DOCUMENT RESUME

ED 374 973 SE 054 926

AUTHOR	Masingila, Joanna O.
TITLE	Making Mathematics Learning In and Out of School Complementary.
PUB DATE	6 Apr 94
NOTE	15p.; Paper presented at the Annual Meeting of the American Educational Research Association (New Orleans, LA, April 4-8, 1994).
PUB TYPE	Reports - Research/Technical (143) -- Speeches/Conference Papers (150)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	*Experiential Learning; High Schools; *High School Students; *Mathematical Applications; *Mathematics Instruction; Models; *Problem Solving

ABSTRACT

Students need in-school mathematical experiences to build on and formalize the mathematical knowledge they gain in out-of-school situations. This paper presents illustrations from research that was conducted to better understand how mathematics practice and problem solving in everyday work situations compared to secondary students' solutions of the same problems. A research framework for studying the interplay between sociocultural and cognitive developmental processes is described. The framework consists of three analytic components: (1) goals that emerge during activities, (2) cognitive forms and functions constructed to accomplish those goals, and (3) interplay among the various cognitive forms. The paper concludes by linking the framework to classroom practice. Contains 23 references. (MKR)

Making Mathematics Learning In and Out of School Complementary

Joanna O. Masingila

Syracuse University
Department of Mathematics
215 Carnegie Hall
Syracuse, NY 13244-1150
(315) 443-1471
jomasing@sued.syr.edu

American Educational Research Association
New Orleans, LA
April 6, 1994

Some of the research in this paper was supported by a grant to the author from the Fund for Innovation, Office of the Vice Chancellor of Academic Affairs, Syracuse University.

Introduction

A variety of researchers in the last fifteen years have described how people use mathematics in out-of-school situations to solve problems and achieve goals (e.g., Lave, 1988; Masingila, 1992a; Millroy, 1992; Scribner, 1985). Furthermore, it is generally accepted that mathematics learning "is not limited to acquisition of the formal algorithmic procedures passed down by mathematicians to individuals via school. Mathematics learning occurs as well during participation in cultural practices as children and adults attempt to accomplish pragmatic goals" (Saxe, 1988, pp. 14-15).

However, there are differences between mathematics *practice* in and out of school, as well as mathematics *learning* in and out of school. Lave (1988) has found evidence that mathematics practice in everyday settings differs from school mathematics practice in a number of ways. In everyday settings: (a) people look efficacious as they deal with complex tasks, (b) mathematics practice is structured in relation to ongoing activity and setting, (c) people have more than sufficient mathematical knowledge to deal with problems, (d) mathematics practice is nearly always correct, (e) problems can be changed, transformed, abandoned and/or solved since the problem has been generated by the problem solver, and (f) procedures are invented on the spot as needed.

Researchers who have investigated how persons solve problems in school-like situations and solve mathematically-similar problems in everyday contexts found that in the former situation people "tended to produce, without question, algorithmic, place-holding, school-learned techniques for solving problems, even when they could not remember them well enough to solve problems successfully" (Lave, 1985, p. 173). When the same people solved problems in situations that appeared different from school, they used a variety of techniques and invented units with which to compute (Lave, 1985).

These differences in mathematics practice appear to be explained by the fact that: (a) problems in everyday situations are embedded in real contexts that are meaningful to the problem solver and this motivates and sustains problem-solving activity (Lester, 1989), and (b) "the mathematics used outside school is a tool in the service of some broader goal, and not an aim in itself as it is in school" (Nunes, 1993, p. 30)

Just as mathematics practice in and out of school differs, so does mathematics learning. Whereas school learning emphasizes individual cognition, pure mentation, symbol manipulation and generalized learning, everyday practice relies on shared cognition, tool manipulation, contextualized reasoning and situation-specific competencies (Resnick, 1987).

Knowledge constructed in out-of-school situations often develops out of activities which: (a) occur in a familiar setting, (b) are dilemma driven, (c) are goal directed, (d) use the learner's own natural language, and (e) often occur in an apprenticeship situation allowing for observation of the skill and thinking involved in expert performance (Lester, 1989). Knowledge gained in school all too often grows out of a transmission paradigm of instruction and is largely devoid of meaning (lack of context, relevance, specific goal). Furthermore, Resnick (1987) has argued that "[t]he process of schooling seems to encourage the idea that . . . there is not supposed to be much continuity between what one knows outside school and what one learns in school" (p. 15).

In some instances, the difference between mathematics practice in and out of school may be inherent. Sometimes a mathematical concept is understood and used differently in everyday situations than the way it is taught in school (e.g., de Abreu & Carraher, 1989). For example, percentage of change is a common concept in retailing and in school mathematics. In school, percentage of change is understood to be the amount of change from the original amount. A typical textbook exercise involving this concept might be the following:

> *Find the percent of change for a video game system that costs $29 in 1980 and $99 in 1990. (Davison, Landau, McCracken & Thompson, 1992, p. 262)*

A student finding the answer to this exercise would subtract $29 from $99 to get a $70 increase, then divide $70 by $29 to get an increase of approximately 241%. Percentage of change in retailing, however, is understood to be the amount of change from the retail price. Thus, for the situation in the textbook exercise above, a retailer would divide $70 by $99 to get an increase of approximately 71%. Since the final result in retailing is sales, all percentages of change are based

on retail prices. In this case, the solution process in the everyday context is different because of the different conceptual understanding of percentage of change (Masingila, 1993b).

I believe that while some differences may be inherent in mathematics learning and practice in and out of school, the differences can be narrowed so that instead of being disjoint activities that do not influence each other, mathematics learning and practice in and out of school can build on and complement each other. In this way, students can bring to bear their mathematical knowledge gained in out-of-school experiences on their school mathematics. Likewise, students can use their school mathematics in solving problems that occur in everyday situations. Acioly and Schliemann (1986), in their study of lottery game bookies in Brazil, found that the bookies with school mathematics experience were able to understand and solve novel problems while bookies who had not attended school were unable to do this. In this case, the schooled bookies seemed able to draw on their school mathematics to use in an out-of-school situation.

Individuals need both in-school and out-of-school mathematical experiences in our society. Without everyday mathematical experiences, in-school learning is solely for the sake of learning. Students need in-school mathematical experiences to build on and formalize their mathematical knowledge gained in out-of-school situations. This occurs in the classroom through the activity of constructing knowledge that is subject to "explanation and justification as students participate in the intellectual practices of the classroom community" (Cobb, Yackel & Wood, 1992, p. 7).

The question, then, is how can teachers use students' in-school and out-of-school experiences so that mathematics learning and practice in these contexts can be complementary? Before discussing and elaborating on a framework that I think sheds some light on this, I will present some research from several studies (Masingila, 1992a, 1993b) that illustrates some of the points made above and lays the groundwork for discussion of the framework.

Illustrations from Research

Structure of the Studies

In each of these studies, mathematics practice was examined in an everyday work situation. Workers in each context were observed and informally questioned as they worked. Data were collected from carpet laying estimators and installers, an interior designer, and a retailer. I analyzed the data through a process of inductive data analysis, looking for the concepts and processes that were involved in the mathematics practice in these contexts.

Selected problems that occurred in these contexts were then given to pairs of secondary students. I observed and informally questioned the pairs of students as they solved these problems. I analyzed the data by examining how each pair solved the problems, and then compared these with how the persons in the work context had solved the same problems.

Comparing In-School and Out-of-School Practice

The follow discussion focuses on three key differences that I found in my work comparing in-school and out-of-school mathematics practice. The differences involved the goals of the activity, the conceptual understanding of persons in each context, and flexibility in dealing with constraints.

Goals of the activity. In each context (carpet laying, interior design, retailing, school), the distinction between the goals of the individuals in the out-of-school contexts were in sharp contrast to the goals of the students in the school context as long as the students viewed the problems as school problems. However, when the students were able to place themselves in the everyday situation, they appeared to view the problems differently.

For the individuals in work situations, the goal was to make a decision. In the course of that decision-making process, problems needed to be solved and mathematics was a tool to be used to solve the problems. The students had as their goal to solve the problems I gave them; nothing beyond that was required of them. I offer an example of this distinction.

This illustration of this difference in goals is from the interior design context. The interior designer was deciding on the amount of materials needed for a house that was being refurbished. One aspect of the redecorating involved wallpapering a number of walls of the house. The wallpaper that was chosen was 20" wide and had a repeat length of 9" (i.e., the pattern repeated every 9"). The interior designer told me that what she does in considering repeat lengths is to divide the desired length by the length of the repeat, in this case 96" ÷ 9". This gives the number

of repeats in the desired length. She calculated that there were 10.67 repeats, "so there needs to be 11 repeats in each strip of wallpaper so 11 times 9" means 99" needs to be considered as the length of the wall. So, you figure the number of strips you need for the rooms and figure each strip to be 99" long. Of course, you also have to consider that you have to place the seams for the strips at least two inches past the corner so it stays down better. Also, wallpaper is sold in single, double, and triple rolls so you must figure the best deal for what you need." The interior designer's solution process is aimed toward the goal of making a decision the quantity of materials needed while considering cost efficiency.

I gave a pair of secondary students a problem concerning this same situation. These students were from the same advanced algebra course as the first pair of students who solved the recipe problem. In order to structure the situation a bit more and focus on how the students would deal with the repeat length constraint, I gave the following problem:

Suppose you decide to wallpaper your bedroom. If your room is 10' by 8' and the walls are 8' high, how much wallpaper do you need if the wallpaper has a repeat length of 9"? The wallpaper is 20" wide and a roll has 45 feet of wallpaper on it.

The concept of repeat length was explained and the students indicated that they understood. They began the solution process by calculating that the one wall was 120" long and the other was 96" long. The students decided that they would need 6 strips of wallpaper on each of the 10' walls and 5 strips of wallpaper on each of the 8' walls. They figured that each wall was 96" from ceiling to floor so that 576" of wallpaper were needed for each 10' wall. When asked to explain what the 576" inches meant, they said that there were 6 strips needed across the wall and each strip was 96" long "and 6 times 96 is 576." The students continued in this vein and determined that each 8' wall would need 480" of wallpaper. Totaling the amount needed for the walls, they indicated that 2112 inches of wallpaper were needed for the room.

The students appeared to act as if their solution was complete. Finally, as one student checked back over the statement of the problem, she noted that they had not considered the repeat length: "Well, how would we do this if we were really going to wallpaper a room? Maybe we should draw a picture to see what this looks like." At that point, the students drew a diagram of one 10' wall: "The first strip doesn't matter. The pattern doesn't have to match anything, so all we need is 96". But the next strip has to match that one."

The students decided to divide 96 by 9 to find out how many sections of the pattern are on each wall. They determined that there were approximately 10.7 sections of the pattern in the first strip and so three inches were going to have to be trimmed off the wallpaper before cutting the next strip. When asked to explain their reasoning, one student replied, "Since 99" would be a whole number of 9" sections, we have to cut off three more inches [after the 96"] so that the pattern will be starting again for the next strip." At this point, the students decided that they could treat all the remaining strips as 99" and come out with matching patterns. They totaled the strips and decided that 2175 inches of wallpaper were needed. The students made no attempt to determine the number of feet or number of rolls of wallpaper that would be needed but seemed satisfied to leave their solution in inches.

The approaches to the problem-solving activity were different in the in-school and out-of-school contexts because the persons involved had different goals. For the interior designer, solving problems was a necessary part of her job. She used mathematics as a tool to help her solve problems and not as the goal of the problem. The students, however, seemed to view the problems as mathematical exercises and immediately started using algorithms that they thought would be appropriate. Although the students gained some insight when they tried to put themselves in the everyday problem situation, they did not stick with this perspective totally and did not check the reasonableness of their solution with the everyday context (e.g., converting the number of inches of wallpaper needed into number of rolls).

Conceptual understanding. I observed differences in conceptual understanding between individuals in the everyday work situations and the secondary students. While the students had the procedural knowledge to solve the problems, they were not able to understand the concept

involved and apply the procedures. On the other hand, the workers understood the concept, at least in this context, and had the tools necessary to solve the problems. The following example illustrate the differences between the workers and the students.

In the carpet laying context, the concept of area is pervasive in the all work done by the estimators and installers. All the workers Masingila (1992a) observed converted square feet to square yards by dividing square feet by nine. This algorithm is essential in the carpet laying business since measurements are taken in feet but carpet must be ordered from a supplier in square yards. In a conversation, Dean, an estimator, explained why this algorithm worked.

Joanna:	If you just know the length and width of a room, how do you find how many square yards of carpet you need?
Dean:	Well, if the room is 12' by 8' then you take 12 x 8 ÷ 9.
Joanna:	What does the 9 mean?
Dean:	That's the way you convert square footage to square yardage.
Joanna:	Okay, but where does the 9 come from?
Dean:	I don't know. Maybe I don't understand the question. . . . Where does the 9 come from?
Joanna:	Yeah, why isn't it 8 or 6?
Dean:	Well, when you have square footage (draws diagram with 3 by 3 grid—see Figure 1) . . . each of these squares is a square foot and there are three feet in a yard (puts x's inside the three squares in the right column of the grid) and then three across (puts x's inside three squares in the top row)—so that makes 9. (pp. 114-115)

By using a diagram Dean was able to illustrate, although not fully articulate, that in one square yard there are nine square feet and to convert from square feet to square yards involves dividing by nine.

In contrast, several pairs of secondary general mathematics students were given a similar problem and did not understand that the concept of area was involved. The problem and conversation with two students, Jim and Matt, are given below.

Suppose you need a piece of carpet 12 feet by 9 feet. How many square yards should you order from the carpet supplier?

Matt:	I don't know nothin' about square yards.
Joanna:	Well, let's see. What does a piece of carpet 12' by 9' look like?
Jim:	(draws a rectangle and labels the dimensions 12' and 9')
Joanna:	Alright. Now how would you change that to yards?
Matt:	Divide by 3.
Joanna:	Why?
Matt:	'Cause it takes 3 feet to make a yard.
Jim:	(writes "4 yds" and "3 yds" and scribbles out 12' and 9')
Joanna:	Okay, now how many square yards is that?
Jim:	Square yards? Oh . . . well, there's two 4's and two 3's—one on each side. So that's 4 square yards and 3 square yards.
Joanna:	What does square yards mean?
Matt:	I don't know. (Jim shakes his head.)
Joanna:	What it means is area; finding the square yardage of this carpet is finding the area.
Jim:	So that's 4^2—8 and 3^2—6 and take 8 x 6.
Joanna:	Where did you get the 8?
Jim:	4^2—4 x 4—no, that's not 8. Area is length times width times height. (pause) I'm not sure.

Joanna: Area of a rectangle is length times width. So what's the area of this carpet piece?
Jim: You'd multiply 4 x 3—no, 8 x 6 because those are square yards.
Joanna: So the area or square yards is what?
Jim: 8 x 6.
Joanna: Matt, do you agree with that?
Matt: Yeah. (Masingila, 1992a, pp. 235-237)

None of the six pair of students who worked this problem, including this pair, understood that finding the square yardage of the piece of carpet was the same as finding the area of the carpet piece. However, these students had studied area with square units for several years. In fact, in the textbook they had used the previous year there were exercises that were similar to this problem. The main difference may have been that the exercises were in a chapter on area and in a lesson on area of rectangles so the students knew what procedure to use.

Contrast this with Dean's explanation of how to convert from square feet to square yards. He knew the algorithm, dividing by nine—because he used it regularly in his job. However, Dean also understood that he was dealing with area, and that in one square yard there are nine square feet. It is our conjecture that if Dean had been asked to explain this conversion when he was a ninth grader, his explanation would not have been much different than that of these general mathematics students. However, through his day-to-day experience working with rectangular area, Dean had come to a fuller understanding of this conversion and was able to construct a reason for its mathematical validity.

I attribute the differences in conceptual understanding between the individuals in everyday work contexts and the students to a lack of experience on the part of students in dealing with these concepts in problematic situations where mathematics is used as a tool rather than an object. I observed with the respondents that the workers were able to understand the problems within the context and had the conceptual understanding to solve the problems within that context.

Flexibility in dealing with constraints. Problems that occurred in each of the everyday situations that I examined were filled with constraints. I observed noticeable differences in the ways that workers in these contexts and the students were able to deal with these constraints.

The carpet laying context contains a variety of constraints: (a) floor covering materials come in specified sizes (e.g., most carpet is 12' wide, most tile is 1' by 1'), (b) carpet pieces are rectangular, (c) carpet in a room (and usually throughout a building) must have the nap (the dense, fuzzy surface on carpet formed by fibers from the underlying material) running in the same direction, (d) consideration of seam placement is very important because of traffic patterns and the type of carpet being installed, (e) some carpets have patterns that must match at the seams, (f) tile and wood pieces must be laid to be lengthwise and widthwise symmetrical about the center of the room, and (g) fill pieces for both tile and base must be six inches wide or more to stay glued in place. Some particular situations have more constraints, such as a post in the middle of a room that is being carpeted (Masingila, 1992a).

The ninth-grade general mathematics students who worked problems from the carpet laying context often had difficulty dealing with the constraints involved in the problems. For example, in a problem involving the installation of tile, the students struggled to figure our a way to install the tile so that the constraints about lengthwise and widthwise symmetry and fill pieces being at least six inches wide were fulfilled (see Masingila, 1992a, for more discussion about the students' problem-solving work).

The students were also not as flexible as the experienced workers in seeing more than one way to solve a problem. In a pentagonal-shaped room that needed carpeting, the students were able to see only one way (without guidance) to install carpet. The estimator, on the other hand, was able to visualize how the carpet would be laid if it were installed with the nap running in the direction of the maximum length of the room and with the nap running in the direction of the maximum width of the room. By having more than one solution, he was able to weigh cost efficiency against seam placement and make a decision while considering these constraints (Masingila, 1992a, 1992b).

One of the interior design problems I gave to pairs of students was as follows:

You need to purchase some materials for upholstering some chairs. If you buy the whole bolt, which has 60 yards of material, it will cost $5.00 per yard. If you purchase less than a full bolt, it will cost an additional $1.50 per yard. At what point does it become more economical to purchase the whole bolt of material?

One pair of second-year geometry students approached the problem through trial and error. They first found that 30 yards of material would cost $195, 40 yards would cost $260, and 50 yards would cost $325. At this point, one of the students said, "Well, the whole bolt only costs $300 so it must be less than 50 yards." After some more calculations they decided that 46 yards was the cutoff point: "Since 46 yards costs $299, anything more than 46 yards would cost more than that—so it would be cheaper to buy the whole thing."

The interior designer, when faced with this problem, found that the whole bolt cost $300 and then divided by $6.50 to find that 46 yards and 5 inches is approximately the amount at which it becomes more economical to buy the whole bolt. However, she decided that if she needed an amount close to 46 yards, "like if I need 44 yards, I will buy the whole thing because I'm spending less than 5% over what I need and I can most likely use the material for something." For the students, this problem had only one answer; for the interior designer, the answer depended upon the situation.

My interpretation of this difference in flexibility on the part of the students and the workers is that the students, for the most part, have not been exposed to problems with real-life constraints that must be considered and addressed in order to find solutions (Masingila, 1993b). Although there are many exercises in school textbooks that are set in these contexts, the exercises are typically devoid of real-life constraints and, as a result, do not require students to engage in the type of problem solving required in the everyday contexts (Masingila & Lester, 1992).

Interplay Between Mathematics In and Out of School

Saxe (1991) has delineated a "research framework for gaining insight into the interplay between sociocultural and cognitive developmental processes through the analysis of practice participation" (p. 13). The theoretical underpinnings of the framework are based on both Piaget and Vygotsky, but the framework moves beyond them in considering this interplay. Saxe explains:

> The framework shares the underlying constructivist assumptions of the Piagetian and Vygotskian formulations, and, with respect to core constructivist assumptions, the model . . . is consistent with both approaches. However, the framework . . . targets a level of analysis that is not addressed by either of these formulations. Unlike the Piagetian approach, my concern is to treat cognitive development on a level of analysis in which activity-in-sociocultural context is a critical focus and cognitive developmental processes are analyzed with reference to these contexted activities. Unlike the Vygotskian writings, which do not develop core developmental and sociocultural theoretical constructs with reference to systematic analysis of core domains of knowledge, the present approach is concerned with a systematic analysis of mathematical cognition that integrates cognitive developmental and sociohistorical perspectives. (pp. 13-14)

Although Saxe's framework is a method for studying the interplay between sociocultural and cognitive developmental processes, I find it helpful in thinking about working towards in-school and out-of-school mathematics learning and practice being complementary. Thus, I discuss his framework with illustrations from our my research, and then elaborate on ways to make this interplay between in-school and out-of-school contexts more deliberate.

Saxe's (1991) framework consists of three analytic components: (a) goals that emerge during activities, (b) cognitive forms and functions constructed to accomplish those goals, and (c) interplay among the various cognitive forms. Goals are "emergent phenomena, shifting and taking new form as individuals use their knowledge and skills alone and in interaction with others to organize their immediate contexts" (p. 17). Forms are "historically elaborated constructions like

number systems, currency systems, and social conventions." As these forms are "acquired and used by individuals to accomplish various cognitive functions" (e.g., counting, measuring), they become cognitive forms (p. 19). Interplay among the various cognitive forms occurs as individuals, "in order to accomplish goals in one setting, . . . appropriate and specialize cognitive forms linked" to another (p. 22).

Emerging Goals

Saxe outlines four parameters that influence the emergence of goals: (a) the goal structure of activities, (b) social interactions, (c) conventions and artifacts, and (d) an individual's prior understandings. Figure 2 illustrates this four-parameter model. I will use examples from my own research to illustrate these parameters.

The goal structure of an activity consists of the tasks that must be accomplished in the activity. For example, in order to run a store a retailer must buy and reprice items for sale. A principal concern for the retailer is to sell an item for as much money as possible while selling as many of the item as possible. Thus, mathematical goals that emerge in marking items up and down are guided by this economic concern.

Social interactions that occur during activities may also influence the emerging goals. In the carpet laying context, installers worked with helpers in a master-apprentice relationship. The discussion and interaction that occurred between installers and helpers often allowed helpers to engage in activities they would not have been able to unassisted (Masingila, 1992a).

Saxe (1991) writes of conventions and artifacts as "cultural forms that have emerged over the course of social history, such as . . . the Oksapmin indigenous body-part counting system and . . . a particular currency system" (p. 18). Sometimes individuals within a culture develop a set of conventions that may be unique to their particular situation. For example, Prus-Wisniowska (1993) found in her work that a restaurant manager developed a notation system to keeping track of the restaurant's inventory. In counting items for inventory purposes she used different units for different items (e.g., pound, 5 pound, box, case, pack, each). These units were usually the same as the units used for delivery purposes.

However, she adopted a different unit for French fries. French fries were only delivered in full cases, where 1 case = 10 boxes and 1 box = 10 pounds of French fries. The restaurant manager found it difficult to operate with case as the unit for French fries since the restaurant was rather small and 10 pounds of French fries would often be more than was needed for a particular meal. So she decided to use zero to denote a case less than half full; a zero indicated to her not that there were no French fries, but rather that it would soon be time to order more.

After using this convention for some time, the restaurant manager found it, too, was inconvenient because sometimes five boxes of French fries sufficed for one week; other times it did not, and so the distinction between zero and one became critical. In the end, the restaurant manager decided to change her notation to using box as the unit. Even though the French fries continued to be delivered in cases, she reported each case as ten boxes and so from this time on her inventory indicated the amount of French fries with an accuracy of one box. Thus, the convention used by the restaurant manager influenced emerging mathematical goals of activities associated with the inventory.

The prior understandings that "individuals bring to bear on cultural practices both constrain and enable the goals they construct in practices" (Saxe, 1991, p. 18). In solving the carpet problem that involved converting from square feet to square yards, the students' prior understandings about area as a formula, dependent upon the geometric shape appeared to constrain their goals. However, for another problem that involved a pentagonal-shaped room to be carpeted, one student knew from personal experience that the room had to be treated as a rectangle and this enabled him to construct goals that were different from students who tried to determine how to lay carpet in a five-sided room (Masingila, 1992a).

Form-Function Shifts

The second analytic component of Saxe's (1991) research framework is the dynamic in the "shifting relations between cultural forms and cognitive functions as they are interwoven with the socially textured goals linked to practice participation" (p. 19). He describes how the cultural form of body counting shifted in function as individuals' levels of economic participation changed.

This phenomena also occurred in the carpet laying context as the helpers gained experience through participating in the practice of installing floor coverings. For example, one convention that was present in this context was an algorithm for laying tile. The algorithm was an agreed-upon procedure for laying tile so that the tile was lengthwise and widthwise symmetrical about the center of the room and that fill (partial) pieces were at least six inches wide (Masingila, 1992a). However, as the helper participated in the tiling process and, as was sometimes the case, became an installer, the procedure (form) became a cognitive tool (function) to be used for making decisions when complicating factors compounded the installation.

Interplay Among Various Cognitive Forms

In studying Oksapmin schoolchildren, Saxe (1985) found evidence that the children used out-of-school cognitive forms to bring to bear on in-school problems. Other researchers have determined that persons in out-of-school contexts may use knowledge gained in school to address problems they encounter (Acioly & Schliemann, 1986). Thus, there can be interplay between cognitive forms that may be appropriated and specialized in one setting and their use in another.

Saxe (1991) has specified a generalized portrayal (see Figure 3) of how cognitive developmental and sociocultural processes are "interwoven with one another in complex ways" (p. 186). Saxe notes:

> As the figure shows, in our daily lives, we are engaged with multiple practices. Within practices, goals emerge that must be accomplished, avoided, or reckoned within the achieving of larger objectives. Across practices, the understandings we generate in one may be appropriated and transformed to structure and restructure goals in another. (p. 186)

Linking the Framework to Classroom Practice

As mentioned previously, although Saxe's intent was to outline a framework for conducting research to better understand the interplay among various cognitive forms through practice, I find the framework useful in thinking about ways to bring about more and deliberate interplay between developmental processes in different settings.

I have discussed ways in which mathematics learning and practice often differ in school and everyday contexts. However, individuals do make use of knowledge in one context that was situated in another context when they view the problem situations as being similar (Stigler & Baranes, 1988). I suggest that if we, as teachers: (a) can create situations where students experience their mathematics learning and practice in school as similar to mathematics learning and practice out of school, and (b) encourage students to participate in activities out of school in which the mathematics learning and practice may be similar to their mathematics learning and practice in school, then these experiences can become complementary to each other.

Connecting in-school with out-of-school experiences. First, in order to create in-school experiences similar to out-of-school experiences, the goal structures of activities must be similar for in-school and out-of-school activities from which students may construct similar mathematical knowledge. This means that the curricula includes a wide variety of problem situations that engage students in doing mathematics in ways that are similar to doing mathematics in out-of-school situations. Thus, problems are embedded in situations that are real and meaningful to students, and mathematics practice can be structured in relation to these problematic situations. It also means that mathematics is a tool to be used and that procedures and processes are learned as they are needed in the midst of accomplishing emerging goals.

I further suggest that in order to structure classroom experiences like this instruction should be via problem solving. In teaching via problem solving, "problems are valued not only as a purpose for learning mathematics, but also as a primary means of doing so. The teaching of a mathematical topic begins with a problem situation that embodies key aspects of the topic, and mathematical techniques are developed as reasonable responses to reasonable problems" (Schroeder & Lester, 1989, p. 33). Teaching via problem solving deviates from the traditional instructional approach of the teacher presenting information and then assigning exercises in which students practice and apply this information. Using a teaching via problem solving instructional approach means that

mathematical understandings are constructed by students as they seek to accomplish emerging goals through problematic situations.

Second, social interactions are an essential part of this classroom mathematics practice. In working individually and collectively to accomplish emerging goals, mathematical knowledge is developed within a meaningful context and cognitive development occurs as students work together with peers and teacher to negotiate shared meanings. As Saxe (1991) noted, social interaction is a key influence on the emerging goals of an activity.

Third, in-school activities should make use of cultural artifacts and conventions that students can use to interpret problems and make sense of them. Students should also be encouraged to generate conventions that may be helpful to them in the course of accomplishing their emerging goals. For example, students may invent notation to indicate when objects are the same size and shape, in the course of working in a measurement context, before they have formalized the concept of congruence.

Finally, teachers can build on students' prior understandings. All students bring to school mathematical knowledge acquired in other contexts. This knowledge is often hidden and unused by students in school as they learn to use the mathematical procedures that teachers demonstrate and evaluate (Masingila, 1993a). If teachers engage students in conversation about their everyday experiences, listen to them, and encourage and observe their informal methods of mathematizing, they can learn much about students' prior understandings. Similarly, teachers can encourage students to bring to bear their prior understandings by having students: (a) create their own problem situations, (b) solve problems in more than one way and share their solution methods with each other (Lester, 1989), and (c) focus on semantics rather than syntax.

Connecting out-of-school with in-school experiences. Besides creating experiences in school that may complement out-of-school mathematics learning and practice, teachers can guide students in reflecting on how in-school learning and practice are used out of school. In a study examining middle school students' ideas about their out-of-school mathematics practice (Masingila, 1994), I observed that with encouraged reflection students were able to note a number of ways that they used mathematics outside of school. Sixth- and eighth-grade students were interviewed before and after keeping a log for a week in which they recorded their use of mathematics. Although students reported ways they used mathematics they classified as "non-school math," they also indicated many instances where they used knowledge they categorized as "school math."

I suggest that an important aspect of in-school and out-of-school mathematics experiences becoming more complementary is to encourage students to be aware of their mathematics learning and practice outside of school. This involves having students discuss their out-of-school experiences and what mathematics concepts and processes they used in those experiences. Additionally, teachers can have students reflect on how their in-school mathematical experiences influence this learning and practice. Teachers can also ask students to think of out-of-school experiences that are similar in some aspects to mathematical problem situations they have encountered in the classroom. Students and teachers can have a good discussion concerning similarities and differences between these situations that can help students to see the value of mathematics practice in both contexts.

In both in-school and out-of-school experiences, students participating in mathematics practice will become engaged with novel mathematics goals that require form-function shifts. Teachers who observe these gradual and complex shifts, gain valuable assessment information about students and can serve to facilitate the process of students acquiring mathematical knowledge to use as cognitive tools.

Concluding Remarks

Mathematics learning and practice in school and out of school differ in some significant ways. Some of these differences may be inherent because a concept is learned and used differently in school than out of school. However, I believe that many of the differences can be narrowed by creating experiences that engage students in doing mathematics in school in ways similar to mathematics learning and practice outside of school. The framework Saxe (1991) outlined for examining the interplay between sociocultural and cognitive developmental processes targets

cultural practices as important contexts for study. Similarly, this discussion has used Saxe's framework to suggest how more and deliberate interplay can be encouraged between these developmental processes by focusing on mathematics learning and practice in everyday contexts as starting points. I believe that by making in-school and out-of-school mathematics experiences more complementary, student learning and practice in both of these situations can be enhanced.

References

Acioly, N. M., & Schliemann, A. D. (1986). Intuitive mathematics and schooling in understanding a lottery game. In *Proceedings of the 10th Annual Meeting of the International Group for the Psychology of Mathematics Education* (pp. 223-228). London: University of London, Institute of Education.

Cobb, P., Yackel, E., & Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics Education, 23* (1), 2-33.

Davison, D. M., Landau, M.S., McCracken, L., & Thompson, L. (1992). *Pre-algebra.* Needham, MA: Prentice Hall, Inc.

de Abreu, G. M. C., & Carraher, D. W (1989). The mathematics of Brazilian sugar cane farmers. In C. Keitel (Ed.), *Mathematics, education, and society* (pp. 68-70), (Science and Technology Education Document Series No. 35). Paris: UNESCO.

Lave, J. (1985). Introduction: Situationally specific practice. *Anthropology and Education Quarterly, 16* (3), 171-176.

Lave, J. (1988). *Cognition in practice: Mind, mathematics and culture in everyday life.* Cambridge, UK: Cambridge University Press.

Lester, F. K., Jr. (1989). Mathematical problem solving in and out of school. *Arithmetic Teacher, 37* (3), 33-35.

Masingila, J. O. (1992a). Mathematics practice and apprenticeship in carpet laying: Suggestions for mathematics education. (Doctoral dissertation, Indiana University-Bloomington, 1992). *Dissertation Abstracts International, 53A*, 1833.

Masingila, J. O. (1992b). Mathematics practice in carpet laying. In W. Geeslin & K. Graham (Eds.), *Proceedings of the 16th Annual Conference of the International Group for the Psychology of Mathematics Education* (Vol. II, pp. 80-87). Durham, NH: University of New Hampshire.

Masingila, J. O. (1993a). Learning from mathematics practice in out-of-school situations. *For the Learning of Mathematics, 13* (2), 18-22.

Masingila, J. O. (1993b). Problem solving in and out of school. In J. R. Becker & B. J. Pence (Eds.), *Proceedings of the 15th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. II, pp. 40-46). San Jose, CA: San Jose State University.

Masingila, J. O. (1994). Middle school students' perceptions of their everyday mathematics usage. *Submitted for publication.*

Masingila, J. O., & Lester, F. K., Jr. (1992). Experience-based knowledge vs. school-based knowledge. *Paper prepared for the Seventh Conference of the International Congress on Mathematical Education* (Topic Group: Ethnomathematics and Mathematics Education), Quebec City, Quebec, Canada.

Millroy, W. L. (1992). An ethnographic study of the mathematical ideas of a group of carpenters. *Journal for Research in Mathematics Education, Monograph No. 5.*

Nunes, T. (1993). The socio-cultural context of mathematical thinking: Research findings and educational implications. In A. J. Bishop, K. Hart, S. Lerman & T. Nunes, *Significant influences on children's learning of mathematics* (pp. 27-42). Paris: UNESCO.

Prus-Wisniowska, E. (1993). *Examining mathematics practice in restaurant management,* Research Apprenticeship Report. Unpublished manuscript, Syracuse University.

Resnick, L. B. (1987). Learning in school and out. *Educational Researcher, 16* (9), 13-20.

Saxe, G. B. (1985). The effects of schooling on arithmetical understandings: Studies with Oksapmin children in Papua New Guinea. *Journal of Educational Psychology, 77* (5), 503-513.

Saxe, G. B. (1988). Candy selling and math learning. *Educational Researcher, 17* (6), 14-21.

Saxe, G. B. (1991). *Culture and cognitive development: Studies in mathematical understanding*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Schroeder, T. L., & Lester, F. K., Jr. (1989). Developing understanding in mathematics via problem solving. In P. R. Trafton (Ed.), *New directions for elementary school mathematics* (pp. 31-42). Reston, VA: National Council of Teachers of Mathematics.

Scribner, S. (1985). Knowledge at work. *Anthropology and Education Quarterly, 16* (3), 199-206.

Stigler, J. W., & Baranes, R. (1988). Culture and mathematics learning. In E. Z. Rothkopf (Ed.), *Review of Research in Education, 15* (pp. 253-306). Washington, D.C.: American Educational Research Association.

Converting Square Feet to Square Yards

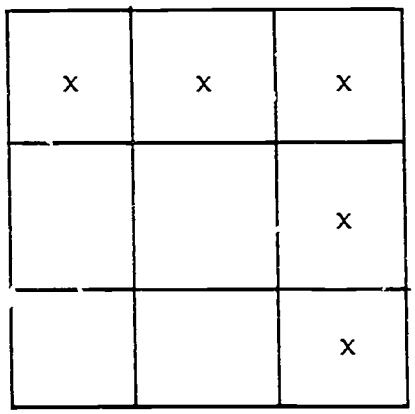

Figure 1

Four-Parameter Model*

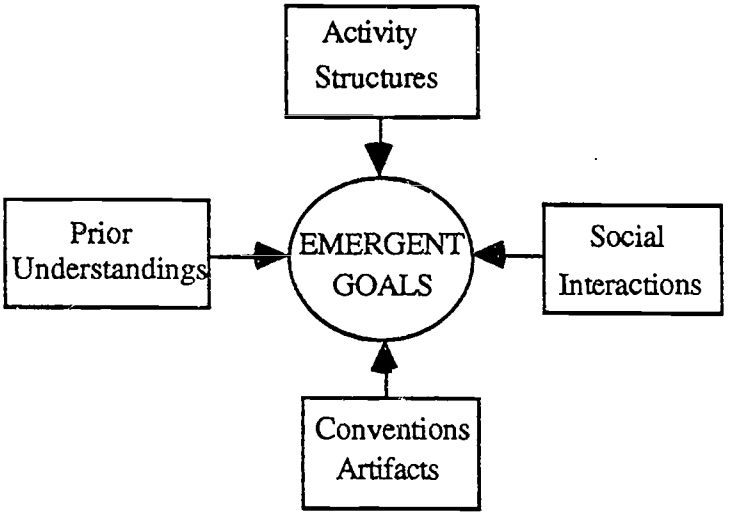

Figure 2

*(Saxe, 1991, p. 17)

Expansion of Four-Parameter Model*

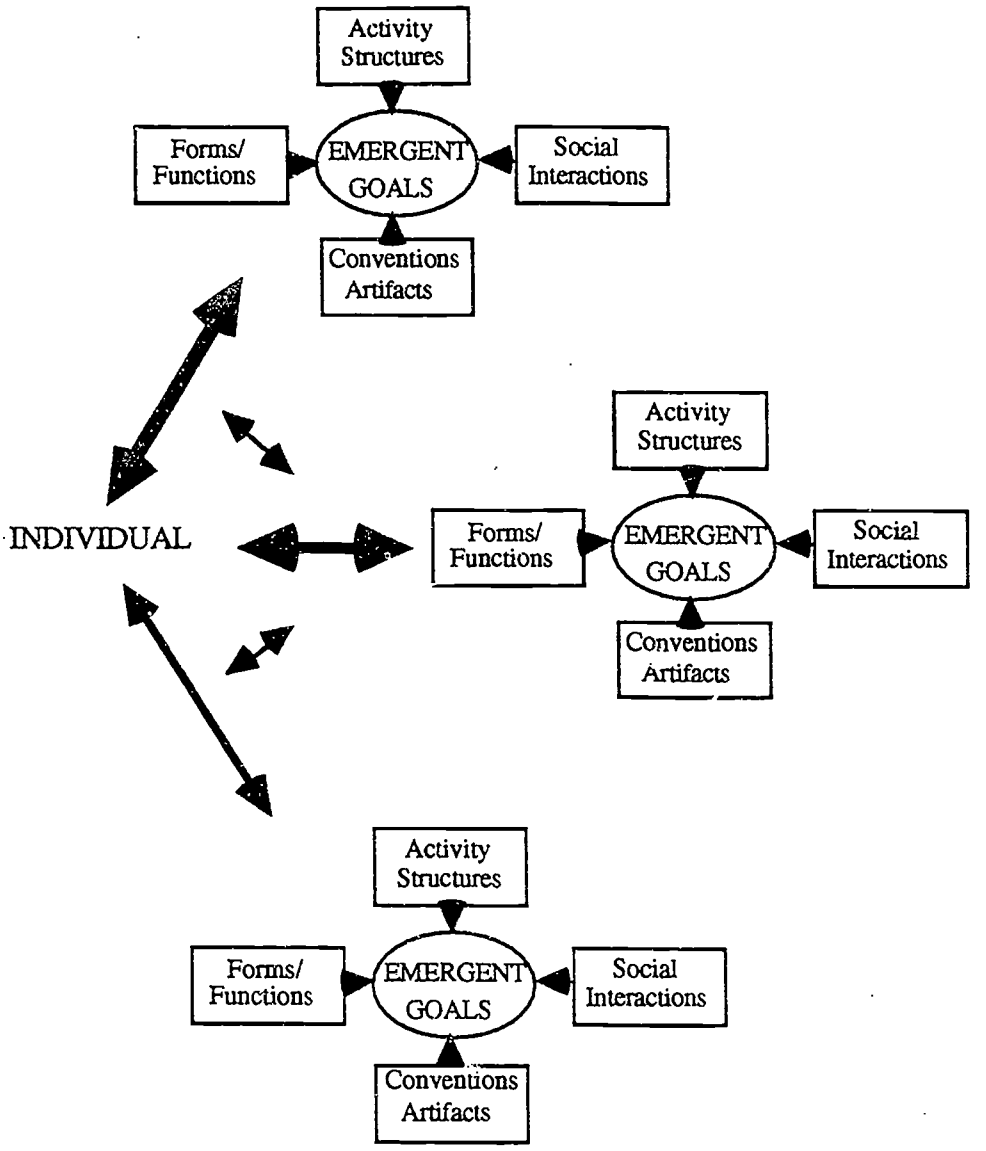

Figure 3

*(Saxe, 1991, p. 185)