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ABSTRACT

Considering teachers' thinking about a specific mathematical topic allows one t› better understand the broader domain of teachers' mathematical thinking and its influence on teaching and learning. This paper explores the meanings and understandings communicated by one middle school mathematics teacher about mathematics and mathematics teaching in general, and fractions in particular, in the context of his attempt to use an innovative set of curriculum materials. Data were collected using interviews, observations, and students' and teacher's written work and plans. The teacher's view of mathematics was of a correct set of rules and concepts and this contributed to his insistence on maintaining a teacher-dominated classroom environment. Also, his own flexible understanding of fractions allowed him to adjust his instruction to accommodate mathematical ideas that were not at the forefront of his own thinking and to emphasize important connections among mathematical representations when provided with curriculum materials to support such an emphasis. Contains 20 references. (MKR)

Implications for Teaching of One Middle School Mathematics Teacher's Understanding of Fractions

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Implications for Teaching of One Middle School Mathematics Teacher's Understanding of Fractions

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This paper describes how one middle school mathematics teacher understands an important part of the mathematics he teaches. It explores the meanings and understandings he communicated about mathematics and mathematics teaching in general, and fractions in particular. In the context of him attempting to use an innovative set of curriculum materials, the paper explores relationships between his conceptions and teaching practice. Results reported in this paper build on the findings of other studies related to students' and teachers' understanding of rational numbers (Behr, Harel, Post, & Lesh, 1992; Lehrer & Franke, 1992; Ohlsson, 1988; Post, Harel, Behr, & Lesh, 1991) and other mathematical topics (e.g., Ball, 1990, Stein, Baxter, & Leinhardt, 1990, Simon, 1993; Wilson, in press a). It also supplements a growing body of research related to teachers' beliefs about mathematics and mathematics teaching (Thompson, 1992) and a smaller but equally important body of literature reporting teacher change (e.g., Fennema, 1991, 1992).

Considering teachers' thinking about a specific mathematical topic, such as fractions, allows one to better understand the broader domain of teachers' mathematical thinking and its influence on teaching and learning (Cooney & Wilson, 1993). In other words, study of a specific case can enrich one's conceptualization of the general. The teacher described in this paper is participating in an ongoing project (currently in its second year) that is exploring the mathematical and pedagogical understandings of four middle school teachers. The study reported here (as well as the larger project) is based on an assumption that knowledge and beliefs are relative, personal and can best be revealed through methods that promote communication between researchers and those being researched. This perspective guided each phase of the study, including the design of data collection instruments, the choice of analysis procedures, and the interpretation and reporting of results. The perspective was also consistent with my desire to obtain a detailed, in-depth understanding of the participant's conceptions.

Theories based on previous research on students' and teachers' knowledge of rational number contributed significantly to the conceptualization of the current study. For example, this study made use of a theory by Ohlsson (1988) that suggests that rational number topics lie in a complex semantic field referred to as *quotient terms*. Ohlsson contends that trouble associated with understanding rational number concepts may be caused as much by terminology applied to distinct mathematical ideas as by difficulties of individual procedures and concepts. For example, the same terminology is typically used to refer to fractions as ratios of whole numbers (e.g., the baseball player hit safely in 9 of 20 at bats ($\frac{9}{20}$)), division of whole numbers (e.g., 11 cookies are shared fairly among 4 children ($\frac{11}{4}$)), and as parts of wholes (e.g., $\frac{5}{6}$ of the pie remains).

While Ohlsson's theory focuses on the semantics of fractions, Lesh's model for translation between modes of representation (Behr, Harel, Post, & Lesh, 1992) focuses on the issue of translating among common representations of fractional ideas. For an individual to demonstrate a given fractional quantity using Cuisenaire rods when shown the quantity using fraction circles would be an example of translation within a particular mode (manipulatives). For an individual to reconceptualize a given idea in a different mode would be translation between modes. An example of such translation would be to draw a picture of $\frac{2}{3}+\frac{1}{4}$ (numerical to pictorial). Lesh contends that translations within and between various modes of representation (e.g., pictorial, verbal, numerical, manipulatives, real-world) make the important mathematical ideas meaningful.

Also influencing the conceptualization of this study were general documents describing the recommended mathematical knowledge of middle school teachers and students (e.g., National Council of Teachers of Mathematics [NCTM], 1989). Within this literature there is general consensus regarding what it means to understand fractions deeply. Identified areas that were included in the current project's analysis include

(1) identifying and representing fractions in a variety of forms (e.g., as operators, ratios, parts of units, division of integers), (2) translating among common representations of fractions (e.g., graphical, pictorial, numerical, verbal, manipulatives), (3) modeling real-world situations using fractions, (4) ordering fractions, (4) understanding how basic operations on fractions are related, (5) computing with fractions, (6) estimating results of fraction computations, and (7) connecting an understanding of fractions to whole number and rational number concepts and operations.

Research on mathematics teachers' beliefs (e.g., Cooney, 1985; Thompson, 1984, 1992) also provided empirical, theoretical, and methodological impetus for the design of this study. Most of the previous research in the area of teachers' beliefs has had a general focus; it has not concentrated on specific mathematical topics. This study builds on previous research in two major ways: (a) it provides greater depth of information concerning teachers' knowledge of fractions than has previously been available, (b) it documents the influence of a teacher's specific *and* general mathematical understandings on his teaching practice.

Method

Participant and Research Site. An ethnographic case study design (Stake, 1978) was employed for this study. The participant, Mr. Burt, has taught nearly every elementary and middle school grade during his 20-year teaching career but for the past few years has taught only sixth grade. He teaches in a diverse (racially and otherwise) middle school located in a Northeast urban community. He teaches mathematics during three 45-minute periods each day and language arts during two others. Mr. Burt was excited to participate in the study. Although he understood that I would not be directive about his teaching, he saw our association as an opportunity for him to interact with an interested professional and thus perhaps help him improve his teaching through personal reflection.

Data Collection. Data were collected between September 1992 and May 1993 using interviews, classroom observations, and students' and teacher's written work and plans.

Two one-hour interviews, conducted during the first month of the school year (September 1992), investigated Mr. Burt's views about mathematics, teaching, and his specific understanding of fractions. During our second interview, Mr. Burt sorted and commented on the contents of 24 index cards representing various fractional ideas and situations. One of Burt's mathematics classes was observed for a total of 26 days, including 20 consecutive days while he taught a unit on fractions. Approximately half (12) of these observations were followed by half-hour stimulated recall interviews in which Burt commented on classroom events of that or the previous day. His written notes and plans during the 20-day period were also collected. Two one-hour interviews at the end of the observation period assessed Mr. Burt's reflections on his experiences. During one of these final interviews, Burt was asked to sort and comment on statements he had made during previous interviews (Wilson, in press b). Three students, chosen to be representative of the class's racial, gender, and achievement diversity, were identified as target students and observed and interviewed periodically during class. Observations of bi-weekly meetings of the sixth grade mathematics teachers constituted another data source. Teacher interview data were audio recorded and transcribed for ongoing analysis. Classroom observations were also audio recorded. Detailed fieldnotes were made of observations (classroom and teacher meetings) and student interviews. Photocopies were made of written artifacts.

Analysis. Data were analyzed using a modified constant comparison approach (Glaser & Strauss, 1967; Lincoln & Guba, 1985; Strauss, 1987). Initial data analysis (open coding) involved reading interview transcripts and fieldnotes holistically and writing memos describing general impressions and overall tone. For interviews, such coding occurred soon after the interviews had been transcribed. Open coding of fieldnotes was conducted immediately after each observation. Analyses of previous interviews and recent observations were used in planning the stimulated recall interviews. Selective coding included a line-by-line analysis of the interview transcripts. Diagrams (i.e., graphic illustrations) were created throughout this phase to help organize and synthesize data. The final phase of coding involved analyzing ways in which Burt commented on and sorted his statements during the next-to-last interview. It also involved reviewing in detail the other interview transcripts (particularly that of the second interview), observation and student interview fieldnotes, and written artifacts. During this final phase categories were merged within and across data sources and emergent categories were defined by referring to quotations from the interviews and examples from fieldnotes and teacher's written plans.

Results

This section contains descriptions of (1) Mr. Burt's views of mathematics and mathematics teaching, (2) his understanding of fractions, and (3) his approach to teaching about fractions. The description of his general views is intended to form a backdrop or context to help the reader interpret Mr. Burt's specific conceptions of fractions and his approach to teaching about fractions.

Mr. Burt's conceptions of mathematics and mathematics teaching. The aspect of mathematics that appeals most to Mr. Burt is its usefulness. In our first interview he stated, "math problems are everywhere." During this and later conversations he elaborated on this claim by describing several consumer applications of mathematics (e.g., the use of fractions in cooking, woodwork and general construction), as well as ways in which mathematics connects to other curricular areas (e.g., a social studies application in which one might consider what percent of a given group might be of a certain religion or racial make-up). Throughout his teaching he also stressed the practicality of mathematics. For example, to emphasize fraction and decimal concepts he frequently described situations involving money. Mr. Burt believes that practical applications help motivate students. In other words, they provide "[a] reason for all that pain and suffering."

Related to this issue of practicality, Mr. Burt believes that it is essential for students to master basic skills such as whole number multiplication, so they will have tools to solve meaningful problems. To emphasize this theme, a traditional task for students was, upon finishing a quiz or test, to write a specified portion of the multiplication table (e.g., the "sevens") on the back of their quiz or test paper.

Mr. Burt also believes that it is the teacher's responsibility to teach correct concepts and procedures in an organized fashion, explaining exactly which procedures students are expected to use, so there is no confusion. His primary objective is to bring all of his students to the same (correct) understanding of important mathematical ideas. To accomplish this objective, each day during the study he carefully explained new procedures and concepts before allowing students to work independently on assigned exercises and problems. He also carefully reviewed solutions to most assigned problems. Mr. Burt involved students in these classroom discussions by calling on them individually to give answers or explain how they arrived at their solutions, but he generally allowed for little discussion or debate of the ideas considered.

Mr. Burt's understanding of fractions. During our second interview, Mr. Burt demonstrated a strong desire to discuss his own understanding of fractions in the context of teaching about fractions. That is, he had difficulty separating his own mathematical understanding from his ideas of how to teach. This is consistent with his predilection toward practicality. Why would anyone want to know about his specific understanding of mathematical ideas except as they relate to his teaching, a practical application of that understanding? This inclination to discuss his teaching rather than his own mathematical understanding persisted even after I reminded him (on more than one occasion) that during this particular interview (the second interview) I was interested in exploring his thinking about fractions, not necessarily his thinking about how fractional ideas should be taught. For example, immediately after such a reminder, he reacted to an index card containing the diagram in Figure 1 by stating, "That's an interesting card. . . . I don't think I've ever used that [in my teaching]." His responses throughout the second interview were similar in tone; he repeatedly referred to how the different fractional situations we discussed related to his teaching.

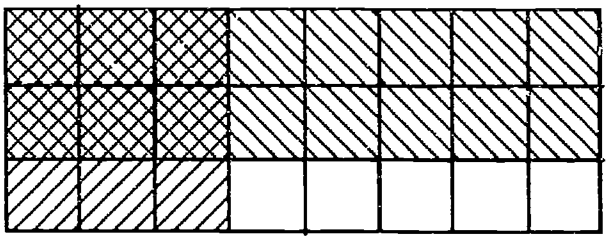

Figure 1. Pictorial representation of $\frac{3}{8} \times \frac{2}{3}$.

Another prominent theme of our interviews before Mr. Burt began teaching a unit on fractions was that he conceptualized fractions principally as division problems. His statements, "I relate fractions to division," and "I want my students to understand that all fractions are division problems," are typical of his emphasis on a quotitive interpretation of fractions (Ohlsson, 1988). When asked to provide an example of a meaningful problem involving the use of fractions, he suggested one in which three pizzas (12 slices each) are shared among seven friends. His suggested solution to this situation involved the fraction $\frac{36}{7}$, which he interpreted to mean 36 divided by 7.

Although Mr. Burt's preferred interpretation of fractions was quotitive, his understanding also included a partitive interpretation, and he understood that fractions could also represent ratios. His ability to interpret fractions in a partitive way is illustrated by his reaction to the diagram in Figure 3: "I always internalize [fractions] as being parts of a standard unit we call a whole."

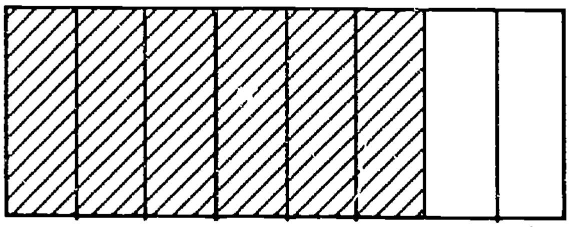

Figure 2. Pictorial representation of $\frac{6}{8}$.

His interpretation of fractions as ratios is similarly illustrated by his discussion of the diagram in Figure 4: "The way you have it divided here is kind of nice. You could say two out of eight students preferred McDonald's, three out of eight preferred Big Boy, and three of them preferred Taco Bell."

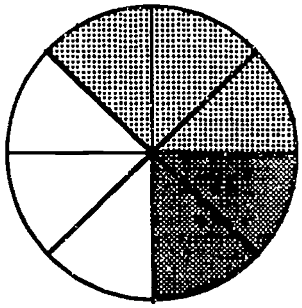

Figure 3. Pictorial representation of $\frac{3}{8}+\frac{2}{8}$.

Mr. Burt's interpretations of the diagrams in Figures 2 and 3 also illustrate his flexibility in translating among various modes of representation (in these examples, between pictorial, numerical, verbal, and real-world modes). His ability to easily move between and within common representations was evidenced throughout our early interviews as well as during his teaching. On several occasions he alluded to the importance of representing fractional ideas in multiple ways: as "pictures," using "numerical representation," with "word problems," and with "concrete materials."

Mr. Burt's experience teaching about fractions. The curriculum materials (Towsley, Payne, & Payne, 1992) used during this study place heavy emphasis on pictorial and physical representations of fractions, as well as on connections among pictorial, physical, numerical and verbal representations, and traditional algorithms for operations. Mr. Burt took about three months to teach the 20 lessons suggested by the materials, although some of this time was spent reviewing topics covered before the fractions unit. Mr. Burt's approach to teaching about fractions was significantly influenced by his views of mathematics and the teaching of mathematics. This is illustrated by analyzing two teaching episodes. The first is taken from a lesson taught on November 18, 1992, the second from a lesson taught on November 10, 1992.

He began the November 18 discussion by stating the day's objective, "labeling the missing [fractional] part." Referring to the exercise shown in Figure 4, he stated, "this is how *I* would do the first one."

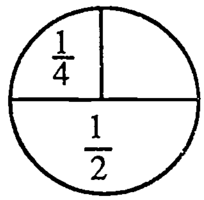

Figure 4. An exercise involving identifying a fractional part

He then explained that since the missing piece is the same size as one of the identified pieces, the correct answer would be $\frac{1}{4}$. He proceeded to model several similar exercises before instructing students to work on the next few problems independently. In the midst of his explanation, he again commented that students should note that "this is the way I [Mr. Burt] would do it." He finished his demonstration by telling students that they would be most successful if they did the problems the way he had shown them.

The second episode (which actually occurred *before* the first episode) illustrates the nature of Mr. Burt's response when students used alternative solution methods. During this lesson, Mr. Burt modeled a solution to the exercise shown on the left of Figure 5 by drawing a vertical altitude to the middle (isosceles) triangle. One of the students asked if she could show a different way to do the problem. Mr. Burt invited the student to the front of the class and she sketched a solution like the one shown in the right part of Figure 5.

The Problem: Rosalind's Solution:
Draw lines to show fourths:

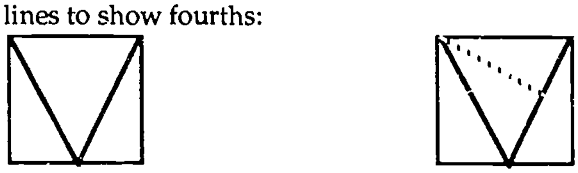

Figure 5. Fraction problem and one student's solution

Upon seeing the student's solution, Mr. Burt responded, "I don't know if that would be right or not. I'd have to really look at it. I can't tell, I'd have to measure it." He then measured the lengths of the two segments along the base of the triangle she had divided, and announced while pointing to the diagram, "Nope, that won't work because this is longer this way that it is this way. But that was a good idea, you were thinking. Thank you Rosalind." No further mention was made of Rosalind's proposed solution.

Mr. Burt's teaching approach was also significantly influenced by his specific understanding of fractions. Because this was Mr. Burt's first attempt to use these particular curriculum materials (he had used another textbook in previous years), and since Mr. Burt's main interpretation of fractions is quotitive, one would expect a certain amount of dissonance between his understanding and the predominantly partitive interpretation of fractions emphasized in the lessons and problems of the curriculum materials. This was not the case, however. As pointed out previously, despite his focus on a single interpretation, he was *aware* of multiple interpretations of fraction concepts. This flexibility allowed him to encourage his students to develop an understanding of mathematical relationships and concepts. Whereas his experience in previous years had involved emphasizing traditional rules for operating on fractions, rather than on understanding fraction concepts, because he had curriculum materials that supported a more conceptual emphasis, he was able to devote the increased time and attention he believed was necessary to promote a deep understanding of fractions among his students.

Both of the episodes discussed earlier in this section illustrate that Mr. Burt was extremely directive in his instruction. Rarely in his teaching did he involve students in deciding the appropriateness of strategies or solutions; for the most part, he was the sole arbiter of correctness. However, we point out that throughout the fractions unit, though not student-centered, Mr. Burt's teaching was conceptually oriented. The episodes presented here only partially show how Mr. Burt emphasized connections among common representations of fractions (e.g., both episodes emphasize a connection between numerical and pictorial modes of representation). Other lessons better illustrate this fact. For example, during a lesson taught on November 2, 1992, Mr. Burt used paper strips to illustrate the idea of fractions as parts of wholes. Later during this lesson, he extended connections between manipulatives (paper stips), numbers, words, and pictures, to "real-world" objects such as "a bunch of bananas," "a bouquet of flowers," "a whole wall."

The two episodes discussed earlier in this section do not adequately illustrate Mr. Burt's attention to conceptual understanding of operations on fractions (as opposed to memorization of rules associated with these operations), either. During lessons on addition, subtraction, multiplication, and division of fractions Mr. Burt did not introduce procedures until students first spent several days on activities designed to help them understand concepts underlying the procedures. Before introducing an algorithm to add and subtract fractions, he taught a lesson on estimating sums and differences. In the case of comparing and ordering fractions, he never introduced a procedure (e.g., cross multiply), but based his instruction entirely on connections to pictorial and physical models. This accent on concepts contrasts not only with his *claimed* approach to teaching fractions in previous years, but also with his *observed* instructional emphasis immediately before and after the fractions unit, when attention was placed primarily on correct procedures. Although he has recognized for some time that an essential aspect of fractions involves being able to "visualize them," he said he has in past years emphasized rules for operating with fractions because his textbooks have not supported a more conceptual emphasis.

Mr. Burt claimed that his current students understood fraction concepts more clearly than in past years. Both during and after the fractions unit, his students were for the most part successful in making important connections among common representations of fractions, and in basing their understanding on conceptual and visual representations of fractional ideas. For example, during interviews two months after completing the fractions unit, all three of the target students, without being prompted, constructed pictorial diagrams to explain how to order three fractions (1/2, 1/3, and 5/8).

Discussion

Mr. Burt's focus on fractions as division problems was representative of his larger view of mathematics as a correct set of rules and concepts (i.e., the most "correct" interpretation of fractions is as division problems). This view of mathematics contributed to his insistence on maintaining a teacher-dominated classroom environment; it was his way of making sure that his students clearly understood correct concepts and procedures. But this view of mathematics and mathematics teaching prevented him from allowing students to participate in the development of mathematical ideas, notwithstanding the open approach suggested by teacher's guide accompanying the curriculum materials. On the other hand, his own flexible understanding of fractions (i.e., his *ability* to see fractions in multiple ways) allowed him to adjust his instruction to accommodate mathematical ideas that were not at the forefront of his own thinking and to emphasize important connections among mathematical representations when provided with curriculum materials to support such an emphasis.

Thus, Mr. Burt's case provides evidence that teachers who have a flexible understanding of important mathematics can adjust their teaching to accommodate ideas not traditionally emphasized. On the other hand, it appears that changing teaching practice to accommodate student exploration or other similar practices currently favored in the mathematics education community is a more difficult endeavor (at least in Mr. Burt's case). Perhaps this is because teachers' orientations to these practices are closely tied to deeply-held beliefs about the nature of mathematics and mathematics teaching, as well as to their understanding of specific mathematical content.

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