, with each student being given one shoebox containing the fraction strips. Fraction strips showing halves, fourths, eighths, thirds, sixths and twelfths were identified. Teacher discussed the importance of the "unit". She felt the lesson was not "crisp" enough because too many boxes and strips on the childen's desks distracted the children. She said that next time she would use two boxes per group of four or five children.

Lesson 6 (Monday, April 22, 1991) - Comparing fractions.

Using the appropriate symbol, " >, < and =" between common fractions. The idea of a chocolate bar being shared was introduced. Some of the fractions discussed were 2/10 &1/10, 1/4 & 1/3, 1/12 & 1/20, 2/10 & 1/5, 1/4 & 2/8 (once again, fraction symbols were not used). Students used mainly diagrams on the board to explain the relationship. Some did use the fraction strips, at the teacher's suggestion. Students were then given the opportunity to find out as many relationships/number sentences they could and then explain some of the more interesting ones to the class.

Lesson 7 (Wednesday, April 24, 1991) - Writing fraction symbols.

Different names for the whole unit were revised. The overhead projector was used to introduce common fraction symbols and the names numerator and denominator. Worksheet 1 was handed back to be checked and completed. Worksheet 2, prepared by the teacher (on writing fraction symbols, given shaded regions), was distributed.

Lesson 8 (Thursday, April 25, 1991) - Mixed numbers.

Fractions greater than one and mixed numbers were introduced. Worksheet 2 on mixed numbers was distributed. Except for the terminology "fraction name" and "mixed number name" which seemed to be unclear to the children, they seemed to have little difficulty with mixed numbers.

Lesson 9 (Monday, April 29, 1991) - Review of fraction symbols, mixed numers and fractions as parts of a region.

Discussion of Question 5 (equal parts), Question 6 (different names for whole unit) and Question 7 (writing fraction symbols, given fraction names in words) from Worksheet 1. Worksheet 2 on mixed numbers was continued.

Lesson 10 (Tuesday, April 30, 1991) - Word problems involving common fractions.

Discussed Question 4 (Worksheet 2) on sharing chocolate bar divided into fourths equally among 3 persons. Four students came up with different ways of doing this. Discussed questions prepared by another class. Students then constructed questions of their own (in pairs), together with the solutions. Some students had trouble devising problems that not only involved division (sharing), but also involved fractional parts less than one. For example, the question " if 18 pizzas are shared equally among 3 people, how many will each get?" did not exemplify clearly the idea of fractions.

Lesson 11 (Wednesday, May 1, 1991) - Comparison of fractions.

Worksheet 3 was used as a starting point to compare fractions. Students worked in pairs, to decide which fraction was bigger or smaller, from a given pair of fractions. They also had to decide how to order fractions in increasing order of magnitude, given three fractions. Some students concentrated on the denominator and ignored the numerator when deciding on the larger or largest of a given number of fractions. Two students said that they thought it a waste of time to draw diagrams to explain their decision as to which was the larger (or smaller) fraction. The teacher also asked the students to prepare questions on mixed numbers and exchange these with others.

Lesson 12 (Monday, May 6, 1991) - Review comparison of fractions.

Discussed students' answers to Questions 1 and 2 (comparing two fractions) and Questions 3 and 4 (ordering three fractions), from Worksheet 3. There were still a few students unsure about this topic. Many still did not use the manipulatives to decide on their answer. Rather, many drew pictures (quite inaccurately-drawn, some of them), and because some of the units drawn were not the same for a given pair of fractions, wrong decisions as to the relative magnitude of the fractions were made.

Lesson 13 (Wednesday, May 8, 1991) - Review mixed numbers.

Exercises on changing common fractions to mixed numbers from Worksheet 2 were discussed. The fraction strips were used to explain the answers. One student said that to change 12/10 to a mixed number, one just had to "move the one to the left", so as to get 1 2/10. Another said that this would work only for tenths.

Lesson 14 (Friday, May 10, 1991) - Completing a given diagram of a fraction to its whole unit.

Regions showing fractions such as 3/12, 3/4 and 3/7 were drawn on the chalk board. Students were to draw the unit for each fraction shown on the board. Then the 3/12 fraction strip was taken and students were asked to imagine that this strip was 3/5 and show the unit. After a few more examples, Worksheet 4 (drawing the unit, given a drawing of a fraction) was put on the OHP and discussed. Then students tried to complete the worksheet.

Lesson 15 (Monday, May 13, 1991) - Review (test) on common fractions.

The following topics were tested: identifying a fourth of a given region; drawing the whole unit, given the part; equivalent fractions; diferent names for one whole unit; writing common fraction symbols, given the word name; mixed numbers; identifying the larger of two fractions; ordering common fractions; and word problem. Except for the topic "equivalent fractions", for which item the students scored 66%, students scored more than 70% for all other topics, with an average score of 82% for the test items/topics. The teacher conducted the test and no video or audio-recording was done. The test was scored by the researcher.

Lesson 16 (Wednesday, May 15, 1991) - Equivalent fractions.

Worksheet 5 (on equivalent fractions, based on shaded regions) was distributed. The first picture, consisting of a square divided by its lines of symmetry and a fourth of it (shaped as a small square) shaded, was discussed in detail. Students came up to the chalkboard, explaining how they could get different names for the shaded part, for example, by inserting and removing lines. Students could see that 1/4 = 2/8 = 4/16 etc. However, they were only seeing the "additive" pattern resulting from doubling and so on. For instance, they could see that 4/16 = (4+4)/(16+16), and not that 4/16 = (2x4)/(2x16). The researcher later pointed this out to the teacher and suggested that working towards the multiplicative relationship would be advantageous, because of its generalizability to a procedural rule.

Lesson 17 (Wednesday, May 22, 1991) - Introduction to decimal fractions.

Introduced the "long" (a 10 cm rectangular strip divided into 10 equal square regions) as the unit and used it to demonstrate 1 tenth, 2 tenths etc. Most students could see that the shaded parts need not be contiguous/adjacent. Students gave various names for fractions, for example, they said that 4/10 = 8/20 = .4 = 0.4 and they could also see that 5/10 = 0.5 = 1/2, and suggested that 1 whole unit = 10 tenths = 1.0 = 1 0/10. Worksheet 6 (on naming common fractions as decimal fractions) was done quite fast by the children. Those who had completed that worksheet were asked to design their own questions and share them with their friends/neighbours. Worksheet 7 (on rewriting fifths as decimal fractions via tenths) was also disributed.

Lesson 18 (Thursday, May 23, 1991) - Decimal and common fractions, including numbers greater than 1.

Continued discussion of equivalent fractions, including the use of multiplication and division for getting equivalent fractions. Students used strips and pictures to show decimal fractions such as 0.7 and 1.1, initially working in pairs and later demonstrating to the class. Worksheet 8, where pictures of "longs"(with some squares shaded and some where the students were expected to shade) were to be used to write both the common fraction as well as the decimal fraction names, were distributed.

Lesson 19 (Friday, May 24, 1991) - Common fractions to decimal fractions and vice versa, using the large square made up of 10 (1 x 10) strips put together as the unit.

Worksheet 9 (changing common to decimal fractions and vice versa, using the "flat" as a unit) was distributed. The teacher asked the children to take their time completing the worksheet. In the meantime, she revised the algorithm for equivalent fractions. The researcher realised that the children did not relate the "whole" to the number "1". So he discussed this relationship with the class, using the large square and giving examples such as 30 tenths = 30/10 = 3.0 = 3 (where the 3 is from the sequence of whole numbers 1, 2, 3, 4,....). One student (who had been away for some time) had trouble identifying the unit when two wholes were involved.

Lesson 20 (Monday, May 27, 1991) - Common fractions to decimal fractions and vice versa, using the "flat" as the unit.

Some questions from Worksheet 9 were discussed and eight students explained their thinking, using drawings on the board. Then Worksheet 10 (where the flat was used as the unit) was distributed. Most students completed this pretty quickly. The teacher then went through the questions and spent some time discussing whether 0.4 was the same as 0.04.

Lesson 21 (Wednesday 29, 1991) - Using the longs as tenths

The research assistant conducted the class. An attempt was made to use the manipulatives more than diagrams/pictures. The students were told that what was important was whether they could explain their thinking, using manipulatives, rather than just geting the answer. Then Worksheet 11 A was distributed, without much explanation. It was hoped that the worksheet would generate discussion on the students part. The lesson did not go as hoped, mainly because the students were not clear as to what to do (previously they had been told in detail what and how to do the questions), and also because they found it difficult to treat the long as a tenth when all this while they had used it as a unit.

Lesson 22 (Friday, May 31, 1991) - Common fractions to decimal fractions and vice versa, using the longs as tenths, and the cubes as hundredths.

The teacher did a lot of oral work on units, tenths, hundredths using flats, longs and cubes. Questions such as "Could you show 2.31 and 2.13 with your manipulatives?" were asked, to emphasize the use of the manipulatives to enhance understanding. Subsequently, the students completed Worksheet 11 (changing common to decimal fractions and vice versa, using the longs and cubes as tenths and hundredths respectively) quite comfortably. Worksheet 12 (different names for the same decimal fraction, using different groups of tenths, hundredths etc and identifying the larger/smaller of a pair of decimal fractions) was also distributed.

Lesson 23 (Monday, June 3, 1991) - Different ways of naming decimal fractions, using different groups of tenths, hundredths etc. Also fraction of a discrete number of objects.

Questions from Worksheet 12 (different ways of naming decimal fractions, depending on ways of grouping) were discussed. For example, 1.23 = 1 unit, 2 tenths and 3 hundredts = 12 tenths and 3 hundredths. The idea of fractions of a discrete number of objects was introduced, using a group of children in the class. For example, 1/2 of 8 children = 4 children etc. Then cookies of different sizes were distributed, to bring home the point that, for instance, 1/4 of 8 cookies = 2 cookies and that in this case the size of the cookies did not matter (compared to the case of fractions of regions).

Lesson 24 (Wednesday, June 5, 1991) - Equivalent fractions, fraction of discrete sets and word problems from text book.

After working only from worksheets prepared for the project so far, wanted to know how students would fare when faced with questions from the text book. Students were asked to work in pairs on pages 204 (Grade 4 "Journeys in Mathematics") and (Grade 5 "Journeys in Mathematics") on equivalent fractions, fractions of discrete sets and word problems. They were to be prepared to explain their thinking/work to the class. Plenty of discussion was generated and students came up with various good explanations for their answers. They were also asked to prepare their own questions that involve comparing two fractions.

Lesson 25 (Friday, June 7, 1991) - Relationshp between common and decimal fractions (tenths, hundredths, fourths and fifths); ordering common and decimal fractions; word problems involving common and decimal fractions.

Students explored the strips representing 25/100 and 20/100 and were asked to write as many different ways as they could (equivalent fractions), including using decimal fraction notation for these strips. They also came up to the front of the class and explained how they figured out the names. Then Worksheet 13 (relationship between common and decimal fractions, ordering common and decimal fractions; word problems involving common and decimal fractions) was distributed.

Lesson 26 (Tuesday, June 11, 1991) - Relationship between common and decimal fractions; word problems involving addition of common and decimal fractions.

A number of students explained their reasoning for Questions 3 and 4 (word problems involving decimal and common fractions) from Worksheet 13. Then Worksheet 14 (relationship between common and decimal fractions; word problems involving addition of common and decimal fractions) was distributed. Question 1 (relationship between common and decimal fractions) was discussed.

Lesson 27 (Wednesday, June 12, 1991) - Subtraction of decimal fractions.

Some questions from Worksheet 14 (e.g. Is 4/10 + 3/10 = 7/20?) were discussed. Many disagreed that 4/10 + 3/10 = 7/20, but there were a few who believed it to be true. Those who disagreed said that we were considering tenths and not twentieths. Those who agreed were actually misled by diagrams, as they considered the shaded parts of two wholes, and did not relate the question back to one whole unit. Worksheet 15 (subtraction of decimal fractions) was then distributed. Most students completed the questions procedurally, using their experience with whole number subtraction, without using any manipulatives to explain their answers.

Lesson 28 (Friday, June 14, 1991) - Subtraction of common fractions and decimal fractions, including word problems.

Worksheet 16 (subtraction of common and decimal fractions, including related common and decimal fractions and word problems) was distributed and discussed. There was an animated exchange between two students as to whether 1/3 was the same as 0.3. This exchange generated a lot of interest among their fellow-students. This was the last lesson for the project.

APPENDIX B

Project Evaluation by Researchers and Teachers

The project on common and decimal fractions involved a Grade 6 teacher, A, (with 22 pupils) and a Grade 4/5 teacher, B, (with 26 pupils, 16 of whom were in Grade 4) from two different schools in Richmond, B.C. The video-taping of lessons and interviews lasted from January 23, 1991 to June 14, 1991 (Instructional days coveredJanuary 23 to March 27 for Grade 6 and April 10 to June 14 for Grade 4/5). Towards the end of July, an evaluation of the project was done by the researcher, the two teachers and the research assistant, after viewing some video tapes and a discussion guided by some focus questions. A summary of some of the main points that arose out of the discussion are described below.

Interviewing

One thing that stood out for both the teachers concerned was the importance of the interview as a technique to assess children's understanding. According to them, they became very much aware of individual differences in the learning and understanding of Mathematics when they viewed the interview tapes. As teacher A put it, "I didn't imagine that they were that confused!" She felt that "kids need to respond one-to-one" and that "our teacher expectations colour our view". Teacher B said she learned "more about what children do and do not understand" by viewing the tapes, saying that "students who are good at computation can sure fool a lot of mathematics teachers". The general feeling was that feedback from the interview tapes would have helped future planning and teaching. In future, the researcher should arrange for the teachers to see the interview tapes as soon as possible. Also, they felt that somehow interviewing had to be made a part of their teaching. They agreed that this would be extremely difficult to do, but suggested that during journal writing, one child could be drawn aside to be interviewed by the teacher, as a kind of "oral journal". They were also impressed with the way the researcher was able to elicit information from the children being interviewed and felt that this was something they would like to work toward. Teacher B also felt that such interview skills could be utilised advantageusly for classroom questioning as well.

Student attention

One point of concern was the apparent lack of active involvement by all pupils when two pupils were involved in a dialogue or when one pupil was trying to explain how he or she arrived at an answer. The teachers were surprised and disappointed to see this lack of involvement by the pupils when they viewed the video tapes of classroom lessons. They had been under the impression that almost everyone was paying attention to explanations by their classmates. The teachers suggested that such verbalizing/explanations should be confined to groups (unless there was something of general interest/importance), without having explanations by students coming to the front of the class. The reasoning behind students' answers was important, not verbalizing per se, in front of the class.

Journal writing

As far as journal writing was concerned, it was felt to be important, especially in terms of the feedback that could be obtained. For example, the Grade 6 pupils wrote that preparing their own questions was a very interesting and beneficial task, while some the Grade 4/5 pupils wrote that the lessons on fractions were boring. However, both teachers felt that they did not optimise the potential of the journal and would welcome help to make this more effective. The research assistant (whose interest was in this area) suggested some relevant resources.

The teachers also felt that written responses by the teachers in the student journals were essential. As teacher B said,"You become more of a person to the pupil when you respond". Even so, teacher B felt a bit "inhibited" about responding (as she usually did to previous student journals) because of "affecting" the project. Teacher B felt that though she raised this question of affecting the results of the project early in the project , she did not get a definite answer. And because of this, the pupils, too, felt inhibited and responded more perfunctorily than usual, thereby not revealing insights or doubts etc. She felt her role in the student journal writing was not clear: for example, was she to respond or was the research assistant going to respond, or were neither supposed to respond?

The Grade 6 pupils, however, seemed to have got more out of writing the journal, possibly because "that was the only journal they were writing at that time", whileGrade 4/5 pupils writing other journals at that time. As well, because the Grade 6 students were not just asked to explain what they had understood (i.e. as an "evaluative" tool), but rather to have some sort of ownership on the lessons by trying to help the teacher answer the question "how can you help me teach you?", or "how can you help me be a better teacher?". Both teachers agreed that the time involved in reading the journals and writing responses would be quite demanding, but could not see how to overcome this difficulty.

Another matter that cropped up was the possible advantage of the teachers keeping a journal as a means of remembering what had happened and also as a line of communication between the teachers and the researcher. All agreed that this would have to be done in future projects. Integration

As regards the integration of common fractions and decimal fractions, all four of us felt that most pupils had integrated these ideas quite well. This was borne out especially in the Grade 6 class during the multiplication of decimal fractions. Though the teacher did not explicitly teach the multiplication algorithm, some of the pupils converted the decimal fractions to common fractions to estimate the product or explain their rule (for placing the decimal point) for the product of decimal fractions.

Unfortunately, this integration seemed to be specific to the context of the classroom lessons and exercises. For example, the remainder 2/5 was never considered as 0.4, when doing division. Also, the pupils had forgotten the multiplication algorithm (which they had actually discovered themselves), after a few weeks, as evidenced by their responses during the interviews. It looks as if there has to be consolidation of what had been understood plus varied type of exercises in different contexts. The teachers suggested that pupils write what they had learned, do a problem and explain how they did the problem in their journals. This would then serve to "arrest the curve of forgetting" as well as provide reinforcement of the concepts learned by application to new situations.

Student-constructed questions

Regarding the student-constructed questions, teacher A felt that that they were very beneficial. They motivated the students and provided opportunities for deeper understanding and more challenging questions. Since the questions were generated through group discussion, and each member of the group had to understand the solution, they were a valuable source of peer and social interaction. Moreover, the students felt a sense of ownership. In spite of all these advantages, there was one main drawback: the lack of focus on the mathematical topic under discussion. For example, there were a number of good questions, but they were only cursorily concerned with fractions and more with money.

Teacher B agreed that student-generated questions were valuable, but unfortunately she did not get much opportunity to use them because of being caught up with trying to complete the worksheets prepared for the project., as she felt completing the worksheets was "exactly what I thought you wanted me to do!" Somehow student-constructed questions had not been stressed by the researcher to teacher B even though it had been seen as a successful technique.

Worksheets

As regards worksheets, Teacher B felt that they were too much like the text book exercises and test sheets. She suggested that we should use larger fonts, more pictures and put fewer exercises per sheet. Also, her pupils had got accustomed to thinking of worksheets as something that had to be completed and not something to start a discussion. The researcher did suggest to teacher B that students stop and discuss what they had done after completing certain questions in each worksheet. There were some efforts in this direction a few times, but somehow such discussions never quite caught on.

Teacher B, though committed to making the project a success, did not assert herself to obtain more ownership of the project, by, for example, saying that working through worksheets was quite foreign to her and her class. According to her, "I believed my job was to use your materials to see if using these materials - and thereby a different format - would enhance student understanding. Was there a hidden agenda?". Moreover, the teacher's earlier practice (i.e. in mathematics classes before this project was initiated) of allowing pupils to try "today's brain teaser" or working on something they liked after completing the mathematics exercises promoted the idea of the worksheets as something that had to be completed as quickly as possible. She suggested the use of the blackboard examples to initiate discussion before going on to similar examples and(fewer) exercises in the worksheets.

Teacher A felt that because she stopped pupils at different stages of the worksheet to discuss certain points, they treated the worksheets as somethimg to initiate discussion rather than something that had to be completed as quickly as possible. Her lack of emphasis on getting the right answer also helped pupils to feel comfortable about "messing around", without necessarily completing the question or getting the right answer. In fact, sometimes she celebrated a different answer by saying " Aha, here we have someone with a different answer, let's hear this!" or even expressed dissapointment when there was no disagreement!

Manipulatives

Regarding the effectiveness of manipulatives, all four of us agreed that manipulatives helped the pupils' understanding of common and decimal fractions. Even so, many of the Grade4/5 pupils did not use it very much, because they were too caught up with just getting the answer. Even the "better" pupils considered the manipulatives a "waste of time" as they "knew" how to get the right answer, as evidenced by their answers to questions put to them about the usefulness of manipulatives. They did not see a need to eithe explain or check their answers with manipulatives.

Teacher B was quite surprised that her pupils seemed to view the manipulatives in this light and also that they did not use it as much as she would have liked. This was especially puzzling because she had always considered herself a very "hands-on" type of person where the teaching-learning of mathematics was concerned. Perhaps the emphasis on explaining or checking the answer with manipulatives was counterproductive(as was the earlier explained attitude of trying to just complete the worksheets as quickly as possible). Rather, there could have been more time "playing around" with the manipulatives and developing concepts. In other words, manipulatives should have been used prior to, and concurrently with the exercises (i.e. during the processing stage), rather than as something to be used after the fact as it were (i.e. after the production stage). The researcher was in agreement with this view, and had thought all along that this had been clearly communicated to the teachers concerned. He realizes now that there had been a breakdown in communication and he would have to work at this in future.

From the researcher's (and graduate assistant's) point of view, another factor that might have brought about this disappointing attitude to manipulatives could have been the rather late introduction to, and sporadic use of, manipulatives both by the teacher and her pupils. For example, manipulatives were first used only in the fifth lesson. Also, prior to this, (and even very many times after this) freehand drawings of rectangular and circular regions were the main (visual) aids utilised.

In contrast, teacher A found that the manipulatives had another role to play, namely as a "communication tool, after learning the concept". She elaborated on this by saying that both she and her pupils used the manipulatives to explain something and that the manipulatives became a common language as it were. For example, in trying to explain to fellow classmates which of two fractions was bigger, pupils used the manipulatives as a matter of course, without any prompting by the teacher or anyone else.

Profesional development

In terms of professional development, both teachers said that they had grown as a result of their involvement in the project. They were unanimous that they would never teach common and decimal fractions as separate topics in the future.

For teacher B, the use of manipulatives in this project further affirmed her belief in their effectiveness. Her only problem was to get over the reluctance of her pupils to use them, even though many of them could understand better once they used them. For example, in adding 3/10 and 4/10, many pupils just wrote down 7/20, but when urged to represent the addition using manipulatives, they could at once see why the answer was 7/10 and not 7/20. As explained earlier, the attitude that mathematics consisted of just writing the answer down using previously-learned algorithms/procedures was difficult to overcome.

In retrospect, teacher B was disappointed that she did not take more ownership of the project, though according to her "obviously, this goal was not clear to me". She felt that her role was to carry out the project as planned by the researcher, and felt unsure as to how much she could modify the plans and still maintain the objectives of the researcher. Moreover, the time spent planning lessons together was very much less than with teacher A. This could have been because teacher B had lots of experience teaching mathematics whereas teacher A had very little experience in teaching mathematics and had, right at the outset, declared that she would need a lot of help. Hence, we spent a lot of time in planning lessons at different stages of the project with teacher A, but spent much less time with teacher B. But as teacher B pointed out, she "was not an experienced researcher and would have appreciated and benefitted from being given a similar amount of support regarding planning time as teacher A".

For teacher A, there were two "watersheds": one, after seeing what the pupils could do about preparing their own questions and two, when the children got involved and excited about finding and testing their rules for placing the decimal point in the product of two decimal fractions.

She realised that motivation was very high during periods when they were asked to prepare questions for their classmates. She also found out that they were capable of asking and answering questions which were more difficult than the ones they were usually accustomed to.

In trying to find the rule for the placement of the decimal point, very often the children were so involved that they forgot the time. Moreover, there were times when the teacher herself had to respond to quite unexpected and unplanned situations. But rather than feeling dismayed by this, she began to enjoy the experience of "learning together" with the pupils. In fact, she became quite used to the idea of the children setting the pace and direction of the lesson. In short, she began to assume more ownership of the project as days passed.

Then. on completion of the project, she was prepared to teach division of decimal fractions (a topic she had not felt any great confidence in teaching), using some suggestions from the researcher as well as some resources given to her by the reseacher. Also, she said she felt positively "evangelical" and more like a "born again Mathematics teacher", after her experiences in the project, so much so that she wanted to know how to share her experiences with others.

Teacher B, too, felt that her experience with the project had changed her for the better. For instance, she felt that although she had been very good at mathematics as a student, she felt surprised that there were a number of things in the Grade4/5 lessons on fractions that she had a "re-vision" of, especially when the manipulatives were used.

She too was pleasantly surprised at what her pupils could do without explicit instruction. For example, when a "pizza problem" was presented, she was initially unsure whether her pupils could solve it. When they came up with not just one method of solution, but many different ways, she was surprised and pleased.

She was also happy to see her pupils willing to spend an extended period of time trying to solve a problem that arose outside the mathematics period. According to her, she realised that we should have had more problems for the pupils to "struggle" with than the rather formal-looking worksheets they were given during the project, and had brought up the matter earlier, but somehow nothing much had been done about it While agreeing that there was a lack of word problems, the researcher felt that there could be two reasons for such a lack: one, that we had been so caught up with the worksheets that we had overlooked preparing word problems for the pupils to attempt; and two, we had thought that the worksheets would have been used to stimulate discussion, thereby acting as a source of problem solving, but somehow that had not taken place.

She also started letting the pupils decide which questions they would like to discuss after we were some way into the project, rather than going through every question as she used to earlier. However, her one big regret was that her role as a teacher-researcher was not clear, so much so that she assumed the more passive role of trying to carry out someone else's agenda/project, rather than actively pursuing her own agenda for fear of "interfering" with the "expected" outcomes of the project.

Also, she felt that having a student teacher doing her practicum during the duration of the project brought about special problems. For example, the student teacher taught mathematics on the two day a week that the project lessons were not being done (the project lessons were usually on Mondays, Wednesdays and Fridays). This proved rather disruptive, especially in terms of the students having to get used to two different "approaches" to mathematics. On the other hand, because the same teacher (teacher A) taught mathematics on the "non-project" days, she could complement and integrate her lessons, even though she had different topics (measurement and problem solving) on the non-project days.

Both teachers also felt that they would be able to use the teacher's guide and text books more productively. Also both believed that it would be worthwhile sharing their experiences with other teachers, in the form of a workshop at least. Teacher B especially felt that "Just publishing it is not enough. We have to get it to the teachers, those in the 'trenches' ". So they have decided to run just such a workshop shortly.

Teacher-researcher role

Regarding the role of the teacher as a researcher, there was initially a consensus among the teachers. Both thought that they were primarily assisting the researcher's project, and secondarily trying to learn about some current methods that could make their teaching more effective.

But now they view their role rather differently. As teacher A put it, "when we have a nagging uncertainty, that's the place to start ", so that the research emerges out of a felt need of the teacher rather than someone else's agenda. However a lot of support would be needed and doing it all by oneself would be problematic. This is where a collaborative research (and not only collaborating with the university, but also with colleagues) would prove fruitful.

Another role for the teacher was as a student, learning as the project progressed and allowing time to reflect on what had and was taking place. In that sense, it would be difficult to specify right at the outset the exact direction and role of the participants in the project. As teacher B pointed out, "every time I view the video tape, I pick up something different", and so "what I have to say now is certainly not what I would have had to say at the beginning or even halfway into the project". Teacher A agreed, saying that "I don't know whether I could have talked sensibly to you about this at the beginning of the project. It's only after going through the whole project can I begin to think about my role and what I would do differently and what I have learned".

Another point of interest was the matter of "how were we selected to take part in this project?" The researcher said that this was done through a school district resource person who was aware of teachers who might be "willing" to take part in such a project. This resource person was someone known to the researcher as well.

Conclusion

In summary, then, all four learned a lot from the project. One was that common fractions and decimal fractions could be taught in an integrated way, using manipulatives. Secondly, there was no one way of going about doing the research and how much one got out of it seems to be a function of how actively one pursued one's own agenda (though teacher B felt strongly that she "did not know that having 'one's own agenda' was one of the researchers' major goals!"). Thirdly, learning was emergent and had to be given time for reflection. Fourthly, student journals and student constructed questions seem to have good potential, but a lot depended on how they were utilised. Fifthly, experience in the project gave greater confidence to participate in similar projects and also to share such experiences with others. Finally, student interviews are very informative, and video tapes of such interviews should be viewed prior to the planning of future lessons. Also, such interviews should somehow be incorporated in the classrooms as part of the teaching-learning process.

Project Evaluation by Grade 6 Teacher

UNDERSTANDING CHILDREN'S DEVELOPMENT
OF RATIONAL NUMBER CONCEPTS

Being involved in this research project has been challenging and exciting. Intially I was interested in the project because division with decimals was the most difficult concept I tried to teach my grade 6 class last year. I was very frustrated with my attempts to help the students who had difficulty, and hoped that this year could be more successful. I believed that using manipulatives to strengthen students understanding of common and decimal fractions could make a difference.

Throughout the course of the project I have had to re-examine my approach to teaching math. I have always enjoyed teaching math and thought that I explained concepts well. My students were usually successful and I believed I was teaching for understanding. Now I'm not so sure.

PART ONE: Common Fractions Jan. 21 - Feb. 3
Using fraction strips, number lines and diagrams to develop concepts of equal sized parts, number of parts in whole, size of whole unit, comparing

1. Fraction concepts/symbols
   standard notation
   worksheet p.168

2. review equal sized parts
   # of parts in whole
   mixed numbers
   p.169
   p.170 move comparing question #7 to later section
   extra page

3. review
   equal parts p.171
   size of whole unit p.172 are all halves the same? Important discussion

draw the whole unit p.173
mixed #s p.174 #7 & 8

4. comparing

p.173 #5 p.174 #6 p.170 #7 p.178 #1
review p.179 #2 & 3

5. use of the number line. Very important to
transfer understanding and represent a new way
trace a fraction strip use p.183
p.180

6. review p.184 185, 186, 187

These six lessons seemed to take a very long time. Especially considering all these concepts are covered in one lesson in the math textbook. However, the pretest showed that in spite of the fact that these students have had previous experience with common fractions, they did not really understand the concepts of equal sized parts of a whole unit. This was a surprise to me and to them.

The lessons in this section were presented in a very lock step manner. Everyone did the same thing at the same time in the same way. This may seem to be tedious. However, on reflection, I believe that this was important because it established a very firm basis of common understanding and success. I feel this was important because both the students and I were uncertain about what was going to happen in the project. I was very dependent on the materials that were provided. As the project progressed, we began to take risks, make mistakes and learn. But first we needed to establish a sense of trust based on confidence.

One benefit of using manipulatives that became apparent almost immediately, was that I had an instant method of monitoring the student's understanding. It was obvious as soon as someone became confused because they had the wrong "stuff" in front of them.

During lesson five, the students were asked to work in groups to write their own problems using common fractions. These problems became very important to the success of the project. Not only were they a bridge to the real world and a challenge to the students, but they helped us to break away from the structure of the materials and started us thinking about alternate ways to think and solve problems.

PART TWO: Decimal Fractions Feb. 6 - 22

Using flats, longs, cubes and number lines to understand equivalence, compare and rename common and decimal fractions.

1. naming points on # line connection with common fraction concepts. Use fraction strips and p.182 to mark 4ths, 5ths, 3rds on # line. Use long the mark tenths.

   p.188 - 190 easy

2. renaming p.190 - 192 easy

3. equivalence tenths to hundredths and hundredths to tenths

   p.193 - 196 did not need so many exercises. 2 of each kind might have been enough. Charts on p.195 #2 and p.199 #4 should be used in this lesson.

4. comparing p.197 very important

   p.198?

   p.199 #5 unnecessary

5. re-naming using # line p.200

   p.201

   p.202?

6. expanded notation p.203 and 204?

   Show me 0.25. How else could you show that? Could this be accomplished using manipulatives instead of written representation?

7. ordering and comparing

There was more discussion and decision making involved in comparing, ordering and renaming. The students began to "construct" their own understanding and to make sense of tasks for themselves. It was important to work in partners or groups to use language to justify answers through discussion.

   p.205 compare very important

   p.206 largest & smallest very important

   p.207 #1 discovery in groups was good. Explain how you decided. Develop list of strategies from the students.

8. strategies

close to 1 strategy p.207 - 210

near a half p.210 - 213

p.211 #7 # line good

p.212 #14 approximate, eyeball

good for estimating

p.213 #17

I would suggest that next time I would just have more ordering tasks. I wouldn't try to teach students these strategies but rather have them discover, develop, and share their own successful strategies.

9. Renaming

Name the parts in the mystery bay. Use flats, longs, cubes, metre stick to develop a rule for renaming common and decimal fractions.

renaming 5ths and 10ths p.216 - 219

renaming 4ths as hundredths p.219- 223 use metre stick

renaming 4ths and 8ths as thousandths p.223 - 225

10. odds and ends p.226, 227 omit

p.228 O.K.

p.232 numbers between

By this point the students were very confident about using the number line, ordering and renaming equivalent common and decimal fractions. The post test showed that this confidence was justified. There was a dramatic improvement of understanding of common and decimal fractions for most students. However, I had increasing concerns about meeting the needs of all the students. Some needed more challenge; some were being left behind.

PART THREE: Operations March 11 - 22

1. Addition and subtraction of decimal fractions

Students were presented with a problem and asked to try to solve it. We then discussed their procedures for solving the problem. Since everyone was successful I presented an incorrect solution

$$\begin{array}{r} 0.2 \\ \underline{+\,0.34} \\ 0.36 \end{array}$$

Students were able to explain why this was incorrect. We used problems from the real world - distance in kilometres, litres of gas, metres of fabric, time in ski races and track andfield. Students used a think/pair/share strategy where they tried to solve the problem themselves and then compared and justified their procedure and solution with a partner.

2. Addition and subtraction of common fractions

Students used fraction strips to show how they would add 1/2and 1/4. There was no need to "teach" them a rule about re-naming with a common denominator. They understood what they needed to do, used manipulatives to do it and then quickly transferred to using symbols.

These were very good lessons. The problems were based in real world situations and the students used different methods and strategies to think about and solve them.

3. Multiplication of decimals

My teaching method underwent radical change during these lessons. In the past when I "taught" this concept, I would present and explain the algorithm, guide the practice and assign some independent practice. This time I did not tell the students how to multiply decimal fractions. They were asked to struggle and mess around with the problems and try to make a little sense out of multiplying decimals. They had no trouble understanding and solving the problems but were unable to manipulate the numbers to find the answer they knew should be correct. For example, they knew that the G.S.T. on a $30 pair of jeans would be $2.10 but when they multiplied 30 x 0.07 the answer was $210.

The next step involved organizing the experience to help the students discover the"rule." We rounded, estimated, and checked with the calculator. Along the way students suggested several interesting rules, but they would not work in all situations. Eventually almost all students discovered their own version of the rule.

We dealt with confusions and consolidated and verbalized understandings and then re-visited the problems to apply this new understanding.

During these lessons I was never sure what direction the lesson would take or what the outcome might be. I needed confidence to be able to go with the flow. One student reported in her log that this search for the rule made her frustrated and angry at first, but excited and proud when she was successful. The quiet murmer of OH!s was quiet wonderful.

CONCLUSION

As the project progressed my thinking about teaching math was reaffirmed in some ways and changed radically in others. The use of manipulatives was important to some students. They benefited from having a concrete visual tool to help them understand the concepts. Many of them used a manipulative tool when they needed to solve a problem. Some strengthened their understanding using a manipulative and then were able to work with just symbols. This was very apparent in the section on adding common fractions.

I was reminded constantly of the importance of "talk" in clarifying thinking. Student need to justify and share their thinking. At first when one of the students gave me a different answer than the one I expected, I wasn't always sure if it was correct or how they had arrived at that idea. That dialogue explaining our thinking improved as my understanding improved.

We all learned from our mistakes. Taking risks is very important in learning. The climate that allows, encourages, and celebrates risk taking leads to success. We learned the most when there were different answers and ways of approaching a problem.

I am convinced that the approach we used to learn to multiply decimals leads to greater understanding. As teacher, my understanding of the concepts became clearer. I felt more confident and less dependant on the worksheets. I really had never thought of 0.2 x 0.3 as 2/10 x 3/10. I was taught

    0.2
x 0.3   count the numbers after the decimal and
  0.06   count the same number of places in the answer

At this time I have a few lingering concerns. One is meeting individual needs. Some students need more of a challenge. They did improve their understanding and they benefited from writing problems but they did not need as much time as the rest of us and need other challenging tasks. Others lack of confidence made this approach very difficult and unsettling. They worried because they knew they didn't understand

and they were used to producing pages of correct exercises. There is also a small group of students who still have only marginal understanding. They need small group instruction in a special tutorial session. This concern over individual differences is not any different in any other situation but it is an unresolved concern.

The final concern is my hesitancy to try the same instructional strategy again on my own in another situation. I would like to try to teach division with decimals by starting with problems, organizing the experience and consolidating and verbalizing understanding. The problem is I don't understand the concept of division of decimals well enough to do that. I also suspect that the students do not understand the concept of division.

So . . . It was fun, we learned a lot, but now what?

Project Evaluation by Grade 4 Teacher

re: U.B.C. Project: "Understanding Children's Development of Rational Number Concepts"

I participated in this project from April 10 - June 14, 1991, as the classroom teacher of 26 grade 4 and 5 students.

It is my opinion that there were many positives gained from this experience:

1. use of inexpensive and readily available materials
2. easy storage of materials
3. unique learning experiences for both students and teacher
4. planning sessions for the teacher
5. Math lessons involved a great deal more discussion
6. lack of focus on textbook exercises
7. teaching techniques were coming closer to what is done in Science (test, analyze, discuss, re-test, discuss, share, prove, etc.)
8. lessons took on "Whole Math" aura
9. much greater student involvement in lessons - time for students to share their thinking
10. lack of pressure on students to "always finish"
11. change in attitude - realization that it is OK to struggle and to make mistakes - enjoyment and eagerness when Math problem found in reader, then willingness to spend an extended period of time solving this and sharing ideas.
12. focus on understanding rather than rote recall
13. Learning Logs helped some students clarify their thinking.

The following are some suggestions for improving future similar projects.

1. lessons should be less than an hour for this age student
2. the teacher should feel confident to digress at appropriate moments

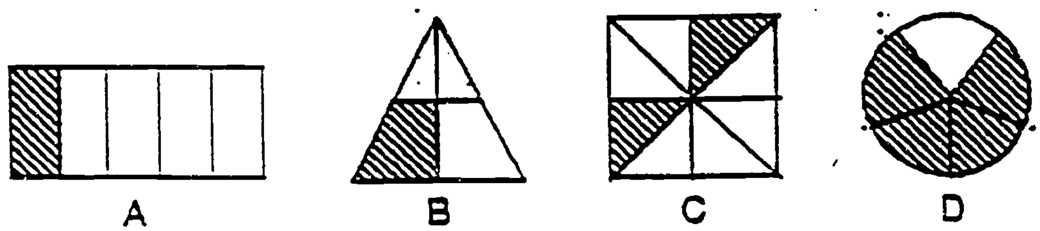
3. variety of lessons and materials to keep interest and excitement at a high level
4. diagrams on overhead should be made up ahead of time to ensure accuracy
5. more focus on problem solving, especially student-generated
6. teachers should have an opportunity to view the first videos fairly early on in the project
7. the teacher should **NOT** have a student teacher doing a final practicum while doing a similar research project
8. more review and practice of concepts needed - to be done on "off days"
9. student Learning Log work needs more careful structure
10. the teacher should have a predetermined organization for what should be recorded in his/her Learning Log
11. there needs to be a constant connection made between the Math being done for the project and all other Math
12. communication links between teachers doing similar projects

It is difficult to say what the long-reaching effects of this project will be. I believe we need to follow the work of these students for the next two or three years in order to determine the extent to which we have had success in their understanding of Math. The intangible rewards, the ones that count the most, are certainly there. When a teacher sees excited, happy students facing her when they've been given Math to do, especially problem solving work, one has to believe we've made some progress.

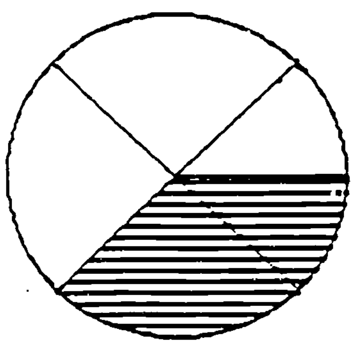
APPENDIX C

Grade 6 Pretest Questions

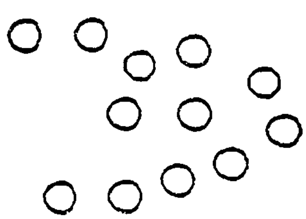
1. Which of the figures has 1/4 shaded?



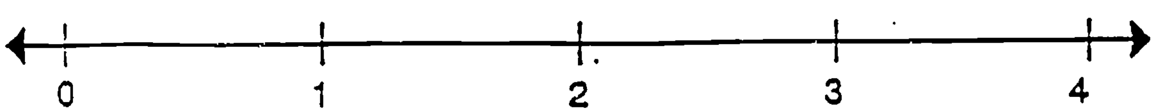
2. What fraction of the circle is shaded?



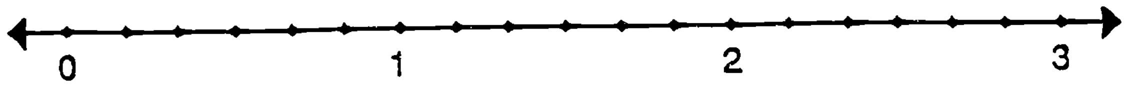
3. Shade 2/3 of the circles.



4. Mark an X on the number line where 3/4 should be.



5. Mark an X on the number line where 1 2/3 should be.



6. Mark an X on the number line where 1 should be.

In questions 7 - 10 write the <u>greatest</u> and the <u>least</u> of the three given fractions.

				Greatest	Least
7.	1/8	1/7	1/6	_____	_____
8.	6/7	8/9	7/8	_____	_____
9.	3/7	4/9	4/5	_____	_____
10.	4/7	3/8	1/2	_____	_____

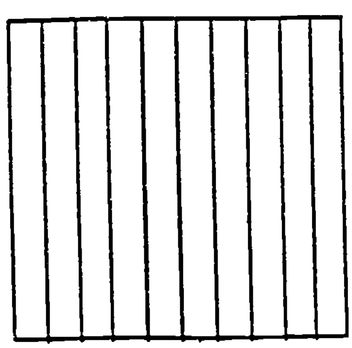
In questions 11 - 13 write a fraction that is <u>somewhere between</u> the two given numbers.

11. 1/3 _______ 3/4

12. 3/5 _______ 4/5

13. 0 _______ 1/7

14. Shade 0.12 of the square.



15. What number is equal to 6 tenths + 3 ones + 4 tens?

16. Which number is equal to 31 tenths?
A. 31.0 B. 3.1 C. 0.31 D. 30.1

17. Round to the nearest hundredth: 0.0984

18. Which is another name for 0.3?
A. 0.03 B. 3.0 C. 0.303 D. 0.300

19. Mark an X on the number line where 0.65 should be.

20. Place the decimal point in the number so that the 7 is in the thousandths place.
4 6 9 3 7 8

In questions 21 - 23 write the greatest and the least of the three given decimals.

				Greatest	Least
21.	0.5	0.47	0.613	____	____
22.	0.004	0.4	0.04	____	____
23.	0.32	0.302	0.3	____	____

In questions 24 - 26 write a decimal that is somewhere between the two given decimals.

24. 0.3 ______ 0.4

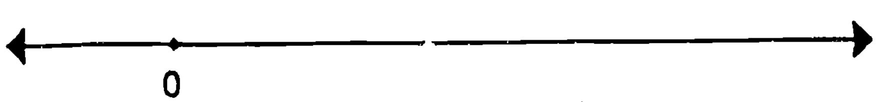
25. 3.7 ______ 3.71

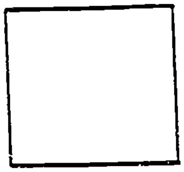
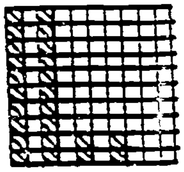
26. 0 ______ 0.1

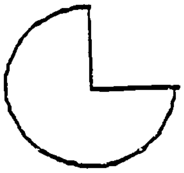
27. Which decimal is equal to 1/5?
A. 0.2 B. 0.15 C. 0.5 D. 0.51

28. Which fraction is equal to 0.75?
A. 5/7 B. 7/5 C. 3/4 D. 7 1/5

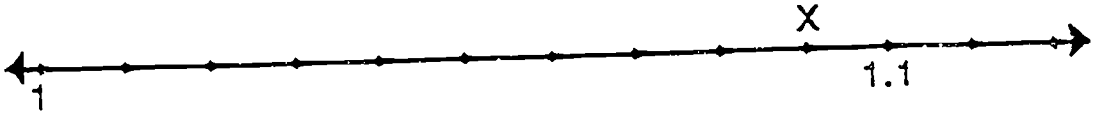
29. Mark this line to show 2/3.



30. If  is one unit, what decimal number is represented by  ?

31. If  is one unit, what fraction is  ?

32. Write a decimal fraction for the point marked X.



33. Choose the best estimate for the answer to 12/13 + 7/8.
A. 1 B. 2 C. 19 D. 21

34. Estimate the answer. Then place the decimal point in the given "answer".
7.342 X 0.5 = 3 6 7 1

In questions 35 - 42 show all your work in the space provided.

35. Jack ate 1/2 of a cake and Mary ate 1/3 of the same cake. What fraction of the cake was eaten?

36. A carton contained 1 1/4 L of milk. Jill drank 1/2 L. How much milk was left in the carton?

37. 1/2 of the students rode to school. 1/4 of the students who rode came by bus. What fraction of the students came by bus?

38. Joe bought 2 kg of ground beef. He repackaged it in 1/3 kg containers. How many packages did he have?

39. Beth walked 0.2 km to a store and 0.34 km from the store to school. What is the total distance she walked?

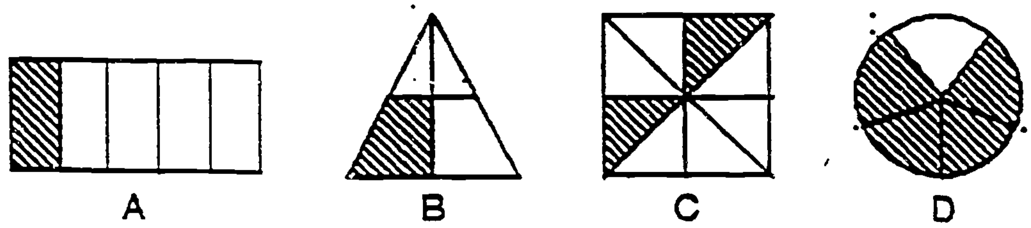
40. Victoria received 3 cm of rain on Tuesday and 1.2 cm of rain on Wednesday. How much more rain fell on Tuesday than on Wednesday?

41. A variety of bamboo grows 0.3 m each day. How much does it grow in 5 days?

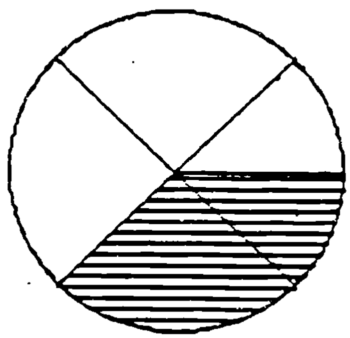
42. A rope measures 4.2 m. Alan wants to cut it into 3 equal pieces. How long should each piece be?

Grade 6 Posttest Questions

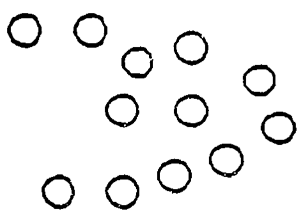
1. Which of the figures has 1/4 shaded?



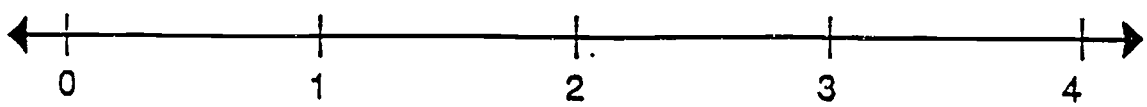
2. What fraction of the circle is shaded?



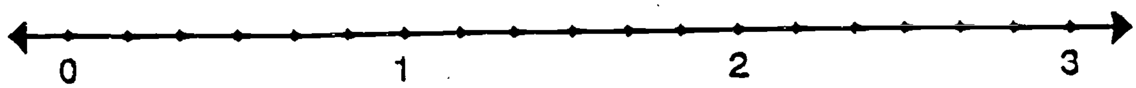
3. Shade 2/3 of the circles.



4. Mark an X on the number line where 3/4 should be.



5. Mark an X on the number line where 1 2/3 should be.



6. Mark an X on the number line where 1 should be.

In questions 7 - 10 write the <u>greatest</u> and the <u>least</u> of the three given fractions.

				Greatest	Least
7.	1/8	1/7	1/6	_____	_____
8.	6/7	8/9	7/8	_____	_____
9.	3/7	4/9	4/5	_____	_____
10.	4/7	3/8	1/2	_____	_____

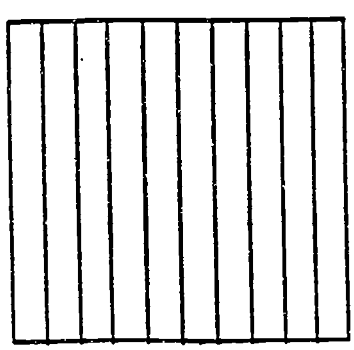
In questions 11 - 13 write a fraction that is <u>somewhere between</u> the two given numbers.

11. 1/3 _______ 3/4

12. 3/5 _______ 4/5

13. 0 _______ 1/7

14. Shade 0.12 of the square.



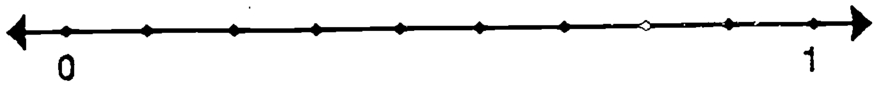
15. What number is equal to 6 tenths + 3 ones + 4 tens?

16. Which number is equal to 31 tenths?
A. 31.0 B. 3.1 C. 0.31 D. 30.1

17. Round to the nearest hundredth: 0.0984

18. Which is another name for 0.3?
A. 0.03 B. 3.0 C. 0.303 D. 0.300

19. Mark an X on the number line where 0.65 should be.



20. Place the decimal point in the number so that the 7 is in the thousandths place.
4 6 9 3 7 8

In questions 21 - 23 write the greatest and the least of the three given decimals.

				Greatest	Least
21.	0.5	0.47	0.613	______	______
22.	0.004	0.4	0.04	______	______
23.	0.32	0.302	0.3	______	______

In questions 24 - 26 write a decimal that is somewhere between the two given decimals.

24. 0.3 ______ 0.4

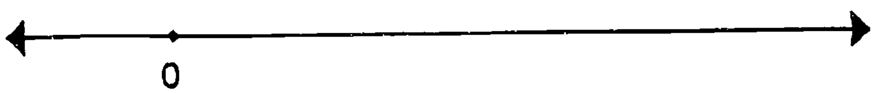
25. 3.7 ______ 3.71

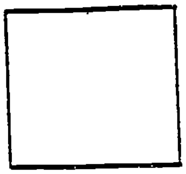
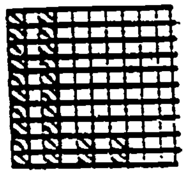
26. 0 ______ 0.1

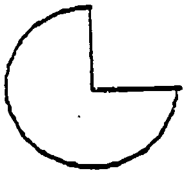
27. Which decimal is equal to 1/5?
A. 0.2 B. 0.15 C. 0.5 D. 0.51

28. Which fraction is equal to 0.75?
A. 5/7 B. 7/5 C. 3/4 D. 7 1/5

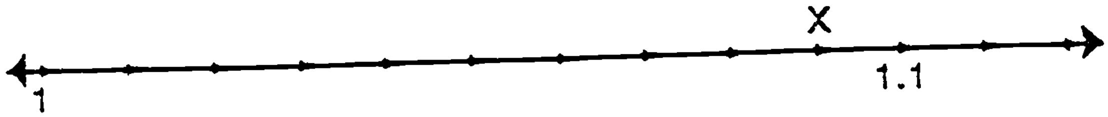
29. Mark this line to show 2/3.



30. If  is one unit, what decimal number is represented by  ?

31. If  is one unit, what fraction is  ?

32. Write a decimal fraction for the point marked X.



33. Choose the best estimate for the answer to 12/13 + 7/8.
A. 1 B. 2 C. 19 D. 21

34. Estimate the answer. Then place the decimal point in the given "answer".
7.342 X 0.5 = 3 6 7 1

In questions 35 - 42 show all your work in the space provided.

35. Jack ate 1/2 of a cake and Mary ate 1/3 of the same cake. What fraction of the cake was eaten?

36. A carton contained 1 1/4 L of milk. Jill drank 1/2 L. How much milk was left in the carton?

37. 1/2 of the students rode to school. 1/4 of the students who rode came by bus. What fraction of the students came by bus?

38. Joe bought 2 kg of ground beef. He repackaged it in 1/3 kg containers. How many packages did he have?

39. Beth walked 0.2 km to a store and 0.34 km from the store to school. What is the total distance she walked?

40. Victoria received 3 cm of rain on Tuesday and 1.2 cm of rain on Wednesday. How much more rain fell on Tuesday than on Wednesday?

41. A variety of bamboo grows 0.3 m each day. How much does it grow in 5 days?

42. A rope measures 4.2 m. Alan wants to cut it into 3 equal pieces. How long should each piece be?

In questions 43 - 44 write the largest and the smallest of the three given numbers.

				Largest	Smallest
43.	7/10	0.68	4/5	_____	_____
44.	0.3	3/8	1/3	_____	_____

In question 45 - 46 write a number that is <u>somewhere between</u> the two given numbers

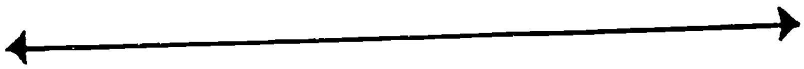
45. 3/8 ______ 0.6

46. 1/100 ______ 0.1

47. a. Write the mixed number 1 3/4 as a common fraction. ______

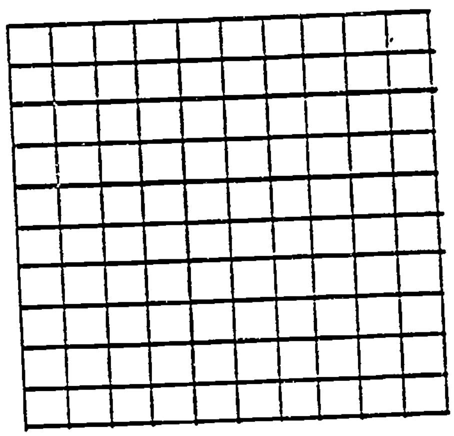
b. Draw a picture to explain the common fraction name for 1 3/4.

c. Mark a number line to explain the common fraction name for 1 3/4.



48. Draw a picture to explain why 0.2 = 0.20. Write a sentence to explain your picture.

49. a. Shade 0.6 of the flat.



b. Write 0.6 in fifths. ____

c. Write five more common fraction names for 0.6.

____ ____ ____ ____ ____

d. Write five more decimal fraction names for 0.6.

____ ____ ____ ____ ____

Grade 6 (Brighouse Elementary) Pre and Posttest Results, 1991 (by Student)

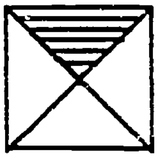
Name	Pretest	Posttest	Gain
1. Adam	12	31	19
2. Adrian	31	48	17
3. Amrit	52	83	31
4. Andrea	17	50	33
5. Anita	24	56	32
6. Carmen	29	48	19
7. Carmen Luisa	38	59	21
8. Chad	21	35	14
9. Cindy	45	89	44
10. David	21	70	49
11. Elexis	45	61	16
12. Francine	24	63	39
13. Greg	48	46	-2
14. Justin	67	80	13
15. Karen	36	74	38
16. Keegan	50	83	33
17. Lance	50	72	22
18. Richelle	31	52	21
19. Rosario	19	67	48
20. Shannon	52	80	28
21. Stephen	71	91	20
22. William	7	15	8
Mean Score	35.9	61.5	25.6 (71.3%)
Standard deviation	16.9	19.4	

Grade 4/5 Common Fraction Review Test Questions

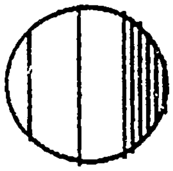
Review

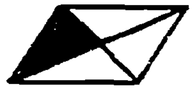
Name: Grade:

1. Which of the shaded parts show one fourth?

a) 

b) 

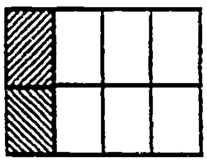
c) 

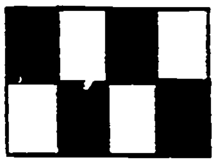
d) 

e) 

2. If □ shows one fifth, draw 2 different pictures for the whole unit.

3. Give 2 different common fraction names for the shaded parts





a) ______

______

b) ______

______

4. Write 3 different common fraction names for one whole unit.

__________ __________ __________

5 Write the common fraction number for each number name

a. 3 fourths _____ b) 5 tenths _____

c. 7 thirds _____ d) 9 fifths _____

6. Write as common fractions or mixed numbers

a. 7/5 ______ b) 3/2 ______

c. 9/4 ______ d) 12/10 ______

e. 1 10/12 ______ f) 5 1/2 ______

7. Circle the larger common fraction in each pair.

a. 1 tenth 1 hundredth

b. 1 fourth 1 eighth

c. 1/2 1/3

d. 3/5 4/5

e. 12/10 7/10

f. 10/10 12/10

g. 3/4 6/4

8. Arrange the three fractions in order from smallest to largest:

a. 1/2, 1/4, 1/3 ____ , ____ , ____

b. 2/8, 2/10, 2/12 ____ , ____ , ____

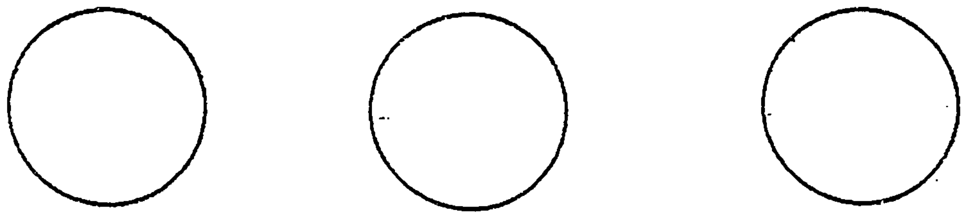
c. 5/6, 3/6, 1/6 ____ , ____ , ____

d. 5/12, 9/12, 7/12 ____ , ____ , ____

e. 6/5, 1/10, 1/100 ____ , ____ , ____

9. Three pizzas are shared equally among 8 children.
a. Draw a picture to show how much each child would get.
b. Write the fraction name for each person's share.
c. Explain in your own words how you worked out the answer.

Picture:



Fraction name:

Explanation:

Grade 4/5 Common Fraction Review Test Results Summary (May 13, 1991)

There were nine questions altogether. The topics tested and the item score (total score of the 26 students) for each are listed below:

Question #	Topic	Score	%
1	Identifying a fourth of a given region	40	80
2	Drawing the whole unit, given a part	37	74
3	Equivalent fractions	66	66
4	Different names for one whole unit	61	81
5	Writing common fraction symbol, given word name	96	96
6	Mixed numbers	128	85
7	Identifying the larger fraction in each pair of given fractions	153	87
8	Ordering common fractions	111	89
9	Word problem (dividing pizzas)	130	74
	Average score (%)		82

Mean score = 33 Standard deviation = 5.5

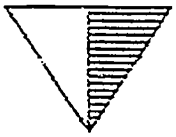
Maximum score = 40 # of students = 25

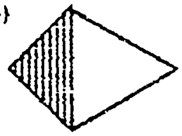
Except for the topic "equivalent fractions", where students scored 66%, students scored more than 70% for the other eight topics. The average score of 82% for the review test on common fractions indicates that Division 6 has good knowledge of common fractions.

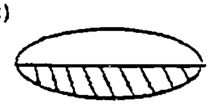
Grade 4 Pretest and Posttest Questions

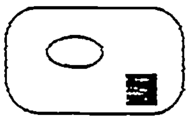
For questions 1, 2, and 3 answer by circling the letters a to h.

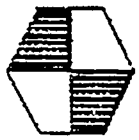
1. Which shaded parts show the fraction one half?

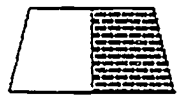
a) 

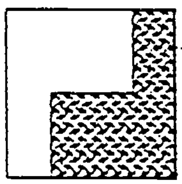
b) 

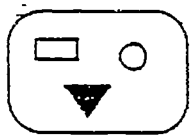
c) 

d) 

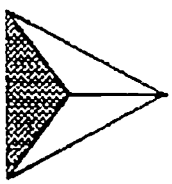
e) 

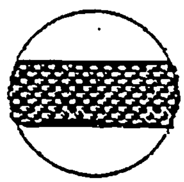
f) 

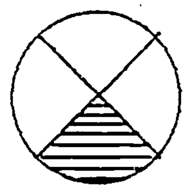
g) 

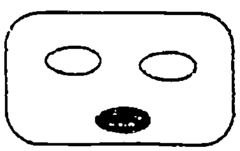
h) 

2. Which shaded parts show the fraction one third?

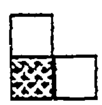
a) 

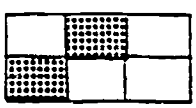
b) 

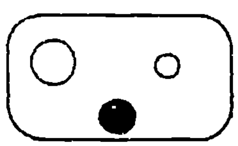
c) 

d) 

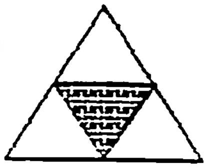
e) 

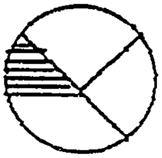
f) 

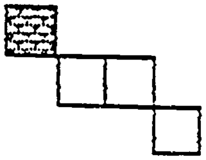
g) 

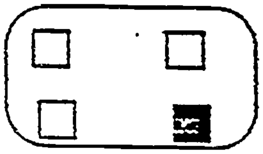
h) 

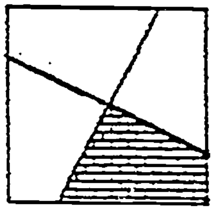
3. Which shaded parts show the fraction one fourth?

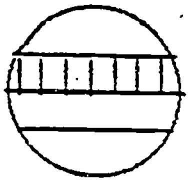
a) 

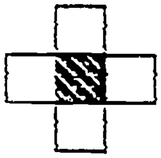
b) 

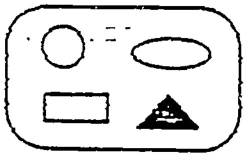
c) 

d) 

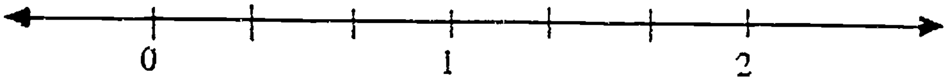
e) 

f) 

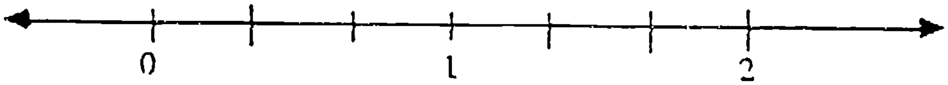
g) 

h) 

4. a) Mark one third on the number line given below.



b) Mark four thirds on the number line given below.



5. Write the common fraction symbols for the following:

a) one half b) one third c) one fourth d) one sixth

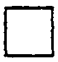
e) one tenth f) four twelfths g) one hundredth h) three fifths

6. Arrange the following from smallest to biggest:

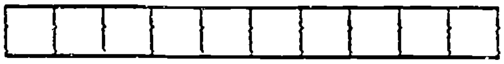
a) one sixth, one eighth, one fotfourths ____________, ______________, ______________

b) three sixths, two thirds, three twelfths ____________, ______________, ______________.

7. Adam and Cheryl share a granola bar. Adam eats half of it and Cheryl eats one fourth of it. How much of the granola bar do they eat altogether? If you need to, use a drawing to help you work out the answer.

8. If  is one fourth of a whole unit, draw two different diagrams to show the whole unit.

9. Using the rectangle divided into ten equal parts as the whole unit, shade in the decimal fractions shown.



a) 0.5

b) 0.3

10. If the rectangle divided into ten equal parts is the whole unit, write the common fraction and also the decimal fraction for the shaded parts.

a) common fraction ______

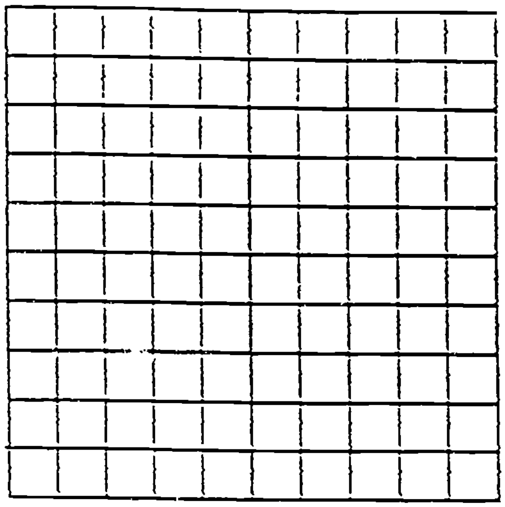
decimal fraction ______

b) common fraction ______

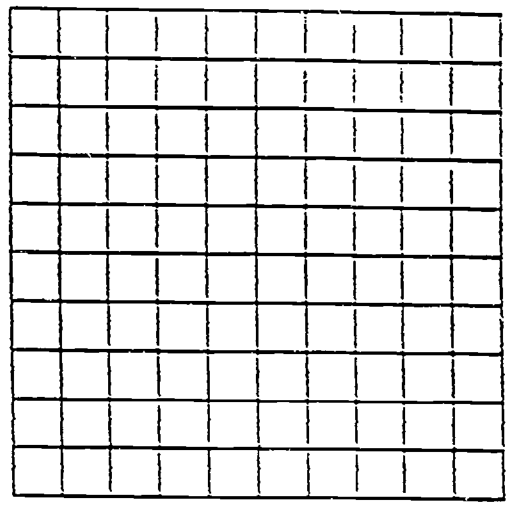
decimal fraction ______

11. Using the square divided into 100 equal parts as the whole unit, shade in the decimal fractions shown.

a) 0.25



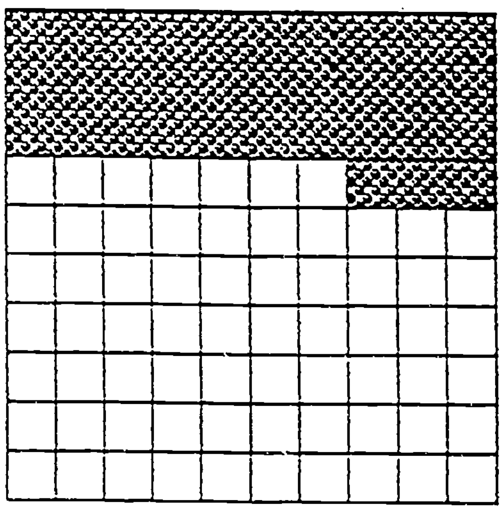
b) 0.4



12. If the square divided into 100 equal parts is the whole unit, write the common fraction and also the decimal fraction for the shaded parts.

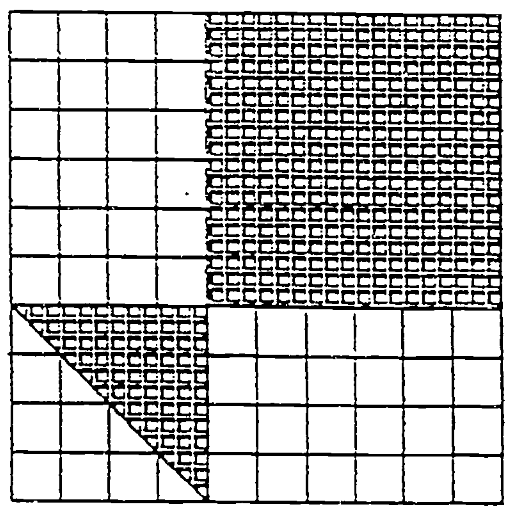
a) common fraction ____

decimal fraction ____



b) common fraction ____

decimal fraction ____



13. Add or subtract the following:

a) 1.23 + 1.89

b) 2.39 + 1.8

c) 0.9 + 3.1

d) 3.23 - 1.79

e) 4.31 - 1.6

f) 0.7 - 0.58

14. One third of a Division Four class is made up of girls. There are altogether 24 children in the class. How many are a) girls? b) boys? Show your work below. You may use a diagram or picture to explain your work.

15. A litre of gasoline cost 59.4 cents in February. In March it cost 39.9 cents. How much more did it cost in February?

16. One half can also be written as 0.5 or 0.50. Write the following as common fractions, and also as decimal fractions, in tenths and hundredths:

a) one fourth ______, ______, ______ b) two fifths ______, ______, ______

17. Arrange the following in order, from smallest to largest:

a) 0.9, 0.10, 1.0 ______, ______, ______ c) 0.3, 0.27, 1.2 ______, ______, ______

b) 1.08, 8, 1.1 ______, ______, ______

18. Make a common fraction question of your own which is different from the ones you have already done in this test. Then answer your own question, showing all the work.

19. Make a decimal fraction question of your own which is different from the ones you have already done in this test. Then answer your own question, showing all the work.

Grade 4 (Division 6)   Fraction Pre and Post Test Results (by Topic)

There were nineteen questions altogether. The topics tested, the maximum score per item and the mean item scores for the pretest and the posttest are listed below:

Question	Topic	Max.	Pre	Post
1	Identifying a half, given some regions and a set of objects	6	4.4	4.7
2	Identifying a third, given some regions and a set of objects	6	3.3	4.3
3	Identifying a fourth, given some regions and a set of objects	5	2.8	3.4
4	Identifying fractions on a number line	2	0	0.3
5	Writing common fraction symbol, given the word name	8	1.0	7.8
6	Ordering common fractions	6	1.5	5.3
7	Word problem (addition of common fractions)	8	2.7	5.1
8	Drawing the whole unit, given a part	2	0.7	1.3
9	Shading regions, given the decimal fraction symbol (tenths)	2	1.1	2.0
10	Writing common and decimal fractions, given the shaded region (tenths)	4	1.0	3.8
11	Shading regions, given the decimal fraction symbol (hundredths)	2	0.6	1.3
12	Writing the common and decimal fraction, given the shaded region (hundredths)	4	0.4	2.8
13	Addition and subtraction of decimal fractions	6	1.8	2.9
14	Word problem (discrete set)	8	1.4	4.1
15	Word problem (decimal subtraction)	4	1.3	3.0
16	Equivalent fractions (common and decimal)	6	0.3	2.3
17	Ordering decimal fractions	9	1.5	3.2
18	Construction of common fraction word problem by students	6	0.5	4.2
19	Construction of decimal fraction word problem by students	6	0	4.0

Grade 5 (Division 6)   Fraction Pre and Post Test Results (by Topic)

There were nineteen questions altogether. The topics tested, the maximum score per item and the mean item scores for the pretest and the posttest are listed below:

Question	Topic	Max.	Pre	Post
1	Identifying a half, given some regions and a set of objects	6	4.0	4.5
2	Identifying a third, given some regions and a set of objects	6	4.1	4.1
3	Identifying a fourth, given some regions and a set of objects	5	2.6	3.0
4	Identifying fractions on a number line	2	0	0.3
5	Writing common fraction symbol, given the word name	8	4.8	8.0
6	Ordering common fractions	6	1.2	4.4
7	Word problem (addition of common fractions)	8	2.6	5.7
8	Drawing the whole unit, given a part	2	0.2	1.5
9	Shading regions, given the decimal fraction symbol (tenths)	2	1.7	2.0
10	Writing common and decimal fractions, given the shaded region (tenths)	4	1.4	3.9
11	Shading regions, given the decimal fraction symbol (hundredths)	2	0.8	1.6
12	Writing the common and decimal fraction, given the shaded region (hundredths)	4	0.4	3.1
13	Addition and subtraction of decimal fractions	6	2.0	3.0
14	Word problem (discrete set)	8	2.4	3.9
15	Word problem (decimal subtraction)	4	1.7	2.7
16	Equivalent fractions (common and decimal)	6	0.6	2.3
17	Ordering decimal fractions	9	3.5	4.9
18	Construction of common fraction word problem by students	6	0.4	3.7
19	Construction of decimal fraction word problem by students	6	0.1	2.4

Grade 4. Fraction (Pre and Post) Test Results (by Students)

Name	Pretest	Posttest	Gain
1. B, Jodi	22	74	52
2. C, Sandy	25	61	36
3. C, Fraser	13	53	40
4. C, Dallas	18	51	33
5. D, Ainsley	42	79	37
6. H, Travis	39	79	40
7. H, Myles	15	52	37
8. J, Kristine	17	51	34
9. J, Derrick	10	61	51
10. M, Treva	16	50	34
11. M, Lyndsay	48	88	40
12. M, Adam	28	77	49
13. N, Tyler	11	70	59
14. P, Stephen	36	63	27
15. S, Ian	24	68	44
16. Y, Roberta	58	72	14
Mean	26.4	65.6	39.2 (148.5%)
Standard deviation	13.9	11.7	

Grade 5. Fraction (Pre and Post) Test Results (by Students)

Name	Pre	Post	Gain
1. C, Peter	56	93	37
2. E, Jayson	36	85	49
3. G, Linda	29	48	19
4. M, Steven	27	49	22
5 P, Ruzed	59	88	29
6. S, Ramandeep	22	51	29
7. S, Patricia	33	55	22
8. S, James	12	72	60
9. S, Colleen	35	42	7
10. T, Nathaniel	31	63	32
Mean	34	64.6	30.6 (90%)
Standard deviation	13.5	17.7	

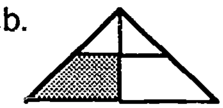
APPENDIX D

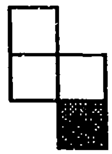
Grade 6 Interview Questions

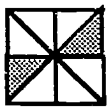
Interview 1

1. Draw a picture to show three fourths.
2. Which of these diagrams show one fourth? (Key: C, D, F, H)

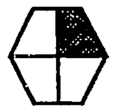
a. 

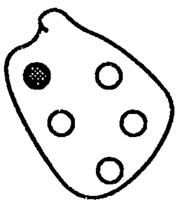
b. 

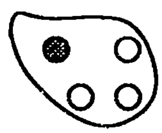
c. 

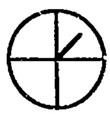
d. 

e. 

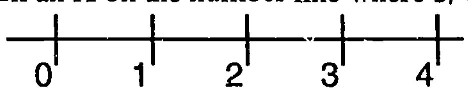
f. 

g. 

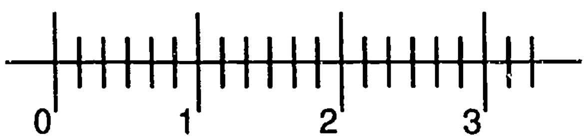
h. 

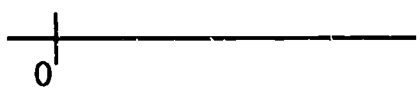
3. What fraction of this circle is shaded? 

4a. Mark an X on the number line where 3/4 should be.



4b. Mark an X on the number line where 1 2/3 should be.



4c. Mark this line to show 2/3 

5a1. Which one of these fractions is the greatest? least? 4/8, 4/7, 4/6

5a2. Which one of these fractions is the greatest? least? 3/7, 2/7,5/7

5b1. Can you think of a fraction between 1/3 and 3/4?

5b2. Can you think of a fraction between 3/5 and 4/5?

5b3. Can you think of a fraction between 0 and 1/7?

6. Which answer is closest to 12/13 + 7/8? A. 1 B. 2 C. 19 D. 21

7. 1/2 of the students rode to school. 1/4 of the students who rode to school came by bus. What rraction of all the students came by bus?

8a. Can you shade in 0.12 (12 hundredths)?

8b. Can you draw 1.3? (One point three)

9a. Which is the greatest? least? 0.5, 0.47, 0.613

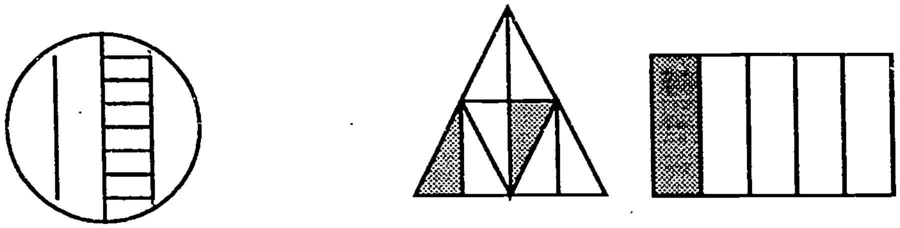
9b. Are these numbers same or different? 0.3, 0.03, 0.30

10 a. Mark 0.65 on the # line.

10 b. Find 1 on this # line.

Interview 2

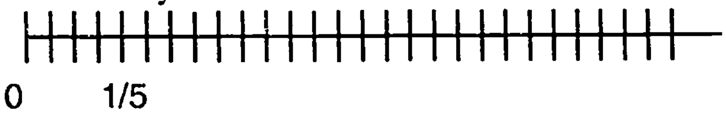
1. Is one fourth of each diagram shaded?



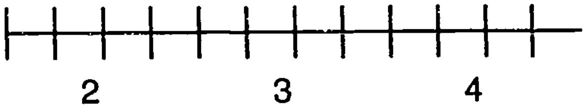
2a. [Student is showed a fraction strip divided into four equal parts] If this is two fifths, show me three fifths.

b. How much is a beige 12 strip, in terms of the above (two fifth) strip?

3. Show where 3 fifths would come on this number line.
Show where 1 tenth would come on this number line.
Show where 7 tenhs would come on this number line.
Show where one would come on this number line.
Is it easier to work with the 1st (marked) number line or the 2nd (unmarked) number line? Why?

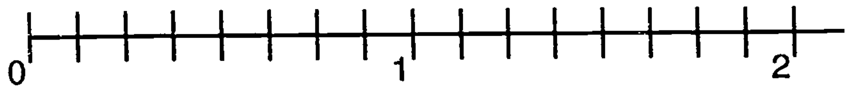


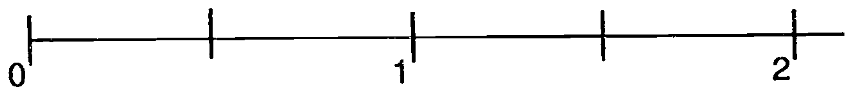
4. Show where one and 2 fourths would come on this number line.
Show where 3 and 1 half would come on this number line.
Show where 4 and 1 fourth would come on this number line.



5a. Show where 0.5 would be on this (1st) number line.

b. Show where 0.5 would be on this (2nd) number line.





6a. Show where 0.7 would be on this (1st) number line.
Show where one fourth would be on this (1st) number line.
Show where one and one half would be on this (1st) number line.
Which is the smallest: 0.7, one fourth or one and one half? Why?

6b. Show where 0.7 would be on this (1st) number line.
Show where one fourth would be on this (1st) number line.
Show where one and one half would be on this (1st) number line.
Which is the smallest: 0.7, one fourth or one and one half? Why?

6c. Can you write one and one half as a decimal fraction?

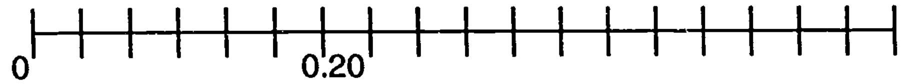
7a. This "long" is one tenth. Show me one whole unit.

b. How much is one small cube?

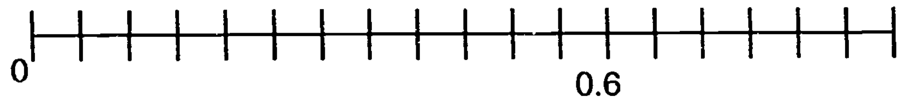
Interview 3

1. On the number lines given, mark a) 4/10 b) 20/100 c) 2 d) 0.15 e) 1.4 f) 70/100

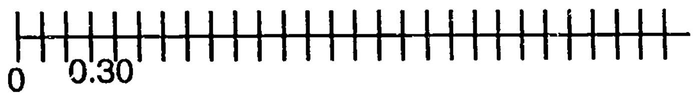
a.



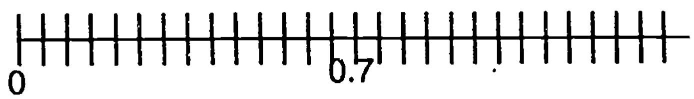
b.



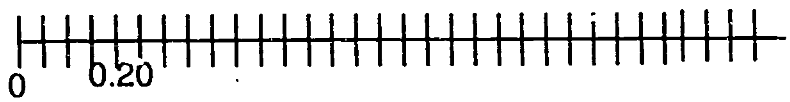
c.



d.



e.



2a. Give 5 different names for the shaded part
How much is one of the smallest ones (rectangles)?
How many thousandths are shaded?

2b. How much of the diagram is shaded?

3a. Put these in order: 1/9, 0.1, 1/12. Which is the smallest?

3b. Which of these is closest to one? 8/9, 11/12, 0.9

3c. Order these: 36/100, 1.23, 1.9, 1.007, 0.997, 9/10, 24,1, 4, 0.125

4. Which is larger, 0.7 or 0.47? How do you know?

5. [With a metre rule provided].

a. Where is one half? Some other names for one half? Some names for 1/2 metre?

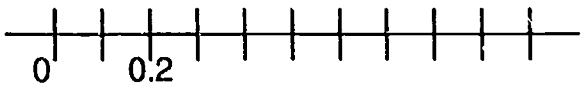
b. Where is half of a half? What are some other names for it?

c. Where is half of a half of a half? What are some other names for it?

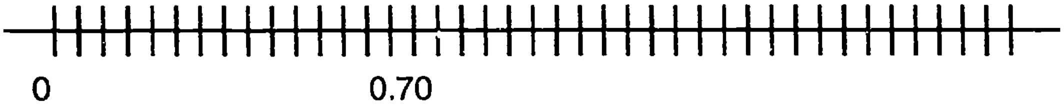
Interview 4

1. Can you show me where one-eighth of the meter stick would be?
   What is a decimal fraction name for one-eighth?
   Can you show me where five-eighths of the meter stick would be?
   Can you give some different names for 1/8 and 5/8?
2. (Referring to the "flat" marked in thousandths)
   Please shade one-eighth of the flat. Give some other names for one-eighth.
   Please shade three-eighths of the flat. Give some other names for three-eighths.
3. (Referring to actual balls of different sizes)
   Can you show me one-eighth of the balls? But the balls are of different sizes! Does it matter?
   Can you show me three-eighths of the balls?
4. (Translating fraction word problem involving tenths greater than one and multiplication into diagram and solving it; may use fraction strips, "longs" etc.)
   A bamboo plant grows 3 tenths of a meter each day.
   a. How many tenths of a meter would it grow in 7 days?
   b. Draw a diagram to explain your solution.
   c. Write the results in decimal and common fraction form.
5. (Translating fraction word problem involving fourths, eighths and addition into diagram and solving it; may use fraction strips, longs etc.)
   Wes ate 1/4 of a granola bar and Elexis ate 3/8 of the same granola bar.
   a. How much of the granola bar was eaten? Use a diagram such as the one below or some materials to help you solve this problem.

6 a. Show and mark where 0.15 would be on the following number line:



b. Show and mark where 1.8 would be on the following number line:



Interview 5

1. Multiply : 0.4 x 0.2

2. Anita walks 0.7 km a day. How many kilometres would she have walked in 5 days?

3. Wes had a calculator which would NOT display decimal points. Help Wes place the decimal point in the "answer" displayed by the calculator for the multiplication below :

$$7.342 \times 0.5 = 3\,6\,7\,1$$

4. A teacher paces up and down a classroom. Each pace is 0.74 metres in length. How many paces will she make in walking the full length of the classroom, 9.2 metres? Choose from the answers below. {There may be more than one correct answer.}

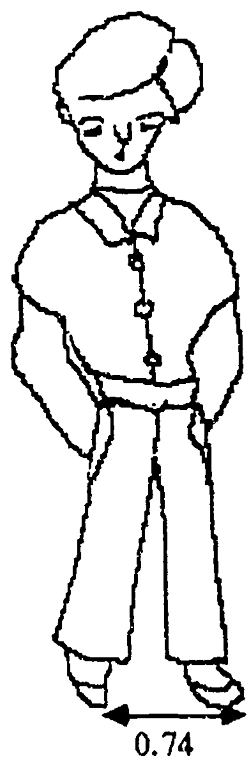
a) 0.74×9.2

b) $0.74 \div 9.2$

c) $9.2 \div 0.74$

d) $\frac{9.2}{0.74}$

e) $\frac{0.74}{9.2}$



5. To make a wallet size picture of Michael Jackson from a poster, it has to be decreased to 0.052 [redacted] of its original height. If the poster of Michael Jackson was 1.2 metres high, what will the height of the picture be? Choose from the answers below. {There may be more than one correct answer.}

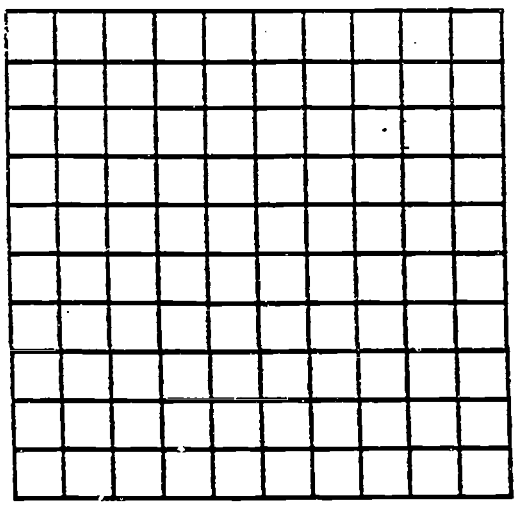
a)	0.052×1.2	c)	$1.2 \div 0.052$	e)	$\frac{0.052}{1.2}$
b)	$0.052 \div 1.2$	d)	$\frac{1.2}{0.052}$		

6. Steve spent $ 900 for 0.75 kg of platinum. What would be the price of 1 kg bar of platinum? Choose from the answers below. {There may be more than one correct answer.}

a)	0.75×900	c)	$900 \div 0.75$	e)	$\frac{0.75}{900}$
b)	$0.75 \div 900$	d)	$\frac{900}{0.75}$		

7. On a highway a four - wheel drive car can go 7.5 km on each litre of gasoline. How many kilometres can the car be expected to go on 1.3 litres?

8. Shade the diagram below to show 0.3 of 0.2



Fill in the blank : 0.3 of 0.2 = ________________

Interview 1

Grd 6, 1991 (21 & 23 Jan, 1991)

Question 1: Draw a picture to show three fourths.

Jan 21,1991.

Cindy: Draws circular region, divides into 4 (equal) parts, and shades in 3 parts.

Lance: Same as Cindy, except that he used a square rather than a circle.

Richelle: Same as Lance

David: Same as Cindy

Jan 23, 1991.

Justin: Same as Cindy

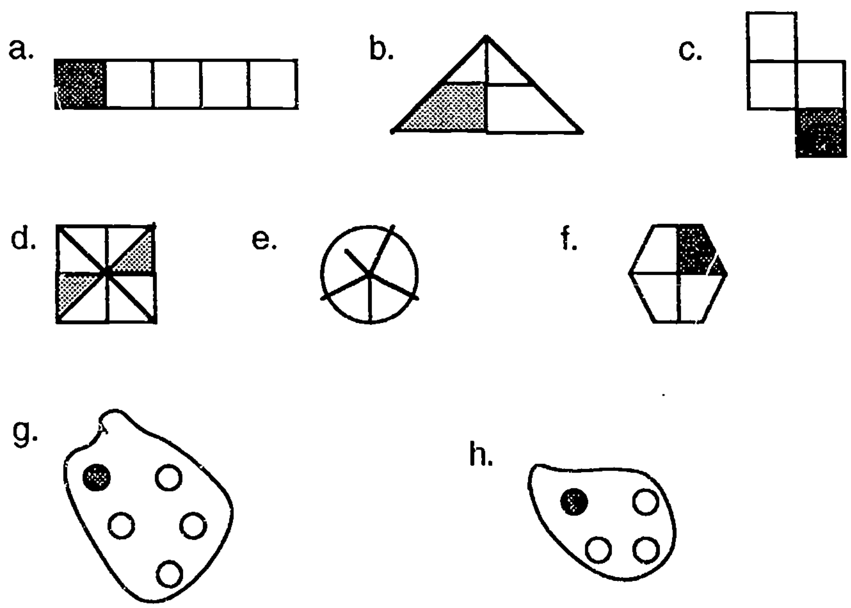
Francine: Same as Cindy

Adrian: Same as Cindy

Andrea: Drew 4 small circles, coloured in 3 (Only one to use discrete case)

Steven: Didn't ask him the question, as already "covered # 1 in class".

Question 2: Which of these diagrams show one fourth? (Key: C, D, F, H)



Cindy: B, C, F, H. Ignored different sized parts (B, H)

Lance: C, D, F, H. "If you took off the lines", for D; not B because different sized parts; for H, doesn't matter if sizes different.

Richelle: C, F, H initially, & then after was asked whether she could see the 2/8 as 1/4, agreed to D as well. Similar reasons as Lance, for H & for not choosing B.
David: C, F, H. Thought all same size in H. Didn't choose B because not equal parts. Didn't choose D because "8 parts, no 4ths". Agreed it might be 1/4 if the lines are erased, but was unsure whether that would be correct.
Justin: C, B, F, H. Ignored differentt sized parts in B and H. D was not included because it was 2/8, not fourths. Agreed that you could make it 1/4 by taking out "the cross marks".
Francine: B, C, H, F, and later added in D. Then cancelled B (& also H) later, after deciding parts have to be same size.
Adrian: C, B, E, F, H. Was unsure about B later. Afterwards, cancelled E, "not 1/4 because 5 pieces..and so 1/5". Maintained H, even though agreed size not same. Said D couldn't be 1/4.
Andrea: C, D, F, H. Chose D because "these 2 halves go together as one". Didn't choose B, because different sizes, but felt size didn't matter for H.
Steven: C, D, F, H. Chose D because "2/8 = 1/4; if you put these side by side, you get 1/4" Said he didn't need a picture to see that 2/8 = 1/4. Chose H, in spite of different size, because "it's not one shape altogether".

Question 3: What fraction of this circle is shaded?
Cindy: Didn't know at first, but agreed it could be 3/8 "if we don't put the cross in", but was unsure.
Lance: Immediately drew 2 additional lines to indicate 8ths and wrote 3/8.
Richelle: Saw the fourths, but was unsure what fraction the "small part" was, although did say it was "a half of one of these (fourths)", and said it was "1.5 fourths". After the question "If you could draw some more lines,..."got 3/8. When asked "Does drawing more lines change the value or the way you look at it?", said "The way I look at it".
David: 1 and a half fourths. When asked "Any other way to se it?", put in more lines and got 3/8. Agreed 1 and half fourths is 3/8.
Justin: Wrote $\frac{1\frac{1}{2}}{4}$, saying "That's 1/4 and half more is shaded in". Also suggested $\frac{1.5}{4}$, but said "that's decimal fractions". Couldn't see 3/8.
Francine: "One and a half fourths, but if you make more lines, it could be 3/8".

Adrian: Wrote $1\frac{1}{2}$, and said "There are 4 pieces and each piece is a one", then changed to "each piece is a half", and later "0.5 and 0.5 equals up to half". Didn't get 3/8. Difficult to follow his reasoning.

Andrea: Wrote $\frac{1\frac{1}{2}}{4}$, and couldn't see any other way of doing it.

Steven: Immediately said 3/8, and said "one half of a quarter is 2/8". Also said he needn't physically draw in the lines, that he could do it in his head.

Question 4a: Mark on the # line where 3/4 should be.

Cindy: Initiallly marked the 3, but after discussion, marked it correctly.

Lance: Marked point 3/4 of the way between 3 and 4. Insisted 3/4 > 1, referring to # line.

Richelle: Didn't mark any point. Said she had trouble with # lines.

David: Marks the point 3, and when asked whether 3/4 > 1, said "Ya, I would have to say that".

Justin: Similar to Cindy

Francine: Divided space between 3 and 4 into 10 parts, counted off 4, and marked her 3.4 as 3/4. When asked how she would know what's larger on a # line, wasn't sure.

Adrian: Divided space between 3 and 4 into 10 parts "in my head", & marked off 1/3 of the way between 3 & 4, as 3/4.

Andrea: Thought for a while and marked a point about 1/3 of the way between 3 and 4.

Steven: Asked "of the whole line or...?". Correctly marked the point, saying "I split it into 4 in my head and picked the one before one".

Question 4b: Where would 1 2/3 be on this # line?

Cindy: First marked 1 and 1/3, then corrected herself. Explained that she used the fact that there were 6 spaces between each marked whole number.

Lance: Put mark in middle of 2nd and 3rd space after 1, but couldn't explain.

Richelle: Marked 1st mark after 1, and explained that she counted 1 for the mark 1, 2 for the 1st mark after 1, and 3 for the 2nd mark after 1.

David: Locates it between 2nd and 3rd space after 1, closer to 2nd space. Could locate 1 and 1/2, but said 1 and 2/3 < 1 and 1/2.

Justin: Indicated correctly, saying "2/3 is about 3/4 ".

Francine: Marked the 2nd mark after 1 (which was actually 1 and 1/3).

Adrian: Initially marked at beginning of 3rd space after 1 (i.e. at 2nd mark), then changed to exactly midway between 1 and 2.
Andrea: Said she didn't understand, hadn't been taught this, but guessed it to be at 1 and 2/6.
Steven: Correctly identifies and marks 1 and 2/3. Used 2 spaces for 1/3.

Question 4c: Mark this line to show 2/3.
Cindy: Marks it correctly.
Lance: Marks middle of 3rd space after 1, but couldn't explain why he did so.
Richelle: Not asked
David: Not asked
Justin: Decided on where to place one whole, and placed 2/3 in relation to the whole.
Francine: Marks off 1, 2, 3, & 4. Then marks off spaces between consecutive whole numbers on # line. Finally marks off 2nd mark after 3 (which actually denoted 3 and 1/3) as 2/3, but couldn't explain why.
Adrian: Considered each space between two consecutive whole numbers as one whole, and therefore talked about 1/3, 2/3 and 3/4 as lying in between 2 and 3, and also in between 1 and 2. What was 2 and 1/3 for us was just 1/3 for him.
Andrea: Not asked.
Steven: Not asked

Question 5a1: Which one of these fractions is the greatest? least? 4/8, 4/7, 4/6
Cindy: Initially says 4/8 greatest, saying "in 4/6 there are bigger boxes". Then says it's hard to tell without drawing, so draws # lines one below the other, saying 4/6 is biggest because "4/8 has smaller boxes & its the same # of boxes", and 4/8 is smallest.
Lance: From greatest to smallest: 4/6, 4/7, 4/8. "If you pick 4/8, is 4 away, 4/7 is 3 away amd 4/6 is 2 away (Subtracted numerator from denominator, or how much left over to make one whole?).
Richelle: Said largest was 4/6 and smallest was 4/8. Explained by referring to two rectangles shaded in to represent these fractions. She had made the (whole) rectangle for 4/8 bigger and had coloured in semmingly similar areas, but maintained 4/6 > 4/8. Didn't seem to understant "one whole".
David: 4/6 greatest because 3/6 = 1/2, so 4/6 > 1/2; least was 4/8, because "8 bars", also 4/8 = 1/2. Said 4/7 > 1/2

Justin: Greatest 4/6. "4/8 may seem bigger because it's got an 8, but it's divided up into smaller parts. Like if you had 6, the 6 would be bigger, because these (# of points) would be less". Smallest was 4/8.
Francine: Drew 2 (unequal) rectangles, one below the other,shading in 4/6 & 4/8, and said 4/8 was biggest and 4/6 was smallest, refering to her diagram.
Adrian: Related the fractions to circles, saying "6 spaces, so colour 4, 4/6, 2 left over". Similarly "3 left over" for 4/7 and "4 left over" for 8. His rule seemed to be "the more left over, the greater", so 4/8 was greatest and 4/6 was smallest.
Andrea: Greatest, 4/8 because "8 > 6".
Steven: Largest was 4/6; 4/8 "is like a half, but 4/6 > 1/2, 4/7 is a little tiny bit over 1/2, so 4/6 os a little bit bigger than 1/2". He knew 3/6 = 1/2

Question 5a2: Which one of these fractions is the greatest? least? 3/7, 2/7,5/7
Cindy: Largest, 5/7 & smallest, 2/7. Similar justification as in question 5a1.
Lance: Not asked this question
Richelle: Correctly ordered these fractions.
David: 5/7 largest, 2/7 least. Didn't ask for explanation
Justin: Same as David's answer.
Francine: Said 5/7 was greatest and 2/7 the smallest.
Adrian: Not asked
Andrea: Largest, 5/7; smallest, 2/7
Steven: Correctly ordered the fractions. Also correctly ordered 6/7, 8/9 and 7/8, using the idea of "missing pieces" to make the whole, e.g "8/9, 1/9 missing".

Question 5b1: Can you think of a fraction between 1/3 and 3/4?
Cindy: Uses # lines to indicate 2/3 lies between 1/3 and 3/4.
Lance: "I don't know".
Richelle: Didn't know how to do this, but made a guess (2/3). Didn't seem to understand meaning of "in between" these fractions.
David: Probably 2/3 because 1<2<3. Also was sure 2/3 < 3/4, and said that it would be a bit smaller.
Justin: 2/4 and drew a diagram to explain. Said 3/4 > 1/4 but < 1, so 1/2 would be between 1/4 and 3/4. When asked why he first said 2/4, and now said 1/2, replied that 1/2 = 2/4.
Francine: 2/3, because "betwen 1 & 3 is 2, and between 3 & 4, you can't do that, so 2/3".

Adrian: 2/3 because "I had to presume 2 at the top, 3 & 4 can't put a 2 there & that will mix it all up".
Andrea: Couldn't
Steven: 1/6, because "1/3 would be 2/6, so if you take away 1/6, so ot would be more than this.., or is it the other way around?". After further discussion, decided on 2/3

Question 5b2: Can you think of a fraction between 3/5 and 4/5?
Cindy: Was not asked this question.
Lance: "I don't know", but wrote $\frac{3\frac{2}{3}}{5}$ on paper.
Richelle: Not asked
David: Not asked
Justin: 3.5 or $\frac{3\frac{1}{2}}{5}$
Francine: Not asked
Adrian: Not asked
Andrea: Not asked.
Steven: 5/10, "changed to 10ths & took away 1/10, no, I should add" , so 7/10

Question 5b3: Can you think of a fraction between 0 and 1/7?
Cindy: Was not asked this question.
Lance: Not asked
Richelle: Not asked
David: Not asked
Justin: Gave a few answers first, including 1/2, 1/2 of 1/7. Then said 1/8, because "when fraction is smaller, divide into smaller parts, more lines".
Francine: Undecided between 1/4 & 1/5, but seemed to favour 1/5. Couldn't explain clearly. Was also unable to find a decimal fraction between 3.7 & 3.71.
Adrian: Not asked
Andrea: Not asked.
Steven: 1/14 because "1/14 is half of 1/7, and that is less than 1/7 but bigger than 0"
Also was asked for fraction between 0.3 and 0.4 , and he came up with 0.35

Question 6: Which answer is closest to 12/13 + 7/8? A. 1 B. 2 C. 19 D. 21
Cindy: Was not asked this question

Lance: Not asked

Richelle: Not asked

David: Not asked

Justin: Chose A, after lengthy deliberation. "19/21, that's not one whole...fraction name for a whole is 8/8, inthis case you have got 21 (so 21/21 = 1)".

Francine: Not asked

Adrian: Not asked

Andrea: Not asked

Steven: Not asked

Question 7: 1/2 of the students rode to school. 1/4 of the students who rode to school came by bus. What fraction of all the students came by bus?

Cindy: Was not asked this question

Lance: Not asked

Richelle: Not asked

David: Not asked

Justin: Tried 1st with 500 students, then "250 rode, that would be quarter of .. but I don't know a quarter of 250". When suggested that 400 would be more suitable to start with, got 200 as 1/2 and 100 each for the remaining quarters. Anyway didn't answer the original question.

Francine: Not asked

Adrian: Not asked

Andrea: Not asked

Steven: Not asked

Question 8a: Can you shade in 0.12 (12 hundredths)?

Cindy: Tries dividing the 10th strip into 12 parts. Thought 0.12 could mean 1/12.

Lance: Shades 0.12 correctly, but couldn't explain place value of the 2, saying it's 2 ones, though he did say the 1 stood for 1 tenth.

Richelle: Couldn't

David: Shades (incorrectly) 1 and a half 10ths strips. Was unsure as to its correctness.

Justin: Shaded it correctly, saying "Oh, we did it in class".

Francine: Drew horizontal lines, to get 100, saying that there were only 10 lines, & then showed 12 hundredths

Adrian: Drew horizontal lines on given diagram and showed 0.12 correctly. Said "coloured in 12, and zero is not coloured". Looks as if he was trying to represent the zero in 0.12 as well!
Andrea: Shaded in 1 and a 1/2 strips, because "that's 1 and a 1/2".
Steven: Shaded a tenth strip, and a vertical slice (about 1/5), adjacent to the tenth strip.

Question 8b: Can you draw 1.3? (One point three)
Cindy: Draws a line, marks 1 and 2, divides space between 1 & 2 into 3 equal parts, & marks the end of 1st space after 1 as 1.3 - indicating 1.3 is 1 and 1/3 for her.
Lance: Initially writes 1/3, then corrects himself. Shades 1.3 correctly.
Richelle: Thought it was 1/3 initiallly. Then shaded one strip of 1/10, saying that ws a whole, and divided adjacent strip into10 equal parts, shading in 3 of them.
David: Correctly draws a rectangle (signifying the whole), and 3 of the 10ths strips.
Justin: Showed it correctly. Also said 30 hundredths = 3 tenths. Then some inconsistency, saying "3 hundredthe same as 3 tenths".
Francine: Said another way of reading 1.3 was 1 and 3/4, and so drew 1 whole and shaded in 3 parts out of 4 parts of another whole.
Adrian: First wrote 1/3, then corrected himself. Then drew a square (5 x 4 grid)and shaded in a 1 x 5 strip for 1.3, indicating he wasn't sure what 1.3 meant.
Andrea: Drew 3 small circles, and shaded in one circle. Looks like 1.3 = 1/3 for her.
Steven: Drew it correctly.

Question 9a: Which is the greatest? least? 0.5, 0.47, 0.613
Cindy: Correctly ordered them, looking at 1st digit after decimal point, of each number.
Lance: 0.613 greatest, because "more numbers", then 0.5 because "can put 0.50 for 0.5" and smallest was 0.47.
Richelle: 0.613 was greatest because "613 .> 47 and also > 5";).5 was smallest because "5 < 47 and also < 613". When asked to consider in terms of 10ths and 100ths, said "I think 47 is smallest, because 4 tenths and 7 hundredths". When asked how many hundredths in 0.5, first said there were zero hundredths, then 50 hundredths, but was not too sure. Anyway, finally agreed 0.47 < 0.5
David: 0.5 largest because "more numbers, gets smaller". Indicated 1000ths are smaller. His rule seems to be "more digits after the decimal point, smaller the number".
Justin: Greatest 0.613, because 0.5 < 0.6. Also said 0.5 > 0.4, and).5 > 0.47.

Francine: 0.613 was greatest because "it's 3 spaces over, so 100 is higher than 10 or 1", and 0.5 was smallest , because "one space".

Adrian: 0.613 "has more numbers, so biggest".

Andrea: Largest was 0.613, because "613 > 5, 613 > 47", and 0.47, in the middle.

Steven: 0.613 is largest, because "6 > 5, & 4 is in the tenths spot".

Question 9b: Are these numbers same or different? 0.3, 0.03, 0.30

Cindy: Said 0.3 = 0.30, but 0.03 was diffferent. Didn't know about place values of 10ths etc, but did say that she could "add zeroes" to maintain equality.

Lance: 0.3 = 0.30, because "doesn't matter if you put zero", but 0.03 was different. (Was also asked to find a number between 3.7 & 3.71, but couldn't).

Richelle: 0.3 = 0.30, but 0.03 was different. Could correctly identify the place values of 3 in 0.3 and 3 in 0.03

David: 0.3 = 0.30, because "you can cross out zeroes". 0.03 was different because "in front of number can't cross out zero".

Justin: Not asked

Francine: 0.03 is different because "only zero at the end matters"

Adrian: Similar to Francine's.

Andrea: All same size for her.

Steven: 0.03 is different, 0.3 = 0.30

Question 10 a: Mark 0.65 on the # line.

Cindy: Initially draws her own # line, and marks off 7 spaces beteen 0 and 1. Then marks 0.65 at what should be 0.55 on her # line. Then uses the given # line and correctly marks 0.65, saying that the mark on her # line was incorrect.

Lance: Initially didn't know, but on being asked to guess, thought for a while and marked it correctly, saying "counted 6, & guess this is 5 (referring to point midway between 0.6 & 0.7)".

Richelle: Got it correct, by counting in hundredths.

David: Correctly marks the point, counting 6 spaces and "5 in the middle". Frequently used the word "probably", perhaps showing lack of confidence.

Justin: Noted that "65 not over 1", counted in hundredths, and marked the point correctly.

Francine: Used the 6 marks as 6 spaces, & put 0.65 between 6th & 7th mark.

Adrian: Counted from zero (above the zero mark), up to 6, and then correctly marked 0.65

Andrea: Correctly marks the point.

Steven: Correctly marks the point, without any hesitation.

Question 10 b: Find 1 on this # line.

Cindy: Marked the point correctly, without any problem.

Lance: "This is 1.1, so this is 1.0"; had no difficulty in marking the point 1.

Richelle: Not asked

David: Not asked

Justin: Correctly marked the point.

Francine: Correctly marked in 1.

Adrian: Correctly marked in 1.

Andrea: Correctly identifies the X as 1,0, but not sure why. (She was asked to identify the point marked X).

Steven: Not asked.

Interview 2 Feb 4 1991

Question 1

Lance: He picked the triangular figure as the one having 1/4 of it shaded because putting the 2 shaded parts together yielded one of the 4 parts of a triangle. When asked about the other figures, he said the circular one has 1/4 of it shaded while the rectangular one has 1/5 of it shaded.

Andrea: She picked the triangular diagram as having 1/4 of it shaded because it has 4 smaller triangles divided into halves with two of the halves shaded, giving 2/8 "halves", or 1/4. She identified the whole triangle as the unit under consideration. She said the shaded part of the circle does not represent 1/4 of the circle because the pieces are not the same and for the rectangular figure, she said it is rather 1/5 of it that is shaded but not 1/4.

Adrian: He said none of the diagrams has 1/4 shaded because the pieces are not equal. For the triangle, he said there are too many parts and that the two shaded parts are smaller than the other 6 parts. For the circle, the pieces are not of the same shape and size. And for the rectangle, the shaded part is 1/5 of the whole. (Note: the student looked very confident.)

Justin: He said the circular figure has unequal pieces, the rectangular one has too many pieces and only the triangular one has 1/4 shaded. When he was asked to explain, he said he could not.

Question 2

Lance: When he was given two fifth strips and asked to pick three fifths from several others, he picked the correct one by counting 3 fifths. He was able to give the length of another strip as 6 fifths by using 2 fifth strips.

Andrea: She had difficulty figuring out what to do. She could not see how a fifth should be considered as "one whole" for her to count "two wholes" as 2/5 and "three wholes" as 3/5. She could not use 2/5 strip to figure out another strip.

Adrian: Given 2/5, he added another piece which he thought to be 1/5 length to give him 3/5. That did not give him 3/5 and when he was challenged on his choice, he pondered for a while and said he did not know what to do.

Justin: Given a 2/5 strip, he was to identify a piece that is 3/5. he added a 1/5 strip to the 2/5 strip and aligned several of the other strips along the two until one of them was equal in length to the other two put together. He then claimed that one piece to be equal to 3/5. Similarly, he was able to identify a piece that was 6/5 in length.

Question 3

Lance: Starting from zero, he counted 5 points to come to the 1/5 mark. He then continued to count 5 points for each 1/5 and then located 3/5 on the 15th point from zero. He could not locate 1/10 on this number line, and for 5/5, he counted 10 more points from where he had earlier on located 3/5. He could not suggest another name for 5/5 but agreed to the interviewer's suggestion that it is also 1.

Andrea: Starting from zero, she counted 5 points which coincided with the 1/5 already located on the number line, so she continued to count in fives for each 1/5 to locate a position for 3/5, even though the divisions between fifths are 4. She could not locate 1/10 on the same number line. As a hint, the interviewer located zero, 1/5, 3/5 and then asked her to locate one whole which she did, but she went ahead to locate 1/10 where 5/5 or one would be located.

Adrian: To locate 3/5, he counted 10 spaces from 1/5. For 1/10, he counted 10 points from zero and even though this coincided with a 2/5 position he had earlier marked, he did not change his mind and still located 1/10 there. to locate a unit given the position of a 3/5, he had to count in fives to a position which he labelled 5/5. He said if he has to locate one unit given the position of 1/10, he will have to count in tens and that the line will have to be extended to make that possible. When he expressed 1 as 10/10 and its position coincided with 5/5, he changed the position of 10/10 and said that it must be further away from 5/5. When he was reminded that 10/10 = 1 just as 5/5 = 1, he said we could have two ones and that the one for 10/10 should be further away from the one for 5/5.

Justin: To locate 3/5 on the given number line, he started counting from zero and represented 5 divisions with a 1/5 and located positions for 2/5 and 3/5. But he changed his mind saying he should have started counting from 1, taking 4 divisions to represent a 1/5, and he correctly located 3/5 on the given number line. To find 1/10 on the same number line, he divided the space between 0 and 1/5 into half and located 1/10 there. When he was asked to locate 1 on the same number line, he did that correctly by counting to 5 fifths. For 7/10, he counted from 1/10 and located it correctly.

<u>Question 4</u>

Lance: She said she would not be able to locate one and 2 fourths on the given number line unless the line was renumbered. When pressed for some other way of doing it, she said she did not have any.

Andrea: She located one and two fourths at the end of the given number line. She was then asked to identify the numbers marked on the line and the spaces between them and she did that correctly. When she was then asked to identify 1 2/4, she hesitated, identified 1 first and then later 1 2/4.

Adrian: He said he did not know if 1 2/4 is a mixed number. He was asked to find how many division s there are between 2 and 3 and he said 4. He was then asked to indicate what fraction each division is and he first said 2/4, but later changed it to 1/4. He gave up at this stage saying that he could not do it.

Justin: He started counting from 2 and located the position of 2 1/4. He was then asked to locate the position of 1 1/2 if he started counting from zero. He argued that 1 and 2 would be to the left of the line and that 1 1/2 should be half way between 1 and 2 and he located 1 1/2 there. He also correctly located 3 1/2 and 4 1/4.

<u>Question 5</u>

Lance: For a), she could not show 0.5 on the given number line because she did not know what 0.5 meant. After the interviewer told her that 0.5 is the same as 5/10, she started counting from zero, counted 5 points and located 0.5 there. For b), she again counted 5 divisions and located 0.5 at the end of the 5th division.

Andrea: She did not know 0.5 is the same as 5/10 and she could not locate it on either of the given number lines. With step by step help from the interviewer, sometimes telling her what it is, she identified 1/10 on both number lines.

Adrian: For a) where there are 8 divisions between whole numbers on a number line, when he was asked to locate 0.5, he counted 5 divisions and located 0.5 there. For b) which has 8 divisions between whole numbers on the number line, he still counted 10 divisions and located 0.5 there. He said however that it is easier to locate 0.5 if the divisions are 10 between whole numbers. When he was challenged that the zeros and the ones on both lines coincide, he said no because one line has 10 divisions while the other has 8 divisions.

Justin: For a), he correctly located 5/10 on the given number line after counting 4 of the 8 divisions that make one unit on this line and he also correctly indicated 1/4 on this number line. For b), he found it easier counting 5 divisions out of 10 to locate 5/10 there.

Question 6

Lance: For a), she correctly located 0.7 on the given number line. to locate 1/4 on the same number line, she counted 4 points and ;labelled the 4th point 1 and then the first point 1/4, ignoring the 1 provided on the number line. For b), she located 0.7 at the end of the 7th division and 1/4 at the end of the first division, just like she did for a). The student did not seem comfortable with the interview. When asked to show one half on the same number line, she said it would come half way between 1 and 2. She could tell 1/4 is smallest among 0.7, 1/4, and 1/2 but she could not tell why.

Andrea: Note: her responses to this question are on diagrams labelled 5. She identified the positions for 1/4 and 1 1/2 on both number lines. First, she said 1/4 is the largest of 5/10, 1/4, and 1 1/2, but then changed her mind to say 1 1/2 is the largest. She did not seem confident. When she was asked to locate 0.7 on the two number lines, she counted 7 divisions on the first number line (a) which has 10 divisions for a unit and then located 0.7 there. She then located 0.7 on the second number line(b) directly under 0.7 on the first number line. It fell between the 5th and the 6th point from zero on this number line which has 8 divisions to a unit.

Adrian: For a) he said he could not show 1 1/2 on the given number line. When 1 1/2 was explained to him as 1/2 more than 1, he was able to locate it on the number line. When he was asked which is smallest among 0.7, 1/4 and 1/2, he first said 5/10 but he quickly changed his mind and said 1/4. He arranged them in descending order of magnitude as 1 1/2, 0.7 and 1/4. When he was to locate 0.7 on the number line having

10 divisions between whole numbers, he did that easily, but when he was to do the same thing on the number line having 8 divisions between whole numbers, he said that would not be possible because there are 8 divisions.

Justin: For a), he was able to locate 0.7 and 1 1/2 on the number line. He did the same for b). He picked 1/4 as the smallest among 0 7, 1/4 and 1 1/2, saying that 5/10 = 1/2 which is double 1/4. He also said 1 1/2 is the largest because it is farthest on the number line.

<u>Question 7</u>

Lance: Given 1/10, she was able to identify one unit as 10 of the 1/10s. For the cubes given her, she identified the smallest size as 1/100.

Andrea: Given 1/10 strip, she was to show 1 unit. She picked one containing 10 parts of 1/10. When she was given another piece that was 1/10 of the 1/10 pieces, she said it would be 1/100 of a unit.

Adrian: Given 1/10 strip, he was able to identify one unit easily as that having 10 pieces. But when he was given 1/10 part of the 1/10 piece, he said that should be 0.5. After a while, he said he could not do it.

Justin: Given a 1/10 strip, he correctly identified one unit strip and said that 1/10 of a 1/10 strip is 1/100 of a whole.

Interview 3, grd 6, Feb 20 & 21, 1991

Question 1: Where do you use common fractions, decimal fractions in daily life?
Cindy: Making poster, lettering, pictures, measuring
Richelle: Grocery store, when you are weighing something
Adrian: Measuring, height of swimming pool, gym, building.
Andrea: For cutting (measuring).
Steven: Carpenter, mathematician, electrician ("how much wire"), high jump, long jump; (time?) "Ya, like bottom of tv screen" (when watching games/sports).
Justin (Feb 21): Carpenter, engineer, painting, measuring walls.
Lance: Not asked

Question 1a: Mark 4 tenths on the # line
Cindy: Correct. "I counted how many boxes before it is 20/100 or 2/10, & then I continued, to get 40/100 or 4/10
Richelle: Thought of it as 40/100, counted from 0.10 to 0.40 and got 4 tenths.
Adrian: Correct. Counted off 6 spaces between 0 & 0.20, recognized that 3 spaces represented 10 hundredths (after renaming 4/10 as 40 hundredths).
Andrea: Initially said that she couldn't, but after some discussion, located 4 tenths.
Steven: Located and wrote 0.4 correctly, without hesitation.
Justin (Feb 21): Correctly marked.
Lance: Wrote 4/10 at correct spot. Explained "Like, that's 2 tenths & there's 5 between, & so added 2 more".

Question 1b: Show 20/100 in the # line
Cindy: Correct. "I counted to 6/10, it's 12 boxes, so 2 boxes for 1/10, & 4 boxes to get to 2/10 or 20/100
Richelle: Made guess and check or trial and error method, and got it right
Adrian: Got it correct after initially making a mistake.
Andrea: Took more than 10 minutes, but finally got it, after guidance. "There's 12 in 6 tenths. Divide into half.....Well, if I change this to 100ths, it would be 20 hundredths....I am trying to cut the 6 tenths into half...I can't do it..."
Steven: Located and wrote 0.2 correctly, without hesitation.
Justin (Feb 21): Marked correctly, after counting off 2 spaces, 3 spaces, etc.

Lance: Had a bit of trouble locating 20/100 (took about 5 minutes), but finally did get it, after much discussion.

Question 1c: Where is 2 on this # line?
Cindy: Correct. "3 boxes to make 30/100, so 10 boxes for one unit & another 10 boxes for 2"
Richelle: Marked 2 correctly, by counting.
Adrian: Marked 2 correctly, by counting up to 1 etc.
Andrea: She counts 30/100, 60/100, 90/100. Then she says "Actually, I need one more". Continues, 0.60 for the 6th space, then 0.90 for the 9th space, & 12th space was 2 for her, instead of 1.2
Steven: Correeect, "Since there are 3 spaces between 0 and 30/100, every space is 10/100"
Justin (Feb 21): Marked correctly, saying "0.3 not one yet, so go 40, 50, 60,....100 & that would be one,....and so this would be 2".
Lance: Located it correctly.

Question 1d: Show 0.15 on the # line
Cindy: Correct. "Counted how many boxes to 7/10 or 70/100, so it was 14 boxes, so...one box and a half". Also said "One and a half tenths".
Richelle: First extended the line, saying "it won't fit" (because she counted 7, 8, 9,....14, 15 on the # line). Found a discrepancy when 15/100 was placed to right of 70/100, and after discussion could see that 1/10 and 2/10 could be renamed as 100ths. Then located 15/100 correctly.
Adrian: Incorrect. Counts 1, 2, 3 (every 2 spaces), finds not enough space, adds on some more spaces to get 15.
Andrea: Not asked.
Steven: After a false start "14 spaces to 7, which means can't really do that", he corrected himself and quickly and correctly located the point.
Justin (Feb 21): Knew that 0.15 was between 0.1 and 0.2, but initially located 0.15 midway between 0.1 and the line indicating 0.15, but corrected himself later, after doing question 1f.
Lance: Tried a few times, but was successful eventually.

Question 1e: Show 1 and 4/10 on the # line
Cindy: Correct. "First, I am going to figure out what's one box".

Richelle: Not asked.

Adrian: Renamed 4/10 to 40/100 & located 1and 4/10 correctly.

Andrea: Not asked.

Steven: Correct. Counted 1 tenth, 2 tenth etc until 1 and 4 tenth.

Justin (Feb 21): Did everything mentally, and wrote down 1.4 at the correct spot. Explained that he counted every 2 spaces as 0.1, and so "...0.8, 0.9, 1, 1.1, 1.2, 1.3, 1.4"

Lance: Not asked.

Question 1f: Show 70 hundredths on the # line

Cindy: Correct. "Counted boxes before 1 and 2 tenths, 24 boxes, so 24 is twice 12, so to make 1/10, need 2 boxes". She had no trouble moving back and forth between 10ths and 100ths.

Richelle: Marks it correctly, after some time. Initially guessed 2 boxes to 1, then 3 boxes to one, but semed to go wrong. When urged to check, finally got it correct.

Adrian: Marks it correctly, renames 70 hundredths as 7 tenths.

Andrea: Couldn't.

Steven: 7/10 or 70/100, as "for every one tenth, 2 spots.."

Justin (Feb 21): Marked correctly, again counting every 2 spaces as 0.1; then went back to question 1d (on being asked to "revisit" it), and corrected the answer.

Lance: Couldn't do it.

Question 2a: How much of the diagram is shaded? Other names?

Cindy: 40/100, 4/10,0.4, 0.40 (forty hundredths), 400/1000, 0.400

Richelle: After a few false starts, managed to get all correct. Initially had thought there were 500 small squars in all, but later corrected herself. She thought the decimal name 400 thousandths "sounds too big".

Adrian: Wrote 400/1000 first; needed a bit of help before he could see it as 40/100 and 4/10. Then gave decimal names 0.4, 0.40 and 0.400 (which he had initially written as 0.004).

Andrea: Not asked.

Steven: Had no problem writing various names: 4/10, 40/100, 0.4, 0.40, 0.400, 400/1000; Said "what you do is multiply by 2 over 2, 10 over 10, 4/10 x 10/10 = 400/1000'. Used grid to explain, too.

Justin (Feb 21): Wasn't sure which square was the whole, but was not given any clarification. After getting 40/100, wrote other names for it: 4/10, 0.4, 0.40, 400/100, but was using the rule of just "adding zeroes". Had difficulty in relating the symbols 400/1000 with the grid, especially as he had computed the total number of small rectangles as 100 x 100 = 10000. But after some discussion did see that the total was 1000.

Lance: Got 4/10, 40/100, 0.4 and 0.40 quite easily. Managed 0.400 and 400/1000 after some discussion, but had slight difficulty with 0.400.

Question 2b: What is the fraction name for the part shaded?

Cindy: 134/1000; 0.134; 1 tenth & 34 thousandths? 13 hundrdths and 4 thousandthds.

Richelle: Not asked.

Adrian: Not asked.

Andrea: Not asked.

Steven: Wrote 134/100, 0.134 and could name tenths, hundredths and thousandths place.

Justin (Feb 21): Didn't have much difficulty in getting 0.134 and 134/1000; could match the digit with its appropriate place value, just by looking at the symbol, but had more difficulty in matching the digit with its diagrammatic representation. For example, couldn't immediately see that the 3 stood for 30 hundredths in the diagram.

Lance: Not asked.

Question 3a: Put these in order: 1/9, 0.1, 1/12

Cindy: 1/9, 0.1, 1/12; "1/9 would have bigger boxes than 10ths"; used # lines to explain.

Richelle: Initially drew 2 rectangles, one below the other, with parts shaded to indicate 1/10 1nd 1/9. Then said "Oh, I don't have to draw pictures now" & used the idea of sharing, saying "sharing betwen 12 people, smaller piece". So could order correctly.

Adrian: 1/12 largest and 1/10 smallest, because "draw 10 equal parts, & you will need 9/10 to make a whole", while "12 equal pieces, you need 11/12 to make a whole". Maybe he was thinking the more pieces needed to make a whole, the bigger the fraction?

Andrea: Correct. Talks about number of pieces being bigger or smaller.

Steven: Initially said 1/12 was largest, but quickly corrected himself and said 1/12 was smallest. Also could identify the fraction closest to zero, among the three.

Justin (Feb 21): Ordered them correctly, saying "a chocolate bar, split up among 9 people rather than 12 people because 12 pieces would be smaller".

Lance: Initially said 1/9 was greatest, and 1/12 was smallest, but reversed it almost immediately. (Incomplete recording, video tape ran out)

Question 3b: Put these in order: 0.9, 11/12, 8/9
Cindy: 11/12, 0.9, 8/9 (smallest); "12 has smaller boxes"; used "nearest to 1" idea.
Richelle: In descending order: 8/9, 0.9, 11/12. Used her idea of sharing, and compared sharing with 9 people and 12 people, saying there will be smaller pieces for the 12 people. Didn't consider the number of pieces (the numerator).
Adrian: Said it was the same answer as for previous question, and so 11/12 was greatest and 9 tenths was smallest. "Do reversing, all you do is add 1 of each to make a whole".
Andrea: Thinks in terms of strips, says 8/9 is biggest as "9 pieces, more space left over and 12 pieces are smaller, & there is a little piece left". Incorrect, because sems to be concentrating on denominator.
Steven: 11/12 is largest "because space not shaded is smaller". Said that thinking about missing pieces helps.
Justin (Feb 21): Initially arranged them (in descending order) 8/9, 0.9, 11/12, but when reminded of the sharing of chocolate bar idea, reversed the order. Then, when asked to explain, reverted to the original (incorrect) order. After further discussion, said 11/12 was the largest, but couldn't explain why.
Lance: No video recording

Question 3c: Order these: 1.007, 1.23, 0,47, 0.7, 1.9, 4, 24.1, 0.125, 0.997, 9/10. 36/100
Cindy: Correct. Looked for those bigger than 1, first, then ordered largest to smallest. Had no problem identifying fraction closest to 1, 2 etc.
Richelle: Not asked.
Adrian: Not asked.
Andrea: Not asked.
Steven: Correct. Similar to Cindy/s approach. Could give 60/100 as a fraction between 7/10 & 47/100, by renaming 7/10 as 70/100.
Justin (Feb 21): Similar to Steven's approach. Also had no problems identifying which was closest to zero etc. Said 0.64 would be between 0.47 and 0.7,after explaining that 0.7 = 0.70
Lance: Not asked

Question 4: (with metre rule) Show 1/2; 1/2 of a 1/2; 1/2 of a 1/2 of a 1/2. Any other names?

Cindy: Didn't have any difficulties, except a little for 1/8. Even for 1/8, could explain why 1/8 = 25/1000, "If you multiply 125 by 8, get 1000, so 1000 times 8 = 8000, so 1/8". Gave many different names, e.g. , for 1/2, she said 50/100, 5/10, 500/1000, 3/6; for 1/4, she said 25/100, 250/1000, 2 & 5 tenths (explained she meant 2 tenths and 5 hundredths), 2 and a half tenths. For 1/2 of 1/4 of the metre rule, she said 12 and 1/2 hundredths, 125 thousandths, etc.

Richelle: Other names for 1/2 were 6/12, 5/10, 3/6, 4/8 and so on. Could see that 1/2 of 1/2 of the metre rule was 25, but couldn't immediately see it as 25/100. After some discussion, managed to see 1/4 = 25/100, as well as "1/2 of a 1/2 of a 1/2" of the metre rule as 12 and a 1/2 and as 12.5, but couldn't give a common fraction equivalent.

Adrian: 1/2 metre, 1/2, 3/6, 4/8, 50/100 "and probably, that's it". For 1/2 of 1/2, he said 25 cm, but couldn't give the fraction 25/100. Neither could he locate 1/4 of the metre stick, though he agreed 1/4 = 25/100, but seemed tired and/or unconvinced.

Andrea: Could give different names such as 5/10, 50/100, 0.5, 0.50; 25/100, 0.25 but "can't do with 10ths". For 1/2 of a 1/2 of a 1/2, she didn't quite understand, and gave 15/100, 0.15 as answers.

Steven: Had no difficulty identifying and naming the fractions. For 1/2 of a 1/2 of a 1/2, initially had 12 and a half cm, then later as 1/8, 125/1000, 1250/10000; Decimal fraction was given as 0.125, common fraction as 125/1000, and the simplest common fraction was 1/8.

Justin (Feb 21): Had no difficulties with 1/2 and 1/4, and could give various common and decir al fraction names for them. Had a litle bit more problems with seeing the different names for 1/8, but managed it,after some time.

Lance: Said "everybody was telling me about the metre rule question". Didn't have problems with 1/2 and 1/4 and the various names. Had quite a lot of difficulty with 1/8, especially as he said that he was not familiar with 1000ths.

Interview 4 March 11, 13, 1991

Question 1

Richelle: The student was asked to indicate 1/8th of a meter stick. She used her fingers to figure it out, pointed to the mark corresponding to 14 or 15 on the stick as 1/8th, fidgeted for a while and said she did not know how to do it. The interviewer gave her a hint "do you remember how we did it the other time, starting with half? She started with the mark corresponding to 50 as the half point on the meter stick, figured out 3/8th of the stick (37 1/2) and then 1/8th as the mark corresponding to 12 1/2. She was asked to give anc ame for 1/8th, she hesitated for a while, looked around the room as if she could find the answer somewhere on the wall and then gave 12.5 (hundredth) as another name. When she pointed to a mark and said that was 625 thousandth, she was asked "where is one whole?" and she pointed to the whole meter stick. (It seems linking student tasks with what they already know helps the student to perform the tasks better.)

Chad: When asked to show 1/4 of a meter stick, he pointed to a position between 75cm and 80cm on the stick. He was then asked to show 1/2, 1/4 of the stick or indicate how many pieces of the stick would make 1/2, 1/4. For 1/4, he said there would be 4 pieces but pointed to the 20cm mark as representing 1/4. He later found out that using the 20cm mark as representing 1/4 divided the meter stick into 5 parts. Student had to start all over again by being referred to the original question. When he was still having problems finding 1/4 of the meter stick, the interviewer asked him to find 1/ of a dollar and this he did very easily. Having done this, he was then able to find 1/4 of the stick as 25cm. After several trials he came up with 12 1/2cm as 1/8 of the meter stick. He also recognized that 1/8 is 125/1000 in millimeters.

Lance: When the student was asked to show 1/8 on a meter stick, he pointed too what looked like 12cm mark (12/100) and when he was asked to be precise, he said 12 1/2. When the student was asked if he could be more precise he said he was not sure. But when the interviewer asked him how many centimeters and millimeters are in a meter, he answered these correctly and got the idea that he could express his answer in millimeters also. He then gave an answer of 125mm or 125/1000 as representing 1/8 of the meter stick. For 5/8, the student started counting in fifths from 12 1/2, 25, ... until the came to 62 1/2 as 5/8. When he was asked to write that in thousandths, he wrote 625/1000, but he could not come up with other names for 625/1000.

Justin: The student said he solved a problem like that before, he was asked to find 7/8. So instead of 1/8, the interviewer asked him to find 7/8 of a meter stick. He pointed to 78 1/2cm and said 12 1/2cm represents 1/8 of the stick so 78 1/2cm would represent 7/8 of the stick. Explaining what he did further, he said he identified the positions for 2/8, 3/8, and so on and then added on till he got 78 1/2cm for 7/8.

Cindy: The student pointed to 12 1/2cm (or 125mm) mark as representing 1/8 of the meter stick. When the student was asked to explain doing it by hundredths, she pointed to 50/100, 25/100, and 12.5/100 as representing 1/2, 1/4, and 1/8 of the meter stick respectively. For thousandths, she said she could do it because she knows a meter equals 1000mm. When she was asked about 5/8, she pointed to it correctly saying it could be represented by 625mm, 625/1000, or 62.5cm.

Stephen: For 1/8 of the meter stick, the student pointed to 125mm and gave other fractions as 2/16, 125/1000, and 1250/10 000. When he was asked to find 5/8, he added 125 and 125 to get 250 for 2/8, multiplied by 2 to get 500 for 4/8, the added 125 to get 625 for 5/8.

David: He found 1/8 to be represented by 12 1/2cm on the meter stick because he remembers that a quarter (of a dollar) is 25 cents (25/100), so half of it would give 12 1/2. Fro 5/8 of the meter stick, the student pointed to 62 1/2cm and said he did that by representing 1/2 by 50cm and then added 12 1/2 as 1/8 to get 62 1/2 as representing 5/8.

Adrian: He could not find 1/8 of a meter stick and said he did not have any strategy to figure it out. So with some help from the interviewer, he found 1/2 a meter to be 50cm and then found 1/4 of a meter to be 25cm. But when he was asked for 1/8 he pointed to 20cm mark on the meter stick. When the interviewer did not approve of his solution, he started counting in 15cm as representing 1/8 but found out that 8 of the 15cm did not give 100cm. He got confused and said so and then decided to leave that problem. (13th March)

Francine: She located 1/8 of the meter stick at the 12 1/2cm mark, saying she started with a quarter as 25 (because she knows 1/4 of a dollar is 25 cents) and then found 1/2 of the 25 to get 12 1/2 as representing 1/8. When she was asked what units she used, she said 12 1/2 hundredths. For another name for 1/8, she wrote 2/16 and for expressing it in thousandths, she wrote 125/1000. For 5/8 of the meter stick, she first guessed 24 1/2 and then said she wanted to check her answer. She started counting and then changed her answer to "60 and 1/2", apparently referring to 625. (13th March)

Kegan: The student located 1/8 of the meter stick at the 12 1/2cm mark, saying that he thought of 1/4 as represented by 25cm and then used this knowledge to get 1/8 as 12.5cm, which he also wrote as 125/1000. He recognized one meter is 1000mm. In locating the position of 5/8 on the meter stick, he started with 500mm as 1/2 and then added 125 as 1/8 to get 625 as representing 5/8 of the meter stick. (13th March)

Andrea: The student was able to point to 50cm as representing 1/2 of the meter stick but when she was asked to show 1/8 of the meter stick, she pointed to 10cm. The interviewer explained to the student that 50cm represents 1/2 of the meter stick and that 25cm represents 1/2 of the 1/2 (or 1/4). She was then asked how many parts of the 1/4 parts would fit one whole meter stick and she said 4 and pointed to their respective positions on the meter stick. She was then able to say that 1/4 of the meter stick would be represented by 25cm, but when she was asked to find 1/8 of the stick, she still could not come up with an answer. She was asked to come up with half of the first quarter and she got 12. She found it difficult getting the answer and the interviewer had to virtually provide the answer. (13th March)

Question 2

Richelle: She was asked to shade 1/8th of the diagram, which she did. She was asked to give another name for 1/8th and she said 125 thousandth or 0.125 in decimal fraction. She was asked to shade 3/8th of the diagram and this took her a while, apparently because of a communication gap between her and the interviewer. When asked to give other names for 3/8th, she gave 375 thousandth and 0.375 as a decimal fraction. She found found 375 by multiplying 3 and 125. When still pressed for some other names for 3/8th she said she couldn't find any. (It looks like the student transferred her knowledge from finding 1/8th in problem 1 to shading 1/8th in problem 2. She might have established a connection between the two problems.)

Chad: shaded 1/8 of the flat by countin squares, saying he used his knowledge of the number line to help him. He could not provide satisfactory answers to the other questions the interviewer asked him.

Lance: When the student was asked to shade 1/8 of a flat, he first divided it into halves horizontally and vertically and then drew 2 other horizontal lines to divide the first pieces into halves again, getting 8 pieces in all. He then shaded 1 of the pieces as representing 1/8. When he was asked to 1/8 in another form, he wrote 125/1000, or 0.125 but could not any more. When he was asked to shade 3/8 of the flat, he shaded 3 of the pieces and gave it other names such as 375/1000 and 0.375.

Justin: When asked to, the student shaded 12 1/2 of the square units as representing 1/8 of the flats and gave 125/1000 or 0.125 as other names for 1/8, but he could not express it as a number over 100. When he was asked to shade 3/8 of the same flat, he did it saying he used 25cm to represent 2/8 and then added 12 1/2cm as 1/8 to get 37 1/2cm to represent 3/8. He gave other names as 375/1000 of 0.375.

Cindy: When she was asked to shade 1/8 oı the flat, she divided the diagram into 8 parts and shaded 1 part, given it other names as 12 1/2cm, 125mm, 0.125, 125/1000, or (12 1/2)/100. When asked to , she also shaded 7/8 of the same diagram and called it 375/100, (37 1/2)/100, 0.375, or 0.37 1/2.

Stephen: When he was asked to shade 1/8 of the given diagram, he said he would first "put" the diagram into quarters and then shaded part of it that he referred to as 1/8. When he was asked to convince that what he shaded represents 1/8 he said he found 1/2 of a quarter, that is 10 + 2 1/2 of the 25 squares in a quarter to get 12 1/2 out of 100 squares which is 1/8. For other names, he wrote 2/16, 125/1000 and for decimal fractions, he wrote 0.125 and 0.1250. When he was asked to shade 3/8, he first identified 1/4 of the area and then added 1/8 to it by counting squares. for other names of 3/8, he wrote 6/16, 0,375, 0.3750, and 375/1000.

David: When student asked if the divisions were in thousandths, he was asked how he would figure it out. He paused for a while and shaded 12 1/2 of the squares saying he made use of his knowledge from problem 1. For other names, he wrote 125/1000, 2/16/ 4/32, and 0.125. He was then asked to shade 3/8 of the same diagram. He did it saying that he took 25 squares for 1/4 and then added on 12 1/2 more as 1/8 to get 37 1/2 squares for 3/8. For other names of 3/8, he gave 0.375, 375/1000, 6/16, 9/24, and 12/32.

Adrian: The student noted there were 10 smaller squares on each side of the larger square and when he was asked to shade 1/8 of the larger square, he divided it vertically into 2 parts and one of these 2 parts horizontally into 5 parts. When the interviewer asked him to explain what he was doing,he realized that dividing the larger squares horizontally into 5 parts would give him too many parts, so he rather divided it into 4 parts to get 8 parts in all. He then shaded 1 of the 8 parts as representing 1/8. When he was asked for another name for 1/8, he said 1/4. The interviewer then asked him to use the diagram to illustrate that 1/8 = 1/4. He said he would not be able to do it without the number line. Using the meter stick, he pointed to 20cm as representing 1/4, but after counting by 20cm he got 5 parts instead of 4. The interviewer then started him with 1/2 of a meter as 50cm and then asked him for 1/4 of a meter and he was still stuck. But when a dollar was used instead of a meter stick, he was able to get 1/4 of it as 25 cents. He was then able to find 1/4 of a meter stick as 25cm or 25/100. (13th March)

Francine: She shaded 12 1/2 of the smaller squares as representing 1/8 of the flat. For other names she gave 2/16 and 4/32, and for decimal fraction equivalents she gave 125/1000. For 3/8 of the same diagram, she counted 3 of the regions shaded for 1/8 and shaded them for 3/8. For other names of 3/8, she gave 6/16 and 400/100. When she was asked to explain why 3/8 = 400/100, she said she gets confused when the numbers become larger. She later guessed that 3/8 should be "125/1000 three times" and got 375/1000 or 0.375. (13th March)

Kegan: The student used his knowledge from the meter stick to shade 1/8 of the flat. He referred to the shaded region as 125/1000 and also called it as 0.125, 2/16, or 4/32. For 3/8 of the flat, he shaded 3 times the area he shaded for 1/8 and called 3/8 as 6/16, 0.375, 12/32 or as 120/320. (13th March)

Andrea: She divided the flat into 8 parts and shaded 1 of it as representing 1/8. When she was asked for another name for 1/8, she said 1/8 and she could not explain how she got that answer, saying it was hard and gave up.(13th March)

Question 3

Richelle: She was asked to indicate 1/8th of balls of different sizes and colors. She claimed the balls were not equal so she couldn't do it. When the number of students in her class was supposed to be 24 and she was asked how many students will be in half the class, she said 12. She was asked if the students in her class are equal before she was able to find half the class. She couldn't continue at this stage and the interview ended. (Fraction as a ratio of numbers (how many?) needs to be made clear to students.)

Chad: picked 1 of the 8 balls and said that was 1/8. He was asked if the size matters in the case of the balls and in the case of the meter stick. The student said the size matters for the meter stick so the balls must be of equal size also. He got confused. Interviewer then asked him what will be 1/2 of a class of 24 students. He answered 12 and the interviewer then asked him if the students are of the same size. He answered no but said for people the sizes are not the same but for the balls they all need to be round (the same- one of them was oval).

Lance: The student counted 7 balls so he was asked to show 1/7 of the balls. He first picked 1 of the balls but later changed his mind to say that he can't pick 1 of the balls because they are not of the same size. the interviewer then asked him to find 1/24 of a class of 24 students and he said he can not do that because the students are of different heights and different weights.

Justin: At first, the student could not show 1/8 of the 8 balls. Given that there were 22 students in his class, he was asked to show 1/22 of the class and he said even though it seemed possible, he could not do it because the kids in his class are not all of the same size as some are tall and others are not.. The student paused for a while and when he was asked what he was thinking of, he picked 1 of the balls saying that would represent 1/8 of the balls if they were of equal sizes, but he could not do that if the sizes are different.

Cindy: She said she could not find 1/8 of the 8 balls unless they were all of the same size. The interviewer then asked her to suppose there were 22 kids in her class and then asked her to find 1/2 the class. She said 11 and then agreed to the interviewer's suggestion that 1/22 of the class would be 1 kid. When the interviewer reminded her that the kids are also not of the same size, she said they were talking of only *one* kid. To another suggestion from the interviewer that she seemed not to think about size when talking about kids but considered size when talking about balls or diagrams, she responded in the affirmative.

Stephen: He said it would not be possible to find 1/8 of the balls because they were of different sizes. The interviewer asked the student to suppose there were 22 students in his class and he was to find half of the class and he said 11. The interviewer then reminded him that the students were not of the same size and the student quickly changed his mind and picked one of the balls as representing 1/8 of the balls. And for 3/8 he picked 3 of the balls.

David: He said he could not find 1/8 of the balls because the sizes were not equal. But when he was asked to find 1/2 of his class of 24, he gave an answer of 12. The interviewer then reminded him that the students of his class were not of equal size, he replied that it would be easier to find 1/8 of the balls if they were of equal sizes but for the kids, it did not matter.

Adrian: When he was asked to show 1/8 of the 8 balls, he picked one of them as representing 1/8. When the interviewer reminded him that the balls were not of the same size, he said they were. The interviewer then asked him to find 1/2 of the 22 kids in his class and he said 11. The interviewer then reminded him that the kids in the class were not of the same size and he replied they were not born on the same date so they don't have to be of the same size. Now back to the balls and he said they don't have to be of the same size. (13th March)

Francine: She said she could not find 1/8 of the balls since they were not of the same size. But when she was to find 1/2 of the 24 kids in her class, she got 12. The interviewer then reminded her that the kids were also not of the same size and she agreed but insisted that for the balls one needed to have same size. (13th March)

Kegan: The student initially gave 1/8 of the balls as 1, but later changed his mind saying that the balls need to have the same size before one can find 1/8 of the balls. When he was asked to find 1/2 of his class of 22, he got 11. The interviewer then reminded him that the kids were not of the same size and he said he was confused and was no more sure of his solution so he had to think about it. (13th March)

Andrea: The student was not able to show 1/8 of the balls so she was asked to show 1/2 of the balls and she said there would be a problem because of the unequal sizes of the balls. She was asked to find 1/2 of the 22 students in her class but she could not do it. (13th March)

Question 4

Richelle: She was to find how many tenths of a meter a bamboo plant would grow in 7 days if it grows 3 tenths of a meter each day. She made counts on a ruler, did some multiplication and came up with 3m 50cm or 350 cm. She was asked to explain how she got it, she paused for a while without explaining so the interviewer, recognizing that her solution was wrong asked her to figure out how much growth took place daily. She used correspondence between the number of days and daily growth in cm. to find that the bamboo grows 210 cm in 7 days. She then changed her original answer. (This student seemed to have problems with multiplication and got the solution wrong. One may infer that certain basic operations (skills) need to be articulated to enhance a student's problem solving ability. Also, by using other strategies that work, a student could detect mistakes earlier made. The question "where did you make an error?" which was quickly changed to "did you make an error?", to which the student responded in the affirmative, might have suggested to the student there was an error without she herself realizing it that she made an error.)

Chad: multiplied 3 by 7 to get 21 and claimed the bamboo would grow 2 1/10 or 2.1 of a meter in 7 days.

Lance: The student said the bamboo would grow 210m in 7 days. He was asked to explain how he got his answer and he said he multiplied 3 by 7 and added zero to get 210m. (Note: may be the student read 3 tenths as 3 tens.) To explain his solution, the student drew 7 bars each of which was to represent a length of 3/10m. This time he changed his answer to 2 1/10m, saying that it is the same as 210m because 210 is the same as 2.10 except for the decimal point. When he was asked to explain his reasoning, he said he could not.

Justin: The student said a similiar problem was discussed in class and gave the solution to the problem as 1.8m. However, when he began counting 3/10, 6/10, 9/10, ... he changed his answer to 2.1m growth in 7 days. When he was asked to draw a diagram to represent his solution, he drew a stack of rectangular bars with each bar 0.3m high.

Cindy: Initially, the student solved the problem by multiplying 0.3 by 7 to get 2.1m to represent the growth of the bamboo plant. When she was asked to draw a diagram to explain her solution, she drew a bar to represent a length of 0.3m and then added on 4 similar bars; she did not draw all the rest.

Stephen: He multiplied 3 by 7 to get 21 and made it 2.1 or 21/10. When he was asked to draw a diagram, he drew 7 rectangular bars with each bar representing 0.3m.

David: The student wrote down 2.1 as the growth of the bamboo plant in 7 days. When he was asked to draw a diagram to illustrate what he had done, he drew a vertical line and divided it into 7 parts, with each part apparently representing a growth of 0.3m in a day.

Adrian: He said he probably had to write down 3/10 a day for 7 days and he got 21/10 for 7 days. He was asked to write that as a decimal fraction and he said he first had to write it as a mixed fraction 2 1/10 and then converted it to 2.1 as a decimal fraction. When he was asked to draw a diagram to illustrate his solution, he drew 30 rectangular boxes and shaded 21 of them as representing 2 wholes and 1/10. (13th March)

Francine: She drew 3 rectangular figures and divided each into 10 parts, shaded all 10 of two of them and one of the third figure to get 2 1/10m for the growth of the bamboo plant in 7 days. when asked to express her answer as a decimal fraction, she wrote 210/100. (13th March)

Kegan: He said one would have to multiply 3/10 by 7 (but he wrote 0.3 x 0.7) to get 21/10 or 2.1m, saying that 2.1 could be separated into 2 whole and 1/10. For other names, he wrote 2 1/10, 2 10/100, 2.10, and as common fraction, he wrote 21/10. (13th March)

Andrea: She multiplied 3 by 7 to get 21m. She was asked if the bamboo grew by 3m everyday. After reading over the question again, he said the bamboo grew by 3/10m daily and she wrote down 21/10 or 2 1/10 as the growth in 7 days. She could not write the solution as a decimal fraction when she was asked to. When she was asked to draw a diagram to explain her solution, she drew a rectangular figure and divided it into several parts but could not use it to explain her solution; she seemed confused. (13th March)

Question 5

Richelle: She was to find out how much granola bar was eaten if Wes ate 1/4 of the bar and Elexis ate 3/8th of the same bar. She was to use a rectangle to aid her. She took time to work on the problem, dividing the rectangle into halves and the into 4 parts to have 8 parts. She shaded 5 of the eight parts, which she did not write as 5/8. When asked to name some possible operations, she mentioned addition, multiplication, etc. Then when she was asked to "write a number sentence of what happened in this problem", she did not know what to do. When asked how she could combine 1/4 and 3/8 to get the parts she shaded, she wrote 1/4 + 3/8 = 4/8. She did not realize the 4/8 was different from the proportion of the parts she shaded. After further probing, she changed her answer to 5/8 but she could not explain why she was wrong. The interviewer offered an explanation that she might have added the numerators and she agreed. (Words not understood in a problem can inhibit successful solutions to the problem.)

Chad: said 1/4 is the same as 2/8 and then added 1/4 and 3/8 to get 5/8. He then divided the rectangular bar into 8 parts and shaded 5 parts to illustrate his solution.

Lance: The student divided the rectangular horizontally into 2 parts and then vertically into 8 parts to get 16 pieces in all. He shaded 4 pieces as representing 1/4 of the bar and 3 other pieces as representing 3/8 of the bar, but he could not offer any further explaination as to how to find the solution to the problem.

Justin: The student divided the given diagram into 8 parts and shaded 5 parts as representing the fraction of the granola bar eaten. However, when he was asked to write a number sentence for what he did, he wrote 1/4 + 3/8 = 4/12. He said he found it difficult adding these types of fractions and despite step by step help from the interviewer, he could still not figure out how to add 1/4 and 3/8 to get 5/8, even though he was able to solve the problem using a diagram.

Cindy: The student divided the diagram into 8 parts, shaded 2 parts as representing 1/4 and then 3 parts as representing 3/8 to get 5 parts of the 8 shaded to represent 5/8. When she was asked to write a number sentence to illustrate her solution, she said she could not. But when the interviewer reminded her of how she multiplied 0.3 by 7 to get 2.1 in problem 4, she wrote 1/4 + 3/8 = 5/8, claiming that 1/4 is the same as 2/8. When she was asked whether she learned to add fractions in school, she replied no.

Stephen: The student first divided the diagram into quarters and then divided the quarters into halves and then shaded 3 of the resulting 8 parts as representing 3/8. When he was asked to write a number sentence to illustrate what he had done, he had some difficulty but later came up with 2/8 + 3/8 = 5/8. When he was asked why he did not get 5/16, he said he was dealing with eighths, not sixteenths.

David: He divided the diagram into 8 parts and shaded 5 parts as the parts eaten. When he was asked to identify the parts eaten by each individual, he did that successfully. When he was asked to write a number sentence for what he did, he seemed to understand "number sentence", but when the interviewer asked him what operation was involved in what he did, he said he changed 1/4 to 2/8 and added it to 3/8 to get 5/8; he wrote it as "add 2/8 and 3/8 together to get 5/8".

Adrian: He first guessed 1/4 of the bar might have been eaten and then said this might be a hard one to figure out. He was asked to use the diagram provided and he only shaded part of it and said it was too hard for him. The interviewer then asked him to find 1/4 of the diagram and then he suddenly said the part eaten should be 5/8, explaining that he changed 1/4 to 2/8 and added 3/8 to get 5/8. When he was asked to write what he did as a number sentence or an equation, he said he did not know what those words mean. The interviewer then asked him to write down how he solved the problem and he wrote down 2/8 + 3/8 = 5/8. <u>(13th March)</u>

Francine: She divided the given diagram into 8 parts and shaded 5 of them as representing the part of the granola bar eaten. When she was asked to write a number sentence for what she did, she wrote 2/8 + 3/8 = 5/8 and she responded in the affirmative when she was asked if her solution makes sense to her. When she was reminded that some kids would get 5/16 because they would add the denominators, she replied that one should not be adding eights but the parts of 8 which are 2 and 3. <u>(13th March)</u>

Kegan: He said he divided the bar into 8 parts, changed 1/4 to 2/8 and then added 3/8 to 2/8 to get 5/8 as the part of the granola bar eaten. When asked what operation he used, he said he used addition and wrote down the operation as 2/8 + 3/8 = 5/8. When he was reminded that some kids would get 5/16 by adding the numerators and the denominators, he said that would be wrong since the bar has been divided into 8 parts (but not 16). When he was asked to illustrate his solution by using a diagram, he divided the diagram into 8 parts, shaded 2 parts at one end and then 3 parts at the other end to get 5 parts shaded in all out of the 8. When he was asked if it mattered which of the 8 parts were shaded, she said no, so long as they represent 1/4 and 3/8 of the bar. (13th March)

Andrea: She added 1/4 and 3/8 to get 5/8. When she was asked to draw a diagram to explain her solution, she drew a rectangular figure, divided it into 8 parts and shaded 5 parts as representing the part eaten, 5/8. (13th March)

Question 6a and b

Richelle: She was to mark 0.15 on a number line having intervals of 0.10 and the positions zero and 0.20 already marked. She did that correctly. Her reasoning was that 15 lies between 10 and 20 and therefore 0.15 which is 15/100 must lie within 10/100 and 20/100. For 6b, she was to mark 1.8 on a number line having intervals of 0.05 and the positions zero and 0.70 already marked. She started counting the intervals in pairs and found out that seven counts brought her to the 0.70 position. She became convinced of counting in pairs and correctly marked the position for 1.8.

Chad: located 0.15 between 0.1 and 0.2 and gave it a name of 15/100. He said 0.2 is the same as 2/10 or 20/100. When he was to locate 1.8 on a number line on which 0.70 is already marked, he first identified 1 and then counted 16 more divisions to get 1.8 (two divisions on this number line make 0.1).

Lance: For a), the student easily marked 0.15 and explained that 0.15 is the same as 15/100 and that it lies between 10/100 and 20/100. For b) student wanted to find out how many divisions add up to 0.7 on this number line but made a mistake in counting so he gave up. When the interviewer suggested to him to count 2 divisions as 1/10, he was able to correctly locate 1.8 which he also called 1 and 8/10, on the number line.

Justin: The student correctly located 0.15 between 10/100 and 20/100. For another name for 0.15, he he gave 15/100. For the position of 1.8 on the second given number line, he counted 2 divisions as representing 0.1 and correctly located its position.

Cindy: She located 0.15 correctly on the number line explaining that it should lie between 10/100 and 20/100. For 1.8 on the second number line, she counted 2 divisions for 0.1 and correctly located its position.

Stephen: He easily located 0.15 saying that each unit is 1/10 and that half way between units would be 5/100, so 0.15 would lie half way between 0.1 and 0.2. for the numbers in hundredths, he wrote 10/100, 15/100, and 20/100 for 0.1, 0.15 and 0.20 respectively. He said 1.8 is 1 8/10 and he located it correctly on the given number line, using 2 divisions to represent 1/10.

David: He located the position of 0.15 (which also he called 15/100) between 10/100 and 20/100. For 1.8 or 1 8/10, he located its position incorrectly on the given number line because he missed his counting of the divisions.

Adrian: He called 0.15 "15 tenths" and said he needed another number line to be able to locate 0.15 on it. When asked if he could not locate it on the given number line, he said no (because the given number line ended at 10/10 and he needed 15/10 as the location for this number). For 1.8 which the student also wrote as 1 80/100, he started counting 3 divisions as representing 1/10 and his counting did not tally with 0.70 which was already located on the number line, so he started counting by twos and he was able to locate the position of 1.8 correctly. (13th March)

Francine: She referred to 0.15 as 15/100, but when she was to locate it on the given number line, she located it between 0.5 and 0.6. After a while, she said she was wrong and she located 0.12 between 0.1 and 0.2. For 1.8 on the given number line, she counted 2 divisions as representing 1/10 and then correctly locate 1.8. (13th March)

Kegan: He called 0.15 15/100 and located its position between 10/100 and 20/100. For 1.8, he correctly identified its position on the given number line. (13th March)

Andrea: She called 0.15 15/100 and then asked the interviewer if 0.2 is the same as 2/10. When the interviewer answered in the affirmative, she went ahead and located 0.15 half way between 0.1 and 0.2. When she was to locate 1.8 on the number line, she counted two divisions for 1/10 and correctly located its position on the number line. However, she was confused about whether she should have counted the points of the number line or the divisions between the points. The interviewer told her the counting is of the intervals, not the points. (13th March)

Interview 5, May 1, 1991 (Grd 6, LL)

Question 1: Multiply 0.4 x 0.2

Six of the 7 interviewees wrote the vertical form of it and then multiplied. Only one, Andrea, gave an answer (0.8), without using the vertical form. She said 2 times 4 is 8. She wrote both 0.8 and 8/10.

3 of them (Adrian, David, & Justin) corrected themselves after doing # 8 (0.3 of 0.2), and rewrote the answer as 0.08. All three initially wrote 0.8, & looked as if they "lined up the decimals",.though only Justin specifically used the phrase "bring down the decimal point". The 3 who initially wrote 0.08 (Stephen, Cindy, & Richelle) had differnt reasons for their answer: Steven said "tenths multiplied by tenths is hundredths"; Cindy said"because when we did in class, I remembered, when you multiply, you're supposed to get bigger, but when it's less than 1, it gets smaller, so if I picked 8 tenths, it would be bigger, & so it's wrong"; Richelle said"because 2 numbers before the decimal, so 2 numbers before the ". She seemed to count the **total** number of places after the decimal point, as evidenced by her placement of the decimal point in # 3, 7.342 x 0.5 = .3671.

Question 2: Anita walks 0.7 km a day. How many kilometres would she have walked in 5 days?

All 7 got the correct answer, though Andrea had to correct herself, after writing 35. 6 used multiplication immediately, 1 (Adrian) used repeated addition. Andrea, who wasn't too sure of her "multiplied" answer (35), used repeated addition as another method, and got 3.5, and finally multiplied 0.7 & 5 to get 3.5 also. Yet she wasn't sure of her answer, though she had done 3 "different" approaches. Richelle continued her rule of counting the total number of digits after the decimal point. Justin kept aligning the decimal points to get his answer. Adrian could use multiplication as an alternative to repeated addition, while Cindy checked her answer using addition. Both David & Steven could explain why multiplication would give the right answer. Justin and Richelle, however, just used their rules to get the answer, and couldn't really explain the answer conceptually. (Interestingly enough, David kept saying zero point 8 tenths for 0.8, and 3 point 5 tenths for 3.5, rather than 3 and 8 tenths, while all the others used the equivalent common fraction verbal names, without using the word "point").

Question 3: Wes had a calculator which would NOT display decimal points. Help Wes place the decimal point in the "answer" displayed by the calculator for the multiplication below:

7.342 x 0.5 = 3671

Justin: Arranged the numbers for multiplying (vertically), insisting that he had to "times" it, & gets 36.710 (he had put 0.5 right below 7.3). Looked as if he was using his rule of "bringing down the decimal point".

Cindy: 3.671, because "half of 7 is about 3 and a half".

Andrea: 36.71, by rounding 7.342 to 7 and multiplying 7 by 5 (ignored the decimal point in 0.5); but admitted she was not sure.

Richelle: .3671, because "well, there is 1, 2, 3, 4 numbers before the decimal" (i.e. she added the total number of digits after the decimal point).

5.Adrian: Initially said he guessed the answer would be 3.671; then multiplied 7.342 & 0.500 and got the answer 03671.000 (looked as if he was lining up the decimal point), and said the answer could be 03.671000 or 03671.000, depending on whether we wanted the answer with or without the zero. Then he tried 7.342 x 0.5 and got 36.700, cancelled the zero in 0.5 and the zero between 7 and 0, and got 36.70; finally decided on 36.71, saying that it would be better to "do without the zeros".

David: Initially wanted to multiply, but when asked to estimate, rounded 0.5 to 1 & so 1 x 7 = 7 gave him the answer 367.1 (seemed to concentrate on the 7); didn't want to check by multiplying.

Steven: Said that the decimal point should be placed after the 3, thus, 3.671 "because that would be like dividing in half, so 3 is about half of 7" and "5 tenths is half of 1, so it would be like multiplying by 1/2, so you would get 1/2 of 7"

Question 4: A teacher paces up and down a classroom. Each pace is 0.74 metres in length. How many paces will she make in walking e full length of the classroom, 9.2 metres? Choose from the answers below. {There may be more than one correct answer.}

a) 0.74 x 9.2 b) 0.74 ÷ 9.2 c) 9.2 ÷ 0.74

d) $\frac{9.2}{0.74}$ e) $\frac{0.74}{9.2}$

Justin: First chose B, then changed his mind, and chose C. When pressed, replied it was hard. After about 3 minutes on the question, gave up.
Cindy: (asked for meaning of "pace") First chose A, then B, then finally C. Thought it had to be a division question, but could not explain clearly why she chose that operation. (It seems that in France the usual notation for long division was read in reverse order, compared to English-speaking countries).

Andrea: Chose C, "because full length is 9.2 & each pace is 0.74, & I want to see how many 74 hundredths is in this number". Said B could not be the answer because "this number (9.2) is bigger than this (0.74)" Similar reasoning to that of Stephen, but was unfamiliar with the vinculum /fraction notation..

Richelle: Chose A, because "want to know how many paces.....classroom is 9.2......, so you have to multiply it".

Adrian: Said it was too hard, but would choose multiplying over dividing. Then multiplied, to get 68.08 (Used lining up decimal points rule)

David: Took a while to do this, but said he would multiply, choosing A because "one step is 0.74, 2 steps is 2 x 0.74..."

Steven: First chose A, then changed to B "because you have to figure out how many times 74 hundredths would go into 9 & 2 tenths". When asked about writing this in symbols, changed answer to C, because "B asks you to put 9.2 into 0.74, you can't do that". (Interesting, because division seems to imply an answer >1). Also agreed that E might be like C, "because it is like 0.74 into 9.2".

Question 5: To make a wallet size picture of Michael Jackson from a poster, it has to be decreased to 0.052 of its original height. If the poster was 1.2 metres high, what will the height of the picture be? Choose from the answers below. {There may be more than one correct answer.}

a) 0.052 x 1.2 b) $0.052 \div 1.2$ c) $1.2 \div 0.052$

d) $\frac{1.2}{0.052}$ e) $\frac{0.052}{1.2}$

Justin: Noticed a typo "0.52 cannot be correct because 0.52 was not in the answer". Then chose B. Said multiplying "makes it grow, & division makes it less". Agreed it could also be E.
Cindy: Guesses C, tries a few times, draws a rectangle and continues drawing smaller rectangles within the big one, but finally gives up, without being sure whether it is a division or multiplication problem.

Andrea: Asked "does decrease means make bigger?", after which says that "the one I want is not here", and writes it as a subtraction, 0.052 - 1.2, lining up the decimals. Thought 0.052 was original height, then changed to 1.2 - 0.052, and said she used subtraction to "make smaller".

Richelle: Also did not know meaning of "decrease". Guessed B, then later changed answer to subtraction, writing 1.200 - 0.052, in vertical form.

Adrian: Because more than one answer is possible, thought A, B & C were all correct. Finally chose A "because easy for me to do, but division I cannot do - but probably it will come out the same answer"

David: Chose B because "when decreasing must divide". When asked why not subtract, said that it could be either subtract or divide, but when pressed to make a choice, chose divide.

Steven: Chose C, "because it is kind of like # 4, a # going into another #."

Question 6: Steve spent $ 900 for 0.75 kg of platinum. What would the price of 1 kg bar of platinum? Choose from the answers below. {There may be more than one correct answer.}

a) 0.75×900 b) $0.75 \div 900$ c) $900 \div 0.75$
d) $\frac{900}{0.75}$ e) $\frac{0.75}{900}$

Justin: Knew platinum was expensive. Chose A "because this is higher than that, so you multiply". Then changed his mind, saying "if that was 1, 900 times 1 is still 900, so that will be lower". When asked whether the amount of $ for 75 hundredths is smaller than 900, agreed it would be, and said "probably it is a dividing question or a fraction". Checked by multiplying 900 and 0.7, obtaining 675.00, and seemed puzzled why "timesing" decimals could give a lower number. Guesses B & E, then doesn't agree they are the same Finally, chooses B, and shows 0.75 as the divisor of 900, in the usual long division notation.

Cindy: Had to be told platinum is an expensive metal. Drew a rectangle, and divided it into equal parts, saying "$900 would be 3/4, so 1/4 would be $300, & 4/4 would be $1200". Wasn't too sure whether it was a division or multiplication problem, then chose E, but later said "900 divided by 0.75 would give you $300, not the whole", and finally chose C as the "closest".

Andrea: Chose C, & said could not be E because "this # is bigger" meaning the 900 in 0.75 ÷ 900.

Richelle: Chose A, because "will have to be more". Looks as if "dividing makes it less"for her.

Adrian: Had to be told platinum is an expensive metal. Chose A, and said this was just like question # 5. Also thought B & C are the same, "just reversed".
David: Also had to be told platinum was an expensive metal. Chose C, saying that "...for $ 100, how much platinum , so can find out how much for 1 kg", and "keep on adding more & more until we get 1 kg". Could see that division was not commutative, but thought D & E were the same as C, but wasn't sure what D & E meant. When he was told that these were like fractions, then he said "probably"could be the same as C.

Steven: Chose B, saying he wanted to figure out how many times 0.75 would "go into" 900. He hesitated quite some time, saying he was "trying to figure out how much money..", but seemed pretty uncomfortable, as he seemed to be getting nowhere. So he was asked to skip it & go on to the next question.

Question 7: On a highway a four - wheel drive car can go 7.5 km on each litre of gasoline. How many kilometres can the car be expected to go on 1.3 litres?

Justin: Shows 15 = 2 litres, 7.5 = 1 litre, 3.75 = 1/2 litre, and says 1 1/2 litres would give around 10 km, because "if you add 3.75 to 7.5, that will be around 10". Shows the addition gives 11.25, and feels it is "probably multiplication or division", and tries multiplying 7.5 with 1.3, obtaining 9.75.

Cindy: "can go up to 10 km, because 1 litre, 7.5 km, 7.5 divided by 3 is 2.5, because 2.5 + 2.5 + 2.5 = 7.5, and 2.5 + 7.5 = 10 km". When it was pointed out that she had done thirds, rather than 3 tenths, she divided 7.5 by 5 and got 1.3 (computational error?), saying 1.3 would be 1/5, so it would be 0.65 x 2 = 1.30, plus .65, giving 1.95. Then adding 1.95 to 7.5 she got 9.45. When she was asked to just choose between multiplication & division, rather than actually compute, she said divide, 7.5 divide by 10. Then she changed her mind and said "no, you have to use both multiplication & division".

Andrea: She initially wanted to divide 7.5 by 1.3, but corrected herself, reasoning that for every litre, it was 7.5 km, so for 1.3 litres it should be 1.3 divided by 7.5, but that this was too difficult for her to do.

Richelle: Says "a little bit more", so adds 7.5 to 3, putting the 3 directly below the digit 5, to get 7.8.

Adrian: Said he couldn't do it, but thought it would be less than 7.5 km. Later changed to more than 7.5 km.

David: Initially thought it was "a littlebit more than 1 km", then wrote 7.8, but later thought it would be 1.3 divided by 7.5.

Steven: Thought it a division problem initially, but later said it would be a multiplication problem, because it would be "1.3 times as much".

Question 8: Shade the diagram below to show 0.3 of 0.2 (diagram of 10 x 10 grid)

Justin: After some help, shades in 0.06, checks by multiplication, then following a suggestion, re-does # 1 again, correctly.

Cindy: Shades 2 tenths first, then 2 hundredths. Then shows 0.2 = 0.20 = 0.200 & similarly for 0.3, and continues shading the remainder of the shaded 0.2, & then stops, saying "it's impossible". After a little help, writes 0.3 of 0.2 = 0.06. When asked to write an equation for what she nas done asked "what's an equation?", & when asked whether this was a multiplication or division problem, said it was division. Then, after looking at question 1, said that it could be both multiplication & division, as "you have to divide 2 tenths into hundredths & then multiply 1 tenth into 3".

Andrea: Didn't understand at first, but after some help, showed 5 tenths, then corrected herself & showed 2 tenths. Then she said she was having trouble dividing into 3, but when reminded it was 3 tenths, drew a line horizontally to represent 3/10 (of the whole), and on being asked to show 3 tenths of 2 tenths, drew a vetical line to represent 2 tenths of the whole, and then shaded in the intersection of 3/10 & 2/10. She then correctly wrote 6/100 and 0.06 as her answer. When asked whether this was a multiplication or division question, was unable to answer, and tried doing addition followed by multiplication. On multiplying, she got 0.6 or 6 tenths. She had difficulty reconciling the answer from the diagram with that from the multiplication.

Richelle: Also didn't understand until it was explained to her that she had to take 0.2 first. Then she shaded 2/10, followed by 3/10 of 2/10 by shading 6 small squares. She wrote the answer 0.06 for 0.3 of 0.2, but thought the operation was division. When told it was multiplication, she seemed astonished, but seemed to get the idea when this was related to finding area of a rectangular region.

Adrian: Didn't understand the notation until it was explained to him. Finally, after some help, did manage to shade 0.06 correctly. Then he corrected # 1 as well, but couldn't tell the reason why.

David: With some help, got the answer 0.06, but after multiplying, got the answer 0.6 (using lining up the decimal points rule). Finally seemed to see that it would have to be 6 hundredths. Then corrected question # 1 to 0.08.

Steven: Though he did get 0.06, still thought the operation was division. After some discussion, decided it was a multiplication problem. Finally, he wrote 0.3 x 0.2 = 0.06.

Summary of Grade 6 interview results

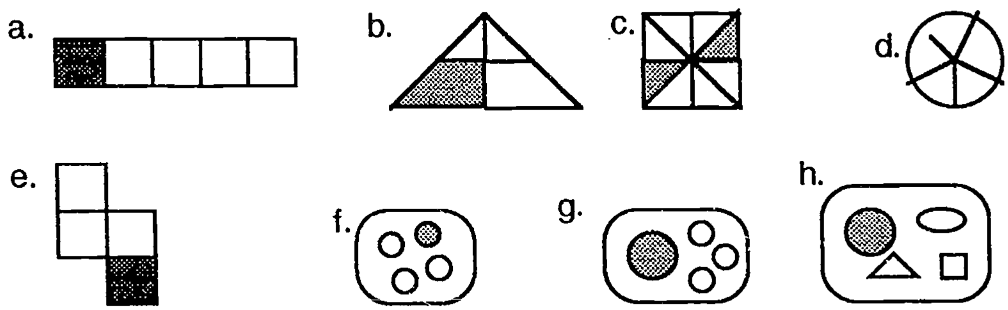
From interview 3, we can gather that most of the students have integrated common and decimal fractions. For example, the question on marking 4/10 on a number line marked so that the end of the sixth space from zero was 0.20, did not pose much of a problem. Most students read 0.20 as "20 hundredths" and/or "2 tenths". Some even wrote this as 2/10. The question asking for 5 different names for the shaded part of a 1000 grid also indicated the students facility at moving from common fraction notation to decimal fraction notation and viceversa, as they gave both common and decimal fraction names. The results of interview 4 confirmed that students had indeed integrated common and decimal fractions (as did the lessons on developing decimal fraction multiplication).

Grade 4 Interview Questions

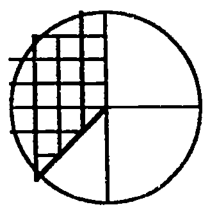
Interview 1

1. Draw a picture to show three fourths.

2. Which of these diagrams show one fourth? Circle the letters a to h to show your answer.

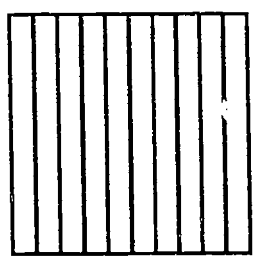


3. What fraction of the circle is shaded?



4a. Draw a picture of 1.3

4b. Shade 0.15 of the square below



5. Circle the larger number in each pair of numbers given below:

a. 0.5, 0.47   b. 0.1, 0.09   c. 0.3, 0.30

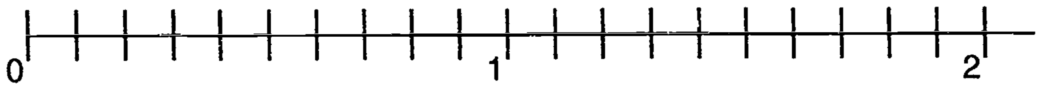
6. There are 24 students in a class. One fourth of them ride to school. How many students ride to school?

7. Circle the larger number in each pair of numbers given below:

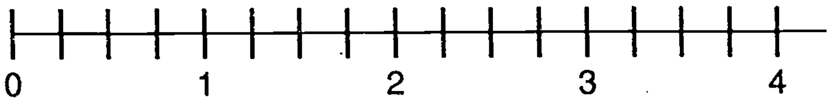
a. $\frac{3}{5}, \frac{4}{5}$
(three fifths, four fifths)

b. $\frac{2}{3}, \frac{2}{4}$
(two thirds, two fourths)

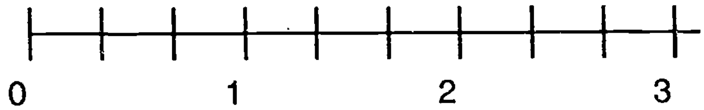
8a. Mark 0.75 on the number line below:



8b. Mark 3/4 (three fourths) on the number line below:

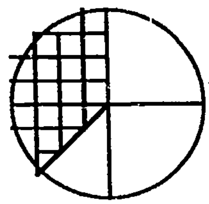


8c. Mark 1 2/3 (one and two thirds) on the numbe line below:



Interview 2

1. What fraction of the circle is shaded?



2. Draw a picture of 1.3

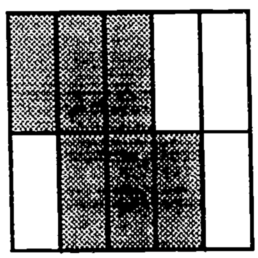
3. Twelve students in a class completed three hours in a " Jump Rope for Heart" charity competition. One fourth of these students were boys. How many of the twelve students were boys?

4. Arrange the following from smallest to largest :

a. 7/12 , 11/12 , 5/12 _____ , _____ , _____

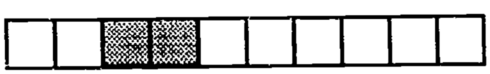
b. 4/3 , 4/5 , 4/10 _____ , _____ , _____

5. What common fraction is shown by the shaded part of the figure below? Give some other names for this fraction.



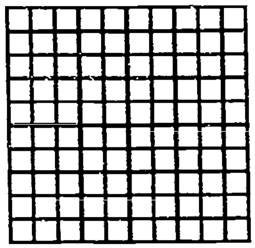
6. Fraser and Treva share a granola bar. Fraser eats half of it and Treva eats one fourth of it. How much of the granola bar do they eat altogether? If you wish, use the fraction strips or a drawing to help work out the answer.

7. The rectangle divided into ten equal parts is one whole unit here. Write down as many different fraction names as you can for the shaded parts.



8. Of the 27 children in a class, one third are girls. How many are girls? How many are boys? Show your work below. You may use the fraction strips or a picture to explain your work.

9. The square divided into 100 equal parts is one whole unit here. Shade in the decimal fraction 0.4 in the figure below :



10. [Metre rule shown] Where is one half? Half of a half? Half of a half of a half? Any other names?

Interview 3

1. Draw a picture of 1.3

2. Arrange the following from smallest to largest:
a) 4/3 , 4/5 , 4/10 ____ , ____ , ____ b) 0.10 , 0.09 , 0.8 ____ , ____ , ____

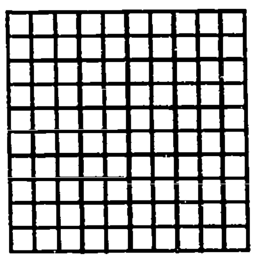
3. Which of the numbers in each pair is larger? Explain your choice.
a) 1/4 , 0.2 b) 0.3 , 1/3 c) 3/4 , 4/5

4. Tyler, Roberta and Dallas share a granola bar. Roberta eats a third of it , Dallas eats 0.25 of it and Tyler eats the rest. What fraction of the granola bar did Tyler eat? Explain how you got your answer.

5. 3/10 + 4/10 =

6. Of the 27 children in a class, two thirds are girls. How many are girls? How many are boys? Show your work below. You may use the fraction strips or a picture to explain your work.

7. The square divided into 100 equal parts is one whole unit here. Shade in the decimal fraction 0.4 in the figure below :



Give another name for 0.4

8. Show 1/4 of the 8 balls [Balls of different shapes and sizes were shown].

Summary of Grade 4 Interview Transcripts

Interview 1

Question 1

Lindsay: The student was to draw a picture of three fourths and she did.

Steve: The student was to draw a picture of three fourths and he drew a rectangular figure, divided it into 4 parts and shaded 3 of the parts, claiming the shaded part represents 3/4.

Jodi: She drew a circle and shaded part of it as 3/4.

Fraser: He said he did not know much about what was required, but he would guess. He drew 4 lines and crossed out 3 of them to illustrate what he guessed to be 3/4.

Kristine: She drew a circle and shaded part of it.

Tyler: He said he did not know how to so it and when he was asked if he knew anything about fractions at all, he said no.

Derrick: He drew a rectangular figure, divided it into 4 parts and shaded 3 parts.

Roberta: She drew a circle, divided it into 4 parts and shaded 3 parts.

Treva: She drew a circle, divided it into 4 parts and shaded 3 parts.

Dallas: She couldn't draw it.

Question 2

Lindsay: The student was to select from a group of diagrams those that were showing one fourth. When the student was asked to justify her selections, she said there were four spaces and one of them was colored (shaded), so they must be one fourth. When she was asked why she did not select a particular picture even though it had four parts and one was shaded, she said that the parts (sizes) were not equal. Also, she did not select diagrams whose parts were of different shapes.

Steve: The student was to select from a group of diagrams those that were showing one fourth. When he checked some of the diagrams he was asked to explain why he made those choices. One of his choices has 5 parts with one part shaded. The student wasn't sure if the shaded part forms 1/4 of the whole, even though he said there were 5 parts in all. He said he did not know fifths (suggested by the interviewer), so he will go with 1/4 as the fraction equivalent of the part shaded. He selected most of the diagrams having one of four parts shaded as representing 1/4, even those having different *shapes* (e. g., h). However, he did not select (g), saying that the circle shaded has a larger *size* than the rest of the circles, even though there are four of them with one shaded. He was referred to (b) which he chose as representing 1/4 but whose shaded part is equal to one other part but larger than the other two parts. He said he did not notice the different sizes earlier on and that he was changing his mind that (b) represents 1/4.

Jodi: The student was to select from a group of diagrams those that were showing one fourth. When she checked some of the diagrams she was asked to explain why she made those choices. She did not know why she chose e) but it looked like 1/4 to her. She chose f) because there are 4 parts of the same size and 1 is shaded. When asked why she chose b), she said she was not sure, and after a pause, she said she was changing her mind that b) is 1/4 because of the different shapes. When the interviewer asked whether she meant different sizes or different shapes, she changed her mind to different sizes. She did not choose a) and c) because they do not have 4 parts. Even though g) and h) are of 4 parts, she did not choose them because for g) the parts are not of the same size and for h) the parts are not of the same shape and size.

Fraser: The student was to select from a group of diagrams those that were showing one fourth. When he checked some of the diagrams he was asked to explain why he made those choices. He chose a) saying that 4 parts are not shaded and 1 part is shaded, giving 1/4. He did not choose b), e), g) and h). His explanation is that for these, there are 3 unshaded parts and 1 shaded part so that is not 1/4. He also noted that for g) the shaded circle is bigger than the rest while for h) all the shapes are not the same.

Kristine: The student was to select from a group of diagrams those that were showing one fourth. When she checked some of the diagrams she was asked to explain why she made those choices. She did not choose a) because there are supposed to be 4 parts but there are 5. She chose e) because there are 4 parts and 1 is shaded. She did not choose c) because there are 8 parts and 2 are shaded. When he was asked if that is not equivalent to 1/4, he said no. She did not chose g) because the circle shaded is bigger than the rest, but she marked b) even though the part shaded is bigger than the other parts, saying he *just* marked it. For h), she did not choose it as 1/4 because the shapes are different.

Tyler: The student was to select from a group of diagrams those that were showing one fourth. He said he did not know what to do. The interviewer pointed to a particular diagram and asked him if that was representing 1/4 and he answered yes, but he could not explain why. When the diagram with 5 parts was shown to him and he was asked if it was different from the one previously shown him, he said yes, explaining that one has 5 parts with 1 part shaded while the other has 4 parts with 1 part shaded. The interviewer told him those are 1/5 and 1/4 respectively. He was then able to identify some of the diagrams having 1/4 part shaded. He was asked if the shape of the parts matter in deciding the fraction shaded and he said he was not sure but he did not think the shape of the parts affected his thinking. He was asked if c) was 1/4 and he said no. When asked if he could make c) 1/4 and he attempted to by drawing a diagram but this wasn't clear.

Derrick: The student was to select from a group of diagrams those that were showing one fourth. When he checked some of the diagrams he was asked to explain why he made those choices. For e) he said there are 4 parts and 1 part is shaded so that gives 1/4. For a) there are 5 parts with 1 shaded so that can't be 1/4. In responding to questions asked him by the interviewer, he said both size and shape are not important in deciding the fraction that has been shaded. For c) he said there are 8 parts and if you take away 2 parts, you are left with 6 parts which cannot be 1/4 and he cannot think of it as 1/4.

Roberta: The student was to select from a group of diagrams those that were showing one fourth. When she checked some of the diagrams she was asked to explain why she made those choices. She did not choose g) and h) because she thought the parts should be of the same size but they were not. She did not choose c) also saying that only 2 of the 8 parts were shaded, giving 2/8 which she did not recognize immediately to be 1/4. After a while, she said if you put those 2 parts together you might get 1/4, but he did seem to be sure, saying it is a tricky one. Since the 2 parts are not together, she did not want to call the fraction 1/4.

Treva: The student was to select from a group of diagrams those that were showing one fourth. When she checked some of the diagrams she was asked to explain why she made those choices. She chose b), e), f), g), and h) because they have 4 parts with 1 part shaded. She did not choose a) because there are 5 parts with 1 shaded, and this fraction she identified as 1/5. She did not choose c) because there are more parts (8) with 2 shaded. To make it 1/4, she drew 2 diagonals to get 4 parts and shaded 1 of them.

Dallas: The student was to select from a group of diagrams those that were showing one fourth. When she checked some of the diagrams she was asked to explain why she made those choices. She did not choose those with different sizes or different shapes. She did not choose c) because to have 1/4, there must be 4 parts with 1 part shaded but c) has 2 parts shaded so it could not be 1/4.

<u>Question 3</u>

Lindsay: The student was to determine what fraction of a circle was shaded. She said she was not sure; it looked like 1/3 but she was not sure. When asked if she could draw a line to help her provide an answer, she paused for a while, drew some line and then said the shaded portion must be 3/8.

Steve: The student was to determine what fraction of a circle was shaded. He said the shaded part is 1/4 and 1/2 (of 4). When asked to write it down, he said he could not.

Jodi: The student was to determine what fraction of a circle was shaded. She said she was not sure what part was shaded, but when asked to make a guess, she guessed 2/5. Her explanation was that there are 5 parts and 2 are shaded.

Fraser: The student was to determine what fraction of a circle was shaded. He guessed 1/3 saying that there could be 6 parts in all with 2 shaded, given 1/3.

Kristine: The student was to determine what fraction of a circle was shaded. She said she did not know but he will try. She could not come up with anything.

Tyler: The student was to determine what fraction of a circle was shaded and he said 1/4. His explaination was that half of the circle was not all shaded so the shaded part should be 1/4. When the interviewer asked him how parts of the circle are shown, he said 4 and identified the 4 parts. He then said the shaded part must be a 1/4 and a 1/2, but he could not explain. At least, he recognized that it must be more than a 1/4.

Derrick: The student was to determine what fraction of a circle was shaded. He said 1/4 and 1/2, where the 1/2 is of the 1/4, but he did not know how to write it.

Roberta: The student was to determine what fraction of a circle was shaded. She said it would be 1/4 and something else, may be 1/3, but her guess would be 1/4 and 1/5.

Treva: The student was to determine what fraction of a circle was shaded. She called the shaded portion "half without a quarter" or (half with a piece missing).

Dallas: The student was to determine what fraction of a circle was shaded. She gave an answer of 3/4 explaining that the two diameters divide the circle into quarters and the one shaded can then be divided into 2 parts. These two parts and the other part shaded together make 3/4.

<u>Question 4a</u>

Lindsay: She drew a picture to represent 1/3.

Steve: He drew a picture to represent 1/3.

Jodi: She drew a picture that I could not see.

Fraser: He drew a picture similar to that in question 3.

Kristine: She drew a square, left it to draw a circle, saying it is difficult to get a square into 3 parts, but easier with a circle. She shaded part of the circle.

Tyler: He drew a rectangle and divided it into 3 parts and shaded one.

Derrick: He drew a picture of 1/3. To him, 1.3 is 1 3/4.

Roberta: She drew a picture of rectangle, divided it into 3 parts and shaded 1 part.

Treva: She drew a rectangular figure, divided it into three parts and shaded 1 part.

Dallas: She couldn't do it.

<u>Question 4b.</u>

Lindsay: She was to shade 0.15 of a given square and she said she had no idea how to do that. When asked if it was because of the way the picture was drawn or it was because of the numbers, she said she could not do it because of the numbers.

Steve: He was to shade 0.15 of a given square but he said he could not do it.

Jodi: She was to shade 0.15 of a given square. She said she was not sure how to do it because she has not seen that type of picture and number before.

Fraser: He was to shade 0.15 of a given square. He counted 5 of the columns and shaded them as 0.15.

Kristine: He was to shade 0.15 of a given square. She said she did not know of a square like the one provided but she had seen a number like before. She could not do it.

Tyler: He was to shade 0.15 of a given square and he said he did not know what to do. When asked if it was the square or the number he did not know, he said both.

Derrick: He was to shade 0.15 of a given square. Starting with the second column, he shaded alternate columns and labelled them 05, 010, and 015 respectively. He could not explain what he did.

Roberta: She was to shade 0.15 of a given square. she counted 10 columns and said if there had been 5 more, she would have counted all and shaded them as representing 0.15. She thought the question was a tricky one and she hadn't seen that type before.

Treva: He was to shade 0.15 of a given square. She said she could not do it.

Dallas: She was to shade 0.15 of a given square. She said if there were 15 columns, she would have shaded all of them, but since there were only 10, she did not know what to do.

Question 5.

Lindsay: She was to choose the larger number in each of a pair of numbers and she did. It was difficult to tell which numbers she picked.

Steve: He was to choose the larger number in each of a pair of numbers. For a) 0.5, 0.47, he chose 0.47 as larger than 0.5; for b) 0.1, 0.09, he chose 0.09 because 9 is bigger than 1; for c) 0.3, 0.30 he chose 0.30 explaining that 30 is larger than 3.

Jodi: She was to choose the larger number in each of a pair of numbers. For a) 0.5, 0.47, she chose 0.47 as larger than 0.5 because 47 is larger than 5; for b) 0.1, 0.09, he chose 0.09 because there are more numbers in 0.09 than in 0.1; and for c) 0.3, 0.30 he chose 0.30 explaining that 30 is larger than 3.

Fraser: He was to choose the larger number in each of a pair of numbers. For a) 0.5, 0.47, he chose 0.47 as larger than 0.5; for b) 0.1, 0.09, he chose 0.09; and for c) 0.3, 0.30 he chose 0.30.

Kristine: He was to choose the larger number in each of a pair of numbers. For a) 0.5, 0.47, he chose 0.5 as larger than 0.47; for b) 0.1, 0.09, he chose 0.09 saying 0.09 has more numbers than 0.1; and for c) 0.3, 0.30 he chose 0.30.

Tyler: He was to choose the larger number in each of a pair of numbers. For a) 0.5, 0.47, he chose 0.5 as larger than 0.47; for b) 0.1, 0.09, he chose 0.1 saying that 1 which comes directly after the decimal is larger than 0 which also comes directly after the decimal in 0.09; and for c) 0.3, 0.30 he chose 0.3, and when he was asked if they could be the same he said they both have 3 after the decimal point so they could be the same but he felt 0.3 was lager than 0.30.

Derrick: He was to choose the larger number in each of a pair of numbers. For a) 0.5, 0.47, he chose 0.47 as larger than 0.5 because 7 is larger than 5; for b) 0.1, 0.09, he chose 0.1 saying that double zero is lower than single zero, so 0.09 must be less than 0.1; and for c) 0.3, 0.30 he chose 0.30 as larger.

Roberta: He was to choose the larger number in each of a pair of numbers. For a) 0.5, 0.47, he chose 0.5 as larger than 0.47 because 0.5 is half and it is bigger than 0.47; for b) 0.1, 0.09, he chose 0.09 saying that 0.1 is probably 1 and 0.09 is probably 9 and 9 is bigger than 1; and for c) 0.3, 0.30 he chose 0.30 because 0.3 is probably 3 and 0.30 is probably 30 and 30 is bigger than 3.

Treva: She was to choose the larger number in each of a pair of numbers. For a) 0.5, 0.47, she chose 0.5 as larger than 0.47; for b) 0.1, 0.09, she chose 0.09; and for c) 0.3, 0.30 she chose 0.30.

Dallas: She was to choose the larger number in each of a pair of numbers. For a) 0.5, 0.47, she chose 0.5 as larger than 0.47; for b) 0.1, 0.09, she chose 0.09 because 9 is larger than 1; and for c) 0.3, 0.30 she chose 0.30 since 30 is larger than 3.

Question 6.

Lindsay: She provided an answer of 6 to this problem and when asked how she thought about the problem, she said she figured out what number multiplied by 4 would give her 24 and that is 6.

Steve: He shook his head and said he did not get it. He asked if he could proceed to the next problem and he was allowed by the interviewer.

Jodi: She requested to draw a picture, she made 24 marks to represent the 24 students and then said she did not know how to do it because she was confused. The interviewer asked her what was confusing her and she said she did not know how to find 1/4 of the children. She was asked if she could do it if there were 4 students and nodded her head and gave 1 as the answer. After a while, she could still not provide an answer to the original question and even when the interviewer suggested she suppose there were 8 students, she could not provide an answer to this one also.

Fraser: He made 24 marks to represent 24 students and then circled 4 of them as representing 1/4.

Kristine: She got an answer of 4. When asked how she got it, she said she did not know but went on to say that she thought 1/4 could mean 4 or 14 and that was why she was not sure of how to solve the problem.

Tyler: He said he was not sure how to answer the question. The interviewer then asked him how many will be 1/2 of 24 and he said 12. But when he was asked again how many will be 1/4, he said he did not know.

Derrick: He asked if he could draw a picture and when encouraged to do so, he drew 24 lines to represent 24 students. He shaded 4 regions of his diagram and subtracted 4 from 24 to get 20 as the number of students who ride to school.

Roberta: She said that's the type of question she likes. For an answer to 24/4, she first found 24/2 to get 12 and then 12/2 to get 6, saying if you put all the sixes together, you get 24.

Treva: She wanted to draw a picture and she was allowed to do so. She drew 24 lines to represent 24 students and then gave an answer of 4 after dividing 24 by 6. She was asked how she got the 6? She said she got confused and that she could not continue.

Dallas: She said she couldn't do it. When the interviewer asked her to suppose there were 24 students in her class and then asked her to find 1/2 the class, she gave an answer of 10 saying 10 is closer to half of 24, without giving what that value is.

Question 7.

Lindsay: She circled one of each of the pairs of numbers given as the larger of the pair. It was difficult to tell which ones were circled.

Steve: For a) 3/5, 4/5 he chose 4/5 as larger; for b) 2/4, 2/3 he chose 2/4 as larger because "it is bigger", after making a picture in his head.

Jodi: For a) 3/5, 4/5 she chose 4/5 as larger because 4 is larger than 3; for b) 2/4, 2/3 she chose 2/4 as larger explaining that for 2/4, you take away 2 from 4 to get 2 and for 2/3 you take away 2 from 3 to get 1, and since 2 is larger than 1, 2/4 is larger than 2/3.

Fraser: For a) 3/5, 4/5 he chose 4/5 as larger than 3/4; for b) 2/4, 2/3 she chose 2/4 as larger than 2/3.

Kristine: For a) 3/5, 4/5 he chose 4/5 as larger than 3/5 because the denominators (5) are the same but 4 is larger than 3; for b) 2/4, 2/3 she chose 2/4 as larger than 2/3 because the numerators (2) are the same but 4 is larger than 2.

Tyler: For a) 3/5, 4/5 he chose 4/5 as larger than 3/5 because the fives are the same and 4 is larger than 3; for b) 2/4, 2/3 she chose 2/4 as larger than 2/3 because the twos are the same and 4 is larger than 2.

Derrick: For a) 3/5, 4/5 he chose 3/5 as larger than 4/5 because 4-5=-1 and 3-5=-2, but -2 is larger than -1 so 3/5 must be larger than 4/5; for b) 2/4, 2/3 she chose 2/4 as larger than 2/3 argueing similarly that 2-4=-2 and 2-3=-1 and since 2- is larger than -1, 2/4 must be larger than 2/3.

Roberta: For a) 3/5, 4/5 she drew a picture and explained that 4/5 should be 1 more than 3/5 so she chose 4/5 as larger than 3/4; for b) 2/4, 2/3 she said one has to draw boxes of same sizes otherwise one of the numbers may seem lager even if it's not, so after drawing 2 rectangles of the same size and dividing one into 3 parts and the other into 4 parts, she chose 2/3 as larger than 2/4.

Treva: For a) 3/5, 4/5 she chose 4/5 as larger than 3/5; for b) 2/4, 2/3 she chose 2/4 as larger than 2/3.

Dallas: For a) 3/5, 4/5 she chose 4/5 as larger than 3/5 because 4 is larger than 3; for b) 2/4, 2/3 she chose 2/4 as larger than 2/3 because 4 is larger than 3.

Question 8a.

Lindsay: When she was to mark 0.75 on a given number line, she started counting for a while and said she was not sure where to mark it. She was asked if she were to mark 10 1/2 on the given number line, she circled a point on the line.

Steve: When she was to mark 0.75 on a given number line, he started counting by tens to get 75 and he did not find it possible because 75 was not on the given number line.

Jodi: When she was to mark 0.75 on a given number line. She said she was not sure of the number and the number line.

Fraser: When he was to mark 0.75 on a given number line, he counted 7 divisions and then counted another 5 (corresponding to 1.2) and located 0.75 there.

Kristine: When she was to mark 0.75 on a given number line, she said she did not know about both the number line and the type of number.

Tyler: When he was to mark 0.75 on a given number line, he said he was not sure about the number and the number line.

Derrick: When he was to mark 0.75 on a given number line, he counted 7 divisions and then 5 more to locate 0.75 there.

Roberta: When she was to mark 0.75 on a given number line, she said she did not know adding "this one is hard, oh boy" and she could not do it.

Treva: When she was to mark 0.75 on a given number line, she said she does not understand the number line.

Dallas: When she was to mark 0.75 on a given number line, she said she did not understand.

Question 8b.

Lindsay: When she was asked to mark 3/4 on a given number line, she could not do it. She was then asked what she thought the numbers 0, 1, 2, 3, ... meant and she said she was not sure.

Steve: When she was asked to mark 3/4 on a given number line, he counted past 3, then counted 3 of the 4 divisions between 3 and 4 and located 3/4 there (this rather turns out to be 3 3/4).

Jodi: When she was asked to mark 3/4 on a given number line. She located it at 3 on the number line saying that you count 3 out of 4 numbers to get 3/4.

Fraser: When he was asked to mark 3/4 on a given number line, he counted the divisions marked 1, 2, 3, 4 and located it at 4 on the number line.

Kristine: When she was asked to mark 3/4 on a given number line, she said she did not know.

Tyler: When he was asked to mark 3/4 on a given number line, he said he could not do it

Derrick: When he was asked to mark 3/4 on a given number line, he counted 4 divisions and shaded 3 of them saying that was 3/4 He referred to a rectangular which when divided into 4 parts, 3 parts of it gives 3/4.

Roberta: When she was asked to mark 3/4 on a given number line, she said it depends on which one is the 4, so you just count 1, 2, 3 and locate 3/4 at 3. Pondering for a while, she said "three fourths are twelves" so she counted to the 12th mark and located 3/4 there.

Treva: When she was asked to mark 3/4 on a given number line, she located it somewhere that I couldn't tell.

Dallas: When she was asked to mark 3/4 on a given number line, she said she did not understand.

<u>Question 8c.</u>

Lindsay: She was to mark 1 2/3 on a given number line. She said she was not sure because the number is 1 2/3; she could have done it if it were only 2/3 and she knew the position of 1. She was shown the positions of 1 and 2 on the number line and then asked to locate 1 2/3. This time, she pointed to a spot on the number line, but there was no indication as to whether it was correct.

Steve: He was to mark 1 2/3 on a given number line. He counted 1, 2, and 3 and located 1 2/3 at 3. he was asked to explain his answer and he said he did not know and that he was only trying his best.

Jodi: He was to mark 1 2/3 on a given number line. She located it at 2 of the 3 numbers given saying that she did not know what to do with the 1.

Fraser: He was to mark 1 2/3 on a given number line. He counted the divisions 1, 2, 3 and located it at 3 on the number line.

Kristine: She was to mark 1 2/3 on a given number line. She counted 1, 2, 3 and located 1 2/3 a t 3, repeating she hadn't heard of the number line before.

Tyler: He was to mark 1 2/3 on a given number line and he said he could not do it.

Derrick: He was to mark 1 2/3 on a given number line. He counted 0, 1, 2 and located 1 2/3 there.

Roberta: She was to mark 1 2/3 on a given number line. There are 3 divisions for a unit on this number line, so after counting 1 unit, she counted 2 more divisions and located 1 2/3 there.

Treva: She was to mark 1 2/3 on a given number line, she located it somewhere that I couldn't tell.

Dallas: She was to mark 1 2/3 on a given number line. She said she did not understand; she had not learnt about the number line.

Grade 4, Interview 2, May 22, 1991

Question 1: What fraction of the circle is shaded?
Dallas: One third, no, 1/3 and a 1/2 of something.....1 and 1/2 fourths.....(draws in more lines) looks like 3/9, I think, oh, 3/8 (confidently).
Treva: One & a half out of 4. Then, after being asked for a fraction, drew in more lines and replied "3/8". Also said that 1 and a 1/2 fourths didn't "sound right".
Steve P: 4ths and a half. Took about 4 minutes to finally get 3/8, from the drawing.
Jodi: Got 3/8 after drawing in lines.
Tyler: Pauses about 5 seconds, then says "3/8", and then draws in line to explain.
Roberta: Draws in lines counts the parts and says "3/8".

Question 2: Draw a picture of 1.3
Dallas: Drew
Treva: Read it as 1/3, started with circular region, was dissatisfied with sectors not looking equal, and switched to a rectangular region, shading in 1/3 correctly.
Steve P: Not asked.
Jodi: Not asked.
Tyler: Not asked.
Roberta: Though it was 1/3, so skipped the question.

Question 3: Twelve students in a class completed three hours in a "Jump Rope for Heart" Charity competition. One fourth of these students were boys. How many of the twelve students wee boys?
Dallas: I'm thinking, 1/4 of these students were boys...I'm not sure. If I draw a picture,....can't figure that out. Can I draw a picture of stick men? Compares 1/4 of rectangle she has drawn with 1/4 of the stick men and comes up with 6 boys (incorrect), and the fraction 6/12 (even though the question did not ask for a fraction)
Treva: Said "Have to divide here; we did this before, but I can't remember". Thought there would be 6 boys and 6 girls. Had initially said "6 boys and 6 girls, that's half ..., 6/12 ", and thought it still answered the question even though was asked "Does it match the question?". Seemed to ignore the 1/4 in the question.
Steve P: Drew a picture "12 boxes", but even after discussion could only come up with 1/12, and not the answer to the question asked.

Jodi: Drew picture of 12 stick people, in a 3 (rows) by 4 (columns) array. Circled one in each row, to indicate 1/4, then said "1/4 + 1/4 + 1/4 is 3/12", not 3/4 or 1/4, and that "3 students are boys, 9 are girls". As a fraction, could see 9/12, but could not see 9/12 = 3/4, and when asked why 3 groups, said "it said 4, so I drew 4 people".
Tyler: Thinks (without writing anythin) for about 7 secs, then says "3", because "quarter of the people, 12, see how many in 1/4, that is, 1 boy in 3, therefore 12 students, that will be 3, because 4 goes into 12, thre times". Also explains "1 plus 1 plus 1 is 3, & 4 plus 4 plus 4 is 12....". Easily gave 9 as the # of girls, but takes a bit of time to see that 3/4 of the students were girls.
Roberta: Said 3 with no hesitation, and explained "There's 12 & 12 divided by 4 is 3", and when asked why divide by 4, replied "because 1/4 of the boys, so you got to divide by 4". Answered correctly that there were 9 girls, and that would be 3/4 of the students.

Question 4: Arrange the following from smallest to largest:
a. 7/12, 11/12, 5/12 b. 4/3, 4/5, 4/10
Dallas: a) 5/12. 7/12, 11/12 because 11 > 5 & 7. Says she sees a box with say 7 of the 12 parts shaded.
b) 4/10, 4/5, 4/3; explained by using a rectangle with parts shaded in.
Treva: a) 11/12 is biggest and 5/12 is smallest because "all 12".
b) 4/10 , 4/5, 4/3; drew a square and divided it to illustrate. Said "..room for people, it would be easier to put 4 tenths than 4 fifths because 4 tenths is smaller". Her subsequent explanation was hard to follow. She said "If you had like a box, some amount of erasers in it, you had to fit 10 lines into it, 5 lines or 3 lines, not so hard. If it takes up a lot of room, it's probably smaller. If you had a cake,, therefore if you had 10 people, if 10 people, you would get a larger piece. If 4/5 and 4/3, 4/3 larger, 5ths you could space them out", then draws 4/3. (Time on tape: 8.48 to 8.51)
Steve P: a) 5/12 smallest as "doesn't take up as much space as 7/12 or 11/12".
b) 4/10, 4/5, 4/3, by similar reasoning as part a), and knew 4/3 > 1
Jodi: a) 5/12 < 7/12 & 11/12 because "5 < 7 & 11", and all were "same fraction".
b) 4/10 is smallest as "10 larger # and more in it - if you draw a picture, it takes up more space, there's less space in a little piece; 4/10 = 2/5, so smaller space. 4/3 is one whole unit & a third, & thirds don't take up space & there's more space left".
Tyler: a) Orders correctly, and says 5/12 is smallest because "only 5 pieces of the 12" and compares with pieces of a cake.

b) Orders correctly, and explains with reference to cake, without recourse to a diagram, but then follows suggestion to use drawing to explain. Uses circles with sectors to explain, correctly.

Roberta: a) Ordered correctly, and said she visualized a square divided into 12.

b) Ordered correctly, saying "10th < 5th < 3rd & there's same amount of each". But was quite positive that 1 whole was different from the # 1.

Question 5: What common fraction is shown by the shaded part of the figure below? Give other names for this fraction.

Dallas: 6/10. then added a few lines to get 12/20. Said "I can imagine in my mind that the line is not there, & move this.." to get 3/5

Treva: 6/10, then 12/20 for another name, and 24/40 as well, later. Could not rename to 5ths initially, and thought that shaded parts of one half of the figure showed 3/5, that is considered that half to be one whole. Finally, did see that the shaded parts represented 3/5 of the whole figure.

Steve P: 6/10 (by realigning shaded parts) but couldn't get 3/5, or other names.

Jodi: 6/10; also had no trouble seeing 3/5 ("remove line, slide them over"), 12/20 (drew the lines in her mind), and could see the pattern.

Tyler: 6/10, then 3/5, from the figure, referring to only the top portion of the figure. Says "It doesn't show 3/5 in this picture, but it adds up to 3/5". Seems to think that 3/5 at the top of the figure and 3/5 at the bottom of the figure make up 3/5 of the whole figure.

Roberta: Had no problem identifying 6/10, and renaming as 12/20 and 3/5. Could see a # pattern, and could get 6/10 and 3/5 without need to move parts of figure in the mind.

Question 6: Fraser and Treva share a granola bar. Fraser eats half of it and Treva eats one fourth of it. How much of the granola bar do they eat altogether? If you wish, use the fraction fraction strips or a drawing to help work out the answer.

Dallas: Used strips ("they help me"), and got 3/4. Could see 1/2 = 2/4

Treva: (Was tickled to seee her name on the problem, as well as Fraser's "he sits beside me, too"!). Drew a rectangle and found 3/4 was eaten and 1/4 was left over. She knew 1/2 = 2/4.

Steve P: "3/4 was eaten", but only after some probing could see that 2/4 = 1/2

Jodi: Drew a rectangle, got 3/4 ("can pretend there is a line here"), and knew 1/2 = 2/4

Tyler: Takes some time thinking until asked to draw. Then gets 3/4 from the drawing.

Roberta: In 3 secs she worked it out mentally and gave the correesct answer. Sa:d she drew a picture in her mind.

Question 7: The rectangle divided into ten equal parts is one whole unit here. Write down as many different fraction names as you can for the shaded parts.
Dallas: 2/10, then "if you stuck a line right in between (indicating an imaginary horizontal line with her hand), would get 4/20". When asked whether could be put in 5ths, said "I don't think so, unless you go 1, 2, 3, 4, 5 (ignoring half of the rectangle)". Then she saw that it could be done.
Treva: Not asked.
Steve P: Not asked.
Jodi: Not asked.
Tyler: Not asked.
Roberta: 2/10, 1/5 & said "I could go on forever" and said "I sort of thought back to the question on 6/10". Interestingly, she had been absent for the lesson on equivalent fractions.

Question 8: Of the 27 children in a class, one third are girls. How many are girls? How many are boys? Show your work below. You may use the fraction strips or a picture to explain your work.
Dallas: Used fraction strip 1/3, then says could be 6 boys, and tries repeated addition of 6 but does not get 27, so tries 8, is unsuccessful again and finally gets 9 boys and 18 girls. Used fingers to add and was very slow at subtracting 9 from 27.
Treva: Drew 27 strips, tried 3, circling every 3 strips, and found she was wrong. After some time said "9 of them are girls" and that "I didn't do the fraction thing, I just divided 27 by 3". Checked with diagram when asked to do so. Could see that 18/27 was the fraction of boys, but didn't see any connection between 1/3 and 9/27 or 2/3 and 18/27.
Steve P: Discarded fraction strips which he had initially taken, and drew. Tried 1, 3 ("you don't usually get 3 girls in a class"),6; Then "6/27 are girls and 21 are boys"
Jodi: Drew 27 'people', muttered "groups of 3", and put the 27 in groups of 3. Then "so 9 of them are girls & 18 of them are boys" because "you can go 3 times 9 is 27, then 27 can be broken into 3 equal parts of 9". Said this was easier than the 'Jump Rope' question, which had 1/4 in it, because "3 times 9 is 27".
Tyler: After thinking for about 4 secs, says "9". Explains "One third are girls, so if 3 is divided by 27, is 9, & 9 divided by 1 is or if 1 divided by er..(pauses), if 3 times 9 is

27, so would be 9, I think". When asked for the # of boys, thinks for about 10 secs, then answers "18" and explains "if 1/3 are girls, that's 1 of 3 people, so there will be 2 boys, so 2 times 9 is 18". When asked how he thought about groups of 3, said "because 3 divided by 27 is 9". Explained also "...there's 2 in each one (group of 3), there has to be 9 groups (of 3), so if each group there is 1, then 9 girls".
Roberta: Draws 27 lines. Three secs later says "I think 9 are boys, 18 are girls". Explains "27 divided into 3 is 9; if 1/3 girls, 1/3 of 27 is 9 & what is left over is 18". Explains also that 2/3 are boys because "if 1/3 are girls, 2/3 are boys".

Question 9: The square divided into 100 equal parts is one whole unit here. Shade in the decimal fraction 0.4 in the figure below.
Dallas: couldn't see on the tape!
Treva: Shades in 4 small squares, and says "4 over zero, I guess". Then "Oh, 100 boxes, so 4/100". Can't decide from tape whether she knew 0.4 = 4/10 or whether she shaded in 4/10 correctly.
Steve P: Says "4 boxes out of 100", writes "4/100, 0.4", and says "no wholes and 4 hundredths". Incorrect.
Jodi: Shaded 4 small squares , because "zero stands for no whole units & 4 out of hundredths". Was not sure about 4/10 and 4/100
Tyler: Read 0.4 as 4 out of 100, then when told that was wrong, thought for a while and came up with 4 tenths. Then shaded 4 small squares (horizontally). When asked to shade in the 100ths, began to shade downwards until 4 strips were completely shaded. Then takes about 3 minutes to get 40/100 and see a # pattern. Said that he preferred to have pictures than to think without them.
Roberta: Thinks she had to shade in 40, but that was 4/10, though was not too sure about it. Compared it with question where she divided by 3 (the 27 students), saying "so here divide 100 into 10 & so divide into 10, & we need 4".

Question 10: metre stick
Dallas: Had no difficulty with 1/2 but took a long time to show 1/2 of 1/2 was at the 25 cm mark. Even so was unsure.
Treva: Initially identified the 50 cm mark as 1/2 of the metre, but subsequent probing with 1/2 of 1/2 led her to 45, and being confused.
Steve P: Had a lot of trouble understanding the question (took more than 5 minutes to answer the questions here), before finally getting the answers to 1/2, 1/2 if 1/2 etc.

Jodi: Had no difficulty with 1/2 the metre stick, but initially thought the 1/2 meant the point rather than the space, but corrected herself later. Took a bit of time to see 25/100 = 1/4; when asked whether she thought of $ and quarters, said she didn't.
Tyler: Had no trouble seeing 1/2 was at 50 cm mark, 1/2 of 1/2 was at 25 cm mark, and that was 1/4 of the metre stick, 3/4 was at 75 cm mark, and 1/4 = 25/100.
Roberta: Had no trouble with this question. Thought of half as the space between 0 and 50 cm rather than the 50 cm mark.

Interview 3

Question 1

Treva: She drew two rectangles and divided one into 3 parts to represent 1.3. which she called "one whole and three parts." Later, she said she "screwed up" because she drew a diagram for 1 1/3 instead of 1.3.

Roberta: She drew a rectangular figure, divided it into 10 parts and shaded 3 of them as representing 1.3 which she call 1 1/3. She later realized what she drew represents 3/10 but she could not continue to solve the problem.

Dallas: She read 1.3 as "one third" and drew some diagram which I could not see.

Tyler: He said 1.3 is a whole and 3/10. He drew a picture and explained part of as representing a whole and the other part 3/10.

Steve: He drew a rectangular figure, divided it into 3 parts and shaded one of them which he called a third (his representation for 1.3).

Jodi: She drew a picture and said one part stands for a ten (one part) and the other part stands for 3/10.

Question 2

Treva: For a), she drew a diagram to explain her answer which was not clear to me. For b), she got the arrangement 0.8, 0.09, 0.10. (She pronounced 0.10 as "zero point ten").

Roberta: For a), she got 4/10, 4/5 and 4/3 and drew a picture to explain why 4/5 is larger than 4/10. For b), she said she was not sure but for a best guess, she would go for 9/100 as the smallest because it's not tenths but hundredths and hundredths are smaller than tenths. She had some difficulty deciding between 0.10 and 0.8. with the help of tenths and hundredths strips from the interviewer, all she could do was to come up with 0.10 as 10/100 and 0.8 as 8/100, thereby making 0.10 larger than 0.8.

Dallas: Her arrangement for a) was 4/10, 4/5, and 4/3, using diagrams to explain. For b), she got 0.8, 0.10, and 0.09, but she could not explain her answer.

Tyler: For a), he got 4/10, 4/5, and then 4/3, explaining that it is the same box into 10 and 4 pieces for 4/10 and 4/5, and the same box plus 1/3 of the box for 4/3. For b), he converted 0.10 as 10/100, 0.09 as 9/100, and 0.8 as 8/10 and got 9/100, 10/100 and 8/10 as the order in magnitude. When given strips to explain his solution, he used tenths and hundredths and got confused, he was mostly nodding at what the interviewer said.

Steve: For a), he first said 4/3 is smallest followed by 4/5 and then 4/10 because one would not have "as much room" for 4/3 as one would have for the rest of the numbers. When he was asked by the interviewer to draw a diagram of the fractions, he drew some diagrams and after a pause changed the order to 4/10, 4/5, and 4/3. For b), he changed to fractions 0.10 as 10/10, 0.09 as 9/100, and 0.8 as 8/10 and the said 9/100 is largest because of the 100, followed by 10/10 and then 8/10.

Jodi: For a), she got 4/10 followed by 4/5 and then 4/3, saying that 4/10 "takes less room" while 4/3 "takes more room." For b), after drawing some pictures, she got 9/100 followed by 10/100 and then 8/10.

Question 3

Treva: For a), she first said they were equal. When asked to explain her answer, she asked for fourths and tenths strips and after comparing them, she changed her mind and said 1/4 is bigger than 0.5. For b), she said 1/3 is larger, saying that it will be easier to divide a box into 3 parts than into 10 parts, using fraction strips to explain why 1/3 is larger. For c), she said 3/4 is larger but after using strips she changed her mind and said 4/5 is larger.

Roberta: For a), she used 1/4 and 1/10 strips to conclude that 1/4 is larger than 0.2. For b), she similarly used tenths and thirds strips to decide that 1/3 is larger than 0.3. And for c), she used hundredths strip to convert 4/5 to 80/100 and 3/4 to 75/100 and concluded that 4/5 is larger than 3/4. She recognized 4/5 as 0.80 and 3/4 as 0.75.

Dallas: For a), she chose 1/4 to be larger, saying "2/10 will be squeezed and 1/4 will be larger." For b), she chose 3/10 as lager because she imagined a picture of 3/10 to be larger than a picture of 1/3. But when she was given strips to prove her case, she changed her mind and said 1/3 is larger than 3/10. For c), she guessed 3/4 might be larger, but after using strips, she said 4/5 is larger than 3/4.

Tyler: For a), he said 1/4 is larger than 2/10, but argued that 1/4 of 4 is 1 while 1/4 of 10 is 2.5; his reasoning was not clear. For b), he said 1/3 is larger than 0.3 because the diagram he drew showed 1/3 to be larger than 0.3. For c), he first said 4/5 is larger than 3/4 but when he was asked to explain his answer, he changed his mind and said 3/4 is larger than 4/5. His explanation was that dividing a unit into 5 parts will be smaller than dividing a unit into 4 parts, so 3/4 must be larger than 4/5.

Steve: For a), he could not use his diagrams to decide, but when he was given fraction strips to use, he came up with 1/4 as larger. For b), he found 3/10 and 1/3 to be the same but when he used fraction strips, he found 1/3 to be larger. Fro c), he did not draw diagrams again, but instead asked for fraction strips to use. He first guessed that 3/4 will be larger than 4/5 but after using the strips, he realized 4/5 is larger than 3/4.

Jodi: For a), she used a diagram she drew to claim that 1/4 equals 2/10. She was given a strip in tenths and then asked to locate 1/4 on it but she couldn't. She was then asked to locate 1/2 on it and she did, and when later she was asked to locate 1/4 on it, she did that with some difficulty. With several other strips to aid her, she came up with 1/4 as larger than 2/10. For b), she used her diagram to claim that 1/3 is larger than 0.3 and she was able to use the fraction strips to confirm her answer. For c), she first used her diagram to claim 4/5 is larger than 3/4, but when using the flats to illustrate her solution, she changed her mind and said 3/4 is larger than 4/5. After some few manipulations with the flats and realizing that 3/4 is 75/100 and 4/5 is 80/100, she changed her mind again saying that 4/5 is larger.

Question 4

Treva: She drew a rectangular box and divided it into 3 parts but had difficulty shading 0.25 of the box. After awhile, she said Tyler had nothing to eat. When challenged to explain her answer, she started all over again after recognizing that 0.25 is 25/100 and it's 1/4. But the diagram she drew was just like the first one and all she could say was that Tyler would eat more than one of the 3 parts of the box.

Roberta: She drew a rectangular figure and divided it into 4 parts noting that Dallas ate 1 of the 4 parts (1/4 or 0.25). She found it difficult illustrating with the diagram the part (1/3) eaten by Roberta. When she was given strips to aid her she said it was easier for her and she wrote down what fraction of the bar Tyler ate but it was not visible.

Dallas: She first tried drawing a diagram but later said she could not do it. She was given fraction strips, she picked thirds and fourths and aligned them along a unit strip, one after the other. She then said the portion that was left of the unit strip represents the fraction of the granola bar eaten by Tyler, without being able to tell what the fraction is. But after trying several other strips, she found out that 5/12 fit the portion of the unit strip left over and she concluded Tyler ate 5/12 of the bar.

Tyler: He started by drawing a rectangular figure which he divided into 3 unequal parts. He labelled one part 1/3, the other part 1/4 (0.25) and the remaining part as the portion eaten by Tyler, but he could not tell what fraction that is. He said he might know by adding 1/3 and 1/4. After using several strips with the help of the interviewer, he concluded that Tyler ate 5/12 of the bar.

Steve: He asked if 0.25 stands for 25/100. The interviewer replied yes and asked him what that fraction stands for and he said 1/4. He drew a diagram and shaded a portion as 1/3, another portion as 1/4, and then said the remaining portion which he called 2/4 was what Tyler ate.

Jodi: She drew a picture and shaded a portion she called a third. She shaded another portion representing 1/4 and said the remaining portion, 50/100, was what Tyler ate, but she could not explain how she came by the value 50/100. She was asked to use fraction strips to help her but she could not do that on her own and it was with extensive help from the interviewer that she finally came up with 5/12 as the portion eaten by Tyler.

<u>Question 5</u>

Treva: She got 7/10. When she was reminded that some kids would get 7/20, she said the whole question is about tenths, not twentieths, so the answer should not be 7/20.

Roberta: She got 7/10 and when she was reminded that some kids get 7/20, she said one has only tenths but not twentieths, so it can't be 7/20.

Dallas: She got 7/20 as an answer. The interviewer asked her to justify her solution by using fraction strips. She aligned one 3/10 strip over a 10/10 strip, did the same thing for one 4/10 strip over a 10/10 strip and said she got 7/20. The interviewer moved one of the 10/10 strips away and with a lot of guidance the student finally agreed that the result should be 7/10.

Tyler: He got 7/10, saying that the unit is tenth, so 3/10 and 4/10 should give 7/10. When he was reminded that some kids would get 7/20, he said he thought of it in the same way initially, but realizing that it should be one unit, the answer should be 7/10.

Steve: He first got 7/10 as the answer but when the interviewer was about to remind him that some kids would get 7/20, he quickly changed his mind and said he would get the same 7/20 because 3+4=7 and 10+10=20. Therefore 3/10 + 4/10 = 7/20. However, when he was given tenths strips, he came up with 7/10 as an answer, but when he was asked to confirm his answer, he again said 7/20. With two tenth strips, the interviewer helped him to claim that with twentieth as unit, 3/20 + 4/20 = 7/20.

Jodi: She got 7/10 because the problem is dealing with tenths and it cannot be 7/20, when this latter value was suggested to her as a possible solution.

<u>Question 6</u>

Treva: She said for problems of this type, she normally divides to find out how many boys and how many girls. When asked what she will divide 27 by, she said she was not sure. He was asked if 2/3 of the class are girls, what fraction are boys and she could not answer. After a while, she divided 27 by 3 to get 9. She then drew a rectangular figure and divided it into 3 parts, with each part representing 9 kids. She finally came up saying 2/3 should be girls and 1/3 should be boys after being quizzed several times by the interviewer.

Roberta: She said she likes this type of problem because it gives her "better ideas" than that with only numbers. Saying she normally works in her head, she divided 27 by 3, paused for a while and gave an answer of 18. Her explaination was that after dividing by 3 she got 9. She then added another 9 to get 18 as representing 2/3 of the class. She noted that 1/3 is the fraction of boys.

Dallas: She paused for a while and asked for third and two-third strips. She compared the strips and said there are 2/3 girls and 1/3 boys, but she could not tell how many of each. The interviewer asked her if there were 5 students for each third, how many students would be there altogether and she said 15. She tried 6, 7, and 8 students per each third and when none of them worked, she said it might not work for 9 also. She was getting frustrated when her several attempts were not providing her with a solution. The interviewer suggested other alternatives, like trying 3 tens to get 30, but she did not seem to get it. Finally, the interviewer had to tell her that 3 nines give 27.

Tyler: After a long pause, he drew a rectangular figure and divided it into 27 parts. He drew another similar figure under the first one and after a while she said there must be 18 girls because "3 divided by 27 gives you 9 and you have to times it by 2 to get 18 girls, leaving 9 boys." So for boys, the fraction must be 1/3 which he got from drawing an analogy from having 3 people and 1/3 being 1 and 2/3 being 2.

Steve: He asked for twenty seventh strips but he was told there was none. He was then asked to make one from cubes that were available, but he said it would be easier if he used flat strips. He got 2 of the tenths and 1 of the sevenths and he put them together. He counted 9 divisions as girls and said there were 18 girls (after counting all 18 divisions) and 9 boys.

Jodi: She made 27 marks, circled 18 of them and said there are 18 girls and 9 boys. When she was asked what fraction are boys, she said 1/3.

Question 7

Treva: She shaded some portion of the diagram to represent 0.4 which she also referred to as four hundredths.

Roberta: She shaded a portion and called it 40 hundredths or "zero point forty". When she was asked for other names for 0.40, she wrote some down, like 2/5, and called them equivalent fractions.

Dallas: She shaded 4 of the 100 squares and called that "4 over 100", not zero point four.

Tyler: He identified 0.4 as 4/10, and after a pause he said 4/100, but he shaded 4 of the tenths, that is 40/100. He gave 2/5 as another name for 0.4.

Steve: He said 0.4 looks like 4/10, or 4 hundredths. But he marked 4 of the tenths to get 40 of the 100 squares. For a common fraction name of 0.4 in fifths, he gave 2/5.

Jodi: She shaded 4 columns and gave 2/5 as another name for 0.4.

Question 8

Treva: She said 1 out of 4 balls will be 1/4 but could not readily find 1/4 of the 8 balls. when she was referred to the diagram accompanying the question, she could still not find 1/4 of the 8 balls. The interviewer tried drawing a diagram to help her find 1/4 of the 8 balls but she was only nodding the head without her being able to find the solution.

Roberta: She counted 8 balls and took 2 of them as representing 1/4. Her explanation was that she divided the balls into 2 sets of 4 and 1 of each set represents 1/4, so for the 2 sets, there will be 2 balls out of 8 to give 2/8 or 1/4. When asked why she picked only small balls, she said she did that to avoid carrying heavier balls.

Dallas: She counted 8 balls and selected 2 of them as 1/4. She argued that taking 2 out of the 8 balls gives 2/8 or 1/4, and that she could choose any two balls, irrespective of size and shape.

Tyler: He counted 8 balls and chose 2 of them as representing 1/4. He was asked if the sizes of the balls mattered and he said he did not think so.

Steve: He first picked one of the 8 balls as representing 1/4, but after the interviewer asked him to identify the unit which he gave as 4, he came up with 2 of the 8 balls as representing 1/4. When he was asked why he chose only big balls, he said he could "even it up" by choosing one big and one small ball.

Jodi: She said with 4 balls, she will pick 1 as representing 1/4, but with 8 balls, she did not know what to do. But when the ball s were put into fours, she was able to pick one ball for each group to have 2 balls out of 8 as representing 1/4.

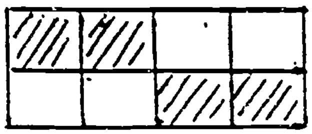
APPENDIX E

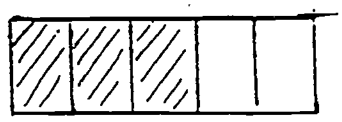
LESSON 1

COMMON FRACTIONS: CONCEPTS & SYMBOLS

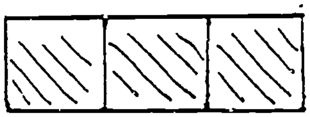
NAME: ______________________

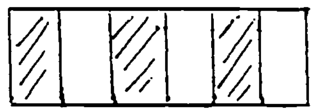
1. NAME THE COMMON FRACTION THAT TELLS HOW MUCH OF THE SHAPE IS SHADED.





A. ______________________ B. ______________________

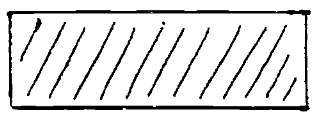




C. ______________________ D. ______________________

2. BELOW IS A PICTURE OF ONE WHOLE UNIT
DRAW A PICTURE FOR EACH FRACTION OF THIS UNITS.

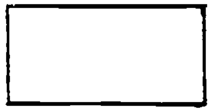
ONE WHOLE UNIT



2 THIRDS OF THE UNIT

3 SIXTHS OF THE UNIT

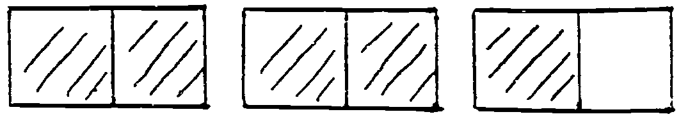
4 FOURTHS OF THE UNIT

3. IF  IS ONE WHOLE UNIT,

WHAT IS THE <u>FRACTION NAME</u> AND <u>MIXED NUMBER NAME</u> FOR THE AMOUNT OF THE UNITS SHADED

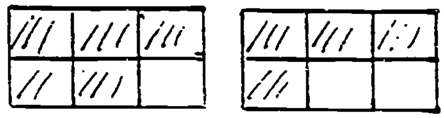
A. ____________________

____



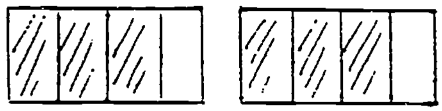
B. ____________________

____



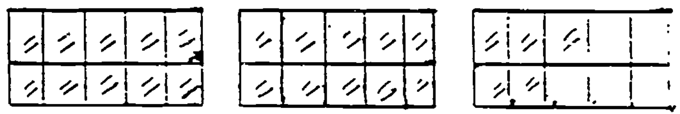
C. ____________________

____



D. ____________________

____



4. IS  <u>1 THIRD</u> OF THE RECTANGLE?

WHY OR WHY NOT?

____

____

5. IS  <u>5 FOURTHS</u> OF THE RECTANGLE?

WHY OR WHY NOT?

____

____

6. WRITE 3 DIFFERENT COMMON FRACTION NAMES FOR ONE WHOLE UNIT.

____________ ____________ ____________

HOW MANY DIFFERENT COMMON FRACTION NAMES ARE THERE FOR ONE WHOLE UNIT?

____

7. CIRCLE THE BIGGER COMMON FRACTION OR MIXED NUMBER IN EACH PAIR.

A.	1 FOURTHS	1 FIFTH
B.	4 SIXTHS	2 SIXTH
C.	1 AND 3 FOURTHS	1 AND 1 FOURTH
D.	1 TENTH	1 HUNDREDTH
E.	1 AND 1 HALF	3 HALVES

WORKSHEET 2

COMMON FRACTIONS

NAME: ______________________________

1. WRITE THE COMMON FRACTION NUMBER FOR EACH NUMBER NAME.

A. 2 TWELFTHS ________ B. 3 FOURTHS ________

C. 4 TENTHS ________ D. 3 FIFTEENTHS ________

E. 7 THIRDS ________ F. 9 FIFTHS ________

2. DOES EACH PICTURE SHOW THE AMOUNT OF THE COMMON FRACTION IN THE SENTENCE. WHY OR WHY NOT?

A.

JOHN ATE $\frac{3}{4}$ OF THE GRANOLA BAR.

____

B.

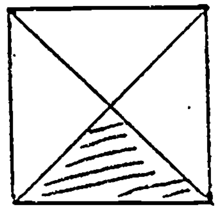
CAROLINE IS 3 FOURTHS THE HEIGHT OF MARY.

____

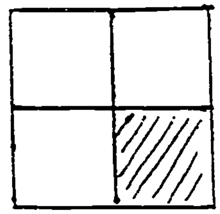
3. EACH OF THE TWO PIECES OF CLOTH WERE CUT UP IN DIFFERENT WAYS. THE SHADED PARTS WERE USED TO MAKE COSTUMES.

WHAT FRACTION OF EACH PIECE OF CLOTH WAS USED? WERE THE SHADED PARTS THE AMOUNT OF CLOTH, OR WAS ONE MORE THAN THE OTHER? HOW CAN YOU TELL?

A.



a.

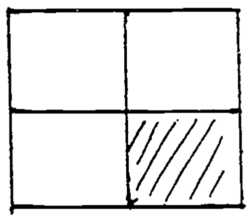


b.

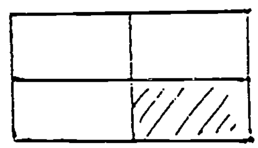
FRACTION NAME. ____________ ____________

SHADED PARTS,SAME AMOUNT? YES/NO
IF NO, WHICH IS MORE? _____

B.



a.

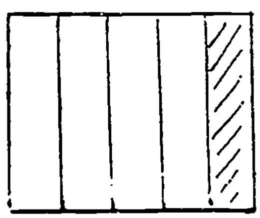


b.

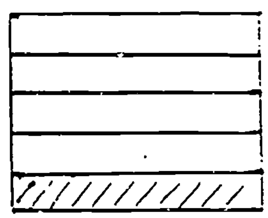
FRACTION NAME. ____________ ____________

SHADED PARTS,SAME AMOUNT? YES/NO
IF NO, WHICH IS MORE? _____

C.



a.



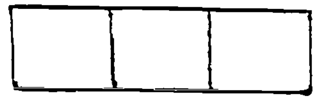
b.

FRACTION NAME. ____________ ____________

SHADED PARTS,SAME AMOUNT? YES/NO
IF NO, WHICH IS MORE? _____

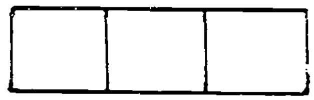
4. FOR EACH OF THE FOLLOWING, DRAW A PICTURE OF ONE WHOLE UNIT.

A. $\frac{3}{4}$



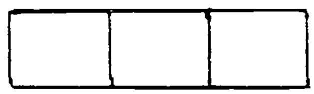
ONE WHOLE UNIT?

B. $\frac{3}{5}$



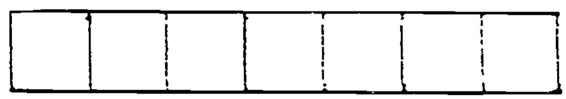
ONE WHOLE UNIT?

C. $\frac{3}{6}$



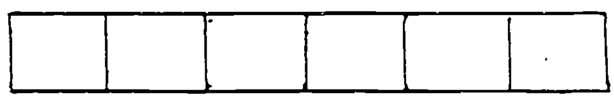
ONE WHOLE UNIT?

D. $\frac{7}{4}$



ONE WHOLE UNIT?

E. $\frac{6}{3}$



ONE WHOLE UNIT?

5. CIRCLE THE BIGGER COMMON FRACTION IN EACH PAIR

A. $\frac{4}{5}$ $\frac{3}{5}$ B. $\frac{1}{5}$ $\frac{1}{3}$

C. $\frac{2}{8}$ $\frac{2}{10}$ D. $\frac{7}{10}$ $\frac{12}{10}$

6. IDENTIFY THE LARGEST AND THE SMALLEST COMMON FRACTION. DRAW A PICTURE TO EXPLAIN YOUR DECISION.

		LARGEST	SMALLEST	PICTURE
A.	$\frac{1}{8}$ $\frac{1}{6}$ $\frac{1}{3}$	________	________	
B.	$\frac{2}{8}$ $\frac{2}{6}$ $\frac{2}{4}$	________	________	
C.	$\frac{3}{8}$ $\frac{6}{8}$ $\frac{2}{8}$	________	________	

7. RENAME THESE COMMON FRACTIONS TO MIXED NUMBERS.

(USE YOUR FRACTION STRIPS TO HELP YOU)

A. $\frac{7}{5}$ ________ B. $\frac{3}{2}$ ________

C. $\frac{5}{2}$ ________ D. $\frac{5}{4}$ ________

E. $\frac{7}{3}$ ________ F. $\frac{12}{10}$ ________

8. RENAME THESE MIXED NUMBERS TO COMMON FRACTIONS.

(USE YOUR FRACTION STRIPS TO HELP YOU)

A. $1\frac{4}{5}$ ________ B. $2\frac{1}{2}$ ________

C. $1\frac{5}{10}$ ________ D. $2\frac{3}{4}$ ________

WORKSHEET 3

COMMON FRACTIONS

NAME: ____________________

1. IDENTIFY THE LARGEST AND THE SMALLEST COMMON FRACTION. USE THE STRIPS TO HELP YOU.

				LARGEST	SMALLEST
A.	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{5}$	________	________
B.	$\frac{2}{8}$	$\frac{7}{8}$	$\frac{4}{8}$	________	________
C.	$\frac{3}{12}$	$\frac{3}{6}$	$\frac{3}{8}$	________	________
D.	$\frac{3}{4}$	$\frac{6}{4}$	$\frac{2}{4}$	________	________

2. RENAME EACH TO A COMMON FRACTION OR MIXED NUMBER.

(USE YOUR FRACTION STRIPS TO HELP YOU)

A. $\frac{7}{5}$ ________ B. $\frac{3}{2}$ ________

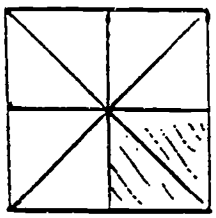
C. $\frac{5}{2}$ ________ D. $\frac{9}{4}$ ________

E. $\frac{7}{3}$ ________ F. $\frac{12}{10}$ ________

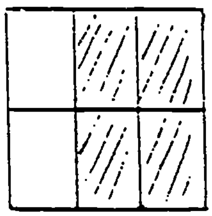
A. $1\frac{4}{6}$ ________ B. $2\frac{1}{3}$ ________

C. $1\frac{5}{12}$ ________ D. $2\frac{3}{8}$ ________

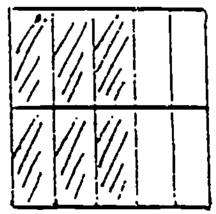
3. WHAT COMMON FRACTION IS SHOWN IN EACH PICTURE?



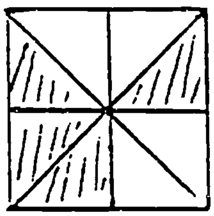
A. _____ FOURTHS



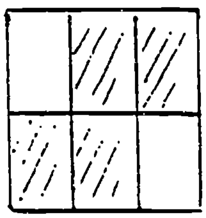
B. _____ THIRDS



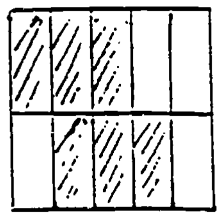
B. _____ FIFTHS



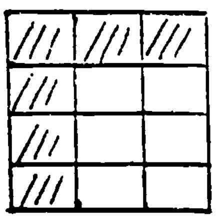
D. _____ FOURTHS



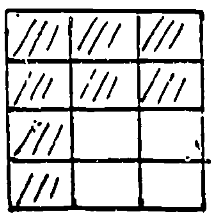
E. _____ THIRDS



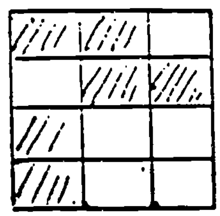
F. _____ FIFTHS



G. _____ FOURTHS



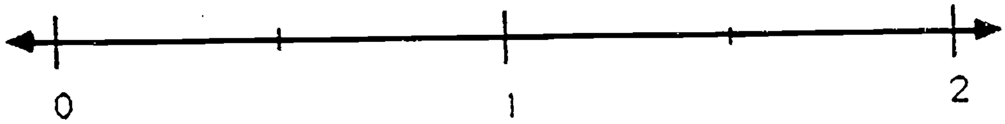
H. _____ THIRDS



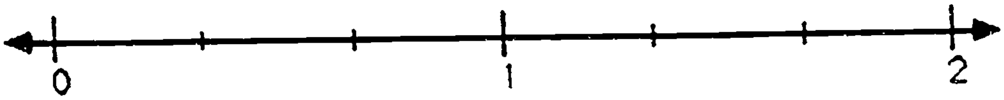
I. _____ SIXTHS

Mark where each pair of common fractions would be on the number line.

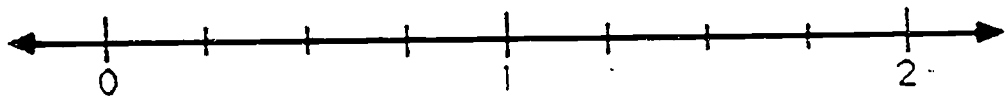
a. $\frac{1}{2}$, $\frac{3}{2}$



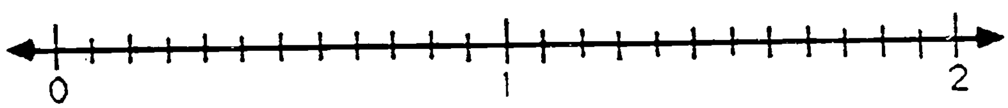
b. $\frac{1}{3}$, $\frac{4}{3}$



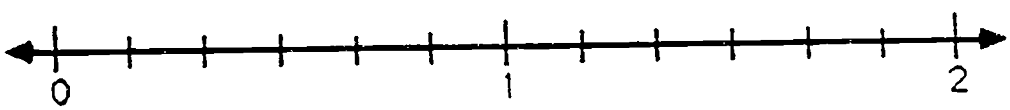
c. $\frac{1}{4}$, $\frac{6}{4}$



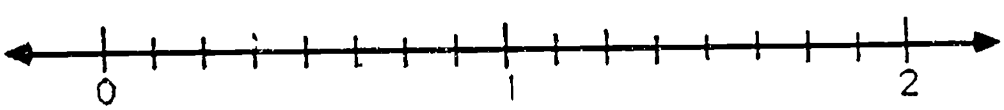
d. $\frac{2}{6}$, $\frac{15}{12}$



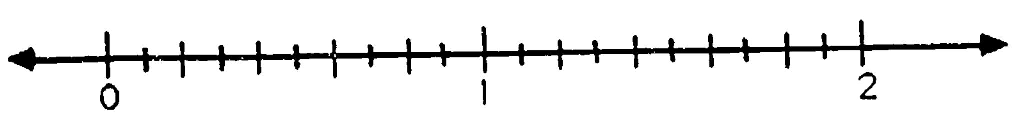
e. $\frac{2}{3}$, $\frac{7}{6}$

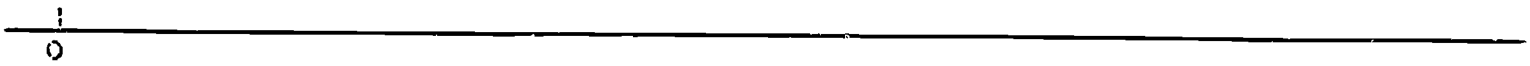


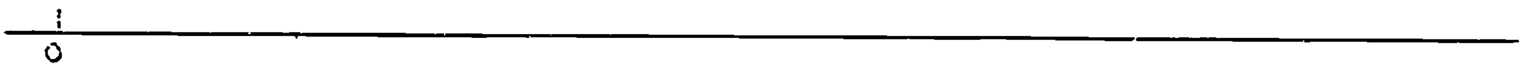
f. $\frac{2}{4}$, $\frac{5}{8}$

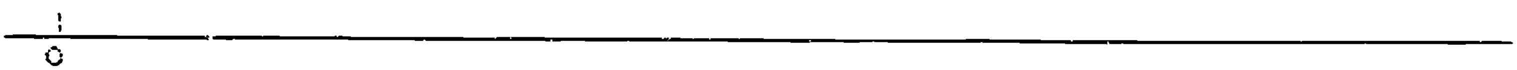


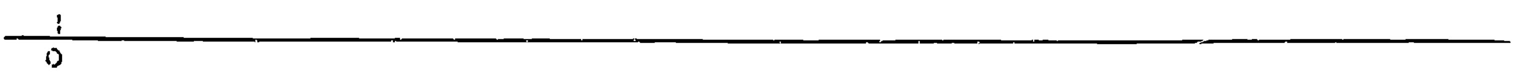
$\frac{6}{5}$, $\frac{9}{10}$



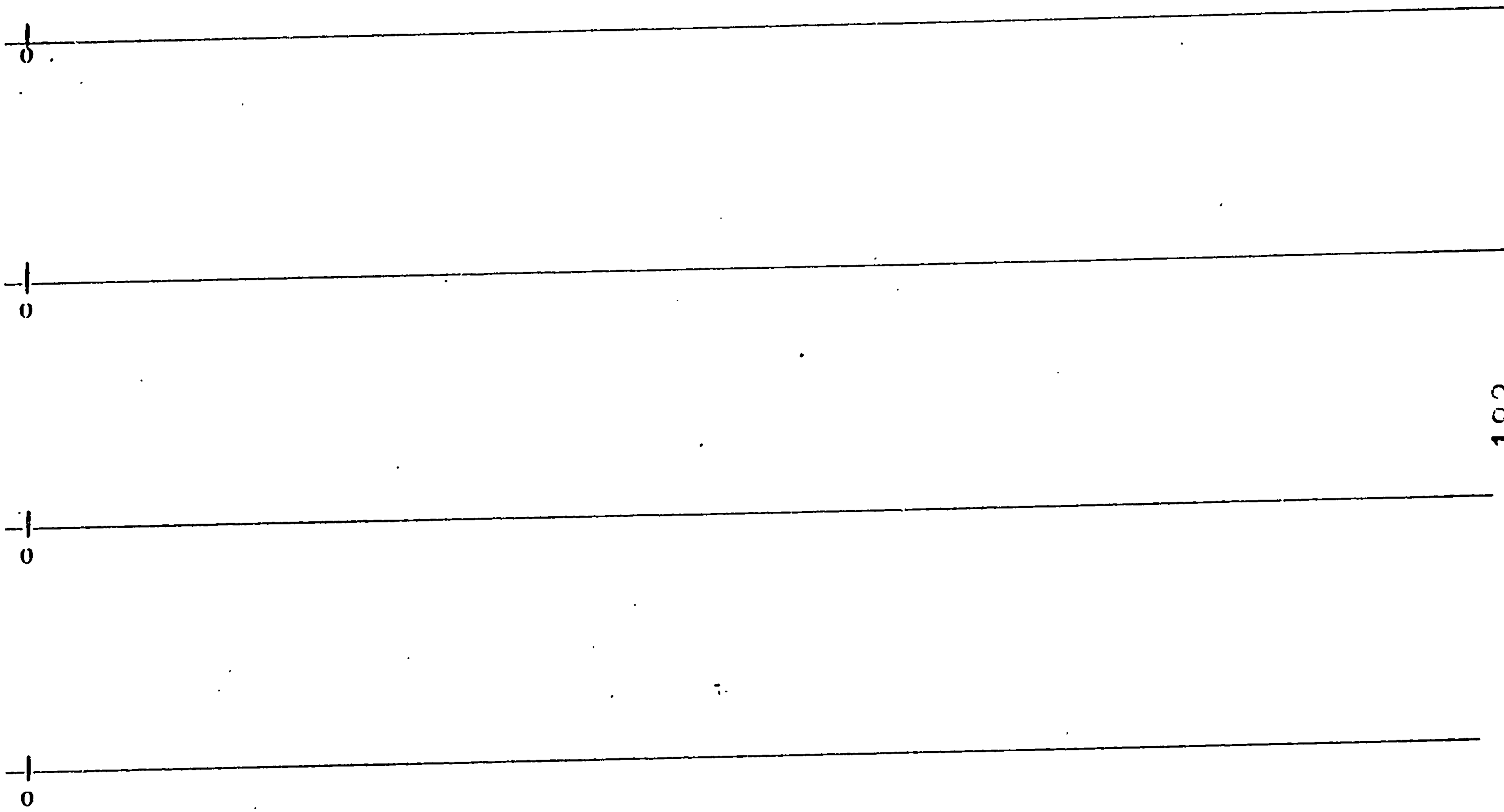

0


0


0


0

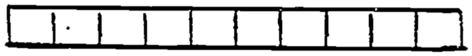
NUMBERLINES RECORD SHEET 2



WORKSHEET 4

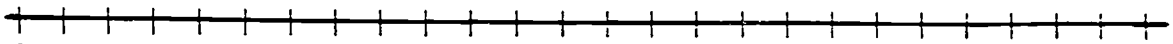
COMMON FRACTIONS

NAME: ____________________

1. IF  IS A UNIT WITH STRIPS, HOW MANY SPACES MAKE A SIMILAR UNIT ON THE NUMBER LINE?

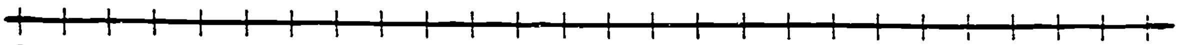
NUMBER OF SPACES = ________

MARK THE END OF 3/10, 5/10, 16/10, 2 1/10,
WRITE THE FRACTION NAME BELOW THE MARK



0

MARK THE END OF 3/5, 5/5, 6/5, 2 1/5,
WRITE THE FRACTION NAME BELOW THE MARK



0

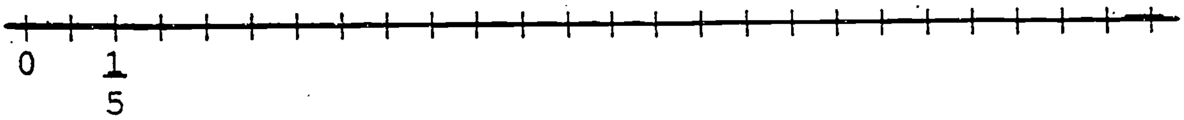
LOOK AT 3/5 AND 3/10 ON YOUR NUMBER LINES. WHICH IS BIGGER?
WRITE A WAY TO TELL WHICH IS BIGGER WITHOUT A PICTURE.

____

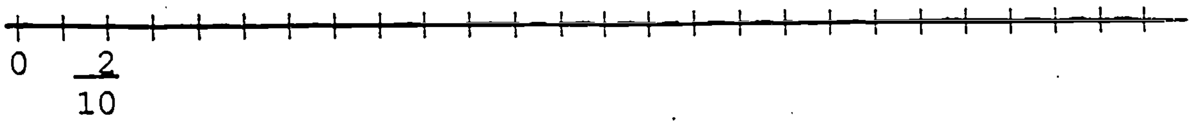
____

2. EACH NUMBER LINE HAS THE END OF A COMMON FRACTION MARKED.
WRITE THE OTHER COMMON FRACTIONS OR MIXED NUMBER AT THE PROPER END POINTS

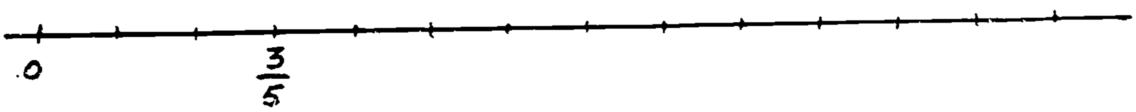
A. MARK THE END POINTS OF $\frac{5}{5}$, $1\frac{3}{5}$, AND $\frac{12}{5}$



B. MARK THE END OF $\frac{1}{10}$, $\frac{5}{10}$, $\frac{10}{10}$, $1\frac{3}{10}$.

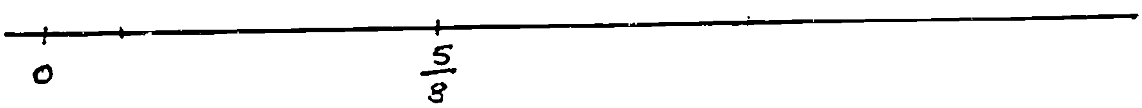


3. A. MARK THE POINT SHOWING THE WHOLE UNIT

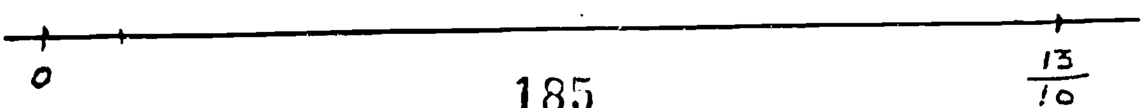


B. MARK THE POINT SHOWING THE WHOLE UNIT

C. MARK THE POINT SHOWING THE WHOLE UNIT



D. MARK THE POINT SHOWING THE WHOLE UNIT



COMMON FRACTION REVIEW

NAME: ______________________________

1. CIRCLE THE BIGGEST COMMON FRACTION
   DRAW A PICTURE TO EXPLAIN YOUR DECISION

PICTURES

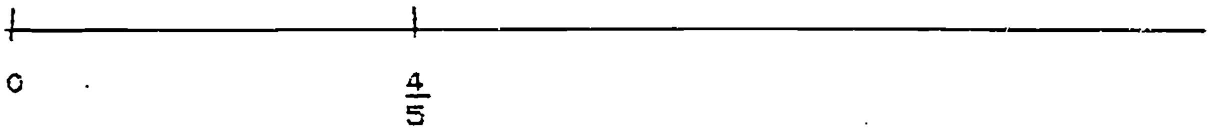
A. $\frac{2}{6}$ $\frac{2}{3}$

B. $\frac{4}{8}$ $\frac{7}{8}$

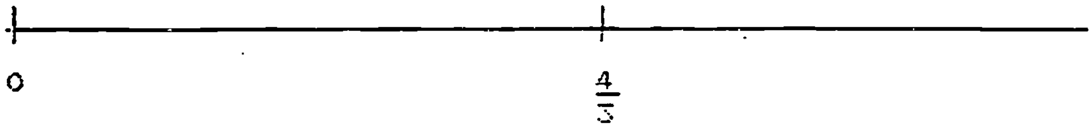
2. A TAILOR BOUGHT 10 METERS OF CLOTH. HE NEEDS $\frac{2}{5}$ OF THE 10 METERS OF CLOTH FOR ONE COAT. HOW MANY METERS OF CLOTH DID HE NEED FOR ONE COAT? HOW MANY COATS COULD HE MAKE WITH THE 10 METERS? DRAW A PICTURE TO SOLVE THE PROBLEM.

3. FIND AND DRAW ONE WHOLE UNIT IN EACH QUESTION.

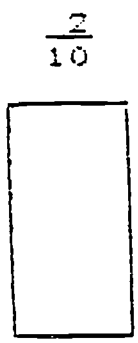
A.



B.



C.



ONE WHOLE UNIT

4. CHANGE TO MIXED NUMBERS OF COMMON FRACTIONS (USED YOUR FRACTION STRIPS TO HELP YOU THINK ABOUT THE PROBLEM)

A. $\frac{6}{3}$ ______ B. $\frac{10}{4}$ ______ C. $\frac{5}{2}$ ______

D. $2\ \frac{2}{10}$ ______ E. $10\ \frac{4}{5}$

WORKSHEET 5

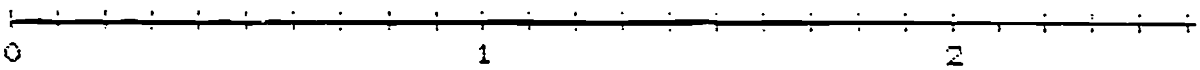
COMMON FRACTIONS AND DECIMAL FRACTIONS

NAME: ______________________________

1. FOR EACH QUESTION, MARK THE LENGTH OF THE FRACTION ON THE NUMBER LINE WITH A COMMON FRACTION NAME

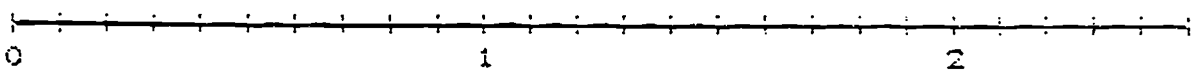
   WRITE THE COMMON FRACTION NAME AS A DECIMAL FRACTION NAME

   A. 7 TENTHS



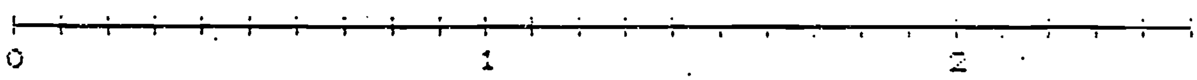
   DECIMAL FRACTION NAME ____________________

   B. 2 AND 3 TENTHS



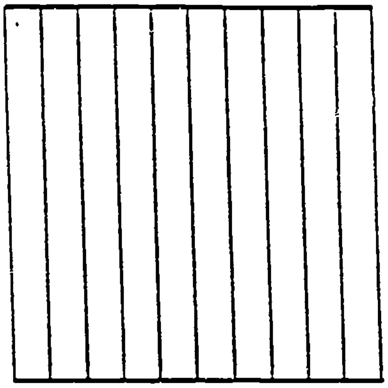
   DECIMAL FRACTION NAME ____________________

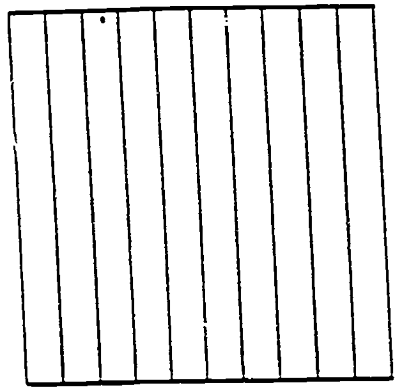
   C. 1 AND 7 TENTHS



   DECIMAL FRACTION NAME ____________________

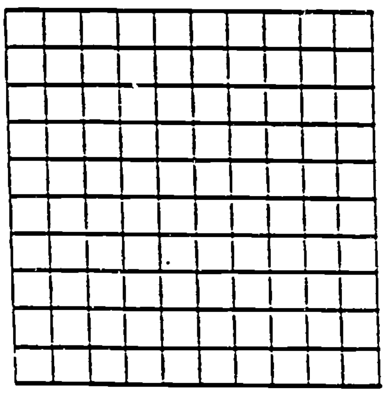
E. <u>1.7</u>

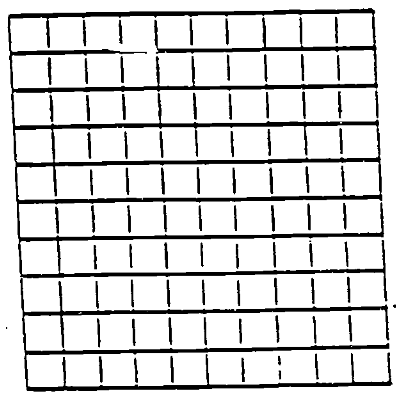




COMMON FRACTION NAME ______

F. <u>1.1</u>





COMMON FRACTION NAME ______

3. WRITE EACH WITH A <u>COMMON FRACTION NAME</u> OR A <u>DECIMAL FRACTION NAME</u>

	COMMON FRACTION	DECIMAL FRACTION
A. 9 TENTHS	______	______
B. 2 AND 3 TENTHS	______	______
C. 12 AND 4 TENTHS	______	______
D. 5 TENTHS	______	______
F. 20 AND 3 TENTHS	______	______

4. WRITE EACH OF THE FOLLOWING WITH A COMMON FRACTION NAME OR A DECIMAL FRACTION NAME

A. $2\frac{5}{10}$ ________ B. 0.9 ________ C. $\frac{8}{10}$ ________

D. $\frac{1}{10}$ ________ E. 3.7 ________ F. 5.5 ________

5. WHERE IS THE DECIMAL POINT?? IN EACH, PUT IN THE DECIMAL POINT TO MAKE THE STATEMENT TRUE.

A. 25 AND 5 TENTHS = 2 5 5

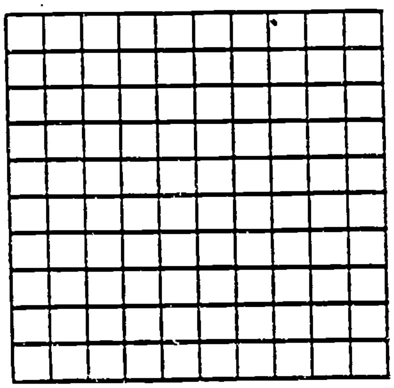
B. 1 2 4 4 = 124 AND 4 TENTHS

C. $25\frac{6}{10}$ = 2 5 6 D. $1555\frac{5}{10}$ = 1 5 5 5 5

6. FOR EACH COMMON FRACTION, SHADE THE AMOUNT IF THE FLAT IS ONE UNIT

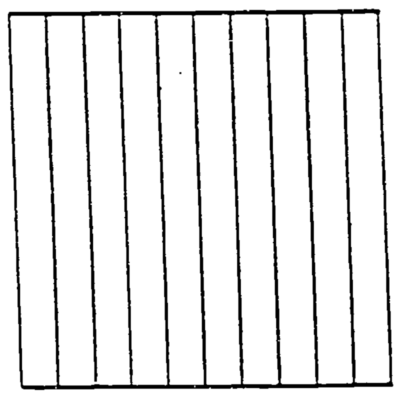
WRITE THE DECIMAL FRACTION NAME

A. $\frac{9}{100}$



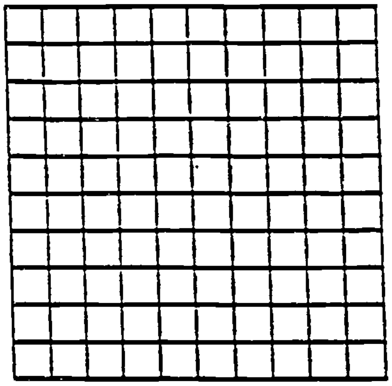
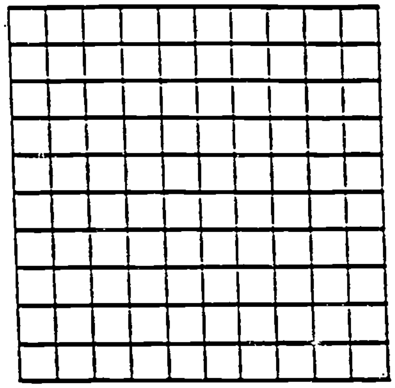
DECIMAL FRACTION NAME ______

B. $\frac{5}{100}$



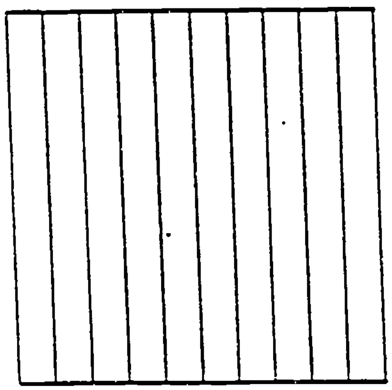
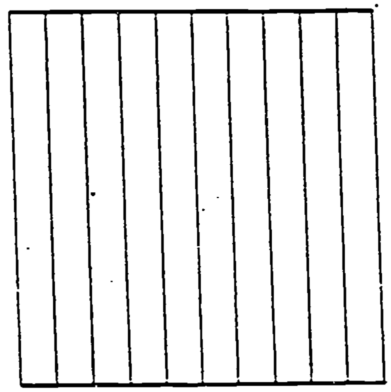
DECIMAL FRACTION NAME ______

C. $1 \frac{9}{100}$

DECIMAL FRACTION NAME ______

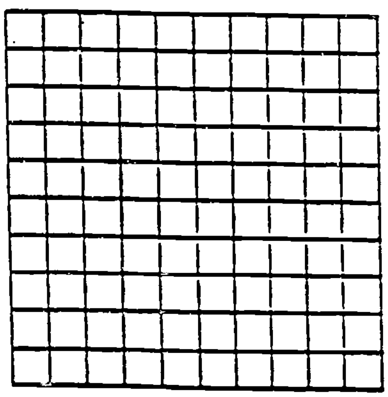
D. $1 \frac{6}{100}$

DECIMAL FRACTION NAME ______

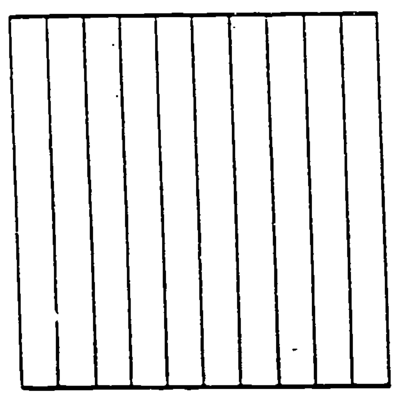
7. FOR EACH DECIMAL FRACTION, SHADE THE AMOUNT IF THE FLAT IS ONE UNIT. * WRITE THE COMMON FRACTION NAME

A. 0.07



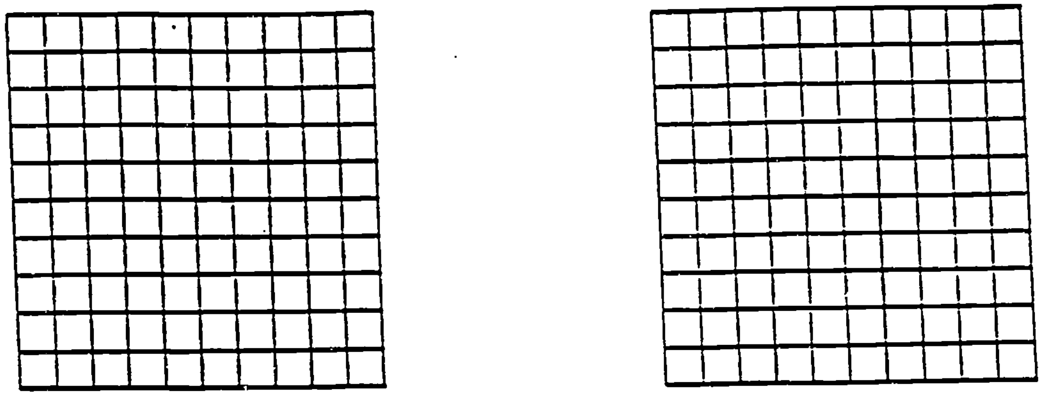
COMMON FRACTION NAME ______

B. 0.03



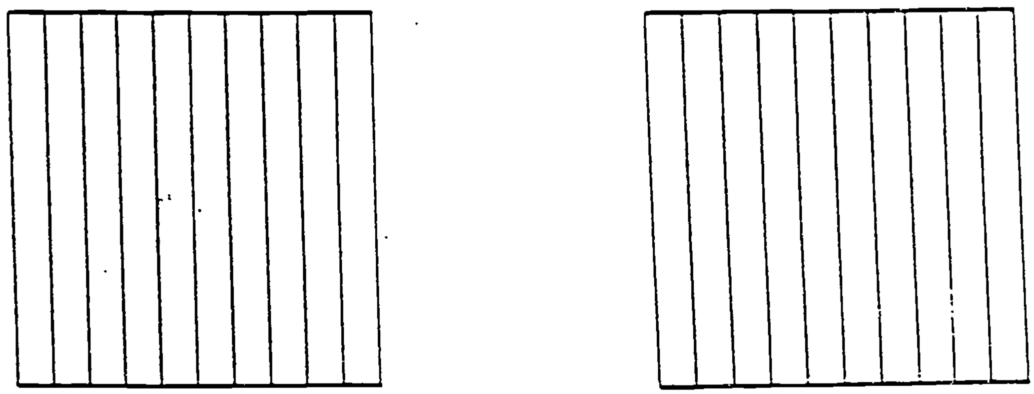
COMMON FRACTION NAME ______

C. <u>1.07</u>



COMMON FRACTION NAME _____

D. <u>1.04</u>



COMMON FRACTION NAME _____

8. WRITE EACH WITH A <u>COMMON FRACTION NAME</u> OR A <u>DECIMAL FRACTION NAME</u>

	COMMON FRACTION	DECIMAL FRACTION
A. 9 HUNDREDTHS	_____	_____
B. 2 AND 3 HUNDREDTHS	_____	_____
C. 12 AND 4 HUNDREDTHS	_____	_____
D. 5 HUNDREDTHS	_____	_____
F. 20 AND 7 HUNDREDTHS	_____	_____

WORKSHEET 6

COMMON FRACTIONS AND DECIMAL FRACTIONS

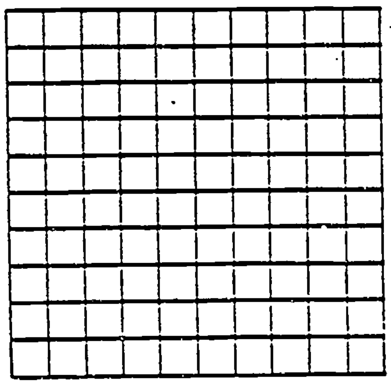
NAME: ______________________________

1. RENAMING TENTHS TO HUNDREDTHS

FOR EACH QUESTION

A. DRAW THE FRACTION IN THE SQUARE
B. SEE THE HUNDREDTHS IN THE SQUARE
C. RENAME THE FRACTION AS HUNDREDTHS.
D. WRITE THIS IN COMMON FRACTION AND DECIMAL FRACTION FORM

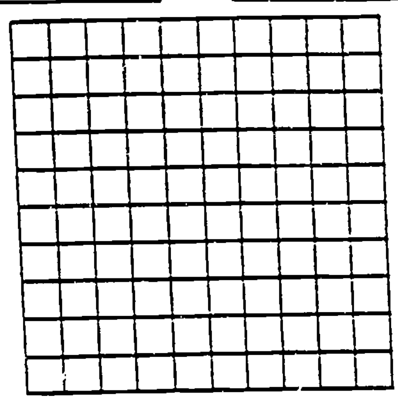
A. DRAW 5 TENTHS = ______ HUNDREDTHS



WRITE AS A COMMON FRACTION: $\frac{5}{10}$ = ______

WRITE AS A DECIMAL FRACTION 0.5 = ______

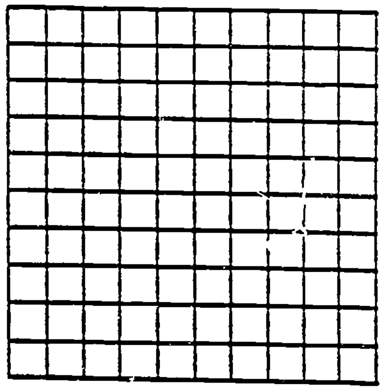
B DRAW 4 TENTHS = ______ HUNDREDTHS



WRITE AS A COMMON FRACTION: $\frac{4}{10}$ = ______

WRITE AS A DECIMAL FRACTION ______ = ______

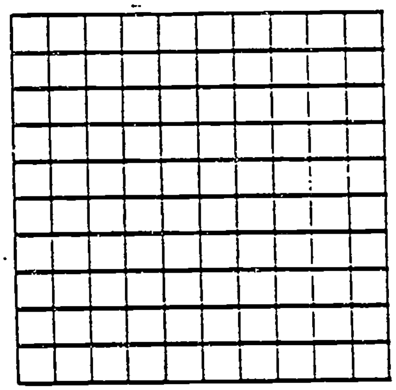
C. DRAW 7 TENTHS = ______ HUNDREDTHS



WRITE AS A COMMON FRACTION: ______ = ______

WRITE AS A DECIMAL FRACTION ______ = ______

D. DRAW 10 TENTHS = ______ HUNDREDTHS



WRITE AS A COMMON FRACTION: $\frac{\_\_}{10} = \frac{\_\_}{100}$

WRITE AS A DECIMAL FRACTION ______ = ______

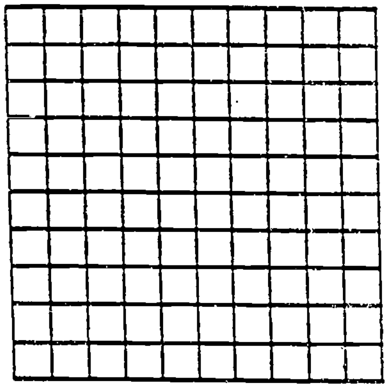
2. USE YOUR FLATS, LONGS, AND CUBES TO HELP YOU RENAME THESE FRACTIONS AS COMMON FRACTIONS AND DECIMAL FRACTIONS IN HUNDREDTHS.

	COMMON FRACTION			DECIMAL FRACTION		
	TENTHS		HUNDREDTHS	TENTHS		HUNDREDTHS
3 TENTHS	______	=	______	______	=	______
2 TENTHS	______	=	______	______	=	______
5 TENTHS	______	=	______	______	=	______
8 TENTHS	______	=	______	______	=	______

3. RENAMING HUNDREDTHS TO TENTHS

FOR EACH QUESTION
- A. DRAW THE FRACTION IN THE SQUARE
- B. SEE THE TENTHS IN THE SQUARE
- C. RENAME THE FRACTION AS TENTHS.
- D. WRITE THIS IN COMMON FRACTION AND DECIMAL FRACTION FORM

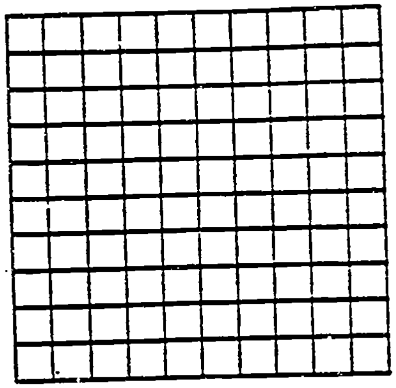
A. DRAW 50 HUNDREDTHS = ______ TENTHS



WRITE AS A COMMON FRACTION: $\frac{50}{100}$ = ______

WRITE AS A DECIMAL FRACTION: 0.50 = ______

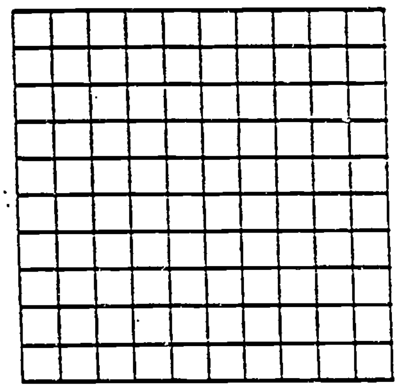
B DRAW 90 HUNDREDTHS = ________ TENTHS



WRITE AS A COMMON FRACTION: $\frac{9}{100}$ = ________

WRITE AS A DECIMAL FRACTION ________ = ________

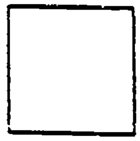
C. DRAW 20 HUNDREDTHS = ________ TENTHS



WRITE AS A COMMON FRACTION: ________ = ________

WRITE AS A DECIMAL FRACTION ________ = ________

8. IF THE WHOLE UNIT FOR EACH OF THESE COMMON AND DECIMAL FRACTIONS IS THE FLAT:



DECIDE IF ONE OF THE FRACTIONS IS BIGGER, OR IF BOTH FRACTIONS ARE THE SAME AMOUNT OF THE UNIT.

CHECK YOUR DECISION WITH THE FLATS AND LONGS.

A. $\frac{30}{100}$ 0.40

THE SAME AMOUNT? ______ WHICH IS BIGGER? ______

B. 0.8 0.20

THE SAME AMOUNT? ______ WHICH IS BIGGER? ______

C. $\frac{4}{10}$ 0.40

THE SAME AMOUNT? ______ WHICH IS BIGGER? ______

D. $\frac{9}{10}$ $\frac{90}{100}$

THE SAME AMOUNT? ______ WHICH IS BIGGER? ______

E. $\frac{30}{100}$ $\frac{8}{10}$

THE SAME AMOUNT? ______ WHICH IS BIGGER? ______

F. 0.3 0.30

THE SAME AMOUNT? ______ WHICH IS BIGGER? ______

9. WRITE EACH OF THE FOLLOWING WITH A COMMON FRACTION NAME OR A DECIMAL FRACTION NAME

A. $2\frac{5}{100}$ ________ B. 0.09 ________ C. $\frac{8}{100}$ ________

D. $\frac{1}{100}$ ________ E. 3.07 ________ F. 5.05 ________

G. $\frac{5}{10}$ ________ H. 30.7 ________ I. 5.9 ________

10. WHERE IS THE DECIMAL POINT?? IN EACH, PUT IN THE DECIMAL POINT TO MAKE THE STATEMENT TRUE.

A. 19 AND 5 TENTHS = 1 9 5

B. 1 2 4 0 4 = 124 AND 4 HUNDREDTHS

C. $25\frac{6}{100}$ = 2 5 0 6 D. $1555\frac{5}{100}$ = 1 5 5 5 0 5

BEST COPY AVAILABLE

4. USE YOUR FLATS, LONGS, AND CUBES TO HELP YOU RENAME THESE FRACTIONS AS COMMON FRACTIONS AND DECIMAL FRACTIONS IN TENTHS.

	COMMON FRACTION			DECIMAL FRACTION		
	HUNDREDTHS		TENTHS	HUNDREDTHS		TENTHS
30 HUNDREDTHS	______	=	______	______	=	______
20 HUNDREDTHS	______	=	______	______	=	______
60 HUNDREDTHS	______	=	______	______	=	______
80 HUNDREDTHS	______	=	______	______	=	______

5. WRITE EACH FRACTION IN 5 DIFFERENT WAYS.

IF A LONG IS ONE WHOLE UNIT, DRAW A PICTURE FOR EACH FRACTION

A. 3 TENTHS ______________________________

____

PICTURE:

B. $\frac{20}{100}$ ______________________________

____

PICTURE:

C. 0.70 ______________________________

____

PICTURE:

BEST COPY AVAILABLE

D. $\frac{6}{10}$ ______________________________

____

PICTURE:

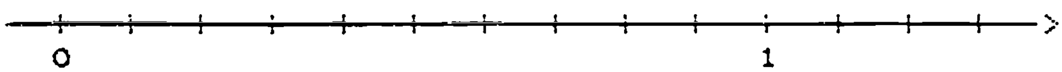
E. 40 HUNDREDTHS ______________________________

____

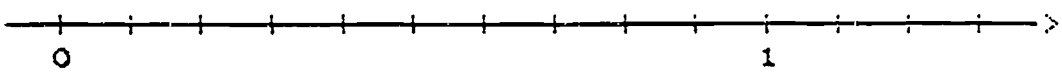
PICTURE:

6. MARK EACH FRACTION ON THE NUMBER LINE AND WRITE TWO OTHER NAMES FOR THE FRACTION.

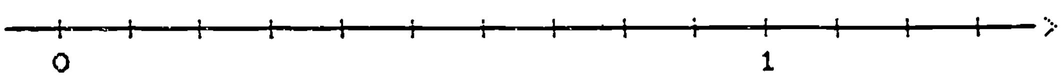
A. $\frac{6}{10}$



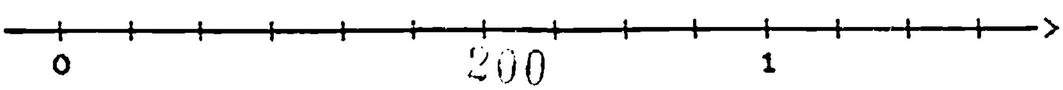
B. $\frac{50}{100}$



C. 0.3



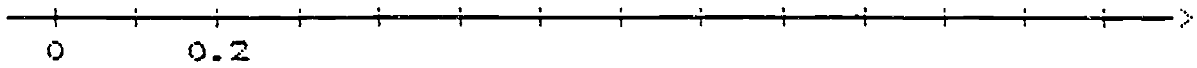
D. 0.70



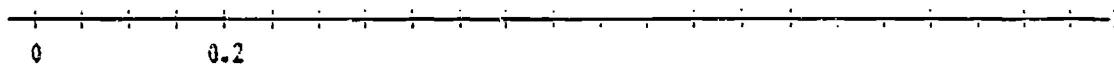
200

7. A FRACTION IS SHOWN ON EACH NUMBER LINE. MARK WHERE THE OTHER FRACTION WOULD BE ON THE NUMBER LINE.

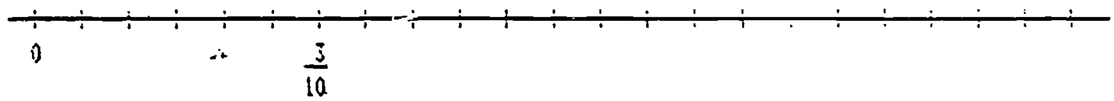
A. MARK <u>END OF 0.40</u>



B. MARK <u>END OF 0.40</u>

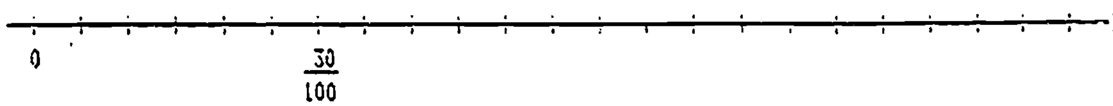


C. MARK <u>END OF 0.60</u>

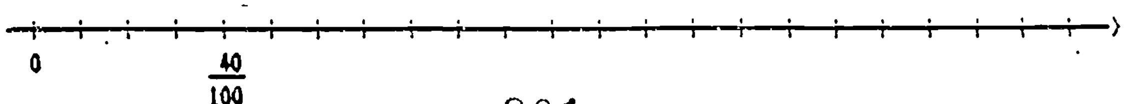


D. MARK <u>END OF</u> $\frac{8}{10}$

E. MARK <u>END OF</u> 0.7



E. MARK <u>END OF</u> 1.9



11. WRITE EACH OF THESE SUMS AS A DECIMAL FRACTION (USE YOU FLATS, LONGS AND CUBES TO HELP)

A. $3 + \frac{4}{10} + \frac{9}{100} =$ ________

B. $\frac{6}{100} + 4 + \frac{5}{10} =$ ________

C. $2 + \frac{12}{10} =$ ________

D. $1 + \frac{20}{10} =$ ________

C. $2 + \frac{7}{100} + \frac{14}{10} =$ ________

9. USE YOUR FLATS, LONGS AND CUBES TO HELP YOU RENAME THE FRACTIONS.

A. 0.23 = __ TENTHS AND __ HUNDREDTHS = ___ HUNDREDTHS

B. 0.29 = __ TENTHS AND __ HUNDREDTHS = ___ HUNDREDTHS

C. 0.14 = __ TENTHS AND __ HUNDREDTHS = ___ HUNDREDTHS

D. 0.04 = __ TENTHS AND __ HUNDREDTHS = ___ HUNDREDTHS

E. 0.20 = __ TENTHS AND __ HUNDREDTHS = ___ HUNDREDTHS

F. 0.53 = __ TENTHS AND __ HUNDREDTHS = ___ HUNDREDTHS

G. 0.87 = __ TENTHS AND __ HUNDREDTHS = ___ HUNDREDTHS

10. USE YOUR FLATS, LONGS AND CUBES TO HELP YOU RENAME THE FRACTIONS.

A. 0.23 = 20 HUNDREDTHS AND 3 HUNDREDTHS = __ TENTHS AND __ HUNDRETHS

B. 0.15 = 10 HUNDREDTHS AND 5 HUNDREDTHS = __ TENTHS AND __ HUNDRETHS

C. 0.29 = __ HUNDREDTHS AND _ HUNDREDTHS = __ TENTHS AND __ HUNDRETHS

D. 0.14 = __ HUNDREDTHS AND _ HUNDREDTHS = __ TENTHS AND __ HUNDRETHS

E. 0.03 = __ HUNDREDTHS AND _ HUNDREDTHS = __ TENTHS AND __ HUNDRETHS

F. 0.20 = __ HUNDREDTHS AND _ HUNDREDTHS = __ TENTHS AND __ HUNDRETHS

G. 0.56 = __ HUNDREDTHS AND _ HUNDREDTHS = __ TENTHS AND __ HUNDRETHS

11. USE YOUR FLATS, LONGS AND CUBES TO HELP YOU RENAME THE FRACTIONS.

A. 0.23 = 20 HUNDREDTHS AND 3 HUNDREDTHS = TENTHS AND HUNDRETHS

WRITE THIS WITH DECIMAL FRACTIONS

0.23 = 0.20 + 0.03 = ______ + ______

WRITE THIS WITH COMMON FRACTIONS

____

B. 0.19 = 10 HUNDREDTHS AND 9 HUNDREDTHS = TENTHS AND HUNDRETHS

WRITE THIS WITH DECIMAL FRACTIONS

0.19 = 0.10 + 0.09 = ______ + ______

WRITE THIS WITH COMMUN FRACTIONS

____

C. 0.45 = HUNDREDTHS AND HUNDREDTHS = TENTHS AND HUNDRETHS

WRITE THIS WITH DECIMAL FRACTIONS

0.45 = 0.40 + 0.05 = ______ + ______

WRITE THIS WITH COMMON FRACTIONS

____

D. 0.29 = HUNDREDTHS AND HUNDREDTHS = TENTHS AND HUNDRETHS

WRITE THIS WITH DECIMAL FRACTIONS

0.29 = 0.20 + 0.09 = ______ + ______

WRITE THIS WITH COMMON FRACTIONS

____

E. $\frac{99}{100} = \frac{\quad}{10} + \frac{\quad}{100}$

F. $\frac{37}{100} \quad \frac{\quad}{10} + \frac{\quad}{100}$

12. IF THE WHOLE UNIT FOR EACH OF THESE COMMON AND DECIMAL FRACTIONS IS THE FLAT:

DECIDE IF ONE OF THE FRACTIONS IS BIGGER, OR IF BOTH FRACTIONS ARE THE SAME AMOUNT OF THE UNIT.

CHECK YOUR DECISION WITH THE FLATS LONGS AND CUBES.

A. 2 TENTHS AND 6 HUNDREDTHS OR 26 HUNDRETHS

THE SAME AMOUNT? _______ WHICH IS BIGGER? _______

B. 1 TENTH AND 3 HUNDREDTHS OR 11 HUNDREDTHS

THE SAME AMOUNT. _______ WHICH IS BIGGER? _______

C. $\frac{25}{100}$ 0.56

THE SAME AMOUNT? _______ WHICH IS BIGGER? _______

D. $\frac{8}{10}$ $\frac{35}{100}$

THE SAME AMOUNT? _______ WHICH IS BIGGER? _______

E. $\frac{85}{100}$ 0.9

THE SAME AMOUNT? _______ WHICH IS BIGGER? _______

F. 0.9 0.46

THE SAME AMOUNT? _______ WHICH IS BIGGER? _______

13. DECIDE WHICH NUMBER IS THE LARGEST AND THE SMALLEST. USE THE FLATS, LONGS AND CUBES TO CHECK YOUR DECISION.

				LARGEST	SMALLEST
A	0.4	$\frac{23}{100}$	$\frac{5}{10}$	________	________
B.	0.56	0.9	0.3	________	________
C.	1.23	$\frac{45}{100}$	$\frac{2}{10}$	________	________
D.	0.60	$\frac{6}{10}$	0.23	________	________
E.	2.45	$\frac{99}{100}$	3	________	________
F.	23.1	$\frac{23}{10}$	4.23	________	________
G.	0.9	$\frac{70}{100}$	0.71	________	________
H.	$\frac{25}{100}$	$\frac{6}{10}$	0.3	________	________

WORKSHEET 7

COMMON FRACTIONS AND DECIMAL FRACTIONS

NAME: ______________________________

1. PUT EACH GROUP OF FRACTIONS IN ORDER FROM LARGEST TO SMALLEST

   USE THE STRIPS YOUR FLATS & LONGS TO CHECK

   A. $\frac{3}{6}$ 0.3 $\frac{3}{25}$ 0.03 $\frac{3}{2}$

   ____

   B. 0.35 $\frac{12}{100}$ $\frac{15}{10}$ 1.50 0.7

   ____

2. PUT THESE FRACTIONS IN ORDER FROM SMALLEST TO LARGEST.

   DRAW A ROUGH PICTURE TO SHOW THE SIZE OF EACH FRACTION.

   FOR EACH FRACTION, WRITE HOW MUCH MORE WOULD BE NEED TO MAKE ONE

   A. $\frac{11}{12}$ $\frac{7}{8}$

   __________ __________

   YOU NEED TO MAKE 1 __________ __________

B. $\frac{19}{20}$ 0.9

YOU NEED
TO MAKE 1

C. $\frac{5}{6}$ 0.99

YOU NEED
TO MAKE 1

3. PUT THESE FRACTIONS IN ORDER FROM <u>LARGEST TO SMALLEST</u>

WRITE A <u>SHORT SENTENCE</u> TO SAY <u>HOW YOU CAN TELL WHICH IS LARGEST</u>. (DISCUSS WITH A PARTNER)

A. $\frac{5}{4}$ 1.01 $\frac{15}{18}$ ____________________

WHY: ____

B. 0.8 $\frac{98}{100}$ $\frac{6}{8}$ ____________________

WHY: ____________________

____

C. $\frac{119}{120}$ 0.99 $\frac{56}{57}$ ____________________

WHY: ____________________

____

4. <u>DECIDE</u> WHICH IS THE LARGEST AND THE SMALLEST FRACTION.

<u>USE THE THINKING STRATEGIES</u> DRAW PICTURES TO HELP

				<u>LARGEST</u>	<u>SMALLEST</u>
A.	$\frac{2}{3}$	0.2	$\frac{3}{4}$	______	______
B.	$\frac{5}{3}$	1.2	$\frac{6}{4}$	______	______
C.	$\frac{13}{12}$	0.9	$\frac{8}{9}$	______	______
D.	$\frac{11}{12}$	0.98	$\frac{9}{10}$	______	______

5. IF THE FLAT IS THE UNIT,

CIRCLE THE FRACTIONS THAT ARE THE SAME AMOUNT AS 5 LONGS?

$\frac{4}{8}$ $\frac{3}{5}$ $\frac{1}{2}$ 0.50 0.6 $\frac{3}{6}$ $\frac{4}{9}$ $\frac{7}{14}$

WRITE:
1. THREE MORE COMMON FRACTION NAMES
2. ONE MORE DECIMAL FRACTION NAME
THAT ARE THE SAME AMOUNT AS 5 LONGS

____

6. COMPLETE EACH OF THESE FRACTIONS SO THAT THEY ALL EQUAL A HALF

DRAW A PICTURE OF EACH, IF THE RECTANGLE IS ONE

A. $\frac{\_\_}{4}$

B. $\frac{\_\_}{8}$

C. $\frac{3}{\_\_}$

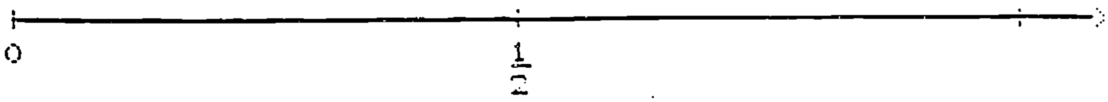
D. $\frac{\_\_}{10}$

E. 0._

F. $\frac{\_\_}{6}$

7. PLACE ALL OF THESE NUMBERS ON THE NUMBER LINE.

1.00 0.50 $\frac{15}{30}$ $\frac{8}{8}$ 0.5 1 $\frac{3}{4}$



8. CIRCLE THE FRACTIONS WHICH ARE CLOSE TO ZERO.

A. $\frac{6}{3}$ $\frac{6}{8}$ 0.06 B. 0.2 $\frac{2}{8}$ $\frac{3}{6}$ $\frac{2}{3}$ $\frac{3}{12}$

CHECK WITH YOUR FRACTION STRIPS

9. CIRCLE THE FRACTIONS WHICH ARE CLOSE TO ONE

A. $\frac{6}{3}$ $\frac{7}{8}$ 0.9 B. 0.6 $\frac{6}{8}$ $\frac{7}{6}$ $\frac{3}{5}$ $\frac{10}{12}$

CHECK WITH YOUR FRACTION STRIPS

10. WRITE THREE COMMON FRACTIONS CLOSE TO BUT LARGER THAN ONE

____

11. WRITE THREE DECIMAL FRACTIONS CLOSE TO BUT LARGER THAN ONE

____

12. WRITE THREE DECIMAL FRACTIONS CLOSE TO ZERO

____

13. WRITE THREE COMMON FRACTIONS CLOSE TO ZERO

____

14. SHADE EACH BAR TO SHOW THE APPROXIMATE AMOUNT TO EACH FRACTION.

A. $\frac{8}{15}$

B. 0.89

C. 0.56

D. $\frac{3}{5}$

E. $\frac{3}{7}$

F. $\frac{3}{6}$

15. CIRCLE THE FRACTIONS CLOSE TO A HALF

PUT AN X UNDER THE FRACTIONS THAT ARE LARGER THAN A HALF

0.45 $\frac{4}{7}$ 0.2 $\frac{4}{9}$ $\frac{8}{10}$ $\frac{7}{12}$ $\frac{4}{10}$ 0.55 0.6

16. CIRCLE THE COMMON FRACTION WHICH IS CLOSEST TO ONE HALF?

CHECK WITH YOUR FRACTION STRIPS

A. $\frac{3}{5}$ $\frac{4}{6}$

B. $\frac{5}{12}$ $\frac{5}{8}$

C. $\frac{6}{10}$ $\frac{3}{5}$

D. $\frac{2}{5}$ $\frac{2}{3}$

E. $\frac{6}{10}$ 0.51

F. 0.4 0.62

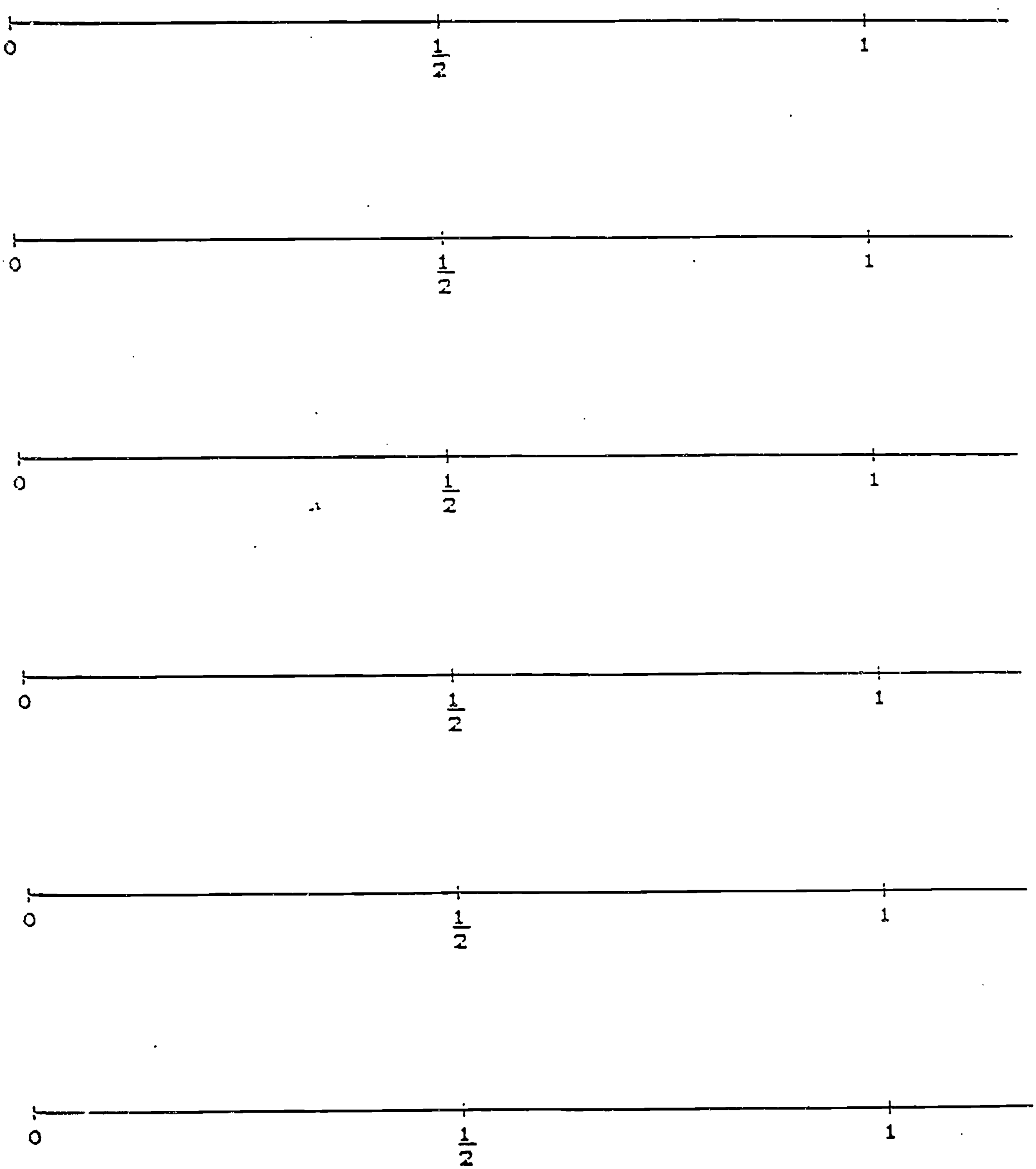
17. ESTIMATE WHETHER EACH FRACTION IS CLOSER TO 1, 0, OR A HALF.

USE YOUR ESTIMATES TO DECIDE WHICH IS LARGER.

			LARGER
A.	$\frac{1}{9}$	$\frac{7}{8}$	________
B.	$\frac{5}{8}$	$\frac{3}{40}$	________
C.	$\frac{5}{9}$	0.13	________
D.	0.57	$\frac{3}{4}$	________
E.	$\frac{3}{5}$	0.8	________

RECORDING SHEET 1

FRACTIONS AS APPROXIMATE AMOUNTS



RECORDING SHEET 2

FRACTIONS AS APPROXIMATE AMOUNTS

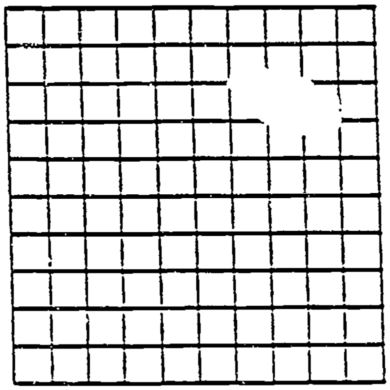
WORKSHEET 8

COMMON FRACTIONS AND DECIMAL FRACTIONS

NAME: ______________________________

1. SHADE THE FRACTION
   RENAME THE FRACTION

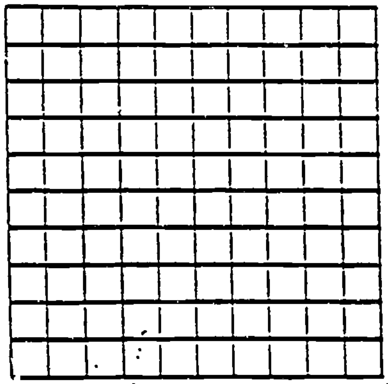
A. SHADE $\frac{3}{5}$ OF THE FLAT



WRITE THIS AMOUNT AS TENTHS ______________________

WRITE THIS AMOUNT AS HUNDREDTHS ______________________

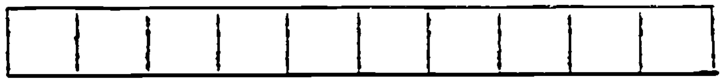
B. SHADE 0.8 OF THE FLAT



WRITE THIS AMOUNT AS FIFTHS ______________________

WRITE THIS AMOUNT AS HUNDREDTHS ______________________

C. SHADE $\frac{3}{5}$ OF THE STRIP



WRITE THIS AMOUNT AS TENTHS ______________________

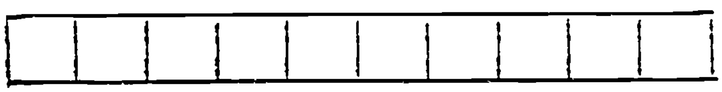
WRITE THIS AMOUNT AS HUNDREDTHS ______________________

D. SHADE $\frac{4}{5}$ OF THE STRIP

WRITE THIS AMOUNT AS TENTHS ______________________

WRITE THIS AMOUNT AS HUNDREDTHS ______________________

E. SHADE $\frac{6}{10}$ OF THE STRIP



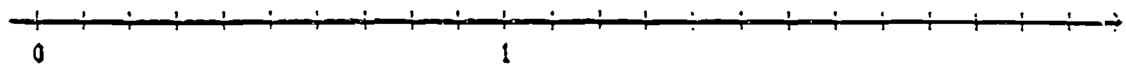
WRITE THIS AMOUNT AS FIFTHS ______________________

WRITE THIS AMOUNT AS HUNDREDTHS ______________________

2. MARK THE FRACTIONS ON THE NUMBER LINE

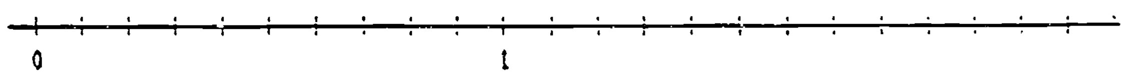
A. <u>MARK THE END</u> OF $\frac{2}{5}$, $\frac{4}{5}$, $\frac{7}{5}$ ON THE NUMBER LINE.

<u>WRITE FOUR OTHER NAMES</u> FOR THE FRACTION UNDER EACH END POINT



B. <u>MARK THE END</u> OF $\frac{1}{5}$, $\frac{6}{10}$, 1.6 ON THE NUMBER LINE.

<u>WRITE FOUR OTHER NAMES</u> FOR THE FRACTION UNDER EACH END POINT



3. RENAME THESE FRACTIONS. USE YOUR MATERIALS TO CHECK YOUR WORK.

A. $\frac{2}{10} = \frac{\_}{5}$

B. $\frac{3}{5} = \frac{\_\_}{10}$

C. $0.4 = \frac{\_}{5}$

E. $\frac{6}{5} = 1.$____

4. CIRCLE THE FRACTIONS OR MIXED NUMBERS THAT CAN BE RENAMED AS FIFTHS. WRITE A NUMBER SENTENCE THAT RENAMES THE FRACTION TO FIFTHS.

0.5   1.2   0.60   0.75   0.4 $\frac{8}{10}$

$\frac{55}{100}$ $\frac{15}{10}$

A____ = ____ B____ = ____ C____ = ____

D____ = ____ E____ = ____ F____ = ____

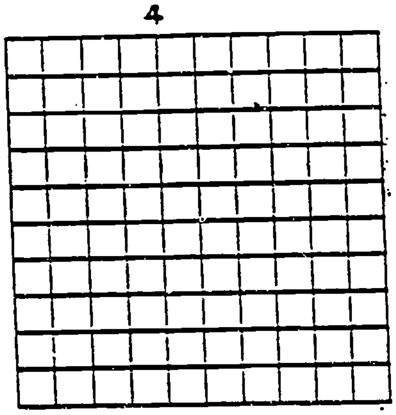
5. PUT THESE FRACTIONS IN ORDER FROM LARGEST TO SMALLEST. USE YOUR MATERIALS TO CHECK YOUR DECISIONS.

A. $\frac{3}{5}$ 0.10 $\frac{5}{10}$ $\frac{5}{100}$ $\frac{1}{5}$ 0.45 $\frac{6}{5}$

____

6. SHADE THE FRACTION
RENAME THE FRACTION

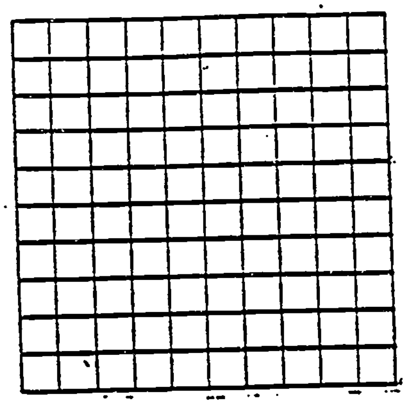
A. SHADE $\frac{3}{4}$ OF THE FLAT



WRITE THIS AMOUNT AS TENTHS ____________________

WRITE THIS AMOUNT AS HUNDREDTHS ____________________

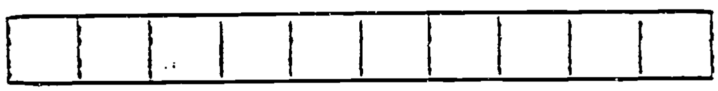
B. SHADE 0.25 OF THE FLAT



WRITE THIS AMOUNT AS FOURTHS ______________________

WRITE THIS AMOUNT AS TENTHS ____________________

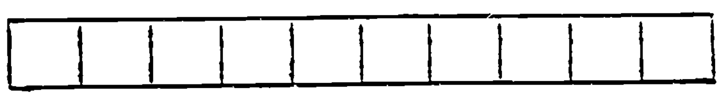
C. SHADE $\frac{1}{4}$ OF THE STRIP



WRITE THIS AMOUNT AS TENTHS ______________________

WRITE THIS AMOUNT AS HUNDREDTHS ____________________

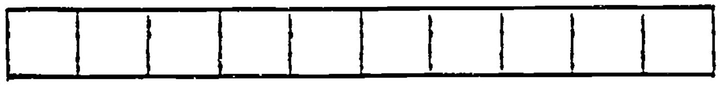
D. SHADE $\frac{3}{4}$ OF THE STRIP



WRITE THIS AMOUNT AS TENTHS ______________________

WRITE THIS AMOUNT AS HUNDREDTHS ____________________

E. SHADE $\frac{5}{10}$ OF THE STRIP



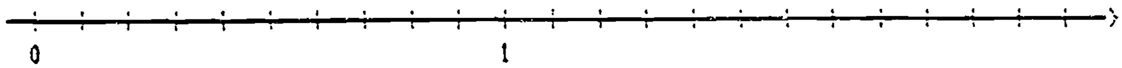
WRITE THIS AMOUNT AS FOURTHS ____________________

WRITE THIS AMOUNT AS HUNDREDTHS ____________________

7. MARK THE FRACTIONS ON THE NUMBER LINE

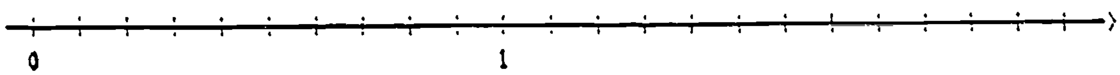
A. MARK THE END OF $\frac{3}{4}$, $\frac{5}{4}$, $\frac{6}{4}$ ON THE NUMBER LINE.

WRITE DECIMAL FRACTION NAMES FOR EACH



B. MARK THE END OF 1.25, 0.75, $\frac{2}{4}$, $\frac{7}{4}$,
ON THE NUMBER LINE

WRITE ALL DECIMAL OR COMMON FRACTION NAMES FOR EACH. RENAME WITH FOURTHS WHEN POSSIBLE.



8. CIRCLE THE FRACTIONS OR MIXED NUMBERS THAT CAN BE RENAMED AS FOURTHS. WRITE A NUMBER SENTENCE THAT RENAMES THE FRACTION TO FOURTHS.

0.5   1.2   0.60   0.75   0.4 $\frac{8}{10}$

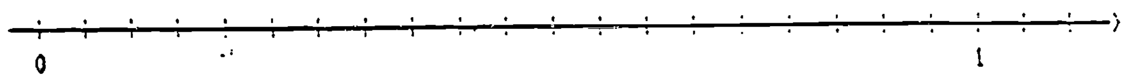
$\frac{125}{100}$ $\frac{15}{10}$

A____ = ____ B____ = ____ C____ = ____

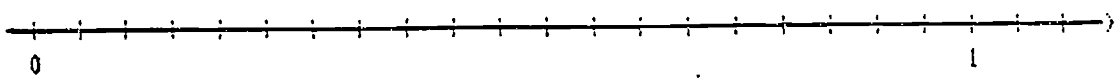
D____ = ____ E____ = ____ F____ = ____

9. MARK THE END POINT FOR EACH FRACTIONS ON THE NUMBER LINE. RENAME THEM AS TENTHS AND HUNDREDTHS

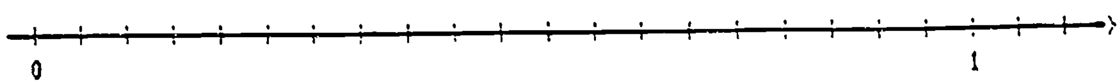
A. $\frac{2}{4}$ AND $\frac{4}{5}$



B. $\frac{3}{5}$ AND $\frac{3}{4}$



C. $\frac{1}{4}$ AND $\frac{6}{5}$



10 RENAME THESE FRACTIONS OR MIXED NUMBERS

A. $\frac{3}{4} = 0.\_\_\_$ B. $\frac{1}{2} = \frac{\_}{4} = \frac{\_\_}{10}$

C. $1.25 = \frac{\_}{4}$ D. $0.6 = \frac{\_}{5} = \frac{\_\_\_}{100}$

11. RENAME THESE FRACTIONS

A. $0.8 = \frac{4}{\_\_\_}$ B. $0.75 = \frac{3}{\_\_\_}$

12. PUT THESE FRACTIONS AND MIXED NUMBERS IN ORDER FROM LARGEST TO SMALLEST

$\frac{4}{5}$, 0.75, $\frac{3}{2}$, 1.2, $1\frac{1}{4}$, 2 0.4 $\frac{1}{5}$

____

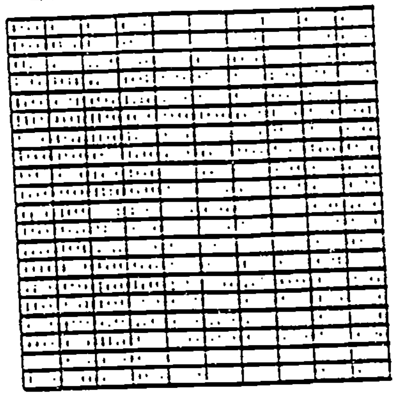
13. RENAME EACH FRACTION INTO EIGHTHS OR FOURTHS. USE YOUR STRIPS TO HELP YOU.

A. $\frac{4}{8} = \frac{\_}{4}$ B. $\frac{3}{4} = \frac{\_}{8}$ C. $\frac{7}{8} = \frac{\_}{4}$

D. $\frac{1}{4} = \frac{\_}{8}$ E. $\frac{10}{8} = \_$ F. $\frac{6}{4} = \frac{\_}{8}$

14. <u>SHADE</u> THE FRACTION
<u>RENAME</u> THE FRACTION

A. SHADE $\frac{1}{4}$ OF THE FLAT



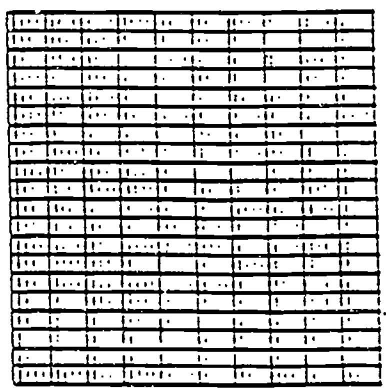
WRITE THIS AMOUNT AS EIGHTHS ____________________

WRITE THIS AMOUNT AS:

$\frac{\quad}{10} + \frac{\quad}{100} + \frac{\quad}{1000}$

WRITE THIS AMOUNT AS A DECIMAL ____________________

B. SHADE $\frac{1}{8}$ OF THE FLAT



WRITE THIS AMOUNT AS FOURTHS ____________________

WRITE THIS AMOUNT AS:

$\frac{\quad}{10} + \frac{\quad}{100} + \frac{\quad}{1000}$

WRITE THIS AMOUNT AS A DECIMAL ____________________

15. RENAME EACH OF THESE FRACTIONS AS EIGHTHS AND FOURTHS. USE YOUR MATERIALS TO HELP YOU.

A. $0.250 = \frac{\_}{4} = \frac{\_}{8}$

C. $0.750 = \frac{\_}{8} = \frac{\_}{4}$

B. $0.125 = \frac{\_}{8} = \frac{\_}{4}$

D. $1.500 = \frac{\_}{8} = \frac{\_}{4} = \frac{\_}{2}$

16. COMPARE THESE FRACTIONS. DECIDE WHICH IS THE LARGEST AND THE SMALLEST.

CHECK YOUR DECISIONS WITH THE MATERIALS.

				LARGEST	SMALLEST
A.	$\frac{1}{5}$	$0.\overline{33}$	$\frac{3}{10}$	________	________
B.	0.23	$\frac{1}{4}$	$\frac{2}{5}$	________	________
C.	$\frac{3}{4}$	$\frac{6}{8}$	0.8	________	________
D.	1.50	$\frac{7}{4}$	$\frac{125}{100}$	________	________
E.	0.60	$\frac{4}{5}$	$\frac{3}{4}$	________	________

17. RENAME THESE FRACTIONS INTO THOUSANDTHS. USE YOUR MATERIALS TO CHECK.

A. $\frac{7}{10}$ = ________

B. 0.91 = ________

C. 0.6 = ________

D. $\frac{7}{100}$ = ________

18. WRITE EACH FRACTION IN WORDS, THEN IN EXPANDED FORM (TWO WAYS)

EXAMPLE: 1.25 = ONE AND 25 HUNDREDTHS

$$1.25 = 1 + \frac{20}{100} + \frac{5}{100}$$

$$1.25 = 1 + \frac{2}{10} + \frac{5}{100}$$

EXAMPLE: 1.256 = ONE AND 256 THOUSANDTHS

$$1.256 = 1 + \frac{200}{1000} + \frac{50}{1000} + \frac{6}{1000}$$

$$1.256 = 1 + \frac{2}{10} + \frac{5}{100} + \frac{6}{1000}$$

A. WRITE 2.345 = ______________________________

(1) 2.345 = ______________________________

(2) 2.345 = ______________________________

B. WRITE 4.750 = ______________________________

(1) 4.750 = ______________________________

(2) 4.750 = ______________________________

19. CIRCLE THE FRACTIONS THAT ARE CLOSE TO ONE HALF

0.48	0.05	0.051	0.501
0.309	0.409	0.049	0.6

20. CIRCLE THE FRACTION IN EACH QUESTION THAT IS <u>CLOSEST TO ONE HALF</u>

CHECK WITH YOUR MATERIALS

A. $\frac{3}{5}$ 0.601 B. 0.629 0.631

C. $\frac{49}{100}$ 0.492 D. $\frac{3}{4}$ 0.72

21. CIRCLE THE FRACTION IN EACH QUESTION THAT IS <u>CLOSEST TO ONE</u>

CHECK WITH YOUR MATERIALS

A. 0.7 0.601 B. 0.71 0.699

C. $\frac{84}{100}$ 0.902 D. $\frac{4}{5}$ 0.79

22. CIRCLE THE FRACTION IN EACH QUESTION THAT IS <u>CLOSEST TO ZERO</u>

CHECK WITH YOUR MATERIALS

A. 0.2 0.199 B. 0.35 0.361

C. $\frac{24}{100}$ $\frac{1}{4}$ D. $\frac{1}{5}$ 0.125

23. COMPARE THESE FRACTIONS. DECIDE WHICH IS THE LARGEST AND THE SMALLEST.

CHECK YOUR DECISIONS WITH THE MATERIALS.

				LARGEST	SMALLEST
A.	$\frac{45}{100}$	0.9	0.345	________	________
B.	0.56	$\frac{1}{2}$	0.501	________	________
C.	$\frac{3}{4}$	$\frac{1}{3}$	0.3	________	________
D.	0.5	$\frac{1}{8}$	$\frac{125}{1000}$	________	________
E.	0.621	0.9	0.81	________	________

Worksheet

1. A pair of jeans costs $30 before G.S.T. If the G.S.T. is 0.07. what is the amount of G.S.T. you have to pay? If G.S.T. is included, what is the total cost of the pair of jeans?

2. If gasoline costs 54.9 cents per litre, how much would 25 litres cost (excluding G.S.T.)?

3. If each cornflake cookie has 4.29g of margarine, how many grams of margarine would be needed for 150 cornflake cookies?

4. A car travels 80 km in one hour, and keeps going at the same speed for another 0.125 hours. How many kilometers would it have gone in this 0.125 hours? What would the total distance travelled be, in 1.125 hours, at this speed?

5. From one 1 kg of wheat we get 0.75 kg of wheat flour. How much wheat flour can we get from 15 kg of wheat?

6. The length of a chocolate bar is 15 cm. What is the length of 0.75 of this chocolate bar?

7. For every one litre of gasoline, a car can travel 15 km. How many kilometers can the car travel on 0.75 litres of gasoline?

8. Stephen had 90 marks out of 100 marks for his Math test. Justin had 0.9 of the marks that Stephen had. How many marks did Justin get out of 100?

9. There are 24 children in a Grade 6 class. If 0.375 of the class are girls, how many girls are there in the class?

10. Tickets for a show at the Coliseum were priced at $16 for an adult. A ticket for a child between 5 years and 12 years old was 0.6 of the price of an adult ticket. What was the price of a ticket for a child aged 10 years of age?

11. Write, and solve a word problem using multiplication and the numbers 32 and 0.125

12. Write, and solve a word problem (as <u>different</u> as possible from the ones you have just done in this worksheet) involving <u>multiplication</u> of two numbers (or more) with <u>at least one</u> of the numbers a <u>decimal fraction less than one</u>.

Worksheet

1. For each of the following, WITHOUT working out the actual answer, circle the best estimate. Give a reason for your choice.

	Estimate			Reason
a) 19x32	300	450	600	
b) 48x28	800	900	1500	
c) 71x18	700	1400	1600	
d) 26x27	500	600	900	
e) 69x9	600	630	700	

2. Circle the correct answer (or answers) for each of following, WITHOUT working out the answer. Give a reason for your choice:

	Answer			Reason
a) 3.9x12	36.108	46.8	46.80	
b) 3.9x1.2	4.68	3.18	46.80	
c) 39x1.2	046.80	46.8	39.78	
d) 3.9x0.12	0.108	4.680	0.468	
e) 0.39x1.2	4.680	0.468	0.780	
f) 0.39x0.12	0.4680	0.468	4.6800	

3. For each of the following, WITHOUT working out the answer, put the decimal point in the proper place to make the "answer" correct. Give a reason for your answer.

	Answer	Reason
a) 76x2.3	1748	
b) 7.6x23	1748	
c) 7.6x2.3	1748	
d) 0.76x23	17480	
e) 7.6x0.23	1748	
f) 0.55x3.6	198	
g) 0.55x0.36	198	

4. For each of the following, first give an estimate of the answer. Then work out the actual answer. Finally, find the difference betwen the actual answer and your estmate.

	Estimate	Actual Answer	Difference
a) 0.9x29			
b) 31x0.25			
c) 0.33x36			
d) 0.125x88			
e) 27x0.75			
f) 0.375x16			
g) 0.896x0.50			

grade 4

Worksheet 1

1. Each of the two pieces of cloth was cut up in different ways. the shaded parts were used to make costumes.

a. What fraction of each piece of cloth was used? Were the shaded parts the same amount of cloth, or was one more than the other? How can you tell?

Fraction name. ________________ ________________

Shaded parts, same amount? yes / no.
If no, which is more? __________

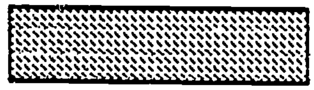
b. What fraction of each piece of cloth was used? Were the shaded parts the same amount of cloth, or was one more than the other? How can you tell?

Fraction name. ________________ ________________
Shaded parts, same amount? yes / no.
If no, which is more?

c. What fraction of each piece of cloth was used? Were the shaded parts the same amount of cloth. or was one more than the other? How can you tell?

Fraction name. ________________ ________________
Shaded parts. same amount? yes / no
If no, which is more? __________

2. Below is a picture of one whole unit. draw a picture for each fraction of this unit.

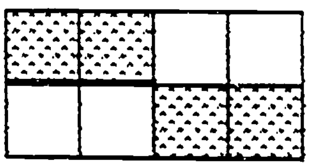
one whole unit 

2 thirds of the unit

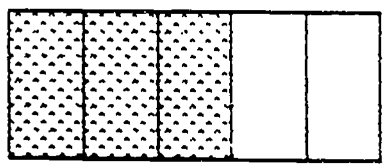
3 sixths of the unit

4 fourths of the unit

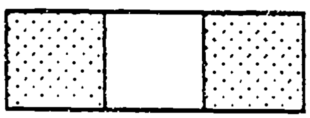
3. Name the common fractions that tell how much of the shape is shaded.



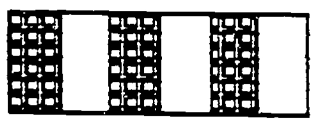
a. ____________________



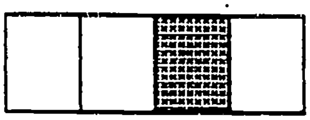
b. ____________________



c. ____________________



d. ____________________

4. Is  1 third of the rectangle?

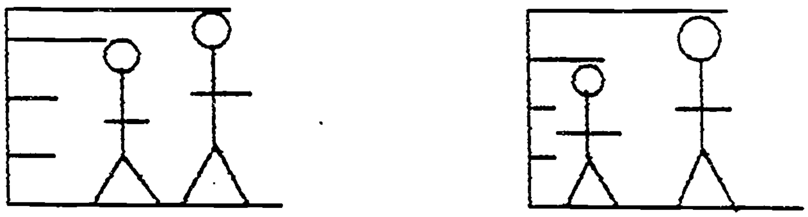
Why or why not?

5. Does each picture show the amount of the common fraction in the sentence. Why or why not?

a.

John ate 3 fourths of the granola bar.

b.



Caroline is 3 fourths the height of Mary.

6. Write 3 different common fraction names for one whole unit

__________ __________ __________

How many different common fraction names are there for one whole unit?

____

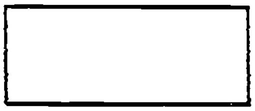
7. Write the common fraction number for each number name.

a. 2 twelfths ________ b. 3 fourths ________

c. 4 tenths ________ d. 3 fifteenths ________

e. 7 thirds ________ f. 9 fifths ________

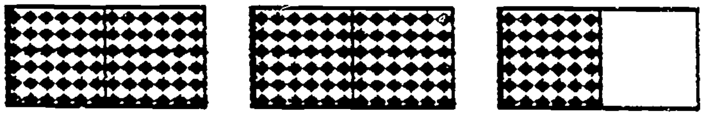
Worksheet 2

1. If  is one whole unit.

What is the fraction name and mixed number name for the amount of the units shaded.

a. ________________

____



b. ________________

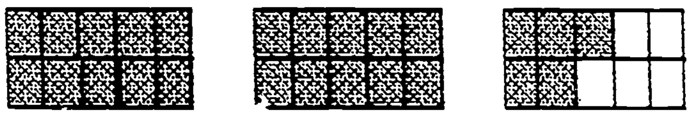
____

c. ________________

____

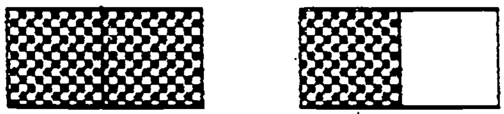
d. ________________

____



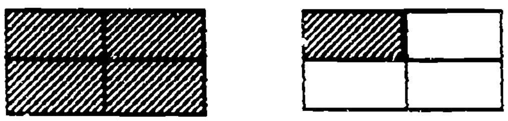
e. ________________

____



f. ________________

____



2. Rename these common fractions to mixed numbers. Use the fraction strips to help you.

a. 7/5 ________ b. 3/2 ________

c. 5/2 ________ d. 5/4 ________

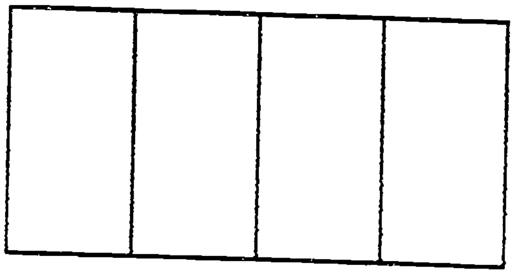
e. 7/3 ________ f. 12/10 ________

3. Rename these mixed numbers to common fractions. Use the fraction strips to help you.

a. 1 4/5 ________ b. 2 1/2 ________

c. 1 5/10 ________ d. 2 2/4 ________

4. Here is a chocolate bar:



a. Show how it would be shared among three people.

b. How much does one person get?

c. Can this bar be shared equally in more than one way? Explain this.

Worksheet 3

1. Circle the larger common fraction or mixed number in each pair.

   a. 1 fourth 1 fifth

   b. 4 sixths 2 sixths

   c. 1 and 3 fourths 1 and 1 fourth

   d. 1 tenth 1 hundredth

   e. 1 and 1 half 3 halves

2. Circle the larger common fraction in each pair below.

   a. 4/5 3/5 b. 1/5 1/3

   c. 2/8 2/10 d. 7/10 12/10

   e. 5/8 5/6 f. 10/10 11/12

3. Identify the largest and the smallest common fraction. Use the strips to help you.

		largest	smallest
a.	1/8 1/4 1/5	______	______
b.	2/8 7/8 4/8	______	______
c.	3/12 1/6 3/8	______	______
d.	3/4 6/4 2/4	______	______

4. Identify the largest and the smallest common fraction. Draw a picture to explain your decision.

		largest	smallest	picture
a.	1/8 1/6 1/3	______	______	
b.	2/8 2/6 2/4	______	______	
c.	3/8 6/8 2/8	______	______	

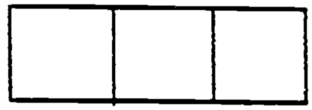
5. Rename each to a common fraction or mixed numbers [use your fraction strips to help you]

a. 7/5 ________	b. 3/2 ________
c. 5/2 ________	d. 9/4 ________
e. 7/3 ________	f. 12/10 ________
g. 1 4/6 ________	h. 2 1/3 ________
i. 1 5/12 _______	j. 2 3/8 ________

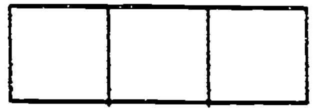
Worksheet 4

1. For each of the following. Draw a picture of one whole unit

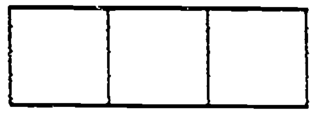
a. 3/4
one whole unit?



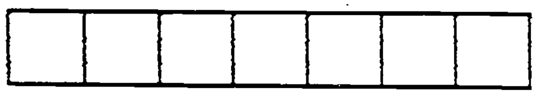
b. 3/5
one whole unit?



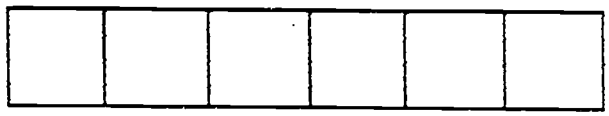
c. 3/6
one whole unit?



d. 7/4
one whole unit?

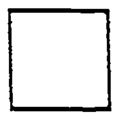


e. 6/3
one whole unit?



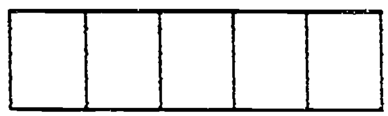
f. 1/3

one whole unit?



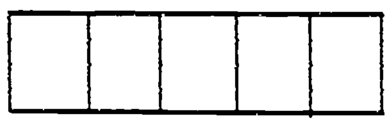
g. 5/3

one whole unit?



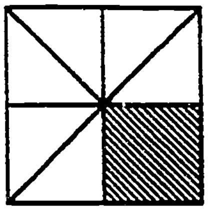
h. 5/8

one whole unit?

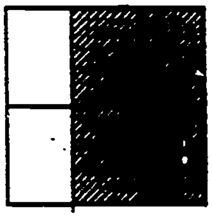


Worksheet 5

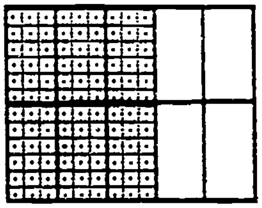
1. What common fraction is shown in each picture? Give as many different names as you can for these common fractions.



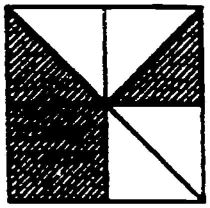
A. ________ fourth



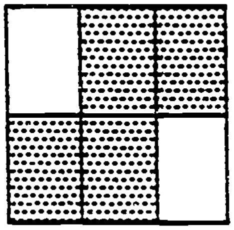
B. ________ thirds



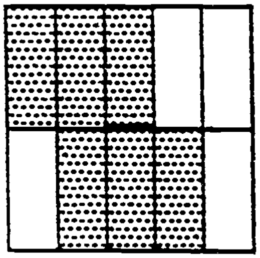
C. ________ fifths



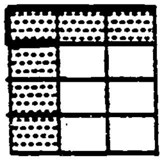
D. ________ fourths



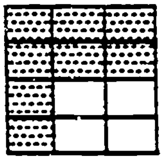
E. ________ thirds



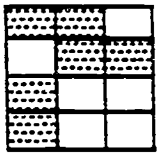
F. ________ fifths



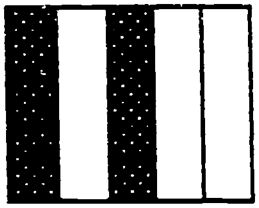
G. ________ fourths



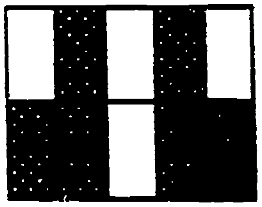
H. ________ thirds



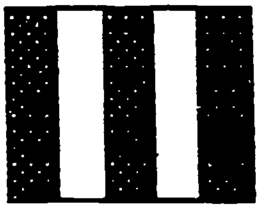
I. ________ sixths



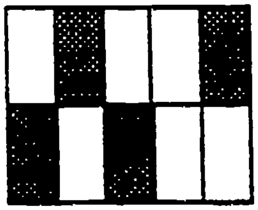
J. ________ tenths



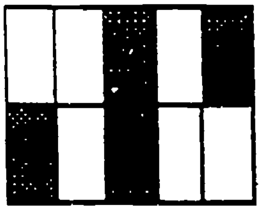
K. ________ fifths



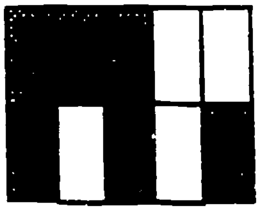
L. ________ tenths



M. ________ fifths



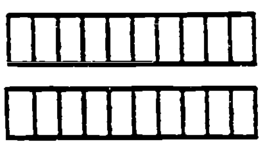
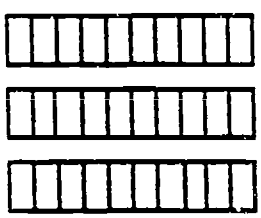
N. ________ tenths



O. ________ tenths

Worksheet 6

1. Each strip below which is divided into 10 equal parts represents a whole unit. Shade the parts showing the common fraction name given. Then write the common fraction name as a decimal fraction name.

	common fraction name	picture	decimal fraction name
a)	3 tenths (3/10)		0.3
b)	7 tenths (7/10)		
c)	9 tenths (9/10)		
d)	10 tenths (10/10)		
e)	1 and 4 tenths (1 4/10)		
f)	2 and 3 tenths (2 3/10)		

2. Write each with a <u>common fraction name</u> and a <u>decimal fraction name.</u>

		Common Fraction	Decimal Fraction
a)	6 tenths		
b)	8 tenths		
c)	5 tenths		
d)	1 and 7 tenths		
e)	2 and 3 tenths		
f)	12 and 4 tenths		
g)	20 and 3 tenths		
h)	11 tenths		

Worksheet 7

1. Write each as fifths, tenths and decimal fractions, as shown in the example

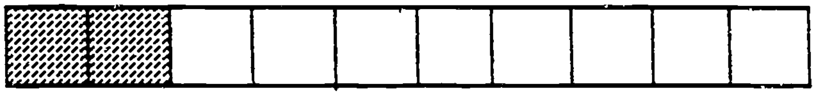
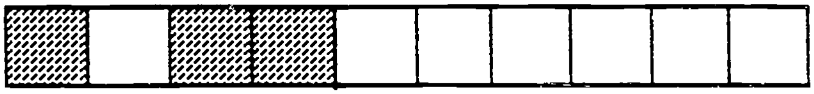
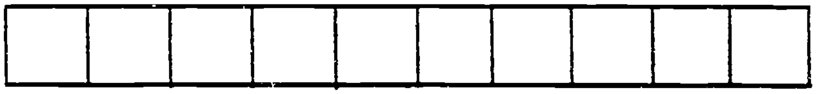
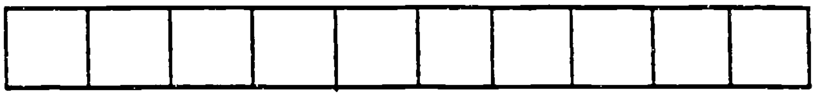
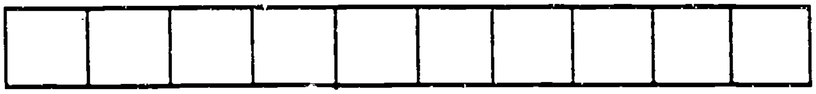
	Example	Fifths	Tenths	Decimal fraction
		1 fifth (1/5)	2 tenths (2/10)	_____
a)		__ fifths ( )	__ tenths ( )	_____
b)		__ fifths ( )	___ tenths ( )	_____
c)		__ fifths ( )	__ tenths ( )	_____
d)		__ fifths ( )	__ tenths ( )	_____

2. Use the fraction strips to decide which symbol " >, < or = " to use between these fractions:

a)	2/10	1/5	b)	4/10	2/5
c)	5/10	1/2	d)	11/10	3/5
e)	3/5	6/10	f)	1/10	2/5
g)	1 2/5	1 1/10	h)	2 5/10	2 2/5

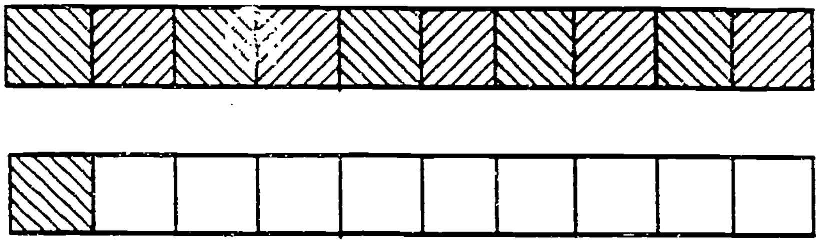
Worksheet 8

The "long" represents one whole unit here (It is a 10 cm rectangular strip, divided into 10 equal parts.) Complete the following, using the example given:

	Long	Common Fraction	Decimal Fraction
Example		2/10	0.2
1.			
2.		4/10	
3.			0.5
4.		9/10	

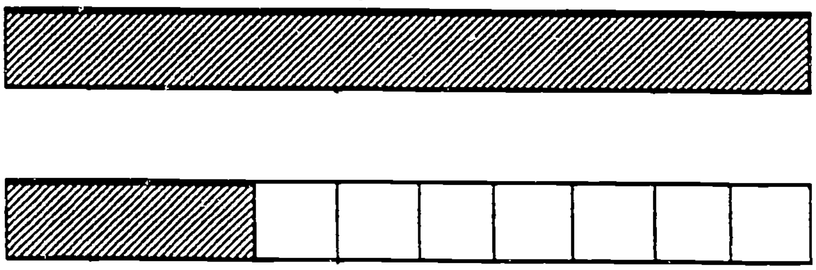
	Long	Common Fraction	Decimal Fraction

5.



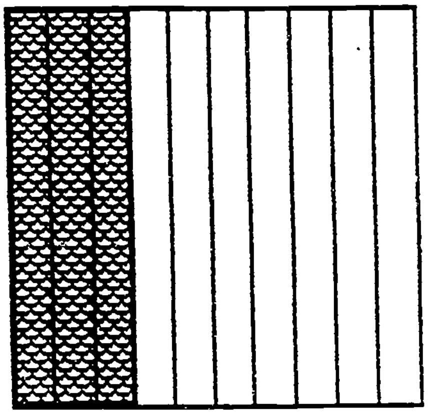
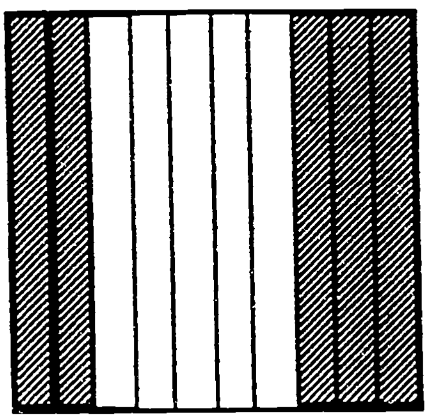
11/10

6.

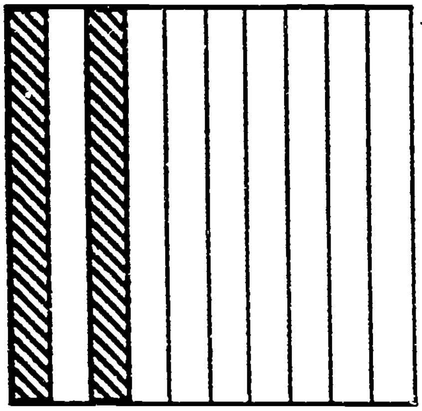
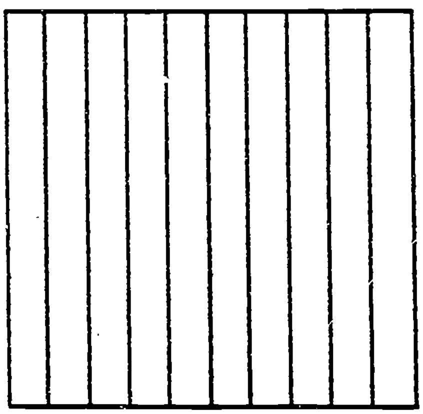
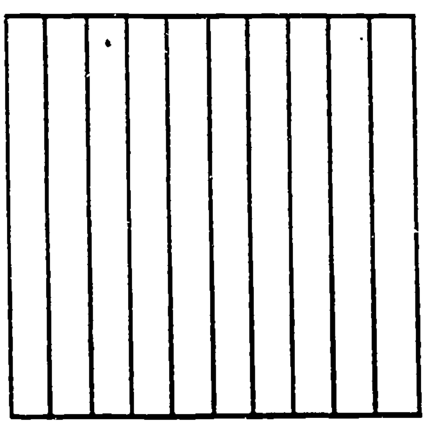


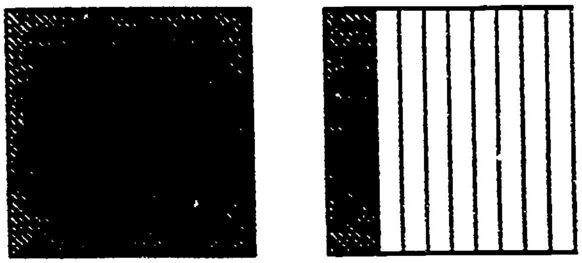
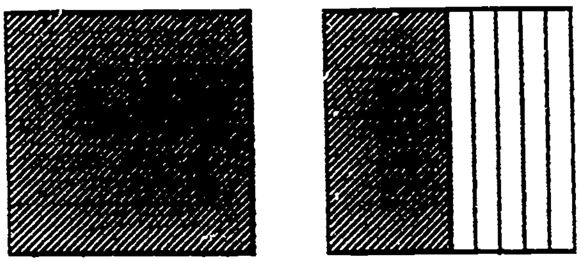
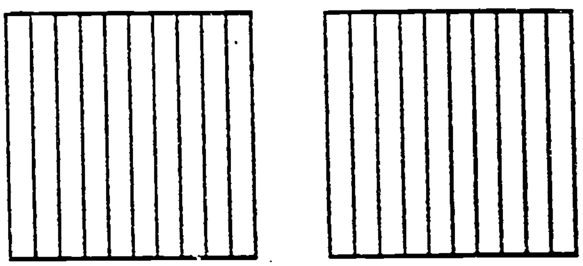
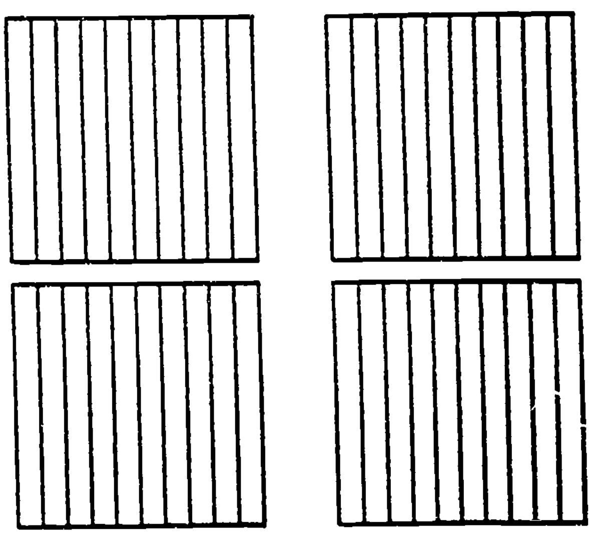
Worksheet 9

The large square represents one whole unit here. Complete the following, using the example given.

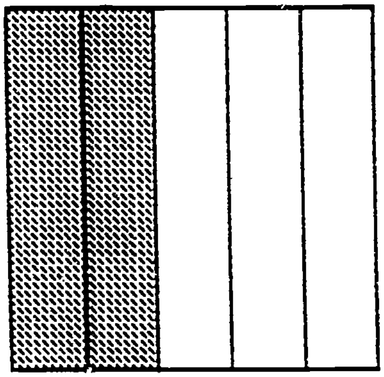
Example:	Common Fraction	Decimal Fraction
	3/10	0.3
1. 	5/10	

247

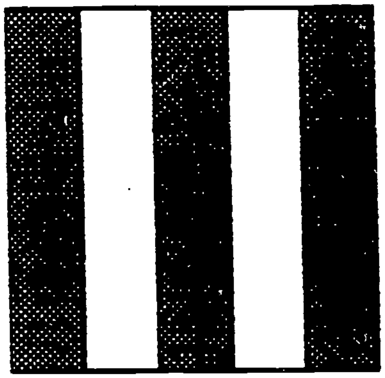
	Common Fraction	Decimal Fraction
2. 		
3. 	7/10	
4. 		0.8

	Common Fraction	Decimal Fraction
5. 	12/10	
6. 		
7. 		1.8
8. 	33/10	

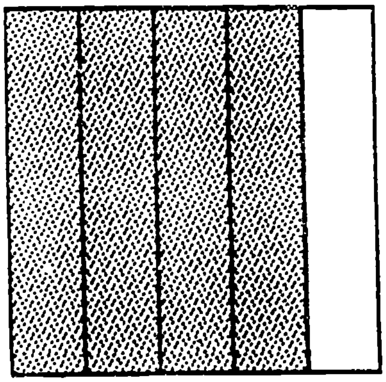
In questions 9 to 12, the large square divided into 5 equal parts represents a whole unit. Complete the following, using the example given:

Example	Fifths	Tenths	Decimal Fraction
	2/5	4/10	0.4

9.

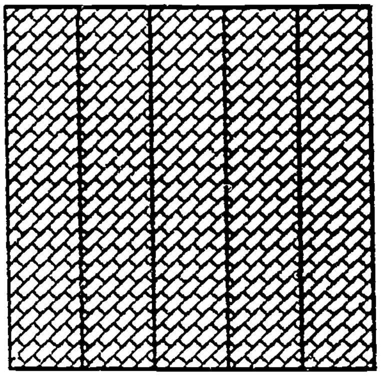


10.

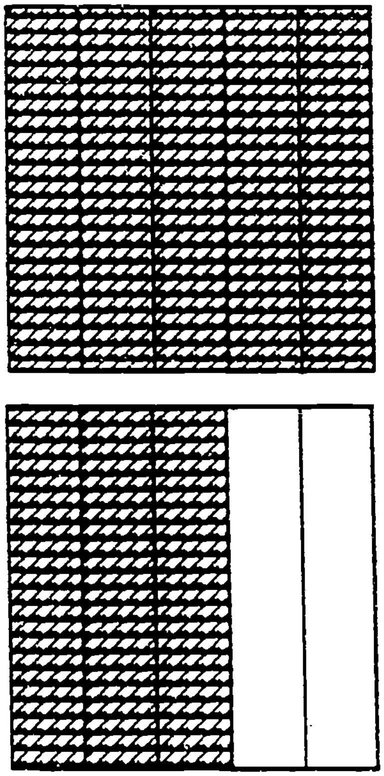


	Fifths	Tenths	Decimal Fraction

11.

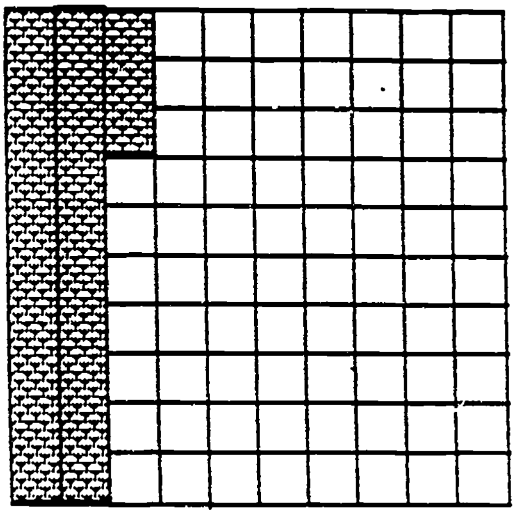
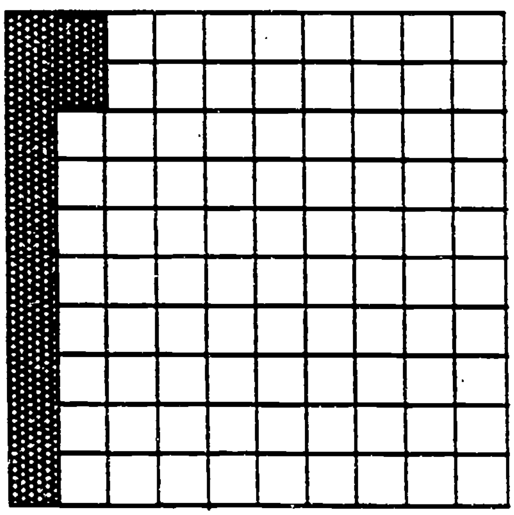


12.



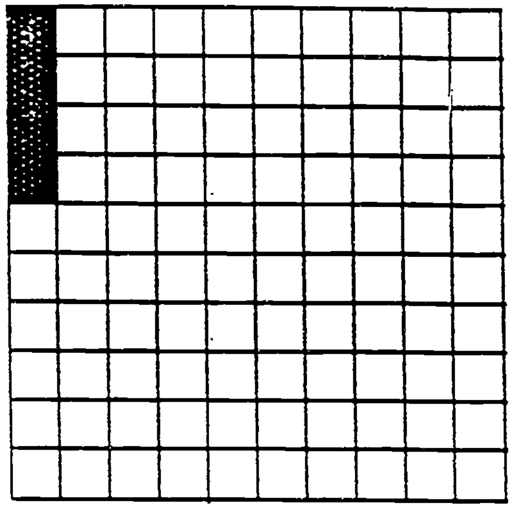
Worksheet 10

The "flat" represents one whole unit here (It is a 10cm square divided into 100 equal parts). Complete the following, using the example given:

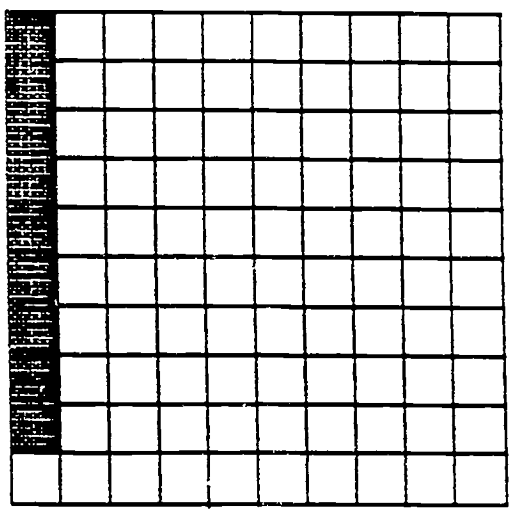
Example	Flat	Common Fraction	Decimal Fraction
		23/100	0.23
1.			

252

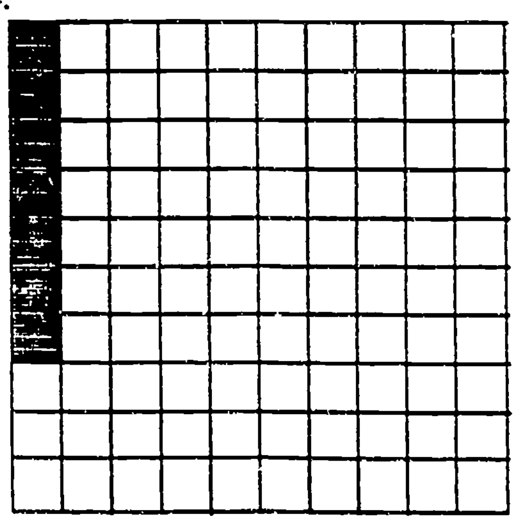
	Common Fraction	Decimal Fraction
2.		0.04



3.



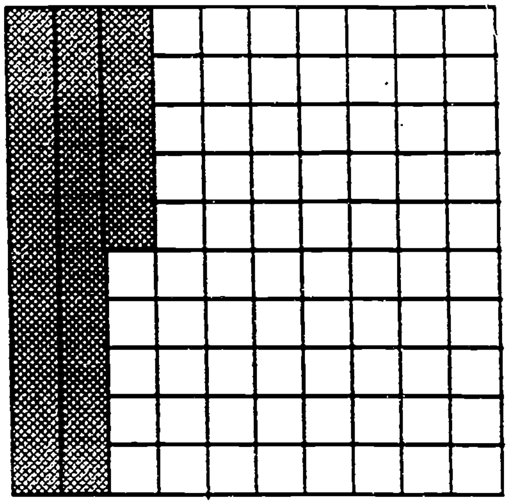
4.



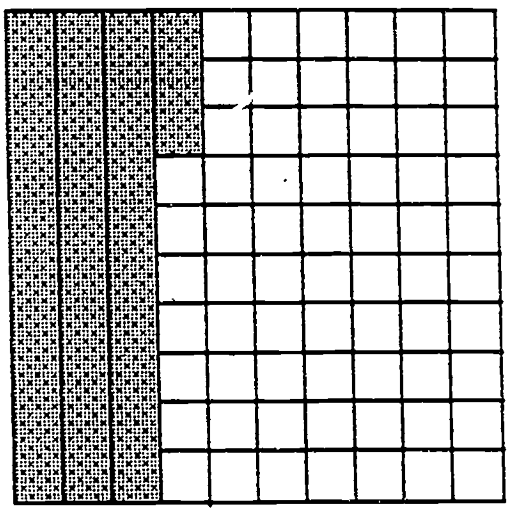
Common Fraction

Decimal Fraction

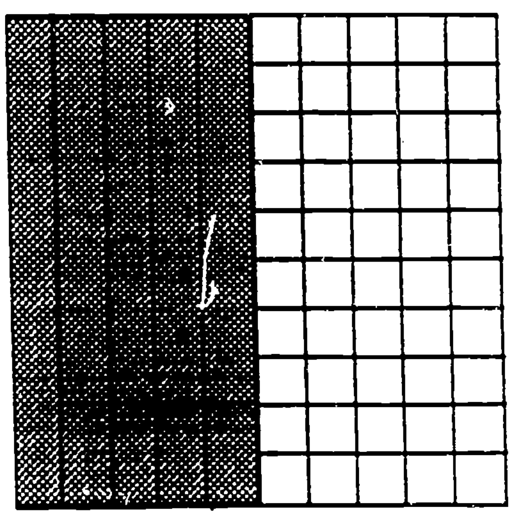
5.



6.

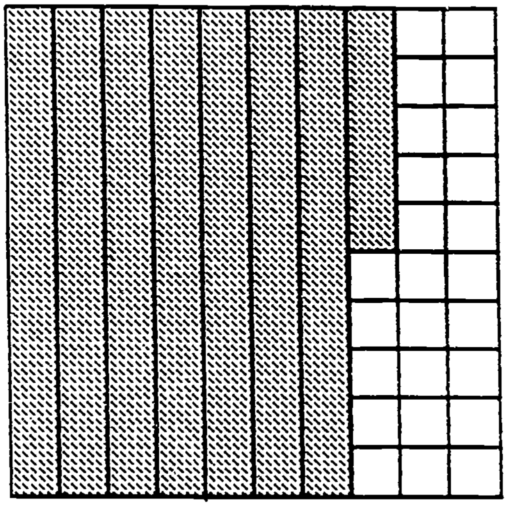


7.

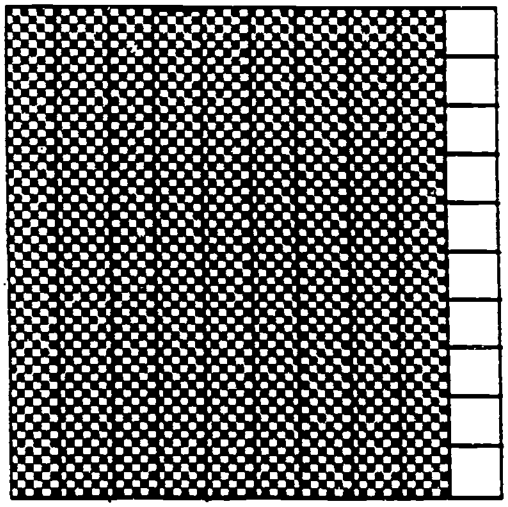


	Common Fraction	Decimal Fraction

8.

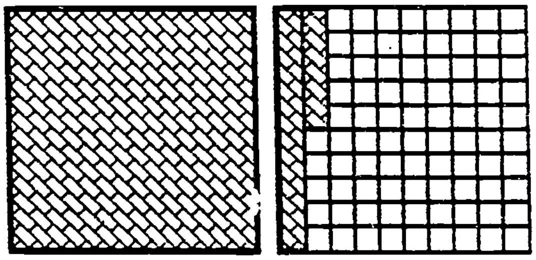


9.

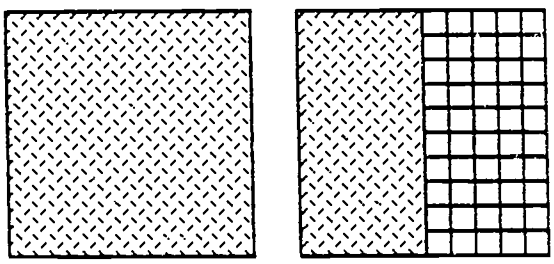


	Common Fraction	Decimal Fraction

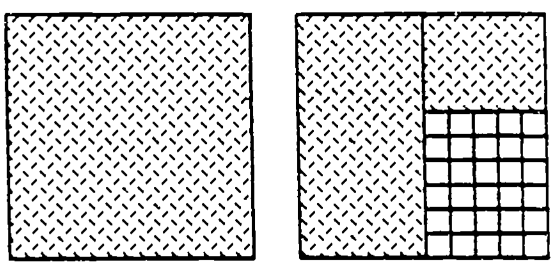
10.



11.

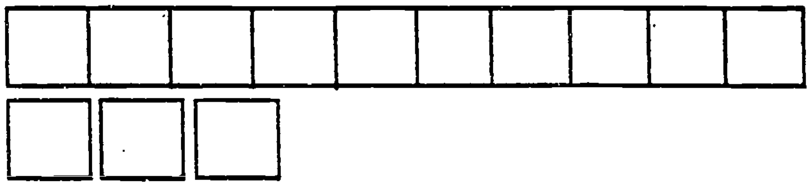
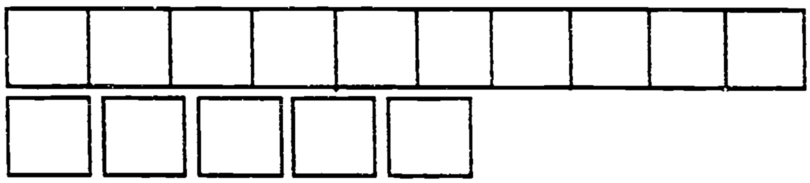
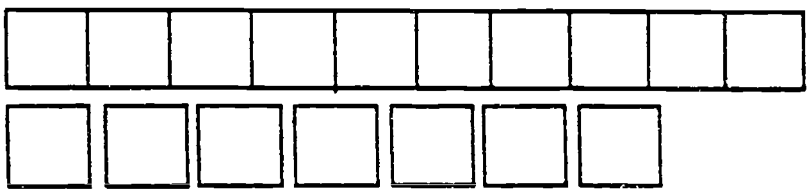


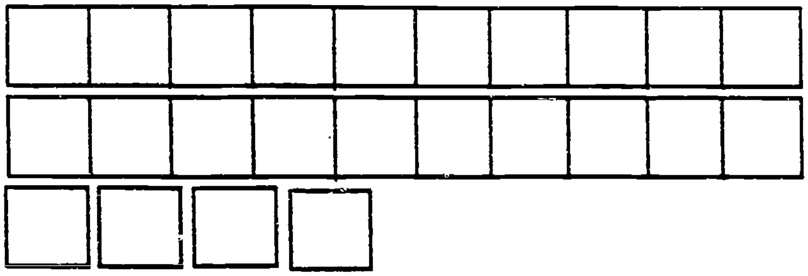
12.



Worksheet 11

The flat is the unit. So,here, the longs represent tenths and the cubes represent hundredths. Complete the following, using the given example:

Example	Picture	Common Fraction	Decimal Fraction
		13/100	0.13
1.			
2.			
3.		19/100	

		Common Fraction	Decimal Fraction
4.			
5.		27/100	
6.			0.31
7.		35/100	
8.			0.40

Worksheet 11 A

Use the longs (tenths) and the cubes (hundredths) to show the following:

1. 2 tenths and 3 hundredths
2. 23 hundredths
3. 1 tenth and 5 hundredths
4. 15 hundredths
5. 2 tenths
6. 20 hundredths
7. 1 tenth 3 hundredths = 13 hundredths
8. 30 hundredths = 3 tenths
9. 0.2 > 0.19
10. 0.10 < 0.9
11. 0.24 = 2 tenths and 4 hundredths
12. 0.17 = 0.1 + 0.07

Worksheet 12

1. First, use the flats (units), longs (tenths) and cubes (hundredths) to show the following. Then write the decimal fraction name for each of the following:

Decimal Fraction

a. 2 units, 2 tenths and 2 hundredths

b. 22 tenths and 2 hundredths

c. 2 units and 22 hundredths

What do you notice about the decimal fraction names above? Explain what you noticed.

2. Use the flats, longs and cubes and complete the following:

1.23 = __ unit, __ tenths and __ hundredths = __ tenths and __ hundredths

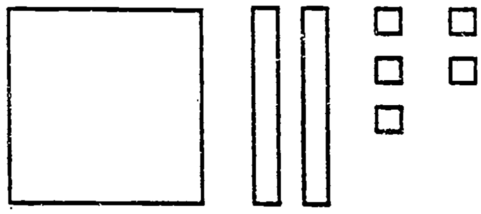
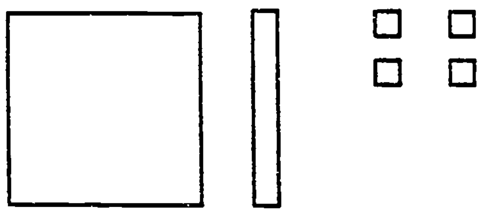
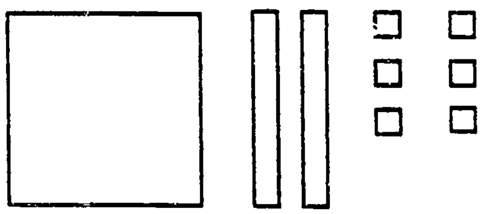
3. Tanya said that 0.06 + 0.4 + 1 = 0.146 but

Sonya said that 0.06 + 0.4 + 1 = 1.46 Do you agree with either of them? Explain your thinking using the flats, longs and cubes.

4. Look at the following pairs of decimal fractions. Use the flats, longs and cubes to decide the appropriate symbol < or >, for each pair of decimal fractions. Then write the appropriate symbol < or >, in 4a, 4b and 4c.

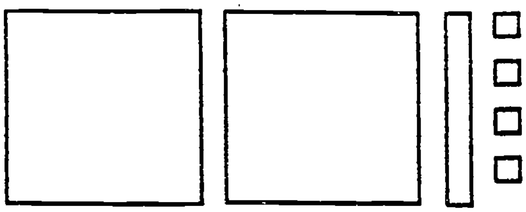
a) 1.1 1.09 b) 0.9 1.0 c) 1.10 1.5

The flat represents one whole unit, the long represents a tenth and the cube represents a hundredth here. Complete the following, using the example given:

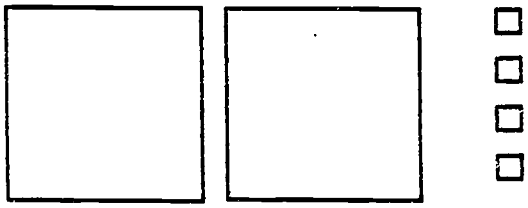
Picture	Decimal Fraction	Common Fraction
	1.25	125/100
1. 	1.14	
2. 		
3.		132/100
4.	1.03	

	Decimal Fraction	Common Fraction

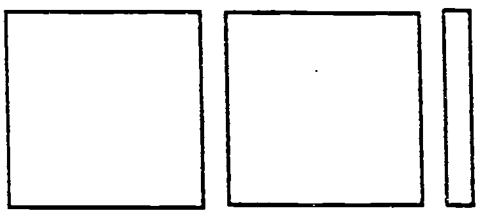
5.



6.



7.



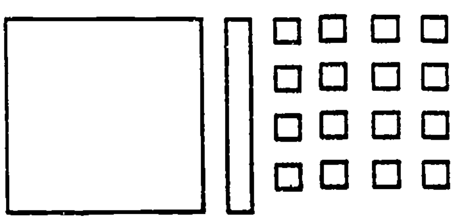
8.

1.40

9.

2.3

10.

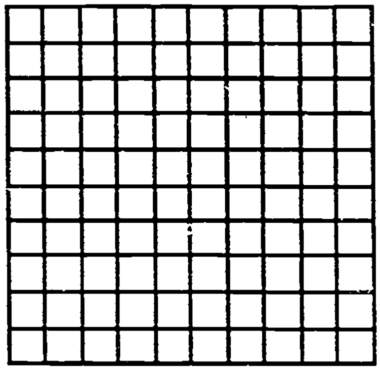


262

<u>Worksheet 13</u>

1. The flat is the whole unit (that is, 1) here.
   Shade the fraction. Then rename the fraction.

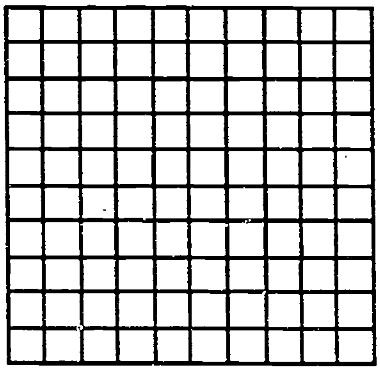
   a. Shade 3/5 of the flat below:



Write this amount as tenths ________

Write this amount as hundredths ________

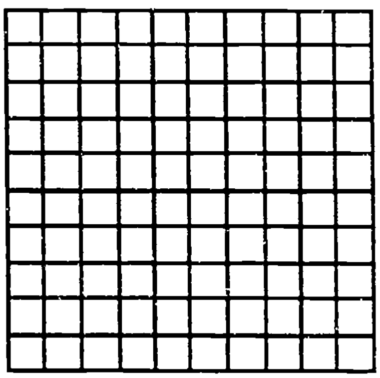
   b. Shade 0.8 of the flat below:



Write this amount as fifths ______

Write this amount as hundredths ______

   c. Shade 0.25 of the flat below:



Write this amount as fourths ________

Write this amount as tenths ________

2. Use the appropriate symbol (<, >, or =) between the numbers below:

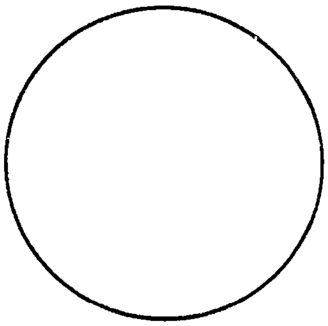
a) 1/3 1/4 b) 1/2 2/3 c) 3/4 2/3

d) 0.2 0.20 e) 0.10 0.09 f) 0.3 0.25

g) 1/5 0.4 h) 0.25 1/4 i) 1.5 7/4

3. Of the 100 immigrants in the last month, 0.5 were Chinese, 0.2 were Filipinos and Asian Indians, and the rest were Europeans. How many were Chinese? Filipinos and Asian Indians? Europeans? Use your flats and longs to help you.

4. Five children share a pizza. Sandy eats 1/12, Jodi eats 1/6, Ruzed eats 0.25, James eats 1/3 and Ainsley eats the rest. Shade the circle below to show each child's portion.



Worksheet 14

Use the fraction strips, flats, longs and cubes to help you do the following:

1 a) 2/10 + 7/10 = b) 0.2 + 0.7 =

2 a) 1/5 + 2/5 = b) 0.2 + 0.4 =

3 a) 3/5 + 1/5 = b) 0.6 + 0.2 =

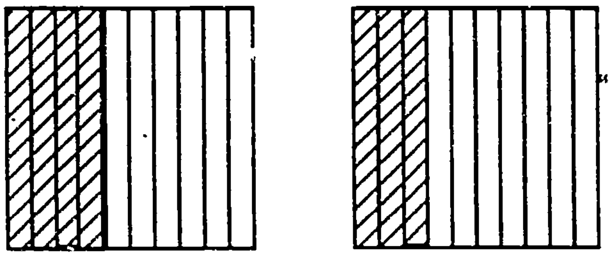
4 a) 1/4 + 2/4 = b) 0.25 + 0.50 =

5 a) 1/2 + 1/2 = b) 0.5 + 0.5 =

What do you notice about the common fraction answer and the decimal fraction answer to each pair of questions above? Explain what you noticed.

6. The length of a rectangular room is 5.4 metres and its width is 4.6 metres. What is the perimeter of the room? Use manipulatives or a picture to explain your work.

7. John said that 4/10 + 3/10 = 7/20. He used the following diagram to explain his thinking. Do you agree with him? Explain why or why not.



"7 equal parts out of a total of 20 equal parts, so 7/20 "

Worksheet 15

Here, 1 flat = 10 longs = 100 cubes. That is, 1 = 10 tenths = !00 hundredths

Use the manipulatives (flats, longs and cubes) and a picture to explain how you would do the following decimal fraction subtractions.

1) 0.43 - 0.21 =

2) 1.35 - 1.23 =

3) 1.04 - 1.02 =

4) 2.51 - 1.41 =

5) 0.52 - 0.39 =

6) 0.43 - 0.28 =

7) 2.36 - 1.85 =

8) 2.12 - 0.71 =

9) 2.07 - 1.96 =

10) 2.22 - 0.49 =

11) 2.01 - 1.69 =

12) 2.1 - 1.07 =

Worksheet 16

1. Use the fraction strips to help you do the following:

a) 3/4 - 1/4 =	b) 7/8 - 3/8 =
c) 10/12 - 5/12 =	d) 5/6 - 1/6 =
e) 9/10 - 3/10 =	f) 1 - 1/3 =
g) 3/2 - 1/2 =	h) 4/3 - 1 =

2. Use the fraction strips, flats, longs and cubes to help you do the following:

Common Fraction	Decimal Fraction
a) 7/10 - 2/10 =	0.7 - 0.2 =
b) 75/100 - 50/100 =	0.75 - 0.50 =
c) 43/100 - 28/100 =	0.43 - 0.28 =
d) 4/5 - 2/5 =	0.8 - 0.4 =
e) 3/4 - 1/4 =	0.75 - 0.25 =

3. Fraser runs the 100 metres in 17.2 seconds. Ben Johnson does the 100 metres in 9.9 seconds. How much faster does Ben Johnson run the 100 metres than Fraser? Show your work below.

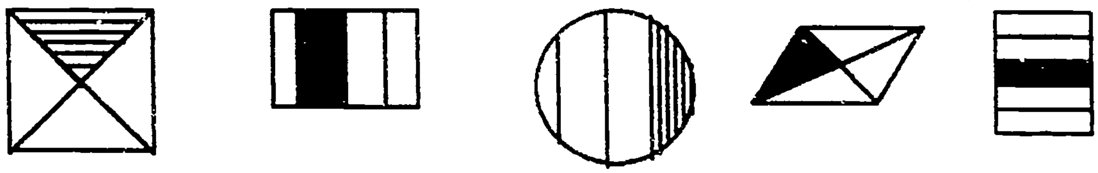
4. Lisa thought that since 1/10 was the same as 0.10, 1/3 was the same as 0.3; explain why you agree or disagree with Lisa.

<u>Review</u>

Name: Grade:

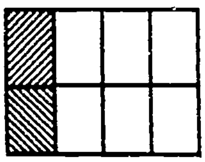
1. Which of the shaded parts show one fourth?

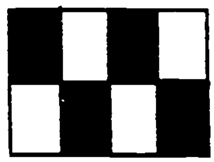
a) b) c) d) e)



2. If  shows one fifth, draw 2 different pictures for the whole unit.

3. Give 2 different common fraction names for the shaded parts





a) ______ b) ______

______ ______

4. Write 3 different common fraction names for one whole unit.

________ ________ ________

5 Write the common fraction number for each number name

a. 3 fourths _____ b) 5 tenths _____

c. 7 thirds _____ d) 9 fifths _____

6. Write as common fractions or mixed numbers

a. 7/5 ______ b) 3/2 ______

c. 9/4 ______ d) 12/10 ______

e. 1 10/12 _______ f) 5 1/2 ______

7. Circle the larger common fraction in each pair.

a. 1 tenth 1 hundredth

b. 1 fourth 1 eighth

c. 1/2 1/3

d. 3/5 4/5

e. 12/10 7/10

f. 10/10 12/10

g. 3/4 6/4

8. Arrange the three fractions in order from smallest to largest:

a. 1/2, 1/4, 1/3 ____, ____, ____

b. 2/8, 2/10, 2/12 ____, ____, ____

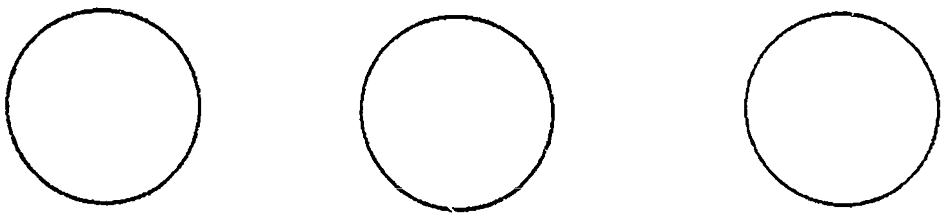
c. 5/6, 3/6, 1/6 ____, ____, ____

d. 5/12, 9/12, 7/12 ____, ____, ____

e. 6/5, 1/10, 1/100 ____, ____, ____

9. Three pizzas are shared equally among 8 children.
a. Draw a picture to show how much each child would get.
b. Write the fraction name for each person's share.
c. Explain in your own words how you worked out the answer.

<u>Picture</u>:



<u>Fraction name</u>:

<u>Explanation</u>: