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ABSTRACT

The implementation of the National Council of Teachers of Mathematics' "Curriculum and Evaluation Standards for School Mathematics" implies fundamental shifts in the teaching and learning of mathematics. Intended for those individuals who will be instrumental in the implementation of those changes--teachers, administrators, and parents--this document presents 10 ideas for transforming mathematics teaching and learning based on research and successful practical experience: (1) All students can and must learn mathematics, which should serve as a "pump," not a "filter"; (2) Teachers need to listen to students and incorporate into their instruction what they learn from listening; (3) Students learn mathematics best when they construct their own mathematical understanding; (4) Students need to learn more and different types of mathematics; (5) Mathematical discussion should be a daily part of classroom activity; (6) Teachers need to become "informed guides" to the learner; (7) Calculators, computers, and related technology can be effective tools in the teaching and learning of mathematics; (8) Students need shared learning experiences; (9) Curricular and pedagogical change in mathematics cannot occur without accompanying change in student assessment; and (10) Lasting change takes broad support. (Contains 14 references.) (MDH)

# State of the Art

# *Transforming Ideas for Teaching and Learning*

# Mathematics



OFFICE of RESEARCH
*U.S. Department of Education*
*Office of Educational Research and Improvement*

# State of the Art

# *Transforming Ideas for Teaching and Learning* Mathematics

Carole B. Lacampagne

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The illustrations in this booklet, taken together, represent a transformation of four congruent shapes rotating about fixed points in ten increments to create a new and unified shape.

# *Background*

"State of the art" is a goal that every school mathematics program in the United States would say it strives for. It is a goal owed our children and it is attainable. State of the art depends upon curriculum reform. In the current national curriculum reform movement, initiated by the National Council of Teachers of Mathematics and supported at all levels by those involved with mathematics education, it is particularly exciting to be engaged in transforming mathematics teaching and learning to reach the state of the art.

Major curriculum reform is not new in the field of school mathematics. The last such reform was the "new math" of the late 1950s and 1960s which emphasized the unifying mathematical concepts of logic and set theory. For a variety of reasons the new math did not receive widespread acceptance. Specifically, it did not pay close attention to how students learn and what they are capable of learning at different ages. It also did not address what teachers know about mathematics and pedagogy or how they can best enhance their own knowledge.

The new math was followed by the "back to basics" movement which emphasized rote memorization of arithmetic facts and the learning of paper-and-pencil algorithms. The current reform movement grew out of the inability of the back to basics movement to address key issues, including

- Neglect of higher order thinking and problem-solving skills;
- Disquieting findings about American students in recent international studies on mathematics achievement, despite the return to basics;
- Changing mathematical skills needed in the work force;
- New research findings on teaching and learning mathematics; and
- The mushrooming of inexpensive calculators and computers.

In an effort to systematically address these issues the National Council of Teachers of Mathematics (NCTM) established in 1986 the Commission on Standards for School Mathematics. This commission comprised a cross section of mathematics educators, including classroom teachers, supervisors, educational researchers, teacher educators, university mathematicians, and PTA representatives. Over the next three years, the commission developed a document, *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), which incorporates the suggestions of the mathematics community and is now accepted as the world class standard for mathematics. By a similar process, NCTM also developed the *Professional Standards for Teaching Mathematics* (NCTM, 1991). Both sets of standards have been endorsed by groups representing the mathematics community from kindergarten through graduate school, as well as by many other groups with a stake in mathematics education.

The new standards imply fundamental shifts in the teaching and learning of mathematics toward a classroom environment that promotes the development of every student's capability. To create such an environment, the NCTM recommends five major shifts:

- toward classrooms as mathematical communities—away from classrooms as simply a collection of individuals;
- toward logic and mathematical evidence as verification—away from the teacher as the sole authority for right answers;
- toward mathematical reasoning—away from merely memorizing procedures;
- toward conjecturing, inventing, and problem solving—away from an emphasis on mechanistic answer-finding; and

- toward connecting mathematics, its ideas, and its applications—away from treating mathematics as a body of isolated concepts and procedures.

  (National Council of Teachers of Mathematics 1991, p. 3.)

In addition, NCTM recommends a shift in the early years away from emphasis on the formal, abstract representation of concepts toward their introduction via manipulatives, experiments, and computer simulation.

These shifts make it essential for teachers to acquire new mathematical and pedagogical knowledge. But at a more fundamental level they require changes in many teachers' and parents' beliefs about the nature of mathematics and how it can best be taught and learned. The U.S. Department of Education has identified 10 ideas for transforming mathematics teaching and learning that are backed up by research or by promising practical experience. These ideas are intended to be useful to teachers—the key agents in the transformation process—for making the fundamental changes needed to help every student realize his or her mathematical potential.

This document is also addressed to parents and school administrators who share with teachers the common goal of educating children for excellence. Toward this end they can and must support teachers in their endeavors to transform the teaching and learning of mathematics to state of the art.

# *All students can and must learn mathematics, which should serve as a "pump," not a "filter."*

**Myth: Learning mathematics requires special ability, which most students do not have.**

**Reality: Only in the United States do people believe that learning mathematics depends on special ability. In other countries, students, parents, and teachers all expect that most students can master mathematics if only they work hard enough. The record of accomplishment in these countries—and in some intervention programs in the United States—shows that most students can learn much more mathematics than is commonly assumed in this country.**

(Mathematical Sciences Education Board 1989, p. 10)

The idea that all students can and must learn mathematics means that the study of mathematics, by serving as a pump—an access to success—can transform the learning of the general population. All students must have the opportunity to learn mathematics.

In the past it was assumed that problem-solving ability was tied to the ability to perform paper-and-pencil calculations. Years of teachers' and students' time were spent trying to remediate children who lacked this ability. The emphasis on remediation was based on the premise that mathematics is linear and hierarchical and must be taught in a prescribed order—rote skills first, problem solving later. But research shows that repeating the same uninteresting tasks in the same unimaginative way is not effective. Students learn best when they are intellectually challenged so that they are motivated to fill in mathematical gaps when necessary. The teacher's role is to provide stimulating problems and environment to motivate mathematical learning. In fact, research points out that certain teaching strategies can help all students develop "mathematical power." Providing students with real-life problems to investigate is just one strategy for helping them develop an understanding of the mathematical concepts that underlie a variety of problems.

Tracking, on the other hand, when it is used to filter students out of mathematics, is antithetical to the development of mathematical power: little learning is expected of students in lower tracks and, as a result, they produce little and lose the opportunity to work in mathematics-related occupations; teachers of these tracks often feel like second-class citizens too and lose their enthusiasm and creativity over the years; tracking fosters an elitism that contributes to the underrepresentation of women and non-Asian minorities in mathematics fields and careers that rely on a solid mathematics background; and tracking is a poor substitute for implementing a wide variety of the enrichment activities at the pre-high school level and of mathematics courses at the high school level, including advanced placement courses, that can stimulate the quickest students to greater achievement.

All school mathematics courses should be of high quality and challenge all students to high achievement. Parents and students must be shown that achievement in mathematics does not depend on an accident of birth such as innate talent, but that it is attainable through hard work—the same way all skills are successfully accessed.

## *Teachers need to listen to students and to incorporate into their instruction what they learn from listening.*

**The first several years of teaching I really was into "This is the section of the book that we're doing today, and here's the practice problems, and now we'll go over homework, and then I'll teach you how to do it, then you'll practice, and then you'll have some to try before you go home," and that kind of thing. I teach very differently now.**

(Middle school teacher Becky Wickham as quoted in Philipp et al. 1992, p. 30)

Teachers who listen to students, and who plan instruction based on what they learn from listening, transform student learning. For example, two children may arrive at the same solution of a problem but with different strategies. These strategies may reflect different levels of understanding and suggest different follow-up activities. Moreover, teachers who listen carefully to students' mathematical explanations often find that their students know a great deal of mathematics at an informal level. By building upon this informal knowledge, teachers can help their students construct more sophisticated concepts.

Effective teachers listen carefully to how students go about solving problems. They know their students' mathematical strengths and weaknesses and they can develop a teaching strategy based on this understanding. Research shows that when teachers act upon their knowledge of student thinking, their beliefs about learning and instruction, their classroom practices and, most importantly, their students' learning and beliefs can be affected profoundly.

# *Students learn mathematics best when they construct their own mathematical understanding.*

**Ms. M.: "The African elephant ate 37 peanuts. The Indian elephant ate 43 peanuts. How many fewer peanuts did the African elephant eat than the Indian Elephant?"**

**Ms. M.: "Got it? How many fewer did the African elephant eat. . .?"**

**Ubank: "Six."**

**Ms. M.: "Does everyone agree with that? . . . How did you figure it out, Ubank?"**

**Ubank: "Well, I had 43 here (pushing out 4 stacks of ten cubes and 3 additional cubes joined together), and I had 37 here (pushing out 3 stacks of ten cubes and a stack of 7). I put 30 on top of these 30. I took 3, and I put them here. There were 4 left, so I took 4 off, and there were 6 left. . ."**

**Ms. M.: "Did he do it a good way? . . . Did anyone do it a different way?"**

**Marci: "I took 37, and I needed 43. So I counted up 3 more. That was 40. Then I took 3 more to 43."**

**Ms. M.: "Good. Does her way work well? . . . It sure does. Did anybody do it differently?"**

(Carpenter and Fennema 1992, pp. 462–463)

Students who construct their own mathematical understanding transform their mathematical potential. It takes courage to begin using the "constructivist" approach in the classroom, but the rewards can be great. Teachers often start with an experiment—a somewhat ill-defined but interesting mathematical problem or application for students to solve. They resist pleas to solve the problem for their students. They often find that their best students resist the change in teaching and learning at first—after all, the best students have succeeded in the old mode, even if they found the mathematics boring. The teachers give the experiment time to succeed.

One of the most difficult shifts for teachers is to relinquish their role of keeper of "the right answer." As students grapple with constructing their own knowledge, they may ask questions that the teacher cannot answer. They may go down mathematical paths that their teacher had not trod. They may devise algorithms that are unknown to their teacher. Teachers too need to construct their own mathematical and pedagogical knowledge. As teachers become learners they model the mathematical behavior they expect of their students.

Teachers must assume a new role if students are to construct their own mathematical understanding. Rather than just being the information givers—pouring mathematical knowledge into the student's head—teachers must provide stimulating mathematical problem situations that encourage mathematical learning. Students must change from being passive recipients to becoming active seekers of knowledge. Students must also learn to verify their own mathematical knowledge.

# *Students need to learn more and different types of mathematics.*

**It is now possible to execute almost all of the mathematical techniques taught from kindergarten through the first two years of college on hand-held calculators.**

(Mathematical Sciences Education Board 1990, p. 2)

The need for a work force equipped with more and different mathematical concepts is transforming the mathematics curriculum. Nonroutine problems rarely involve ideas from just one part of mathematics. Just as the printing press made calligraphy obsolete as a common writing tool while, at the same time, it increased the need for people to read and write, so too technology is making pen-and-pencil calculations obsolete while, at the same time, increasing the need for people to model and solve complex problems. Thus the curriculum at all grade levels needs to include geometry and measurement, probability and statistics, pre-algebra or algebra, patterns, relations, functions, and discrete mathematics.

This suggested curricular reform is not as radical as it first appears. Many countries have used an integrated curriculum successfully for years, and teachers across the United States have already begun to develop instructional units based on problem situations that involve a variety of mathematical content areas and that may take two to five weeks to investigate.

Some teachers worry that teaching more and different types of mathematics will crowd the mathematics curriculum. Constructing one's own mathematical understanding and solving complex mathematical problems and applications are very time consuming. It may not be possible to cover the same ground using this approach as one would using the lecture method. Yet research indicates that the mathematical understanding students construct themselves is deep and enduring—that students taught this way can score as well as their peers on low-level mathematics skill items and better on problem-solving and conceptual items. Orchestrating the major mathematical concepts that students should understand and eliminating from deep coverage those items of less importance are difficult new roles for teachers.

# *Mathematical discussion should be a daily part of classroom activity.*

**If a child asks you if this answer is right, and you say yes, you've robbed him of the real learning. It's a question of when you say good, not *if* you say it. Once you've probed for understanding, and you're sure that the child knows, *then* to say, 'You've convinced me, that's terrific, what you said really made sense to me. Why don't you share it with the rest of the class?' But I'd wait until the last moment when I'm really sure that the child really knows it.**

(Ball and Wilcox 1989, p. 15)

Mathematical discourse transforms student learning. In offering praise too quickly teachers sometimes lose the opportunity for productive mathematical discussions, a key ingredient for building mathematical power. The lecture mode of instruction also discourages mathematical discourse in the classroom. Recent research shows that classrooms where students engage in a rich mathematical dialog with their peers as well as with their teachers are effective learning environments. Students need to be actively involved in questioning, conjecturing, defining, and explaining.

Teachers can shape the classroom environment to encourage mathematical discussion. They can encourage the participation of all students by valuing each student's contribution, by reducing the risk of ridicule for being wrong, by encouraging honest disagreement, and by making sure that all students are included in the discussion. Mathematical discussion that is rigorous but open minded should be a regular and valued part of classroom activity.

When teachers openly discuss their own mathematical thinking and demonstrate the process by which they solved a problem, they encourage this active mathematical behavior in their students. Teachers cannot expect students to tackle difficult mathematical problems, to discuss, question, define, and conjecture if they do not do so themselves. They cannot expect students to be curious and excited about mathematics unless they are.

# *Teachers need to become "informed guides" to the learner.*

**Within the short space of these few minutes of classroom time, I faced a series of issues: how to get and maintain all my students' engagement, how to make sense of what Sean and Riba [students] were thinking, how to help them move toward appropriate and connected understandings of fractions. From moment to moment I was having to consider whether to praise, explain, solicit others' ideas, let an issue grow, or even stir up trouble in order to press on a crucial mathematical point. . . . Day after day in my classroom, students say things I had never considered. Day after day they have trouble with ideas I used to think were simple. And day after day, these eight-year-olds catch me off guard with what captures their interest and what they reach for.**

(Ball 1992, pp. 14–15)

Teachers who "guide" rather than "tell" transform student learning. The role of informed guide is much more difficult to assume than that of the lecturer. As teachers focus more on guiding their students' learning they need to know more mathematics. According to the Mathematical Association of America's *A Call for Change*,

> Teachers need to recognize the relationship between what they teach and what is taught at other levels of school mathematics. They need, for example, to understand the close parallel among the development of integer arithmetic in the elementary grades, the algebra of polynomials in the middle and early high school curriculum, and the ideas of number systems explored later in high school. . . . They should explore the relationships between geometry and algebra and the use of one to investigate the other. (Leitzel 1991, p.3)

Case studies indicate that teachers who have a good background in mathematics also add a richness to their lessons, involve students extensively in mathematical dialog, and capitalize on students' questions and discussions to weave and extend mathematical relationships. Such teachers guide their students to discover mathematical concepts and procedures. They do not list definitions and step-by-step procedures for students to memorize without understanding their meaning and function. Research indicates that classroom behavior is affected by an interplay among teachers' general and content-specific knowledge of mathematics, their understanding of how children think about mathematics, and their beliefs about mathematics and about how children learn it.

# *Calculators, computers, and related technology can be effective tools in the teaching and learning of mathematics.*

**Failure to introduce and to use calculators and computers in school creates a needless barrier between what is happening in students' everyday lives and what they are being taught in school. . . . For mathematics education to remain viable in the future, it must include a major role for the computer now.**

(Shane and Tabler 1981, p. 107)

Calculators, computers, and related technology used as tools in the teaching and learning of mathematics transform the learner from calculator to critical thinker. Technology implies a shift from using brain power for computational tasks to using brain power to think critically, to communicate clearly, to solve mathematical problems, and to apply mathematics to complex scientific and social problems. Research shows that the proper use of calculators and computers can in fact enhance mathematics learning at all stages. Calculators and computers can take the drudgery out of mathematics by handling routine arithmetic and algebraic calculations, freeing the learner to concentrate on the problem that requires such calculations. Calculators and computers can be used to illustrate mathematical concepts graphically and this kind of visual representation can help understanding. Computers can simulate a variety of modeling options, freeing the learner to determine the most appropriate model to use in a given application.

The continual development of new technology—graphing calculators; computer-based exploratory tools such as spreadsheets, LOGO, the Geometric Supposer, and the Geometers' Sketch Pad; and hypermedia—requires teachers to continually enhance their technological skills. Professional mathematics and computer science education journals and inservice workshops can help provide this enhancement for more effective mathematics teaching and learning.

# *Students need shared learning experiences.*

**Steven Kirsner [interviewer]: What's that been like—to work in groups instead of what you're used to?**

**Penelope [sophomore, special education student, referring to her first opportunity to study meaningful mathematics]: Well, first of all, . . . when I didn't work in groups, it was harder to get to know people. . . We just worked separate and we actually didn't learn practically anything, but what we learned from the teacher. Here we learn from everybody. We learn how they do it, how they understand it, and we share our ideas with each other.**

(Kirsner and Bethell 1992, pp. 17–18)

Cooperative learning transforms the teaching and learning of mathematics to model the work force environment. In the work force, teams of people collaborate to solve difficult problems. The expertise of each team member adds a dimension to the solution process. Students need to learn to work cooperatively, too. Students working together help each other learn. Together, students can often tackle challenging situations that would be beyond the capacities of the individuals who comprise the group. The group situation can motivate students and stimulate mathematical discussion, thus helping each student realize her or his own potential.

In order for this group process to work effectively, the teacher must carefully prepare the learning environment. Problems presented to the group should be too difficult or too complex for one child to solve alone. The problems should also pique the group's interest and curiosity. The teacher must ensure that all children participate in the group work and learn cooperative skills. Teachers themselves may need inservice education in using cooperative learning strategies so they can successfully implement them in the classroom.

Research indicates several positive effects of cooperative learning in mathematics education. When coupled with individual accountability, cooperative learning leads to greater academic achievement. Cooperative learning also can increase the self-esteem and self-confidence of the learners and lead to positive intergroup relations—including cross-racial and cross-cultural friendships and social acceptance of mainstreamed children—and greater ability to use social skills.

# ***Curricular and pedagogical change in mathematics cannot occur without accompanying change in student assessment.***

**Through assessment, a better understanding should be obtained of how students are relating mathematical ideas to each other and if they are building an integrated notion of mathematics. . . . Making sure that assessment is integral to instruction should mean that the information obtained is directly useful for guiding instruction. In short, good assessment is good instruction.**

(Webb and Briars 1990, p. 117)

Curricular and pedagogical changes in mathematics must transform how students are assessed. As mathematics curricula and pedagogy are changed, the instruments for measuring student achievement must also be changed. It is not fair to students, teachers, or school districts to be measured by outdated standards.

The majority of standardized tests our children take are still overly reliant on multiple choice items that measure predominantly low-level mathematics skills. Although they are beginning to reflect the changes in mathematics teaching and learning, these tests include few types of questions that require higher order problem-solving skills. School districts should analyze standardized tests and use the test that most closely assesses meaningful standards that are in place, such as the NCTM standards.

Researchers are developing alternative assessment tools that both measure student achievement and promote learning. Performance assessment, student interviews, group project reports, and portfolios are a few in the wide range of new assessment tools that researchers are investigating and teachers are beginning to use.

# *Lasting change takes broad support.*

**It costs state legislators and bureaucrats relatively little to fashion a new instructional policy that calls for novel sorts of classroom work. These officials can easily ignore the pedagogical past, for they do not work in classrooms, and they bear little direct responsibility for what is done in localities—even if it is done partly at their insistence. However, teachers and students cannot ignore the pedagogical past, because it is their past. If instructional changes are to be made, they must make them. And changing one's teaching is not like changing one's socks. Teachers construct their practices gradually, out of their experience as students, their professional education, and their previous encounters with policies designed to change their practice. Teaching is less a set of garments that can be changed at will than a way of knowing, of seeing, and of being.**

(Cohen and Ball 1990, p. 163)

Broad support from the educational community is needed to advance the reform effort and transform it to state of the art. Teachers willing to risk making the recommended shifts in classroom practices are at the forefront of the reform in teaching and learning mathematics. Yet systematic change cannot occur unless the members of the learning team—students, parents, school administrators, and policymakers—are also key participants in the process. Past reform efforts have died out because the whole learning team was not involved. The rationale for changing mathematics teaching and learning and plans for implementing the changes should be disseminated to all of these groups. The learning team needs to be involved in the construction of the new school mathematics environment.

Although research on the current reform movement in mathematics is ongoing and as yet incomplete, several components of the reform's success have emerged. It is evident that teachers cannot accomplish it alone. A coordinated school-based reform effort guided by world class standards in mathematics is necessary to transform the mathematics curriculum, teaching methods, and student assessment. The reform's success will also depend on the availability of greater opportunities for all students to learn mathematics and to use new technology. In addition, since the reform movement asks much of teachers, extensive and continuous staff development is needed. This includes courses in content to develop new and deeper knowledge of mathematics, in skills for facilitating learning, in new assessment methods, in implementing cooperative learning, and in working with diverse student populations.

In the end, the appropriate organizational structures must be in place to support the professional cooperation, planning, and school governance that in turn promote risk taking and reform and lead to a new state of the art in mathematics.

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