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ABSTRACT

Traditional elementary statistics instruction for non-majors has focused on computation. Rarely have students had an opportunity to interact with real data sets or to use questioning to drive data analysis, common activities among professional statisticians. Inclusion of data gathering and analysis into whole class and small group activities acknowledges that students learn in different ways. This paper discusses seven data gathering problems that have been successful in facilitating student understanding of elementary statistical concepts common to introductory courses. The problems are: (1) "Breaking the barriers to success with surveys"; (2) "Which exam should Prof. Dee Viation scale?"; (3) "Probability distributions from Mars - M&M, that is"; (4) "Making cents of the Central Limit Theorem"; (5) "Descriptive statistical summaries - what about the data?"; (6) "Body correlation"; and (7) "Telephones: a necessary expense, but is the expense necessary?" (MKR)

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Students Learn Statistics When They Assume a Statistician's Role[1]

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For many people, learning mathematics is a painful task. When asked about previous instances in mathematics that affected them, individuals reveal traumatic incidents that highlight their struggle to learn a difficult subject.

The proliferation of information necessitates that it be condensed into a form that individuals can peruse, understand, and assimilate quickly. A statistical presentation, which includes charts, graphs, and summary statistics, is often the preferred mode in business publications, research reports, news weeklies, and daily newspapers. Recognizing the need for statistical literacy, undergraduate faculty across disciplines are encouraging their students to study statistics. These students are often the same ones who experienced mathematical trauma in earlier mathematics classes. Discovering how to best teach these students so that they gain the statistical understanding necessary for literacy is no small task.

Statistics is an active discipline. Statisticians gather data; they study it, to discover obvious patterns and anomalies; they seek solutions to data problems, such as missing data; and only when they are satisfied that the data is the best available, they analyze it, using tools that have developed over time. Statisticians get a 'feel' for the data before they 'crunch numbers'; estimation skills for data, regardless of its presentation, are essential to that process.

1 Presentation at the Nineteenth Annual Conference of the American Mathematical Association of Two-Year Colleges, Boston, MA, November, 1993.

Traditional elementary statistics instruction for non-majors has focused on computational tools, probability, selected distributions, and hypothesis testing. Rarely have students had an opportunity to interact with real data sets. Rarely have they approached their study of statistics from a contextual perspective that was either familiar or related to their academic interests. Consequently, many students viewed statistics as a dull, boring course, something to be endured, and it became a dreaded course among their degree requirements. Some students reported that they memorized rules and formulas but had no idea why the rules worked or when the formulas were appropriate. Their understanding of statistical concepts was minimal. The availability and use of technology simplified some of the computational difficulties, but it did not appear to increase student conceptual understanding of the discipline (Gilligan, 1990; Stephenson, 1990; Ware & Chastain, 1989).

The current reform movement proposes that teachers aim for mathematical understanding in their instruction; that teachers conduct their classes so that students develop conceptual knowledge. Statistics education literature recommends alternatives to traditional lectures in the introductory course. For statistics courses that serve non-majors on the undergraduate level, reform leaders urge less lecture and more active learning experiences (Moore, 1993; Snee, 1993). Currently there are six National Science Foundation (NSF) funded projects which focus on improving the introductory college level course (Amstat News, June 1993, No. 199). One NSF project is developing a curriculum based upon activities.[2] Within the materials, there is an emphasis on student as

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2 Schaeffer, R. An Activity-Based Introductory Statistics Course. Ongoing NSF sponsored project. For information, contact Prof. Richard Schaeffer, Department of Statistics, University of Florida, Gainesville, FL 32611 email: schaeffe@stat.ufl.edu

active constructor of his/her understanding with guiding support from the teacher, which is very different from student as passive recipient of teacher transmitted knowledge that is common in traditional lecture courses.

Constructivist approaches, of which concrete activities that can be completed within small groups are one example, acknowledge that students build their understanding of the subject. When planned and presented so as to include connections to the theory, activities can enhance statistical understanding and provide a context from which students can think about the concepts.

In light of research and their personal experiences, some faculty question whether use of a statistical package enhances conceptual understanding in statistics. Consequently, the data gathering activities and analyses presented here do not require computer analysis. However, once the data has been collected, recorded, discussed, and the appropriate analysis decided, the computer can be a useful tool to minimize computation and routine processing. Advocates of the technology can rightly argue that the computer can be useful to uncover hidden data patterns.

Using questions to drive data analysis, common among professional statisticians, is not common in the traditional elementary statistics course for non-majors. Adding that component to the course assists students to learn and use the traditional tools of descriptive and inferential statistics when the need for them emerges naturally in the process of answering questions. Students become familiar with data variable type and assumptions for the analysis, areas that beginning students often fail to integrate, as they consider data. Statistical questions based on personal experiences offer opportunities for individual and small group projects. They may be motivated to learn statistical software to facilitate data handling and analysis.

The data analysis, once completed, forms the basis for making connections. Rarely is any activity used in isolation. Constantly during an activity, questions arise that take the student back to earlier ideas and that set the stage for concepts still to be presented: What do you think the shape of the distribution is? What will be the best measures of center and spread? What patterns do you observe? Should the data be partitioned? Do you think the sample is representative of the larger population? When questions like this occur throughout the semester, students are able to connect topics The activities presented here begin with student data collection. Much of the data can be collected within the classroom, in a reasonable amount of time. Although each activity is geared toward a particular aspect of the elementary course, the data may be reused in a variety of areas. Students prefer to work with data they have collected; it is more real to them than exercises in the textbook.

Inclusion of activities that incorporate data gathering and analysis into whole class and small group discussion also acknowledges that students learn in different ways. Recognition that faculty need to attend to the learning style of students is increasing within the statistics education community (Snee, 1993). There are many different ways to present statistics in addition to the traditional lecture model, which does not support many learning style types. Data gathering activities are one approach; video and project orientations provide others, but they are topics for another time.

Many student who populate two year schools prefer active, concrete learning experiences. Activities that require students to collect and use their own data facilitate their conceptual understanding. The process of working in small collaborative groups and participating in full-class, consensus-building discussions empowers students to believe they can do mathematics. Spoken and written communications, aside from calculations, support their mathematical and statistical thinking. Using situations that are familiar facilitates their grasp of new ideas because they already have a mental framework to which they can attach new material. When students have to create a framework, the process of understanding can take longer, require more mental energy, and not always happen before the instructor moves on to another topic.

This presentation includes seven illustrations of data gathering activities that have proven successful in facilitating student understanding of elementary statistics concepts common to introductory courses. Their titles are presented here; brief descriptions follow.

1. Breaking the barriers to success with surveys.
2. Which exam should Prof. Dee Viation scale?
3. Probability distributions from Mars--M & M, that is.
4. Making cents of the Central Limit Theorem.
5. Descriptive statistical summaries--what about the data?
6. Body correlation.
7. Telephones: a necessary expense, but is the expense necessary?

1. Breaking the barriers to success with surveys.[3]

Create a list of questions that elicit information about students in the class. Avoid including questions that may cause students to feel uncomfortable; for example, in a class that contains non-traditional students, a question regarding

3 A similar activity can be found in the Activity-Based Statistics Project materials.

age. Include questions which call for responses at different levels of measurement. A question about gender illustrates nominal level; questions that ask students to rate their level of anxiety in mathematics or level of satisfaction with courses offered at the institution on a scale of 1-5 are instances of ordinal level data; a question that asks students to find their pulse rate for one minute illustrates interval level; and the cost of their last haircut is at the ratio level.

Through discussion, classify the variables according to their level of measurement. Choose one item whose variable is interval or ratio level of measurement and, using student responses, introduce visual data representation techniques, such as stem and leaf plot, dotplot, boxplot, or histogram. After measures of center and spread have been introduced, use this data again and ask students to choose the best measure of center and spread for the variables in each question, and explain why. Students often disagree during this process. Rather than be their authority, encourage them to convince each other, and in doing so, contribute to their feeling empowered to do statistics. Pick one variable, and ask students to provide a descriptive analysis. If technology is available, create a class profile or a description of an 'average' student.

2. Which exam should Prof. Dee Viation scale?

For statistical understanding it is necessary but not sufficient that students can compute summary statistics and create visual representations of data. Students should also be provided with opportunities to make decisions concerning data and be supported to justify their decision. As a result of sharing student decisions and justifications, they appreciate the subjective nature of the statistician's role.

Below are the scores for 22 students in three exams. The teacher has promised to scale ONE of the exams. Analyze the data to decide which one should it be, and justify your choice. Explanations usually include a discussion of shape, center, and spread.

Exam 1	Exam 2	Exam 3
27	30	25
33	51	28
45	55	30
57	56	34
62	72	35
62	72	36
62	73	36
64	74	37
65	75	71
66	75	72
67	85	74
72	85	89
74	85	89
75	93	89
75	93	89
75	94	89
75	94	89
78	95	92
83	97	92
85	100	92
96	100	92
96	100	98

Unlike some of the other activities that require in class collection of data, students are given the data and asked to analyze it, usually out of class, in order to answer the question. They must present reasons for their choice. Grading, if done, is on the basis of thoroughness and correctness of the descriptive analysis, degree to which students use exploratory data analysis and/or visual representations, and the justification of their decision.

Usually, class members do not agree on the choice of exam to be scaled. Often, in the justifications for their choice, they reveal personal beliefs about scaling, which sometimes overrule the analysis they have just performed. The results of the students' decisions are presented in class, usually with the dilemma faced by Prof. Dee Viation that there is no clear decision about which exam to scale. A lively debate follows in which class members struggle to take a side and argue it until consensus is reached. On behalf of Prof. Dee Viation, I thank the students for their assistance in resolving the dilemma, and facilitate a discussion on the issue of bias and subjective interpretations that exist in statistics.

3. Probability distributions from Mars--M & M, that is.

Students find any venture into probability beyond simple dice, card, and coin illustrations very confusing. For that reason, many reformers in statistics education argue that the introductory course should omit probability altogether or scale down its importance (Moore, 1993; Schaeffer, 1992; Snee, 1993), but few textbooks have taken that approach. Regardless of depth of presentation in probability, students rarely connect the concepts of frequency distribution and simple probability on their own, yet without such connections, probability distributions become another 'new topic' to learn. A successful activity that connects these ideas uses M & Ms. The activity allows a review of variability, and lays a preliminary foundation for a treatment of chi-square goodness of fit later in the course.

Students should organize the contents of a small size bag of M & Ms by color. They neea to be prepared to report the frequency of each color and total piece count, which the instructor records on an overhead. Ask students whether they think the Mars candy company has a method for putting M & Ms into packages.

Ask a student about the probability of red, P(R), for his/her sample. Ask that same student what s/he he would expect for the P(R) in the next bag, if the only information available is his/her sample. Students realize that empirical probability, based on the samples, can vary considerably. (This understanding is useful later when the instructor discusses drawing inferences from samples.) Ask students what data patterns they observe; for example, the number of identical samples, the colors that appear with greatest/least frequency. Ask students to compute the probability for each color in their sample. Rounding to a single digit is sufficient accuracy. Ask them to note the sum of the individual probabilities.

Brainstorm means by which they could discover the theoretical distribution, which I tell them does exist. They sum the sample frequencies for each color and compute the total number of pieces; then they use these values to calculate the class theoretical probability distribution. Information about the actual theoretical distribution from two sources[4] often surprises them.

Ask students to justify whether their sample is a good illustration of the theoretical distribution. Tell them that means to statistically determine goodness of fit will be presented.

Shift gears, at this point, to define a discrete random variable, for example, total number of M & Ms in the sample package. This shift away from color to a discrete random variable prepares students for the binomial probability distribution, which tends to be a difficult concept for them. Create a chart of the probability distribution for the random variable, and verify that $\Sigma P(x) = 1$.

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4 David Moore and George McCabe, Introduction to the practice of statistics, p. 317. Rebecca Corwin and Susan Friel, Used Numbers Series: Statistics: Prediction and sampling, p. 40.

Discuss the mean of the distribution, and demonstrate the connection to the mean of the frequency distribution for grouped data. When students see the connection that exists for the mean of the probability distribution and the mean of the frequency distribution, they realize they are not learning another new topic. They tend to have less difficulty with the standard deviation for a probability distribution after this activity.

Sketch the frequency histogram and the probability histogram for the data, and connect the probability to a measure of area in the bars of the histogram. This discussion helps students who have no calculus background to accept the same idea when they encounter the normal distribution.

Provide each student with a copy of the class-generated data.

4. Making cents of the Central Limit Theorem.[5]

The Central Limit Theorem and sampling, as often presented, leave students confused and forced to assume a rote processing methodology for any topic that depends on those concepts, especially hypothesis tests. In spite of text examples that illustrate the idea, students have difficulty with the abstraction. For many, their understanding of the normal distribution is fragile. This successful activity involves age of pennies. Students need 30 pennies each. Request that students save the pennies they receive in change for about ten days, until they have 30. It is important that they not bring a roll of brand new pennies!

In the initial description of the activity, explain that the variable of interest is age of pennies, which is not directly given, but is easily computed. Transforming the information given, year on the coin, to the desired

5 Adapted from Activity-Based Statistics Project materials.

information, age of the coin, enable them to appreciate that data may need to be transformed before it can be used in analysis. Ask students to take two random samples of size 5, with replacement, write down the year of the penny, transform the year to age, and calculate the mean age for the samples. Repeat the process for two samples of size 10, and for the sample of size 30.

Create a histogram for the ages of individual pennies. It is worth the time expended in order that students see the distribution's skewness emerge. Hypothesize about the mean age of the class' population of pennies. Discuss the spread of the distribution. You will want to calculate the mean and standard deviation for later use. It is a worthwhile assignment to ask students to create other visual data displays so that they gain experience in making connections among shapes that result from different visual representations of the same set of data.

Using the same axis, plot the means from the five-penny sample, and ask what differences they notice (minimally, you want them to comment on shape, center, and spread) apart from the number of data points. Repeat with the samples of size 10 and 30.

Students see the shape of the distribution of sample means becoming more like that of a normal distribution. Calculate the means and standard deviations from the three distributions of sample means, and compare them with the mean and standard deviation for the entire distribution. Create a chart of student's observations that are covered by the statement of the Central Limit Theorem in your text (e.g., means are similar, standard deviation decreases as the sample size increases, etc.).

In preparation for understanding confidence intervals and hypothesis testing, ask them whether the mean of their sample of size 5 would be a good estimate of the mean of all the pennies; repeat with samples of size 10 and 30. Project their thinking toward a similar consideration for shape and standard deviation. Ask them what they think would happen if a sample of size 50 or 75 were drawn. Conclude with a question: How could you study a variable of interest; for example, posted car mileage on a new GM design, if 10,000 autos were produced? When they acknowledge the utility of random sampling, ask them to identify a useful sample size. Ask them how they might deal with the difference between the true population mean (which will be unknown) and the sample mean. It is not necessary that the instructor provide answers to these questions!

5. Descriptive statistical summaries--what about the data?

Many faculty give students experience in calculating means from raw data. Many texts provide data from actual studies to capture student interest and remove the sterility of the numbers. How many of us ask students to process the statistical information in the opposite direction? I suggest that you give students a median and a sample size and ask them to construct a distribution; do the same thing with a mean (which provides insight into degrees of freedom). Extend the task to include range and standard deviation. Expand their understanding by presenting them with a visual representation such as a boxplot or histogram, and ask them to construct a possible distribution for a specified sample size.

You may be wondering about the rationale for these tasks, when it is difficult enough for students to understand measure of center from a given distribution, without adding to their existing confusion. I contend it is like driving a car: knowing how to drive forward is a necessary but not sufficient skill to say one knows how to drive. In order to get a driver's license, one must also learn to drive backward. Processing in an opposite direction, for example, subtraction instead of addition, division instead of multiplication, root extraction instead of exponentiation, always seems to present more difficulties than the original, but learning to do so leads to a fuller understanding of the whole concept.

It is also worthwhile to provide opportunities for students to match variables to visual representations, match summary data to visual representations, and match different forms of visual representation,[6] and discuss reasons for choices. These activities work very well in small groups.

Many students who enroll in the introductory statistics course are majoring psychology, management, nursing, and biology. Faculty in those disciplines want their students to understand statistics so that they can read the professional literature and interpret charts and summaries in the texts. Few of our courses prepare students for that role. Given the ease with which the computer handles multivariate statistics, fewer published studies feature the elementary statistics we teach in our courses. Consequently, students wonder why they take statistics. When students encounter the literature in their field, they often have difficulty reading the tables and interpreting the charts. Rarely do they consider whether the statistics used by the researcher make sense.

It is appropriate to incorporate illustrations of the use of statistics into the introductory course, to facilitate their making connections to other disciplines. First, ask students to read a general interest article that contains averages and ranges, and comment on the statistical information reported. Many articles compare current and previous numerical values, and often report very small gains or losses as significant. Asking students to explain these statements paves

6 Some examples appear in Activity-Based Statistics Project materials.

the way for a discussion of statistical significance and practical importance. Another approach is to take a chart from a research article and ask students questions that require them to interpret the information.[7] In practice, some researchers actually report means and standard deviations on demographic data variables, such as gender. It is useful for students to see statistics in practice, and to critique reports in which tools they have studied have been used in a sloppy fashion. When I have asked students to interpret a chart that reported means and standard deviations for several nominal level of measurement variables, they have expressed anger that I wasted their time in reading such garbage! The final task in this phase requires students to bring in an article of interest from their field that contains statistical information in chart or table form. Students are asked to summarize the article and to describe the chart/table. Very often the reported information includes statistics that they do not know, which has provided motivation for continuing their study in statistics.

6. Body correlation.

Most students realize that certain variables are connected. They know that there are relationships between grades and ability, and grades and effort, for example. This data-gathering activity explores the relationship between two body parts that allows students to connect statistics to other areas of mathematics and to other disciplines.

A supply of measuring tapes is needed for this exercise. The ones that measure in inches on one side and centimeters on the other provide more

7 There is a textbook that contains a collection of excerpts from journal articles and questions of the type I am describing. See: Holcomb, Z. (1992). Interpreting basic statistics. Los Angles: Pyrczak.

interesting results. Students group themselves by threes and rotate the tasks of measurer, measuree, and recorder.

The measurer makes two measurements on the measuree: width of shoulder and length of arm, and reports values to the recorder. Groups rotate roles until each person has held each job. The instructor describes each role, and refrains from giving detailed instructions about measuring. If asked, define length of arm to be distance from shoulder to wrist.

When all groups have recorded their information, the instructor transfers it to the overhead. It is useful to keep track whether the measurements belong to a male or female by use of different color transparency pens, without explanation. Facilitate a discussion of the experience. (Usually the data indicate that there have been different interpretations of the instructions: some students may have used centimeters and others inches, some may have included the hand as part of the arm, different interpretation about width of shoulder, and location of boundary between arm and shoulder.) Uncover difficulties that may have resulted from variability in the measuring techniques and ask how that might affect results. Ask for suggestions to minimize measurement error. It may be necessary to remeasure and record results again if students think their process was flawed. If so, it is worthwhile to make a comparison between the two charts so that students can see that measurement error can be considerable.

Perform a visual analysis of the data for patterns. Many students note differences between males and females. Build a scatterplot, again using color to distinguish gender. Discuss shape; ask whether they think the two measures are related. Ask how an 'average person' should be described.

Create a new variable for the data by taking a ratio of arm length to shoulder width, and record the value on the data chart. Plot the ratio values on a dotplot. Ask students to describe the shape of the ratio data; hypothesize as to the mean value of the ratios. Many will see that the ratios are normally distributed and may recognize that the mean of the distribution is the golden ratio. Build upon their existing knowledge. If the golden ratio is not familiar to them, give them an assignment that requires their researching the golden ratio and their discovering other ratios in the human body that are in similar relationship. An assignment like this give students the chance to relate statistics to art and physical features of the human body.

This activity offers a realistic introduction to regression, which need not happen in the same class as the data collection. Ask students whether, based on the activity, they think they could predict the arm length or shoulder width if they knew the other measurement.

7. Telephones: a necessary expense, but is the expense necessary?

Those who use a computer in the introductory course might want to use other collection situations that are a little more involved. After each student has entered his/her own data, the instructor can aggregate the data into a single document, a copy of which can be placed in each student's file. Students can analyze the data as a laboratory activity. It has been my experience that a visual inspection of a hard copy of the data gives students experience in acquiring a 'feel' for the data which they can confirm through analysis, underscores the importance of checking for data anomalies or missing values, and illustrates the value in forming hypotheses.

Ask students to estimate the number of minutes they spend on the phone on long distance calls in a billing period.[8] Record their estimates. Ask students to bring in a copy of their long distance phone bill. They need to mark off the calls that they made during a billing cycle.

Create a file with the following variables: student name; long distance carrier; total number of long distance minutes in the cycle; total number of long distance calls; number of call minutes in the day, evening, and night rate periods, respectively; total cost; and original estimate.

If this data collection occurs early in the course, students have many variables with which to practice creating graphical data representations and preparing descriptive analyses. Consideration of level of measurement for the variables included, comparison of estimates and actual values, and source of data lead to issues that underlie data analysis, including most appropriate measures of center and spread, measurement error, and sampling types.

This data set provides opportunities for student writing assignments. If students agree to consider this data as a population, they can write a response to the question, "How would you construct simple random, stratified, cluster, or systematic samples using these data?" The data set also offers opportunities for students to generate testable hypotheses, such as: "Should one believe the claims by MCI that it is cheaper than AT & T?"

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8 This exercise was created during a Faculty Advancement in Mathematics workshop in Statistics held at Virginia Commonwealth University in June, 1992 by two other participants and myself. All exercises from this workshop are available through the Consortium for Mathematics and Its Applications (COMAP) as one of the FAIM modules. The FAIM module in which this exercise appears is titled <u>Statistics</u>.

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