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ABSTRACT

The purpose of this study was to find out whether students perform differently on algebra word problems that have certain key context features and entail proportional reasoning, relative to their level of logical reasoning and their degree of field dependence/independence. Field-independent students tend to restructure and break stimuli into parts and to perceive details more readily than field-dependent students. The underlying theoretical view is that context may be an important factor in how students approach, analyze, and restructure word problems. The sample included university students (n=37) and secondary school students (n=193) from two large high schools in two cities. The Gottschaldt Hidden Figures Test was used to assess field dependence/independence. Selected items from the Equilibrium Balance Test were used to assess Piagetian stages of logical reasoning. A 2 x 3 MANOVA was used to analyze the effects of cognitive style (dependence, independence) and operativity (concrete, transitional, formal). Overall, field-independent subjects who were formal operational reasoners performed highest across all the problem features. The results supported the influence of cognitive style, together with cognitive development, in mediating a student's ability to solve algebra word problems. Contains 34 references and 2 test references. (Author/LDR)

"The Effects of Cognitive Style and Piagetian Logical Reasoning on Solving a Propositional Relation Algebra Word Problem"

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ABSTRACT

The purpose of this study was to find out whether students perform differently on algebra word problems that have certain key context features (i.e., clothing to the problem's structure) that entail proportional reasoning relative to their level of logical reasoning and their degree of field dependence/independence. A convenience sample of 37 students was obtained from a comprehensive university and a second convenience sample of 193 secondary school students was obtained from two large high schools in two cities.

Three instruments were used. The Gottschaldt Hidden Figures Test (HFT) (Crutchfield, 1975) was used to assess field dependence/independence. Selected items of the Equilibrium Balance Test (EBT) (Adi, 1976), was used to assess Piagetian stages of logical reasoning. A domain referenced set of 16 algebra word problems which systematically varied key problem features was used to measure the dependent variables.

A 2x3 (cognitive style x operativity) multivariate analysis of variance MANOVA was used to analyze the data. Main effects were examined for cognitive style (field-dependence and field-independence) and operational reasoning (concrete, transitional and formal).

Main significant effects for operatitvity and cognitive style were found for problems with all the key context features with exception to the concrete feature problems. These same effects of operativity and cognitive style were found on the rescored algebra problems with the verbal and pictorial features. There was no significant interaction between operativity and cognitive style on all the problem features. Overall field-independent subjects who were formal operational reasoners performed highest across all the problem features. These results supported the importance of cognitive style (i.e., field independence) as opposed to cognitive development alone in students' ability to solve algebra word problems.

Paper presented at the North England Educational Research Organization in Portsmouth, New Hampshire.

Introduction

It is well documented from the literature on Piaget's developmental levels that proportional reasoning is a strong predictor of formal operational reasoning, even when the task of proportional reasoning is embedded in mathematics problems (Linn, 1978). However, this predictive relation may not be free of the effects of individual differences in cognitive style.

In the past decade, researchers such as Linn (1978) and Lawson and Wollman (1977) have viewed student success on formal reasoning tasks as being bound by other factors found in a problem-solving situation. Some of these factors are embedded in the problem (or item) and are usually considered to be contextualized information; i.e., clothing to the problem's structure.

The factors or elements in the problem-solving situation have a two-fold influence. First, these elements or problem features could help students to disembed elements in a problem that might be hidden or appear to be hidden to students. Second, these contextualized problem features may act as interference elements to students' abilities to approach, disembed, and analyze the problem's elements. These opposing effects of the contextualized features of word problems restrict full reasoning approaches as these effects tend to occur in time before the reasoning processes dominating this stage in the process of solving word problems.

Several tests have been developed to evaluate levels reasoning. Some of these tests have used algebra tasks and in particular math problems which inherently include proportions (Adi, 1978). Furthermore, it has been suggested that a propositional relation problem might be one of another form of a proportional reasoning task because it asks the solver to set up ratios in metric form (see Lawson and Wollman, 1977). Also, the proportion problem has been empirically found to have a strong relationship to proportional reasoning abilities and skills (Niaz, 1989 and Lawson, 1978).

Numerous studies have investigated students' abilities to solve a propositional relational algebra word problem (e.g., Lochhead and Mestre, 1988; Mestre and Gerace, 1986). However, there is a paucity of research in relating performance on the relational proposition problem to Piagetian levels of intellectual development and individual differences in field dependence/independence, which is a key factor or element of cognitive style. Field-independence refers to the tendency of subjects to restructure and break stimuli into parts and to perceive particular details more readily than field-dependent subjects (Witkin and Goodenough, 1977). Furthermore, those studies that attempted to study the effect of cognitive style have usually used content and structure rather than the presentation or context. Also, there are no studies we could find which analyzed the effects of cognitive style on the pictorial, symbolic and verbal propositional relation problems with several key contextual features.

This study, therefore, investigated students' performance on propositional relation algebra word problems as it relates to cognitive style and cognitive development constructs. These domain-referenced set of algebra word problems systematically varied key contextual features of these word problems such as presentation and responding formats (i.e., pictorial, symbolic and verbal) and response modes (generative or passive translations) as well as problem familiarity, imageability and variable type (see below). A student must successfully cope with these problem components to internalize the abstract concepts embedded in the problem in order to transform these abstract concepts into concrete terms. It was hypothesized that the successful solution of a given problem would be affected by an individual's ability to extract from the stimulus important part-whole information concerning a problem's representation and then construct a solution strategy.

Forms of Intellectual Functioning: Differences and Communalities

Broadly classified as analytic abilities, Piaget's logical reasoning and cognitive style are two important psychological constructs. Piaget's logical reasoning is a developmental construct (Inhelder and Piaget, 1958) and maintains a unipolar dimension throughout development such that it progresses in a one-directional continuum. In contrast, cognitive style is argued to be relatively constant and maintains a bipolar construct having field-dependence and field-independence, at either ends of a non-continuous dimension (Messick, 1976).

Piagetian logical reasoning is a structural construct. Piaget states that the complexity of the structure and its understanding is indicative of a stage of development in human intellectual growth, and that the representational mode or character of the structure processed determines the level of logical reasoning (Vosniadou and Brewer, 1987). Since this model tends to view individual differences as being nonexistant for those holding similar structures, it separates the structure from the context and content of the problem. The underlying theoretical view of this paper states that context may be an important factor to how students approach, analyze and restructure algebra word problems. However, the developmental model available and the literature that supports it, seem to be hard pressed to explain intellectual processes when context and content effects are taken into account in algebra problem tasks.

The ability to disembed the context from a problem is at least in part measured by field dependence/independence. Those students who are field dependent approach and restructure a problem in one way and those who are field independent will restructure the problem in a completely different manner. The field-dependent processes are known to be global and are usually compared with a syntactic style approach. Field dependent subjects' approaches are holistic, and there is a general tendency for those field dependent subjects to shy away from complex cues. In contrast, the problem-solving approaches of field independent subjects are more complex and analytical.

The relation between cognitive development and cognitive style can to some degree be sorted out by observing student performances on problems with systematically varied key context features. The relationship between these two variables is clearly defined by Lawson and Wollman (1977), and underscored by the notion that students' field dependence status may be a draw back to the full development of formal operations (Saarni 1973; Lawson 1978; and Lawson and Wollman 1977). Formal reasoning is positively correlated with field independence and as previously stated is strongly related to students' success in solving the propositional relation algebra word problems.

The relation between cognitive style and formal reasoning defined above is based on the principal of differentiation theory (Ausubel, 1978). Differentiation theory is concerned with the complexity of the structure of a psychological system and its relation to its surrounding. Differentiation theory asserts that psychological development progresses from a diffused global level to a more differentiated and highly detached system, which means that with increased cognitive development, there is a tendency for students to be more analytical and focused on specific stimuli and features in a task. Ausubel (1978) used the term progressive differentiation to describe this development process.

Purpose

Over two thousand studies have examined a variety of treatment effects within the field dependence/independence question (James and Moore, 1991). However, there are no studies which examine the relation between field dependence/independence, developmental reasoning level and the solving of propositional relation algerba word problems. In addition, no investigations have studied student performance on the propositional relation algebra word problems which systematically vary key context features of the problems or the relationship between key contextual features of algebra word problems with cognitive development, and cognitive style. The primary objective of this study, therefore, was to compare problem-solving performance on the propositional relation problems with the different key contextual features in terms of reasoning levels (i.e., concrete, transitional or formal reasoning abilities) and cognitive style as expressed in terms of field-independence and field-dependence).

Method

A convenience sample of 80 students from a large commuter university from the eastern part of the United States was used. Only 37 college students completed the two testing periods. All college students were 19 years of age and above.

A convenience sample of 193 secondary school students was obtained from two large high schools which serve two cities in the eastern part of the United States. Each of two cities had populations greater than 60,000 people.

The age of the composite sample ranged from 11 to 40. The mean age was at 17.25 and median age at 17.00.

All the high school and college students had successfully passed their first and second algebra courses.

Two validated instruments were used in the study: the Gottschaldt Hidden Figures Test (HFT) (Crutchfield, 1975) for the field dependence/independence measure and selected items of the Equilibrium Balance Test (EBT) (Adi, 1976), for the measure of Piagetian stages of operational reasoning.

Field dependence and field independence of cognitive style were assessed by the Gottschaldt Hidden Figures Test. The median was used to separate the sample into the lower score of field dependence and the upper into the field-independent. Cognitive development identified by Inhelder and Piaget (1958) was assessed by a balance scale apparatus of paper and pencil test. There are three stages in this task which characterize the developmental levels, connected to the working and understanding of proportions, these are concrete, transitional and formal operations.

Gottschaldt Figures Test

The Gottschadt Figures Test (HFT) seeks to measure ability of individuals to locate simple geometrical figures in more complex figures. The test is group administered with two parts, each having 10 complex figures. Students are instructed to find one or two simple figures embedded in the more complex figures. The time limit is 2 minutes 15 seconds for each part. However, this study increased the time limit to 3 minutes for each pair. The test is to be completed in 4 to 6 minutes.

The total score on the two parts can vary from zero to 20. Reported correlation between the two parts of the test is at .64. There is a tendency for students to perform lower on the first part versus the second explained by the dissipation of stress (Crutchfield, Woodworth and Alberecht, 1958). Important Gottschaldt scores correlates with intellectual efficiency of spatial ability at .80; problem-solving ability at .71 and verbal intelligence at .44. Significant correlation between the Gottschaldt figures test and basic algebra problem-solving was at .40; space relations at .46; abstract reasoning at .306 (Schonberger, 1982). No reliability scores for the HFT test are reported by the authors.

Equilibrium in a Balance Test

The Equilibrium in a Balance Test (EBT) is a paper and pencil test. This test evaluates subjects' ability to balance various combinations of weights at different locations along a beam. For example, given a weight of 20 units, at a location away from the fulcrum, the student is asked to balance the scale by maintaining equilibrium with a 5 unit weight. A correct response would indicate understanding of the inverse proportion, required to solve the problem.

For the purpose of assessing late concrete operations i.e., stage IIB and formal operational reasoning i.e., stage IIIA. This study used a sample six items from this test. These 6 items were selected based on the difficulty index provided by Adi (see Table 1). The three most difficult items were chosen from the ages of 6 through 10 and 11 through 15.

Table 1: Difficulty Index for Each Problem of the EBT

Item	Difficulty Index
6	.59 *
7	.61 *
8	.73
9	.56 *
10	.64
11	.35
12	.17 *
13	.29
14	.21 *
15	.24 *

A reliability measure for the three sets of items (i.e., concrete, transitional and formal operations) are given by Adi, from item 6 through 10 a reliability was reported at .70 and items 11 through 15 a reliability measure was reported at .48. The Guttman Scalogram analysis of the 15 items yielded three cummulative scales with a coefficient of reproducibility at .96 (Adi and Pulos, 1980, p. 152). The Cronbach alpha measure was at .87 (Adi et al.).

The Algebra Problems Instrument

Twenty algebra problems were constructed. The problems were reduced to 16 having different presentation formats; i.e., pictorial, verbal and symbolic representations.

Students cross translate these problems into all combinations of pictorial, verbal and symbolic outcomes or answers.

The presentation format types (or forms), were crossed with the answers types or forms in these 16 word problems (see Table 2). The empirical importance of presentation and answer format to solving the algebra word problems has been demonstrated by Clarkson (1978) based on Brunner's notions of

Table 2: A Descriptive and Conceptual Characterization of the Domain of Algebra Word Problems.

Mode of Relational Representation and Cross Translation	Key Contextual Features			
	FI/D	UI/D	FU/C	UU/C
Verbal to Symbolic	1	1	1	1
Symbolic to Verbal	2	1	2	1
Pictoral to Symbolic	2	1	2	1
Symbolic to Pictoral	2	1	2	1
Verbal to Pictoral	1	1	1	1
Pictoral to Verbal	2	1	2	1

FI/D= familiar-readily imageable-discrete
UI/D= unfamiliar-readily imageable-discrete
FU/C= familiar-not readily imageable-continuous
UU/C= unfamiliar-not readily imageable-continuous
1= First set of word problems to be develope
2= Expansion set of word problems to be developed

problem representations.

Numerous studies have identified the key contextual features (e.g., familiarity and imageability) that significantly effect students a' 'lity to solve algebra word problem involving the translation of inherent proportions into algebraic formulae (see Caldwell, 1977; and Sims-Knight and Kaput, 1983). Logical analysis revealed that the mode of representation (pictoral, verbal, and symbolic) is also a key feature of algebra word problems, particularly in terms of the translation of relations from one mode to another. The construction of six modes (or types) of "cross translation" are possible (see Table 2 for details). For example, the relationship in the problem is stated in verbal terms, but the answer must be expressed in symbolic terms (i.e., an equation).

The key contextual features of familiarity, imageability and variable type were the main constructs of interest in these 16 algebra word problems. Studies have shown that these attributes individually affect performance on arithmetic and algebra problems (e.g., Sims-Knight and Kaput, 1983; Lyda and Franzen, 1945; Sutherland, 1942; Brownell and Stretch, 1931; Washborne and Osborne, 1926 and Horwitz, 1980). No previous studies have employed algebra word problems, with more than one of these key contextual features.

Table 2 indicates that all of the verbally presented problems were given the attributes of familiar, unfamiliar, readily imageable, and not readily imageable. Attached to the readily imageable features are discrete quantities. Continuous quantities are attached to the not readily imageable feature. Combinations of these attributes allowed triads to be formed. The triads were: (1) familiar-readily imageable-discrete (FI/D); (2) familiar-not readily imageable-continuous (FU/C); (3) unfamiliar-readily imageable-discrete (UI/D) and (4) unfamiliar-not readily imageable-continuous (UU/C). The verbal presented problems were created to have all of these features. All of these features, however, could not be assigned to the pictorial and symbolically-presented problems. The symbolic and pictorial problems in this study were limited to the following attributes unfamiliar-readily imageable-discrete quantities and unfamiliar-not readily imageable-continuous. A completely crossed, nested set of features for problems may be constructed by researchers as needed.

The problem features were validated by six mathematics educators. Overall, the results were very positive; only 6.5% of the ratings of the key context features were incorrect. Based on the 17 algebra problems the raters, therefore, agreed with each other and were correct on 93.5% of the ratings. All six judges rated received 8 problems correctly. Of the 9 remaining problems, there were three or less raters who disagreed on the key contextual features present in the problem. When one item which had a high incorrect response judgement among raters was removed from the analysis, no statistical significance difference was found among raters, across all items. The results indicates both strong reliability and validity for the 16 algebra problems.

For the six raters interrater reliability score among 51 ratings for all the attributes was at R=+.95. Haggard's (1958) ANOVA procedures was used in computing the interclass R. The interrater reliability for familiarity, imageability and variable type quantities classifications was at +.93, +.95 and +.97 respectively. As expected, the lowest interrater reliability was observed on the familiarity classification because of a spattered ratings in the profile between different raters within different items. Coefficients indicates that both correctness and agreement levels were extremly high.

Results

A 2x3 (cognitive style x cognitive development) MANOVA was performed on total correct scores on the subset of items for FI/D, FU/C, UI/D and UU/C. Each item of the subset score was assessed by scoring a 1 for a correct solution and a 0 for an incorrect solution. Each correct item in the subset was summed and divided by the number of items in each subset or grouping.

The MANOVA focused upon the central issue of the study; namely, could performance on the propositional relation algebra word problems be understood in terms of cognitive development and cognitive style.

Table 3 presents the means (proportion correct levels) of the subset scores for the FI/D, FU/C, UI/D and UU/C type problems by the levels of cognitive development and cognitive style. Table 4 presents the 2x3 (cognitive style x cognitive development) MANOVA results for the four subset scores of key context features. Significant main effect for cognitive development level were found for FU/C, UI/D and UU/C problems. The F-ratios were $\underline{F}(2,203)=5.58$, $\underline{p}<.01$ for the FU/C problems, $\underline{F}(2,203)=8.01$, $\underline{p}<.001$ for the UI/D problems and $\underline{F}(2,203)=4.69$, $\underline{p}<.01$ for the UU/C problem. No significant main effect of cognitive development was found on the FI/C subset of scores.

A significant main effect for cognitive style was found on three subscores. These three subscores were at $\underline{F}(1,203)=16.00$, $\underline{p}<.001$ for FU/C; $\underline{F}(1,203)=15.54$, $\underline{p}<.001$ for UI/D and $\underline{F}(1,203)=4.25$, $\underline{p}<.05$, for the UU/C problems. No significant main effect of cognitive style was found on the FI/D subset scores. Overall, field-independent subjects performed significantly higher than field-dependent ones on all the problem subset with FU/C, UI/D and UU/C features.

Overall on each subset of items, formal reasoning subjects performed above their concrete operations and transitional counterparts. Similarly, those subjects who were transitional performed higher than their counterparts the concrete operational subjects. Those subjects who were formal reasoners and field-independent performed the highest on each subset of items. The superior performance of these subjects was in order from low to a high: FU/C, FI/D, UI/D and UU/C.

No interaction effects were found between cognitive style and cognitive development on the problem subset of FI, FU, UI and UU. Cognitive development showed its effects on those problems that were more "abstract" (i.e., UU) than those problems that were more "concrete" (i.e., FI/D).

Formal reasoning subjects obtained the higher scores on "abstract" problems than those who were "concrete." (scheffe' post hocs revealed a 4.68, $p<.05$ for formal operational subjects between FI and UU key contextual features).

The main effects of cognitive style on the items with "abstract" features (i.e., FU, UI and UU) can be explained directly from the theory Witkin and Goodenough (1977) presented. They assert that for perceptual representations that are too complex, field dependent subjects will have greater difficulties than field independent ones. On those problems that were "worldly" and "concrete" (i.e., FI features), there was no significant difference between levels of cognitive style. The high scores for those students who were field independent versus the low scores of field dependent subjects on the "abstract" features explain what Witkin and his associates established; namely, that subjects who disembed these features of the problem and focus on the structural elements necessary for the solution may be more able to frame these problems in their knowledge structures for the appropriate problem-solving processes.

The high scores for those subjects who were formal reasoners on the "abstract" problem features, in contrast to their lower scores on the "concrete" features may be due to the fact that cognitive style is "confounding" or mediating development levels.

Table 5 presents the means (proportion correct levels) for the three presentation modes by cognitive development and cognitive style. Table 6 presents the 2x3 (cognitive style x cognitive development) MANOVA for the pictorial, symbolic

Table 3: Raw Score Means (Proportion Correct Levels) on the Performance of Subset of items with Key Context Features for Cognitive Development and Cognitive Style.

Cognitive Style and Cognitive Development		FI/D k=2	FU/C k=2	UI/D k=6	UU/C k=6
Field-depen.	(n=115)	.307	.200	.304	.387
Field-indep.	(n=94)	.372	.431	.433	.457
Concrete	(n=76)	.303	.178	.298	.355
Transitional	(n=62)	.328	.331	.349	.452
Formal	(n=72)	.375	.410	.444	.456
Field-dependent					
Concrete	(n=52)	.308	.144	.266	.333
Transitional	(n=30)	.311	.233	.306	.433
Formal	(n=33)	.303	.258	.364	.429
Field-independent					
Concrete	(n=23)	.304	.261	.355	.406
Transitional	(n=32)	.344	.422	.391	.469
Formal	(n=39)	.436	.538	.513	.479
For Entire Sample		.337	.304	.362	.419

Scheffe' post hocs (N=209)

	FI	FU	UI	UU
FI		2.3	1.3	14.1**
FU			7.3*	27.6**
UI				6.8*

*=Significant at the .05 level

**=Significant at the .01 level

Table 4: Summary of F-ratios for a 2x3 (Cognitive Style x Cognitive Development) MANOVA on the Four Key Context Features.

	Cognitive Style df(1,203) (A)	Cognitive Development df(2,203) (B)	AxB df(2,203)
FI/D	2.93	1.45	1.74
FU/C	16.00***	5.58**	.98
UI/D	15.54***	8.01***	.60
UU/C	4.25*	4.69**	.17
Mult. F-Ratio	5.90***	3.10*	.78

*=Significant at the .05 level
**=Significant at the .01 level
***=Sigificant at the .001

and verbal presentation problem subsets. No significant interaction effects were found between cognitive style and cognitive development levels on the presentation mode scores. However, significant main effects for cognitive style and level of cognitive development were obtained on the pictorial and verbal presentation format. For cognitive style, the results were at an $F(1,203)=9.18$, $p<.01$ and $F(1,203)=23.96$, $p<.001$, respectively. For the levels of cognitive development, the results were $F(2,203)=14.39$, $p<.001$ and $F(2,203)=6.31$, $p<.01$ for the pictorial and verbal presentations respectively. No significant main effects on the translation from the symbolic presentation were found for either cognitive style or levels of cognitive development.

Table 5: Raw Score Means (Proportion Correct Levels) on the Presentation Mode of items by Level of Cognitive Development and Cognitive Style.

Cognitive Style and Cognitive Development		From Pictorial k=4	From Verbal k=8	From Symbolic k=4
Field-dependent				
Concrete	(n=52)	.322	.233	.389
Transitional	(n=30)	.475	.279	.417
Formal	(n=33)	.523	.284	.417
Field-independent				
Concrete	(n=23)	.424	.310	.435
Transitional	(n=32)	.555	.391	.383
Formal	(n=39)	.635	.487	.404
For Entire Sample		.481	.328	.404

Table 6: A 2x3 (Cognitive Style x Cognitive Development) MANOVA on the Three Presentation Formats.

	Cognitive Style (A) df(1,203)	Cognitive Development (B) df(2,203)	AxB df(2,203)
From Pictorial	9.18**	14.39***	.09
From Symbolic	.00	.07	.64
From Verbal	23.96***	6.31**	2.10
Mult. F-Ratio	8.61***	5.13***	.96

*=Significant at the .05 level
**=Significant at the .01 level
***=Significant at the .001 level

To investigate the effects of cognitive development and cognitive style on the cross translation variable, the items were regrouped and a score was determined for the translations <u>to</u> a pictorial, <u>to</u> a symbolic and <u>to</u> a verbal format. Table 7 presents the means (proportion correct levels) of the cross translation problems by the levels of cognitive development and cognitive style. Table 8 presents the 2x3 (cognitive style x cognitive development) MANOVA results on the cross translation variables. No significant interactions are found on the cross translations. However, significant effects were obtained for the levels of cognitive development on the cross translation <u>to</u> the pictorial and <u>to</u>

the symbolic form, at $F(2,203)=2.50$, $p<.05$ and $F(2,203)=11.52$, $p<.001$, respectively. A main effect for cognitive style was found on the cross translation to the pictorial $F(1,203)=11.04$, $p<.001$ form and on the cross translation to a symbolic form $F(1,203)=14.79$, $p<.001$.

Table 7: Raw Score Means (Proportion Correct Levels) on the Performance of Subset of items for the Translation to a Presentation Type for Cognitive Development and Cognitive Style.

Cognitive Style and Piagetian Level of Cognitive Development		To Pictorial k=6	To Verbal k=4	To Symbolic k=6
Field-dependent				
Concrete	(n=52)	.380	.500	.065
Transitional	(n=30)	.471	.583	.117
Formal	(n=33)	.458	.576	.189
Field-independent				
Concrete	(n=23)	.511	.543	.114
Transitional	(n=32)	.508	.555	.266
Formal	(n=39)	.606	.558	.362
For Entire Sample		.481	.548	.184

Table 8: Summary of F-ratios for a 2x3 (Cognitive Development x Cognitive Style) MANOVA F-Ratios on the Three Cross Translations.

	Cognitive Style (A) df(1,203)	Cognitive Development (B) df(2,203)	AxB df(2,203)
To Pictorial	11.04***	2.5*	1.16
To Symbolic	14.79***	11.52***	1.39
To Verbal	.00	.96	.51
Mult. F-Ratio	6.61***	4.06**	1.10

*= Significant at the .05 level
**= Significant at the .01 level
***=Significant at the .001 level

Tables 6 presented the MANOVA results on the pictorial, symbolic and verbal presented problems. The significant effects for the pictorial and verbal problems by cognitive development level were high; particularly on problems of the pictorial format. A significant cognitive style effect was also found on the pictorial and verbal form.

The high difference between field independent and field dependent subjects on the verbal and pictorial presented problems explains the effects of cognitive style on the complex presentations of the verbal and pictorial forms. This theory of cognitive style comes directly from Witkin and Goodenough (1977) and Witkin and Berry (1975). Similarly, the difference between the field independent and field dependent subjects on the translation _to_ a verbal format was not significant, whereas significant and high F-ratios between levels of cognitive style were found on problems translated _to_ a symbolic form. The above findings are very significant and extremely important in terms of understanding the effects of cognitive style on the problem-solving processes.

These findings suggest that cognitive style as an intellectual functioning construct does _not_ have a moderately large influence on the functional processes of problem-solving. Well supported is the general premise that any field independent subject's ability to articulate, analyze and disembed important information in the problem is directly related to the perceptual structure of the stimulus more so than the tacit and underlying cognitive procedures needed for the problem-solving solution. The generalizability of this view is evident in the significant difference found between the levels of cognitive style on the translation _from_ a verbal format and non-significant difference levels found on the translations _to_ a verbal format. Similarly, the non-significant differences between levels of cognitive style found on the translations _from_ a symbolic form and the high and significant difference found on the translations _to_ a symbolic format supports the view mentioned above; namely, that complex perceptual presentations seem to hinder field dependent subjects' ability to perform at the same levels of field independent subjects. Where the perceptual presentation seems to be static and ordered in appearance, field independent and field dependent subjects tend to perform equally well on those type of presentations.

The hypothesis that a perceptually more complex presentation will be easier for those who are formal reasoners and field independent than those who are concrete reasoners and field dependent comes from Pascual-Leone's (1977) theory and research on field independence/dependence. The lack of interactions found in this study do not support this view. Significant main effects were found on the pictorial and verbal presentations. Those students who were formal reasoners and field independent tended to perform higher on all the problem types than their counterparts; namely, field dependent subjects who were concrete reasoners. We had no explicit method to characterize the exact complexity of each word problem, nor does such a method currently exist to the best of our knowledge. Logical analysis of each problem feature, however, suggests that the verbal presentation problems might be more complex because of the number of elements and meanings that are contained within the verbal presentation. The pictorial and symbolic modes of representation have a reduced number of elements and meanings present in the problem that make them less complex for students. The complexity of algebra word problems, therefore, may have to be conceptualized as to have at least two major dimension; namely, a surface or clothing dimension and a deep structure dimension. Further, the surface or clothing dimension seems to be related to field independence traits and skills, whereas the deep structure seems related to cognitive development; namely, the development of logical reasoning skills.

Discussion

Field dependence/independence is a construct that differentiates a person's ability to perceive analytically embedded stimuli. At one extreme, field dependence focuses perception on the surrounding field and the whole gestalt. At the other extreme of the field independence the individual experiences the field separate from the stimulus (Kagan and Kogan, 1970). The multivariate analysis of variance focused upon one of the central issues of the study: can problem-solving and effects of key context features be understood in terms of formal reasoning and cognitive style? This analysis revealed a significant multivariate F-ratio for the main effect of reasoning level and cognitive style on problems of the FU/C, UI/D and UU/C key context features. The interaction of cognitive style with level of reasoning did not yield any significant differences on the subset problems. Therefore, when taking the constructs of cognitive style with operational reasoning, it did not provide any additional information for understanding performance on the algebra problems assigned with key context features.

These results challenge some of the alternative views in the literature (e.g., Linn, 1978 and Lawson and Wollman, 1977) on cognitive development and cognitive style. That students who are judged as concrete reasoners may in fact be at formal levels is perhaps due to their having a high degree of field dependence. This hypothesis suggests that performance on algebra problems with abstract key context features (i.e., UU, UI and FU) may be lower for those who are field dependent and concrete operational; this was evident in the results presented on Table 3.

In summary, this study attempted to ascertain whether problem-solving performance would be better for those students who are field independent than field dependent and for those students who operate at formal reasoning levels as compared to those whose operativity levels are transitional or concrete reasoning levels. Second, it was anticipated that field-independent and formal reasoners students would perform better on the unfamiliar-not readily imageable problems than its contrast the familiar-readily imageable problems. Third, when reaggregating the key context features to obtain a score on the pictorial, symbolic and verbal representation, we expected that those students who are field independent will tend to perform higher on problems whose elements are structurally more complex (i.e., verbal and pictorial forms) than their counterparts their field dependent students.

Field-independent and formal reasoning students scored consistently higher across all the problems with the key context features, important main effects of cognitive development and cognitive style are reported on the FU/C, UI/D and UU/C key context attributes at the .05 level. Lowest performance scores are reported on the FU/C attributes with higher scores on the FI/D, UI/D, followed by UU/C attributes. No main significant effects on the FI/D problems were found. Overall students at different reasoning levels and cognitive style seemed to perform equally on this type of problem.

The influence of the familiarity key context feature of problems on their solution points to a set of educational implications. Overall, the familiarity key context feature had a major influence on student problem-solving choices and strategies. Students performed markedly lower on problems of the familiar-readily imageable-discrete (FI/D) and familiar-not readily imageable-continuous attributes (FU/C). Specifically, those subjects who were formal reasoners and field-independent were found to have most of the difficulty on the readily imageable attributes with familiar and unfamiliar attributes and on problems presented in the verbal type. However, those students scored consistently higher than the field dependent subjects.

The results of this study do not support the developmental model. This model states that as children progress intellectually from concrete to abstract modes of thinking, the knowledge structures that are deeply embedded in their cognitive structure are predominantly concrete because of the reliance on the sensory-motor perception of the environment. At one end of the continuum, formal operational reasoning is fully developed where symbolic knowledge is completely integrated with general knowledge. At the lower end, concrete operational subjects are driven by the perceptual field. Concrete operations are acts that reflect only minimal schematizing and abstract generalizations. Thus, one can argue that those students who are at formal levels of logical reasoning tend to treat abstract information in the same way as concrete information. Those subjects who are formal reasoners and field independent should show comparable profile scores on all the problems with the key context features. This outcome would be especially the case on problems which contrast key context features of the FI/D and UU/C respectively. We found scattered profile scores for those who were formal reasoners and field independent. Those field dependent students tended to show a different performance profile as compared with those students who were field independent. The concrete reasoners and field dependent group performed the lowest on a set of problems, whereas the formal reasoners who were field dependent performed the highest on those problems. For those students who were concrete reasoners and field-dependent, their performance profile on the algebra problems followed the following profile pattern from low to high: FU/C, UI/D, FI/D and UU/C. This latter profile had a reversed performance profile as compared with those students who were formal reasoners and field independent.

Performance among students who were field dependent and concrete reasoners was lower than those who were field-independent and formal reasoners. It could be concluded that the validated key context features that are deeply embedded in one's cognitive structure and contained in the problem seem to interfere with retrieval or recognition of procedures needed for the solution of problems. This view, when analyzed from a developmental perspective, gives no clear-cut evidence to support the developmental model. The high-embedded key context features or familiar features that are readily imageable appeared to influence problem-solving performance. These results are explained by students over generalizing the familiar or imageable key features of the problem which cause them to approach the problem in a strictly static way. The problem does not illicit schemas needed to obtain the overall solution, and student's static approach to the problem does nothing to correct the failure to arouse the schemas needed.

Results of this study do not cross-validate some of the empirical results reported by Caldwell (1977) and Quintero (1980). These researchers found that student performance was highest on the concrete-factual problems. However, the results of this study indicate just the contrary. Those problems that had the concrete key context features of the familiar and readily imageable type were the problems on which students performed the lowest. The problem, moreover, do not operate independently of the problem format. Scribner (1984) and Bernardo and Okagaki (1992) have shown that the context in which the problem is embodied is not detached from the actual operation of cognitive skills. Any further research in this area needs to control the format of the key context features in order to understand the interactions between these features and the format.

Both field independent and field dependent group subjects performed lowest on the verbally presented problems followed by the symbolic presented type. Highest performance was found on the pictorial presented problems for both field dependent and field independent groups. The concrete and formal operational students performed lowest on the verbal presented problems followed by the symbolic. These students performed highest on the pictorial problems. Because these results did not give clear cut support for the developmental model, we turned to the qualities of the problem to give some theoretical basis to our results.

In this analysis, we viewed the translation type to be indicative of the underlying structure of the performance model. As reported in the results section, the items were rescored with respect to translation type. It was found that the problems translated to a symbolic representation followed a translation to a pictorial and lastly to a verbal presentation. Some inferences can be made from these results that reflect on the features of the problems. Performance levels on the symbolic represented problems were much higher for the formal who were field independent than the concrete reasoners who were field dependent subjects. One interpretation of these results is that general symbolic knowledge improves when problem solvers reach levels of formal operations. Significant main effects of cognitive style on the verbal and pictorial representations was found on the algebra problems. However, the nonsignificant main effects of cognitive style on the symbolic representations was consistent with the view that pictorial or verbal representations may have more than one entity in the representation.

Because symbolically presented problems tended to have static entities, disembedding organized perceptual information would seem to be redundant in the task. As expected, nonsignificant effects of cognitive style were reported on the symbolically represented problems. Evidence indicates that students who were performing the lowest on problems translated to the symbolic forms were also performing the lowest on the problems presented in the verbal and pictorial form. This generalized finding may indicate that those items which are cross translated from a representative form were more a function of the cognitive process that underline the problem-solving act than the encoding of the representative form of the problem.

In conclusion, the cognitive style factor produced a marked effect in that students classified as field-independent and those classified as field dependent showed very significant differences on the propositional relation algebra problems presented in different formats. Similarly, main significant effects were found for cognitive development factor. However, the F-ratios for cognitive style were much higher than those for cognitive development, indicating that cognitive style may be confounding or overlapping the effects of cognitive development found in many previous studies.

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