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ABSTRACT

This study investigated the effects of Oregon's Lane County "Problem Solving in Mathematics" (PSM) materials on middle-school students' attitudes, beliefs, and abilities in problem solving and mathematics. The instructional approach advocated in PSM includes: the direct teaching of five problem-solving skills, weekly challenge problems, and guided discovery lessons. Two 6th-grade teachers and four 7th-grade teachers, and their students, participated in the study. Half of the teachers at each grade level received training in PSM and used the heuristic materials with their students over a 1-year period. The others did not. The study contained both qualitative and quantitative components. Quantitative results indicated that the heuristic students placed less emphasis on the role of memorization and expected their teachers to ask thoughtful questions and not answer questions when the students did not know the answers; believed real mathematics problems could be solved by common sense; and were less dependent on the teacher and the textbook for identifying incorrect answers to mathematical problems. Qualitative results indicated that heuristic students preferred problems that made them think and believed that mathematics was useful, regardless of ability. The study concludes that the process of engaging in mathematical problem solving may positively affect students' attitudes and beliefs. Contains 35 references and the student interview guide. (MDH)

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An Investigation of the Effects on Students' Attitudes, Beliefs, and Abilities in Problem Solving and Mathematics After One Year of a Systematic Approach to the Learning of Problem Solving

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An investigation of the effects on students' attitudes, beliefs, and abilities in problem solving and mathematics after one year of a systematic approach to the learning of problem solving.

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Abstract

The purpose of this study was to investigate middle-school students' attitudes, beliefs, and abilities, in problem solving and mathematics after one year of instruction using Lane County's Problem Solving in Mathematics (PSM) materials. The study was conducted from April 1990-October 1990.

The instructional approach advocated in PSM includes: the direct teaching of five problem-solving skills, weekly challenge problems, and guided discovery lessons. The content of these lessons encompasses most of the topics taught in the elementary school curriculum.

Two 6th-grade teachers and four 7th-grade teachers, as well as their students, participated in the study. Half of the teachers at each grade level received training in PSM and used the materials with their students over a one-year period. (I herein refer to their students as the heuristic students.) The other teachers did not. (I herein refer to their students as the nonheuristic students.)

This study is both descriptive and causal-comparative in design, including quantitative and qualitative components. The quantitative component consisted of the results from a student questionnaire and a problem solving inventory. These two instruments were given to one hundred forty students. The qualitative component consisted of eighteen student interviews, which included the solving of four nonroutine problems; and an interview with the PSM project director.

The results of the quantitative component of the study revealed differences between the heuristic and nonheuristic students in perceptions of school mathematics and classroom practice. The heuristic students placed less emphasis on the role of memorization, a finding which supports the findings from the interviews. The heuristic students expected their teachers to ask thoughtful questions and not answer questions when the students did not know the answers. The heuristic students believed real math problems could be solved by common sense and were less dependent on the teacher and the textbook for finding out about incorrect answers to mathematics problems. When students were sorted by gender, no statistical significance was found in any of their responses.

The results of the qualitative component also revealed differences in several areas. The heuristic students preferred problems that made them "think" and believed that mathematics was useful, regardless of ability. Whereas many of the nonheuristic students equated how fast they could solve a problem with how much they understood a problem, the heuristic students claimed they could tell they understood a problem when they could solve the problem in different ways and explain the answer to someone else. The heuristic students had greater task persistence in solving problems than the nonheuristic students.

Many of the heuristic students considered the problem-solving skills they had learned to be rules or "steps" to solve all problems. In fact, many of the heuristic students equated problem solving in mathematics with the problem-solving skills they had learned at the beginning of the school year; problem solving was PSM.

This study indicates that the process of engaging in mathematical problem solving on a regular basis may positively affect students' beliefs and attitudes in a way encouraged by the vision of the NCTM's Curriculum and Evaluation Standards.

An investigation of the effects on students' attitudes, beliefs, and abilities in problem solving and mathematics after one year of a systematic approach to the learning of problem solving.

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Most people concerned with mathematics education today are talking about problem solving. Curriculum guides list problem-solving skills as key objectives at all levels (e.g., Oregon, California), professional organizations are advocating problem solving in the mathematics classroom (NCTM, 1989a; 1991), and conferences of mathematics educators' have numerous problem-solving sessions throughout their agendas.

In 1980, the National Council of Teachers of Mathematics issued a set of recommendations in the form of An Agenda for Action: Recommendations for School Mathematics of the 1980s. The first recommendation of this report is that "problem solving be the focus of school mathematics in the 1980s" (p. 1). More recently, In March 1989, the National Council of Teachers of Mathematics (NCTM) published Curriculum and Evaluation Standards for School Mathematics. This document contains a set of standards for mathematics curricula in North American schools, K-12. Once again, problem solving "rears its head." One of the five major goals for students is "that they become mathematical problem solvers" (NCTM, 1989a, p. 5). At the heart of these documents and most other calls for reform is the view that most mathematics curricula express a narrow view of mathematics.

> Current elementary school mathematics curriculum overemphasizes efficient computational arithmetic skill at the expense of understanding and problem solving. Most researchers and mathematics educators agree that there is more to mathematics than computational proficiency. But beyond this agreement exist diverse views about what it means to know, understand, and learn mathematics. (Putnam, Lampert, & Peterson, 1989, p. 304)

The results from the Fourth Mathematics Assessment indicate that most of the progress in mathematical performance of students at ages 9, 13, and 17 has occurred in the domain of lower-order skills and that the majority of the students failed to demonstrate skills and understanding in mathematical problem solving. An increase in age only widens the gap between students' expected and actual proficiency in this area. Dossey claims, "A nation that wants to continue to reap the benefits of modern technology and to compete in the future global economy depends on the skills of the young, and it appears that our students are ill-prepared to meet these challenges" (NCTM, 1989b, p. 134).

The purpose of this study was to investigate middle-school students' attitudes, beliefs, and abilities, in problem solving and mathematics. Half of the students in the study had one year of problem- solving instruction using Lane County's Problem Solving in Mathematics (PSM) materials; the other half did not. The study was conducted from April 1990-October 1990 with six different teachers and their students. These six teachers were from three different schools; five of the schools were rural schools; one was from an urban white-collar neighborhood. Comparisons were made between students that did and did not use the PSM materials. Specifically, I was interested in the following questions: Does one year of instruction with PSM make a difference in students' (a) definitions of and beliefs about problem solving in mathematics? (b) ability to solve nonroutine problems in mathematics? (Do they deliberately use any of the skills they have had training in while working problems?) and (c) attitudes towards and/or beliefs about mathematics?

The Teaching, Learning, and Assessing of Mathematical Problem Solving

PSM was developed by a team of mathematics educators led by Oscar Schaaf, a former mathematics educator at the University of Oregon and the PSM Project Director. According to Schaaf, it was the belief of the PSM writers that most textbooks did very little to develop students' conceptual understandings in mathematics. Whereas most textbooks used a direct approach to the teaching of mathematical skills, the writers of the PSM materials chose to use an instructional approach that guided students to discover mathematical relationships and algorithms.

This group of writers was influenced by theorists and teachers who advocated a method of teaching mathematics which evolved around discovery and problem solving (Young , 1924; Brownell,1942; Polya, 1957). Brownell (1942) discussed problem solving from various perspectives in the Forty-first Yearbook of the National Society for the Study of Education. Of special interest is his definition of a "problem" as compared to other learning situations. This distinction "lies in the peculiar relationship which exists between the learner and his task. It is a matter of common observation that what seems objectively to be the same situation may constitute for one person a puzzle, for another a problem, and for a third a condition with which he is thoroughly acquainted" (1942, p. 416).

If one accepts this idea of the personal nature of problem solving, then one must likewise accept the idea that a solution to a "problem" lies at the very heart of discovery--discovery by the person who attempts to find a solution. Young (1924) eloquently places this idea in his "Heuristic Method" of teaching mathematics. This method, as described by Young, is

> dominated by the thought that the general attitude of the pupil is to be that of the discoverer, not that of passive recipient of knowledge. The pupil is expected in a sense to rediscover the subject, though not without profit from the fact that the race had already discovered it. The pupil is a child tottering across a room, not a Stanley penetrating into the heart of Africa. The teacher stands before him; with word and smile entices him on; selecting the path, choosing every spot where he is to plant his foot, catching him when he stumbles, raising him when he falls, but when he has crossed the room he has done it himself and had made more progress towards walking whither he would than if he had been carried across the room and across hundreds of rooms, or even into the heart of Africa. It is the function of the teacher and of the text so to present the things to be done, so to propose the problems to be solved that they require rediscovery on the part of the pupil. (pp. 69-70)

Young claimed that the execution of any piece of mathematical work consisted of several parts, none of which may be neglected:

1. Grasping the problem, getting a clear idea of what is known and what is required.
2. Planning the work, deciding how to ascertain the desired information from the known facts. The first plan made may not be successful, but there should always be an intelligent plan.

3. Execution of the plan. This is carried so far that one is convinced either that the plan will not work (in which case he tries another), or until the result is attained.
4. Testing the results. Compare the result with the data of 1, and make certain that one has really done what he set out to do. (1924, p. 208)

Although George Polya is often called the "father of mathematical problem solving," one can see the similarities in Young's four steps, to Polya's four well used and documented phases of the problem solving process: 1. understanding the problem, 2. devising a plan, 3. carrying out the plan, and 4. looking back. (1957, pp. xvi-xvii) These four phases form the basis of strands in problem solving evident in many mathematics textbooks used today.

Discussions on the teaching of problem solving are entwined with controversies, especially those surrounding the pros and cons of explicitly teaching specific problem-solving skills and/or strategies (often called "heuristics"). With as many research studies that claim the teaching of heuristics does positively affect one's problem-solving ability (Bruner, 1963; Lester, 1985; Kantowski, 1974; Webb, 1975; Schoenfeld, 1985; Putt, 1978; Peacock, 1979; Mayes, 1980) there are an equal number of studies that claim they do not. (See Begle's summary of 75 empirical studies on problem-solving strategies, 1979.)

One of the major challenges facing mathematics education is the assessment of problem-solving attitudes and abilities. There are very few, if any, reliable "paper and pencil" tests for measuring problem-solving abilities. As with all higher-level thinking skills, they are very difficult to measure. Another reason given to the complexity of assessing mathematical problem-solving ability is the type of knowledge involved in the problem-solving process. Previous mathematical knowledge, affects, beliefs, resources, and socio-cultural conditions are just a few of the factors (Lester, 1990; Schoenfeld, 1985). The multi-dimensionality of the problem-solving process is made evident as attempts are made to look at <u>all</u> thinking done to solve a problem.

Throughout the past fifteen years, there has been an increasing amount of research directed toward linking problem solvers to the cognitive processes they employ. One of the most common data gathering techniques used in problem-solving process tracing is the "think aloud" technique (Schulman and Elstein, 1975; Kilpatrick, 1967; Lucas, 1972; Kantowski, 1977). These are used to obtain verbal protocols of ideas and thoughts that occur to a problem solver. Usually, an audiotape recording is made and some coding procedure is used to record key cognitive behaviors of the subject.

One of the most common paradigms of protocol codings is based on Polya's phases of the problem-solving process and involves an analytic scoring scale. Analytic scoring is an evaluation method that assigns point values to various dimensions of the problem-solving episode (Charles, Lester, & O'Daffer, 1987). At the Oregon Department of Education, an analytic approach is being developed to score written protocols of open-ended problems in mathematics. This scoring assesses four dimensions of the problem-solving process: conceptual understanding, procedural knowledge, problem-solving skills and strategies, and communication. Each dimension can receive a score ranging from 1 (low) to 5 (high). Papers are assessed by trained raters who are normally classroom teachers.

The Instructional Approach Used in <u>PSM</u>

<u>PSM</u> is a program of problem-solving lessons and teaching techniques for grades 4-8 and (9) algebra. The instructional approach utilized throughout the

program entails the following: the direct teaching of five problem-solving skills, weekly challenge problems, laboratory work, small-group discussions, nondirective instruction, individual work, and guided discovery lessons. The content of these lessons encompasses most of the topics taught in the elementary school curriculum.

Each grade-level book contains approximately 80 lessons and a teacher's commentary with teaching suggestions and answer key for each lesson. PSM is not intended to be a complete mathematics program in itself. Neither is it supplementary in the sense of being extra credit or to be done on special days. Rather it is designed to be integrated into the regular mathematics program.

Five fundamental problem-solving skills are presented in the "Getting Started" section of each PSM book. The skills taught to the students used in this study were the following: guess and check, look for a pattern, make a systematic list, make a drawing or model, and eliminate possibilities. In teaching these skills, a direct mode of instruction was recommended. Problem-solving skills were to be demonstrated by teachers as a method for solving a specific puzzle-type problem. Students on five succeeding days were asked to practice this skill in solving other puzzles. The intention was to focus on skills rather than problems, spending no more than ten minutes each day on this phase of instruction.

By concentrating on these skills during the first few weeks of school, it was the hope of the PSM writers that the pupils would have confidence in applying them to problems that occurred later on, especially the "Weekly Challenges" and laboratory-type problems. The challenge problems left the choice of the problem-solving method up to the pupil.

Design of the Study

This study is both descriptive and causal-comparative in design, including quantitative and qualitative components. The quantitative component consisted of the results from a student questionnaire and a problem solving inventory. These two instruments were given to one hundred forty students. The qualitative component consisted of eighteen student interviews, which included the solving of four nonroutine problems; and an interview with the PSM Project Director.

Description of Subjects

Two sixth-grade teachers, four seventh-grade teachers, and their students participated in the study. Half of the teachers at each grade level had received training in PSM and used the materials with their students over the 1989-1990 school year. These teachers had been using the materials for at least five years prior to the study. (I herein refer to their students as the "heuristic" students.) The other half of the teachers had not received training on the use of the materials, and did not use the instructional approach as advocated in the PSM program with their students. (I herein refer to their students as the "nonheuristic" students.)

Description of Instruments

During April and May of the 1990 school year, two assessment instruments were given to all students in each teacher's class; a total of 140 students. The first was a "Problem Solving Inventory" which consisted of six problems. Five problems were taken from the "Getting Started" section of the PSM materials, one per problem-solving skill. A multiplication problem was given as the sixth problem. At the end of the inventory, students were asked to rank the six problems from their most favorite to their least favorite. The purpose of this was to find out if there were differences in where the nonheuristic students and heuristic students placed the computation problem.

Two major issues were considered in the scoring of the inventory problems: (a) Did the student get the correct answer? (b) Did the student use the problem-solving skills associated with the problem in the PSM materials? The scoring of the answer comprised three categories: 1 = correct answer, 0=incorrect answer, Ø= no solution attempt. The scoring of the problem-solving skill comprised the following three categories: 1=the student appeared to use the intended skill, 0=the student appeared to use a skill other than the one indicated by the PSM materials, Ø=no evidence of a problem-solving skill (answer only). A χ^2-test of independence was calculated for these data.

The second instrument was a student questionnaire which contained 39 multiple-choice questions related to students' perceptions of mathematics and school practice, views of school mathematics, motivation, and personal and scholastic performance. This questionnaire was adapted from Schoenfeld (1989). The purpose of the questionnaire was to explore the relationship between students' beliefs about mathematics and their problem-solving instruction (i.e. through PSM).

The questionnaire was analyzed using the statistical program StatView 512+. Means and standard deviations were recorded for both heuristic and nonheuristic students. T-tests were calculated for unpaired comparisons, with p-values corresponding to a two-tail test of significance.

Student Interviews

After the two instruments were given to each teacher's class in the spring of 1990, the classes were sorted by student assessment of ability. This information came from the questionnaire. Three students per teacher were chosen by randomly picking their questionnaires out of the total class. This stratified sampling technique was used so that each ability level (low, average, high) would be represented. Teachers verified that the students were indeed of the indicated ability.

To conceptualize and measure some of the more metacognitive components of problem solving and mathematics for this study, I chose to use structured interviews, with necessary probes, as a major source of data. The interview questions examined six areas: (a) memories of mathematics classes; (b) issues directly related to problem solving in mathematics, including PSM; (c) issues related to school mathematics; (d) issues related to beliefs of mathematics; (e) attitudes toward different types of problems in mathematics; and (f) abilities in solving nonroutine problems in mathematics. At the ends of the interviews, students were asked to solve four nonroutine problems. (See Appendix for interview questions and problems.) I told the students that the problems were the kind that everyone solved differently, and that I was interested in their thoughts as they worked on the problems rather than if they got the problems right or wrong. Students were asked to "talk aloud" as they worked on the problems and to record their work without erasures on the paper. Interviews lasted from one hour and fifteen minutes to almost two hours. Interviews were held during August and September of 1990. All interviews were audio-recorded and field-notes were taken throughout the interviews.

Students' solutions to the nonroutine problems were analyzed using the analytic scoring scale being developed at the Oregon Department of Education for the scoring of open-ended problems in mathematics. After the interviews were conducted, I listened to the audiotapes and recorded all students' comments as they worked on the problems. These comments were considered when I rated the communication dimension, and also provided insights into the problem-solving skills and strategies used by the students.

Results of the Study

Summaries of the results of the quantitative and qualitative components of the study are discussed separately in this section. I have elaborated further in the qualitative summary by (a) highlighting what I believe to be the three key findings, and (b) discussing the findings as to their impact on the teaching and learning of problem solving and mathematics.

Results of Quantitative Component of Study

The results of the Problem Solving Inventory would indicate there were little if any, differences in problem-solving ability between the heuristic and nonheuristic students. Although some significance was found with the solutions of the problem involving the skill of eliminating possibilities for seventh-grade students, there was a similar significant finding with the same problem for the sixth-grade nonheuristic students.

The results of the Inventory indicate that students do and can use problem-solving skills which have been taught to them. The greatest evidence of this came from the nonheuristic students, though, rather than the heuristic ones. A working backwards algorithm for solving certain types of number puzzles and the use of logic charts for solving logic problems had been taught by the sixth-grade teacher of the nonheuristic students. The nonheuristic students used these strategies in solving the problems when the strategies were applicable to the problem. The problem-solving skills of guess and check and look for a pattern seemed to be intuitive skills for all students, regardless of training in these skills.

To draw conclusions regarding problem-solving ability through the use of this instrument alone was not possible. The differences I found could have been the result of the students' ability levels or certain teachers' behaviors. (The sixth-grade teacher of the nonheuristic students claimed her group of students were of a higher ability than normal.)

The results of the Student Questionnaire, in relation to the questions which were statistically significant (using a χ^2-test of independence, $p<.05$), revealed differences between the heuristic and nonheuristic students in perceptions of school mathematics and classroom practice. The heuristic students placed less emphasis on the role of memorization, a finding which supports the findings from the interviews. The heuristic students expected their teachers to ask thoughtful questions and not answer questions when the students did not know the answers. The heuristic students believed real math problems could be solved by common sense and were less dependent on the teacher and the textbook for finding out about incorrect answers to mathematics problems. When students were sorted by gender, no statistical significance was found in any of their responses.

Results of Qualitative Component of Study

Overall, the heuristic students recalled aspects of their PSM instruction, especially the specific skills of guess and check and look for a pattern. Many of the heuristic students considered the problem-solving skills they had learned to be "steps" to solve problems. All of the heuristic students recalled the weekly challenge problems.

In comparing the heuristic students to the nonheuristic students, several differences were found. The heuristic students preferred problems that made them "think" and believed that mathematics was useful, regardless of ability. They also placed less emphasis on the role of memorization in learning mathematics. Whereas many of the nonheuristic students equated how fast they could solve a problem with how much they understood a problem, the heuristic students claimed they could tell they understood a problem when they could solve the problem in different ways and explain the answer to someone else. The heuristic students had greater task persistence in solving problems than the nonheuristic students.

Very few differences between both groups of students were found in the following areas: likes and dislikes of their mathematics classes, definitions of problem solving in mathematics, willingness to work on nonroutine problems, and beliefs that mathematics was a discipline in which one could be creative and make personal discoveries.

While working on problems at the ends of the interviews, differences between heuristic and nonheuristic students were noted in several areas. The heuristic students tended to do the following: (a) generalize their solutions more, (b) verify, through estimation, the reasonableness of their results, (c) verbalize their solution strategies using the vocabulary they had learned through the PSM materials, and (d) approach problems deductively.

Some specific instances of creativity in problem solving were noted with the nonheuristic students which were not observed with the heuristic students. This was especially true with problems that could most efficiently be solved by making a systematic list.

There are many areas of this study that have implications for the teaching, learning, and assessment of problem solving and mathematics. Three findings are highlighted that are especially unique to the present study and have the greatest potential to impact the teaching of problem solving.

1. Problem-solving skills were internalized by heuristic students as rules or steps to solve all problems.

Many of the heuristic students considered the problems-solving skills they had learned to be steps or rules to solve all problems. Students informed me that if they got stuck on a problem, either at home or at school, all they had to do was go through the skills and it would usually end up that one of them would work. Students reported doing this at home before they went to their parents for help. One teacher also tested his students on the five problem-solving skills. Many of the heuristic students equated problem solving in mathematics with the problem-solving skills they had learned at the beginning of the school year; problem solving was PSM.

2. Working on weekly challenge problems may have increased heuristic students' perseverance in solving problems.

All of the heuristic students in this study recalled the weekly challenge problems which were part of their PSM instruction. Some students called them challenge problems; other students referred to them as weekly problems.

When students were asked what was the longest they had worked on a problem, the majority of the heuristic students answered in terms of hours, days, and weeks. On the other hand, the majority of the nonheuristic students answered in terms of minutes.

When asked how long they would work on an impossible problem (By impossible, I meant a problem which has no solution.), further evidence strengthened this difference. Overall, the responses from the heuristic students ranged from four hours to one week. The range in responses from the nonheuristic students was five minutes to two periods.

Another finding of this study was that the heuristic students did not equate speed with which one could do a problem to understanding as did many of the nonheuristic students. I believe this finding is related to the issue of perseverance. The heuristic students realized that the understanding of and solution to a problem would come through working on the problem during an extended period of time.

3. All heuristic students perceived mathematics to be useful.

All of the heuristic students claimed that mathematics was useful and gave various examples of situations where one uses math. These included money, estimation, shopping, building, sewing, measuring, architecture, percents, and baking. The three high-level nonheuristic students gave answers similar to the heuristic students. On the other hand, the average-level nonheuristic students claimed it was useful only because you will need to know it when you get older and have a job. The responses of the low-level nonheuristic students indicated that they did not find mathematics useful, especially where daily applications were involved. One low-level student said, "It is useful for next year--the beginning of the year review. No other way."

When students in this study were asked if their problem-solving instruction helped them solve problems in their other classes, all the heuristic students indicated many areas of application. The low-ability heuristic students gave answers in relation to mathematics rather than problem solving, but the point is that they held a perception of problem solving and mathematics as being useful.

I had to word the problem differently for the nonheuristic students by asking them about their mathematics instruction rather than their problem-solving instruction. Whereas the high-ability students gave me responses similar to the heuristic students, the average-ability students could only think of situations where they had measured, and the low ability students could not think of any other classes where their mathematics instruction had helped them solve problems.

Another finding of this study related to the concept of usefulness came out when students were asked about problems they did not like to work on. One of the main reasons the heuristic students did not like to work on certain problems was because they found the problems irrelevant and saw no applications in the "real world." On the other hand, the nonheuristic students did not mention irrelevance at all with regard to problems they did not like. They claimed the problems were boring or else they blamed their teachers' instruction for the reasons they did not like the problems.

The findings of this study indicate that all heuristic students perceived mathematics as being useful, regardless of ability. The high-ability nonheuristic students perceived the usefulness of mathematics, but this perception was not consistent with the average and low-ability nonheuristic students where beliefs toward the utility of mathematics decreased with ability level. In relation to the usefulness of mathematics, Fennema and Sherman (1977, 1978) found that perceived usefulness of mathematics correlated positively with achievement. This seemed to be true with the nonheuristic students, but not true with the heuristic students, where all students perceived mathematics as being useful.

Discussion: Impact of These Finding on the Teaching of Problem Solving and Mathematics

Finding One: Problem-Solving Skills as "Rules"

There are both positive and negative aspects of this finding. Knowing they had a set of skills or rules to fall back on when faced with a problem they did not immediately know the solution to gave the heuristic students confidence in solving problems. The skills also gave them a place to start in the solution process. This worked for them on daily problems, test problems, and homework problems. Students were also able to state specific instances where their problem-solving skills had transferred to other classes as well as situations outside the school environment. The deliberate use of problem-solving skills appeared to empower the students in their own learning.

I question what could happen if students are faced with a problem where none of the five skills they have learned are appropriate as a starting point for solving the problem. By viewing problem-solving skills as rules, students might limit not only their problem-solving ability, but their creativity in solving problems. Although the present study does not give evidence that learning problem-solving skills interfered with the heuristic students' abilities to solve problems, it did give some evidence, when comparing heuristic to nonheuristic students, of a lack of creativity in solving problems. An example of this phenomenon is shown with Figures 1 and 2. Figure 1 is a typical heuristic student's solution to the dartboard problem which can be efficiently solved by making a systematic list, a problem-solving skill explicitly taught in the PSM materials. (Notice that the student forgot the possibility of a single dart landing in each of the rings.) Figure 2 is a more creative solution to the problem, though equally efficient, given by a nonheuristic student.

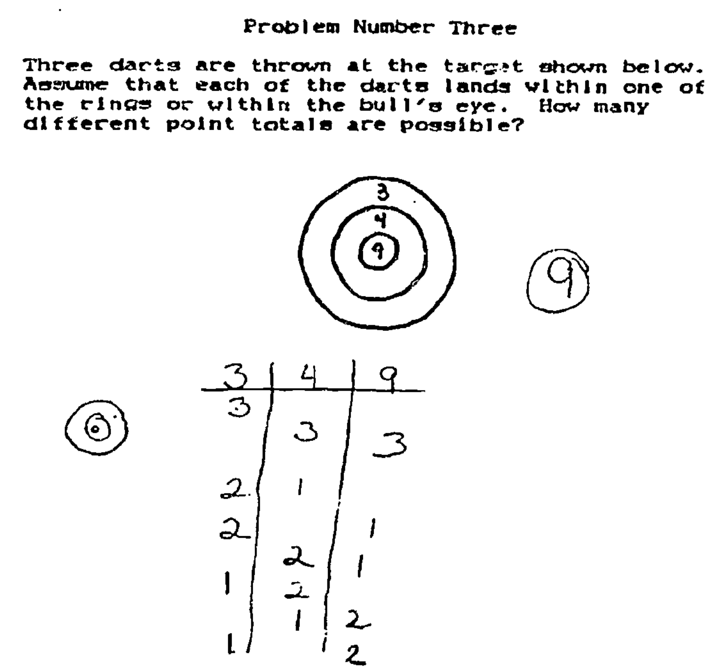

Figure 1. Example of systematic list strategy used by 6th grade heuristic student.

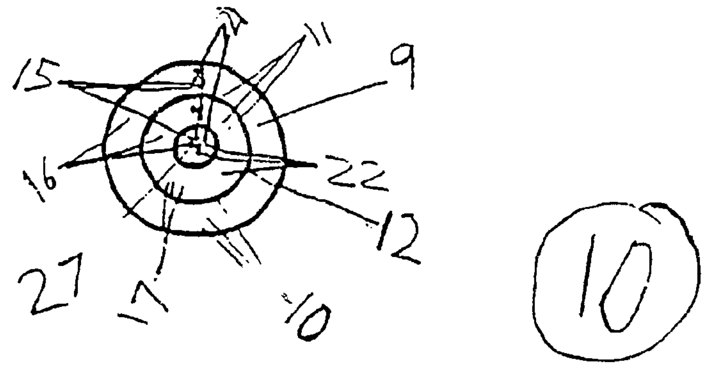

Figure 2. Example of systematic recording scheme used by 6th grade nonheuristic student.

According to a constructivist view of learning and knowledge, children do not simply absorb mathematical knowledge as it is presented, but impose their existing frameworks of knowledge to incorporate and invent new knowledge (Putman, Lampert, & Peterson, 1989). Some researchers believe that many children appear to develop a framework which views mathematics as a collection of rules and procedures (Ginsburg, 1977; Resnick 1986; 1987). Kleinman claims, "Learning mathematics is seen as memorizing facts, formulas, and procedures--as simply absorbing knowledge that has been handed down for generations" (1991, p. 48).

If students do internalize this view of learning mathematics as researchers claim, and the initial introduction to problems solving in mathematics is through the direct teaching of problem-solving skills, (which would fit into their belief structure or framework of how mathematics should be taught), then it makes sense that the students would consider these skills to be rules or steps to solve all problems.

If we want students to view certain problem-solving skills as more than rules or steps to solve <u>all</u> problems, then the skills which are directly taught must be placed into the perspective of being only a subset of many skills one can use to solve problems. Perhaps the metaphor of placing these skills into a "toolkit" of problem-solving skills and strategies would be helpful.

The skills from this toolkit that are directly taught to the students should be the ones that are less intuitive, such as the skills of making a drawing or model, simplifying a problem, and working backwards. As teachers solve problems and model the problem-solving process, they should discuss why they used particular skills and strategies to solve the problems. Teachers should make explicit the more intuitive skills, such as guessing and checking and looking for patterns, as students discuss how <u>they</u> solved their problems. This would help increase the students' repertoire of skills, thus, making students less apt to consider certain problem-solving skills to be rules or steps to solve all problems.

I believe that students' explanations of how problems are solved as well as why particular skills and strategies are used, would also help students realize that different skills and strategies can solve the same problems. The writers of the PSM materials held the philosophy that teachers can teach skills but not strategies. This philosophy strengthens the notion that different perspectives to the same problem would help students realize the importance and necessity of this personal relationship between the problem and the problem solver. Perhaps these explanations would also help students develop strategies to select among skills and not blindly apply skills as rules.

The writers of the PSM materials wanted the teaching of skills to give students a language with which students and teachers could talk about the strategies they used to solve problems. This certainly happened, as the heuristic students used the vocabulary of the PSM skills as they solved problems. This vocabulary was completely lacking in problem-solving explanations from the nonheuristic students.

Over forty problem-solving skills are listed in the Introduction of the PSM books. Problem-solving skills, other than the ones taught at the beginning of the school year, are listed in the teacher commentaries for each PSM activity and problem. In the Introduction to the materials, though, discussions on the use of skills other than the ones specifically taught in PSM are mentioned almost in passing: "Such skills as 'guess and check,' 'make a systematic list,' 'look for a pattern,' or 'change a problem into one you can solve' are seldom made the object of direct instruction. These skills, as well as many more, need emphasis" (LCMP,1983, p. iv). Was the message intended by the writers of the materials, through listing other skills, strong enough to be learned by the users of the PSM materials? Perhaps the use of alternative strategies and skills could be emphasized more by explicitly asking students throughout the PSM materials to find more than one way to solve the problems.

Finding Two: Challenge Problems and Perseverance

This finding on perseverance has implications not only for the teaching of problem solving, but for the teaching of mathematics in general. Perseverance in solving problems is important, as students need to realize that some problems just take time to solve. This attitude will "better prepare them to solve problems they are likely to encounter in their daily lives" (NCTM, 1989a, p. 76).

Working on a problem throughout the course of a week is an important aspect of the instructional approach to PSM. According to the writers of the materials, the challenge problems "leave the choice of the problem-solving method up to the pupil. The intention is to allow for and encourage individual differences, creativity, and cooperation" (LCMP, 1983, p. 215). Although the intent of using the challenge problems was not to increase students' perseverance in solving problems, the results of the present study indicate the challenge problems may have done just that. Further studies need to be performed to verify this claim.

The importance of perseverance in solving problems is supported through the Standards: "In addition to cooperative effort, real-world problems often require a substantial investment of time. Students should be encouraged to explore some problems as extended projects that can be worked on for hours, days, or longer" (NCTM, 1989a, p.77). The use of weekly challenge problems also supports a strong push away from the belief that mathematics is nothing more than a set of problems that can be solved within a few minutes or less and with very little thinking. Because of this, embedding weekly challenge problems into the mathematics curriculum has the potential to positively change students' perceptions of mathematics.

Finding Three: The Usefulness of Mathematics

As with the area of perseverance, this finding has implications for the teaching of problem solving and mathematics. The students who had the teaching of problem solving integrated into their school mathematics experience perceived mathematics as being more useful than those who did not have this experience. I believe one of the reasons students become disenchanted with mathematics is because they do not see its usefulness or relevance in their daily lives. I also claim that students are more motivated to learn a content area if they perceive it as being useful. I believe perceptions towards the usefulness of mathematics are also directly related to the area of transfer. It is important for students to perceive mathematical relevance and application in arenas outside the mathematics classroom.

Unfortunately, I do not know what it was within the heuristic students' mathematics instruction that caused this perception. Many of the heuristic students considered their problem-solving instruction to be an opportunity to think and "use their brain," especially in relation to the challenge problems. Is the amount of "thinking" necessary in a content area related to its perceived usefulness? If so, then it could be that the weekly challenge problems or other aspects of their PSM instruction affected their perceptions towards the usefulness of mathematics as well as their perseverance in solving problems.

Recommended Applications

Based on the results of this study, I make the following recommendations for the teaching and assessment of problem solving.

Recommendations for the Teaching of Problem Solving:

1. Teachers continue teaching problem-solving skills, but continuously stress that these skills are not exhaustive, nor are they rules or procedures for solving all problems.
2. Teachers take every opportunity possible to use students' discussions of problem-solving strategies to capitalize on and make explicit many different problem-solving skills.
3. Teachers model how they decide which strategy they use to solve mathematical problems.
4. Teachers encourage their students to explain not only how they solved mathematical problems, but why they chose the particular strategies they used.
5. Teachers use weekly challenge problems as a way of increasing students' perseverance in solving problems.
6. Teachers place more emphasis on the "looking back" phase of the problem-solving process--to include verification, making conjectures and generalizations, and problem posing.

Recommendations for the Assessment of Problem Solving:

1. Teachers use individual interviews within their classrooms to find out what their students are thinking as they solve problems.
2. Teachers as well as others interested in the assessment of students' problem-solving abilities do not rely solely on written work.
3. Teachers embed the assessment of problem-solving ability and attitudes regularly into their mathematics instruction.

Concluding Thoughts

I believe this study has important implications for the assessment of mathematical problem solving. Although I had written protocols from almost 140 students, I certainly learned more from listening to the eighteen students solve problems than I did from the 140 written protocols. From the eighteen, I learned the importance of studying what a person does and is thinking, rather than just what they produce. We may be able to assess procedural skills through written work alone, but to truly get at some of the other dimensions of the problem-solving process, there is nothing that surpasses sitting down with a child and communicating with him or her, particularly when considering some students' difficulties with writing.

Obviously, we can not assess problem-solving ability through a multiple-choice test. One must solve problems. But, it is very important for those involved in assessing students' work through written protocols alone, to realize the limitations of that assessment, and place them into their proper perspectives. One is not assessing just problem-solving ability; one is also assessing written communication. If students have not been given opportunities to communicate their metacognitive thoughts in writing, then it may be unfair to assess them solely in this way.

There is certainly a hope that by giving open-ended problems on state assessments, teachers will begin incorporating more open-ended problems into their instruction. Perhaps the greatest power in implementing performance-based assessments lies in the potential to bring about this change within the classroom. Studies in Holland have found that because their tests contain problems that are rich in structure and that require students to perform a wide range of mathematical actions, mathematics instruction tends to emphasize such problems and make similar demands on students (Schwartz, 1991). Schwartz further claims that if we are going to use assessment to constructively influence the teaching and learning of mathematics, at least two conditions must prevail. "First the assessments we use must not contradict, either explicitly or implicitly, our pedagogic goals....They must not convey, as they do now, an image of mathematics that is at odds with the nature of the discipline....The second condition is that the test questions must be, at a minimum, mathematically interesting" (1991, p. 139).

But, Cizek (1991) believes "it is wrong to promote the false notion that simply changing the form of the assessment will ensure better classroom instruction or make assessments immune to the corruption we wish to avoid" (p. 151). According to Zessoules and Gardner (1991),

> Just as standardized testing has driven curriculum and instruction in our schools, so too the implementation of new measures must influence and shape the daily life and activities in the classroom. Unless new modes of assessment reach deep into school culture, incorporating pedagogical approaches, expectations and standards of performance, and the education of students' own capacities for self-critical judgment, new forms of testing will be as discontinuous with teaching and learning as they have ever been. (p. 50)

The goal of assessment should be to improve teaching and learning. If it is possible to structure the assessment of problem solving in such a way that one cannot tell when learning or teaching stops and assessment begins, then we would truly be on our way to the authentic assessment of problem solving.

Observations from this study, as well as related research studies, indicate that there is still much to be learned about the teaching and assessing of mathematical problem solving. There are also many other questions that need to be raised, many of which I am just now beginning to appreciate and to realize are important. The question of whether or not the teaching of specific problem-solving skills makes a difference in one's problem-solving ability may or may not be important. The research in this area needs to be expanded to include not only skills in problem solving, but the affective domains of attitudes and beliefs in both mathematics and problem solving. Does the very process of engaging in mathematical problem solving affect those beliefs and attitudes? This study does give indication that this may indeed be the case.

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Appendix

STUDENT INTERVIEW GUIDE

1. What kinds of things stand out most in your mind about your mathematics class with ______________?
2. Were there things you especially liked about the class?
3. Were there things that you especially did not like about it?
4. If I had been a little bird observing one of your math classes with _______, what kinds of things would I have seen happening during a typical math lesson?
5. Were there any types of problems that you especially liked to work on in your math class with ______________? (Describe.)
6. Why did you like those types of problems?
7. Were there any types of problems you did not like to work on? (Describe.)
8. Can you tell me why you didn't like those types of problems?
9. Did your teacher last year give you any problems like logic puzzles, brain teasers or mind stretchers--problems that were kind of thought provoking?
10. Do you remember any of these problems?
11. How often would you say you were given these problems?
12. Did your teacher ever discuss these problems with your or your classmates?
13. Can you describe to me how the teacher went over these problems? (Did the teacher ask lots of students how he/she did the problems?)
14. Were you allowed to work with someone else on the problems?
15 Do you like those types of problems? Why/why not?
16. When I say "problem solving in math," what does that mean to you?
17. Do you remember receiving any math problem solving instruction?
18. (IF YES) What do you remember about it? (If necessary, probe on problem-solving skills such as guess and check, look for patterns, etc.)
    IF NO) Do you remember learning anything about "guess and check," or "look for a pattern?"
19. (If appropriate) Can you remember any other problem solving skills you might have learned about in ___________ class?
20. How would you describe one of your mathematics problem solving sessions to your best friend?
21. Do you feel that your problem solving instruction in your math class has helped you solve problems in any of your other classes--not just math, but any classes? (Describe.)
22. Did you feel that your problem solving instruction has helped you solve any types of problems outside of school? (Describe.)
23. Do you think mathematicians work alone on problems or together? Which do you think is better, and why?
24. Are the different mathematics topics you've studied related to each other in any way? If so, how?
25. How much of your ability to DO math shows up when you take math tests?
26. What can you do if you get stuck while doing a math problem?
27. In what way, if any, is the math you've studied useful? The arithmetic, the geometry?
28. Do you think that students can discover mathematics on their own, or does all mathematics have to be shown to them? Please explain.
29. If you understand the material, how long should it take to solve a typical homework problem?

30. Do you think your math teacher should ever give you a problem to work on that is impossible? Why/Why not? (If yes: If you didn't know that a problem was impossible, how long do you think it would be reasonable to work on the problem before you would know it's impossible?)
31. How can you know whether you understand something in math? What do you do to measure (test) yourself?
32. How important is memorizing in learning mathematics? If anything else is important, please explain how.

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How would you feel working on some math problems for me right now?
Will you talk to me about how you are solving the problem?

(Show student three different types of problems and ask him/her to solve them. Stress to the student that I am only interested in how he/she thinks as she/he goes about solving the problem--not just the answer. Stress that everyone does them differently!)

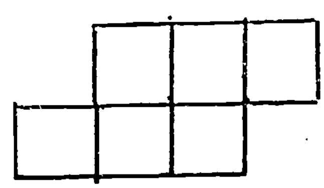
(After the student has spent some time on the problems they were asked to rank them from their most favorite to their least favorite.)

INTERVIEW PROBLEMS

1. I want you to pick five digit-cards from the stack. There are ten cards altogether with the digits 0-9 on them.
   Digits _____ _____ _____ _____ _____
   Use these five digits to form a 2-digit and a 3-digit number so their product is the largest possible. Then find the arrangement that gives the smallest product.
   You may use the calculator.

2. A group of 8 people are going camping for three days and need to carry their own water. They read in a guide book that 12.5 liters are needed for a party of 5 persons for 1 day. How much water should they carry?

3. Can you add tiles to this figure to make a new figure with a perimeter of 18 units? Tiles much touch each other along an entire edge.



4. Three darts are thrown at the target shown below. Assume that each of the darts lands within one of the rings or within the bull's eye. How many different point totals are possible?