DOCUMENT RESUME

ED 372 948 SE 054 434

AUTHOR Kamii, Constance
TITLE Equivalent Fractions: An Explanation of Their Difficulty and Educational Implications.
PUB DATE 12 Apr 94
NOTE 15p.; Paper presented at a National Council for Teachers of Mathematics Research Pre-session (Indianapolis, IN, April 12, 1994).
PUB TYPE Speeches/Conference Papers (150) -- Reports - Research/Technical (143)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS Arithmetic; *Cognitive Development; Elementary School Mathematics; *Fractions; Grade 4; Intermediate Grades; Learning Theories; *Mathematics Instruction; *Piagetian Theory
IDENTIFIERS *Partitioning Procedures

ABSTRACT

Equivalent fractions are usually introduced in fourth grade and reviewed repeatedly in the subsequent grades as the four arithmetical operations are taught. In spite of this repeated instruction, the results are disappointing. This paper reviews some data from previous research documenting the difficulty of equivalent fractions, explains this difficulty in light of Piaget's theory, and suggests ways in which instruction might be improved. The discussion is limited to fractions of wholes and does not include fractions of sets of objects. Contains 24 references. (Author)

Equivalent Fractions:

An Explanation of Their Difficulty and Educational Implications*

Constance Kamii

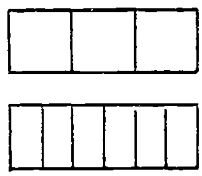
University of Alabama at Birmingham

Equivalent fractions are usually introduced in fourth grade and reviewed repeatedly in the subsequent grades as the four arithmetical operations are taught. In spite of this repeated instruction, the results are disappointing. This paper reviews some data from previous research documenting the difficulty of equivalent fractions, explains this difficulty in light of Piaget's theory, and suggests ways in which instruction might be improved. The discussion will be limited to fractions of wholes and will not include fractions of sets of objects.

Findings from Previous Research

Behr, Lesh, Post, and Silver's Study

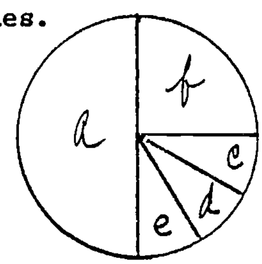
Behr, Lesh, Post, and Silver (1983) cited data collected by Bezuk, who asked 77 fourth graders to show 2/3 of shapes such as the following two:



These subjects in a suburban school "had undergone normal fourth-grade instruction dealing with fractions" (p. 111). While none of them gave an incorrect answer in response to the first rectangle, 25% responded incorrectly to the second one. These and other similar data show that 2/6 is not the same thing as 1/3 for many fourth graders.

Behr, Lesh, Post, and Silver went on to give the following excerpt from an interview with a fourth grader. This account is instructive from the standpoint of understanding the nature of children's difficulty.

> Problems arose when students were asked to give more than one name to either b or cde. Portions of an interview sequence with a fourth-grade student indicate the general nature of these difficulties.



> I: b is what fraction of the whole?
> S: One-fourth . . . .
> I: cde together is what fraction of the whole?
> S. One-fourth . . . .
> I: Is there another way you can tell me what fraction this [b] is of the whole?
> S: I don't think so.

*Paper presented at the NCTM Research Presession jointly sponsored with the AERA SIG/RME, Indianapolis, April 12, 1994.

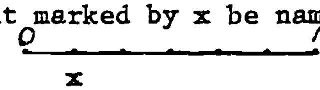
S: [As I(nterviewer) points to c, d, and e in turn] One-twelfth, one-twelfth, one-twelfth.
I: So what fraction is this [cde] altogether?
S: One-fourth.
I: Now count with me.
S: One-twelfth, two-twelfths, three-twelfths.
I: So what fraction is this of the whole circle?
S One-twelfth . . . oh! Hold it . . . one-fourth.
I: Now count with me again [pointing in turn to c, d, and e].
S: One-twelfth, two-twelfth, three-twelfths.
I: Now how can I say another name besides one-fourth for all of this [e, d, and c]?
S: One-fourth, two-fourths, three-fourths [counting while pointing to e, d, and c].
I: Let's see, what was this [e] again?
S: One-twelfth, two-twelfth, three-twelfths [while I(nterviewer) points to e, d, and c].
I: How can I describe the whole thing?
S: Three-twelfths . . . because there are three twelfths, so we can call it three-twelfths.
I: Is there another name for this [b]?
S: No. (p. 114-115)

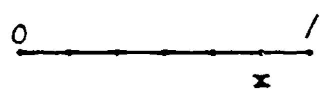
Behr et al.'s purpose in designing this interview was "to assess children's flexibility in regarding a part of a whole as an unpartitioned region and as a partitioned region" (p. 114). More specifically, they stated, "of interest was whether the child could ignore the partition lines in cde to consider it as one-fourth and imagine partition lines placed in b to consider it as three-twelfths" (p. 114). As I will explain shortly, I do not think children's difficulty is due merely to the inflexibility of their thought or to an inability to ignore or imagine partition lines.

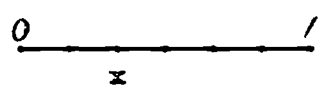
Larson's Study

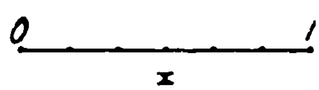
To study children's understanding of equivalent fractions, as well as other aspects of fractions, Larson (1980) gave a multiple-choice group test to 382 seventh graders. Below are two examples of the kinds of questions she asked:

14. On which number line can the point marked by x be named by the fraction $\frac{1}{3}$?

A) 

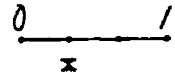
B) 

C) 

D) 

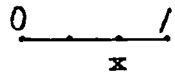
E) None of the above

16. On which number line can the point marked by x be named by the fraction $\frac{1}{3}$?

A) 

B) 

C) 

D) 

E) None of the above

By comparing the frequency of correct answers to such questions, Larson concluded that, for her subjects, finding simple fractions such as 1/3 was significantly easier ($p < .001$) when the number of segments was the same as the denominator than when it was twice the denominator. For these seventh graders, 2/6 was clearly not the same thing as 1/3. (Research has repeatedly shown that fractions of number lines are much more difficult than fractions of areas. However, this conclusion may be due to the fact that children were asked about a "point marked by x" rather than an interval (length) from zero to x.)

Larson's interpretation of this difficulty was also that these seventh graders' concept of fractions was not flexible. She also stated that these children did not know "as part of their fraction concept that a fraction represents a number that has many names and that each of these names can be associated with the same point on the number line regardless of the number of segments in each unit" (p. 427). I will argue shortly that I do not think equivalent fractions are <u>different names</u> for the same number.

<u>National Assessment of Educational Progress (NAEP)</u>

Recent National Assessments do not give information concerning equivalent fractions as such. However, the data available on addition and subtraction with easy unlike denominators (such as 1/2 and 1/3) shed light on the difficulty of equivalent fractions. The percentages of 13-year-olds who gave correct answers to simple computational problems in four national assessments are given in Table 1 (Carpenter et al., 1976; Carpenter et al., 1980; Lindquist, 1983; Kouba et al., 1988). It can be seen in this table that only about a third of the national samples, even at age 13 or seventh grade, have been successful since the 1970s. Something is clearly wrong with the way equivalent fractions and/or common denominators have been taught.

- - - - - - - - - -
Table 1 about here
- - - - - - - - - -

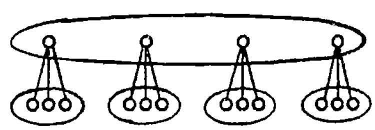
<u>Explanation of Children's Difficulty</u>

As stated earlier, researchers have generally viewed knowledge of equivalent fractions as the ability to call the same number by different names, the ability to ignore or imagine partition lines, and/or the manifestation of flexible thought. Piaget (1977), however, made a distinction between the figurative aspect of knowledge (based on shapes, which are observable) and the operative aspect (based on relationships, which are not observable). For example, half of a rectangle can be either rectangular or triangular. While the triangular half may look bigger than the rectangular half from a figurative point of view, our operative knowledge enables us to deduce that the two halves have the same area.

Equivalent fractions involve two related aspects of operative thinking identified by Piaget: (a) Multiplicative thinking (Piaget, 1983/1987) and (b) the conservation of the whole and of the parts (Piaget, Inhelder, & Szeminska, 1948/1960; Parrat-Dayan & Vonèche, 1992).

Multiplicative thinking is characterized by the hierarchical, simultaneous structure illustrated below. It can be seen in this illustration that repeated addition, such as the repeated addition of 3, is successive and involves thinking on only one level (3 + 3, followed by 6 + 3, followed by 9 + 3). By contrast, multiplicative thinking, such as 4 x 3, involves thinking on two hierarchical levels, simultaneously.

4 x 3



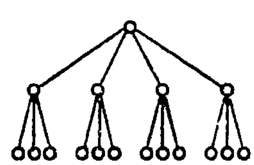
3 + 3 + 3 + 3

The difference between additive and multiplicative thinking was empirically documented recently by Steffe (1992) and Clark and Kamii (1994). Equivalent fractions, too, involve hierarchical, simultaneous thinking as shown by Olive (1993). The structure of this hierarchical thinking, illustrated below, explains why it was so hard to get the fourth grader in Behr et al. (1983) to say "three-twelfths" when he or she was thinking about one-fourth.

1

1/4

3/12



To think about one as three-twelfths and as one-fourth, children have to conserve the whole and the parts (4/4). Those who cannot conserve the whole and the parts think about one-fourth only as "one piece." Since the whole and the other parts (3/4) disappear from their minds, what is three-twelfths for us can only be three-thirds for these children. Likewise, in the task requiring the comparison of a rectangular half with a triangular half, the children who cannot conserve the whole and the parts compare only a rectangle with a triangle.

A Study of the Operative Aspect of Equivalent Fractions

The following study was conducted to find out whether or not fifth and sixth graders can reason operatively when asked about simple fractions such as halves, fourths, and eighths that could not be judged figuratively. The study had two parts inspired by Parrat-Dayan's (1980) research and involved 120 children in two suburban schools near Birmingham, Alabama in January through March. All the subjects were individually interviewed and videotaped and consisted of 61 fifth graders and 59 sixth graders. The fifth graders had been taught equivalent fractions in fourth grade and were interviewed before fractions were covered in fifth grade. The sixth graders, on the other hand, had just finished studying equivalent fractions and were in the midst of working on problems such as 3 1/3 - 1 1/5.

Children's thinking about two halves. The first part of the study was designed to find out whether or not the children thought that a half made by cutting a rectangle vertically (a in the figure below) had the same area as a half made by cutting the same rectangle diagonally (c in the figure below).

Materials

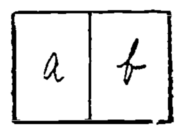
Two rectangles (4.25 x 5.5 inches) made by cutting a standard sheet of paper (8.5 x 11 inches) into four parts, a ruler, a pencil, and scissors.

Procedure

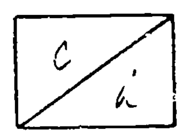
Each interview consisted of the following three parts:

1. Making sure the child believed that the two rectangles were the same size

2. Asking whether or not two halves were "the same amount to eat"



   a. The interviewer folded and cut one of the rectangles as shown asking the child, "Am I cutting this piece into two halves?" (The children always replied, "Yes.")



   b. The interviewer then drew a diagonal line on the other rectangle and asked the child as she cut on the line, "Am I cutting this piece into two halves?" The children always answered "Yes," and the interviewer showed the congruence of the two triangles saying, "Yes, these are the same."

   c. The interviewer then asked, "If these were chocolate instead of paper, and I gave you this piece (giving a to the child) and gave myself this piece (c), but not these (pushing b and d away), would you and I have the same amount of chocolate to eat?" Whatever the child said, an explanation was always obtained.

3. Making a counter-suggestion if the child said that a and c were not the

same amount
The interviewer said, "Another boy (or girl) in another school said that this (a) and this (c) were exactly the same amount. His/her argument was that these (a and b) are both halves, and these (c and d) are both halves. So he/she said that these (a and c) are the same amount because they are both halves. Do you think he/she was right, or do you think you are right?" A justification of the answer was always obtained.

Results

As can be seen in Table 2, only 44% of the fifth graders said that a and c were the same amount to eat (the "+" category), and 38% said that the triangular half was more (the "-" category). The group in between (±) said that they were not sure and/or that a and c were "about the same but not exactly." These children were in conflict because their operative knowledge led them to think that a and c were the same amount, but their figurative evaluation led them to think that the triangular half was more. The counter-suggestion was made only to the children who clearly said that c was more to eat. Twenty-three percent of the fifth graders rejected the counter-suggestion and stated that a and c were both halves but that the amount depended on how the paper was cut.

Table 2 about here

The proportion of the correct answer increased in sixth grade, with 51% saying that a and c were the same amount to eat because they were both halves of two wholes which were the same. Forty-four percent initially gave an incorrect answer, but most of them changed their minds upon hearing the counter-suggestion that a = c. However, 17% persisted in thinking that the triangular half was more.

The preceding findings show that even half is a relationship, which is not figurative knowledge. If fifth and sixth graders have such difficulty with half, the easiest of all fractions, we can surely expect equivalent fractions to be hard for them.

Children's thinking about three-fourths and six-eighths. The second part of the study was designed to find out about children's multiplicative thinking and their conservation of the whole and the parts when they could not use the figurative aspect of their knowledge.

Materials

Two rectangles (8.5 x 5.5 inches) made by cutting a standard sheet of paper (8.5 x 11 inches) in two and scissors

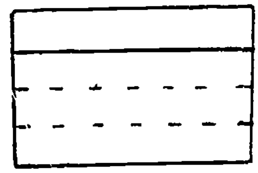
Procedure

Each interview consisted of following three parts:

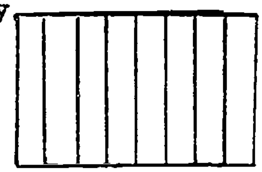
1. Making sure that the child believed the two rectangles were the same size.

2. Asking how many eighths were necessary to make "the same amount to eat" as 3/4.

   a. The interviewer folded one of the rectangles horizontally twice and asked, "How many parts did I fold my rectangle into?" (The children always replied, "Four.") She then cut off 1/4 saying, "I am cutting off one of the parts."



   b. The interviewer then folded the other rectangle vertically twice and asked, "Do I have four parts now?" (All the children answered "Yes.") She then folded the paper once more asking, "How many parts do I now have?" (The great majority replied, "Six!") The paper was then unfolded. and the child was asked to count the eight parts. The interviewer cut the rectangle on all the folds and ascertained that the child knew there were eight strips.



   c. Offering the eight strips to the child, the interviewer asked, "If these were chocolate instead of paper, and I gave myself this piece (3/4) but not this piece (pushing 1/4 away), how many of your strips would you use to give yourself exactly the same amount to eat?" An explanation of the answer was always obtained.

3. Making a counter-suggestion <u>if</u> the child gave an incorrect answer. The interviewer said, "Another boy (or girl) in another school said that he/she would use six strips because we divided this rectangle into four parts (reassembling the horizontally cut rectangle) and the other rectangle into eight parts (pointing to the eight strips). That's why he/she thought two strips (2/8) would make exactly the same amount as one like this (1/4)." The two-to-one correspondence (of "one, two, three" and "two, four, six") was then demonstrated, and the child was asked to evaluate the counter-suggestion and to explain his/her reason.

<u>Results</u>

The findings are presented in Table 3. The first point to be noted is that, in spite of the fact that the sixth graders were learning to add and subtract fractions with unlike denominators, only 32% got the correct answer with the correct explanation (the "+" category). Many (the 46% in the "−" category) approached the task spatially and figuratively, trying to fit the strips onto the 3/4 piece. Others (22%) were categorized in between (±) either because they answered "<u>about</u> six strips" or because they gave the correct answer at one point but changed their minds upon being asked, "How do you know that two of these strips (2/8) would make the same amount as one like this (1/4)?" These children could sometimes reason correctly, but their reasoning was not solid enough to overcome their perceptual judgment. Twenty-four percent of the sixth graders even rejected the counter-suggestion that 3/4 = 6/8.

- - - - - - - - - -

Table 3 about here

- - - - - - - - - -

While the sixth graders did not do well, they did much better than the fifth graders. It can be seen in Table 3 that only 13% of the fifth graders reasoned correctly on their own that 3/4 = 6/8. Forty-nine percent rejected outright the counter-suggestion that 3/4 = 6/8.

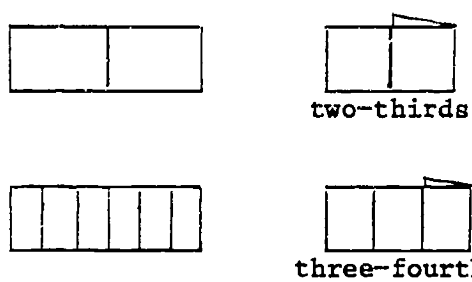
The percentage of 32 in sixth grade who solidly demonstrated their knowledge of equivalent fractions is similar to those in the National Assessments (Table 1) about 13-year-olds or seventh graders. The Alabama sixth graders were a year younger than the National Assessment samples but members of a middle-class, advantaged group attending a suburban school. These percentages lead us to conclude again that something is wrong with the instruction children are receiving in equivalent fractions in grades 4, 5, 6, and 7.

Educational Implications

It can be said in light of findings from research and Piaget's theory that traditional instruction in equivalent fractions has at least the following three shortcomings:

1. It teaches equivalent fractions perceptually and figuratively with pictures and manipulatives (as well as spoken words, written symbols, and algorithms). Even the NCTM Standards (NCTM, 1989) gives advice such as the following:

> Children need to use physical materials to explore equivalent fractions and compare fractions. For example, with folded paper strips, children can easily see that 1/2 is the same amount as 3/6 and that 2/3 is smaller than 3/4.



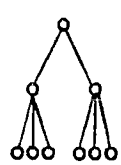
(p. 58)

Showing that 1/2 = 3/6 in this way does not foster the development of the hierarchical thinking (the operative aspect) illustrated below:

1

1/2

3/6



Likewise, perceptually showing that 3/4 > 2/3 leaves children's reasoning (the operative aspect) untouched. If children are not encouraged to make such perceptual comparisons, they are more likely to reason that 9/12 > 8/12 or that since 1/3 (the complement of 2/3) > 1/4 (the complement of 3/4), 3/4 > 2/3.

2. By telling children that certain fractions are equivalent, traditional instruction deprives children of the possibility of thinking and inventing equivalent fractions.

3. Traditional instruction teaches proper fractions first and then improper fractions and mixed numbers. All of these should be involved from the beginning so that children will think about parts and wholes at the same time.

The most promising approaches I have found so far are those of Streefland (1991, 1993) and Mack (1990). Streefland (1993) gives realistic problems such as "Divide 3 pizzas among 4 children" (p. 291) that encourage operative thinking in the following ways:

1. "Teaching" starts with realistic problems and encourages children to invent their own solutions so that fractions can grow out of children's own thinking. Encouraging children to logico-mathematize their own reality is much better than presenting a chapter titled "Fractions" with pictures of circles, squares, and rectangles that have already been partitioned.

2. Ready-made pictures or manipulatives are not given, and children have to put their own thinking on paper. If children think about how to represent parts of a number of pizzas, this thinking will further their reasoning much more than ready-made pictures and fraction circles. Children may draw circles that look like those found in today's textbooks, but the figurative knowledge they put on paper represents their own work and understandings as opposed to the circles presented in textbooks, which represent someone else's thinking.

3. Equivalent fractions can be invented from the very beginning in relation to whole numbers. This is in contrast with traditional instruction that waits for a long time to present mixed numbers and addition with unlike denominators. Streefland's approach, however, involves halves and quarters, which are easy for children to invent.

The following story problem suggested by Streefland (1993) is another example that fosters reasoning:

> A family, consisting of father, mother, Peter, and Ann, have pizzas for lunch. The first one is shared fairly. In the meantime, the second one is prepared in the oven. Mother divides this one in four equal parts, too. Then she says: "Oh, how silly of me, I've had enough. You three can share this one." "No," said Ann, "one of these pieces is enough for me," and, turning to Peter and her father, she added, "you two can share the rest." Peter and his father did not have to be told a second time. Divide the pieces. How much do each of the family members get? (p. 294)

Mack, too, advocates encouraging children to invent their own procedures in addition and subtraction, and to invent equivalent fractions in the process. Working with sixth graders from middle to upper-middle income families who were having difficulty with fractions, she posed questions such as "Which one is more, 2 2/3 or 2 5/6?" An especially important point made in Mack's article is that prior "knowledge of rote procedures frequently interfered with students' attempts to build on their informal knowledge" (p. 16).

As far as fractions of lines are concerned, it seems best to "teach" them in the context of measurement. As stated elsewhere (Kamii, 1989, 1994), an essential principle of teaching drawn from Piaget's constructivism is the importance of the exchange of points of view among children. In measurement as well as in all the other activities, children should be encouraged to agree or disagree among themselves and to defend their ideas. Piaget (1932/1965b, 1965a, 1976) argued that social interactions are essential for children as well as scientists to decenter and construct higher-level thinking.

The ineffectiveness of traditional instruction can further be seen in Table 4. This table shows a comparison of fourth graders in May, 1993, who had and had not been taught equivalent fractions in two other suburban schools near Birmingham, Alabama. It can be seen that the fourth graders who had received this traditional instruction with a textbook did slightly better on the aforementioned tasks in the short run. Fifty percent said that the rectangular and triangular halves were the same amount as compared to 43% of those who had not received instruction. Twenty-seven percent figured out that 3/4 = 6/8 as compared to 11% of those who had not been instructed. Eight months later, however, albeit in another school, the fifth graders in Table 3 looked like the fourth graders who were never taught these topics. Only 13% of the fifth graders in Table 3 said that 3/4 = 6/8, and this percentage is very close to the 11% of the fourth graders who were never taught equivalent fractions.

- - - - - - - - - - -

Table 4 about here

- - - - - - - - - - -

Further research is necessary to determine <u>when</u> to teach equivalent fractions and <u>how</u>. It is necessary to know how well seventh graders do on the tasks described above. We must also know which equivalent fractions are easy and which ones are not. For example, 1/4 = 2/8 is much easier than 3/4 = 6/8. Likewise, 1/2 + 1/4 seems easier than 1/4 + 1/8. We plan to experiment in fourth and fifth grade with an approach that emphasizes the operative aspect of fractions.

References

Behr, M. J., Lesh, R., Post, T. R., & Silver, E. A. (1983). Rational number concepts. In Lesh, R., & Landau, M. (Eds.), Acquisition of mathematics concepts. New York: Academis Press.

Carpenter, T. P., Coburn, T. G., Reys, R. E., & Wilson, J. W. (1976). Notes from National Assessment: Addition and multiplication with fractions. Arithmetic Teacher, 23(2),137-141.

Carpenter, T. P., Kepner, H., Corbitt, M. K. Lindquist, M. M., & Reys, R. E. (1980). Results and implications of the second NAEP mathematics assessments: Elementary school. Arithmetic Teacher, 17(8),10-12.

Clark, F. B., & Kamii, C. (1994). Identification of multiplicative thinking in children in grades 1-5. Manuscript submitted for publication.

Kamii, C. (1989). Young children continue to reinvent arithmetic, 2nd grade. New York: Teachers College Press.

Kamii, C. (1994). Young children continue to reinvent arithmetic, 3rd grade. New York: Teachers College Press.

Kouba, V., Carpenter, T. P., & Swafford, J. O. (1989). In Lindquist, M. M. (Ed.), Results from the fourth mathematics assessment of the National Assessment of Educational Progress. Reston, VA: National Council of Teachers of Mathematics.

Larson, C. N. (1980). Locating proper fractions on number lines: Effect of length and equivalence. School Science and Mathematics, 53(5): 423-428.

Lindquist, M. M., Carpenter, T. P., Wilver, E. A., & Matthews, W. (1983). Third national mathematics assessment: Results and implications for elementary and middle schools. Arithmetic Teacher, 31(4),14-19.

Mack, N. K. (1990). Learning fractions with understanding: Building on informal knowledge. Journal for Research in Mathematics Education, 21(1),16-32.

National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: Author.

Olive, J. (1993, March). Children's construction of fractional schemes in computer microworlds, Part II: Constructing multiplicative operations with fractions. Paper presented at the Research Presession of the National Council of Teachers of Mathematics, Seattle, WA (jointly sponsored with AERA SIG, RME).

Parrat-Dayan, S. (1980). Etude génétique de la notion de moitié. Geneva (Switzerland): J. L. de Rougemont.

Parrat-Dayan, S., & Vonèche, J. (1992). Conservation and the notion of "half." In Bideaud, J., Meljac, C., & Fischer, J.-P. (Eds.), Pathways to number. Hillsdale, NJ: Erlbaum.

Piaget, J. (1965a). Etudes sociologiques. Geneva, Switzerland: Librairie Droz.

Piaget, J. (1965b). The moral judgment of the child. New York: Free Press. (Original work published 1932)

Piaget, J. (1976). Ecrits sociologiques. Revue Européenne des Sciences Sociales. 14, 44-123.

Piaget, J. (1977). The role of action in the development of thinking. In Overton, W. F., & Gallagher, J. Mc. (Eds.), Knowledge and development, Vol. 1, Advances in research and theory. New York: Plenum.

Piaget, J. (1987). Possibility and necessity. Minneapolis: University of Minnesota Press. (Original work published 1983)

Piaget, J., Inhelder, B., & Szeminska, A. (1960). The child's conception of geometry. New York: Basic Books. (Original work published 1948)

Post, T. R. (1981). Fractions: Results and implications from National Assessment. Arithmetic Teacher, 28(9), 26-31.

Steffe, L. P. (1992). Schemes of action and operation involving composite units. Learning and Individual Differences, 4(3), 259-309.

Streefland, L. (1991). Fractions in realistic mathematics education. Dordrecht, Netherlands: Kluwer Academic Publishers.

Streefland, L. (1993). Fractions: A realistic approach. In T. P. Carpenter, E. Fennema, and T. A. Romberg (Eds.), Rational numbers. Hillsdale, NJ: Erlbaum.

Table 1

Percentages of 13-Year-Olds Giving Correct Answers in Four National Assessments of Educational Progress (NAEP)

	First NAEP (1972-73)	Second NAEP	Third NAEP	Fourth NAEP (7th graders)
1/2 + 1/3	42*	33		
3 1/3 - 3 1/4		35	36	
3 1/2 - 3 1/3				33**

*30% gave the answer of 2/5.
**Since 53% of the 62% who responded chose the correct answer, the actual percentage giving the correct answer was 33.

Table 2

Percentages of Fifth and Sixth Graders Responding to the Question about the Equality of the Rectangular and Triangular Half

	5th graders n = 61	6th graders n = 59
Initial judgment		
+	44	51
±	18	5
-	38	44
Judgment after the counter-suggestion		
-	23	17

Table 3

Percentages of Fifth and Sixth Graders Responding to the Question 3/4 = ?/8

	5th graders n = 61	6th graders n = 59
Initial solution		
+	13	32
±	10	22
-	77	46
Judgment after the counter-suggestion		
-	49	24

Table 4

Percentages of Fourth Graders Who Had and Had Not Been Taught Equivalent Fractions

	Had been taught n = 22	Had not been taught n = 37
Rectangular half = triangular half		
Success in initial judgment	50	43
3/4 = 6/8		
Success in initial response	27	11