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ABSTRACT

Relating natural language to mathematical language is an important component of elementary mathematics education. This paper describes new steps toward a computerized database of addition and subtraction word problems that could provide teachers and students with access to critical natural language terms and expressions for mathematical relationships. On the basis of new results from a computer simulation that is sensitive to slight changes in problem wording, the suggestion is that a database of word problems that reflects a relative order of difficulty is not only feasible but essential if researchers in this area are to increase the practical dissemination of research results. A collection of math stories sensitive to children's text comprehension skills and mathematical development is the foundation of an ongoing effort to implement a knowledge-building environment that can facilitate children's discussions of why one math story is more difficult than another, as well as encourage communities of students to build their own database of stories. An appendix contains an example simulation of EDUCE and SELAH solving a static-non-relational (combine 5) problem. Contains 64 references. (Author/MKR)

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Using a Computer Simulation to Determine Linguistic Demands in Arithmetic Word Problem Solving

or

Is the Time Right for a Database of Word Problems?

by

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Abstract: The mastery or ownership of mathematical language is one view of obtaining competence in mathematics. This view defines a profitable learning trajectory as one that continually exposes students to the rich collection of mathematical expressions that are needed to *verbalize* and *write about* mathematical situations and the quantitative relationships in those situations. The paper describes new steps towards a computerized database of addition and subtraction word problems that could provide teachers and students with access to critical natural language terms of and expressions for mathematical relationships. On the basis of new results from a computer simulation that is sensitive to slight changes in problem wording, the suggestion is that a database of word problems that reflects a relative order of difficulty is not only feasible but essential if researchers in this area are to increase the practical dissemination of research results. A collection of math stories sensitive to children's text comprehension skills and mathematical development is the foundation of an ongoing effort to implement a knowledge-building environment that can facilitate childrens' discussions of why one math story is more difficult than another as well as encourage communities of students to build their own database of stories.

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Introduction

Relating natural language to mathematical language is an important component of elementary mathematics education. The mediating role of natural language in expressing everyday problem situations is increasingly being recognized as a potential focus for both research on children's mathematical problem solving and new classroom approaches which encourage children to construct their own solutions (Steffe, Cobb & von Glaserfeld, 1988). The NCTM Curriculum Standards (1989) emphasize the pedagogical importance of representing and reading mathematics, and both the Curriculum Standards and the Professional Standards (1991) focus on encouraging children to write mathematics and particularly to *talk about* mathematical aspects of situations. Observing how children make their thinking about such situations explicit in terms of natural language also has value for purposes of assessment (Carpenter & Fennema, 1988; Loef, Carey, Carpenter, & Fennema, 1988). Some kind of *verbalization* of mathematical problems enters into all of these activities.

Traditionally, the verbal aspect of mathematics at the elementary level has been incorporated into curricula as "word problems," which have often served as a source of children's continuing frustration with mathematics. The recent emphasis on how children could or do make connections between natural language and mathematical concepts in discourse, whether they proceed from a given verbal representation or use a verbal representation to make precise a problem situation, provides new motivations for a consideration of why certain linguistic forms of mathematical problem situations are difficult. Carpenter, Fennema, Peterson, and Carey (1988) note that most teachers' knowledge of children's thinking is not organized into a coherent network which distinguishes different types of problem difficulty. In addressing this fact, the suggestion inherent in this paper is that teachers would benefit from a computerized database of problems which reflects the relative difficulty or facilitating effect of phrasing of those problems. In a related manner, children's linguistic expression of problem situations might reveal aspects of their comprehension to a teacher who is sensitive to the competence represented by certain means of expression. In attending to different types of language and linguistic understanding, the proposed approach to types of problem difficulty goes beyond the traditional problem classification by semantic structure, considered by some researchers (e.g., Cobb, Yackel & Wood, 1988) to be an incomplete form of pedagogical knowledge.

The idea of focusing on various cognitive dimensions of word problem solution rather than simply on problem classification has received support from interdisciplinary research concerned with, e.g., the recognition of semantic relationships (Booth, 1989; Herscovics, 1989; Kaput, 1987; and many others) and the use of problem-solving strategies (Thompson, 1988; Greeno, 1987) based on understanding as opposed, for example, to superficial "clue-word" approaches. However, linguistic factors (Cummins, 1991; Lewis & Mayer, 1987; Stern, 1993 and others) and presentational factors (Davis-Dorsey, Ross & Morrison, 1991; De Corte, Verschaffel, & DeWin, 1985; Staub & Reusser, 1992) have only recently been recognized as a level of representation at which misunderstandings can occur. These results give insights into isolated sources of difficulty, as do informal classroom observations of children's mathematical discourse. Such approaches highlight the importance of understanding specific "cognitive bottlenecks" as a critical step towards designing effective classroom intervention strategies (e.g., Carpenter & Fennema, 1988; Resnick, 1992).

However, there is as yet no detailed model of children's *complete* solution process (that is, beginning with a left-to-right reading of individual sentences) for *all* problem types (e.g., Change, Combine, Compare), although there is ongoing work which begins to address this (Fuson, 1994b; Reusser, 1990; Weber-Russell, 1994). Most cognitive simulations of word problem solution (Briars & Larkin, 1984; Cummins, Kintsch, Reusser, & Weimer, 1988; Kintsch and Greeno, 1985; Okamoto, 1994; Riley & Greeno, 1988) start directly with artificial propositions. The exception is Reusser's *Situational Problem Solver* (SPS) (1990). SPS possesses an elaborate text-processing component to distinguish the representation of the situation from the representation of the text, as advocated by Kintsch (1986). SPS is thus the first attempt to simulate the progressive and incremental process of transformation from text to situation to equation. In particular, SPS stresses that from a problem solving (and instructional!) point of view, arriving at a representation of the situation in a problem should not be a superfluous, but a necessary process. SPS does not however address static and/or relational language problems, e.g., Combine and Compare types. Thus, the role of situational representations in statically-worded problems (i.e., problems which do not contain significant action language) is not entirely clear (*cf.* Stern & Lehrndorfer, 1992).

The simulation and results presented in this paper address the dual need for a model which starts with the process of reading as well as one which recognizes the critical role of situational representations in statically worded texts. Centered around results from this "bottom-up" computer simulation which is sensitive to changes both in problem wording and presentational structure (LeBlanc & Russell, 1993), the case is presented that the time is right to establish a computerized database of arithmetic and subtraction word problems that includes a multiple number of rewordings for each "traditional" problem wording. Predictor equations from regression analyses are used to confirm previous research on the effects of problem wording and to extend the traditional measures of why one problem is easier or harder than another. The premise is that a synthesis of research results concerning

(i) children's developing mathematical competence; (ii) developing linguistical competence in conjunction with (iii) knowledge of the affects of problem wording in the form of a database can provide real-time instructional suggestions as to the "next best problem to present."

Specific research questions of interest include:

- What kinds of natural language used to convey a problem situation have different effects on the ability to conceptualize the problem in terms of correct mathematical relationships? and

- Assuming that problem sequencing is important in order to encourage progress without making jumps for which the student is not ready, does sequencing word problems (for example, in the form of a database) suggest a more profitably learning trajectory for students?

The paper proceeds as follows. After a brief introduction to the computer simulation, empirical results are presented which show how the simulation defines word problem solving difficulty as a function of memory load and text integration inferences. Three traditional word problem types are then reviewed, including examples which show multiple ways to reword those types of problems. Where possible, empirical studies, including results from the computer simulation, confirm and/or suggest a relative order of difficulty between multiple rewordings of problems.

The Computer Model

The current computer simulation comprehends arithmetic word problems written in English in a word-by-word, sentence-by-sentence, "bottom-up" fashion. The simulation is composed of two components: EDUCE and SELAH. EDUCE is an expectation-driven parser adapted from the work of Schank and Riesbeck (1981) and developed according to principles found to be distinct for word problem solving (Burns, 1993; LeBlanc and Russell, 1989). EDUCE handles a number of language features inherent in word problems, including: (i) quantities (e.g., 3, three); (ii) noun compounds (e.g., *soda cans*); (iii) pronominal reference (e.g., *she*); (iv) time sequence (e.g., *then, in the beginning*) (v) set partitions (e.g., [by *ownership*: "Dick *and* Jane have"], [by *type* of object: "8 cats *and* dogs"]); (vi) ellipsis (e.g., "Ed has 3 cans. He found 2 more." That is, 2 more *cans*) and (vii) reference to previous sets (e.g., 4 *of them*). SELAH is a text integration component which accepts EDUCE's canonical representations of individual sentences and constructs a situational interpretation of the problem based on explicit conceptual actions in the text or direct or implicit references between sets (LeBlanc, 1991). While building a "situation model," SELAH instantiates an arithmetic action and later a counting strategy to solve the problem.

The simulation is currently implemented as a "bottom-up" problem solver. SELAH completely determines the role of each sentence after it is read by EDUCE and then the entire reading and text integration process continues to the next sentence. Bottom-up processing is opposed to a more global, expert-like, "top-down" strategy that potentially involves an instantiated schema in memory for each particular type of problem and/or an associated plan to look for quantities which can fill slots in that schema. Although SELAH has been designed so that it could function in a more expert-like top-down fashion, the focus of the current implementation is a model of the young reader or, at least, a novice word problem solver. At this stage, the simulation is not a developmental model in the sense of Briars and Larkin (1984), Riley *et al.* (1988) and Okamoto (1994). That is, in the current implementation, SELAH does not alternate between varying levels of expertise. More specifically, the focus is to model the *necessary* text comprehension and logico-mathematical requirements of problem solution. Because SELAH solves all of the problems in the Riley *et al.* (1988) benchmark set, it must exhibit expert-like capabilities (e.g., performing a relatively large number of inferences) on some problems but only rudimentary capabilities on the easiest problems. For example, some problems involve higher memory loads than other problems. While it is possible to set a limit on the number of concepts which can be stored in working memory (*cf.* Fletcher, 1986), the simulation currently solves each problem with an unlimited memory capacity. One theoretical interest is the *total* demand on memory that each problem requires when read in a bottom-up fashion. In short, EDUCE and SELAH simulate a problem solver who reads each sentence only once and attempts to maintain the sets and their relationships in memory until deciding on an appropriate arithmetic action which can lead to a solution.[1]

A unique feature of the simulation is that it keeps track of (i) the number of text integration inferences that are required and (ii) the load on working memory while performing text integration across sentences. The fundamental hypothesis of the simulation is that children's ability to follow-up on explicit set references (or infer such references) is a crucial step towards recognizing the conditions that make an arithmetic operation appropriate for a given situation. The empirical results present new measures of text comprehension which determine why a particular word problem may be difficult, especially for young readers.

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[1] The current implementation of SELAH does not simulate solutions which use manipulatives such as blocks on the table (*cf.* the CHIPS model of Briars & Larkin, 1984). In a sense, children who use manipulatives are closer to a strict definition of bottom-up problem solving; that is, for each sentence that is read the blocks are moved. The focus of SELAH is to model the total text comprehension requirements that novices experience as they solve these problems, including the cognitive demands of remembering previous sets rather than off-loading those requirements by depending on physical aids.

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Example of the Simulation Reading and Solving a Problem (see Appendix)

A detailed example of how the parser, EDUCE, is sensitive to changes in problem wording and how SELAH builds a situational representation and solves a problem is given in the Appendix.

Summary of Experimental Results

In a series of exploratory regression analyses, children's probability of solution on a benchmark set of problems from Riley *et al.* (1983, 1988) were correlated with the monitored performance of the simulation on those same problems. Global measures of text integration and local sentence-level variables which account for a significance proportion of the variance in children's probability of solution were isolated. (A thorough presentation of the experimental method and results are beyond the scope of this paper and would detract from the focus on problem rewording. The interested reader is directed to LeBlanc and Russell (1993) for an abbreviated view and/or LeBlanc (1993) for a complete discussion).

One significant result emerging from the experiments deserves mention here. A combined measure of the number of concepts to remember and the number of inferences to make while remembering those concepts is a consistent predictor of children's problem solving success (for all grades K through 3, the variance accounted for (R^2) is at least 50%, $p<.001$). If one considers a child's total processing space as fixed (as suggested by Case, 1982), a word problem that requires a relatively large number of concepts to be held in memory also limits the resources that can be devoted to executing basic operations, including the making of inferences. In the context of arithmetic word problems, the inferences necessary for text integration are critical. An ability to arrive at a correct arithmetic operator is directly dependent on establishing the relationships between sets, which in turn is dependent on an integrated representation of the text. Unlike narrative texts, word problems tend to be very brief and consecutive sentences contain limited degrees of overlap (*cf.* Haviland & Clark, 1974). Children who are unable to make the inferences which lead to "mathematical connections" are confronted with independent sets in memory and must resort to ad hoc strategies (e.g., a *keyword* or *first-number-given* strategy) as confirmed in the literature (Cummins *et al.*, 1988; De Corte & Verschaffel, 1985; and others).

The empirical results suggest that children's difficulties with arithmetic word problems are due in part to an inability to make text integration inferences, especially when a relatively high number of concepts occupy memory. The implications for instruction are two-fold. First, the processes of text comprehension and mathematics are tightly coupled in arithmetic word problem solutions; however, there are fine-grained methods of altering a problem's probability of solution in each area. Because making "mathematical connections" is so critical, these results suggests that rewording problems in ways which imply or even explicitly state the relationships between sets is a critical step towards helping those students who can not yet make the necessary inferences, as recently shown by Davis-Dorsey *et al.* (1991) and others. Children are often expected to make complex inferences required by sparsely worded problems *while* they are just beginning to read. The second educational implication, related to the first, is that a more fine-grained classification of word problems is emerging.

Toward a Database of Word Problems

In conjunction with developmental theories of children's addition and subtraction competencies (Fuson, 1994b; Okamoto, 1992), a number of task characteristics individually and in combination have been shown to affect the solution probability of arithmetic word problems:

- **semantic structure**; e.g., change problems are generally easier than compare problems (Riley, Greeno & Heller, 1983; Carpenter, 1985; De Corte, Verschaffel & Pauwels, 1990; and others)

- **misunderstandings of quantification or reference**; e.g., "some" is an adjective like "red", or "altogether" means *each* (Cummins, Kintsch, Reusser, & Weimer, 1988)

- **complex relational phrases**; e.g., "more than" and "less than" (Stern, 1993; Hudson, 1983 and others)

- **"inconsistent" language**; e.g., problem says "less" but requires addition (Lewis & Mayer, 1987 and others)

- **qualifying dependencies**; supersets are partitioned based on *ownership* (Jim, Sally, Jim and Sally), *adjectives* (e.g., big, small, big and small), *arguments* (e.g., boys, girls, children), and *location* (on the table, on the shelf, in the room) (Nesher, 1982)

- **structural format**; *sequence of the given numbers*: problems start with the smaller vs. the larger number (Verschaffel & De Corte, 1990); *extraneous information* (Searle, Lorton & Suppes, 1974); *location of the unknown:* e.g., unknown is the initial set (Hiebert, 1982)

- **general context and affective factors**; e.g., use of favorite objects and the names of friends (Davis-Dorsey, Ross & Morrison, 1992; *cf.* McLeod, 1988)

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- **presentational structure**; *time sequence*: sentences are not in chronological order; *narrative focus*: protagonist is not always in actor role; *related co-actors*: one person is referred to in terms of her family relationship to the other person (Staub & Reusser, 1992)

- **text integration**; *absence of potentially helpful explicit set reference language*;: e.g., "of them", "more", "the rest" (De Corte, Verschaffel, & DeWin, 1985; and others); *integrated propositions in memory* (O'Brien, 1987; Trabasso & Sperry, 1985; van den Broek, 1988); *importance of short-term memory as a bottleneck in the comprehension process* (Cooney & Swanson, 1990; Fletcher, 1986; LeBlanc, 1993).

The next sections introduce example problem types where the presence or absence of specific task characteristics influence solution probability. Because the simulation is sensitive to slight changes in problem wording, the monitored performance of the simulation for a particular rewording can be entered into the predictor equations that were derived from the regression analyses. Other empirical studies are cited which show a relative order of difficulty between two problems and the predictor equations are used to confirm the empirical results and/or predict new effects of a task characteristic on the probability of solution.

Making Static Non-Relational Problems Easier

The following problem is typical of a class of problems which do not contain significant actions (thus the term, *static*) and do not involve relational expressions such as "more than" (thus the term, *non-relational*). (Riley *et al.* (1988) refer to this particular problem as Combine 5 - subset unknown and Fuson (1994a) refers to it as Put Together - unknown part.)

David and Kathy have 8 soda cans altogether. David has 5 soda cans. How many soda cans does Kathy have?

Problem 1 - Traditional (Combine 5) Wording

In this problem, the first sentence describes a superset with a known amount. A subset with a known quantity is then described, followed by a question requesting the amount of the other subset.

This problem has received considerable attention in the literature, mostly due to its level of difficulty. Considering all eighteen problems in Riley's (1988) benchmark set, this problem, on average, is the third most difficult problem. Children's *success rates* across the first four grades in the Riley *et al.* (1983, 1988) studies reflect the difficulty: 22%-K, 33%-1st, 55%-2nd and 75%-3rd.

These relatively low probabilities in even the higher grades are confirmed elsewhere (Davis-Dorsey *et al.*, 1991; De Corte, Verschaffel *et al.*, 1985).

There are two **potential rewordings** which might help those students who are not able to solve this type of problem: (i) the use of language to facilitate text integration between sentences and thereby highlighting the relationship between quantities (e.g., *of them, the rest*), see Problem 2 and (ii) the same changes in (i) along with the removal of the conjunction (*and*) and the removal of the word *altogether* , see Problem 3.

The first potential alternative (Problem 2) to the "traditional" wording (Problem 1) is to include words which facilitate the process of text integration by highlighting the relationship between quantities.

David and Kathy have 8 soda cans altogether. David has 5 of them. The rest of them are Kathy's. How many soda cans does Kathy have?

Problem 2 - "altogether" & "of them" Rewording of Combine 5

When EDUCE parses the second sentence of Problem 2, the phrase *"of them"* in the second sentence generates an explicit reference to a previously mentioned set, i.e., an explicit reference to David and Kathy's set of 8 (see the Appendix for a complete discussion of how the simulation handles this problem). This explicit reference identifies that David's 5 are *part of* the previously mentioned set of eight. In other words, EDUCE is sensitive to the "mathematics embedded in the natural language." On the other hand, in the second sentence of Problem 1's traditional wording ("David has 5 soda cans"), no reference to the previous set of eight is made in the text. Thus, EDUCE's representation of the second sentence of Problem 1 does not contain an explicit link back to the first sentence. When performing text integration, SELAH must make an inference to establish that David's five cans are *part-of* David and Kathy's original set of eight cans, as opposed to five totally unrelated cans.

In addition, the phrase, *"The rest of them"* in the third sentence of Problem 2 marks Kathy's set as part of the eight soda cans that are *left* as the result of a previous separation. In the traditional (Problem 1) wording, no reference is made to Kathy's set so SELAH must *infer* that Kathy's cans are part of the set of eight as well as those remaining after separating out the cans belonging to David.

De Corte, Verschaffel *et al.* (1985) and others have provided empirical support that the reworded version significantly increases children's solution probability. In line with these results, the simulation predicts that including the phrases "of them" and "the rest" will increase solution probability. The average memory loads for these two problems are quite similar although the simulation performs three fewer inferences on the reworded version.

Table 1 compares first and second grade childrens' success on both the traditional and reworded versions as reported by De Corte, Verschaffel *et al.* (1985) and predicted by the simulation. As might be expected, the simulation's predictor equation for first grade is more sensitive to a change in the number of inferences than a later grade, although for both grades (1st and 2nd), the simulation predicts a greater increase in solution probability (from the traditional to the reworded version) than found by De Corte *et al.*

Table 1

Predicted and observed 1st and 2nd grade probabilities for versions Problem 1 & 2

Problem Wording (grade)	Simulation's Prediction	De Corte Data
Traditional (1st) **Problem 1**	0.28	0.43
"altogether" & *"of them"* (1st) **Problem 2**	0.59	0.57
Traditional (2nd) **Problem 1**	0.55	0.71
"altogether"" & *"of them"* (2nd) **Problem 2**	0.81	0.83

Another potential rewording (see Problem 3) involves the use of the phrases "of them" and "the rest" like Problem 2, yet the conjunction ("and") and the word "altogether" in the first sentence have been removed.

There are 8 soda cans. 5 of them belong to David. The rest are Kathy's. How many soda cans does Kathy have?

Problem 3 - No "and/altogether" Rewording of Combine 5

Cummins (1991) used this version in her studies of the factors that influence childrens' interpretations of arithmetic word problems. According to Cummins, the critical difference between this rewording and the other

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rewording (Problem 2) is the absence of the conjunction in the first sentence (the removal of "altogether" is considered secondary). A conjunction of owners (David *and* Kathy) is often misinterpreted by young children as meaning *each*. Thus, in a sentence such as:

David and Kathy have 8 soda cans altogether.

Many children misinterpret this to mean: "David has 8 *and* Kathy has 8." Such a misunderstanding would lead children to answer "8" when asked how many Kathy has. Cummins validates her hypothesis with results from a computer simulation, childrens' recall protocols (Cummins *et al.*, 1988) and the classification of solution error types (Cummins, 1991). In the latter study, children were found to commit *given-number errors* (i.e., the answer to the traditional wording is "8," a number given in the problem) on 46% of the incorrect responses (see De Corte, Verschaffel *et al.*, (1985) for similar results).

Table 2 compares first grade children's success on both the traditional and reworded versions as reported by Cummins (1991) and De Corte, Verschaffel *et al.* (1985) and predicted by the simulation.

Table 2
Predicted and observed 1st grade probabilities
for reworded versions of Problem 1 (Combine 5)

Problem Wording	Simulation's Prediction	De Corte Data	Cummins Data
Traditional **Problem 1**	0.28	0.43	0.30
"altogether" & "of them" **Problem 2**	0.59	0.57	--
no "and" no "altogether" **Problem 3**	0.75	--	0.85

The simulation offers an information processing perspective as to why many children use the wrong operation in their solution (i.e., they add 8 and 3 rather than subtracting 3 from 8). According to the simulation, the critical advantage of Problem 3 is the removal of the word "altogether." In the simulation, the parser performs three tasks upon reading the word *altogether*: (i) partition the current set, although the qualifier which causes the partition (e.g., different owners, different colors) is currently unknown; (ii) attempt to determine the qualifying dependency that partitions the set; and (iii) instantiate the arithmetic action Join.[2] In the traditional wording, after the first sentence is read, the simulation has the following representation in memory: (David & Kathy's 8 soda cans, JOIN). While the "sets to join" are still unknown, the critical point is that the Join action has been activated. Upon reading the second sentence, the simulation infers that David's 5 are part of the previous set of 8. In order to construct David's 5 from the set of 8, the arithmetic action of *Separating-From* is needed. Thus, as the simulation reads the traditional wording of Problem 1, the Join action must be suppressed in favor of the Separating-From action. The simulation suggests that young children may not be able to suppress previously activated information, thus they remain with the initial Join activation. Given the Cummins (1991) reworded version without the presence of altogether in the initial sentence, the simulation is not required to suppress an initial activation of Join.

Making Significant Action Language Problems Harder

In addition to being sensitive to wording changes which make problems easier, the simulation also confirms empirical results for rewordings which cause problems to be more difficult, for example, altering the time sequence of problems containing significant action language. (Riley *et al.* (1988) refer to significant action problems as Change Join and Change Separate problems and Fuson (1994a) refers to them as Change Add To and Change Take From.)

As shown by Staub and Reusser (1992), altering the time sequence of a Change problem such that the sentences are not in strict chronological order makes the problem more difficult. Problem 4 uses the traditional wording of a Change 1 (Change Add To) problem:

> Alex had 5 marbles. Then Bethany gave Alex 3 marbles.
> How many marbles does Alex have now?

Problem 4 - Traditional Change 1 Wording

As shown in Problem 5, a change to the presentational structure causes the first sentence to refer to the transfer of possession action and the second sentence to refer to the initial set that existed prior to the action:

> Today Bethany gave Alex 3 marbles. Yesterday Alex had 5 marbles.
> How many marbles does Alex have now?

Problem 5 - Altered Time Sequence of Change 1

[2] In the simulation, instantiating an arithmetic action of Join is not analogous to picking the arithmetic operator of addition. The Join action simply means that the current set is partitioned and can be formed by joining two other (perhaps as yet unmentioned) sets.

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A critical difference between the traditional wording and the time-altered version is explained through the simulation when it solves these two problems. In the first sentence of the traditional wording (Problem 4), Alex is understood to possess five marbles and the simulation creates a set of five for Alex. The second sentence is understood to be a transfer *to* Alex, thus the three that are transferred are JOINed to his previous five as described by the "transfer-in" situation in the first two sentences. The JOIN action is explicit in the text and subsequent situational representation and need not be inferred.

In the time-altered version (Problem 5), the transfer of three to Alex occurs in the first sentence. Because Alex does not previously have any in his possession, the program simulates the transfer as resulting in Alex's set of 3 today. The second sentence indicates that Alex *had 5 yesterday* and the simulation makes another set for Alex with this amount. Since the program is simulating bottom-up, sentence-by-sentence reading, it does not "rerun" the first two sentences over in a chronological fashion. After interpreting the third sentence to mean a request for the number of marbles that Alex has *now*, the simulation infers that a difference in time differentiates the three sets so it can JOIN Alex's marbles-*today* and Alex's marbles-*yesterday*. In short, when the simulation comprehends the time-altered problem, the transfer of 3 from Bethany *to* Alex in the first sentence does not instantiate a JOIN action. That action must be inferred based on the qualifying dependency of time (today, yesterday, now) at the end of the problem.

Staub and Reusser (1992) have clearly shown that the time-altered version of Change 1 is more difficult to solve than the traditional version. Because the simulation is sensitive to the number of inferences that are required in each of these versions, the predictor equations verify their results. Table 3 presents children's solution probability with the traditional wording (Riley & Greeno, 1988), children's solution probability with the time-altered wording (Staub & Reusser, 1992) and the simulation's predicted solution probabilities.

Table 3
Predicted and observed <u>1st grade</u>
probability of solutions for Change 1

Problem Wording	Simulation's Prediction	Staub Data	Riley Data
Traditional Problem 4	0.90	--	1.00
Time-Altered Problem 5	0.70	0.63	--

As expected, the simulation closely predicts the solution probability for the traditional wording.[2] In addition, the simulation does predict a decrease in solution probability (70% success rate) but not as much of a decrease as Staub and Reusser found (63%). One possible reason may be that some children in the Staub study are confused by the difference between *today* and *now*, whereas the simulation is not. If this were the case, a number of children would be expected to give the answer of three (the number Alex had *today*). Staub reports, however, that only a small percentage of children who get it wrong give this answer (personal communication). Another possible reason from the cognitive simulation perspective is that one or both of the inferences required in the time-alteration version may be very difficult inferences for children to make. For example, the inference that marks the set [Alex has ? now] as the superset would appear to be a complex one; i.e., the word "now" is the primary clue that the sets should be distinguished *by time*. Distinguishing superset and subsets by time is in fact a more abstract type of the more difficult Combine (Put Together) problems, where sets are typically distinguished by ownership (John has, Mary has, John and Mary have) or other features such as color (red marbles, blue marbles, red and blue marbles). Because the simulation counts all inferences with the same weight (i.e., +1), the simulation may be underestimating the difficulty of certain types of inferences, such as those due to semantic character (e.g., modality, degree of abstraction of objects and relationships). This is one indication that the types of inferences may be at least as important as, if not more important than the number of inferences.

Making Static Relational Problems Easier

Static (no significant actions) relational (comparative language) problems are the most difficult addition and subtraction problems, as convincingly confirmed in empirical tests of childrens' solutions and recall protocols (Cummins, 1991; Fuson, 1992; Lewis & Mayer, 1987; Stern, 1993), eye-movement experiments (Hegarty, Mayer & Green, 1992; Verschaffel, De Corte & Pauwels, 1992) and studies of childrens' use of relational terms in the home (Walkerdine, 1990). Problem 6 is typical of this class of problems, commonly referred to as Compare problems.

[2] The simulation is expected to closely predict the solution probabilities on the traditional wordings because the predictor equations were derived from childrens' solution probabilities when they solved problems with the traditional wordings. Note, however, that the equations are the result of an analysis using all three types of semantic structure, not just Change problems.

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Jacob has 8 soda cans. Chrissy has 2 soda cans.
How many soda cans does Jacob have more than Chrissy?

Problem 6 - Traditional Wording of Compare 1

In Problem 6 (a difference set unknown problem), the cardinalities of two disjoint sets are compared. The question requests the difference between the sets with a focus on the larger set, i.e., "how many *more*?" One possibility for making the traditional wording of Problem 6 easier is to replace the static relational language (e.g., "more than") with action language which describes a hypothetical act of *making two sets equal* (Carpenter, Hiebert & Moser, 1981 and others). With this "equalize" language, the static relational language (e.g., "how many more than") in the traditional wording of Problem 6 can be replaced with action language which describes a hypothetical act of making the two sets equal (e.g., "how many does one *need to find* to have as many as the other"), as shown in Problem 7.

Jacob has 2 soda cans. Chrissy has 8 soda cans.
How many soda cans does Jacob need to find
to have as many as Chrissy?

Problem 7 - Equalize Rewording for Compare 1

For all grade levels, the simulation predicts that the Equalize version will improve solution probability, especially for the youngest grades, due largely to the fact that the Equalize version provides an explicit action to *Join* ("need to find") and an explicit reference to the result of that Join ("as many as Chrissy") On the other hand, the simulation must make these two inferences when solving the situationally void traditional wording. Table 4 compares the solution probabilities of the traditional Problem 6 as found by Riley *et al.* (1988), the Equalize probabilities as found by Carpenter *et al.* (1981), as well as the simulation's predicted solution probabilities of the traditional and Equalize rewording (Problem 7).

Table 4
Comparison of predicted and observed Compare 1
probabilities and the Equalize version for 1st Grade

Problem Wording	Simulation's Prediction	Carpenter Data	Riley Data
Traditional **Problem 6**	0.48	--	0.28
Equalize **Problem 7**	0.90	0.91	--

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Future Work

At the beginning of this paper, the question was asked:

Does sequencing word problems (for example, in the form of a database) suggest a more profitably learning trajectory for students?

Answer #1: No, if the sequencing leads students toward easier problems and allows them to avoid the difficult problems. In fact, if sequencing in the form of a database is to have any merit, it must expose students to problems that are more difficult than those found in standard textbooks in the United States, as argued by Fuson (1994a). Sequencing must specifically challenge students to deal with complexity in situational descriptions in text of the kind emphasized in algebra instruction (Thompson, 1993).

Answer #2: Yes, if the sequencing leads students to participate in community discussions about *why* some problems are more difficult than others.

Answer #2 is the motivation of our current work.[3] While sequencing problems at levels of detail beyond semantic structure has advantages for the development of questions for text books and achievement tests, the immediate concern is to combine the wealth of linguistical, mathematical, and developmental knowledge with tools which both consolidate research results for classroom teachers and encourage student discussions about and ownership of mathematical language. We believe this is the type of software support that can "foster efficient inquiry" in the classroom (Kaput, 1992). For example, students (and teachers!) could benefit from an environment which encourages them to:

(1) rank order a set of problems according to predicted levels of difficulty
(2) empirically verify within their own classroom the rank order of difficulty
(3) compare the predictions in (1) with the results of (2) and discuss
(4) compare the results of (2) with an "expert" database and discuss
(5) write their own math stories which include the "difficult" language
(6) create their own publicly available "math text" to share on the network

[3] In a second grade class at Merrymount Elementary School in Quincy, Massachusetts, we are engaging students in mathematical discourse by allowing them to rank-order word problems according to both their own predicted and in-class-verified orders of difficulty.

From a software engineering perspective, the construction of a working database of problems is not dependent on the continued development of the simulation (no small task!), although an "expert" reader will be a necessary component for an environment which allows teachers and students to add their own problems to a database, as hinted at in (5) and (6) above.

If the mastery or ownership of mathematical language is a valid view of target competence (and obviously from the above, I believe it is), then a profitable learning trajectory is getting students to a point where they can verbalize and write about mathematical situations and the relationships in those situations. A database of problems would appear to be the next logical (perhaps essential?) step. What do you think?[4]

[4] If you don't catch me in New Orleans, I'm always on-line: Mark_LeBlanc@wheatonMA.edu

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BIBLIOGRAPHY

Booth, L. (1989). A question of structure, in *Research Issues in the Learning and Teaching of Algebra*, Wagner, S. and Kieran, C. (Eds.), Hillsdale, NJ: Lawrence Erlbaum Associates; National Council of Teachers of Mathematics, 57-59.

Briars, D.J., and Larkin, J.H. (1984). An integrated model of skill in solving elementary word problems, *Cognition and Instruction*, *1*, 245-296.

Burns, J.E. (1993). *EDUCE: Explaining Discourse Understanding with Conceptual Expectations.* Unpublished Master's Thesis. Department of Computer Science, University of New Hampshire, Durham, NH.

Carpenter, T.P. (1985). Learning to add and subtract: an exercise in problem solving, in *Teaching and Learning Mathematical Problem Solving: Multiple Research Perspectives*, Silver, E.A. (Ed.), Hillsdale: Lawrence Erlbaum ociates, 17-40.

Carpenter, T . and Fennema, E. (1988). Research and cognitively guided instruction, in *Integrating Research on Teaching and Learning in Mathematics*, Fennema, E., Carpenter, T.P. and Lamon, S.J. (Eds.), Madison, WI: Wisconsin Center for Education Research, University of Wisconsin-Madison, 2-17.

Carpenter, T.P., Fennema, E., Peterson, P.L., and Carey, D.A. (1988). Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for Research in Mathematics Education*, *19*(5), 385-401.

Carpenter, T.P., Hiebert, J. and Moser, J.M. (1981). Problem structure and first-grade children's initial solution processes for simple addition and subtraction problems. *Journal for Research in Mathematics Education*, *12*, 27-39.

Case, R. (1982). General developmental influences on the acquisition of elementary concepts and algorithms in arithmetic, In *Addition and subtraction: A cognitive perspective*, Carpenter, T.M., Moser, J.M., and Romberg, T.A. (Eds.). Hillsdale, NJ: Lawrence Erlbaum Associates, 156-170.

Cobb, P., Yackel, E., and Wood, T. (1988). Curriculum and teacher development: Psychological and anthropological perspectives, in *Integrating Research on Teaching and Learning in Mathematics*, Fennema, E., Carpenter, T.P. and Lamon, S.J. (Eds.), Madison, WI: Wisconsin Center for Education Research, University of Wisconsin-Madison, 92-130.

Cooney, J.B. and Swanson, H.L. (1990). Individual differences in memory for mathematical story problems: memory span and problem perception. *Journal of Educational Psychology*, *82*(3), 570-577.

Cummins, D.D. (1991). Children's interpretations of arithmetic word problems. *Cognition and Instruction*, *8*(3), 261-289.

Cummins, D.D., Kintsch, W., Reusser, K., and Weimer, R. (1988). The role of understanding in solving word problems. *Cognitive Psychology, 20,* 405-438.

Davis-Dorsey, J., Ross, S.M., and Morrison, G.R. (1991). The role of rewording and context personalization in the solving of mathematical word problems. *Journal of Educational Psychology, 83,* 61-68.

De Corte, E. and Verschaffel, L. (1991). Some factors influencing the solution of addition and subtraction word problems. In *Language and Mathematical Education,* Durkin, K. and Shire B. (Eds.), Milton Keynes: Open University Press, 117-130.

De Corte, E. and Verschaffel, L. (1985). Beginning first graders' initial representation of arithmetic word problems. *The Journal of Mathematical Behavior, 4,* 3-21.

De Corte, E., Verschaffel, L., and DeWin, L. (1985). Influence of rewording verbal problems on children's problem representations and solutions. *Journal of Educational Psychology, 77,* 460-470.

De Corte, E., Verschaffel, L., and Pauwels, A. (1990). Influence of the semantic structure of word problems on second graders' eye movements. *Journal of Educational Psychology, 82*(2), 359-365.

Fletcher, C.R. (1986). Strategies for the allocation of short-term memory during comprehension. *Journal of Memory and Language, 25,* 43-58.

Fuson, K.C. (1994a). Children's solving of the whole range of addition and subtraction word problems using schematic drawings. Presented at the 1994 annual meeting of the American Educational Research Association (AERA-94), New Orleans.

Fuson, K.C. (1994b). A developmental model of children's addition and subtraction word problem solving. Unpublished manuscript, Northwestern University, Evanston, IL.

Fuson, K.C. (1992) Research on whole number addition and subtraction. In *Handbook of Research on Mathematics Teaching and Learning,* Grouws, D.A. (Ed.), New York: Macmillan Publishing, 243-275.

Greeno, J.G. (1987). Instructional representations based on research about understanding, in *Cognitive Science and Mathematics Education,* Schoenfeld, A.H. (Ed.), Hillsdale: Lawrence Erlbaum Associates, 61-88.

Haviland, S.E. and Clark, H.H. (1974). What's new? Acquiring new information as a process in comprehension. *Journal of Verbal Learning and Verbal Behavior, 13,* 512-521.

Hegarty, M., Mayer, R.E., and Green, C.E. (1992). Comprehension of arithmetic word problems: Evidence from students' eye fixations. *Journal of Educational Psychology, 84*(1), 76-84.

Herscovics, N. (1989). Cognitive obstacles encountered in the learning of algebra, in *Research Issues in the Learning and Teaching of Algebra,* Hillsdale, NJ: Lawrence Erlbaum Associates; National Council of Teachers of Mathematics, 60-86.

Hiebert, J. (1982). The position of the unknown set and children's solution of verbal arithmetic problems. *Journal for Research in Mathematics, 13,* 341-349.

Hudson, T. (1983). Correspondences and numerical differences between disjoint sets. *Child Development, 54,* 84-90.

Kaput, J.J. (1992). Technology and mathematics education. In *Handbook of Research on Mathematics Teaching and Learning,* Grouws, D.A. (Ed.), New York: Macmillan Publishing, 515-556.

Kaput, J.J. (1987). Towards a theory of symbol use in mathematics. In *Problems of Representation and Translation in Mathematics,* Janvier, C. (Ed.), Hillsdale: Lawrence Erlbaum Associates.

Kintsch, W. (1986). Learning from text. *Cognition and Instruction, 3*(2), 87-108.

Kintsch, W. and Greeno, J.G. (1985). Understanding and solving word arithmetic problems. *Psychological Review, 92*(1), 109-129.

LeBlanc, M.D. (1993). A computer model of the role of text integration in the solution of arithmetic word problems. Unpublished Doctoral Dissertation, University of New Hampshire, Durham, NH.

LeBlanc, M.D. (1991). From natural language to mathematical representations: a model of mathematical reading. In *Mathematical Intelligent Learning Environments,* Nwana, H.C. (Ed.), Intellect Press. Also appears in a Special Issue of *Intelligent Tutoring Media,* 2(3-4), 1991, 149-158.

LeBlanc, M.D. and Russell, S.W. (1993). Making mathematical connections through natural language: a computer model of text comprehension in arithmetic word problem understanding. *Proceedings of the Fifteenth Annual Meeting of the Cognitive Science Society .* Boulder, Colorado, June, 1993, 641-646. Also Technical Report #93-06, Department of Computer Science, University of New Hampshire, Durham, NH.

LeBlanc, M.D. and Russell, S.W. (1989). Using conceptual information to help students obtain equations for algebra word problems. *Proceedings of the Sixth Canadian Symposium on Instructional Technology.* Halifax, Nova Scotia, Canada, 191-194. Also Technical Report #88-47, Department of Computer Science, University of New Hampshire, Durham, NH.

Lewis, A.B. and Mayer, R.E. (1987). Students' miscomprehension of relational statements in arithmetic word problems. *Journal of Educational Psychology, 79,* 363-371.

Loef, M.M., Carey, D.A., Carpenter, T.P., and Fennema, E. (1988). Research into practice: Integrating assessment and instruction. *Arithmetic Teacher, 36*(3), 53-55.

McLeod, D.B. (1988). Research on learning and instruction in mathematics: The role of affect, in *Integrating Research on Teaching and Learning Mathematics,* Fennema, E., Carpenter, T.P. and Lamon, S.J. (Eds.), Madison, WI: Wisconsin Center for Education Research, University of Wisconsin-Madison, 60-89.

ERIC

NCTM - National Council of Teachers of Mathematics (1991). *Professional Standards for Teaching Mathematics*. Reston, VA: Author.

NCTM - National Council of Teachers and Mathematics (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: Author.

Nesher, P. (1982) Levels of description in the analysis of addition and subtraction word problems. In *Addition and subtraction: A cognitive perspective*, Carpenter, T.M., Moser, J.M., and Romberg, T.A. (Eds.). Hillsdale, NJ: Lawrence Erlbaum Associates, 25-38.

Nesher, P. and Katriel, T. (1986). Learning numbers: a linguistic perspective. *Journal for Research in Mathematics Education, 17*(2), 100-111.

O'Brien, E.J. (1987). The role of causal connections in the retrieval of text. *Memory and Cognition, 15*(5), 419-427.

Okamoto, Y. (1994). Computer simulations of children's developmental understanding of quantities and relationships. Paper presented at the Annual Meeting of the American Educational Research Association (AERA-94), New Orleans, April.

Okamoto, Y. (1992). The role of central conceptual structures in children's arithmetic word-problem solving. Paper presented at the Annual Meeting of the American Educational Research Association (AERA-92), San Francisco, CA, April.

Resnick, L.B. (1992). From protoquantities to operators: Building mathematical competence on a foundation of everyday knowledge. In Leinhardt, G. Putnam, R.T., & Hattrup, R.A. (Eds.), *Analysis of Arithmetic For Mathematics Teaching*. Erlbaum: Hillsdale, NJ, 373-429.

Reusser, K. (1990). From text to situation to equation: cognitive simulation of understanding and solving mathematical word problems. In*Learning and Instruction: European research in an international context. Volume 2.2: Analysis of complex skills and complex knowledge domains*, Mandl, H., De Corte, E., Bennett, N. and Friedrich, H.F. (Eds.), Oxford: Pergamon Press, 477-498.

Riley, M.S. and Greeno, J.B. (1988). Developmental Analysis of Understanding Language About Quantities and of Solving Problems. *Cognition and Instruction, 5*(1), 49-101.

Riley, M.S., Greeno, J.G., and Heller, J.I. (1983). Development of children's problem solving ability in arithmetic, in *The Development of Mathematical Thinking*, Ginsburg, H.P. (Ed.), New York: Academic Press, 153-196.

Schank, R.C. and C.K. Riesbeck (1981). *Inside Computer Understanding* (chapter 2). Hillsdale, NJ: Lawrence Erlbaum.

Searle, B.W., Lorton, P. and Suppes, P. (1974). Structural variables affecting CAI performance in arithmetic word problems of disadvantaged and deaf students. *Educational Studies in Mathematics, 5*, 371-384.

Spanos, G., Rhodes, N.C., Dale, T.C., and Crandall, J. (1988). Linguistic features of mathematical problem solving: insights and applications, in *Linguistic and Cultural Influences on Learning Mathematics*, Cocking, R.R. and Mestre, J.P. (Eds.), Hillsdale, NJ: Lawrence Erlbaum, 221-240.

Staub, F. and Reusser, K. (1992). The role of presentational factors in understanding and solving mathematical word problems. Presented at the annual meeting of the American Educational Research Association, San Francisco; also Research Report #10, University of Bern, Department of Instructional Psychology, Switzerland.

Steffe, L.P., Cobb, P., and von Glasersfeld, E. (1988). *Construction of arithmetical meanings and strategies*. New York: Springer-Verlag.

Stern, E. (1993). What makes certain arithmetic word problems involving the comparison of sets so hard for children? *Journal of Educational Psychology*, *1*(1), 1993.

Stern, E. and Lehrndorfer (1992). The role of situational context in solving word problems. *Cognitive Development*, *7*, 259-268.

Thompson, P.W. (1993). Quantitative reasoning, complexity, and additive structures. *Educational Studies in Mathematics*, *25*(3), 165-208.

Thompson, P.W. (1988). Integers as transformations, *Journal for Research in Mathematics Education*, *19*(2), 115-133.

Trabasso, T. and Sperry, L.L. (1985). Causal relatedness and importance of story events. *Journal of Memory and Language*, *24*, 595-611.

van den Broek, P. (1988). The effects of causal relations and hierarchical position on the importance of story statements. *Journal of Memory and Language*, *27*, 1-22.

Verschaffel, L. and De Corte, E. (1990). Do non-semantic factors also influence the solution process of addition and subtraction word problems? In *Learning and Instruction: European research in an international context. Volume 2.2: Analysis of complex skills and complex knowledge domains*, Mandl, H., De Corte, E., Bennett, N. and Friedrich, H.F. (Eds.), Oxford: Pergamon Press, 415-429.

Verschaffel, L., De Corte, E., and Pauwels, A. (1992). Solving compare problems: An eye-movement test of Lewis and Mayer's consistency hypothesis. *Journal of Educational Psychology*, *84*(1), 85-94.

Walkerdine, V. (1990). *The Mastery of Reason: Cognitive Development and the Production of Rationality*. New York: Routledge Press.

Weber-Russell, S. (1994). Modeling stages of arithmetic word problem understanding. Unpublished manuscript. Department of Computer Science, University of New Hampshire, Durham, NH.

Appendix

Example Simulation of EDUCE and SELAH Solving a Static-NonRelational (Combine 5) Problem

The aim of this computer simulation is not to translate directly from the problem statement into mathematical notation (commonly referred to as a problem model), but rather to make precise the tacit steps and processes which mediate a translation. Starting from the beginning, that is, the input to the simulation is identical to that which children see, the simulation "reads" and represents each word and sentence in the problem. Arithmetic actions are selected on the basis of the explicit or implied situational relationship between integrated sets, and final solution processes simulate the tasks of arriving at and carrying out an appropriate counting strategy. Reading comprehension overlaps with mathematical comprehension.

The implementation of these steps in a computer model reflects a particular text comprehension bias that deserves mention. Reading word problems requires a set of domain specific text comprehension strategies that are not utilized in the process of reading other types of texts, as pointed out by Kintsch and Greeno (1985). For example, in natural language, numbers function as predicates of objects whereas in word problems, quantified noun groups must be abstracted to the point where the numbers are the objects of interest (Nesher and Katriel, 1986). Researchers refer to this ability to translate between and/or abstract from natural language and the formal language of mathematics in a number of ways: playing the "word game" (DeCorte and Verschaffel, 1985), understanding "textual presuppositions" (Kintsch and Greeno, 1985), and possessing the "mathematics register" (Spanos, Rhodes, Dale and Crandall, 1988).

Implementation Details

The computer model software is written in Common Lisp. The EDUCE component is implemented as an expectation-based system (somewhat analogous to a traditional rule-based system) and totals approximately 23,000 lines of Lisp. The SELAH component is implemented in a procedural fashion and comprises about 9,300 lines of Lisp. The user has the option to run the model in interpretive fashion (i.e., load uncompiled code for development purposes) or first compile the Lisp. The size of the model when compiled is: EDUCE files (except external lexicon): 262K; SELAH files: 152K. Because the implementation has strictly followed the Common Lisp standard, the model executes on a number of platforms which support Common Lisp, including a RISC mainframe (Lucid Common Lisp/DECsystem, v4.0), SUN workstations

(Sun Common Lisp, v4.0.1) and a number of Macintosh computers (Macintosh Common Lisp; requires at least 4.5 MB disk space and 2 MB RAM).

An Example

Some of EDUCE's sentence-level reading capabilities and SELAH's text integration processes are revealed below in the annotated script of the simulation's output while solving the following word problem:

Jacob and Kathy have 8 soda cans. Jacob has 3 of them. The rest of them are Kathy's. How many soda cans does Kathy have?

As each sentence is parsed, EDUCE passes its conceptualization of that sentence to SELAH. (I will defer a detailed discussion of how EDUCE parses sentences until the more interesting second sentence). Because EDUCE has not detected any pronominal or set reference in the first sentence, SELAH knows that there is no explicit request for text integration. No (implicit) integration is possible, since no other conceptualized sets currently reside in long-term memory (LTM)[4], so SELAH recognizes the conceptual possession of a quantified object as a set and stores the new set in LTM:

```
LTM-(1)
STATE:  possession
        POSSESSOR:  (Jacob Kathy)
        SET:
             OBJECT: (physical (-animate) (function: contain(object: soda)))
             QUANTITY: 8
```

At this point, control returns to EDUCE and the second sentence is parsed. Each word in the second sentence is read one at a time starting from left to right. Upon reading the first word, "*Jacob*," EDUCE recognizes this word as a name and performs a referential search to see if *Jacob* has been previously mentioned. Since *Jacob* has been previously mentioned (in the first sentence), the already existing concept for *Jacob* is accessed and its conceptual meaning is loaded into short-term (working) memory (STM):

```
CONCEPTS:  (1) PERSON -- (physical (+human) (reference: Jacob))

REQUESTS:  nil
```

4 The convention of referring to long-term memory (LTM) as a "location" serves as a convenient method of speaking of "level of activation." LTM represents those concepts which have presumably "decayed" to a state whereby reactivation is necessary in order to reference them.

There are no expectations or requests generated by the word *Jacob*, i.e., the reading of this word does not expect (or request) any particular concepts to follow. The word *"has"* is then read and the conceptual entry for the word *has* is loaded into STM (2) along with its associated requests:

CONCEPTS: (1) PERSON -- (physical (+human) (reference: Jacob))
(2) STATE: possession
POSSESSOR: ?
OBJLCT: ?

REQUESTS: (i) Who possesses? [search for human earlier in sentence]
(ii) What is possessed? [search for an object later on]

Before reading the next word, each of the requests associated with the word *has* is tested to see if it might be satisfied. The first request (i) is of course successful since a search finds the conceptual representation of Jacob in STM (i.e., Jacob's conceptualization is +human) and thus the (1) Jacob-concept is merged into the POSSESSOR-slot of the (2) STATE-possession-concept. The second request (ii) is unsatisfied since no conceptual-object currently resides in STM. Having read and completed the processing for the first two words, *"Jacob has,"* EDUCE has one concept and one outstanding request in STM:

CONCEPTS: (2) STATE: possession
POSSESSOR: ((+human) (DEFINITEref: Jacob))
OBJECT: ?
REQUESTS: (i) What is possessed? [search for an object later on]

The next three words, *"3 of them,"* form a noun group with an explicit reference to a previous set. In short, the "3" causes EDUCE to (i) enter noun group mode; (ii) instantiate a conceptual quantity; and (iii) generate a request to find an object-concept in this noun group. In the context of a quantified noun group, the word *"of"* is interpreted to mean that the conceptual referent to follow (currently unknown) is the whole which possesses as a part the conceptual object in this noun group. In terms of requests, the word *"of"* expects a definite reference of a conceptual object to follow and if that object is found, the quantified object in the current noun group is "part of" a previously known set. The word *"them"* leads to a set reference of the previous set of Jacob and Kathy's 8 soda cans. The noun group *"3 of them"* is eventually merged into an "object-concept" with a quantity of three which is PART-OF a previously known set. The outstanding request (i) from the word *"has"* which is expecting an object is now satisfied. The can-object is merged into the OBJECT slot of the (2) STATE-concept.

A final conceptualization for the entire sentence resides in STM:

```
CONCEPTS: (2) STATE: possession
                  POSSESSOR: ((+human) (DEFINITEref: Jacob))
                  OBJECT: ((physical (-animate)
                                (function: contain(object: soda))
                                (quantity: 3)
                                (PART-OF: "previous set containing
                                           this object")))
```

Of specific importance in this sentence is how EDUCE represents the explicit "part of" reference generated by the "of them" wording. This sensitivity to slight changes in problem wording highlights the importance of "starting from the beginning." As discussed in this body of the paper, such explicit wordings and their associated representations facilitate the processes of text integration which lead to a coherent situational representation of the relationship between sets. As a counter-example, a more traditional wording of the second sentence in this type of problem is:

"Jacob has 3 soda cans."

In this case, EDUCE would encounter no explicit set reference and thus would not be able to represent any connection between the two sentences.

As shown in the detailed parse of sentence two, EDUCE generates a PART-OF slot from the "of them" phrase and thereby indicates an *explicit connection* between the objects in the first sentence and the objects in the current conceptualization (second sentence). SELAH performs the following steps in order to integrate this new conceptualization from EDUCE with the previous set in **LTM-(1)**. First, SELAH notes that the *source* of the objects in the second sentence is a previous concept, that is, the objects from the first sentence. SELAH searches LTM for the set which includes the "soda-can" type object.[5] Finding the previous *set* of eight specific "soda can" objects, SELAH knows that the source of Jacob's three cans is a unique quantified set (as opposed to the universal set of all soda cans). Given that the source of the three cans is the set of eight cans, SELAH associates this qualitative relationship with the mathematical relationship that the set of three is a "member of" or "part of" the set of eight. SELAH makes a new set of three in **LTM-(2)** and links the two sets together, i.e., the set in **LTM-(2)** is PART-OF the set in **LTM-(1)**.

---

5 In this example, the reference from the word "them" insures that the two sets being integrated involve the same *type* of object. In general, when SELAH attempts to determine if two sets are related, the objects in those sets need not exactly match. SELAH performs the following constraint checks: do the objects match exactly (e.g., cans and cans) or are they "like-types" (e.g., infer that cats are pets). If the objects are an *exact* match (e.g., cans and cans), is one object more qualified than the other (e.g., pepsi cans and cans) or are they equally qualified (e.g., big black cats and big black cats).

LTM-(2)
STATE: possession
POSSESSOR: (same Jacob as above)
SET:
OBJECT: (physical (-animate) (function: contain(object: soda)))
QUANTITY: 3
PART-OF: the SET in **LTM (1)**

Having successfully integrated the two conceptualizations, SELAH infers that this static PART-OF connection can be associated with a procedural situation where one set is SEPARATED-FROM another set. More specifically, because **LTM-(1)** is the source of objects in **LTM-(2)**, the set in **LTM-(2)** can be "constructed" *in a manipulative sense* by separating three cans from the previous set of eight in **LTM-(1)**. The transition from statically integrated text to mental actions such as *separating-from* is a critical feature of SELAH's novice problem solving process. The hypothesis is that novice problem solvers who are working without the aid of manipulatives continue to think of relationships between sets in terms of the actions upon those sets. Theoretically, the model proposes that young children must first construct procedural or action-oriented representations in order to arrive at an arithmetic operator (For similar arguments, see Fuson (1994b); Reusser (1990); Stern & Lehrndorfer (1992) and Weber-Russell (1994)). While some problems include explicit actions (e.g., giving) which facilitate children's success (*cf.* Briars & Larkin, 1984), many word problems do not contain action language and require additional processing to infer a procedural interpretation of statically expressed relationships. In the current example, SELAH infers a *separating-from* interpretation of a static *part-of* relationship between two sets.

The third sentence involves a similar link between the third sentence and the first, i.e., Kathy's are PART-OF the set of eight. In addition, the word "rest" implies that a *separating-from* action has already occurred, that is, the "rest" are *left behind*. Thus, the SEPARATED-FROM action that was inferred in the previous sentence is reinforced in this sentence and the objects remaining from that action are explicitly marked as belonging to Kathy. In short, SELAH's current representation is:

LTM-2 SEPARATED-FROM **LTM-1** resulting in **LTM-3**.

In addition to making sets and performing text integration, SELAH monitors some of its cognitive processing such as the number of sets to remember and the number of inferences that must be made. Table 1 summarizes some of SELAH's processing which led to this current representation.

The question in the last sentence focuses SELAH's attention on the unknown amount of Kathy's set. At this point, SELAH's representation has reached a level of abstraction where only the quantities and arithmetic action are of interest. For example, SELAH is no longer concerned with Kathy and Jacob as participants or soda cans as the type of objects. Focusing on the quantities and arithmetic action, SELAH selects an appropriate counting strategy, in this case, *Counting-Down-From* (Carpenter, 1985) to arrive at an answer of 5, e.g.,

"Ok, start at 8: *7* that's 1, *6* that's **2**, *5* that's **3**; the answer is **5** ."

Table 1
Summary of SELAH's processing for
first three sentences of Combine 5

Jacob and Kathy have 8 soda cans. Jacob has 3 of them. The rest are Kathy's.				
	Sentence			
	1	2	3	Average
Processing Summary	make set	explicit part-of; infer SEP-FR	explicit part-of; reinforce SEP-FR; explicit result of of SEP-FR	
Concepts in Memory	[J & K's 8]	[SEP J's 3] [FR J & K's 8]	[SEP J's 3] [FR J & K's 8] [REsult K's ?]	
# of concepts	1	2	3	2.00
# of inferences	0	1	0	0.33

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