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ABSTRACT

There are selected philosophies in the teaching of mathematics that can provide guidance to the teacher in developing the curriculum. This paper discusses four philosophies of teaching mathematics. (1) Idealism stresses that the pupil lives in an idea-centered mathematical world, but not an objective real world. Abstract content is prized more highly than concrete and semi-concrete representations. (2) Realism emphasizes that a person can know the real world in whole or in part as it really is. With pupils attaining precisely measurable stated objectives in mathematics, they become more and more knowledgeable of the real world as it truly is. (3) Experimentalism emphasizes pupils' learning that is useful and utilitarian. Within a given problem area, mathematics is used to solve selected problems. (4) Existentialism stresses individual choices made by a pupil in selecting sequential tasks and experiences in mathematics. The pupil is the chooser. The tasks may involve problem solving as well as other kinds of tasks. Use of diverse philosophies of education to provide for individual differences should assist each pupil to learn as much mathematics as possible. (MKR)

Philosophy of Teaching Mathematics

by
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PHILOSOPHY OF TEACHING MATHEMATICS

There are selected philosophies in the teaching of mathematics which can provide guidance to the teacher in developing the curriculum. Each teacher has selected concepts and generalizations which provide a framework for teaching and learning. A study of the philosophy of education may develop a reservoir from which the teacher may secure the background knowledge, attitudes, and skills to do a quality job If teaching learners.

Idealism in Teaching Mathematics

Idealism is one of the oldest philosophies available which may assist the mathematics teacher to select objectives, learning opportunities, and evaluation procedures for pupils. Plato (427- 347 BC) advocated idealism as a philosophy of education in ancient Athens. Above Plato's academy door, it stated that "no one is to enter unless they know mathematics." He believed mind to be superior to the body. Upon death, the mind/soul survives whereas the body decays. The mind must rule the body so that higher levels of choices are in evidence. It is the body that brings an individual to lower or inferior levels of choices and decisions. In the Forms (heaven), perfection exists in that a perfect something exists, such as different number systems, and geometrical figures, among others. The here and the now on the changing earth is inferior to what is in the unchanging Forms. Thus a triangle, square, parallelogram, and circle, for example, in the here and now are imperfect models of what is perfect in the Forms. The same is true of all things and life on earth. What exists in the stable Forms is much superior to the world of change here on earth.

One only receives ideas about the Forms according to Plato. A person cannot perceive the Forms as they truly are, but receives ideas through thought, mind, meditation, and intellectual endeavors. The well educated and the abstract thinkers have abilities to perceive or receive ideas pertaining to The Forms. Wisdom is a necessary prerequisite to perceive The Forms.

Idealism as a philosophy of education still receives much attention today. A mathematics teacher who is an idealist tends to emphasize mental endeavors as being superior to the physical and its emphasis. The mind is what is truly real about the person. Thus the mathematics teacher needs to stress pupils attaining abstract content in mathematics since this will aid mental development. Higher cognitive level objectives need to be selected and implemented in the mathematics curriculum. These objectives pertain to pupils being able to think critically, synthesize content, and appraise what has been acquired. Mind is real and needs to be developed, according to idealism as a philosophy of teaching mathematics.

Concrete learning opportunities consisting of the use of real objects, and the semiconcrete, emphasizing use of illustrations, should be stressed only if they guide learners to understand abstract ideas in mathematics. The focal point of instruction is ideas and mental development. One receives ideas of the natural and social environment only, not a replica of the real world. All information is developed by the mind. Ideas are then secured about the natural and social facets of life. What is in back of this world is mind and the spiritual, not the physical. Scope and sequence in mathematics emphasizes mental and intellectual development of the pupil pertaining to the following topics:

1. base ten system of numeration, estimating, as well as understanding positive and negative integers.
2. addition, subtraction, multiplication, and division on whole numbers.
3. geometry with its space figures, including plane (squares, rectangles, triangles, parallelograms, and circles, among others) and solid (spheres, cylinders, cones, prisms, and Platonic solids) figures.
4. common and decimal fractions, as well as per cents.
5. measurement including linear, square, and cubic.
6. graphs, tables, statistics and probability.

For the above named topics, pupils with teacher guidance need to study each in depth with emphasis placed upon learners attaching

meaning to content being taught. Mathematics as general education is salient in developing mental maturity to work with numerals and number in the abstract. Critical thinking in the mathematics curriculum stresses mental development. Reason and intelligence are necessary to achieve fully in mathematics. The rational being then becomes increasingly mature mentally to use intelligence in dealing with the world of number and numerals. The teacher stimulates pupils to achieve using a variety of learning opportunities emphasizing inductive and deductive methods of thinking. Idealists stress the concept of purpose for each human being in a purposive world. There is purpose involved in learning mathematics. Th purpose involves, among other things, the development of the spiritual facet of the person. Human beings are not a part of the animal world, according to idealists. Rather they transcend that level and are endowed with rational powers that animals do not possess. Mathematics as an academic discipline can assist pupils to reach out from the finite toward the Infinite, in achieving intellectual and rational goals. Mathematical truths are a priori and thus have always existed. For example, any basic number sentence in mathematics such as 12 x 10 =120 has always been true, prior to human experience. Each pupil must be guided by a competent and academically inclined teacher in discovering preexistent truths in mathematics.

For the idealist, mathematics presents content to pupils to encourage the development of reasoning persons in which the mind achieves in the direction of the Infinite, the unlimited in terms of attaining an ideal. Mind, not matter, represents ultimate reality. The mind and mathematics content stress reaching toward the Ideal or Infinite in achieving a priori content.

Since idealists in mathematics tend to recommend mental development of the learner as a major goal of instruction, a quality series of mathematics textbooks might well provide appropriate scope and sequence in the curriculum. The teacher's role here is to assist pupils to attain optimally in thinking mathematically.

Realism and the Mathematics Curriculum

The mathematics teacher who stresses realism as a philosophy of education believes in using the methods of science in teaching and learning situations. Objective evidence, irrespective of the subjective person, is inherent in mathematics. Thus, subject matter in mathematics is true independent of the observer or person. Precision is a key word to use in teaching mathematics, according to the realist. A realist likes accurate descriptions of what exists. For example, he/she does not care for a person saying that the temperature reading in a room is comfortable. Rather, the exact temperature reading is wanted such as 22 degrees Celsius. If a person states that his/her blood pressure reading is normal, the realist desires to know the precise blood pressure reading using numerals for the systolic and diastolic readings. A teacher who emphasizes that his/he pupils are attaining well does not satisfy the realist critic. Rather numerical results are wanted to ascertain how well learners are attaining, such as grade equivalents, percentile ranks, quartile deviations, as well as standard deviations from the mean and other derived or standard scores. Testing pupils to notice achievement is quite typical of the philosophy of realism. Thus standardized and norm referenced tests may be used to gather data on learner progress. Formative an summative tests are recommended to be given to learners to notice pupil progress in mathematics. The former is given to learners within an ongoing unit of study to monitor achievement along the way. The summative test is given at the end of a unit of study in mathematics so that changes may be made, if evidence warrants, the next time the same unit is taught.

A mathematics teacher then who is a realist desires objectives of instruction to be stated in measurable terms, prior to instruction. The following are examples:

1. The pupil will add correctly ten number pairs, each containing single digit addends.

2. Given four geometrical figures, the learner will accurately compute the area of each.

3. The learner will change five common fractions to decimals and then to per cents.

4. Given three dimensional values for a rectangular prism, a triangular solid, a cylinder, and a pyramid, the pupil will compute accurately the volume of each geometrical solid.

5. Given data pertaining to the federal budget, the pupil will construct a line graph, a bar graph, and a picture graph.

For each of the above named objectives, pupils will reveal as a result of instruction if they have been successful in goal attainment. These objectives are stated with precision so that the mathematics teacher knows exactly what is to be taught. There is no guesswork in terms of what pupils are to learn. There are realists who advocate that the teacher announce prior to instruction what pupils are to learn as stated in the objective(s). Pupils then do not need to out guess the teacher in terms of what is expected of them as learners. When ascertaining how much pupils have learned as a result of instruction, the teacher receives numerical results such as the per cent of correct responses from a test of each pupil. The results from each pupil could also be computed to secure percentile ranks. Derived scores based on the normal distribution curve would indicate the number of standard deviations above and below the mean for each pupil. A mathematics teacher who is a realist in terms of philosophy of education desires precise objectives for learner attainment. He/she matches the learning opportunities with the stated specific objectives so that an increased number of objectives will be achieved by pupils. What is in the learning opportunities then harmonizes with what is stated in the objectives, no more and no less. Appraisal procedures harmonize with the objectives of instruction. Validity in appraisal is then in evidence. Results from the appraisal determine the number of objectives achieved satisfactorily by the learner. Results for the appraisal are objective in that independent of any evaluator, the number of correct responses would be the same each time. Subjectivity is then eliminated in the appraisal process.

Experimentalism and the Mathematics Curriculum

The world of experience represents ultimate reality for the

experimentalist. The realist believes that one can know the real world as it truly is in whole or in part. Also the real world exists independent of any observer or human being. The idealist believes that one can only know ideas about the real world, not as it is truly is.

With knowing what is experienced only, the experimentalist realizes that change is all around us. Our perceptions change in time and place. Life in society continually changes. Thus problems arise which need identification. Each problem is life- like and reality based, not fictional.Clarity in problem selection is relevant. Vague, hazy problems do not lend themselves to solutions. An hypothesis is developed for the identified problem. The hypothesis is actually an educated guess or answer to the chosen problem. The hypothesis is not absolute, but tentative. The hypothesis is then subject to testing in a life-like situation. The consequences of the testing real the correctness or the lack thereof pertaining to the stated hypothesis.

Problems should come from pupils in the world of society. Utilitarian problems are then identified, not textbook story problems. A practical mathematics curriculum is then in evidence. What is useful in the mathematics curriculum is desired in terms of objectives, learning opportunities, and evaluation procedures. The everyday experiences of people in society pertaining to mathematics provides content then for the experimentalist curriculum. Within a mathematics unit being studied, the learners choose problems to solve.

The experimentalist mathematics teacher needs to guide pupils to identify life- like problems. These are purposeful to the learner. The problems are useful to solve and have utilitarian values. Practical subject matter is then being emphasized in mathematics. The following represents a practical problem for pupils to solve within a committee setting:

1. In our school garden, how much of carrots, radishes, potatoes, and other crops should be planted?

2. How far apart should the rows be and how far apart should the plants be within a row?

3. What needs to be done to tale care of the garden after the plants

have come up?

4. Who will be responsible for each of the tasks in gardening?
5. Who will use the harvested crops?

These among other problem areas will need to be solved in producing garden crops. New problems will emerge as the project is carried forward. There is much mathematics involved in establishing a garden for the school. Thus rows need to be measured as to length and distance apart. Space between plants will also need to be measured within each row. The cost of seeds needs to be determined. How much seed for each garden crop needs to be decided upon and implemented Water needs to be used in selected amounts when rainfall is not adequate. Rainfall as well as water used for irrigation can be measured to determine volume. Time spent in taking care of the garden can be measured for each pupil. The amount of garden crops produced can be weighed. Perhaps, the produce will be sold. The price for each fruit or vegetable must then be ascertained. Produce items need to be weighed to determine the total price to be charged for the commodity.
Time schedules for work for each learner should be completed. It is quite obvious that much mathematics is needed in the gardening project.

For the above example, there are selected principles of learning that experimentalism stresses as a philosophy of education. These include the following:

1. much planning by the involved group of pupils is needed.
2. mathematics is useful, not theoretical and abstract.
3. every day experiences in life provide content for the mathematics curriculum.
4. a learning by doing approach is emphasized.
5. pupils are actively involved, not passive recipients of knowledge, in ongoing mathematics lessons and units of study.
6. mathematics is not separated form life outside of school and is related to other curriculum areas, such as agriculture and food production, record keeping and accounting, physical education and in this case working in the gardening project, the language arts (listening,

speaking, reading, and writing activities), science (such as under what conditions do plants grow best), and social studies (including economics and geography).

7. the pupil and the curriculum are not separate, but integrated entities.

8. the learner is heavily involved in curriculum development.

9. problems need identification and solutions.

10. social achievement is stressed in that committee endeavors are in evidence in the total project.

Experimentalism as a philosophy of education is a necessary component in mathematics since application of content is being emphasized. What is learned and acquired is used in life- like problem solving situations. The curriculum becomes and is utilitarian. The learner presently is actively involved in the useful and the utilitarian and does not wait for a future time to be a productive member of society.

There are additional situations in the school and classroom setting which provide experiences for learners in the mathematics curriculum. These are the following, provided as examples:

1. planning how a pupil is to spend his/her weekly money allowance.

2. planning objectives, learning opportunities, and evaluation procedures with pupils for sequential lessons and units in mathematics which stress practical experiences.

3. planning an experimentalist mathematics curriculum which harmonizes with the National Council Teachers of Mathematics (NCTM) Curriculum and Evaluation Standards for School Mathematics (published by NCTM, 1906 Association Drive, Reston, Virginia 22091- 1593).

4. planning how to divide cookies among a certain number of children who are involved in the lesson presentation. When teaching mathematics, there are numerous situations such as this, whereby pupils need to be actively involved in decision- making.

5. planning a class party related to a holiday in which mathematics is heavily used such as how many cookies, cup cakes, and soft drinks to

purchase.

Mathematics teachers need to be creative in thinking about developing and implementing an experimentalist curriculum. There are numerous experiences which can be included in mathematics lessons and units of study emphasizing the practical and the utilitarian in life-like problem solving situations.

Existentialism and the Mathematics Curriculum

Existentialists stress the individual choosing and making decisions. To be sure, it is very salient that each pupil learn to engage in the making of choices. Life consists of making choices. Experimentalism emphasized also that pupils choose and make decisions, but usually within a committee setting. The belief exists in experimentalism that a pupil is a member of society presently and should be actively involved in the mathematics curriculum, but within a committee setting. Existentialism emphasizes the individual as one who should determine his/ her curriculum within a flexible framework. The teacher assists the pupil in achieving the latters goals. I will mention a few other tenets of existentialism which may or may not apply to the mathematics curriculum. One first exists and then determines his/ her essence. Thus the individual pupil should be heavily involved in determining goals, learning opportunities, and evaluation procedures in mathematics. I truly believe this to be a difficult method of teaching, but it certainly has its values and benefits. In all of teaching, it is the learner that is the focal point of instruction. Jean Jacques Rousseau (1712- 1776) in his book Emile emphasized a one on one relationship between teacher and pupil. Thus a pupil would be taught by a mentor or teacher. Here, the teacher could truly provide for individual differences (one pupil and one teacher). The learner asks questions that would be of personal interest. The out of doors or nature provides the necessary curriculum for the pupil, according to Rousseau. The teacher then assists the pupil to find the needed information. Induction as a method of teaching is used here. The pupil does not need to depend upon other pupils for help in

learning, but is to be an independent being, removed from the ills of society. Nor is the learner hindered in optimal achievement since no other pupil is there to hold the former back. The pupil does not need to gauge his (a boy in this case) learning against that of others in making comparisons. Uncomfortable comparisons between learners in achievement then can not be made in the one on one teaching situation. Rousseau's philosophy of instruction had definite tenets of existentialism. Which are selected mathematics experiences for pupils that Rousseau recommenced?

1. estimating the height of a cherry tree so that an appropriate ladder may be found or made to reach and pick cherries.

2. measuring the size of boards to make necessary items and objects.

3. becoming independent as a carpenter so that one does not need to be a servant of others. (Rousseau was very critical of norms in society). In being a carpenter, arithmetic and geometry are salient to learn within the framework of life like situations.

Rugged individualism can be term used to describe existentialism. Soren Kierkegaard (1813- 1855), a theistic existentialist, advocated the the person is first born and then finds his/her essence, meaning the individual must find his/her own purposes in life. These purposes or goals are not given to anyone, but must be found. The individual makes the self in an open ended universe, very limited in restrictions. Prior to this time, most philosophers stressed that the essences or purposes of persons were given to all first, and then the individual would be more certain as to what his/her role in life would be. Idealism was a prominent philosophy during the centuries and emphasized that the Infinite was ultimate reality and had purposes established for all. Kierkegaard was a theistic existentialist who also believed that the Absolute was ultimate reality; however a long struggle was necessary in reaching this goal involving personal choices made. Jean Paul Sartre (1905- 1980) emphasized atheistic existentialism as a philosophy of life. He also emphasized, as did Kierkegaard, that an individual is born and then

must find his/her own essence or purposes. With no Absolute, Sartre stressed that there is no one to manipulate the individual from above to determine purposes in life. Sartre's famous words that "Man is condemned to be free" certainly would make for a world of free choices for the individual. There are no absolutes. An existentialist mathematics teacher needs to give learners as many options as possible in learning. The pupil chooses that option in a very open ended mathematics curriculum. Most teachers of mathematics would tend to feel that existentialist philosophy is too free of borders and boundaries. Mathematics has its own scope and sequence. The scope and sequence has much agreement in and among mathematics educators. I would like to describe a mathematics unit which a few of my student and regular teachers have used. The approach I will describe emphasizes the use of learning stations. The mathematics teacher here needs to decide upon the number of stations needed. Perhaps for twenty- five pupils, there should be at least eight stations. Each station must possess concrete semiconcrete, and abstract materials of instruction. Also at each station, there is a task card which lists possibilities for pupils individually to choose from in terms of learning activities. The pupil may select which station and which tasks to work on sequentially. There should be an adequate number of tasks so that a pupil may omit that which does not possess perceived purpose. Sequence resides within the learner, not textbooks nor the teacher, in that the pupil orders his/her own experiences. If the pupil cannot find a station or task which meets personal purposes, he/she may plan with the teacher which learning opportunities to complete in mathematics. The learner is the chooser in deciding upon these tasks. The teacher encourages, assists, and guides the pupil in finding tasks and materials to complete that which has perceived purpose. Tasks at the different stations should have individual endeavors as well as those which stress committe work. The pupil then can work individually or with others, depending upon perceived purpose. In all cases, pupils individually sequence their very own learning opportunities.

There are mathematics teachers who stress additional tenets of

existentialism in their teaching. The following are examples:

1. having pupils choose extra work to do in mathematics, beyond that which is required.

2. completing a contract with individual learners to indicate what he/she is to complete. The contract lists specifically what a pupil wishes to complete with a due date listed. What is in the mathematics contract represents that which the learner desires to complete with teacher assistance, not teacher direction.

3. using teacher- pupil planning in the mathematics curriculum in which the latter determines what will be learned in sequence with instructor guidance. Thus the objectives, learning opportunities, and evaluation techniques are chose by pupils with teacher guidance in mathematics.

Each of the above three enumerated items contains mathematics learning opportunities which can be incorporated into any classroom. To stress tenets of existentialism, the teacher must lean upon pupils in determining what they wish to learn. From within, the learner is the decision maker in terms of objectives, learning opportunities, and evaluation procedures in the mathematics curriculum.

In Closing

Four philosophies of teaching mathematics were discussed. Idealism stressed the pupil live in an idea centered mathematical world, but not an objective real world. Mental development of the pupil is a number one goal of instruction. The mathematics curriculum is viewed here as a part of the general education curriculum. Abstract content is prized higher that that which is concrete and semiconcrete.

Realism emphasized that a person can know the real world in whole or in part as it really is. With pupils attaining precise measurable stated objectives in mathematics, they become more and more knowledgeable of the real world as it truly is. Each objective attained assists the learner to knowing more and more about the real world as it truly is, not merely ideas of this world.

Experimentalism emphasizes pupils learning that which is useful and utilitarian. Within a given problem area, mathematics is used to solve selected problems. Committee work is emphasized in that in society, people work in groups to solve problem areas.

Existentialism stresses individual choices made by a pupil in selecting sequential tasks and experiences in mathematics. The pupil is the chooser. The tasks may involve problem solving as well as other kinds of tasks.

The teacher needs to select that philosophy to implement which assists a pupil to attain optimally. Pupils differ form each other in numerous ways such as native abilities, past experiences, interests, motivation, and purposes. It behooves the mathematics teacher to prepare well and guide learners individually to attain optimally. Use of diverse philosophies of education to provide for individual differences should assist each pupil to learn as much mathematics as possible.

Selected References

Ediger, Marlow. Teaching Mathematics in the Elementary School (A Collection of Essays), ERIC Clearing House-- Resources in Education, 1985.

Kennedy, Leonard M. and Steve Tipps. Guiding Children's Learning of Mathematics. Seventh Edition. Belmont, California: Wadsworth Publishing Company,1994.

National Council Teachers of Mathematics. Curriculum and Evaluation Standards for School Mathematics. Reston, Virginia, 1989.

National Council Teachers of Mathematics. Student Math Notes. Reston, Virginia, 1989.

National Council Teachers of Mathematics. An Agenda for Action. Reston, Virginia, 1980.

National Council Teachers of Mathematics. The Mathematics Education of Exceptional Children and Youth, an Interdisciplinary Approach. Reston, Virginia, 1981.

Slavin, Robert. "Student Team Learning in Mathematics, in Neil Davidson (Editor), 1990, Cooperative Learning in Mathematics Menlo Park, CA: Addison- Wesley, pp. 89- 97.