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ABSTRACT

This paper presents a technique for applying the Rule Space Model of cognitive diagnosis (Tatsuoka, 1983) to assessment in a semantically rich domain. Responses of 122 architects to 22 architecture test items developed to assess a range of architectural knowledge were analyzed using Rule Space. Verbal protocol analysis guided the construction of a model of examinee performance, consisting of processes for constructing an initial representation of an item (labeled "understand"), forming goals and performing actions based on those goals ("solve"), and determining whether goals have been attempted and satisfied ("check"). Item attributes derived from these processes formed the basis for diagnosis. Successful diagnostic classifications were obtained for approximately 65 percent, 90 percent, and 40 percent of examinees based, respectively, on attributes associated with the "understand," "solve," and "check" processes of the problem-solving model. The findings support the effectiveness of Rule Space in a complex domain and suggest directions for developing new architecture items by using attributes particularly effective at distinguishing among examinees of different ability levels. Nine tables and three figures present study data. (Contains 23 references.) (Author/SLD)

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## EXTENDING THE RULE SPACE MODEL TO A SEMANTICALLY-RICH DOMAIN: DIAGNOSTIC ASSESSMENT IN ARCHITECTURE

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**Extending the Rule Space Model to a Semantically-Rich Domain:  
Diagnostic Assessment in Architecture**

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Abstract

This paper presents a technique for applying the Rule Space model of cognitive diagnosis (Tatsuoka, 1983) to assessment in a semantically-rich domain. Responses to 22 architecture test items, developed to assess a range of architectural knowledge, were analyzed using Rule Space. Verbal protocol analyses guided the construction of a model of examinee performance, consisting of processes for constructing an initial representation of an item (labeled understand), forming goals and performing actions based on those goals (solve), and determining whether goals have been attempted and satisfied (check). Item attributes, derived from these processes, formed the basis for diagnosis. Our technique extends Rule Space's applicability by defining attributes in terms of item characteristics and the causal relations between characteristics and the problem-solving model.

Data were collected from 122 architects of various ability levels (students, architecture interns, and professional architects). Rule Space successfully classified approximately 65%, 90%, and 40% of examinees based, respectively, on attributes associated with the understand, solve, and check processes of the problem-solving model. The findings support the effectiveness of Rule Space in a complex domain and suggest directions for developing new architecture items by using attributes particularly effective at distinguishing among examinees of different ability levels.

Index terms: diagnostic assessment; problems solving; architecture; rule space; item attributes; computer-based testing

### Extending the Rule Space Model to a Complex Domain: Diagnostic Assessment in Architecture

As testing programs begin to employ new forms of assessment, a common goal is to construct tests whose demands are closely related to tasks in the target domain (Wiggins, 1989). While recent research has presented several types of assessment tasks (e.g., simulation) that more accurately capture relevant knowledge and skills, there remains the issue of performance reporting: How can we provide examinees with information beyond scores of overall proficiency, information that captures the richness of knowledge and skills in a domain? In the current work, we employ the Rule Space Model (Tatsuoka, 1983) to generate descriptions of examinee ability that are far richer than those normally derived from large-scale assessment. However, Rule Space has been most successfully applied in the past only to relatively narrow topics in well-defined domains (e.g., mixed number subtraction, single-variable isolation in algebra). This paper presents a technique for applying the Rule Space model of cognitive diagnosis (Tatsuoka, 1983) to a semantically-rich domain in need of more authentic, yet tractable, assessments: architecture.

#### Architecture Assessment

Current architecture assessments consist primarily of short, verbal multiple-choice questions or complex items that mimic the tasks architects normally encounter in the workplace. Because architecture is a complex domain, individuals' scores on relatively simple, verbal multiple-choice tests do not capture the complexity of the knowledge and skills to be assessed. We address these issues by presenting examinees with figural response test items (Martinez, 1991; in press) and by generating diagnostic profiles of examinees based on their performance using the Rule Space model (Tatsuoka, 1983).

The figural response items used in this study differ from standard multiple-choice items in that examinees must construct their answers and the responses consist of the generation or manipulation of figural material (e.g., graphs, pictures). Figural response items are especially suited to domains that are graphical or pictorial in nature; the domain of architecture is a natural candidate for this form of assessment. The approach of using figural response items for architecture assessment has a number of advantages. First, architecture is a graphical domain; designs are drawn, rather than essays being written. Thus, the figural response format provides a natural way for architects to express their ability. Second, constructed response items may be able to tap skills otherwise inaccessible using the multiple-choice format. Martinez & Katz (1992) showed, for example, that different skills are frequently tapped by figural response items compared with their multiple-choice counterparts.

In this study, the figural response items were computer delivered; a sample item is shown in Figure 1. Each item consists of a stem (top of screen), a diagram, and a set of tools for drawing on or manipulating the diagram. The item in Figure 1 requires examinees to move the structures at the bottom of the screen (library, parking lot, and playground) on to the provided site, subject to the explicit constraints stated in the item stem as well as to the implicit constraints that architects associate with libraries, parking lots, and playgrounds (e.g., a playground should not be adjacent to a parking lot; a parking lot must have street access).

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Insert Figure 1 about here

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Architecture brings certain challenges to the practice of large scale assessment. First, much of architectural practice requires design, a notoriously complex cognitive skill. The duration of design projects in architecture are typically measured in days or months, not minutes as with the

usual examination item. Also, design tasks do not typically have "right" or "wrong" answers. Rather, a continuum of designs satisfy the constraints of the task to a greater or lesser extent. Further, in the real world, constraints on a design task are not immutable; often the architect may relax certain initially specified constraints that he or she believes would allow for a better design (Goel & Pirolli, 1991). We do not seek to assess design skills directly. Although some of the figural response items present simple design tasks, most were meant to assess architectural knowledge through subsidiary tasks. For example, two items present a diagram of a building and ask the candidate to specify locations of seismic joints. While a corresponding task set for an architect might not be this simple, the task could come up as part of a larger design task in the real world.

Architecture may be classified as a "semantically rich domain" (Simon, 1984) in that skilled performance involves extensive specialized knowledge. Architecture knowledge is usually gained over several years of intense study. This knowledge comes from a variety of disciplines, including civil engineering, physics, history, psychology, construction, and art. This forms a second challenge for architectural assessment. Optimally, assessment will produce similarly rich descriptions of proficiency based on test performance. In the current work, we employ the Rule Space Model (Tatsuoka, 1983) to generate descriptions of examinee ability that are far richer than those normally derived from large-scale assessment.

Our approach, like that of many emerging test theories, blends traditional psychometric approaches with developments in cognitive psychology (Gitomer & Yamamoto, 1991). Some new approaches including Rule Space build on item response theory (IRT), in which individuals and items are ordered along a proficiency continuum (Lord & Novick, 1969). One well-known shortcoming of IRT is that identical estimates of overall proficiency may be derived from radically different response patterns. If information about response patterns could be simplified and preserved, these rich descriptions of performance could be truly diagnostic (Mislevy, 1993).

### The Rule Space Model

The Rule Space model provides descriptions of examinee performance that extend beyond raw scores or uni-dimensional IRT estimates of overall proficiency. Items are decomposed into attributes, which represent the latent traits that the items assess. Based on an examinee's pattern of correct and incorrect responses, the Rule Space model infers the most likely combination of attributes the examinee has mastered.

The diagnosis of cognitive errors made by examinees is a pattern classification problem. In this study, the patterns are item response vectors, and the vectors are ones and zeroes indicating correct and incorrect responses, respectively. The response vectors are classified as various correct latent knowledge states. The Rule Space model, developed to solve this classification problem, has three steps: (1) determination of classification groups, (2) formulation of a classification space, and (3) classification of examinees' responses.

#### Determination of Classification Groups

We assume that each postulated cognitive attribute—declarative knowledge, cognitive processes, solution strategies, and so forth—is tapped by at least one item in the pool. The relationship between these cognitive attributes and the items is expressed by an incidence matrix  $Q$ , whose order is the number of cognitive attributes  $k$  by the number of items  $n$ . If item  $j$  involves attribute  $k$ , then  $Q_{kj} = 1$ , otherwise  $Q_{kj} = 0$ . Each item is therefore characterized by the cognitive attributes required for its solution.

For example, suppose there are three items whose two underlying attributes are denoted  $A_1$  and  $A_2$ . Further, suppose  $A_1$  is needed to solve items 1 and 3, and  $A_2$  is required in item 2. Then, the incidence matrix  $Q$  ( $2 \times 3$ ) is:

	Items		
Attribute $A_1$	1	0	1
Attribute $A_2$	0	1	0

With three items, there are eight possible response vectors:

$(0,0,0)$ ,  $(1,0,0)$ ,  $(0,1,0)$ ,  $(0,0,1)$ ,  $(1,1,0)$ ,  $(1,0,1)$ ,  $(0,1,1)$ ,  $(1,1,1)$ .

Given two attributes, there are four possible examinee knowledge states:

- State 1. Examinee cannot do  $A_1$ , but can do  $A_2$
- State 2. Examinee cannot do  $A_2$ , but can do  $A_1$
- State 3. Examinee cannot do  $A_1$  nor  $A_2$
- State 4. Examinee can do  $A_1$  and  $A_2$

There are four ideal response vectors conforming to the four states:

- State 1.  $(0,1,0)$
- State 2.  $(1,0,1)$
- State 3.  $(0,0,0)$
- State 4.  $(1,1,1)$

Note that each ideal response vector corresponds to a unique vector of mastered attributes. The remaining possible response vectors— $(1,0,0)$ ,  $(0,0,1)$ ,  $(1,1,0)$ ,  $(0,1,1)$ —do not conform precisely to any of the models. The section entitled Classification of Examinees' Responses discusses Rule Space's treatment of such "non-ideal" response vectors.

Tatsuoka (1991) and Varandi & Tatsuoka (1990) developed an algorithm to produce all possible ideal response patterns, corresponding to all possible latent knowledge states from an incidence matrix  $Q$ . The number of states is determined from the number of attributes, the number of items, and the degree of attribute nesting. In applying Rule Space to other data sets, the number of latent states has often exceeded 1000.

### The Classification Space

In order to preserve continuity with current psychometric theories, the classification space was formulated as a two-dimensional Cartesian product space of the IRT proficiency parameter  $\theta$ , and an index of the unusualness of an item response pattern  $\zeta$ , where "unusualness" refers to the degree to which easier items are answered incorrectly and difficult items are answered correctly (Tatsuoka & Linn, 1981; Tatsuoka, 1984; 1985; 1990; Tatsuoka & Tatsuoka, 1987). When an examinee's response vector conforms well to the average performances on the test items, the absolute value of  $\zeta$  will be nearly zero. When  $\zeta$ -values of a knowledge state are close to zero, that is, close to the  $\theta$ -axis, we can expect that many examinees will be diagnosed to have that knowledge state. If the  $\zeta$ -value associated with a knowledge state is large, positively or

negatively, then we expect that state to be unusual in the sense that few examinees will be diagnosed as having that knowledge state.

### Classification of Examinees' Responses

Examinees' performances on test items are not always consistent with their unobservable patterns of attribute mastery. Responses that deviate from an ideal response pattern are assumed to contain random errors or slips. Under the assumption that occurrences of slips on items are independent across items, Tatsuoka & Tatsuoka (1987) showed that the distribution of the number of slips follow a binomial distribution if the slippage probabilities are the same across the items, and follow a compound binomial distribution if the slippage probabilities differ across items.

When the non-ideal response patterns associated with a particular ideal pattern,  $R$ , are mapped into the Rule Space (by computing their  $\theta$  and  $\zeta$  values), they form a unique subset that swarms around the point  $(\theta_R, \zeta_R)$ . The swarm of mapped points in the Rule Space follows approximately a multivariate normal distribution with a centroid of  $(\theta_R, \zeta_R)$ , and is called the bug distribution or state distribution associated with response pattern  $R$  (Tatsuoka, 1990). When all possible ideal item response patterns are mapped on to the Rule Space, one can apply Bayes' decision rules for determining the minimum errors to classify an examinee's point  $(\theta_x, \zeta_x)$  into one of the possible latent states. More detailed discussions of the classification procedure can be found in Tatsuoka (1990), Tatsuoka & Tatsuoka (1987, 1989), and Sheehan, Tatsuoka, & Lewis (1991).

### Applying Rule Space to Architecture Assessment

The items used in this research were intended to assess a wide range of architectural knowledge and skills across several subdisciplines of architecture. Different items required different problem-solving operations. For example, some items required examinees to specify the properties of structural elements while others required the proper arrangement of architectural elements on the computer. The range of operations used across items implied that defining attributes in terms of low-level operations would produce an attribute set with little overlap across items. This would defeat the purpose of the Rule Space. We therefore analyzed the architecture items at a coarser grain, using attributes descriptive of higher-level processing as suggested by a general model of problem solving. This approach required a modification to the procedure used in other Rule Space analyses. We first defined a cognitive model that was general enough to account for problem-solving behavior on all items. Attribute definitions were then based on the model. In the next section, we describe the cognitive model and our procedure for defining item attributes.

### The Cognitive Model

Our cognitive model was derived in part from a theory of computer interface use (Lewis & Polson, 1990). This model was chosen because of ostensible similarities between problem solving in user interface evaluation and solution of figural response items. Our adaptation of Lewis and Polson's model was based on verbal protocols from one pilot subject who solved all 22 architecture items<sup>1</sup>. The analysis of protocols from a single subject was not used to produce a definitive cognitive model, but a hypothesized model which would guide us in developing reasonable attributes. The reasonableness of this hypothesized model could, in turn, be supported or falsified by our data.

<sup>1</sup>This pilot subject was not part of the test administration discussed in the next section.

The model consists of processes relevant for constructing an initial representation of the item (i.e., understanding the problem stem and provided diagram), forming goals and performing actions based on those goals (i.e., solving the item), and determining whether goals have been satisfied and if they have been satisfied correctly (i.e., checking each problem solving step and the final answer). The model asserts that these processes exist, but makes no claims as to their order. For example, an examinee might come to a new understanding of a problem after attempting to solve it or after checking an initial, incorrect solution. The processes hypothesized by the model are summarized in Table 1.

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Insert Table 1 about here

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**Understand.** The first step in solving any item is to understand what is being asked so that the appropriate knowledge can be invoked. Each figural response item consisted of both a verbal stem and a diagram, the latter of which may contain both graphical and verbal information. Thus, understand processes include: (a) reading and interpreting the verbal stem, (b) scanning and interpreting the diagram, and (c) relating the information in the stem and diagram to one's own knowledge. This processing allows the examinee to form initial goals, and either a plan for solving the item or a set of heuristics. An initial goal might be to apply a strategy learned in the classroom or to invoke a general problem-solving method such as means-ends analysis, in which one chooses at each step an action that will reduce the difference between the current state of the problem and the desired goal state. In specifying the understand processes—read stem, scan diagram, and relate to one's own knowledge—no claims are made as to either the ordering of the processes or the conditions under which they occur. Particular items will be less or more difficult in terms of, say, reading and interpreting the stem, and it is just these sorts of differences which form the basis for the item attribute definitions.

**Solve.** Once an initial representation of the problem has been built, and the initial goals formed, the examinee must perform the actions that lead to solving the problem. Of course, while solving a problem, an examinee may reformulate or refine an initial representation of an item. The processes involved in solving an item are applied to each goal that has not yet been satisfied. Each of these goals may be elaborated by forming subgoals of the currently active goal or the examinee may perform an action that will satisfy the current goal. An action may be physical, such as drawing a line, or cognitive, such as finding a level area on a contour map. These two processes, elaboration of goals and performance of actions, do not determine precisely how a particular item is solved. Certain questions are left open. For example, which subgoals are formed when a particular goal is elaborated? How does the examinee decide on which actions to perform to satisfy a goal? Answering these questions requires a knowledge of the particular strategies used to solve each item. Whatever strategy an examinee uses (whether problem-specific or general), that strategy will determine which goals are attended to and in what order, and what subgoals are formed.

**Check.** Once an action has been performed, the results of that action may be evaluated to ensure that the action was performed correctly and that it satisfies the original goal. If both conditions are met, the examinee may mark that goal as finished (perhaps by saying something to the effect of "Okay, that's done"), and proceed to the next unsatisfied goal. Thus, two types of evaluations may occur: monitoring whether an action has been carried out as planned and noting whether it satisfies the original goal.

### Attribute Creation

Because the figural response items were designed to assess a wide range of architectural knowledge and skill, defining attributes in terms of the actual steps candidates take in solving the items (the approach used in previous applications of Rule Space) was contra-indicated. Instead, we defined attributes in terms of item characteristics or features. Each item has multiple features and could be classified along several dimensions, but for purposes of attribute creation we identified those features with a potential causal connection to examinee performance. The attributes were defined by identifying features of the items that could be expected either to help or hinder problem-solving. For example, we hypothesized that problem solving would be hindered during the process "scan the provided diagram," if the diagram was a specialized graph (e.g., a topographic map) that would not be understood by all examinees. The 38 attributes identified in the task analysis are listed in Table 2. To illustrate the assignment of attributes to items, Table 3 shows the attributes associated with the "library" item of Figure 1 along with an explanation of why that attribute was assigned.

Each attribute is associated with one or more of the three types of processing (understand, solve, and check), and those assignments are shown in Table 4. The assignment of attributes to process was made by two independent judges with an inter-rater agreement of 88%. Disagreements were settled through discussion between the judges. Two independent judges also determined the subset of elementary cognitive attributes needed to solve each question. The inter-rater reliability for this process was again 88%. As before, disagreements were settled through discussion between the judges.

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Insert Tables 2, 3, and 4 Here

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### Method

#### Materials and Design

Twenty-two figural response questions were constructed to draw upon skills needed throughout the broad content of an architectural licensing examination. These questions were developed for presentation on a computer with responses made through mouse movements and clicks. The questions were divided into two eleven-item subsets, and each subset was administered to a random half of the available subjects<sup>2</sup>.

#### Subjects

Subjects (N=122) were selected from three status groups: practicing architects (N=34), architecture interns (N=35), and architecture students (N=53). The eleven item responses provided by each subject were scored correct/incorrect and modeled with a two-parameter logistic IRT model. Maximum likelihood estimates of proficiency ( $\theta$ ) were subsequently obtained for each subject. These estimates were used to classify subjects into three equal-sized proficiency groups. The cross-tabulation of status groups and proficiency groups is shown in Table 5.

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<sup>2</sup>Subjects solved only eleven of the figural response items because they were also administered a set of complementary multiple-choice items. Time constraints did not permit additional testing. Contrasts between item sets are reported in another study (Martinez & Katz, 1992).

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Insert Table 5 about here

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### Procedure

In groups of six, subjects were given a verbal introduction to the item delivery system. Following that, they each attempted the items individually on a computer. Of the 122 subjects, three subjects generated verbal protocols to gather independent support for the cognitive model. To generate the protocols, the subjects were asked to "think aloud" (Ericsson & Simon, 1984), saying anything that they would normally "say" to themselves as they solved the items.

### Rule Space Analyses

Rule Space analyses were conducted separately for each of the three groups of problem-solving attributes identified above. This strategy was chosen for two reasons. One very practical reason is that the combination of attributes made the possible number of knowledge states astronomical for the entire set of 38 attributes, thus the total pool of attributes had to be subdivided. A second reason was to contrast attribute clusters in their ability to classify examinees.

Rule Space was carried out in two steps: First, the BUGLIB computer program (Varandi & Tatsuoka, 1990) was used to determine the set of all possible latent knowledge states associated with the specified stage; second, the RULESPACE computer program (Tatsuoka, Baille & Sheehan, 1990) was used to classify subjects into one of the knowledge states. Three attempts were made to classify each examinee, one for each of the three problem-solving process types (understand, solve, and check).

## Results

### Verbal Protocol Results

Our cognitive model postulated that certain processes would be used as a subject solved the architecture items. One way to gather evidence for the model is to show that these processes are sufficient for explaining the verbalizations made by subjects (Ericsson & Simon, 1984). Eight categories of subject verbalizations were defined, one category for each process in the cognitive model and a "miscellaneous" category. These categories were defined through examining verbalizations of the pilot subject as she solved eleven of the items. The sufficiency of the categories was established by attempting to categorize the verbalizations on the remaining eleven items. One rater categorized all of the subject's verbalizations, while another rater independently categorized a portion of the verbalizations. The inter-rater agreement on the portion scored by both raters was 82%. The final categories are shown in Table 6. The verbalizations encoded as miscellaneous include single words or short phrases ("Okay," "Let's see"), statements concerning the computer interface ("I have to click twice"), and statements irrelevant to the task.

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Insert Table 6 about here

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The categorization scheme was applied to the verbal reports of the three protocol subjects. The cognitive model accounted for 71% of the verbalizations made by subjects; the remaining verbalizations fell into the miscellaneous category. This result suggests that the model adequately

captured subjects' problem-solving performance, and thus supports the validity of the cognitive attributes created from this model.

### Rule Space Results

The projection of examinee response data into the two-dimensional Rule Space is presented in Figure 2. Examinees'  $\theta$  values are plotted along the x-axis;  $\zeta$  values are plotted along the y-axis. The symbols indicate status group membership. The plot shows that practicing architects are located mostly in the medium to high proficiency region and form a cluster that is distinct from the points plotted for interns and students.

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Insert Figure 2 about here

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Each examinee's performance was diagnosed three times, once for each of the understand, solve, and check attributes. For each diagnosis, the examinee's point in the rule space was compared to the points corresponding to the set of knowledge states associated with each attribute group. The item/attribute incidence matrices developed for each problem-solving process type determined the number of possible states: 803 for understand, 1208 for solve, and 121 for check. Within each process type, each knowledge state corresponded to a unique combination of mastered attributes and is represented by a unique point in the Rule Space.

The classification results for each of the three types of problem-solving processes are presented in Table 7. Within each process type, the number and percentage of classified examinees is broken down by IRT-proficiency level (low, medium, and high) and status group (student, intern, architect). Two patterns are worth noting. The first is that the solve attributes are the most powerful in classifying subjects across proficiency levels and status groups; in fact, all 41 low-proficiency examinees were classified. The next most powerful set of attributes is understand, followed by check. A second pattern is that, almost uniformly, examinees in the lower proficiency or status groups were more often classified than those in the higher groups. For example, twice the percentage of low-proficiency examinees (61%) than high-proficiency examinees (30%) were classified under check.

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Insert Table 7 about here

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The low classification rate achieved for the check processes is considered in Figure 3. In this plot, the diamonds stand for latent knowledge states and the boxes indicate the examinees' diagnostic location. The plot shows that the 121 knowledge states deduced from the check incidence matrix do not coincide with the examinees' points. Thus, the attributes defined from the check portion of the model do not capture examinee behavior, suggesting that examinee performance is not greatly differentiated by check processes (or that we need to rework that portion of the model).

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Insert Figure 3 here

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### Attribute Mastery Probabilities

An attribute mastery vector was estimated for each classified examinee. These vectors are composed of zeros and ones, depending on whether the attribute in question was included in the subset of mastered attributes defined for the examinee's state. Attribute mastery patterns were averaged within proficiency and status groups, and analyzed using a repeated measures analysis of variance design, as described in Sheehan, Tatsuoka, and Lewis (1991).<sup>3</sup>

P-values for the analysis of variance F-tests are reported in Table 8. The table provides evidence for three clearly significant effects: proficiency group, attribute, and the attribute by proficiency group interaction. These results are reassuring because they indicate that the attributes associated with each problem-solving stage are differentially difficult and that examinees in different proficiency groups tend to have different attribute mastery profiles.

The results obtained for the status group classification are not as clear-cut. Although the main effect of status group is clearly not significant, the interaction of status group with attribute is marginally significant. This indicates that the average probability of mastery values calculated for some attributes differed among students, interns, and practicing architects, but these differences did not hold up after averaging over all attributes. Thus, on the average, examinees in different status groups did not differ in their mastery of the elementary cognitive skills identified in this study.

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Insert Table 8 about here

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Table 9 presents the mean probability of mastery values estimated for the solve attributes. The different attribute mastery profiles obtained for low, medium and high proficiency examinees are clearly indicated. The table also shows that attributes differ in discrimination. For example, consider the probabilities listed for the "environment" attribute: On average, low proficiency examinees mastered this attribute with a probability of .47; the corresponding probabilities for medium and high proficiency examinees are .60 and .97, respectively. The varying probabilities obtained for low, medium, and high proficiency examinees indicate that this attribute is highly discriminating. By contrast, the three mean values listed for the "learned procedure" attribute are all very similar. Thus, this attribute is not particularly helpful at discriminating among examinees of different ability levels.

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Insert Table 9 about here

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### Discussion and Conclusions

This study exemplifies how an IRT-based model for estimation of overall proficiency can be combined with the diagnostic classification of examinees. The results of the application of Rule Space were satisfying: We were able to classify a large proportion of examinees, especially those of low and medium ability. In principle, these classifications could be reported back to examinees

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<sup>3</sup>A standard analysis of variance design would not have been appropriate for these data because the hypothesis of multisample sphericity—that is, independently observed attributes—is violated. The violation results from the fact that, instead of measuring a single attribute on each examinee, our design involves taking 38 attribute measurements. Thus, non-zero correlations are expected among the attribute measurements associated with a particular examinee.

so that remediation in weak areas could proceed. Traditional psychometrics has served well in discriminating among examinees for selection, placement, or classification on the basis of global estimates of proficiency. Rule Space provides estimates of  $\theta$ , but also yields information that could serve the interests of the examinee in pin-pointing areas of non-mastery. Of course, applications of the technique described in this paper to other complex domains may require a much larger sample size than was used in the current study. Data from a relatively small number of examinees were sufficient for the goal of this paper, which was to demonstrate and explain a methodology for extending Rule Space.

In addition to diagnosis and estimation of  $\theta$ , Rule Space provides a framework for comparing a model of task performance to examinees' response data. There are few well-defined methodologies for comparing models to data (but see Polk & Newell, 1991), especially those that can accommodate a great variety of individual differences in examinees' knowledge, skill, and strategy. Model testing proceeds as follows: On the basis of a cognitive model, items are analyzed into their component cognitive attributes. The resulting item/attribute matrix (or matrices) leads to strong predictions about examinees' response patterns. If the  $(\theta, \zeta)$  position of an examinee's response pattern is close to that of an ideal response pattern, that examinee is classified into the knowledge state that the response pattern implies. To the extent that examinees' response patterns can be classified, the analysis provides support for the cognitive model. There are of course limitations to the Rule Space method. We have already noted that sets of attributes processed together are limited in size. As they approach 25 or so, the combinations of attribute profiles makes the possible number of ideal states unmanageable. Consequently, the attributes must be clustered and run separately as in this study.

One contribution of this work is that we have outlined a methodology for applying Rule Space to complex domains. Generally, a limitation of Rule Space is that at the level of fine-grained analysis, the operations needed to solve items in a complex domain may not overlap a great deal. Many attributes might in fact be unique to particular items within the item set. If this is the case, the cognitive attributes must be cast at a higher-level of generality such as item characteristics (e.g., type of diagram presented) or general problem-solving approach needed to solve each item (e.g., recalling a fact versus applying a learned procedure). Given more general attributes, what can we say about an examinee's performance? From a psychological viewpoint, the attributes tell us little about the examinee's cognitive competence. But from an educational standpoint, the attributes provide examinees with just the information they need to improve their performance on subsequent tests. The attributes allow us to say that an examinee has difficulties with items having certain properties. While we may have little information about the examinee's skill at a fine-grained level, the diagnostic reports (which attributes are mastered and which aren't) does tell the examinee what types of problems they should seek out and practice solving, and what components of problem solving need special attention.

Attributes should be based on an independently constructed problem-solving model. Analysis of verbal protocols, performed in this work, serves as one means for constructing and verifying a cognitive model. The model supports attribute creation by showing which aspects of the items would help or hinder problem-solving performance. In contrast to developing a list of attributes intuitively, a cognitive model provides a rich description of each attribute because the meaning of each attribute is derived from its place in the model. Methodologically, this rich attribute description promotes a fuller understanding of what each attribute means and facilitates the assigning of attributes to items.

Another contribution of this work is that we were able to examine the power of attributes to discriminate among examinees of various levels. Knowing which attributes are highly

**discriminating has value for the construction of items as well as for the design and sequencing of instruction. Differential relevance of attributes across proficiency groups also sheds light on the nature of expert/novice differences in the domain of interest. Rule Space holds a great deal of value for satisfying the requirements of traditional psychometrics and for diagnosis of individual examinees. Through the use of such models, psychometrics has much to offer to learners and teachers beyond estimates of global proficiency.**

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**Table 1**  
**Problem Solving Model**

Attribute Group	Processes
Understanding the Item	Read the item stem Scan the diagram Recall relevant information
Solving the Item	Set subgoals Perform actions
Checking Performance	Is the action correct? Is the current goal completed?

Table 2  
**Attribute Definitions**

Attribute Class	Attribute Name	Description	Relations among Attributes in a Class
Characteristics of Presented Figure	Picture	Presented figure is a sketch of an actual object	The three attributes in this class are <u>mutually exclusive</u> (if an item has one attribute in this class, by definition it does not have another attribute from the same class) and <u>exhaustive</u> (all of the items may be classified as having at least one of the attributes in this class)
	Diagram	Presented figure is an abstract diagram of an object	
	Specialized Diagram	Presented figure is a graph or chart -- a visual representation of some information	
Clarity of General Task	Diagram obvious	Based on just the presented figure, it's possible for someone to understand what task the item is asking them to perform. Details regarding the task included in the item stem might still be needed for correct performance of the task.	Mutually exclusive, but not exhaustive
	Own obvious	Based on the presented figure along with some prior knowledge, it's possible for someone to understand what task the item is asking them to perform. Details regarding the task included in the item stem might still be needed for correct performance of the task	

<b>Problem-solving requirements of item</b>	<b>Declarative</b>	<b>Requires knowing particular architectural symbols and definitions for correct solution.</b>	<b>Mutually exclusive and exhaustive</b>
	<b>Learned Procedure</b>	<b>Requires the application of fairly standard, algorithmic procedures that usually would have been learned previously.</b>	
	<b>Discovered Strategy</b>	<b>Requires the application of knowledge or procedures in a novel way. These items are more puzzle-like.</b>	
<b>Content area</b>	<b>Site Design</b>	<b>The item tests knowledge or skills associated with one of the recognized subdisciplines of architecture listed to the left.</b>	<b>Mutually exclusive and exhaustive</b>
	<b>Structural Technology (General)</b>		
	<b>Structural Technology (Lateral Forces)</b>		
	<b>Materials and Methods</b>		
	<b>Construction Documents</b>		

Particular Architectural Features	Identify Street	Correct problem solving requires that the candidate can recognize a street on a site plan.	Neither mutually exclusive nor exhaustive
	Environment	Correct problem solving requires that the candidate knows about constraints due to environmental factors (e.g., weather, earthquakes)	
	Contour Lines	Requires the ability to read and interpret contour lines.	
	Forces	Requires the ability to recognize, interpret, and use force vectors.	
General Problem-solving Approach	Read and Translate	Problem solving goes through cycles of getting information from the problem stem, using that information to generate part of the answer, and then repeating.	Mutually exclusive, but not exhaustive
	Indicate Location of New Feature	Problem solving involves placing given elements into new positions or adding information to the provided diagram.	
Response Method	Move/Rotate	Requires arrangement of provided elements.	Exhaustive, but not mutually exclusive
	Label	Requires selecting which of a provided set of labels should be placed at various indicated points on the diagram.	
	Draw Line	Requires drawing of lines onto provided diagram.	
	Draw Arrow	Requires drawing of arrows onto provided diagram.	

<b>Misleading Characteristics</b>	<b>Stem Incorrect</b>	Without detailed knowledge of an item type, the item's stem suggests an incorrect problem-solving method.	Mutually exclusive, but not exhaustive
	<b>Diagram Incorrect</b>	Without detailed knowledge of an item type or diagram type, the item's provided diagram suggests incorrect problem-solving methods.	
<b>Relation between Stem and Problem-solving</b>	<b>Stem Independent</b>	The item stem provides practically no information that could not be gained either through prior knowledge or through the provided figure.	Stem independent and Stem dependent are mutually exclusive and exhaustive.
	<b>Stem Dependent</b>	Problem-solving is necessarily based on information presented in the item stem. This category is the union of "Initial Info" and "Interim Info" categories.	
	<b>Initial Information in Stem</b>	While the stem information is necessary for correct solution, that information is not directly required during the course of problem solving.	Initial info in stem and Interim info in stem are mutually exclusive and exhaustive across Stem dependent items.
	<b>Interim Information in Stem</b>	The information in the stem is needed a number of times during the course of correct problem-solving.	

<b>Completion Criteria</b>	<b>Own Knowledge Stop</b>	Examinees must use their own knowledge to decide whether they are finished responding to an item (i.e., if the answer is complete). Neither the stem nor the diagram directly supply this information.	Mutually exclusive and exhaustive
	<b>Diagram Stop</b>	The provided diagram indicates whether an answer is complete.	
	<b>Diagram and Own Knowledge Stop</b>	The provided diagram along with some specialized knowledge indicates whether an answer is complete.	
	<b>Stem Stop</b>	Information provided in the stem indicates whether a given answer is complete.	
<b>Number of Correct Responses</b>	<b>One Correct</b>	The item has only one correct answer.	Mutually exclusive and exhaustive
	<b>Few Correct</b>	The item has two or three correct answers, which are variants of one another.	
	<b>Many Correct</b>	The item has several correct answers, some of which may be qualitatively different from others and some of which may be variants on another answer.	

**Table 3**  
**Attributes Associated with "Library" Item (Figure 1)**

Attribute	Explanation
Specialized Diagram	The provided figure is a site plan, which is an abstract diagram of the actual building site. The site plan diagram contains elements that require specialized knowledge to interpret (e.g., contour lines, property lines, symbols for trees).
Diagram Obvious	Based on the provided elements and the operations available (move, etc.), it is clear that the general procedure for this task is to place the elements somewhere onto the site.
Discovered Strategy	There is no clear, algorithmic procedure for placing the buildings onto the site. The examinees must bring to bear knowledge learned in different situations to the solving of this task.
Site Design	This item presents a prototypical site design task.
Identify Street	Recognizing the street on the site plan is important for correct placement of the parking lot.
Contour Lines	Correctly interpreting the site plan's contour lines is necessary for correct placement of the buildings on the site (e.g., the buildings should not be placed on the steep slope, but on relatively level ground).
Stem Independent	Beyond the general task and the standard "preserve all trees," the stem does not provide any information that is vital to the correct solution of the item.
Many Correct	There are a number of correct solutions to this item, reflecting different arrangements of the buildings on the site.
Move/Rotate	The primary interface operation in this task is moving elements and rotating them to fit better onto the site.
Own Stop	Based on their own knowledge, it is up to the examinees to determine when they are finished responding to the item. Nothing in the stem nor in the diagram provides feedback either on the correctness or completeness of a response.

Table 4  
Attribute Assignments to Processing Types

Attribute Class	Attribute	Problem-Solving Process Type		
		Understand	Solve	Check
Characteristics of presented figure	Picture	X		
	Diagram	X		
	Specialized Diagram	X		
Clarity of general task	Diagram Obvious	X	X	
	Own Obvious	X	X	
Problem-solving requirements of item	Learned Algorithm	X	X	
	Declarative	X	X	
	Discovered Strategy	X	X	
	Site Design	X		
Content area	Structural Technology	X		
	Structural Tech. (Lateral Forces)	X		
	Materials and Methods	X		
	Construction Documents	X		
Particular architectural features	Identify Street	X	X	
	Environment	X	X	
	Contour Lines	X	X	
	Forces	X	X	
Relation between stem and problem-solving	Stem Independent	X		
	Stem Dependent	X		
	Initial Info in Stem	X		
	Interim Info. in Stem		X	
Number of correct responses	One Correct			X
	Few Correct		X	X
	Many Correct		X	X
General problem-solving approach	Read and Translate		X	
	Indicate Location of New Feature		X	
Response method	Move/Rotate		X	
	Label		X	
	Draw Line		X	
	Draw Arrow		X	
Completion Criteria	Own Stop			X
	Diagram Stop			X
	Stem Stop			X
	Diagram + Own Stop			X
Misleading Characteristics	Stem Incorrect			X
	Diagram Incorrect			X

**Table 5**  
**Distribution of Status Groups by Proficiency**

Proficiency	N	Status Group					
		Student		Intern		Architect	
		N	Column %	N	Column %	N	Column %
Low	41	27	51	10	29	4	12
Medium	41	17	32	12	34	12	35
High	40	9	17	13	37	18	53
<b>Total</b>	<b>122</b>	<b>53</b>	<b>100</b>	<b>35</b>	<b>100</b>	<b>34</b>	<b>100</b>

Table 6  
Protocol Encoding Categories

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Understand

**Read stem:** statements involving the reading of the problem stem. Read statements include verbatim readings of the stem as well as partial reading of the problem stem.

**Scan diagram:** statements involving the provided diagram. Diagram statements include verbatim readings of verbal information as well as verbal descriptions of information in the diagram (e.g., "lateral forces coming that way").

**Relate:** statements regarding how the problem or parts of the problem relate to the examinee's own knowledge. Relate statements consist of several types of verbalizations including verbalizations regarding:

- an expectation or the violation of an expectation (e.g., "Normally there would be more lines on this window drawing")
- recognition of the problem (or part of the problem) as of a particular type (e.g., "This is a site vignette," "this is a perspective drawing")
- predictions as to the difficulty of the problem (e.g., "this will take a while")
- the definitions or ambiguity of sections of the problem (e.g., "is it an awning or a hopper?", "most sheathing I know of is...")

Solve

**Goal:** stating an intent or future action. Goal statements are often stated in the future tense or in terms of "should be."

**Perform:** statements regarding the performance of an action. Perform statements are usually stated in the present or "continuing" tense (e.g., "that dips here"). Perform statements relate only to physical actions such as moving a block on the screen or locating a particular item in the diagram (for the latter, e.g., "this is a flat area"). It may be difficult to distinguish between goal and perform statements.

Check

**Evaluate-correct:** statements regarding the correctness of a performed action or the result of that action (e.g., the location of a placed object). Evaluate-correct statements should only refer to the examinee's own actions or answers, not to the problem itself. These statements may either reflect judging the correctness of an action (e.g., "is that right?") or reflect the outcomes of a judgment (e.g., "that isn't what I wanted to do").

**Evaluate-complete:** statements suggesting that some action or goal has been completed. As with evaluate-correct statements, evaluate-complete statements include verbalizations judging if something has been finished (e.g., "is there anything else to be done?") as well as verbalizations concerning the results of such judgments (e.g., "that's it," "that was easy").

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**Table 7**  
**Classification Results for Subjects Grouped by Proficiency and Status**

Group	N	Problem-Solving Process Type					
		Understand		Solve		Check	
		No. Classified	%	No. Classified	%	No. Classified	%
<b>Proficiency</b>							
Low	41	32	78	41	100	25	61
Medium	41	29	71	40	98	13	32
High	40	20	50	33	83	12	30
<b>Status</b>							
Student	53	37	70	52	98	25	47
Intern	35	23	66	32	91	11	31
Architect	34	21	62	30	88	14	41
<b>Total</b>	<b>122</b>	<b>81</b>	<b>66</b>	<b>114</b>	<b>93</b>	<b>50</b>	<b>41</b>

**Table 8**  
**Analysis of Variance Results for Attribute Mastery Data**

Group	Problem-Solving Process Type		
	Understand p-value	Solve p-value	Check p-value
<b>Between Subjects</b>			
Proficiency	.0001	.0001	.0001
Status	.5621	.1948	.3433
Proficiency x Status	.1343	.4707	.7231
<b>Within Subjects<sup>a</sup></b>			
Attribute	.0001	.0001	.0001
Attribute x Proficiency	.0013	.0001	.0130
Attribute x Status	.0885	.0874	.4287
Attr. x Prof. x Status	.4743	.0535	.1029

<sup>a</sup> p-values for within-subject effects were calculated using Wilks' Lambda.

Table 9  
Attribute Mastery Probabilities for Solve

Attribute	Proficiency			Overall Mean
	Low	Medium	High	
Many correct	.35	.28	.35	.34
Draw Arrow	.44	.38	.33	.38
Move/Rotate	.31	.39	.61	.44
Label	.43	.66	.88	.66
Environment	.47	.60	.97	.68
Contour Lines	.61	.78	.83	.74
Forces	.70	.64	.88	.74
Identify Street	.75	.76	.84	.78
Interim Info	.67	.82	.92	.80
Diagram Obvious	.70	.76	.94	.80
Own Obvious	.81	.83	.86	.83
Few Correct	.66	.91	.98	.85
Discovered Strategy	.79	.89	.98	.89
Ind. Location	.75	1.00	1.00	.92
Read + Translate	.86	.98	.98	.94
Declarative	.87	.97	1.00	.95
Learned Algorithm	.92	.97	.98	.96
Stem Independent	.89	.99	1.00	.96
Stem Dependent	.93	1.00	1.00	.98
Draw Line	.98	1.00	1.00	.99
Overall Mean	.69	.78	.87	.78

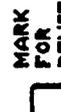
Prepare a siteplan for the Generic City library to accommodate a library, playground and parking lot. All trees must be preserved.



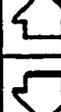
ROTATE



START OVER



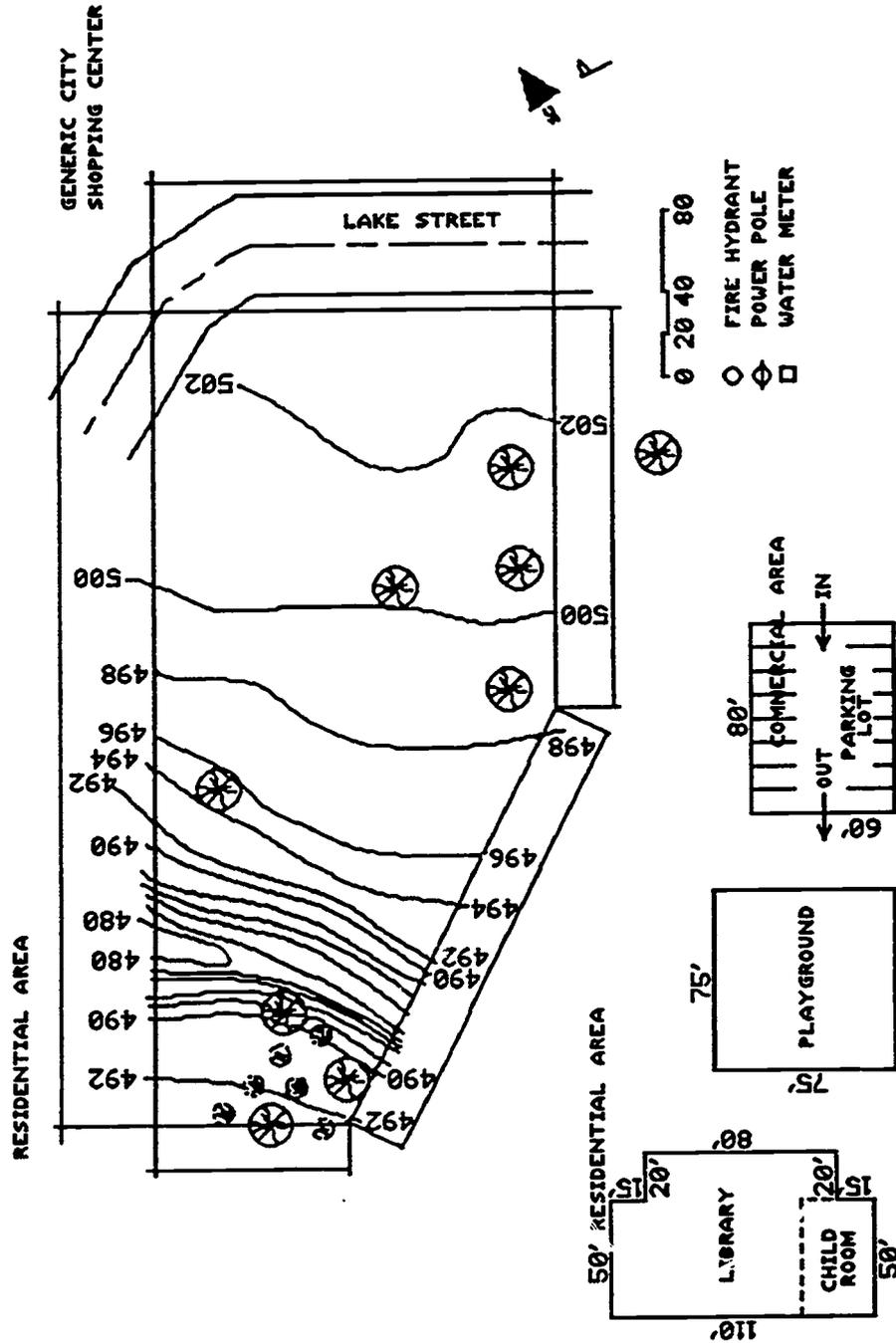
MARK FOR REVIEW



PREV NEXT



NAVIGATE



To move an object, position the crosshairs on the object and click.

Figure 1. Sample figural response item.



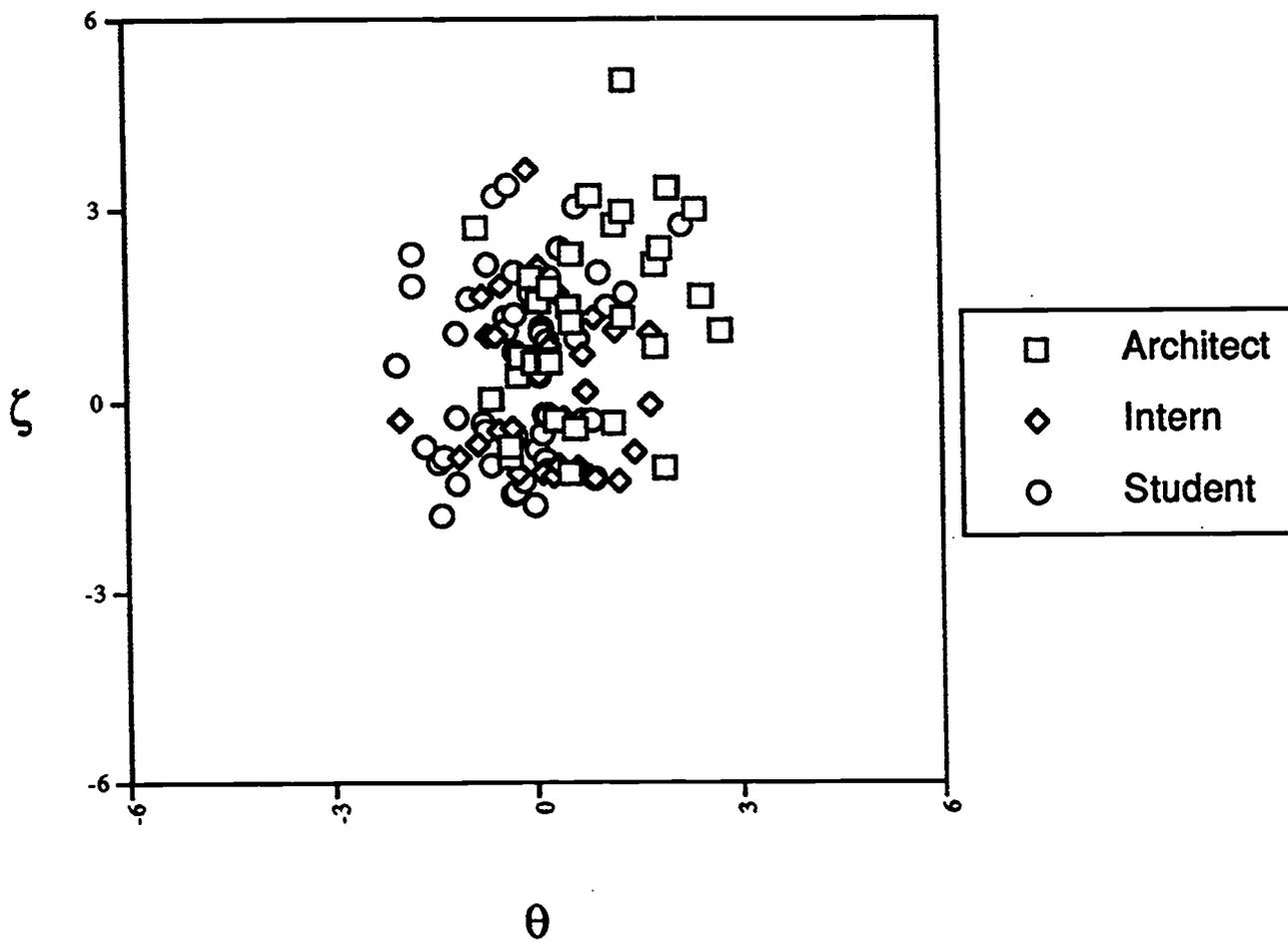


Figure 2. Projection of examinee response data onto  $(\theta, \zeta)$  space.

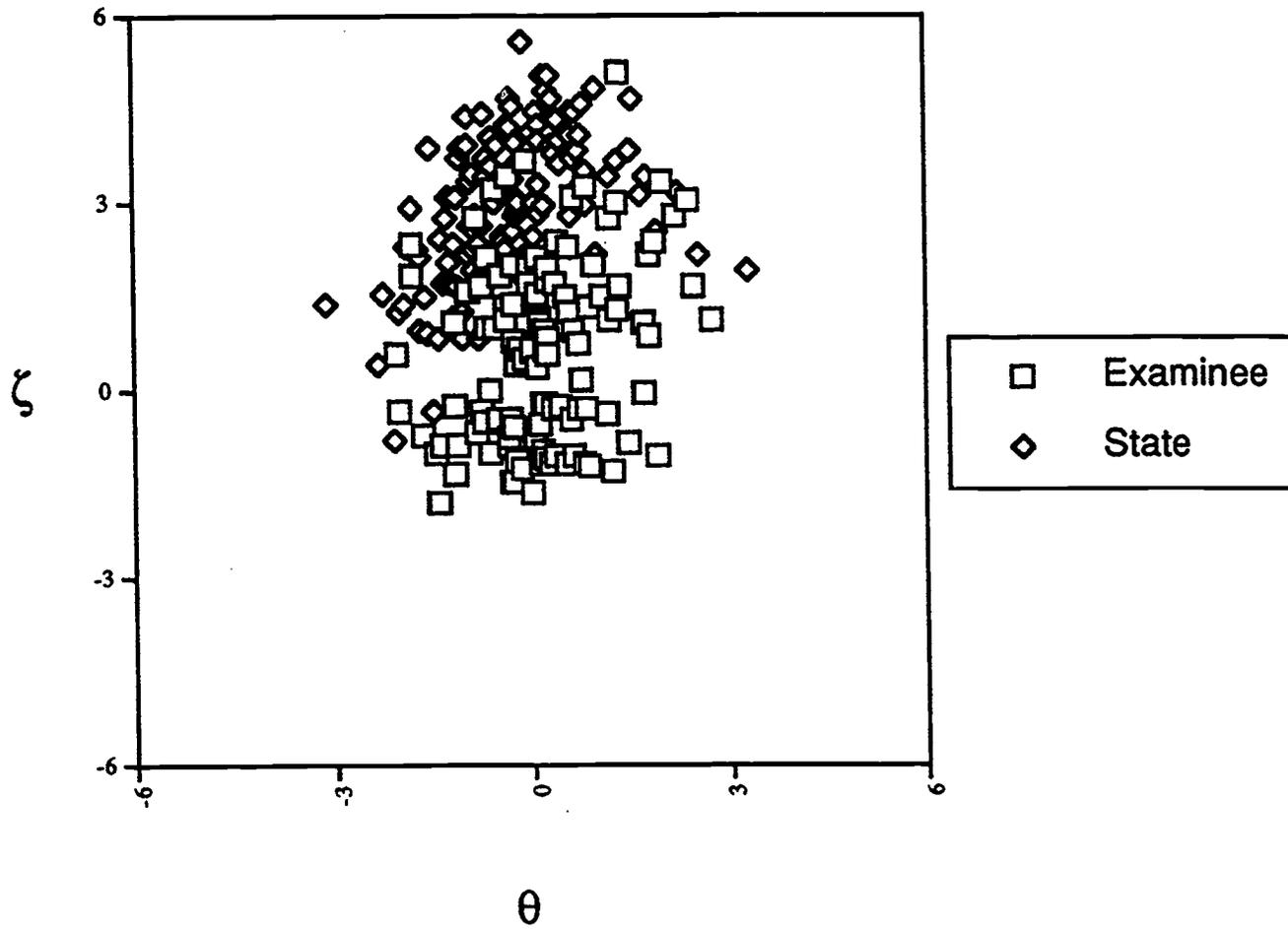


Figure 3. Knowledge states and examinee data for check matrix.

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