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ABSTRACT

This report contains case studies of two teachers who attempt to go beyond traditional mathematics curriculum and instruction. There are important differences in the contexts of teaching and in the approaches to studying teaching and learning in these two classrooms. Case study 1 describes a teacher who draws upon professional experiences to initiate change in the classroom. The authors describe elements of the teacher's mathematics teaching, using NCTM Curriculum and Evaluation Standards as a framework for analysis, then analyze the teacher's views about instructional issues, focusing on connections between beliefs and practice. Case study 2 describes a teacher involved in a Mathematics Study Group in a Professional Development School. The authors chronicle the changes in this teacher's thinking and practice during a 3-year period, focusing on her views about, and understanding of, mathematical content, the nature and role of discourse about mathematics, and what constitutes evidence of mathematical learning. The authors also explore the role of collaboration among teachers and between teachers and university participants in discussing the kinds of changes these teachers made. (Contains 15 references.) (Author)

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CONSTRUCTING TEACHING AND RESEARCH PRACTICE IN ELEMENTARY SCHOOL MATHEMATICS

Center for the Learning and Teaching
of Elementary Subjects

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Institute for
Research on Teaching

College of Education

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of Elementary Subjects

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Center for the Learning and Teaching of Elementary Subjects

The **Center for the Learning and Teaching of Elementary Subjects** was awarded to Michigan State University in 1987 after a nationwide competition. Funded by the Office of Educational Research and Improvement, U.S. Department of Education, the Elementary Subjects Center is a major project housed in the Institute for Research on Teaching (IRT). The program focuses on conceptual understanding, higher order thinking, and problem solving in elementary school teaching of mathematics, science, social studies, literature, and the arts. Center researchers are identifying exemplary curriculum, instruction, and evaluation practices in the teaching of these school subjects; studying these practices to build new hypotheses about how the effectiveness of elementary schools can be improved; testing these hypotheses through school-based research; and making specific recommendations for the improvement of school policies, instructional materials, assessment procedures, and teaching practices. Research questions include, What content should be taught when teaching these subjects for understanding and use of knowledge? How do teachers concentrate their teaching to use their limited resources best? and In what ways is good teaching subject matter-specific?

The work is designed to unfold in three phases, beginning with literature review and interview studies designed to elicit and synthesize the points of view of various stakeholders (representatives of the underlying academic disciplines, intellectual leaders and organizations concerned with curriculum and instruction in school subjects, classroom teachers, state- and district-level policymakers) concerning ideal curriculum, instruction, and evaluation practices in these five content areas at the elementary level. Phase II involves interview and observation methods designed to describe current practice, and in particular, best practice as observed in the classrooms of teachers believed to be outstanding. Phase II also involves analysis of curricula (both widely used curriculum series and distinctive curricula developed with special emphasis on conceptual understanding and higher order applications), as another approach to gathering information about current practices. In Phase III, models of ideal practice will be developed, based on what has been learned and synthesized from the first two phases, and will be tested through classroom intervention studies.

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Abstract

This report contains case studies of two teachers who are attempting to go beyond traditional mathematics curriculum and instruction. There are important differences in the contexts of teaching, and in the approaches to studying teaching and learning in these two classrooms. Case Study 1 describes a teacher who has drawn upon professional experiences to individually initiate change in the classroom. In the first part of Case Study 1, the authors describe elements of the teacher's mathematics teaching, using the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards* as a frame for their analysis. In the second part of the case study, the authors analyze the teacher's views about instructional issues, focusing on connections between beliefs and practice. Case Study 2 describes a teacher involved in a Mathematics Study Group in a Professional Development School (a partnership between school and university participants). The authors chronicle the changes in this teacher's thinking and practice during a three-year period, focusing on her views about and understanding of mathematical content, the nature and role of discourse about mathematics, and what constitutes evidence of mathematical learning. The authors also explore the role of collaboration among teachers and between teachers and university participants in discussing the kinds of changes this teacher made.

CONSTRUCTING TEACHING AND RESEARCH PRACTICE IN ELEMENTARY SCHOOL MATHEMATICS

Center for the Study of Elementary Subjects

Our work in the Center for the Learning and Teaching of Elementary Subjects has provided us with opportunities to enter into various relationships with teachers, students, and other researchers with the goal of better understanding the processes involved in teaching and learning elementary mathematics. Although the case studies included in this report are both about teachers who are attempting to go beyond traditional mathematics curriculum and instruction, our assumptions about these teachers, and our approaches to working with them, differed significantly. Conducting these case studies simultaneously afforded us with occasions to better recognize and understand many of the issues related to conducting research on learning and teaching.

We began our work with Marilyn Anderson and Laura Tate[1] during the fall of 1989. As we indicate in the case studies which follow this introduction, our research work with Marilyn and Laura was guided by the kinds of questions that Shulman (1988) associates with qualitative methods: "How is mathematics instruction carried on?" "What are the experiences and perceptions of teachers and students as they engage in the teaching and learning of mathematics?" What is the underlying or explicit system of rules by which this complex activity is accomplished?" (p. 7). To answer these questions, we employed techniques consistent with interpretive research approaches (Erickson, 1986): continuous narrative description, video- and audiotapes of classroom events, interviews with teachers and students, and collection of artifacts such as lesson plans and student work. At the data collection level, our work with Marilyn and Laura was similar. Beyond these common techniques, however, there were important differences in our approaches to studying the teaching and learning of mathematics in Marilyn's and Laura's classrooms.

Marilyn was identified through a nomination process in which researchers and teacher educators at the university had been asked to provide the names of outstanding elementary school

[1] Names of teachers, school, and community are pseudonyms.

mathematics teachers. Laura was a participant in a Mathematics Study Group (MSG) in a Professional Development School (PDS). Like most of the teachers in this group, Laura did not consider mathematics her favorite subject or a strong area in her curriculum. She was interested in learning about mathematics teaching and learning in discussions with other teachers and university faculty. Heeding Peshkin's (1988) recommendation that "researchers should systematically seek out their subjectivity" (p. 17), we have reflected on the ways that our different introductions to Marilyn and Laura have "shaped our inquiries and their outcomes" (p. 17).

At the time she was observed, Marilyn was teaching first grade in an upper middle class elementary school in a school district near the campus. She had taught at the primary school level for 25 years. During this time, she has remained open to new learning experiences, taking advantage of various professional opportunities (e.g., workshops, participation on a district curriculum committee, involvement in Michigan State University's teacher education program as a mentor of prospective teachers). Although it is always problematic to categorize teachers, Marilyn does strike us as an uncommonly reflective individual who has been able to draw on these professional experiences in "bootstrapping" her way toward improved practice in elementary level mathematics teaching. For this reason, we felt we had much to learn from Marilyn regarding *individually initiated* change and how that might play out in the classroom context.

Laura Tate, the second case study presented in this report, provides a different lens for thinking about mathematics reform. For one, the researchers involved in writing up the case study played a much more collaborative role in working with the teacher to change her mathematics practice; for another, this particular project, which also unfolded in a first-grade classroom, occurred in a markedly different context. Laura Tate was one of six teachers who participated in a study group during the period of time when she was involved in work described in the case study. This group--called the Mathematics Study Group--was one of three projects collaboratively planned and developed at the school level as part of a larger effort to forge closer links between the university and the public schools. The main purpose in establishing these university/school "partnerships" is to create schools in local communities that will serve as sites for "best practice."

It is envisioned that teachers and university researchers in these schools will share responsibility for conducting research, preparing new teachers for the profession and generally working to enhance the quality of K-12 education in the local area and statewide.

Laura Tate teaches in one of the partnership or Professional Development Schools. Her participation in the Mathematics Study Group represented a considerable commitment of time and effort. The MSG met on a weekly or biweekly basis to explore various aspects of teaching for understanding in mathematics. During these meetings, the MSG examined a range of innovative curricular and instructional options. The group deliberately pursued an eclectic agenda during these meetings, recognizing that the possibilities stimulated by thinking anew about mathematics teaching and learning must be viewed against the backdrop of existing political and social realities.

Based on these descriptions of how we approached our work with Marilyn and Laura, one might expect to find important differences in the two the case studies. Our different orientations to our work with Marilyn and Laura *did*, in some ways, influence what we expected to see and the roles we assumed as researchers. With Marilyn, our goal was to document her *existing* thinking about and practice in a naturally occurring classroom situation. With Laura, our goal was to document the *changes* in her thinking and practice as she participated in discussions and planning with other teachers and researchers, and as she experimented with her own practice. With Marilyn, we stayed at the *observer* end of the participant-observer continuum; we took no part in planning mathematics lessons, made no suggestions, took no part in teaching. With Laura, we stayed at the *participant* end of the participant-observer continuum; we planned, made suggestions, taught, and talked together on a regular basis. When it came time to write Marilyn's story, we took our data from observations and interviews and worked at home and in the office, constructing our narrative without her input, her clarification, or her scrutiny. When we wrote Laura's story, we wrote knowing that she would read and respond to numerous drafts, or be there as we constructed them.

Working on both cases simultaneously afforded us the opportunity to draw parallels between aspects of Marilyn Anderson's mathematics teaching and issues about teaching and learning with which the six MSG teachers were grappling. Thus, as the MSG teachers continued to explore alternatives to traditional mathematical instruction, and to co-develop an instructional unit on measurement, an oft-repeated question arose: What are the big ideas? The MSG teachers were seeking ways to operationalize what they had come to understand as an important aspect of teaching for understanding. Because ideas can be "messy" and abstract, it was often difficult for the teachers to identify and articulate just what the key ideas were, much less how to translate this focus on key ideas into classroom practice.

The "big ideas" frame had not figured prominently in our analysis of Marilyn's teaching until we began to engage in discussions about big ideas with members of the MSG. It became increasingly apparent that this way of thinking about the elementary school mathematics curriculum--that is, the notion that a curriculum might be thought of as a network of key ideas--could prove useful in analyzing Marilyn's teaching. One member of the research team decided to "revisit" existing narrative-descriptive data and to gather new data with this issue in mind. This information was integrated into the first draft of the Marilyn case study. This effort to track the ideational content in Marilyn's mathematics teaching over the course of a year, and to test the hypothesis that some mathematical ideas served as a foundation for activities and discussions in other subjects as well, influenced our discussions with teachers in the Mathematics Study Group.

Thus, while researchers involved in the case studies presented in this report approached the study of mathematics teaching in different ways in the two lines of work, the work itself was far from self-contained. On the contrary, the two case studies are a good example of the interactive nature of the work engaged in during the overall research project.

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CASE STUDY I
INNOVATION THROUGH EXPERIENCE: THE CASE OF MARILYN ANDERSON[1]

Richard S. Prawat and Julie J. Ricks[2]

Many studies document existing practice in mathematics. The picture that emerges is not surprising: Individual seatwork tends to predominate, at least at the elementary school level, where students spend approximately 75% of their time working independently at their desks (Denham & Lieberman, 1980; Rosenshine, 1980). Romberg and Carpenter (1986) summarize the typical instructional format as follows:

> The teacher's job is to assign a lesson to [her] class of students, start and stop according to some schedule, explain the rules and procedures of each lesson, judge the actions of students during the lesson, and maintain order and control throughout. (p. 5)

The view of teaching that drives this practice--the so-called transmission view, where teaching is considered to be synonymous with the dispensing of information--appears to be related to equally traditional views about knowledge and learning on the part of teachers (Prawat, 1992). Thompson (1984, 1985), for example, found that teachers who favor a transmission model of teaching in mathematics tend to view the discipline in static as opposed to dynamic terms, as a "finished product" instead of something that is continually undergoing change and revision. Pope and Gilbert (1983) report a similar relationship between teachers' instructional practice and their epistemological views in science.

In the learning domain, traditional instructional practice also appears to go hand in glove with hierarchical views of learning (Prawat, 1989, 1992): That is, with the common-sense notion that learning progresses from the simple to the complex, from a mastery of certain "lower order" facts and skills to more abstract capabilities that mediate transfer and problem-solving. Teachers at the elementary school level who subscribe to this set of beliefs about learning may be more inclined to focus on the transmission of facts and procedures in their mathematics teaching, arguing that

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[1]This paper was presented at the annual meeting of the American Educational Research Association, Chicago, April 1991.

[2]Richard S. Prawat, professor of educational psychology at Michigan State Univeristy, is a senior researcher with the Center for the Learning and Teaching of Elementary Subjects. Julie J. Ricks, doctoral candidate in counseling, educational psychology and special education at MSU, is a research assistant with the Center.

mastery of "the basics" must precede attempts to use knowledge in a more creative way (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). Hierarchical views of mathematics' learning have been contrasted with views that emphasize the relational and constructive nature of mathematics learning. As Hiebert and Carpenter (1990) explain, understanding is now seen as something generated by individual students as opposed to provided by the teacher. The adequacy of that understanding, however, is best gauged by the richness of the internal connections students are able to build: "The greater the number of connections, the better the understanding" (Hiebert & Carpenter, 1990). Teachers who embrace relational, conceptually oriented views of mathematics assign less importance to producing right answers and more importance to constructing new relationships or connections (Carter & Yackel, 1989).

Why raise the issue of teachers' beliefs about teaching, mathematics knowledge, and learning? Because an increasing body of research suggests that they are tightly linked, and perhaps causally connected, to teachers' instructional practice. We may need to attend more carefully than we have in the past to teachers' psychological and epistemological views if we are to change their instructional practice (Cohen et al., 1990). The present study explores this link, using a variety of data from a single classroom, including field notes and audiotapes from weekly observations of a first-grade teacher's mathematics lessons, audiotaped interviews with the teacher and students at key points during the school year, as well as less formal, ongoing "conversations" with participants consistent with the interpretive research paradigm described by Erickson (1986). The exemplary teacher who is the subject of this case study had been identified through a nomination process in which researchers and teacher education personnel at the university had been asked to provide the names of outstanding elementary school mathematics teachers.

The data in this case will be presented in two parts. First, an attempt will be made to describe important elements of Marilyn's mathematics teaching. Criteria derived from the *Curriculum and Evaluation Standards* published recently by the National Council of Teachers of Mathematics [NCTM] (1989) provide a frame for this analysis (e.g., the notion that mathematics teaching should encourage children to explore, discuss, and apply mathematical ideas). In this section of the report, we will also draw on student data to document the effectiveness of Marilyn's teaching as judged by student learning. In the second part of the report, we will analyze Marilyn's views about a host of instructional issues--focusing, in particular, on possible connections between these beliefs and her teaching practice. In a final section of the report, we will draw on additional interview data in our speculations about how Marilyn has come to view the educational world the way she does, and what teacher educators can do to speed up this "developmental" process.

Method

Two observers trained in narrative descriptive research methodology observed Marilyn's classroom teaching at frequent intervals during the 1989-90 school year (late October to May), and then again during the fall of 1990 (August to December). A total of 47 observations were obtained. During the first year of the study, observations were conducted on a weekly basis. In addition to informal interviews throughout the year, two in-depth interviews were conducted with Marilyn at the midpoint of the first year's observations. These interviews were tape-recorded and transcribed for later analysis.

To supplement observational data on student learning, 10 students, randomly selected within equal gender categories, were asked to participate in a clinical interview near the end of the 1989-90 school year. The 30-minute interview protocol included a number of items used by other researchers to assess important aspects of students' mathematical attitudes and understandings (e.g., students' views about what it takes to be "good" at mathematics, their strategies for solving simple addition and subtraction word problems, their understanding of the conventionality of number, their computational ability, their command of place value concepts). For comparative purposes, 10 randomly selected students from each of two more traditional first-grade classrooms were also interviewed.

During the Fall of 1990, the observational pace was accelerated to one to three times per week. In addition, eight students were interviewed using a protocol that included a number of activities designed to investigate students' understanding of, and approaches to, number convention, constructing sets of numbers, story problems, and patterns. Some of these interview activities were novel while others closely resemble the kinds of situations experienced by children during classroom instruction. Based upon observational data and teacher assessment, students included in the fall interview were thought to represent a range of ability in mathematics.

Summary of Classroom Observations and Student Interviews

Marilyn's Classroom

Now in her 25th year of teaching, Marilyn Anderson has taught mostly preschool, kindergarten, first and second grades. Looking around Marilyn's classroom one senses immediately that children are the important people in that environment. The children's desks are arranged in clusters, four in a square pattern and one or two at one end, all facing into the square. On each desk, a child's name, printed on laminated cardboard, stands upright in a wooden holder. Marilyn's corner desk is always stacked with papers, notes from parents, and materials Marilyn has prepared for the day's activities. Marilyn hardly ever sits there; when she does sit down, it is usually in the rocking chair at the edge of a large, open carpeted area which dominates the room. This is the area where children gather for morning activities, for sharing time, for stories, for songs, and for most of the large-group discussions that precede almost every lesson. This area is also where children work with materials during free choice time, or work with a partner or in small groups during lessons.

Around the edges of the room are low shelves and small tables displaying a variety of books, classroom supplies, math materials, and science exhibits, all accessible to children. A bulletin board covers one whole wall near the large open area. One half displays children's artwork, the other half is divided into three sections-- "Who Lost a Tooth?" where children's pictures are displayed on large construction paper teeth labeled June through August; the Math Their Way[3] calendar activities; and "Birthdays", where children's names and birth dates are written on large construction paper cupcakes labeled for each month of the year. There are other colorful and instructive decorations throughout the room--large labeled paper crayons, alphabet letters with

[3] Mathematics Their Way (Baratta-Lorton, 1976) is a program based on the theory that young children learn best from concrete hands-on experiences, then assume a hierarchical approach that moves from concrete to abstract. The calendar activities include counting and grouping objects to represent school days, making temperature and weather graphs, and recognizing even and odd numbers.

accompanying pictures, numeral and tally cards from 0 to 9, a chart that says "How do you travel to school?" with paper buses and cars made by the children, labeled with their names and glued to the appropriate column.

<u>A Typical Day</u>

Children are welcomed into Marilyn's classroom each day with an individual greeting--a comment about new shoes, a question about a pet, a check to see if they rode the right bus today, a hug or a pat on the back. Marilyn often leans down to look in their faces, focused, if only for a moment, on each child. After children arrive, they put their belongings in the coat closet or in their desks, and then choose materials from the shelves, which they use individually or with their friends on the carpet or at their desks. After about 15 minutes, Marilyn typically plays a clean-up song on the piano, and the children quickly put away the materials and gather on the carpet near the rocking chair.

For the next half hour or so, Marilyn and the children complete a familiar routine: the Pledge of Allegiance, songs, calendar activities, attendance and lunch counts. Each day, a different group of children choose jobs, labeled on popsicle sticks: flag holder, closet checker, messenger, line leader, hall monitor, song chart leader. The rest of the jobs are related to the calendar activities: One person tells the day of the week, another the date, one records the weather on a chart, another counts craft sticks representing the number of days they've been in school.

On a typical day in Marilyn's classroom, this routine is followed by periods for reading, math, science, sharing time, art activities, and of course recess and lunch. Two afternoons a week children have gym, two afternoon music, and one afternoon they go to the school library. No matter what the subject or the topic, children are actively involved--moving, talking, singing, working together. Marilyn spends a lot of time listening to what children have to say. She consistently encourages them to explain what they are doing, why they think that, how they know. Getting children to elaborate their ideas is more the norm than the exception in her classroom.

A Typical Mathematics Lesson

This case study focuses on Marilyn's teaching of mathematics. While she plans a specific math lesson each day, mathematical representations and ideas are used in many ways throughout the day. Examples include the calendar activity described above, a lunch chart with places for children to place sticks of different colors representing the different food choices for the day, and identifying patterns in songs and poems. Charts and graphs related to science activities, social studies, and holiday events are used to record estimates, measurements, and counts--estimates of how big the classroom pumpkin is, how many days it took a seed to grow, where children will spend Thanksgiving, how many of each color of valentine candy is in a bag, counting by Chinese numerals during the Chinese New Year celebration. There is an abundance of math materials in Marilyn's classroom, including cuisenaire rods, cubes, pattern blocks, attribute blocks, geoboards, tiles, beads, plastic coins, and found materials.

In almost every math lesson, the children use some of these hands-on materials. Most math lessons begin with all of the children gathered on the large carpeted area around Marilyn's rocking chair. Marilyn introduces activities by demonstrating how to use the concrete materials. She elicits the help of some of the children, asking questions of the group throughout the demonstration, probing to make sure the children understand both the procedures and the object of the activities. Marilyn tends to spend a great deal of time explaining activities, especially when they are new to the children. Her directions are very explicit, and if she thinks children do not understand, she will go through a demonstration several times if necessary.

Often, she draws upon the children' previous experiences to help them see connections between different activities, saying, for example, "How many remember when we stacked up unifix cubes and we compared bigger or smaller or greater or less? Our game today is a little bit like that only we're not going to use the unifix cubes, we're going to use the rods." After Marilyn introduces the activity, the children most often work in pairs or small groups with the materials. Marilyn talks through one more example of how to use the materials after the children are in their groups so that they can practice doing the exercise together. If the activity involves making up story problems, for instance, she either elicits one from the children or gives them one herself. After this, children make up their own problems, and she encourages them to "challenge" themselves and their partners by trying out harder problems if they want to.

For most of the activities, children work on the floor around the room. Occasionally, they work individually at their desks; even then, however, they are in small groups formed by the arrangement of their desks, and talk freely among themselves while they work. Marilyn circulates around the room while the children work--helping, questioning, and discussing. She almost always stops to talk to each child or small group. Children are encouraged to seek assistance from their peers rather than just asking Marilyn. She facilitates this by saying, "I just noticed that Mark and Ben figured out that problem. Ask them to explain it to you." When math lessons include paper work, such as recording patterns, or recording addition problems with cuisenaire rods, Marilyn stamps the children's page with a "good work" or a holiday stamp. Sometimes, after the children work for a while individually, Marilyn calls the large group back to the carpeted area and discusses what they learned. Children who finish with their work can use materials from the math shelf, individually or with others who are finished. This allows children who need more time the opportunity to continue working without feeling rushed. Marilyn tells them not to worry if they don't get done and follows up with children who need extra time the following day.

Elements in Marilyn's Mathematics Teaching

Several elements surface in this general overview of Marilyn's mathematics teaching. Concrete materials are central to Marilyn's teaching of mathematics. Concepts are explored through the active manipulation of these materials. Marilyn's explanations and directions are explicit and detailed, and she draws upon previous experiences to help children make connections between mathematical ideas and concepts. Children have daily opportunities to discuss their ideas with both Marilyn and their peers; apparently classmates are seen as important sources of knowledge. Children are given time to work; they are not hurried to "get their work done". These elements are consistent with the vision of mathematics teaching and learning presented in the *Curriculum and Evaluation Standards* published by the National Council of Teachers of

Mathematics (1989). The vision of school mathematics presented in this document guides the current school reform movement in this domain.

In nearly 50 observations of Marilyn's classroom, we identified many examples of the kind of teaching and learning described in the NCTM *Standards*: Constructing meanings in the context of physical situations; devoting substantial time to the development of understandings; building relationships between the conceptual and procedural aspects of tasks; designing instruction appropriate to the intellectual needs and abilities of children; creating environments where ideas are explored, discussed, and applied; developing mathematical reasoning; relating mathematical ideas. In the examples of Marilyn's teaching that follow, we highlight some of these aspects.

Subtraction With Caves and Cubes

During this lesson on subtraction, the children created "subtraction stories"--they counted out a certain number of cubes, placed some of those cubes in a "cave" (their hand) to represent the number being subtracted and solved the problem by counting the number of cubes left outside the cave. Later in the lesson, the children transferred this experience to completing a worksheet where the "cave" was represented with a symbol (∩), and they wrote in their own subtraction problems on the worksheet. (In this report, the cave symbol, ∩, will be represented with brackets [ ]). At the beginning of the lesson, the children were sitting at their desks, which face together in clusters of four to six. Marilyn asked Max to pass out a "work space" (an 8" x 5" piece of laminated construction paper) to each child, and Will to pass out a small handful of unifix cubes to each child. Marilyn moved to the overhead projector in the middle of the room. She placed six cubes ([] [] [] [] [] []) on the screen. To introduce the lesson, she (T=Teacher) reminded the children of an earlier time when they had used caves (to do addition problems):

> T: This may remind you of some work we did a long, long time ago. Does anybody remember. This was, oh, a really long time ago; reach back in your brain. Remember when we made a cave? Let me show you what a cave looks like. (Cups hand over some of the cubes). Well, we're going to make a cave today, only we're going to make a cave with subtraction.

Marilyn began the lesson by demonstrating how to use the cubes to make the subtraction stories. First she had the class count in unison with her the cubes on the screen, then demonstrated taking away four cubes by putting them in the cave (cupping her hand over them). She asked the children to count how many were left. After this initial demonstration, she worked through several examples with the children:

> T: Let's try some on your own work space now. Let's count out six, just like I have now. Put them kind of in a row and they have to be sort of close together because you'll have to make a cave with your hand.
>
> Charles: This close together? (shows Marilyn his row of cubes)
>
> T: You decide, Charles, what makes sense. Everybody ready? Let's hide one in the cave. We're going to have six and we're going to take away one. How many are left? Let's count them. 1, 2, 3, 4, 5. Most of you can do this in your head without even counting them now, but just in case, you can use your eyes. Just like when we use the rods. The rods are there to prove it, to show us. And in our cave, we can count and see how many. Our number story says this: Six take away one equals five. Now watch. What if I go around on this side? Tell me if this changes the story. (Marilyn demonstrates by taking away one from the other end of the row of cubes.) Six take away one equals five. Doesn't change it at all. It doesn't matter if I take this one away, or this one away, it still says the same story.
>
> Paul: Or the one in the middle.
>
> T: Or the one in the middle, does that change the story?
>
> Children: No

Marilyn solicited additional number stories from individual children. While she demonstrated them on the overhead projector, the children worked individually with cubes at their desks. After working through several problems, Marilyn demonstrated how what they'd been doing with their cubes applied to the worksheet they would do next. She used one of the problems to show them how it would look on paper:

> T: You know there's a special way we can write this. I'm going to give you a paper pretty soon that has a funny looking thing like that (Marilyn writes 3 -[3] = 6 on the overhead screen). That's a cave. And we want the total number in our cave over here. What's that funny sign I put in the middle?
>
> Children: equals
>
> T: Equals . . . Uh, oh, I wrote something impossible, did you see what I did?
>
> Gavin: Oh, geez
>
> Clair: Six should be first.

T: I'm glad you caught me. I wasn't even thinking. Clair says the 6 should be first. That (points to 3 -[3] = 6 on the screen) would be impossible. Remember yesterday when Mrs. Hays was here, did you have some impossible stories? I just wrote an impossible one.

Martin: By accident.

T: By accident cause I wasn't thinking.

Martin: I didn't know teachers could make impossible ones.

Marilyn changed the equation written on the screen to read 6 - [3] = 3. She worked through some additional problems with the cubes, each time writing the equation on the overhead screen the way it would look on the worksheet. Then she wrote some problems on the screen (e.g., 8 -[1] =) and asked the children to solve them with their cubes, "Show me with your hands what the problem would look like." She told the children that they would be making up their own problems:

T: You're going to see that there's a little picture of a cave for each problem. We have to start with how many you had, and inside the cave how many you take away, and on the other side of the cave how many you have left over. You can try any number you wish, they can be big numbers, little numbers, you can try zero, see what happens to your equation. Your equation has to be possible, not impossible. That's what I'm going to be looking for. When I see you working, I'm going to say, "Great job!" or I might say, "That one's impossible. What's wrong with it?. . . How could we make sure it's possible?"

John: Make sure it equals the number

T: Oh, John has the secret. Tell us again.

John: Make the bigger, the biggest number first.

T: That's exactly right. When you do subtraction, you have to be sure you have the biggest number first, cause that's what we're taking parts away.

Many of the children used the cubes to count as they made up their problems. Some children were excited to try some "big numbers" (usually two digits), others stuck with one-digit numbers. Children also tried out various ideas. Martin worked through a pattern of subtracting incrementally higher numbers from 22 (22 -[1] = 21, 22 -[2] = 20, 22 -[3] = 19, etc.); David experimented with the order of the numbers in the equation, simply switching the numbers around (7 -[4] = 3, 7 -[3] = 4). Darlene seemed not to understand the relationship between the numbers and the equations (11 -[9] = 11, 2 -[1] = 2). Marilyn acknowledged each child's ideas, and provided assistance when needed. She asked Darlene to count out 11 cubes. When Darlene miscounted by skipping a cube as she was counting (and touching) them, Marilyn had her count again, this time she held Darlene's finger and touched one cube at a time. Marilyn demonstrated putting 9 cubes in the "cave" and leaving 2 cubes out, thus reinforcing the idea that Darlene's problem (11 -[9] = 11) was an "impossible" problem

Cave and Cubes Summary

Several aspects of this subtraction lesson are worth noting. The children were constructing meanings about subtraction in the context of a physical situation--manipulating the cubes. When they made up their own problems, they could verify the answers by checking the empirical evidence (counting the cubes inside the cave and outside their cave). By constantly moving back and forth between the symbolic and physical representations of subtraction problems, Marilyn built relationships between procedures and concepts. Marilyn also related this math lesson to previous experiences, such as using caves for addition and using rods to check answers. She used her own initial mistake in writing the first problem as model for reasoning mathematically. This was an "impossible problem" because subtracting three from three does not equal six; this could be verified with the cubes. Her mistake allowed her to acknowledge that children, not just the teacher, were important sources of knowledge in the classroom: "I'm glad you caught me," "Clair says the six should be first," "John has the secret." By having children create their own subtraction problems, Marilyn assured that they would work according to their own intellectual needs and abilities; this is evidenced by the different approaches to the task employed by Martin, David and Darlene.

Two Days of Bean Combinations

"We're going to have a little fun right now playing with some beans," Marilyn told the children at the beginning of a math lesson. She passed out a handful of beans to each child at their desk. Each bean was colored red on one side, white on the other. Marilyn asked the children what they noticed about their beans, and told them, "The different colors are going to help us make number combinations." She told the children that they were going to make "random" number stories, and asked them to pick up four beans, shake them in their hand, and toss them on the table. She asked Marcus to read his number story, telling him, "say the red beans first." Marcus said, "Two reds and two whites." Marilyn asked some other children to "read their beans." They continued the activity in this way, eventually adding more beans to the total to see what combinations they would get for five and six beans. For six beans, Marilyn had children read their number stories, then all the children who had tossed that combination stood up. She asked for a different combination, and all the children who had tossed that combination stood up. She recorded each combination on the chalkboard:

6
Reds + Whites
5 + 1
3 + 3
1 + 5
4 + 2
0 + 6
6 + 0

During the next part of the lesson, Marilyn passed out a worksheet which was divided into 8 squares, with a "4" in the bottom right hand corner. Each square had drawn on it four bean shapes. The task was to toss four beans, then record the number of red beans and white beans by coloring in the beans in the square on the paper (i.e., coloring in the reds and leaving the whites as they are). Marilyn had prepared two additional worksheets, one for five beans and one for six beans. These were lying on a chair, and children could come up and get one when they were finished recording their combinations for four beans. While the children worked on these worksheets, Marilyn made three large charts on newsprint. At the top of each one she wrote: How Many Times? and put a "4", "5" or "6" beside each heading.

When all of the children finished the worksheet for four beans, Marilyn asked them to stop what they were working on. She told them they were going to make a chart of how many times they got each combination for four beans. To make the chart, she asked children to count the number of times a particular combination is recorded on their worksheet, then hold up that many fingers in the air. For example, she asked how many times they had tossed the combination four reds and no whites. Each child counted how many times this combination was recorded on their sheet, and held up their fingers. Marilyn then counted the total number of fingers and put a tally mark next to where 4 + 0 is written. This continued until the large chart was completed:

How Many Times? [4]

4 + 0 𝍸 𝍸 𝍸 𝍸 𝍸 (25)
3 + 1 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 (30)
2 + 2 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 / (61)
1 + 3 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 (35)
0 + 4 𝍸 𝍸 (10)

After they completed the chart, Marilyn asked them some questions:

T: Now, let's take a look and see which combination .... came up the most. Can you tell by looking at our graph?

James: Two plus two.

T: It looks like two plus two. Let's count them by fives.

Marilyn and children in unison: 5, 10, 15, 20......50, 55, 60, 61!

T: Sixty-one times we had two plus two! I think we threw in a few extra fingers over here. I think we kind of miscounted a little. Which combination came up the least? The fewest times? Barry?

Barry: Four . . . zero plus four

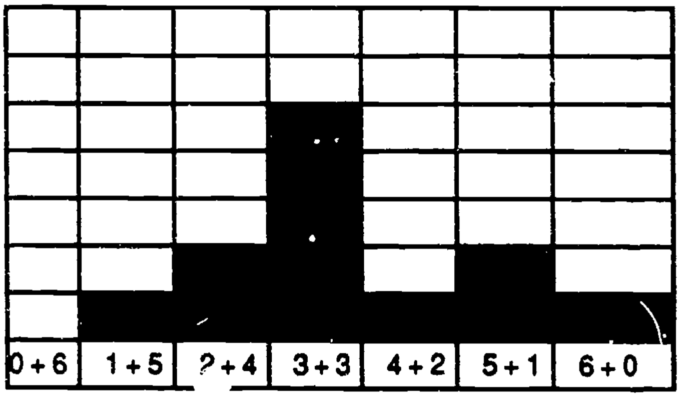
T: Zero plus four, cause it's only ten, and up here we have 5, 10, 15, 20, 25 (points to 4 + 0). So zero plus four came up the least. Let's continue working a little bit, and in a minute we'll stop and see how many times fives combinations came up.

After a few minutes, Marilyn told the children:

T: Boys and girls, if you are finished and waiting, if you would like you could take your pencil and in each box, write the number combination with numerals, like this. (Marilyn shows the children a page with the number combinations written in the boxes) If your box shows this, write this. Write it in pencil.

After about 10 minutes, Marilyn asked the children to read their recording sheets for five beans; on a second chart she recorded a tally for each time that the children had tossed particular bean combinations. She told children they would do the chart for six beans tomorrow, and that if they hadn't finished that worksheet, they would have time tomorrow.

The next day, the children recorded the combinations from their bean tosses in a different way. Marilyn gave each child a piece of graph paper and told them to write in the possible combinations for certain numbers (e.g., 4, 5, 6 etc.) in the squares along the bottom row. The children made a separate graph for each number. They could either think about what the combinations might be for the number five (e.g., 5 + 0, 3 + 2, etc.), or they could toss five beans and write in the combinations as they tossed them. Each time they tossed a given combination, they colored in one square of the graph in the column above where that combination was written. Their graphs looked something like this:



Some children referred to their charts from the previous day or the large chart on the chalkboard to help them determine whether they had listed all the possible combinations of beans at the bottom of the graph. Matt asked Laney, who was finished with her first graph, to help him. Laney came over to his desk, and they shared his chair. Each sat on an edge, dangling one leg over. Laney and Matt both read their equations from right to left instead of left to right. When they tossed 2 red beans and 1 white bean, they said, "two plus one," but they marked a square in the column labeled "1+2".

After the children worked for about 10 minutes, Marilyn asked them to stop for a minute. She said,

> You were doing a good job. Some of you had a hard time getting started because you couldn't figure out how to tell how many combinations there are, and Amy and I have two systems, cause Amy had a lot of trouble with that, and now she says its easy.

Marilyn told the children to get out eight beans and turn them all red side up. She demonstrated how to systematically get all the combinations. The first combination was 8 reds + 0 whites. Next Marilyn told the children to turn over one bean so that the white side was on top; this combination was 7 reds and 1 white. Marilyn and the children continued turning over one bean at a time to list all the possible combinations. Marilyn then introduced the second system:

T: Now, that's one system you could use to figure out how many equations there are. Here's another one. Make them all red, turn them back over again so they're all red. (Waits while children turn over their beans.) Are they all red? How many is your total? We have eight, we didn't change it at all. Here's the second way you can do it. Read the equation first, Cally.

Cally: Um, eight plus zero.

T: Eight plus zero. Let's write that one down. Now, let's write the opposite equation right next to it. What would the opposite be?

Don: Zero plus eight.

T: Very good. Let's write that one down. Zero plus eight. Without flipping, we're just doing it in our head now. Flip one red bean. What's your new equation, Barry?

Barry: Um, seven one.

T: Seven plus one. Now, in your head, what's the opposite equation, Max? The opposite of seven plus one? Helen, do you know? Bob, what's the opposite?

Bob: One plus seven.

T: Isn't he wonderful? Yes, you think in your head, the opposite of seven plus one would be one plus seven.

Marilyn continued to go through the equations by having children turn over one bean at a time, then think of the opposite equation in their head. "That was thinking work!" she said when they got to 4 + 4. She asked the children to get out their papers that they'd done yesterday for six beans, and told them that they'd finish their chart. Again, she tallied the number of times the children had the particular combinations (e.g., 2 + 4, 3 + 3, 0 + 6, etc.). The tally for 3 + 3 was very high. Marilyn said, "What do you think about 3 + 3? Look at it compared to our other charts. It looks like that's going to be a very popular one to show up." When they finished the last chart, Marilyn asked the children to make some comparisons among all the charts (4, 5, and 6). The found the number combination that had the come up the most (2 + 2) and the least (5 + 0).

<u>Beans Summary</u>

These two days of mathematics with beans were actually part of a much longer sequence of activities that started at the beginning of the school year and continued throughout the year.

Marilyn had provided the children with many experiences combining and recombining number. In fact, over several weeks, the children had used nearly every manipulative in the classroom to make sets and combinations of numbers. As is stated in the NCTM (1989) *Standards*, "emphasizing mathematical concepts and relationships means devoting substantial time to the development of understandings" (p. 17). The children in Marilyn's classroom were give time to develop understandings gradually; each day they related the use of new materials or new representations to previous ideas or to concepts they were familiar with, such as the charts and graphs, which they used often in different ways across the curriculum. As she had done with the caves and cubes, Marilyn constantly moved from concrete to symbolic representations of the number combinations. She used a variety of different phrases--number combinations, equations, number stories--to describe the grouping and regrouping process.

Student Interviews

Toward the end of the first year of the study, 10 randomly selected students in each of three first grade classrooms were interviewed in an effort to determine not only how they *felt* about mathematics (e.g., their most and least favorite things in math, their ideas about what it means to be good in that subject), but also what they *know* about ideas like number convention and place value--and how well they could perform on problem solving and computation problems. Ten of the students were from Marilyn's class, the rest were drawn, in equal part, from two other first-grade classrooms at the same school. The teachers who taught these students embraced different, relatively more traditional approaches to the teaching of mathematics. One teacher, for example, named Ann, used a textbook-based, individualized program in her classroom. Assignments were made on an individual basis by the teacher; children worked on a self-pacing basis at their desks, receiving help from the teacher as needed. The second teacher, Betty, relied on a hybrid approach, using traditional, teacher-directed math activities for most of her instruction, but supplementing this work, on occasion, with extra hands-on activities.

For the most part, students in all three classrooms performed similarly on interview tasks. There were some interesting exceptions, however, and these are worth noting. The most apparent difference favoring Marilyn's group falls in the mathematics attitudes and beliefs category: Her students were more likely than those in Ann's class (considered the most traditional in its adherence to the standard curriculum) to name a diversity of students when asked, "Do you know someone, other than yourself, who's really good at math?" Nine of the 10 children in Ann's class named a single child (Ben) on this item, apparently basing their judgment on the number of "math packs" completed by that child at that point in the year. Students in the other two classrooms were more inclined to scatter their choices; thus, a total of 5 different children were nominated by those in Marilyn's group, 4 different children by those in Betty's group.

Marilyn's students were also more prone than those taught by the other teachers to give a diversity of reasons when asked, "What does (the nominee) do to make you think he/she is really good at math?" In the other two classrooms, students invariably mentioned three attributes in accounting for their nominee's success: Speed, quantity of work, and correctness. Students in Marilyn's classroom, by comparison, listed a diverse set of characteristics, including "good problem solving," using manipulative material of various sorts (i.e., "uses bean counters"), employing strategies like counting on and counting back (i.e., "uses fingers to add up"), and so forth. Given Marilyn's commitment to broaden children's perspectives on mathematics as a discipline, this appears to be a highly significant finding. Results on a related question provide further evidence of the effectiveness of Marilyn's approach in affecting how children think about elementary school mathematics. Thus, when asked to describe the "most" and "least" favorite things they did in math, students in Marilyn's class were more likely than those in the other two to diverge from the stock "addition" or "subtraction" sort of response, naming a diverse set of activities ranging from the "use of rods" to "working with money."

Two items borrowed from Ross (1989) were used to assess students' understanding of place value concepts. Both proved extremely difficult for students in all three classrooms. On the first, where students were asked to count objects, record the correct quantity, and then describe the relationship between the objects and the parts of the numeral (25), those in Marilyn's class performed close to significantly better on the tens digit (X^2 (2) = 5.41, p < .06). When asked, "Does this part of your 25 (i.e., the "2") have anything to do with how many sticks you have?" over half of Marilyn's students correctly identified the digit as equaling 20 objects, compared to less than half of the students in the other two classrooms (8/20); however, this difference did not approach significance. The second item used to assess place value knowledge yielded generally disappointing results in all three classrooms, probably due to the nature of the task. Because it deliberately confounded standard and nonstandard groupings (e.g., sorting 13 "candies" into three cups, remainder 1), there was a floor effect which minimized the possibility of detecting classroom-related differences.

Interestingly enough, Marilyn's students performed *just* as well as those in Ann's class on difficult, two-digit addition and subtraction regrouping problems. This, despite the fact that Marilyn's students had received no direct instruction on the relevant computational procedures. Thus, those in the most and the least traditional classrooms significantly outperformed those in what might be characterized as the more mixed or hybrid type classroom (i.e., traditional text supplemented with manipulative activities) [$F (2, 27) = 3.5, p < .05$]. On items measuring number convention (i.e., the use of nontraditional counting procedures), and simple one-digit addition and subtraction, there were no significant differences between students in the three classes.

In the Fall of 1990, eight of Marilyn's students were interviewed. Because children had had many classroom experiences sorting, making patterns and combining and recombining groups of objects, one of the purposes of the interview was to see how the children might apply these experiences to similar and novel tasks in the interview setting. A procedure using finger puppets was used to assess students' understanding of number convention. In this procedure, the interviewer explained that the two puppets were from different countries. One puppet counted three objects in an unconventional manner (i.e., A, B, C or 1, 1, 2). The other puppet counted three objects in the conventional manner (i.e., 1, 2, 3). Only one child failed to understand the arbitrariness of our number system, insisting that "You can't use letters to count." Two of the children thought that counting 1, 2, 3 and 1, 1, 2 were both correct--but judging from their explanations, they may have thought that any use of numbers in this regard was appropriate.

All of the children could use materials to represent sets of different amounts. This was true whether the stimulus was verbal ("Can you make a set of 7?") or visual (cards with written numerals). As expected, some of the children were able to apply their classroom experiences spontaneously to the interview tasks. For example, Michelle, when asked to make a set of five, made three groups of five by sorting toothpicks into groups based on the sharpness of their tips. Max combined sets of four and one to make five, one and one to make two, and six and one to make seven. Both were able to use strategies developed in the classroom.

We were particularly interested in assessing students' use of patterns, an important theme in Marilyn's teaching of mathematics. The procedure was as follows: Using attribute blocks, a pattern was started and children were asked to complete it. All of the children could do this, with one exception: a child who switched the order in the middle of the sequence. Children were also asked to make a pattern of their own with the blocks and to label it. Only two children used more than three attribute blocks in their pattern, but some of the children experimented with the positions of the pieces. All of the children could provide at least one name for their pattern. For example, this pattern: SQUARE / TRIANGLE / DIAMOND / CIRCLE / SQUARE /TRIANGLE / DIAMOND /CIRCLE would be "A B C D A B C D" in "AB language," an approach that children in Marilyn's classroom often used to describe arrays of this sort.

All of the children seemed to have a good understanding of number relationships. For example, they were able to arrange groups of 2, 5, and 8 in order from smallest to largest. All but one of the children named 21 as bigger than 12 and 72 as bigger than 27 when they were shown these pairs of cards. When shown number problems represented symbolically (4 + 2, 5 - 3), which had not been introduced in the classroom, half of the children correctly read the 4 + 2 card and said it equaled 6, but only one of the children correctly read the 5 - 3 card. Further, all of the children interviewed were able to solve two simple word problems using concrete materials.

Marilyn's Views About Teaching, Learning, Curriculum, and Assessment

Teaching Against the Grain

Cochran-Smith (1990) uses the term "teaching against the grain" in a recent paper that contrasts different approaches to reforming educational practice. She argues that teachers who are at the vanguard of the reform movement constitute a distinct minority--and face formidable obstacles in their attempts to change accepted practice: "Often they must raise their voices against teaching and testing practices that have been "proven" effective by large-scale educational research and delivered to the doorsteps of their schools in slick packages" (p. 7). "It is not surprising," Cochran-Smith concludes, "that teachers who work against the grain are sometimes at odds with their administrators and evaluator" (p. 7).

The sort of teaching called for in the NCTM (1989) *Standards* runs counter to many of the prevailing norms in education. This, plus the fact that it generally places greater demands on teacher and student alike (Cohen, 1988), serves as a disincentive for many teachers who otherwise might experiment with more "adventurous," conceptually oriented teaching. Marilyn seems well aware of these difficulties. On several occasions she commented on how hard it was for her to buck the tide: "As long as I've taught-- 24 years--I still get real insecure every now and then," she confided during our December interview. A month later, she again admitted to some continuing anxiety over her decision to depart from standard practice:

> I get a little apprehensive sometimes myself with what they should be doing because it's different than it's been in the last 20 years when you had a book and you just filled in the pages and, "There. We've completed our addition facts up through 20." [Now] the focus is not on addition facts . . . I'm focusing really on the concept of how things fit together.

Marilyn attributes her willingness to stick with a less conventional approach to teaching to two things: A positive self-concept ("I am sturdy enough," she says), and her firm belief in the validity of her own views about teaching and learning. Thus, she indicated on more than one occasion that when she experiences uncertainty about her practice, she says to herself, "'This is what I believe, this is what I'm going to stick with." She acknowledges that many teachers feel pressured to meet certain expectations by state guidelines, school mandates, textbook requirements, testing, and so on, but she argues that this pressure can be resisted when teachers learn to "rely on their own resources, instead of worrying about all the other things."

We attempted to assess the nature of Marilyn's own "resources" in the course of several hours of interviews. What emerges is an amazingly coherent set of beliefs about teaching and learning. Although we may not do justice to the interactive quality of these beliefs by presenting them in a piecemeal fashion, there is little alternative given time and space constraints. In the analysis that follows, four important instructional categories are used to present Marilyn's views: Teaching, learning, curriculum, and assessment. As we will argue shortly, a strong developmental and constructivist bias is evident across all four categories of belief.

Views About Teaching

Marilyn takes issue with the classic distinction between child- centered versus subject-centered instruction. "Teachers spend too much time," she feels, "thinking that what they have to present is much more meaningful than what the child comes up with." This does not mean, she quickly adds, that school should be totally child-directed: "I certainly have a sense of things I want to get across and concepts or units that I want to deal with." The problem, particularly with beginning teachers, is that they focus too much on their own lessons and not enough on how those lessons are being responded to on the part of the learner.

When asked how one might "gently, or not so gently" move beginning teachers away from a focus on lessons, Marilyn responded, "The key is to listen to what the kids are doing and to respond to what you're getting back. . . . Teaching is a give and take situation, and not just the teacher in charge spewing out information." In discussing the difficulties associated with this approach, Marilyn cited a recent student teacher she worked with who could not make that transition. "She was so focused on herself and her lesson," Marilyn said, "that it never was a teaching situation--it was a presentation every time she did something." The transmission mode of teaching is not limited to novices, however. Marilyn thought this view was fairly widespread among all teachers: "Unfortunately, a lot of experienced teachers are still in that realm," she said, characterizing their thinking as, "'There, I presented it, I got through my unit so I'm done.'" It is more productive, she feels, to focus on what students have learned.

Marilyn has developed a term for the kind of teaching she favors. She calls it "real teaching:"

> In "real teaching," you don't look at it from September to June and ask, "What are you going to stuff into their minds?" Real teaching is taking what you think they're coming in with and presenting things to open their minds, show them new avenues. I know this sounds really dumb, but that's carrying them through rather than just thrusting things at them and having them spit it back. That, to me, is not teaching at all.

In her December interview, Marilyn denied that she was exceptional in her teaching: "I don't think that I'm terribly unusual," she said, "I don't think that I'm the only one in the world that thinks this way." She admitted, however, that she was less concerned than many of her colleagues with the "sense of what has to be done," the sheer amount of material that must be covered. Many teachers, she believes are too "uptight"; we must devise a way to lift the content coverage burden if we are to have teachers who "really concentrate on what the child is saying and doing, and then react to that in planning their lesson." Teachers need to be assured that students are "going to learn all right, as long as the teacher has the overall structure in mind--knows where she wants to go from beginning to end."

It is not surprising, given Marilyn's interactive view of teaching, that she places a heavy emphasis on question asking in her instruction--and on the importance of discourse in the student learning process. Regarding the former, Marilyn comments that "asking" is more important than "telling" in her approach to teaching. "You can't think if you're not asking questions," she said, "if you're just telling responses, that's kind of a flat level. You're not having to delve into why and who and wherefore and what if." It is important, therefore, that teachers learn how "to teach with questions": "What would be a good way to . . . ? or Who has an idea how . . . ? or Why do you suppose this happened this way? That kind of technique."

Fostering discourse or dialogue is a high priority in Marilyn's teaching. "Learning is interactive,' Marilyn states, "I don't think it's just individual. . . . When you're here [in her classroom], you hear a lot of really good interactions--kids teaching each other or saying, `Well, no, it can't be that because of this.'" A little later in the interview, (T=Teacher, I=Interviewer) when asked why she thought discourse was so important, the following exchange occurred:

> T: I just think that language has to be there; you have to interact in a verbal kind of way when you're learning something. I don't think you can learn anything in a very quiet, static sort of position.
>
> I: How about the solitary scholar who goes off with his or her stack of books?
>
> T: I think you can think real hard about things, but I really don't think you can learn concepts all by yourself. I think you can go off in different directions on your own, but I think if you're trying to grasp something or master something, it's much better to do that with someone. Now there are children who do like to go off on their own, but I think they're missing something. They miss input. Whether you're the fastest one in the room or not, you still will get a lot of information from other children. Whether you're talking about how to draw a picture or how to do math, by interacting in a verbal sense you get that much more information--that much more input. . . . I think it's important in all grades that kids are allowed to talk with one another. Classrooms where the norm is to be quiet and do your work--I don't think they're doing any work. They're doing individual things, and yes, there are times when you go off on your own and you study and reflect, but I think the best learning comes from interacting.

As this quote suggests, Marilyn appears to embrace a "dialectical" view of knowledge generation: As Cobb (1989) describes it, this view represents a middle ground between the Piagetians and the Vygotskyians because it emphasizes the dialectical relationship between individual knowledge, arrived at by reflecting on one's own activity, and knowledge that is socially mediated or jointly agreed on. This characterization obviously goes beyond the data, but it is consistent with her belief that knowledge is "negotiated" in the classroom.

Given the priority Marilyn places on the discourse process, it is not surprising that she has firm beliefs about the importance of ground rules for discourse in her classroom:

> The other thing that's really important in the classroom related to language is the whole concept of trust. You have to be in a pretty comfortable situation before you're willing to give your ideas--and that's the other thing that teachers have to change. They have to learn how to set up a classroom full of trust because if you have that trust, where the teacher isn't going to jump all over you or the kids aren't going to laugh at you, then you can try ideas, and then learning takes place. But it you're in a situation that is so tight that it's only a "right" or "wrong" or "yes" or "no," you're never going to try--it's just human nature. . . . I try to set up the kind of climate where one is not afraid . . . [one where] they're seeing each other as viable little people who have wonderful ideas.

When pushed about how one develops this attitude in teachers, Marilyn talked about the importance of having models of an a more open-ended style of teaching during one's teacher training. Ultimately, however, she said, it boils down to a question of priorities:

> Teachers have this sense that they are so responsible for this body of knowledge. I don't feel that way at all; I'm responsible for these little people learning. I don't feel the least bit responsible that they do "magic e" by November, but I do feel responsible that they are progressing and learning and reading and writing and questioning.

One further comment should be made about how Marilyn thinks about the discourse process in her classroom. Her views about cooperative learning surfaced in another section of the interview in the context of a question about "thinking skills." (Note: She had expressed reservations about the tendency to isolate skills like problem solving from the other learning that goes on within subject matter domains: "It's not a separate entity, it should happen all the time . . . It's everyday conversation. It's what happens when I say `How many days until the next Zero the Hero Day?' It's `Who knows what number will come next?' It's all of that kind of thing that goes on all the time.") In the January interview, Marilyn made explicit the comparison she saw between the thinking skills approach and cooperative learning:

> T: It's the same thing that people are talking about now in cooperative learning. They're focusing in a cooperative learning lesson on how you interact and all of this. And at the beginning, I thought "Oh, yeah. That's just great--a whole lesson just on how to cooperate and who talks and who responds." Then I started thinking. I said, "That's really unnecessary if you have that kind of structure in the classroom where that cooperative interaction is going on all the time."
>
> I: I don't see the parallel between the thinking skills and the cooperative learning.
>
> T: It's separating them from normal activities--normal things that are going on.

At numerous times during the interview, Marilyn made reference to how her views of learning have influenced her approach to teaching. Because she believes that students never really "master" important concepts, that there is always something more for them to learn, she subscribes to what she calls a "rolling" or "circular" approach to teaching:

> I view myself as teaching in kind of a rolling sort of pattern. It's not linear, I know that, and I know it's not a start and stop. When I've done a unit on reading clocks or counting money, it doesn't just end there. I try to pick it up another time and reintroduce it, or look at it in a different way.

Later in the interview, Marilyn added, "I hope that the next time I come back at least the child who didn't get it all will have an inkling of `Oh, yes, we did talk about that,' and they're start going from there."

Views About Learning

Marilyn would probably be considered a hard-core constructivist. Over the course of several hours of interviews, Marilyn repeatedly came back to a familiar refrain regarding the importance of teachers attending to students' learning--and of crediting them with an active role in that process. Learners don't learn "by just filling in the blanks and just responding to information that's already there." Learning is active, not passive, Marilyn insists. This view about learning is at the heart of her "philosophy of education," Marilyn insists. Learners learn by "sorting, grouping, making things meaningful," and they are not blank slates: "There's a lot out there that they start with. It's in a global sense at first, then they start doing things in finite, refined ways, and you do that by having them point out, by discovering, by talking with others." Marilyn contrasted this view of learning with what she termed the traditional "segmented" view:

> You did this little bit, and once you mastered this then you were ready for that. I think that's totally backwards. I think that's not how people learn. If you're learning a new skill--needlework, for example--you don't come in and practice only your X stitches. You try a variety of things and as you get better, it looks better because you're more practices, you worked on it, someone's helped you; and that's the way learning is in the classroom too. You don't come in and just work on addition facts up to five because we want to master those first. You come in and you do a variety of things and then you see the big picture. Once you see the big picture, than you can start focusing on the smaller parts.

As will become evident in the next section, this "big picture first" image of children's learning maps relates in interesting ways to Marilyn's views about the curriculum. Thus, in an interview conducted a year after the one drawn on above, Marilyn stated that she doesn't look at curriculum guides, preferring instead to look at the "big picture." She tends not to think in terms of "topics" in subjects like science and social studies, focusing instead on important concepts like "change" and "diversity." She applies this same sort of thinking to mathematics:

> I look at math that way too. The important things to teach are sorting, grouping, comparing, and looking at patterns. That's what I think it is important to teach. The rest of it kind of falls into place, or the rest is the vehicle that I'm using, or the outshoot of going that that goes to another level.

As this last quote suggests, Marilyn is able to apply her learning theory to subject matter learning. In this realm, in particular, Marilyn appears to highlight the role of internal representations as evidence of conceptual understanding. Thus, she contrasts rote learning with being able to "see things in your head," which is typical of the person who is good in mathematics:

> They don't just see numbers, unless they see numbers as groups of things--but they're not just repeating an oral kind of a pattern, like the two times one is two, that's not what a good mathematician does. A good mathematician sees pictures of things.

Furthermore, although she is not as explicit as some mathematics educators (see Hiebert & Carpenter, 1990), Marilyn appears to buy into the notion that external representation influences the child's internal representation. Thus, in talking about the importance of "pattern" as the overarching concept in first-grade mathematics, Marilyn cited various activities that help develop that notion as it relates to number: "You can group objects, then you can name that group as a number, and you can compare it to another group." "While it's very concrete," she added, "and you're working with manipulative things, it's also the representation of this basic understanding [e.g., pattern]."

At this point, Marilyn was asked to explain how this type of representation relates to "written representation." "I see it as a hierarchy of representations." She explained that written symbols were the most "abstract." The kinds of representations she uses, and the activities that accompany the representations, fall at some midpoint on the concrete to abstract continuum: "That this piece of paper represents the pattern block, they [her first graders] don't have a lot of trouble with that, and yet for younger children, that would be a big difference. At this level, that kind of representation is very simple." In her teaching, Marilyn added, she tries to move back and forth between the more concrete and the more symbolic:

> I make sure that lessons are designed to be very concrete, and kind of shift them up into that symbolic. At the same time when we're doing concrete, I very often say "Tell me that in AB language," which is very, very abstract. So, I'm always trying to constantly shift them in and out.

The idea that there are many different ways to represent important concepts or ideas fits well with Marilyn's developmental view of the learner. On several occasions, Marilyn emphasized that learning was not a linear process: "My idea of learning [is] that it isn't just a straight line... It's a process of going forward and losing ground." A month later, she elaborated on this notion:

> "Learners don't necessarily master something when you teach it or when you introduce it. . . . I think when you have a group of children, and you present information, some of them grasp it immediately, some of them just are introduced to it, some of them say, "Huh, what?"

For this reason, she frequently revisits important ideas: "I go backwards and forwards and backwards and forwards--that's the way I see it. Maybe," she added, "I see it a little differently." Marilyn's view that students understand concepts at different times and in different ways reinforces her belief that it is always important to provide youngsters with multiple ways of representing ideas. "I think that the more ways you can represent something to a child, or anybody that's learning something, the better it is. With various ways, something is going to click and make sense . . . something's going to fit somebody somewhere."

Marilyn's views about learning, while constructivist in nature, appear to avoid the common pitfall of equating worthwhile learning with being involved in what she calls "cute or fun activities." This distinction was most explicit when Marilyn talked about her involvement in a Math Their Way workshop several years ago. She claimed to learn a lot in this experience, but emphasized that it was not "something that you make and take": that is, the type of workshop where you acquire a repertoire of easy to use learning activities. "I found it very difficult," she said. "The book is just filled with tons of activities." Marilyn paused, then added, "Well, you don't just teach activities. You have to have beginnings, middles, and ends. It has to hook together." Marilyn said that it took her quite a while before she felt like she had a "cohesive" program. As with issues associated with teaching and learning, Marilyn appeared consistent in her adherence to a developmental/ constructivist framework in her views of curriculum and assessment.

Views About Curriculum and Assessment

Marilyn has firm views about mathematics, insisting that, as a subject, "it's not a reactive thing; it's inventive, it's logical, it's creative--it's all those things." She is critical, if somewhat sympathetic, of those teachers who treat mathematics as a "reactive thing." They generally are wedded to a traditional text. Teaching then becomes the management of this material. Teachers probably would do a better job, Marilyn believes, "if they just threw all that stuff out and just taught kids, like you would at home."

Too often, according to Marilyn, teachers teach the "system" in mathematics and not the "concept." The focus is on rules and procedures instead of the important ideas that underlie the rules and procedures:

> What happens is they [students] get trained to look at the top number and the bottom number, and you have the same problem you have in multiplying or adding fractions, cross this one out and do such and so. It doesn't make any sense. Children don't have an understanding of what they're doing if they're just taught to take it at face value.

Marilyn appears to advocate a much more focused and coherent approach to curriculum. For example, she stresses the importance of "numeration" as a key idea at the first-grade level, offering the following definition: "Numeration is an understanding of how our numerical system is set up, and it's a pattern." Her goal in teaching numeration, then, is to get students to appreciate this fact. Concretely, this means that her students, in second grade, will *not* look at a borrowing problem and say, "`I know what to do. You slash this, and you put a one there.'" Instead, Marilyn explains, "I want them to think that, `Oh, I've made a group of 10; that means I now have a 10 and 3, and a 10 and a 3 is 13'." As Marilyn points out, this second, more conceptual approach is "a different way of looking at something."

When students come to view mathematics as a series of patterns, they will be close to the essence of what she thinks the discipline is all about. At one point, Marilyn said, "Mathematics has tons of patterns. Every time you do something, it's a particular pattern. . . . Pattern is one of those steady threads that runs throughout." Marilyn developed this view of mathematics a year later when she was asked to produce a visual representation of the domain at the primary level. She sketched as a sort of umbrella. The top of the umbrella was labeled "concrete patterns of math." Sorting, grouping, and comparing lie along a base line at the bottom of the umbrella, and number, numeration, and symbolic representation constitutes a kind of "handle." The latter are "strands" in the mathematics curriculum, while the concrete activities (e.g., grouping), are the vehicles through which students learn the concepts in the strands. Interestingly enough, Marilyn's way of thinking about mathematics mirrors that of a prominent mathematician, a professor at the Massachusetts Institute of Technology, who also defines mathematics as the science of patterns: "Its aim," he writes, "is to classify, explain, and understand patterns in all their manifestations--whether the patterns have to do with quantity, shape, arrangement, or form. Around this notion," he adds, "a practical philosophy of education can be built" (Hoffman, p. 18).

Marilyn has developed a stance toward elementary school mathematics that she believes allows her to focus on students' learning without having to worry about all the "little individual activities." She relies heavily on her grasp of the "big picture" to chart the course of her instruction in math and science and other subjects:

> I am not real good at lesson plans. I don't very often make detailed ones, but when I am looking at a whole unit in science, for instance, I have a sense of what it is that I want them to know. I suppose I could sit down and, in today's sense, make a concept map. I could very easily, but I don't particularly write that down. I have a sense of what it is they need to know, and I have a few activities that will get us there, and then, in between, I do my best teaching sometimes. "Ah ha, I think I'll do this tomorrow--this makes sense." Or somebody reminds me of something, and I very often change even during the day. I'll come up with an idea that I am going to do one thing, and decide--either because of behavior or what they need--that I am going to do something completely different. So I have this overall sense.

Later, in the same interview, Marilyn endorsed the idea of a "concept map," not, as Lampert (1988) points out, as a course to follow, but rather, as a general map of the terrain to be covered. "You have to have a pretty good idea of where you're going before you ask kids to go there," Marilyn cautions.

Marilyn applies the same fluid, dynamic thinking when asked to discuss issues of assessment and teacher accountability. Much of her instruction is based on the feedback she receives from students: "I take my cues from the children. If they're kind of bored, or if they're confused, I try to represent it [the idea] in a different way." Her approach to teaching, she believes, provides her with ample opportunity to assess what children know. During discussion "you can see those wheels turning," she adds. "These little people are real easy to read, they don't know how to hide when they don't know something." Marilyn also emphasized that her approach to mathematics provided ample opportunity to "test" students knowledge. She frequently circulates and asks children to demonstrate concepts with manipulative materials:

> If they can't do it with cuisinaire rods, maybe they can show me with tiles, or blocks on a box. Some children could show me with all of them, but those children who are sort of hit and miss--you know that they're just sort of grasping it a little bit. We're getting there if they can show me with some of the materials.

She then summed up what she saw to be an important additional argument for her more concrete approach to mathematics: "I think with real concrete kinds of things, you get a better sense of whether they [students] know it or not."

Not surprisingly, given all that has been said so far about Marilyn's views on education, she is not a strong proponent of traditional, written assessment. She feels the concept of pre- and postassessment is based on an erroneous notion: That of "mastery," which she considers a static as opposed to dynamic concept.

> Why do you need to know exactly what skills a child has learned in reading in order to help them read, and help them grow? And I feel the same way for math. Why is it that you need to know exactly how many math facts they have on this particular day in order to watch them grow?

The whole idea of written assessment is overrated, Marilyn believes. Her idea of a good assessment, she said, is a list of what children generally accomplish at a particular age: "I really feel that it should be just kind of a general view. This is what a six- or seven-year-old should be doing . . . and then how does your child fit with that?" Most formal assessments tell parents more than they want to know about their children--and they have a negative influence on instruction.

In several hours of interviews, Marilyn Anderson emerged as a teacher with a firm and coherent set of beliefs about a host of educational issues. A central premise in this case study is that we have much to learn from teachers like Marilyn. It is generally recognized that rich descriptions of exemplary practice can guide us in our efforts to improve on traditional practice. Less attention has been devoted to the sorts of issues focused upon in this section of the case study: How outstanding teachers see their world; that is, what sorts of beliefs about teaching and learning appear to drive their instruction. Marilyn Anderson's constructivist, developmental philosophy of education may or may not constitute an unusual perspective for those committed to a more conceptually oriented type of teaching.

More research intensely probing teachers' beliefs must be conducted before we can hope to resolve such a complex issue. We also need more insight about how it is that teachers like Marilyn have come to think so differently about their work and their students. In the final section of this report, this issue is addressed. These data, based upon the interviews, are, of course, retrospective. Nevertheless, they do shed some light on various factors that have contributed to the development of Marilyn's views about teaching and learning.

Explaining Conceptual Change: Marilyn's Story

Marilyn made several references to her early teaching experience, and how that influenced her thinking about education in general and the teaching of mathematics in particular. She was trained in the 1960's, she said, and at that time great emphasis was being placed on "individualized instruction," or children moving at their own pace through a carefully sequenced set of material. Marilyn had trouble from the start with this approach, she explained. "I just couldn't swing with that," she explained:

> I was taught in that era, where th's is how you're supposed to do it: First, find out where every child is, and then assign them specific things just at their level. And I never could do that, and I didn't know why--but I just knew that I didn't like it and it didn't work for me.

She then added a comment suggesting that reflecting on her own experience as a learner contributed to her doubts about the efficacy of the individualized approach: "I think that as I've grown, it's because . . . I really don't feel you learn alone. You learn with others."

Personal experience has played a key role in shaping Marilyn's attitudes and beliefs about teaching. When asked to talk about her own early experiences in mathematics, she cited a particularly important junior high school experience:

> I remember, as an elementary school student, I was always very good in math. I was fast, and I got my timed tests done well, and I like math a lot. I don't remember thinking much more about it than that. I got a lot of positive feedback, but I think the thing that stands out the most--and I can't remember exactly what age it was. I think it was middle school, which was junior high when I was there--so it was probably seventh or eighth grade. We used an experimental math program developed at the university. It was just a whole bunch of paper stapled together; I

think my grade used it for two years--and I absolutely loved it. We worked with kinds of bases; we worked with all kinds of interesting things that I've never done before--and it wasn't computation at all. We had to learn whole new vocabularies, and that's what I remember most about math.

Marilyn continued to enjoy mathematics in high school, although she felt a little intimidated by the males in her class, and did not complete the senior-year calculus course: "I started taking calculus, and I decided I didn't want to compete with those boys anymore--they kind of overwhelmed me. So I stopped it, and I stopped taking math at that point." Nevertheless, Marilyn insists, her overall experience in mathematics was positive, largely due, perhaps, to the experimental course described above. The interest and enjoyment she derived from this course may have influenced her own expectations as a teacher of mathematics. Certainly, Marilyn was well aware early on that the traditional approach to mathematics teaching was not for her.

On two different occasions, in December and again in January, Marilyn referred back to an important time in her teaching career: She was asked to teach a combination first and second grade after having taught kindergarten for several years. Out of fear or uncertainty, she decided to change her approach--she felt free to explore ideas with her kindergarten children. She selected a mathematics textbook and stayed with it: "I tried to go right through the manual." Unfortunately, according to Marilyn, she did not experience much success using this approach: "I think it was the worst job I've ever done," she admitted. "I felt like I was just plugging through and filling in the day and the students weren't really learning; they were just filling in the blanks." Later, Marilyn used almost the same language to describe this experience, adding that she felt like she was simply "managing" the curriculum.

One theme that Marilyn returned to frequently in her interviews was the importance of early kindergarten teaching. She described this as a situation where she could experiment with her teaching--with a minimum of pressure to cover material and demonstrate the effectiveness of her particular approach: "Kindergarten is good training," Marilyn felt. "There are no manuals in kindergarten that teach you what to do from the time you come to the time you leave." "You're on your own," she stresses, "but you have to have this overall sense of where you're going and how to get there." Marilyn feels that that's the way it should be for all teachers; unfortunately, many of her colleagues would be uncomfortable with that kind of freedom. They cling to their teaching manuals (i.e., their editions of the textbook). As a result, they seldom examine what they are doing in particular subjects. "Even very very good teachers need to rethink what their focus is," Marilyn adds.

Teaching at the kindergarten level gave Marilyn ample opportunity to get to know her students and to appreciate how they think and behave. Marilyn compared this set of experiences to the one she had with her own children. The importance of the latter also constitutes a recurring theme in Marilyn's interviews. Marilyn talked about how upset she was as a beginning teacher with a comment made by a principal, who praised her excellent rapport with children "even though you don't have any of your own." She felt like that was unfair; one can be an excellent observer of students without having children of one's own. She now qualifies this view: "It can make a difference," she admits. (She is quick to add, however, that she knows teachers who operate with a double standard: They are critical of things their children experience in school, but they do those things in their own classrooms.) Being able to observe her own children closely, she feels, to "watch how they learn and grow," has contributed to her views about education. For example, commenting about youngsters' tendency to want to return to things they already have mastered, Marilyn comments, "I see my big ones at home, especially my ninth grader, very often will get very simple picture books and just thoroughly enjoy them--or very simple games that they enjoyed when they were little. I think that kids do this in their learning." A month later she returned to this theme--of learning about students by observing her own children:

> When you have your own children, and watch them and see how they grow and how they learn--if you really think about it, it makes sense for kids in school too. You don't just sit them on a chair at home and teach them fly-fishing. You go out and do something, and you talk about it. . . . I think that's how school should be.

Marilyn appears to be an uncommonly reflective individual, but she has also had ample opportunity to reflect (her kindergarten teaching), and input sufficient to raise questions about the adequacy of standard practice. This, of course, assumes that her own early mathematical experience has played an important role in getting her to think about instruction in that domain. Her more recent exposure to the Math Their Way curriculum is another important experience falling under the "input" rubric. Marilyn attended this weeklong, half-day workshop six years ago, and was very enthusiastic about what was learned at that time. Initially, she said, she was "probably just earning credits" for the salary schedule. Someone had told her that the Math Their Way workshop would be a good one for her to take. In fact, she was one of the first in her district to take advantage of this opportunity.

Marilyn struggled a bit for a year or two because of the richness of the material. The book is "just filled with activities," she indicated, and it took her a while to sort these out and fit them into an overall structure. Marilyn also worked to adapt the Math Their Way curriculum to her own style. She explained, "I tried to do it their way and it didn't fit for me. I just can't operate in the sense that they did." The activities were grouped in "segments," she said. "You work a segment on numbers, you work a segment on whatever." The linearity of that approach posed a problem for her, however. Given her "circular" style of teaching, she felt the need to return to concepts that had been addressed at earlier points in the curriculum. "I ended up trying to kind of plod through a section, and then dropping it a bit, going back and picking it up and then redoing." She thought a lot about the curriculum at that time, she said,

> I thought that's really the way I teach reading and science anyway. You don't just start at the beginning and say, "There, I'm done with that," and go on to the next unit. That isn't the way that I think kids learn. I started thinking real hard about what this means. . . . I'm quite comfortable with it now [her adaptation of Math Their Way].

As with other aspects of her experience, Marilyn seemed intent on making the new mathematics curriculum her own. While this novel approach to mathematics influenced Marilyn's teaching and her views about learning, the relationship was reciprocal: Her views about teaching and learning also influenced the sorts of adaptations she made in the curriculum material.

Implications for Teacher Education

In many ways, Marilyn personifies Schon's (1987) concept of the reflective practitioner. Marilyn has been teaching at the primary school level for 25 years. During this time, she has witnessed many changes in educational practice ("I'm old enough that I've been through the pendulum," she says) and has herself experimented with dramatically different approaches to teaching at the elementary school level. Her attempts to improve on practice have been based on experiences with students, professional course work, and her own past dissatisfaction with traditional, teacher-centered instruction. These influences, along with the precious time afforded her to reflect on this experience at an early point in her career (i.e., her kindergarten teaching experience), have allowed her to construct a coherent and workable "philosophy of education." This philosophy guides Marilyn and serves as a valuable resource in her instructional decision making.

Although she has resolved a number of issues, Marilyn is still open to change: "I know a whole lot more about science than I ever used to 10 years ago because I've been asking questions and trying to figure out," she said. This disposition to be open to new learning experiences may be one of the most important factors in accounting for Marilyn's success. Studies like the present one that seek to understand the thinking and less tangible "world views" that typify exemplary teachers can prove useful. The present study reinforces Schon's (1990) notion that practitioners have much to learn from their everyday "conversations with settings." Schon quotes McClintock to the effect that, to gain from this experience, the practitioner--like Marilyn--"must have the time to look, the patience to `hear what the material has to say to you,' and the openness to `let it come to you'" (p. 17).

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CASE STUDY II

LAURA TATE: BEYOND SURFACE CHARACTERISTICS OF REFORM[1]

Pamela Schram, Julie J. Ricks, and Karen Sands[2]

Recent calls for reforming mathematics curriculum and teaching (e.g., National Council of Teachers of Mathematics [NCTM], 1989, 1991; National Research Council [NRC], 1989) have incited a flurry of activities in both universities and public schools. Revamping teacher education and teacher inservice programs, establishing new policies, evaluating curriculum materials, changing teaching practices, designing new assessments, and doing more and different kinds of research are all instantiations of the ways in which teachers, teacher educators, researchers, policymakers and school and university administrators are responding to the calls for reform in mathematics. Whatever the means, the goals of these reform efforts are to create new opportunities for educators and children, and to engender progress towards a vision of desired changes in mathematics instruction. This vision entails less emphasis on traditional practices such as memorization of facts and procedures and practice of isolated computational skills, more emphasis on understanding, ideas, problem solving, and flexible mathematical reasoning.

In this case study we describe a teacher who is working in collaboration with university faculty to change her mathematics teaching. These collaborative efforts take place in a Professional Development School (PDS)--a setting where the nature of university/school relationships are redefined in the context of a restructured school environment and where teachers and other practitioners collaborate with university faculty to improve teaching and learning for K-12 students, improve the education of new teachers and other educators, and make supporting changes in both the schools and universities as organizations. The main emphasis of this

[1]This paper was presented at a symposium of the American Educational Research Association, San Francisco, April 1992.

[2]Pamela Schram, assistant professor of teacher education at Michigan State University, is a senior researcher with the Center for the Learning and Teaching of Elementary Subjects. Julie J. Ricks, doctoral candidate in counseling, educational psychology and special education at MSU, is a research assistant with the Center. Karen Sands was an intern hired to teach mathematics to third and fifth graders and to assist with project activities. Laura's feedback and participation in the writing of this paper greatly enhanced its quality. The authors are participants in the University-school collaboration described. We identify ourselves but not the teacher in an effort to respect confidentiality. This is a continuing dilemma in this type of work.

collaborative work is to create schools *"for the research and development of the teaching profession"* (Holmes Group, 1990). PDSs are rich environments where teachers and other professionals are provided with time and opportunities to engage in many of the reform activities described above. Assumptions--about what curriculum is most worthwhile for students to learn, about how students learn and what they are capable of learning, and about what forms of pedagogy best promote more meaningful and empowering learning--are continually challenged in PDSs. It is in this context that the teacher we describe in this case study is exploring and experimenting with the changes in mathematics curricula, teaching practices, and teacher preparation recommended by educational research.

The PDS where our story takes place is McGrath Elementary School in a small suburban community just a few miles from Michigan State University. Over the years, teacher candidates from MSU have done their student teaching at McGrath, and several teachers have been involved in research studies and teacher education programs with faculty from the College of Education. In the Fall of 1989, McGrath and MSU entered into a new relationship as a Professional Development School. School and university faculty organized three projects based on mutual interests.[3]

Methodology

There is little consensus about research approaches that are appropriate for PDS collaborative work. Many of the PDS studies (e.g., Nystrand, 1991; Peasley, Rosaen, & Roth, 1992; Rosaen & Lindquist, 1992) have used a descriptive interpretive approach to address their research questions. In PDS settings university researchers are collaborating with classroom teachers. Given the collaborative nature of this work, we became participant observers (Bogdan & Biklen, 1982; Erikson, 1986). Our orientation as participant observers was guided more generally by mathematics education reform movement literature (e.g., NCTM, 1989, 1991; NRC, 1989) and more specifically by the Mathematics Study Group's (MSG) goals.

In the McGrath-MSU Work Plan for 1990-91 one of the goals for the MSG was to develop a community of learners who would "actively participate in making conjectures, gathering

[3]For an elaboration of our beginnings, see Rosaen and Hoekwater (1990) and Schram and Berkey (1992).

evidence, building arguments and justifying their ideas using the language of mathematics." In addition we wanted to support the community by creating an environment in which the goal was "to learn ways of finding out, ways of making sense of mathematics and strategies for inventing procedures to solve problems or for building models to understand mathematical situations." As we continued our work together during the 1991-92 school year trying to bring about change in the teaching and learning of mathematics, additional issues emerged. These issues included "what it means for students and teachers to be mathematically literate; assessment of student learning; development of classroom environment; construction of tasks; and the restructuring and redefining of participants' roles."

In the McGrath-MSU PDS Work Plans for 1990-91 and 1991-92 the following research questions focused our data collection:

- What kind of experiences fosters deeper and more flexible mathematical knowledge and understanding for students and teachers?
- What kinds of classroom environments promote this type of learning for students?
- What are some alternative means of assessment that can provide insight into students' thinking and understanding of mathematical ideas?
- How do the roles of the classroom teachers, university faculty, teacher candidates, and students change to support teaching and learning for understanding?

Data Sources

We analyzed data to develop a case about Laura Tate's development as a mathematics teacher during nearly three years as a participant in one of the three PDS projects at McGrath--the Mathematics Study Group. We examined changes in Laura's thinking, beliefs and practice as she worked with other teachers and university faculty to reflect about and revise her mathematics teaching. In telling Laura's story, we hope to illuminate some of the complexities involved when teachers attempt to translate the vision of mathematics described in reform documents (e.g., NCTM, 1989, 1991; NRC, 1989) into everyday practice.

Sources of data included teacher interviews that were audio-recorded and transcribed, classroom observations that were audio-recorded and included field notes, field notes of MSG meetings, and artifacts (e.g., curricular units jointly constructed, lessons plans, student journals).

The interviews were structured to capture teachers' personal experiences with mathematics; their thinking about mathematics curriculum, teaching, and learning; and their current and envisioned practice. In our field notes we focused on characteristics of the classroom that would enable us to reflect on our research questions (e.g., detailed interactions between participants).

We began our data analysis by reading the teacher interview transcripts, classroom field notes, and MSG field notes. We noted patterns, raised questions, and identified potentially interesting aspects of Laura's teaching to explore further (Bogdan & Biklen, 1982). Three categories emerged as central to Laura's development: (1) views about and understanding of mathematical content, (2) the nature and role of discourse about mathematics, and (3) what constitutes evidence of mathematical learning.

Beginnings of a Mathematical Journey

When Laura Tate joined MSG in the Fall of 1989, she brought with her a personal mathematics history, a wealth of varied teaching experiences, and a host of questions about what it might mean to change her mathematics teaching. As a student herself, she had been fairly successful; she "wasn't used to not understanding" and it was unusual for her through high school not to do well in mathematics classes. She particularly liked high school bookkeeping, "because everything was so neat and orderly." However, in college, she lacked a supportive environment for learning mathematics: "I didn't understand and nobody cared if I understood," and thereafter took as little math as she could (Fall 1989 interview).

As a new teacher, she began in the early 70s by teaching first and second graders "new math," which she "really liked, but the kids were confused." Later she moved to a Montessori School where she taught kindergarten. She had no training in Montessori methods but learned by watching other teachers. She enjoyed her experience at the Montessori School because there were "lots of things for kids to explore with; they could go for hours with the materials" that they had for math (Winter 1992 interview).

When a position opened at McGrath, Laura changed schools and grade levels again. She began by teaching fifth grade. Here, and later when she changed to kindergarten, Laura taught mathematics out of workbooks: "same as when I was taught." This was the school-mandated curriculum, which Laura felt particularly constrained by in kindergarten, where students had no manipulatives whatsoever to work with. When the district began to allow teachers to teach using a program called *Mathematics Their Way* (MTW) (Baratta-Lorton, 1976), Laura jumped at the chance and she continued to use the program when she became the transition teacher (Winter 1992 interview). MTW is an activity-based program which emphasizes children's use of manipulative materials.

Laura's classroom was not the traditional mathematics classroom characterized by much of the reform literature. Her young students were not seated in rows, quietly listening to the teacher lecture; nor did they spend most of their time memorizing facts and procedures and practicing isolated computational skills. Rather, her students often worked with a variety of concrete materials in small groups around the room, exploring and talking with one another. For some educators, Laura's lively classroom of actively engaged students may represent the end product of what is being described in documents such as the *Standards* (NCTM, 1989, 1991). From this view, reform merely consists of adopting a new classroom organization, putting materials in children's hands, and allowing them to talk, without examining or questioning the *nature* of the mathematics being taught, or assumptions about learning and teaching. For Laura, however, her classroom represented a beginning point for developing a deeper understanding of mathematics, of teaching, and of her own and her students' learning.

In the account that follows, we trace Laura's development from the Fall of 1989 through the present, focusing on how her thinking and practice has changed, with particular emphasis on three aspects of mathematics teaching that emerged as central to Laura's development--views about and understanding of mathematical content, the nature and role of discourse about mathematics, and what constitutes evidence of mathematical learning. These aspects of mathematics teaching are, of course, interrelated and interdependent, but we highlight each in an attempt to understand how they influenced Laura's development. In the following sections, we will describe each of the three years we have worked with Laura. In each section we provide one vignette from our classroom observations as a representational snapshot of Laura's mathematics classroom and to frame our discussion about mathematical content, discourse and evidence of student understanding.

Our First Year With Laura

Laura was the transition classroom teacher at McGrath. The transition program was intended to provide an alternative for students who, either socially, emotionally, or academically, needed an extra year of adjustment between kindergarten and first grade. Class size was small, with a maximum of 15 students, so that more individual attention could be focused on each student. Curriculum was largely left to the discretion of the teacher.

Laura's classroom was a colorful, crowded, cozy environment for her six - and seven-year-olds. The children's desks were arranged in a U shape near the front of the room by the door, and around the edges of the room were shelves overflowing with materials that the children used daily. The back wall of the room was lined with windows which looked into the central courtyard of the school. Outside one window hung a bird feeder, and the shelf along this wall hosted several plants, an aquarium, and miscellaneous collections--rocks, shells, leaves. In one corner of the room, there was a small carpet and several oversized pillows to create a quiet reading area; near the center of the room in front of the chalkboard was an open carpeted area where children gathered for large group activities. The walls displayed children's artwork and posters, and other creations hanging from the ceiling danced in the breeze from the windows.

Prior to and during the 1989-90 school year, Laura's mathematics lessons were mostly drawn from the MTW program. Just as MTW had led Laura to change from teaching with a workbook to using more manipulatives, the program also led Laura to the MSG. As plans developed in August 1989, for a MSG, she raised questions about MTW. She said she felt the activities were fun for the students in her classroom, but that she did not have a very good sense about where the overall program was heading, or if students had a strong sense of the mathematics they were learning. Laura was unwilling to accept the assumption that if children were using concrete materials, they would automatically learn mathematics. She thought if she had a better sense of the connections among the mathematical ideas, she might be better able to ask her students questions that highlighted the mathematics that the MTW activities were designed to teach.

Laura joined the MSG, like most of the other teachers, not because mathematics was her favorite subject, or because she felt it was her strength, but because she felt she could learn something about mathematics teaching and learning in discussions with other teachers and university faculty. She thought that the MSG might provide a setting where she could begin to make sense of mathematics. During the first year of the MSG, five teachers, three university participants, and two interns[4] met every other week to explore more conceptually-based approaches to mathematics instruction, curriculum development, and student assessment.[5]

For the first several months the MSG participants read and discussed sections from the NCTM (1989) *Standards* and other literature and research about conceptual approaches to mathematics teaching and learning. In order to explore the practical applications of ideas and issues raised during their discussions, the group decided to design a measurement unit. They began by examining their own understanding of measurement concepts and started to recognize the complexity of the topic. After generating a wide range of related measurement ideas, the participants identified the particular aspects of measurement around which to focus their unit. Collaboratively, the group designed a series of lessons. As they prepared to teach this unit, individuals modified the lessons for their particular grade levels.

Laura was an active participant in the MSG conversations. She thought hard about how to transform the ideas the group generated into appropriate activities for her transition students. Consistent with the plans of the other teachers in the MSG, Laura planned a series of lessons which emphasized some interrelated measurement ideas the group had discussed--the importance of using standard units, the need for accuracy, and identifying useful measurement tools. The

[4]During the first year of PDS work, two recent MSU graduates were hired to teach mathematics in two third- and fifth-grade classrooms. They also participated in the MSG. One of these interns, Karen Sands, is a co-author on this paper. In subsequent years she has taken on increased roles and responsibilities.

[5]For an elaboration about the MSG, see Schram et al, 1991.

teachers introduced their measurement units in May of 1990. The following vignette describes one of the earlier lessons in Laura's measurement unit.

Multiple Measurements

During one of Laura's first measurement lessons, the students measured a table, a notebook, the chalkboard, a wastebasket, the counter, the fish tank, and a balancing beam using their own hand as the unit of measurement. Individual students measurements were recorded on the chalkboard, for example: Table--10, 11, 9, 10, 15, 10; notebook--3, 3, 7, 3, 3, 2, 6, 5, 3, 2; chalkboard--47, 40, 47, 43, 55, 42, 47, 60. In the discussion that followed, the students realized that there were several reasons that these measurements varied: different size hands, the direction and spacing of their hands, bounding the area to be measured.

The next day, Laura introduced the lesson by referring to the previous day's work:

Laura: O.K. Let's try to remember back to yesterday. . . . We're going to think about what you measured with yesterday. Do you remember when you went around the room and you measured with . . .
Tanya: Different things.
Laura: You measured with . . .
John: Unifix cubes.
Laura: You measured with unifix cubes yesterday? You measured with . . . ?
Sue: With our hands.
Chris: We measured our journals.
Laura: We measured our journals. What did we measure them with?
Sue: With our hands.
Laura: Do you agree with that? [Most of the children nod]. . . . Remember, you measured a table, and a notebook, and a chalkboard, and a wastebasket, and counter, and fish tank, and the balancing beam--you had lots of different measurements. And you decided you got different measurements because . . . because . . .
Jenny: We did our hands different.
Laura: Not everybody measured the same way. Maybe some measured sideways, maybe some measured long, some measured on top, some measured big--we measured different ways. Why else do we say maybe we got a few different ideas because . . .
Chris: Some people had big hands and small hands.
Laura: Today we're going to measure with something else. Go sit in a circle, and I'll show you.

[The children gathered near the back of the room on the carpet. Laura placed a yellow sheet of paper and a tub of pattern blocks in the middle of the circle]

Laura: There's a question at the top [referring to the yellow sheet], "How many blocks go across the paper?" What kind of blocks should we try first, Amy?

[Amy picked a red trapezoid-shaped block from the tub]

Laura: Let's see, should I start right at the end or should I start in the middle? [of the sheet of paper]
Class: End.
Laura: Right at the end.

[Amy laid the blocks lengthwise across the paper from left to right]

Laura: Try to get it, Amy, so you can't get any more on.

[Amy laid four blocks and attempted to fit one more on]

Laura: Can you do it?
Class: Nooooo . . .
Laura: So what would my answer be?
Class: Four!
Laura: O. K. And this is where I'm going to put my answer.

Laura demonstrated how the children should record their answers on the recording sheet which had several columns, each marked off into several boxes. At the bottom of each column, a figure and a color word indicated what was to be recorded there. Thus, the first column was labeled with a square and the word "brown," the second, with a triangle and the word "green," the third, a rhombus and the word "blue," and so on. In the column marked with a trapezoid and the word "red," Laura colored in four of the squares, starting from the bottom, with a red crayon, to indicate the measurement they'd agreed on. She asked the students to work with a partner but to each fill in their own recording sheet. Before they got started, she had Ashley demonstrate the measuring procedure using the square shape.

Laura: When you are done, I would like you to come back here and we will compare and see if our answers are a little bit more the same. Yesterday when we did hands we had lots of different answers. Do you think these answers will be more the same?
Several children answered: Yes
Laura: . . . or do you think they'll be different?
A few children answered: More different!
Laura: Do you think they'll be different again? When you come back, we'll find out.

The children moved to their desks, where Laura had set out some pattern blocks prior to the lesson. Although the pairs of children did share a measuring sheet, most of the children worked side-by-side rather than interactively. Many children counted and placed the blocks, removed them, and then let their partner repeat the process for him or herself. The consistency of the class measurements was dependent upon the characteristics of the shape. A number of children oriented the pattern blocks in different ways. The rhombus, for instance, was laid point to point by some and fit tightly together (i.e., treated like parallelograms) by others. Children appeared to differ in the precision with which they carried out the task. Some were careful to line up the blocks to form a relatively straight line while others seemed not to attend to this requirement. In some cases, the blocks were scattered loosely across the paper (even when the unit was standard some of the issues from the previous hand-measuring lesson remained). After the children had worked in pairs for some time, Laura asked the children to form a circle so they could discuss their findings.

Laura: O.K. let's start to get some numbers on the board. Please count and see how many squares it took to go across. Count how many, keep the idea in your head. [Laura called on each pair, recording the following numbers as they responded: 8, 7, 8, 9, 8, 8, 8.]
Amy: Most of them are 8.
Laura: Most of them are 8. Are we a little closer today in our measurement?
Class: Yes!
Tom: Nobody is higher than 9.
Laura: Why do you think somebody might have had a different answer with those? Is there a different way you can turn it?
Andy: I know. Cause they did it big.
Rick: They did spaces.
Laura: Or if they did spaces. . . if they had spaces they might have gotten a different answer.

[Jenny started to offer another a verbal explanation but seemed to hesitate so the teacher suggested that she demonstrate her idea. This student put some squares on her paper in a crooked fashion.]

Laura: They could have had them not exactly straight.
Jenny: This is the right way. [Student straightens up the squares.]
Laura: There are lots of different ways.
Jenny: Maybe some people started off the edge [of the paper].
Laura: What do you think Jason?
Donald: That might have happened to Andy [Andy reported the highest measurement which was a 9.]
Laura: So you think the big number might have started off the paper?

[Later in the discussion, the pairs reported their measurements using the rhombus-shaped pieces: 8, 9, 9, 4, 7, 9, 5, 8, 10].

Yvonne: They're more different
Laura: I agree. I think these numbers are looking a lot different. What's going on with these blues [i.e., rhombus]?
Katie: People must have done it different ways.
Danny: I know how.
Laura: Danny, just show us one way.

[Danny demonstrates putting the rhombuses length-wise, counting 5 across.]

Laura : Are there different ways?

[Liberty demonstrated a second method laying them point to point vertically and counting to 8.]

Laura: Danny said 5. Is he right?... and Liberty said 8, is she right?
Class: Yes!
Laura: So if we told somebody to go across 5 blocks, are we sure they would go across the right way?
Class: No!
Laura: Have we found the best way to measure?
Curtis: No. I know. You should use a measuring stick.
Laura: I think so too.

<u>Mathematical Content</u>

In designing these measurement lessons, Laura had some definite ideas about the kinds of knowledge that she wanted children to develop. In just a few days time, she guided her students through some activities that led to the end result she hoped for--that they would come to the conclusion that to measure accurately, they might need to use a conventional tool, such as a ruler ("measuring stick"). Although the children "discovered" these notions by measuring real objects, the structure of the activities and Laura's questions left little room for other interpretations of the "facts." While Laura was willing to put materials in children's hands that she believed helped them learn, she was less willing to give up her authority for the knowledge that was to be learned.

Compared to her thinking and teaching earlier in the year, however, Laura had made some notable changes. In an interview early in the fall, we asked Laura to identify a mathematics topic that she would be teaching during the early spring, and she chose subtraction. This part of the interview involved responding to a two-part question: What is important to emphasize when you teach subtraction? What are the most important things for students to learn? In describing how she would teach subtraction, Laura outlined a definite sequence of the kinds of experiences she would plan for children--co ng backwards and being proficient in addition (which she considered prerequisite skills); recognizing the minus (-) symbol and learning that "take away" results in a smaller answer; and limiting number choices to single digits.

A belief that guided Laura's teaching decisions in mathematics was that children needed multiple and extended opportunities to work with concrete materials before moving to more abstract tasks (e.g., story problems, paper/pencil work). Laura was concerned that students develop "correct" ideas about subtraction. One of the ways that Laura ensured this was to structure mathematics lessons so that some children could work on their own while she worked with a small group. When asked what she would do if a student was having trouble with subtraction, she said,

> They're going to come back and work with me individually. We're going to make sure they know what "6" is or whatever the number is. If they don't have that then we back up and work on what that number is in lots of different ways. If they've got what the number is then we work through. I'll talk again about what minus is--that we take that many away. I will talk through the problem and do it with them. Do it over and over again and see if they understand that.

During this first year, we frequently observed Laura working with students in the manner she described; when she wasn't working with individual students in the back of the room, she circulated to "check their work" as they used materials individually or in small groups. During the measurement lessons, Laura also interacted with children, asking questions to direct their attention to important features of the measurement tasks. By this time, however she felt less of a need to "rework" through the procedures with children who "didn't get it".

Discourse

Most of Laura's mathematics lesson the first year entailed briefly introducing an activity to the whole group, then having the children work individually, in pairs, or in small groups while she circulated to talk with children. Even during large-group discussions, the dialogue was mostly a two-way interaction between Laura and individual students. In the MSG, Laura had talked about how her lessons might be different from the lessons in the upper grades: Younger students, she felt, could not have extended discussions and could not write their ideas in journals as the older students could. During the measurement unit, however, Laura did make large-group discussions a more central feature of the lessons. Often, lessons began with a large-group discussion to introduce the activity, moved to children working in pairs or small groups, and ended with children coming back to the large group to discuss their findings.

This move to *discussing*, rather than *checking on* students ideas may have been fostered by the kinds of discussions that took place during MSG meetings, where time was devoted to members of the group working on mathematical problems and discussing the "big ideas" in the problems. In response to the teachers' interest in rich problems that would aid in promoting discussion in the classroom, Pam Schram introduced problems, asked questions that helped the group members think about additional aspects of the mathematics, and facilitated group discussions about the ideas. Laura engaged herself in these problem-solving situations, and contributed to discussions where together members of the group worked to make sense of the mathematics. Often, the group members were surprised at how engaged they were in working collaboratively to solve these problems and how much they gained from hearing from the other members about how they'd approached solving the problems.

On several occasions, Laura and other group members commented that what they'd just done--working the problems and discussing their ideas--was exactly what they wanted their students to be doing in the classroom. In learning from and supporting each other in the context of "doing mathematics" themselves, they'd generated a model for introducing and discussing mathematical ideas with their students.

Laura's measurement lessons represent beginning attempts to engage her students in these kinds of discussions. As we mentioned previously, however, her measurement lessons still embodied the belief that in terms of content, there were still specific steps to be taken and correct answers to be learned. We suggest that Laura's notions about content constrained the kinds of discussions that she was able to have with her students. Laura's questions and comments worked to direct kids' thinking; often she paused or restated questions until she got the answer she had in mind. Many of the class discussions were aimed at providing students with a particular set of ideas or way of thinking about those ideas. Because she was the "authority" on the mathematical knowledge to be learned, she was also the "director" of the ways that children talked about this knowledge.

Evidence of Student Understanding

Laura most often used her individual interactions with students as occasions to assess their understanding of the mathematical concepts that the class was working on. Often, Laura asked students directly to solve problems she posed for them. During the Fall 1989 interview when asked how she would know when a student was having trouble with subtraction, Laura replied:

> They'd come back with some wrong answers or be doing some wrong answers as you were watching them work these problems with the blocks. Their answers are not correct so it's real obvious they don't understand what it means.

Evidence of student understanding, then, came not from students being able to explain what they were doing or why something worked, but rather whether or not students could perform certain tasks or answer questions in the right way. Understanding meant having the right answers, either verbally or in using the materials correctly. When she found, through group discussion or with individual students in task-specific questions, that some students did not understand, she would instruct these students in the back of the room. While she retaught, other students worked in activity centers around the room or on mathematical activities Laura had demonstrated.

During the measurement unit, Laura took a less directive approach to both finding out whether students "understood" and to her interventions with them when they did not. But while she less often directly tested or directly taught students, she looked for the same kind of evidence--the answers she had in mind.

Reflections on the First Year

Laura Tate started the journey to reflect on and change her mathematics teaching before she ever joined the MSG. She had abandoned using workbooks and paper/pencil tasks, and had embraced the idea that children needed many hands-on experiences to develop their mathematical competence. As she planned and taught the measurement unit, she took steps to encourage discussions about the measurement ideas, and spent less time assessing and teaching students in one-to-one directed interactions. We feel it is important to emphasize that for the most part, Laura changed her teaching *on her own.* Though the MSG discussions, as reported above, did provide occasions for Laura to reflect about many aspects of mathematics teaching and learning, Laura changed her teaching in response to the ways that *she* made sense of these conversations. Most of the discussions about Laura's lessons took place in the context of the MSG meetings, where she reported on her lessons and shared questions with the group.

Observations of Laura's classroom during the first year of the MSG were limited, and little direct feedback or discussion about her instruction was provided by the university participants or the other teachers. When we did observe lessons, our role was primarily that of an objective documenter. We make this point because this aspect of our work with Laura changed significantly over the three years of the project. In our second and third years, as we describe in the following sections, we began to participate with Laura in new ways as she continued to change her mathematics thinking and practice.

Our Second Year With Laura

During the 1990-91 school year, Laura again taught the transition class of approximately 16 students. Visiting Laura's classroom was like visiting a familiar place--the arrangement of the desks, the shelves brimming with materials, the children's work displayed around the room, and the active, curious children were all similar to last year's classroom environment. As we will see, however, over the course of the year there were some subtle changes in the ways that Laura thought about and taught mathematics in this familiar setting.

The MSG met once a week during this second year, and, like the first year, spent much of the first several months reviewing the NCTM (1989, 1991) *Standards*, examining literature about teaching for understanding, and discussing their classroom practice. Building on the experience of designing the measurement unit the previous year, the group decided it would be worthwhile to again focus collaboratively on a particular mathematical area. We chose to focus on place value, since it is an integral aspect of mathematics both across topics and across grade levels. Instead of designing a series of lessons to be introduced in each classroom, the teachers decided to focus on place value concepts in the context of the mathematics they were currently teaching. As the MSG began to explore place value concepts, the participar spent some of their meeting time working in smaller groups whose interests or focal areas were similar. Thus, the third-grade teacher explored aspects of place value that would help her teach subtraction, the fourth- and fifth-grade teachers applied place value concepts to their teaching of multiplication, division, and fractions. For Laura's students, place value concepts had mostly been introduced in the context of the calendar activities they did daily, where they counted the number of days they'd been in school and represented this number by grouping wooden sticks in bundles of ten.

During the small-group time, Laura met with Kathy Morris, one of the fourth-grade teachers and Julie Ricks, both of whom encouraged Laura to consider introducing her students to different bases as a way to explore place value concepts, and to build upon their familiarity with the base-ten number system. An additional aspect of the collaborative work during this year was that Julie observed on a regular basis in Laura's classroom as she tried out these new activities. This regular interaction gave Laura opportunities to reflect on her lessons immediately afterwards, as well as during the small group time each week. The following vignette describes a lesson during the place value unit.

1, 2, 3, Cato

Laura used a series of activities from the MTW program to introduce her children to different bases. During these lessons, students made up different, nonsense names for familiar numbers--for example, four was called "cato" by Laura's students. This new number was representative of the 10s place in the base-ten number system, and students grouped and regrouped concrete materials (e.g., beans) on a laminated, two-color place value board. In base four, they counted out "singles" on the white side of the board and then grouped these objects and moved them to blue side when they reached the base number. (For example, 4 beans were placed in a small cup) Each time students added an additional object, they "read" their place value board. In base four (cato), for example, students started by reading their blank card, "zero catos and zero". As they added objects one at a time, they would count, "zero catos and 1, zero catos and 2, zero catos and 3," each time adding an object to the white side of the board. When they reached the base number (4) they called out "Cato!" grouped the four objects, and moved them to the blue side of the board. The next sequence began, "One cato and zero" and continued until they had four more singles and then again moved this group to the blue side of the board. Laura spent several lessons familiarizing the students with this procedure. Laura then introduced a new dimension to these lessons--the symbolic representation of what they were "reading" on their boards. Adding the numeric symbols served to raise several issues and ideas that Laura and Julie had not anticipated.

Prior to the lesson, Laura hung a paper chart on the chalkboard which had drawn on it two columns divided into several rows. The lesson began in a similar fashion to many of the previous lessons, with Laura reminding the children to place their place value boards correctly on their desks, then starting the activity by reading their blank board, "Zero catos and zero." At this point, Laura wrote in zeros on the top two boxes on the chart. Laura rang a small bell, which indicated that the children were to add the first object (in this case beans) to the singles side of the board. Laura continued in this manner, each time writing numbers on the chart to correspond to the objects on their place value boards. When they reached "two catos and 3," Tammy called out, "23," and Laura pursued her idea.

> *Laura*: I heard somebody say 23. Is it 23, 2 catos and 3? We'll have to think about that one. We'll have to remember to think about that one.

[Several children begin to talk at once, calling out several numbers, "21, 22, 23."]

> *Donna*: Mrs. Tate, watch. [Donna comes to the front and points the "2" and "1" on the chart.] Twenty-one!

	0	0
	0	1
	0	2
	0	3
	1	0
	1	1
	1	2
	1	3
---->	2	1
	2	2
	2	3

Laura: Looks like that, doesn't it. That's how we write 21 . . . 22, 23. Let's stop while we're right here for a minute. Cause we've got that number right there on your board, don't you? Two catos and 3.
Andy: Yeah.
Laura: Two catos and 3. Is that the same as 23?
A few children : Yes!
Laura: What makes you think it's the same as 23? How can we know it's 23? Mike, how can we know?
Mike: Because it has a 2 in front of it.
Laura: 'Cause it has a 2 in front of it?
Sandy: And a 3 at the end.
Laura: And a 3 at the end. That makes it 23? Mary, how will we know if it's 23?
Mary: I can show you.
Laura: Yes, go ahead.

[Mary comes up to the chart and points at the 1 and 0, 1 and 1, 1 and 2, and 1 and 3 in the boxes on the chart.]

Mary: 10, 11, 12, 13, and that's all.
Laura: 10, 11, 12, 13 . . . I stopped right here at 2 [in the left column] and 3 [in the right column] 'cause this is how many you have on your board. This is 23 cause it's got a 2 and a 3? Um, look at your beans.
Mary: No, it's 4 plus 4 is 8.
Mike: But it means 23 'cause it's 2 right here and 3 right here.
Laura: Two what?
Mike: Two catos
Laura: Two catos . . .
Mike: And 3.
Laura: And 3.
Mike: That makes 23.
Laura: That makes 23. Yes?
Nicole: One and one makes 11.
Laura: OK, yes, 1 and 1, usually when you see 1 and 1 that usually means 11 doesn't it?

[The children were reading the numbers according to the base ten system, the one with which they were most familiar. Laura began to pursue this, when Amy introduced another idea]:

Amy: Mrs. Tate--1, 2, 3, zero, 1, 2, 3.
Laura: Wait a minute--zero, 1, 2, 3, zero, 1, 2, 3--what is that?
Andy: A pattern!
Laura: A pattern. OK, let me circle that pattern for you.

[Laura draws a circle around each set of 0, 1, 2, 3 on the right side of the chart. Nicole comes to the front and points out another pattern--four "zeros", four 1's, four 2s on the left-hand side of the chart, which Laura also circles, so that now the chart looks like this:

	0	0
	0	1
	0	2
	0	3
	1	0
	1	1
	1	2
	1	3
	2	0
	2	1
	2	2
	2	3

Laura : Is that a pattern?
Class : Yes!

Laura pursued the children's interest in patterns by encouraging them to predict what the next numbers on the chart would be. She hung a second chart next to the first, and they continued to play the game, calling out "3 catos and zero". Laura added the numbers 3 and 0 to the second top of the second chart, confirming the pattern the children had predicted. Jason called out, "30," and Laura pursued the idea that had emerged earlier, before some of the children pointed out the patterns.

Laura: Jason says that should be 30. Do you have 30 beans on your board?
Jason: We got three bunches.
Laura: He says we got three bunches.
Sara: We got 12.
Laura: We've only got 12? Well . . .
Mike: Three 4s make 12.
Laura: Oh, so it's not. . . . It's three fours there? Three little 4s.
Mike: No! After 20, it's 30!
Laura: How much would we have to have to have 30? Three what?
Mike : Three beans.
Laura: Three beans would make 30?
Mike: No, a bunch more.
Laura: We'll have to think about that. What would we have to make to make it 30?

Laura left this open-ended, and continued the game with the children. When they reached 4 catos, Laura added 4 and 0, 4 and 1, 4 and 2, and 4 and 3 to the chart. Then she realized that since they were in base four, writing 4 was not correct. She asked the children if they saw a problem with the chart. The children didn't realize the mistake either. Laura reminded them that cato is their word for 4, so they can't say or write a 4 in this game.

Laura: What are we going to when we get 1, 2, 3, *cato* catos?
Josh: Make a bundle.
Laura: Make 'em like a bundle! One, 2, 3, cato (Laura stacks 4 cups--each representing one cato--as she counts)

Laura then drew the children's attention to the chart, pointing out that they shouldn't have written a 4. She asked the children what they would call the new group of 4 catos and after a vote, they decided to call it a "cato bundle," and to put the stack of 4 cups of 4 beans each off the place value board. They decide to mark out the 4s on the chart, replace them with zeros, and put a 1 beside the left-hand column.

0	0
0	1
0	2
0	3
1	0
1	1
1	2
1	3
2	0
2	1
2	2
2	3

	3	0
	3	1
	3	2
	3	3
1	0 ~~4~~	0
1	0 ~~4~~	1
1	0 ~~4~~	2
1	0 ~~4~~	3

For the remainder of the lesson, they continued to add beans to their place value boards and practiced saying the new numbers (e.g., "One cato-bundle, zero catos, and zero," "one cato-bundle, zero catos and one").

Mathematical Content

"I give up, this is too hard!" Laura exclaimed after this lesson. During a conversation following the lesson, and later in the small-group meeting, Laura and Julie tried to sort through the myriad ideas that had emerged during the course of this lesson. They noted that as an observer, Julie had time to reflect on what was going on in the classroom, while Laura had to "think on her feet." Laura wondered if Julie had noticed the mistake when they'd counted to four catos and Laura had added it to the chart. Julie admitted that she hadn't even thought about it until Laura brought it up during the lesson. They realized that the base-ten system is such a strong frame of reference for our society, that it was difficult to "think in other bases," and joked about being the "expert adults": "How can we expect children to do this when we haven't even oriented ourselves to a new way of thinking about number?"

Laura noted concepts that she wanted to pursue in subsequent lessons: making connections between the numeric symbols and the concrete materials on their place value boards; continuing to look for patterns in the sequence of the numbers, and working in other bases. She hoped that eventually, the children (and she) would make sense of the concept of place value. "Making sense" had become an important mathematical goal for Laura, in contrast to her focus the previous year on helping children get right answers. In an interview in the Fall of 1990, Laura revisited some of the questions about subtraction from the Fall 1989 interview. Her responses were somewhat similar, in that she still felt there was a sequence to learning about subtraction, but with an important difference--she stressed the importance of children making sense of what they were doing:

> I would almost think the most important thing that I want the students to learn is to try it themselves, to make an effort and to think, "Does that make sense?" Because I have kids that'll put down 2 plus 2 and the answer is 2. And for them to stop and think, themselves, "Does that make sense?" is the most important thing.

Because working in different bases was new mathematical territory for Laura, her ideas about the kinds of knowledge she expected children to develop were less definite than had been her ideas about knowledge of measurement. During an interview in the Spring of 1991, she reflected about her experiences discussing place value with the MSG members and about teaching place value concepts to her students:

> I never would've tried different bases with six-year-olds and that's been fun. I'm not sure how profitable, but fun. Students feel a great deal of ownership. In the past I would have relied much more on students copying and mimicking. If they did these things often enough, it would sink in and they would be able to do it. I hadn't thought about understanding. I think we're starting to take number apart, think about a variety of ways to think about number and also sense making for me and sense making for the students.

Discourse

During both years, the MSG spent a great deal of time thinking and talking about classroom discourse. Laura thought that participating in the MSG had influenced her:

> I think that our math study group is encouraging me to change. And I think probably in the areas of problem solving . . . how the children become active problem solvers rather than passive problem solvers. To think about their mathematics and talk about their mathematics. (Fall 1990 interview)

During the same interview, she said,

> My thinking's changing faster than my teaching. I suppose that's because there are just so many things in your day that it's hard to change your teaching without some help.

Laura's desire to change, along with a new vision of what discussions *could* be like, may have been the source of her frustration as she wrote the following journal entry in early Spring 1991:

> The discussions we've had in my class about math have been what I consider very basic with little reflection or depth of understanding from the students. This may be because I am not asking the right questions or allowing enough wait time or confusion. Perhaps the students are too young for meaningful math discourse. Anyway. This is an area of little success. I am having students work more in pairs. I wish I could tune in a little to some of these conversations. Might help my questions. But the small group work is successful because they are trying to explain it to each other. . . . The part of math I hadn't thought about before, but really intrigues me now is the discussion and debate it can entail on the way to sense making. Before MSG I had never experienced it myself or seen teachers teach this way.

Laura's students continued to work in smaller groups and pairs during the second year, but whole-group discussions also became an integral feature of her lessons. During all of these interactions, Laura spent considerable time soliciting and discussing children's ideas. In the measurement lessons, Laura's questions and comments gradually guided her students to some predetermined ideas. In the place value lesson described above, Laura was more concerned with following the children's ideas as they emerged in discussion. This aspect of discourse was sometimes uncomfortable for her, however, and she commented frequently that she wasn't sure "where they were going to go with this." This type of teaching can produce many uncertainties and requires teachers to take risks. Opening up the mathematics lesson often leads to different roles for the teacher and the students (see e.g., Prawat, 1989).

Laura reflected on this aspect of her teaching during an interview in the Spring of 1991:

> When I think about students, I think about math thinkers, not just doers. They're puzzling, being thoughtful about what has happened. My role is wondering too. Kids aren't used to that, they're used to the teacher having the answer. I didn't think I'd be able to try wondering with six-year-olds, but it's been fun.

Evidence of Student Understanding

Changes in Laura's thinking about how to assess student learning were closely related to changes in her thinking about mathematical content and discourse. Her view of mathematics had broadened from a focus on procedures and right answers to a focus on sense making and understanding; and Laura began to look for and consider different forms of evidence of student understanding. In interviews and conversations during the 1989-90 school year, Laura noted that she looked for evidence that students were using materials correctly and were able to produce the right answer. In an interview in the Fall of 1990, Laura distinguished between students being able to demonstrate operations using materials and students understanding what the operations meant. Her approach to helping students who were having trouble with particular ideas reflected her concern that the task they were attempting be meaningful to them. For example, she discussed how she might help a student who was having trouble with the minus sign:

> I might [relate it] to a stop sign or a McDonalds sign . . . and say, "What does this sign mean?" . . . and try to make him more aware that signs are meaningful and important.

Laura's experiences engaging students in discussions and hearing their (sometimes unexpected) ideas also seemed to influence the type of evidence of student understanding that Laura considered. During an interview in the Spring of 1991, Laura commented on the interaction between assessment and planning: "Using what students say--we need to see where kids are thinking and the lesson goes from there. When they're working together and one is explaining, that's evidence."

Later during the interview, Laura commented that she felt that the activities she was planning had potential, but felt that she could do a better job of probing and discussing. She noted, however, that "I'm listening to the kids a lot more, more than I ever have done before." Laura was beginning to recognize and appreciate the relationship between students' ideas and her planning and instruction.

Reflections on the Second Year

Laura's own reflections, captured in many of the quotes above, provide perhaps the best portrait of the kinds of changes she was making in her thinking about and teaching of mathematics. Her developing vision of the way she wanted mathematics to be in her classroom was strengthened as a result of her experiences during this second year. She had changed her views of mathematical content, of the kind of discourse she thought was important, and of what she looked for as evidence of student understanding. Near the end of the year, we asked Laura to reflect about where she was in her teaching related to the vision she had, and what kinds of resources might be most helpful to her:

> I think where I've made the most progress are the areas I've had time to discuss with someone from MSG--like place value. I don't think I'd need too many more material resources--I think a resource as a collaborator--it wouldn't matter who it was as long as we had time to think about what would be a meaningful math experience.

Laura had additional support during the second year--more time in small-group conversation, and a graduate assistant who regularly observed and discussed with her the lessons she'd planned for place value. Her hope for a collaborator was to be realized during the third year of the project.

Our Third Year With Laura

This year Laura is teaching in a "one-two team room." The team room is a sprawling, double-sized room made possible by knocking out a wall between two formerly separate rooms. The windows, on the wall opposite the doors, face out on a courtyard serenely filled with pine trees. When the 44 children, between the ages of six and eight, are in the room, hardly any spot of quietude is found. The students are encouraged to spread out anywhere they can get their work done, on either side of the room. Although the desks are in clusters of four, students are not assigned seats and regularly move.

Aside from the removal of the center wall, the physical aspects of the original rooms were not much changed. Both rooms still have coat racks next to the door, and sinks next to the coat racks. And at both ends of the expanded room there is a chalkboard, along with a movie screen and the customary collection of maps. There is space between the chalkboard and the desks, larger on Laura's side than on her partner, Kelly Neal's, side. (On Kelly's side a writing center and the computer take up room.) Therefore, the students typically come together as a whole group on Laura's side of the room. This they do often, whenever a new lesson is being introduced as well as at sharing time and when a story is being read.

As co-teachers of the team room, Laura and Kelly take turns introducing and teaching lessons which they have planned together. If one is speaking to the students, the other might be sitting on the floor with the students, or helping an individual student outside the group. During the time students are working, Laura and Kelly can be found on either side of the room, helping where they are most needed. Both teachers teach all the students in all areas, with one exception: mathematics.

During mathematics, all but a few of the students are separated into groups by grade level. This is due less in part to the subject matter than to the fact that Laura and Kelly work with two different people from the university. Laura works and plans math with Pam whose schedule allows for time to plan Monday afternoons and Friday mornings and teach with Laura on Tuesday; whereas Kelly's university partner is available Tuesday, Wednesday, and Friday afternoons for planning and teaching. Because of this lack of available planning time for all four, Kelly teaches the second graders mathematics on one end of the room while Laura teaches the first graders on the other. This is far from the ideal situations for either teacher; in fact Laura and Kelly have both expressed the hope that in the future, math, too, will be team-taught.

Laura continues to participate in the MSG. The whole group meets less frequently (i.e., monthly) and university and school-based staff have created smaller working groups that meet once or twice each week. Laura is reassigned for one and a half hours two days each week. Pam and Laura meet during that time to plan and reflect about teaching math in first grade. In addition to planning units and day-to-day lessons, they have created a concept map that they continue to expand and revise to help them think about the mathematics curriculum across the school year.

Ideas about ongoing assessment as well as more cumulative assessment are often topics of discussion. Laura and Pam have designed student interviews to try and capture the ways individual students are making sense of various mathematical ideas across the year. They use an anecdotal notebook to record students conjectures and arguments and as a guide to highlight particular students who may be receiving focused attention and those who may be neglected during class discussions. Laura and Pam also exchange an interactive journal to foster ongoing discussions outside of their regular time together. The journal often serves as a stimulus to focus their weekly conversations.

Pam also participates in Laura's first-grade classroom during math on Tuesdays. The two play a variety of roles: Laura may introduce the focus problem/question for the day while Pam takes field notes focused around an agreed upon topic; Pam may lead a class discussion while Laura takes notes; Laura and Pam may share the responsibility for leading the class; Laura may take the lead early in the lesson and Pam later in the lesson. Decisions about roles and responsibilities are determined by Laura and Pam based upon the mathematical task(s) planned for the day and the particular teaching combination that best uses each person's strengths and provides the most appropriate opportunities for students.

An excerpt from Pam's journal provides insight into some ways Laura thinks about teaching and learning mathematics this year. It should be noted that this is not a running account of the entire hour but selected pieces that Pam noted in her journal immediately following the lesson. Following the excerpt we will highlight changes in the way Laura thinks about mathematical content, the nature and role of discourse, and what constitutes evidence of student understanding.

Making 4

The "problem" around which this lesson revolved was "ways to make 4." The first graders had worked on this problem the previous day and had recorded some of their ideas in their math journal. Students were sharing some of their ideas about ways to make 4 and "proving" it to classmates:

Ben wrote 100 - 96 = 4. Students were impressed by the size of Ben's numbers. He used a bead rack (a wire rack of 100 beads with 10 individual beads on a row and 10 rows.) to "prove" his idea. He counted 10, 20, . . . 100 and then pushed the beads in the other direction (starting from the bottom) and counted 10, 20, . . . 90, 91, 92, 93, 94, 95, 96 and then proudly pointed to the remaining 4 beads as "proof" he was correct.

Tom wrote 4 + 4 - 4 = 4. This idea was greeted with, "No way, I disagree." Laura asked students to wait until Tom had a chance to "prove" his idea before agreeing and disagreeing. Tom used his fingers and students registered surprise as it appeared that Tom indeed had a workable idea.

Michael called me over to show me his journal. He said, "Look, I found a pattern that works with Ben's idea."

100 - 96 = 4	101 - 97 = 4
102 - 98 = 4	103 - 99 = 4
104 - 99 = 4	105 - 100 = 4
106 - 101 = 4	107 - 102 = 4
108 - 103 = 4	109 - 104 = 4
110 - 104 = 4	111 - 106 = 4
112 - 107 = 4	

Michael explained to me that he took Ben's idea and added 1 to each of the numbers and that the answer would still be 4. As he continued his pattern, he forgot to add 1 to 99 so now the numbers were off. Laura asked Michael to come and share an idea with the rest of the class. He wanted to use his largest number so he shared 112 - 107 = 4. Laura ask him to "prove" that his idea worked. There was quite a discussion about how to represent 112 with the bead racks--they immediately knew that they needed 2 of the racks. Students were comfortable counting 10, 20, . . . 100 but after 100 there was less agreement. Some wanted to count each row of 10 beads as 1--101, 102, 103 rather than 110, 120, 120, etc. [Reminds me how fragile their knowledge and understanding about number is.] Ben showed them correctly and other students counted and verified that we really only needed 12 more and counted the beads individually. They encountered more problems trying to "take away" 107. . .

Time for recess. 5 students (4 boys and 1 girl) wanted to stay and continue working on the problem--112 - 107 = 4 so I stayed with them. By the time I got to the group, Ben had "proven" that it should be 112 - 108 = 4 and the group apparently was satisfied. Now Steve wanted to share his problem--1000 - 1096 = 4. Ben was quite impressed with Steve's large number and immediately found a scrap of paper to copy the problem down. "We need 1000 to prove it." Ben announced. The group decided that with the three bead racks that were available we could only get to 300 so they suggested unifix cubes, after all we have two big tubs of unifix cubes-- there must be enough to get 1000 they conjectured. Steve looked me in the eye and said, "Are you sure you can count to 1000?" [At least we're making progress towards students not looking to the teacher as the sole authority.] The group started counting each unifix cube and quickly decided that it was quicker to create sticks of 10 and eventually discovered the efficiency of making groups of 100. Ben wanted to count 100, 200, . . . but Steve needed to count 100, 110, 120, . . .We spent recess and activity time creating 1000 unifix cubes with help from various other students--Lindsey, Eric, Tony.

Around 400 unifix cubes, Ben announced to me that Steve's problem was wrong because 1096 was larger than 1000 and that he knew what the answer to that problem would be--zero because 1096 was larger than 1000. So, what will the answer be? I asked. He wasn't sure but he thought 1096 - 1000 might get us 4. The rest of the group was not convinced by Ben's argument so the count to 1000 continued. Interestingly when we finally got 1000 cubes, Steve was counting 100, 200, . . . 1000 (we had recounted several times during the process and he was comfortable counting by 100s now), but when he got ready to "take away" 1096, he needed to count the cubes 10, 20, 30, . . . I wasn't sure whether or not it was the notion of "take away" that made the counting different or that he didn't know where 1096 might come into play and he didn't want to miss it. Time ran out so we carefully put the 1000 cubes [sticks of 10] into a tub and put away so it would be there tomorrow. Steve started to cry because he didn't get to prove his idea but we reassured him that he would have a chance tomorrow. "Oh, sure," he said. (Journal, October 1991).

Mathematical Content

Subtraction, or "take aways" as the first graders refer to it, surfaced in the Laura's classroom in early October. In response to the open-ended problem--show all the ways to make

4--some of the students introduced "take-away" problems. Laura responded to these ideas as she did to others and encouraged students to "prove" it. The students were able to prove it to the group's satisfaction. In contrast to Laura's ideas about teaching subtraction during our first year, she did not express concern that students did not possess the appropriate prerequisite skills (e.g., counting backwards, mastery of addition, recognition of the subtraction symbol) to "do subtraction". Nor did she insist that they work only with single-digit numbers. She was excited that students were working with numbers in flexible ways and was pleased when students combined addition and subtraction as in 4 + 4 - 4 = 4. Once the students had introduced the notion of "take aways" as representations for numbers, many of the first graders began including subtraction ideas in their list of possibilities for various number representations. Perhaps because the possibilities are limitless, many students created subtraction problems.

During an interview a few months after the "take-away" lesson described in Pam's journal, Laura noted that students had realized that when they were restricted to only using addition to find ways to make a particular number, there were a limited number of combinations even though they could use multiple addends (e.g. 1 + 2 + 1 + 2 = 6). But when they were able to include subtraction ideas, they could come up with many, many ways. The interviewer probed if that would lead naturally into working on subtraction next and Laura responded,

> No, I think it is just going to be a natural part of this work . . . addition will be an integral part of ways to represent numbers, but the kids love subtraction because they can get into these huge number like 100 - 96 = 4. (Fall 1991 interview)

During a recent interview Laura was asked to reflect about her views related to a number of different topics including basic skills, problem solving, and creating classroom community:

> *Interviewer*: I'd like to know how you thought about these ideas in the past and how you think about them now. Basic skills, for example, counting backwards before working with subtraction.
> *Laura*: Children need basic skills. My idea about what they are has changed. Before, I would have thought about paper/pencil and memorization. Now, I think about understanding and creating what math is. [laughs] Counting backwards. now, I don't even think about counting backwards.
> *Interviewer*: Why was that skill important before?
> *Laura*: Perhaps, because it was a skill emphasized in MTW as leading to subtraction. I accepted that as a tie in. (Winter 1992 interview)

Later in the interview, Laura was asked to talk about MTW:

> Well, I liked the materials. Now, I haven't cracked the book all year. I never thought it was a Bible but it seemed to have a nice sequence of activities. I never thought it was enough--kids didn't do much thinking. I still like to use it but I like to help kids make more connections.

Laura's notions about the value of students' sense making have continued during this year. She continues to place an emphasis on the students making sense for themselves rather than the teacher trying to do it for them. Communication about their ideas to others seems to be an added emphasis this year. This is illustrated by Laura's ideas about mathematical literacy. Laura said that for children to be mathematically literate means that "they can use numbers and ideas about them to solve problems, create problems and to be able to use them in a way that makes sense to them and others" (Fall 1991 interview).

Classroom observations provide additional evidence that Laura is thinking about mathematics differently. In the excerpt from the journal the range of classroom discussion indicate that the children's ideas regulate the flow of discussion rather than Laura guiding students toward a preconceived "answer." Laura encourages students to make sense of the ideas in their own way rather than in a prescribed way. Unlike last year when Laura noted that her thinking was changing faster than her teaching, classroom observations indicate many links between the way Laura is thinking about mathematics and the kinds of mathematical experiences in which children engage.

However, Laura often comments that she needs more subject matter knowledge. She expresses little confidence in her own ability to understand mathematics:

> I marvel at Pam's understanding of mathematics. I don't have it, never will. She makes me feel that I have some knowledge that can still be used . . . I wish I was more articulate. I wish I had some thought to articulate. Many times I feel like a dunce. I'm better at asking questions, because I have more questions than answers. (Winter 1992 interview)

An increased understanding of mathematics would probably strengthen Laura's mathematics teaching but we suggest that she knows more mathematics than she gives herself credit for knowing. She has worked hard to make sense of the mathematics she knows and to search for connections among mathematical ideas/concepts. Often what she perceives as "new" knowledge may not be new but rather entails making a different connection or recognizing some relationship to another mathematical idea.

Discourse

The image generated from reading the journal excerpt suggests that students in Laura's class freely share ideas and classmates agree and disagree in a routine manner. Other classroom observations support this as well. Interesting "debates" often emerge as students disagree and challenge one another's conjectures/ideas. On numerous occasions Laura expresses surprise that first graders can have "meaningful" mathematical discussions. During one interview she said, "When I thought about first grade students having discussions and creating their own understanding. . . . I didn't think we'd get very far but I've been surprised by kids' thinking about number and being able to make conjectures" (Winter 1992 interview). Laura poses open-ended problems and questions for the first graders recognizing that there are multiple directions in which class discussion may go. Students are encouraged to play around openly with mathematical ideas and conjectures. The mathematics community developing in this first-grade class is a safe environment where intellectual risk taking is encouraged and nurtured.

Laura's vision of having first graders engage in meaningful discussions is becoming a reality. She asks questions that encourage students to think and "wonder" about mathematical ideas. Students eagerly "agree and disagree" with one another's ideas and offer arguments to support their positions. During the fall, Laura described the math community she felt was developing:

> The community seems to be extremely accepting because there isn't anybody that feels like there is a wrong answer or that they have the right answer. You don't have the right answer or wrong answer. Right and wrong are eliminated from their vocabulary very quickly in this environment. The kids might say, "I agree with you" or "I disagree with you" and why they disagree with them. And revising their answers and saying, "I have a thought about that". During discussion they never come out and say, "That's wrong," they'll say, "I don't understand what that is" or "I don't understand what Tom is saying." So we ask them to explain it or can someone else come up and explain, does someone else have an idea that will help so and so? . . . they have no hesitation to go up there with their 1 + 1 = 4. . . they have no hesitation to put that up there and feel comfortable with the discussion that ensues from that. . . . I see children respecting others' ideas. But yet being enthusiastic about making that idea somehow their own. If they get an idea from somebody, they'll expand on it. I don't just see copying like you might see on a

workbook page, somebody just looks over and they copy the other person's answers--they have no idea what those numbers mean. These kids really want to have their own idea so they can have something to share... They are hungry to share their ideas. (Fall 1991 interview)

Later during the interview, Laura contrasted classroom discussions she had during the first year of MSG with the ones she has now:

> Discussions are becoming a much more natural part of math. It wasn't natural before. Even though their ideas might have been accepted, it was, that first year on an individual basis as I was going around and talking with the kids and encouraging them, I accepted their ideas.... And then the next year I was able to get some discussion happening but it still wasn't a regular part of our math activity by any means. I might have been accepting, but I seldom asked the other children's opinion of things or encouraged discussions between participants. And now I see this year [91-92] that math discussion is a major part of our math lessons. And discussion between the children is much more natural than it was in the previous years.

Classroom discourse and the mathematics community in Laura's class is quite different when contrasted to our earlier work with Laura. The teacher's role consists of providing interesting mathematical problems/questions for the students to explore, orchestrating discussions, listening carefully as students express their thinking, and building on students conjectures and ideas to formulate future lessons. Laura values allowing students time to construct their own meanings about particular concepts/topics rather than imposing her ideas or the textbook's methods. Laura also encourages the students to talk to *one another* about their ideas rather than engaging in an isolated student-to-teacher exchange.

During the fall, Laura described what she thought was her role, as the teacher, in building the math community in her classroom:

> To get the children to realize that math is a process that they figure out themselves. That the teacher doesn't give them the answers, they figure it out themselves.... [Laura continues by describing a discussion from class on the previous day about what = means.] One student new to our group said, "Why don't you [teacher] just tell us what it is? We don't have to talk about it, you just tell us what equals is." (Fall 1991 interview).

Laura continues to have ideas about what the "answer" may be but she is open to alternative interpretations and allows the students' ideas to shape classroom conversations and debates. Laura also talked about the student's role in building the math community:

> We need everybody's help to figure out these problems. You may have just the idea that we need today to help us and we need you to help think about it. . . . Students need to listen and see, Does that make sense? what that person just said, did that make sense? So we need them all to listen and help and to think about it and if it makes sense, that's great, if it doesn't make sense let's figure out what is going on. So I really want them to know that their responsibility is to help us, we need everybody's help. (Fall 1991 interview)

Laura's ideas about the nature and role of discourse have changed from "checking on" students' answers in which the teacher is the authority for knowing to a learning community in which students and teachers, together, determine the reasonableness of solutions.

Evidence of Student Understanding

Laura continues to expand the sources she draws from to help her assess student learning. During our first year, she relied upon paper/pencil work, directly questioning students, and small-group interactions. She now draws upon students' math journals, a record of anecdotal notes collected from class discussion, as well as individual conversations with students. Evidence for understanding previously was grounded in whether or not students "got it." Now Laura considers the ways in which students think about an idea. For example, she is interested in the flexibility with which students are able to think about numbers--multiple ways to put numbers together and take them apart as well as multiple ways to represent their ideas.

She wants to think about ways to build on their thinking, to plan problems and experiences that push their thinking and raise new ideas for student to consider. Often she engages Pam in conversations to speculate about why students might be thinking about a mathematical idea in a particular way and where to go with that. She is eager to share ways that students are talking about mathematical ideas, conjectures that they offer, and arguments that they use to support their ideas. The following excerpt taken from an interview earlier this year illustrates:

[Laura described the ways in which one student had taken a pattern developed by thinking about ways to make 4 and had extended it to large numbers]

> One of the students had proven that 100 - 96 = 4 and so Jim went on to figure out, well then if I have more zeros all I have to do is add more nines. But he had to work for a while on the calculator to find out, to see if that was how it was going to work out. He worked for a long time taking away a larger number to get it. He had 100 [100 - 96 = 4] all set and now to get to 1000. Once he got 1000 figured out that was take away 996 and he said, "I think we got a pattern going here, another zero, another 9." And he tried it on the calculator and it worked so he was

writing 10,000 take away 9996 and 100,000 take away 99996. He was just having a ball because he was seeing the pattern. . . . I never would have thought first graders would have. (Fall 1991 interview)

Reflections on This Year (Year 3)

As Laura's thinking about teaching and learning mathematics continues to evolve, her vision about what "can be" continues to be expanded and defined. From the beginning Laura has been able to identify aspects of her mathematics teaching that she would like to change and to think about ways to work towards those changes. Pam remarked to Laura that it was interesting that Laura is able to analyze her own development as a teacher during the process itself. Laura agreed with that assessment but said she does not spend a great deal of time consciously reflecting, however, she does find the more structured interviews to be valuable because they provide the time and opportunity for her to be able to think deeply about where she is and where she wants to go.

The context of a PDS has provided an opportunity for Pam and Laura to spend a significant amount of time together thinking, planning, teaching, and reflecting about that teaching. An interview with Laura and an excerpt from Pam's journal provide insight into their growth and learning related to this collaboration.

> I feel more positive, I enjoy teaching math more. . . . Being with Pam when I plan, makes me think about what we did and what might happen. We think about richer problems. . . . We are able to think about what would make a good learning experience for the kids. . . . It is a true collaboration, we really plan and carry it out together. (Winter 1992 interview)

> I am learning so much working with Laura. I just marvel at the ways in which she interacts with first graders. She provides such a rich but safe environment where children are encouraged to think and be responsible for themself. In most "problem" situations she tries to help the kids identify the options available related to choices to be made but always with an eye towards past choices and outcomes so that they have opportunities to think about making choices that are best for them. It's also a safe environment for me. I can come in and try out ideas without feeling as if everything has to be "perfect." When lessons don't go as planned, Laura doesn't pass judgment or rush to tell other teachers "what happened to Pam" during math today. Instead the focus is on the sense kids seem to be making about the activity or idea and what "we" could do to challenge or broaden that thinking. . . . [After a presentation at NCTM about my undergraduate math course] I realize more and more what an impact teaching in first grade is making on the way I think about many aspects of my work. I often use it to anchor my reactions to ideas about teaching and learning mathematics especially related to younger children. There was a real gap in my experience with primary aged children. I am so lucky to have the opportunity to work with Laura. She brings so much to our conversations and collaboration (Journal, April 1992).

Consistent with previous years, Laura continues to question and challenge the ways in which she is teaching mathematics this year. Some of her questions include:

> Are all of the things I'm responsible to teach being covered without my explicitly teaching them? Would some children benefit more by direct instruction? If so, where does the time to do that come from? If the kids come up with something that is not mathematically true or correct, there is a tension for me if I don't address it in some direct way, it may slip through the cracks. (Winter 1992 interview)

> Am I neglecting certain things that I should be teaching? I feel good about what I *am* doing but I realize there are things that I am *not* doing. (Conversation during a teacher candidate seminar, February 1992)

As one might predict, Laura is already thinking about what she wants to do for next year. This week she told Pam that she wanted to talk about next year--she and two other colleagues want to propose a small group to collaborate with Pam about their math teaching.

Laura did not wish to be involved in the actual writing of this paper but contributed in many other ways. As we continue to work towards providing more restructured opportunities to enable teachers to engage in the collaborative writing about our work, we want to continue to provide options and choices. Laura has engaged in conversations with us throughout the writing process. She read previous drafts and provided feedback and in an effort to provide a more explicit voice, she wrote some reflections about this draft:

Laura's Reflections

> The work on the bases [during the 2nd year] and not knowing the right answer myself really helped me feel how the students were experiencing math and the importance of each of us struggling with the concepts for ourselves. Being allowed the freedom to make mistakes and discoveries.
>
> This is Year 3 and it feels like *Step 3* on an ongoing climb to where I want to be with the students. I hear some very interesting discussions during math and this process permeates most of our discussions all day long. But I see the various students gain and lose interest and then join us again as any large group discussion continues. I would like to have more small-group discussions on specific problems that comes from our studies together. When we do this, I see the quiet ones blossom and more children get a chance to explain their thinking. I would like for my children to feel as I have in MSG when we work together to solve math problems. We have really seen the truth in the saying--the best way to learn something is to teach it to someone else--when we let the children enjoy that type of learning, everyone benefits.
>
> A visitor coming into my classroom on any given day might not think that this is a revolutionary type of math and yet if they were able to stay there for the entire week, the kind of thinking that the students get involved with is *very* different for first graders. They take a lot of ownership. They enjoy that whole process of discovery. Mathematics is almost alive for them. If there was a way to take video

> clips of the processes and have us be able to explain how this came about--it doesn't just happen in one day, or one week or one month--it might give this person a better insight. Most important for me is to be able to share with the other teachers in my building and at this point they are my most eager, receptive audience. Next I would like to be able to share with the other teachers in my district; I'm not sure how interested other teachers would be.

Continuing Challenges

When we met Laura three years ago, her classroom was not the traditional mathematics classroom characterized by much of the reform literature. Nonetheless, given our PDS context--support for innovation, time for reflection and planning, and opportunities to collaborate--Laura made subtle but significant changes in her mathematics teaching.

Changes in Laura's views about mathematical content, the nature and role of discourse, and evidence of student understanding were interrelated and interdependent. Laura's thinking about mathematical content evolved from a focus on thinking about knowledge as fixed to a view of knowledge as dynamic. Early on, discourse was a means for arriving at preconceived conclusions. More recently, discourse has become a means for Laura and her students to jointly develop both the nature and direction of mathematics lessons. Previously, Laura relied on fairly traditional measures--giving "right" answers, performing operations using concrete materials--as evidence of student understanding. Now, she seeks evidence from multiple sources including students' explanations, students' journals, and the teaching team's anecdotal record which highlights patterns and growth in student learning.

Given the complexity, the commitment necessary and the length of time required to make the kinds of changes we have described in telling Laura's story, it is not surprising that Laura questions that teachers outside of a PDS context would be interested in entering into similar reform efforts. The changing nature of our collaborative work has shaped our thinking about mathematics reform. Many questions have emerged from our work together during the last three years and we hope to get smarter about these issues as our work together continues:

•What happens to Laura's growth if she returns to working in isolation?

•What role can Laura play with other teachers and teacher candidates?

•In what ways can we include our school-based colleagues in a collaborative writing process about our work together?

•The school and university cultures and professional structures are different: How can schools and universities be restructured so that aspects of collaborative efforts are recognized and valued in each culture?

•What are appropriate research designs for studying PDS work?

•In what ways can university and school-based personnel collaborate in disseminating what we are learning?

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