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ABSTRACT

This document was designed to help teachers and other school district personnel plan and teach Algebra I and Algebra II in the junior high and high schools. An overview of the Algebra I and Algebra II courses presents the philosophy of algebra instruction for Texas schools, the expected use of technology in mathematics instruction, teacher requirements, and course prerequisites and credits. The following two sections describe the Algebra I and Algebra II courses. The Algebra I course can be offered to students in grades 7 or 8 that are adequately prepared. Issues concerning student needs, entry criteria, high school credits, placement into high school courses, transfer of students back to grade 7 or grade 8 mathematics, and parent communication related to offering the course at that level are discussed. The remainder of the two sections are structured to describe the essential elements of the Algebra I and Algebra II courses, provide sample objectives for related essential elements, and offer a total of 22 sample activities to attain related objectives. Appendix A discusses the issue of evaluation at the local level and at the state level in relation to the Texas Assessment of Academic Skills. Appendix B presents a list of resources for algebra instruction that includes 21 books and articles, 16 state-adopted textbooks, and 6 computer software programs. (MDH)

GUIDELINES FOR TEACHING ALGEBRA I-II

Texas Education Agency
Austin, Texas

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Texas Education Agency
1701 North Congress Avenue
Austin, Texas 78701

FOREWORD

Guidelines for Teaching Algebra I-II is designed to help teachers and other school district personnel plan and teach Algebra I and Algebra II. The publication presents the philosophy and intent of the courses and discusses teacher preparation, credits and prerequisites, the required essential elements of instruction, and the use of technology. Also included are sample objectives and activities to illustrate how the essential elements for Algebra I and II can be taught. School district personnel may want to use these suggestions to develop their own curriculum documents for the courses.

We hope these guidelines will be useful in planning and teaching Algebra I and II and in equipping the algebra classroom.

Lionel R. Meno
Commissioner of Education

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CONTENTS

Overview of Algebra I and Algebra II

Philosophy

The focus of Algebra I and Algebra II is on students' solving relevant and interesting problems and on their applying algebraic principles in a variety of real-world situations. In Algebra I, students learn to understand the use of variables in equations and expressions representing quantities in the real world. In Algebra II, important new emphases are collecting, representing, and processing data, which are major activities of contemporary society. Students learn to apply problem-solving techniques to evaluate alternatives, make predictions, and arrive at informed decisions.

The philosophy of algebra instruction for Texas schools is reflected in the following statement from the *Curriculum and Evaluation Standards for School Mathematics* published by the National Council of Teachers of Mathematics (NCTM) in 1989:

> Algebra is the language through which most of mathematics is communicated. It also provides a means of operating with concepts at an abstract level and then applying them, a process that often fosters generalizations and insights beyond the original context.
>
> Algebraic symbols may represent objects rather than numbers, as in "p + q" representing the sum of two polynomials. This more sophisticated understanding of algebraic representation is a prerequisite to further formal work in virtually all mathematical subjects, including statistics, linear algebra, discrete mathematics and calculus. Moreover, the increasing use of quantitative methods, both in the natural sciences and in such disciplines as economics, psychology, and sociology, have made algebraic processing an important tool for applying mathematics.
>
> The proposed algebra curriculum will move away from a tight focus on manipulative facility to include a greater emphasis on conceptual understanding, on algebra as a means of representation, and on algebraic methods as a problem-solving tool. This represents a trade-off in instructional time as well as in emphasis. Although an appropriate level of proficiency is important, available and projected technology forces a rethinking of the level of skill expectations.
>
> Changes in emphasis require more than simple adjustments in the amount of time to be devoted to individual topics; they also will mean changes in emphases within topics. For example, although students should spend less time simplifying radicals and manipulating rational exponents, they should devote more

time to exploring examples of exponential growth and decay that can be modeled using algebra. Similarly, students should spend less time plotting curves point by point, but more time interpreting graphs, exploring properties of graphs, and determining how these properties relate to the forms of the corresponding equations (e.g., the relationship between the graphs of $y = |x|$ and $y = |x - 5|$). Of course, students should continue to plot critical points to check the reasonableness of graphs.

To emphasize the underlying structure of algebra, five unifying themes are used throughout the sample objectives and activities presented in this document. As they participate in the activities, students should recognize each new concept as an illustration of one of these themes:

- solving equations
- graphs—drawing and interpreting
- constructing models (applications)
- working with expressions
- properties of real numbers
  - distributive property
  - order of operations (inequalities)
  - equivalent fractions ($ab/ac = b/c$)
  - other field properties

Course Implementation

Use of Technology

Mathematics educators believe that effective use of calculators and microcomputers in the teaching of mathematics has become increasingly important. In the *Standards*, the National Council of Teachers of Mathematics recommends that:

- appropriate calculators should be available to all students at all times;
- a computer should be available in every classroom for demonstration purposes;
- every student should have access to a computer for individual and group work; and
- students should learn to use the computer as a tool for processing information and performing calculations to investigate and solve problems.

The textbook proclamation issued by the State Board of Education in March 1988 specified a major new thrust for Algebra I and Algebra II. The use of current technology including calculators, graphing calculators, and computers must be integrated throughout the courses, with students using calculators and computers as problem-solving and discovery tools whenever possible. Textbooks must specifically address the use of these tools, the understanding being that students will have access to scientific calculators, that the teacher will have access to a computer for demonstration purposes, and that students will have opportunities, as needed, to use computer graphing and problem-solving technology.

The use of calculators and computers is not specifically mentioned in the sample objectives in this document because the objectives do not include specific teaching techniques. However, calculators and computers are to be included whenever possible. Some suggested uses have been included in the sample activities.

Teacher Preparation

Algebra I and II must be taught by teachers who are certified in secondary mathematics. Since algebra is fundamental to the understanding of numerous applications and to higher mathematics, exemplary teachers possess a depth of understanding in the subject that goes far beyond what is normally taught in Algebra I and II. They are skillful in applying the content to real-life situations and are consistent in connecting the content to other content areas.

Successful high school algebra teachers not only have a good academic background in mathematics, but they also are knowledgeable about current developments in the teaching of mathematics and technological advances that impact teaching. They are dedicated professionals who attend mathematics conferences and who read professional journals to stay abreast of developments in their field. They never stop learning and always look for new and better ways to teach the topics in their courses. In particular, algebra teachers need to be acutely aware of the technological advances that can facilitate and enhance students' understanding of mathematics. Commitment to the routine use of both handheld calculators and computers in algebra instruction is critical.

Prerequisites and Credits

Students entering Algebra I should have successfully mastered the essential elements of Grade 8 mathematics; students entering Algebra II should have successfully mastered the essential elements of Algebra I. Although Algebra I and II are listed as one-unit courses in the State Board of Education's rules for curriculum, the State Board has ruled that students may receive one-half unit of credit for successfully completing one semester of either course.

Students who take Algebra I in Grade 7 or 8 receive high school graduation credit for successful completion of the course.

ALGEBRA I

Algebra I in Grade 7 or 8

Rationale

Many school districts are now offering Algebra I to students in Grade 8, and some are offering the course in Grade 7. According to State Board of Education rule, districts may offer Algebra I in this way, as long as students who enroll have mastered the required essential elements of instruction for mathematics in Grades 7 and 8. Algebra I can be appropriate for students in Grade 8 if they have sufficient background for the program. Students who complete Algebra I and go on to take four years of mathematics in high school can progress through Calculus. The primary purpose of offering Algebra I in Grade 7 or 8 is to make it possible for students to take more advanced mathematics in high school. Students should take the course with this purpose in mind. If students are interested only in reducing their required mathematics units in high school, this option will not be beneficial to them. In fact, such a program can be harmful to students if the only high school mathematics they take is two years in Grades 9 a[illegible]. If these students enter college mathematics courses, they may have difficulty remembering basic algebra and geometry skills required for college mathematics. If they enter the work force or technical schools, they may also be out of touch with the basic mathematics skills needed.

Issues

School districts that plan to offer Algebra I in Grade 7 or 8 should consider the following important issues:

- **Student Need**

  Before offering Algebra I to Grade 7 or 8 students, school personnel should carefully examine student needs at these levels. Educators cannot assume that students will have sufficiently mastered Grades 7 and 8 mathematics essential elements unless provisions have been provided for such student mastery. They should plan a program to accelerate and/or enrich capable students, beginning at Grade 5 or 6, or even earlier. If sufficient numbers of students have completed such a program, offering Algebra I in Grade 7 or 8 may be justified. Many good students, however, would benefit more from an enriched or honors eighth grade mathematics program, which would give them a sound foundation in pre-algebra skills and would prepare them for success in high school mathematics. If students are placed in Algebra I too early, they will have less chance for success in algebra and other higher level mathematics.

- **Verification of Student Mastery of Essential Elements**

State Board rules require that districts verify that students taking Algebra I in Grade 7 or 8 have mastered, at a satisfactory level, the essential elements for Grades 7 and 8 mathematics. The rules do not specify how this is to be accomplished, and no state-approved examination for this purpose currently exists. Districts must decide how this verification is to be done. Many districts are using locally constructed examinations that assess students' mastery of essential elements for this purpose. The method and results of this verification should be a part of the student's record.

- **Entry Criteria**

Verification of mastery of essential elements for Grades 7 and 8 mathematics certainly should be part of the criteria for allowing a student's entry into Algebra I in Grade 7 or 8. However, this should not be the only criteria for entry. School personnel should use multiple criteria for student entry, which may include some or all of the following: teacher recommendation, prior achievement in mathematics, results of achievement tests in mathematics, result of the student's mathematics score on Grade 7 TAAS, result of an algebra prognosis examination, student attitude, parental approval, student commitment to taking four years of mathematics in high school. Careful attention to entry criteria is crucial to the success of Algebra I in Grade 8. Typically, a school might expect 5 to 10 percent of its students to qualify for this program.

- **High School Credit**

A State Board rule requires that when a student takes a course in Grade 7 or 8 that is designed for Grades 9-12, the student's academic achievement record must reflect that the student has satisfactorily completed the course and has been awarded state graduation credit. This rule makes it mandatory that schools award high school graduation credit for Algebra I taken in Grade 7 or 8. This credit applies to the Advanced High School Program as well as to the regular program. A student can not repeat the course in high school for credit. School personnel should carefully communicate this information to students, parents, and teachers.

- **Placement into High School Mathematics Courses**

Students who pass Algebra I in Grade 8 should expect to continue with Geometry or Algebra II in Grade 9; students who pass Algebra I cannot repeat Algebra I in high school for graduation credit. If school personnel, the student, or the parents feel that the student is not ready for Geometry or Algebra II, the student could be placed in Informal Geometry, Mathematics of Money, or Computer Mathematics.

- **Number of Mathematics Units Required in High School**

According to State Board rule, three units of mathematics are required for the regular high school program; three units of mathematics from a selected list are required for the Advanced High School Program. Algebra I taken in Grade 7 or 8 satisfies one of the units required for either the regular or advanced program. A Board rule requirement allows students who take Algebra I in Grade 7 or 8 to finish their mathematics requirement for either the regular program or the advanced program with only two additional units of mathematics taken at high school. However, school districts may require students to take three or more units of mathematics in high school, regardless of the mathematics taken in Grades 7 and 8, if they make the requirement a part of their local board of trustees' policy.

- **Transfer of Students Back to Grade 7 or 8 Mathematics**

One important consideration for schools planning to implement Algebra I in an Grade 8 program is determining how to place students who are unsuccessful in the program. If a student has difficulty with Algebra I in Grade 8, he or she will likely have even more difficulty in Geometry or Algebra II in high school and may not be able to pass. Teachers should watch carefully for students who appear to be improperly placed in Algebra I in Grade 8 and request that these students transfer to regular or honors Grade 8 mathematics. If the student transfers before receiving credit for Algebra I, he or she may take Algebra I in high school for credit.

- **Parent Communication**

Communication with parents is highly important for the success of the program. Parents should fully understand the purpose of the program, entry criteria, credit to be awarded, and expectations for high school. School personnel should emphasize that placement in the program depends on what would be best for the student. Sometimes it is wiser for students not to enter such a program to better ensure a sound pre-algebra background for their success in Algebra I in high school. Parent communication can be accomplished through meetings and/or letters. Also, the school can develop a fact sheet describing the program and student expectations. Parents could be asked to sign a statement to indicate their understanding and support for the program.

Algebra I Over Two Years

Rationale

With the phaseout of low-level high school courses in mathematics, numerous school districts are offering Algebra I over a two-year period. Such a program has the potential of being successful. However, effective programs involve much more than teaching the Algebra I content at a slower pace.

Issues

School districts that offer Algebra I over two years should consider the following important issues:

- **Instructional Strategies**

  Instruction in a two-year Algebra I program should be activity oriented, incorporating the use of hands-on materials and models whenever possible. Also, real-world applications help students understand the importance of the content and help motivate them in their studies. Graphing calculators, computers, and interactive software are helpful for all students, especially those requiring extra assistance. A variety of resource materials to supplement the Algebra I textbook should be available in the classroom to help students remediate skill deficiencies.

  Teachers may wish to incorporate a variety of instructional approaches in the two-year Algebra I program, as in any classroom. Cooperative learning could be a helpful learning technique for the students in this program. A regular time for tutorial assistance for students needing additional help may be necessary.

- **Entry Criteria**

  Multiple entry criteria, including prior student achievement, test results, teacher recommendation, and student and parent input, should be used to place students in a two-year Algebra I program. Students who could function well in a regular algebra program should not be placed in the two-year program.

as the pace may be too slow to hold their interest. Also, if students devote two years to Algebra I, they will have less opportunity to take higher mathematics courses.

- **Credit**

State Board Rules list Algebra I as a one-unit course. However, the Commissioner of Education may approve waivers for districts requesting to give mathematics credit for each year of a two-year Algebra I program. Details of how these courses are recorded on the Academic Achievement Record (transcript) and how enrollments are reported through the Public Education Information Management System's (PEIMS) data standards are available from the Mathematics Unit, Division of Curriculum Development, Texas Education Agency.

- **Next Course**

Students who complete a two-year Algebra I program may go on to any mathematics course that has Algebra I as a recommended prerequisite. Normally, students follow Algebra I with Geometry, Informal Geometry, or Algebra II.

Essential Elements of Instruction, Objectives, and Activities

Essential Elements for Algebra I

Algebra I (1 unit). Algebra I shall include the following essential elements:

(1) Comparison of the real number system and its various subsystems in terms of structural characteristics including operations. The student shall be provided opportunities to:

(A) classify real numbers as members of the appropriate subset of real numbers;

(B) identify and use properties of the real numbers;

(C) investigate the density property of real numbers: that between every two real numbers there exists another real number;

(D) investigate the order of operations;

(E) evaluate monomials with integral exponents; and

(F) compare algebraic and geometric definitions of absolute value.

(2) Algebraic representation, solution, and evaluation of problem situations. The student shall be provided opportunities to:

(A) write and evaluate linear expressions from verbal descriptions;

(B) use the properties of equality or models to explain and justify the equation-solving process;

(C) determine the solution to problem situations by writing and solving linear one-variable equations and inequalities;

(D) make a convincing informal argument, orally or in writing, justifying the solution to a problem situation;

(E) solve literal equations for a specified linear variable;

(F) solve systems of equations using linear combinations and substitution as appropriate;

(G) use systems of equations in applications and problem-solving situations; and

(H) solve absolute value equations and inequalities.

(3) Graphing as a tool to interpret linear relations, functions, and inequalities. The student shall be provided opportunities to:

(A) investigate and compare the properties of relations and functions;

(B) describe the domains and ranges of various functions and relations;

(C) identify the relationships among a linear equation, a set of ordered pairs of numbers, and a set of points on a coordinate plane;

(D) explore the concepts of slope and intercept by changing the parameters of a linear equation;

(E) graph a line given characteristics such as two points, one point and slope, table, etc.;

(F) graph a line from its equation in point-slope, general, slope-intercept, or nonstandard forms;

(G) design a statistical experiment to study a problem, recording the results using techniques such as scatter plots, and communicating the outcomes;

(H) write an equation of a line given its graph or description;

(I) use linear equations as models of real-world problem situations;

(J) make predictions from scatter plots that fit linear models;

(K) solve systems of linear equations;

(L) graph linear inequalities with two variables;

(M) graph systems of inequalities; and

(N) explore the relationship between the graph of an absolute value function such as $y = |Ax + B| + C$ and the parameters A, B, and C, using computer graphing techniques.

(4) Quadratic equations. The student shall be provided opportunities to:

(A) evaluate quadratic functions for one and for many values of the variable, using a computer or calculator where appropriate;

(B) explore the effects of simple parameter changes on the graphs of quadratic relations, using computer graphing techniques where appropriate;

(C) obtain decimal approximations for the solutions of quadratic equations, using the quadratic formula and a calculator; and

(D) use quadratic equations to make predictions in problem situations.

(5) Polynomials. The student shall be provided opportunities to:

(A) use the definition of polynomial to distinguish between expressions that are polynomials and expressions that are not;

(B) classify polynomials by degree and number of terms;

(C) add, subtract, multiply, and divide polynomials, using concrete models where appropriate;

(D) apply the laws of exponents to include zero and negative integral exponents; and

(E) factor simple polynomials using concrete models where appropriate.

(6) Rational expressions. The student shall be provided opportunities to:

(A) evaluate rational expressions, avoiding division by zero;

(B) apply operations on simple rational expressions (linear or monomial numerators and denominators only);

(C) solve rational equations with linear numerators and denominators;

(D) solve problem situations using ratio and proportion;

(E) use the definition of probability as a ratio of numbers of outcomes to solve problems involving uncertainty;

(F) apply the concept of dimensional analysis (carrying units throughout a computation) in problem situations, to determine appropriate units for denominate numbers; and

(G) perform operations on numbers in scientific notation, both mentally and by calculator, and use these numbers in problem situations.

(7) Properties of and operations with square roots. The student shall be provided opportunities to:

(A) use the calculator to approximate numeric radical expressions involving square roots;

(B) simplify algebraic radical expressions involving square roots;

(C) add, subtract, multiply, and divide numeric and algebraic radical expressions involving square roots;

(D) solve simple radical equations involving square roots; and

(E) use the Pythagorean Theorem in problem situations.

Sample Objectives for Algebra I

District educators should develop or adapt objectives that will ensure that all the required essential elements of Algebra I are taught. The objectives should clarify the essential elements and describe expected student outcomes.

Fifty sample objectives for Algebra I are listed on pages 15-18. (These objectives are not mandated but are intended only to show examples of types of objectives teachers may use for each essential element.) District curriculum planners may use the sample objectives in the development of their own course objectives. The sample objectives are arranged in a logical sequence of instruction, and correlations to the essential elements are noted in brackets. To the left of many of the objectives is a star (☆), which indicates that an activity appropriate for that objective is included among the activities on pages 18-44. Although certain objectives are especially appropriate for using calculators or computers, the teaching and learning of virtually every objective can be enhanced through the use of technology. Finally, objectives marked [H] are appropriate for an honors curriculum.

Essential Element 1

Comparison of the real number system and its various subsystems in terms of structural characteristics including operations.

OBJECTIVES. The student will:

1. distinguish between rational and irrational numbers [EE 1A]

2. apply the density property of real numbers by approximating irrational numbers [EE 1B]

3. form equivalent expressions using:
   - field properties [EE 1B]
   - the definition of integral exponents [EE 1E]
   - the order of operations [EE 1D]

4. define absolute value algebraically and geometrically [EE 1F]

Essential Element 2

Algebraic representation, solution, and evaluation of problem situations.

OBJECTIVES. The student will:

5. write and evaluate linear expressions from verbal descriptions [EE 2A]

☆ 6. use the properties of equality or models to explain and justify the equation-solving process [EE 2B]

7. solve literal equations for a specified linear variable [EE 2E]

8. solve systems of linear equations:
   - linear combinations
   - substitution
   - graphing

   [EE 2F, EE 3K]

9. solve absolute value equations and inequalities involving linear expressions [EE 2H]

☆ 10. determine solutions to problem situations by using appropriate algebraic models (write the equation or inequality) that involve linear equations, linear systems, or linear inequalities [EE 2C, EE 2G]

☆ 11. justify solutions to problem situations by making convincing informal arguments, orally or in writing [EE 2D]

Essential Element 3

Graphing as a tool to interpret linear relations, functions, and inequalities.

OBJECTIVES. The student will:

12. determine if a relation is a function [EE 3A]

13. describe the domains and ranges of functions and relations [EE 3B]

☆ 14. identify the relationships among a linear equation, a set of ordered pairs of numbers, and a set of points on a coordinate plane [EE 3C]

15. explain the changes to the graph of $y = mx + b$ as the values of m and b vary [EE 3D]

☆ 16. graph lines given two conditions or their equations [EE 3E, EE 3F]

17. write the equations of lines given two conditions or their graphs [EE 3H]

☆ 18. draw linear graphs to represent real-world situations and determine the meaning of the slopes and intercepts [EE 3I]

19. solve systems of linear equations by graphing [EE 3K]

20. graph linear inequalities in two variables [EE 3L]

21. graph systems of linear inequalities in two variables [EE 3M]

22. graph $y = | mx + b |$ and compare it to the graph of $y = mx + b$ [EE 3N]

☆ 23. explain the changes to the graph of $y = | Ax + B | + C$ as the values of A, B, and C vary [EE 3N]

☆ 24. gather data, make scatter plots, and estimate lines of best fit for given experiments [EE 3G, EE 3J]

25. make predictions and communicate interpretations of graphically displayed data that fit linear models [EE 3G, EE 3J]

Essential Element 4

Quadratic equations.

OBJECTIVES. The student will:

26. make tables of values for the quadratic relations $y = Ax^2 + Bx + C$ and $x = Ay^2 + By + C$ [EE 4A]

☆ 27. explain the changes to the graphs of $y = Ax^2 + Bx + C$ and $x = Ay^2 + By + C$ as A and C vary [EE 4B]

28. determine decimal approximations of the solutions of quadratic equations using the quadratic formula and a calculator [EE 4C]

☆ 29. make predictions in problem situations involving tables of values, graphs, and solutions of quadratic equations [EE 4D]

[H] 30. solve quadratic equations by factoring, completing the square, or using the quadratic formula

[H] 31. solve quadratic inequalities in one variable

Essential Element 5

Polynomials.

OBJECTIVES. The student will:

32. distinguish between expressions that are polynomials and expressions that are not [EE 5A]

33. classify polynomials by degree and number of terms [EE 5B]

☆ 34. add, subtract, multiply, and divide polynomials [EE 5C]

☆ 35. form equivalent expressions using the laws of exponents (including all integral exponents) [EE 5D]

☆ 36. factor simple polynomials [EE 5E]

Essential Element 6

Rational expressions.

OBJECTIVES. The student will:

37. evaluate rational expressions and determine excluded values [EE 6A]

38. form equivalent rational expressions using addition, subtraction, multiplication, and division [EE 6B]

☆ 39. solve rational equations with linear numerators and denominators [EE 6C]

40. determine solutions to problem situations using ratio and proportion [EE 6D]

☆ 41. solve problems involving probability as a ratio of desired outcomes to possible outcomes [EE 6E]

☆ 42. determine appropriate units of measure in problem situations involving denominate numbers by carrying units throughout the computation [EE 6F]

☆ 43. solve problems involving scientific notation [EE 6G]

[H] 44. solve rational equations involving higher degree polynomials in the numerators and denominators

Essential Element 7
Properties of and operations with square roots.

OBJECTIVES. The student will:

45. approximate square roots using a calculator [EE 7A]

46. perform simple operations on expressions involving square roots [EE 7B, EE 7C]

47. solve simple radical equations involving square roots (solving requires only squaring once) [EE 7D]

☆ 48. determine solutions to problem situations using the Pythagorean Theorem [EE 7E]

[H] 49. solve complex radical equations involving square roots (solving requires squaring more than once)

[H] 50. find the distance between two points using the distance formula

Sample Activities for Algebra I

The 12 sample activities on pages 18-44 illustrate how teachers may address different topic areas by having students perform prescribed tasks. The activities are designed to illustrate methods of teaching and to help clarify what is meant by some of the essential elements. The compilation is not intended to be a comprehensive list of activities to teach the essential elements nor are these activities required to be used. However, the activities demonstrate the kind of work that students should be required to do in Algebra I. The activities relate to some of the sample objectives on pages 15-18.

Activity 1	Solving Equations Using Models

Summary ►
Students solve simple linear equations using models.

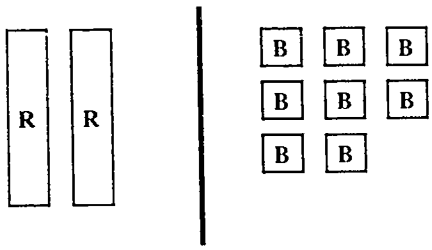
Objective 6 ►
The student will use the properties of equality or models to justify the equation-solving process.

Essential Element ► 2B

Materials ►
Algebra tiles

Concrete Example

Symbolic Recording



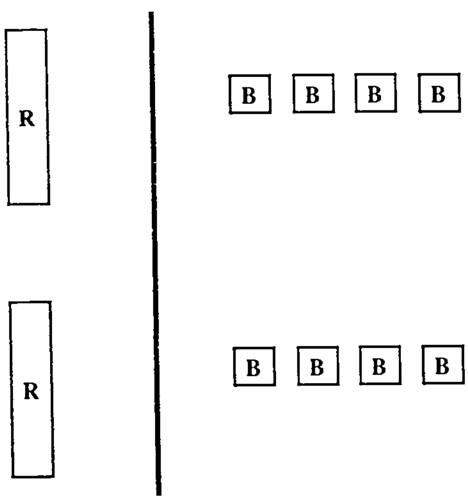
$2R = {}^{-}8$

Figure 3

Now separate the remaining pieces into two equal groups. Thus 1R is equal to $^{-}4$.

Concrete Example

Symbolic Reading



$R = {}^{-}4$

Figure 4

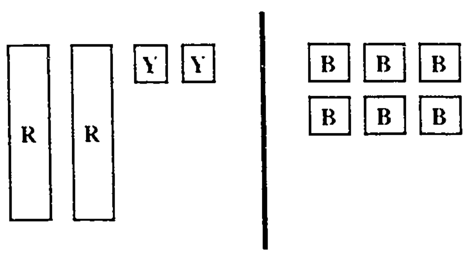
Repeat this process for several more equations being sure to cover all possible combinations of positive and negative numbers. Negative variables can also be covered. If manipulatives are not available to all students, this process can be demonstrated by the teacher using the overhead projector.

Procedure ►

Have students solve the equation $2R + {}^{+}2 = {}^{-}6$. Draw a line on the overhead and have students draw a line down the center of a piece of notebook paper. This line will serve as the equal sign in the equation. The algebra tiles you and the students use in this activity will be rectangles and small squares. Use black pieces to represent negatives. (They will be indicated in the following figures by a B within the squares.)

Place two red rectangles and two yellow squares on the left side of the line you have drawn. The rectangles will represent the variable R, and the yellow squares will represent the positive units. Place six black squares on the right side of the line.

Concrete Example



Symbolic Recording

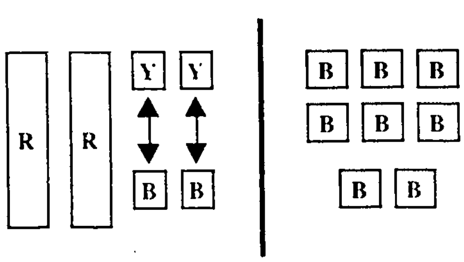
$2R + {}^{+}2 = {}^{-}6$

Figure 1

This model represents the equation $2R + {}^{+}2 = {}^{-}6$. Remind the students that to keep the equation balanced, they must do the same thing to both sides. The object is to discover what the value of R is; therefore, we want to isolate one of the rectangles.

Add ${}^{-}2$ to both sides of the equation.

Concrete Example



Symbolic Recording

$2R + {}^{+}2 = {}^{-}6$

${}^{-}2 = {}^{-}2$

Figure 2

On the left side of the equation the two yellow squares and the two black squares create two zero pairs and thus can be eliminated from the equation.

Activity 2	Convince Me

Summary ►
Students justify solutions to problems.

Objective 11 ►
The student will justify solutions to problem situations by making convincing informal arguments, orally or in writing.

Essential Element ► 2D

Materials ►
None

Procedures ►

A. If n and $n + 2$ represent even integers, could their sum be 65? Why or why not?

B. $y = mx + b$ represents a class of lines when graphed in a coordinate plane. Are horizontal lines included in this class? Why or why not?

C. $y = kx$ and $\frac{x_1}{x_2} = \frac{y_1}{y_2}$ illustrate direct variation.

Does $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ also illustrate direct variation?

Justify your response.

D. Which measure of central tendency (mean, median, or mode) is least descriptive of the data in the following sets (justify your response):

1. 1, 1, 3, 5, 9
2. 1, 2, 3, 4, 5
3. 1, 1, 1, 1

Activity 3	Points, Ordered Pairs, and Linear Graphs

Summary ►
The information necessary to graph a line (and any other curve as well) can be represented by a table of values (points), a verbal description, or an equation. In this activity, students explore these representations. The activity can supplement the usual exercises that involve using a table of values or an equation to determine the graph.

Objective 14 ►
The student will identify the relationships among a linear equation, a set of ordered pairs, and a set of points on a coordinate plane.

Essential Element ► 3C

Materials ►
Coordinate graph paper, straightedge

Procedures ►

A. From words to points and graphs. Draw a straight line using the indicated information:

1. It goes through the origin and it goes through $(-4, 6)$.
2. It is parallel to the x-axis, and the y-coordinate is always 4.
3. It slopes from left to right at a 45° angle and it passes through $(0, -2)$.
4. It slopes down from left to right, and the sum of the coordinates is 8.

B. From points to words and graphs. Plot the points and sketch the graph. Describe the graph in words. Find three more points on the graph.

1. $(1,2)$, $(5,6)$, $(-1,0)$, $(0,1)$, $(3,4)$
2. $(5,-4)$, $(5,3)$, $(5,0)$, $(5,-9)$, $(5,6)$
3. $(8,0)$, $(7,1)$, $(6,2)$, $(4,4)$, $(-2,10)$

C. From graphs to words and points. Describe the graph using words. Write the coordinates of five points that lie on the graph. Find a pattern if you can.

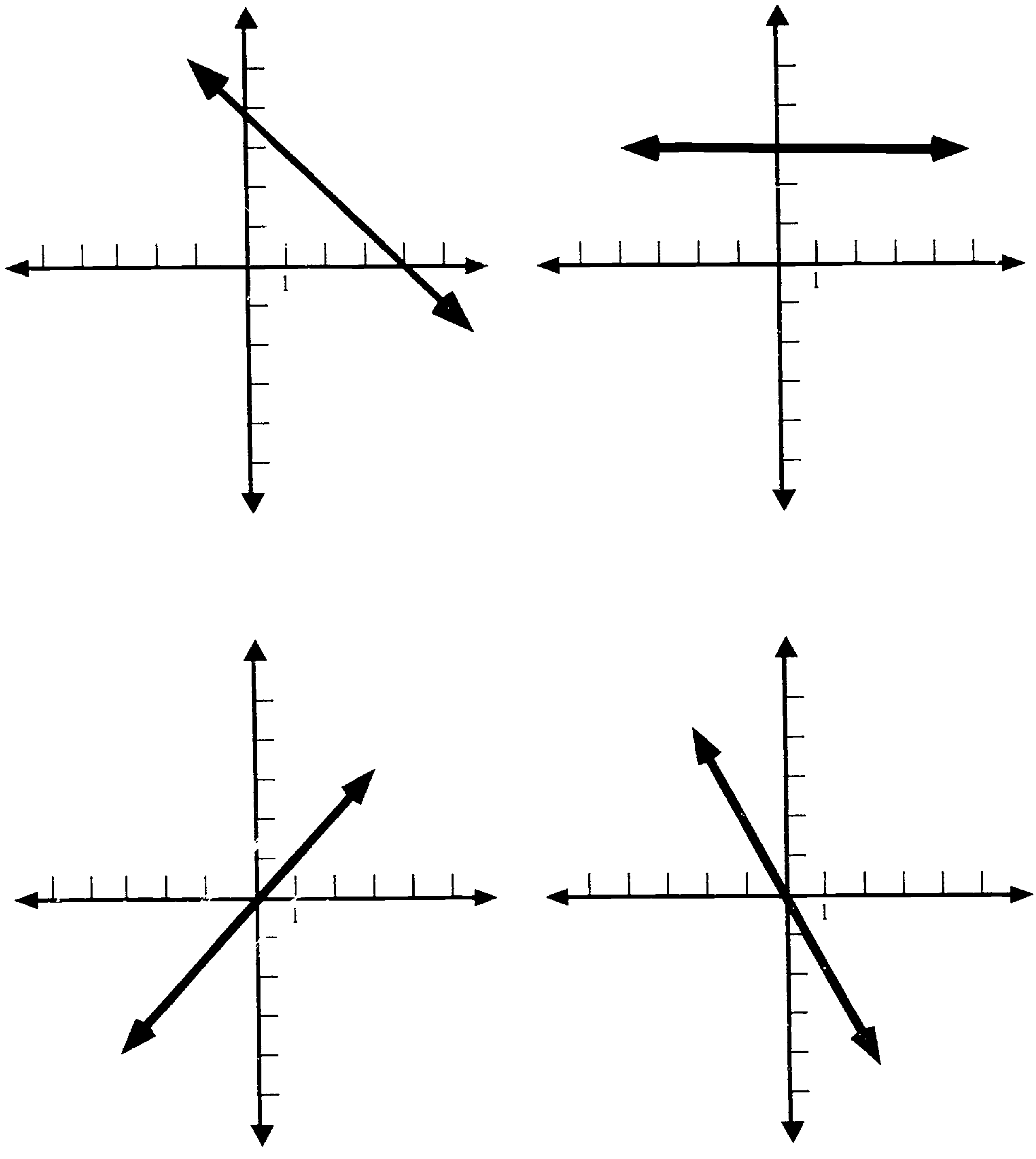


Figure 5

Activity 4	Linear Models: Tables, Graphs, Slopes, Intercepts

Summary ►

A real-world situation can be represented by a table, a graph, or an algebraic equation (function). For each type of representation for a situation that can be modeled by a linear equation, students answer related questions, stating the meaning of the slope and intercepts of the linear graph.

Objective 18 ►

The student will draw linear graphs to represent real-world situations and to determine the meaning of the slopes and intercepts.

Essential Element ► 31

Materials ►

Coordinate graph paper, straightedge

Procedures ►

A. Give the situation as an equation and ask for a graph and other information.

The daily car rental charges for Ajax Rental are given by $C = 30 + .15(m - 100)$, where m is the number of miles driven.

1. What is the cost of driving 250 miles?
2. If the charge for one day was $75, how many miles were driven?
3. Sketch the graph of this equation.
4. What is the y-intercept of the graph? What is its significance?
5. What is the interpretation of the .15?
6. Describe in words how the equation could have been determined.

B. Give the situation as a table and ask for a graph and other information.

The table gives the population of Suburbia at the end of some of the last 12 years.

Year:	1980	1981	1982	1985	1987	1988	1990
Population:	50,000	55,000	60,000	75,000	85,000	90,000	100,000

1. Make a graph to illustrate these data.
2. Find a model (an equation) to describe the data using x as the number of years after 1980 and P as the population.
3. What will be the population in 2000?
4. What is the significance of the coefficient of x?
5. At what points does the graph cross the y-axis? The x-axis? What is the significance of these points?
6. When will the population reach 140,000?

C. Give the situation as a graph and ask for interpretations of the slope and intercepts and other information.

The Ace Widget Company displays its cost and revenue from producing widgets on the following graph. (Make a transparency of the graph. Press the ENLARGE button on your copier several times if you have one.)

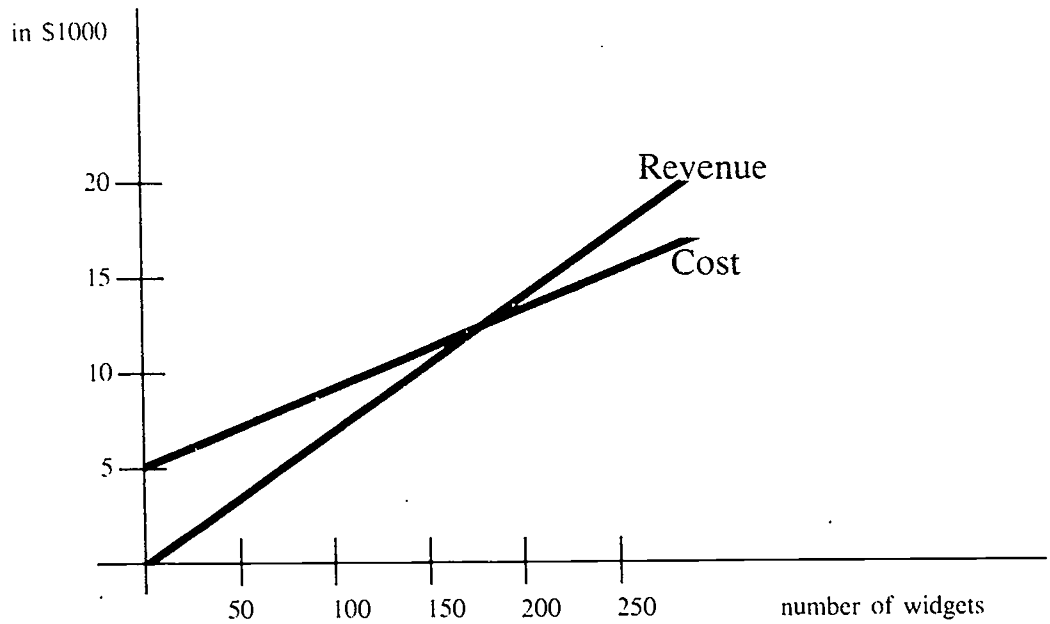


Figure 6

1. What are the start-up costs for producing widgets?
2. What is the approximate revenue for selling 100 widgets?
3. What is the approximate cost of producing 100 widgets?
4. What is the approximate profit or loss for selling 100 widgets?
5. What is the approximate profit or loss of selling 200 widgets?
6. Approximately how much does it cost to produce one widget?
7. Approximately how much revenue is received from the sale of one widget?
8. What is the interpretation of the slope of the Cost line?
9. What is the interpretation of the slope of the Revenue line?
10. What is the interpretation of the y-intercept of the Cost line?
11. Approximately how many widgets must be sold to make a profit of $2500?

Activity 5	Using the Graphing Calculator or Computer

Summary ►

Students use an inquiry approach to determine the effects that various parameters have on the graphs of relations and functions. A scientific approach is used, which consists of identifying the problem, planning the investigation, carrying out the investigation, analyzing the data, developing the conclusion, and testing the conclusion.

Objective 23 ►

The student will explain the changes to the graph of $y = |Ax + B| + C$ as the values of A, B, and C vary.

Objective 27 ►

The student will explain the changes to the graphs of $y = Ax^2 + Bx + C$ and $x = Ay^2 + By + C$ as A and C vary.

Essential Elements ► 3N, 4B

Materials ►

Graphing calculators, computers with graphing software

Procedure ►

Focus. When investigating the effects that parameters have on the graphs of relations and functions or to make any investigation, a systematic, planned approach must be taken. Many students will change values of parameters in a seemingly random fashion with no organization of thought. Unless they receive guidance and modeling in choosing an appropriate approach to investigations, many students will become frustrated with their lack of success.

Allow sufficient time for student investigations. Time spent in this activity should save time in later work since students will understand the process of investigations and will obtain a better understanding of graphs of functions and their parameters.

Investigations. Students should know the meaning of variable and parameter. In addition, they should have discussed these in relation to graphing a linear function.

In cooperative groups, have the students develop an organized approach to the investigation. Plans should include questions that need to be answered and reasons why the approach is suggested. Discuss each plan and develop a class plan to use in the investigations.

The plans for investigation might include the following for Objective 23:

- What parameters should remain constant for a specific investigation? What types of values should be used for the varying parameters? Discuss varying only one parameter at a time, keeping others constant.
- Ho many graphs should be on the same set of axes?
- What happens to the graph of $y = |Ax + B| + C$ when $B = 0$ and $C = 0$? Test the graph of $y = |Ax|$ as A varies. What is the effect of changing the value of A?

- What happens to the graph of $y = |Ax + B|$ when A varies and B remains constant? For example, what happens to $y = |Ax + 2|$ as A varies? Compare the conclusion to the above investigation.
- What happens to the graph when B changes? Let A remain constant and C = 0 (for example, $y = |2x + B|$). Test the graph and determine the effect of changing the value of B.
- What happens to the graph when C changes? Let A remain constant and B = 0 (for example, $y = |-3x| + C$). Test the graph and determine the effect of changing the value of C.
- What happens to the graph when C changes and A and B remain constant (for example, $y = |x - 2| + C$)?

After making their plans, students should carry out the investigations systematically by using a graphing calculator or computer graphing software. They should analyze their data and write their conclusions, discussing each parameter in detail (size, sign, etc.).

The final part of the investigation is to test the conclusions by predicting how the graphs will look for various equations such as:

$y = |2x - 1| - 3$ $\qquad$ $y = |2x - 1| + 3$

$y = |-5x - 1|$ $\qquad$ $y = |-5x + 5|$

$y = |5x + 1| + 4$ $\qquad$ $y = |3x + 1| - 4$

etc.

Students should then test their predictions by comparing them with graphs produced by the graphing calculator or computer. Ask students in each group to share the group's conclusions with the class.

As an extension or enrichment activity, determine what effect a coefficient of the absolute value, other than 1, has upon the graph $y = D - |Ax + B| + C$. What happens as D varies?

If students have made a thorough investigation of the absolute value function, they should be able to design and carry out subsequent investigations as independent groups. Students should always write conclusions in good form (as in a science laboratory investigation) and share them with the class.

The plans for investigation might include the following for Objective 27:

- What parameters should remain constant when determining the effects of the parameters A and C on the graph of $y = Ax^2 + Bx + C$? What types of values should be given each variable?
- How many graphs should be investigated on the same set of axes?
- What happens to the graph of $y = Ax^2$ as A varies; B = 0 and C = 0? What is the effect of A on the graph?
- What happens to the graph of $y = Ax^2 + Bx$ as A varies and B remains constant (for example, $y = Ax^2 + 2x$)? Compare the conclusion with the above investigation.
- What happens to the graph of $y = Ax^2 + Bx + C$ as A varies and B and C remain constant (for example, $y = Ax^2 + 2x + 1$)? Compare the conclusion with the above investigation.
- What happens to the graph of $y = Ax^2 + C$ as C varies and A remains constant? For example, let A = 1. What is the effect of C on the graph?
- What happens to the graph of $y = Ax^2 + Bx + C$ as A and B remain constant and C varies (for example $y = x^2 + x + C$)? Compare the conclusion with the above investigation.

Students should perform the investigation as previously indicated, analyze the data, write and test their conclusions with various quadratic functions, and share the results with the class.

Since graphing calculators and computer graphing software graph functions only, the investigation of $y = Ax^2 + Bx + C$ can be used to study the effects of parameter changes on the graph of $x = Ay^2 + By + C$ by rotating the axes.

Activity 6	Lines of Best Fit

Summary ►
Fitting an appropriate curve to a given set of data is an important aspect of mathematical modeling. Stu lents consider the simplest case where the given data is approximately linear.

Objective 24 ►
The student will gather data, make scatter plots, and estimate lines of best fit for given experiments.

Essential Element ► 3N

Materials ►
Coordinate graph paper, a piece of black thread, graphing calculator (optional)

Procedures ►

Ask students to refer to the column of the table on page 29 that gives the life expectancy for males since 1920.

A. Using the data for 1940 and 1980, find a linear model for the situation.

1. What is the interpretation of the slope and y-intercept of your model?
2. According to the model, what should the life expectancy for males have been in 1984? What was the percent error with the actual data?
3. According to the model, when will the life expectancy of a male reach age 75? Based on the data in this table, is this prediction reasonable? Discuss briefly.

B. Graph carefully the data for 1920, 1930, 1940, 1950, 1960, 1970, and 1980. Use a piece of dark thread to estimate the line of best fit and then find its equation. Answer the three questions in Procedure A using this linear model.

C. (Optional) Using a graphing calculator, enter the seven data points from Procedure B and then find the equation of the line of best fit.

Now answer the three questions using this linear model.

D. Formulate another problem or set of problems you can investigate using this table of data. Solve your problems using some or all of the methods above.

E. Gather your own set of data on a topic of interest to your class.

NOTE: Asking the students to do a paper-and-pencil version of this activity allows them to understand its purpose. It also forces them to decide on appropriate scales for the axes in their graphs—something they must do when setting the parameters while using a graphing calculator or computer graphing program.

Life Expectancy at Birth
1920 to 2080

Year	Total: All	Male	Female	White: All	Male	Female	Nonwhite: All	Male	Female
1920	54.1	53.6	54.6	54.9	54.4	55.6	45.3	45.5	45.2
1930	57.9	58.1	61.6	61.4	59.7	63.5	48.1	47.3	49.2
1940	62.9	60.8	65.2	64.2	62.1	66.6	53.1	51.5	54.9
1950	68.2	65.6	71.1	69.1	66.5	72.2	60.8	59.1	62.9
1960	69.7	66.6	73 1	70.6	67.4	74.1	63.6	61.1	66.3
1970	70.8	67.1	74.7	71.7	68.0	75.6	65.3	61.3	69.4
1980	73.7	70.0	77.4	74.4	70.7	78.1	69.5	65.3	73.6
1981	74.2	70.4	77.8	74.8	71.1	78.4	70.3	66.1	74.4
1982	74.5	70.9	78.1	75.1	71.5	78.7	71.0	66.8	75.0
1983	74.6	71.0	78.1	75.2	71.7	78.7	71.1	67.2	74.9
1984	74.7	71.2	78.2	75.3	71.8	78.8	71.3	67.3	75.2
1985*	74.7	71.2	78.2	75.3	71.8	78.7	71.2	67.2	75.2
1986*	74.9	71.3	78.3	75.4	72.0	78.9	71.4	67.6	75.1
Projections:							**Blacks Only****		
1990	n/a	71.6	79.2	n/a	72.4	79.7	n/a	66.3	75.4
2000	n/a	72.9	80.5	n/a	73.6	81.0	n/a	68.5	77.6
2020	n/a	74.2	82.0	n/a	74.7	82.3	n/a	71.0	79.9
2040	n/a	75.0	83.1	n/a	75.4	83.3	n/a	72.8	81.7
2060	n/a	75.9	84.1	n/a	76.1	84.2	n/a	74.8	83.4
2080	n/a	76.7	85.2	n/a	76.7	85.2	n/a	76.7	85.2

n/a = not available.

*Estimated.
**Data for other nonwhites are not available.

Sources: For 1920 to 1940, the National Center for Health Statistics (NCHS), as cited by U.S. Bureau of the Census, *Statistical Abstract of the United States 1987*, Table 105, 1986; for 1950 to 1986, NCHS, *Monthly Vital Statistics Report*, Vol. 35, No. 13, August 24, 1987; for 1990 to 2080, U.S. Bureau of the Census, "Projections of the Population of the United States, by Age, Sex, and Race: 1983 to 2080," *Current Population Reports*, Series P-25, No. 952, Table B-5, 1984.

Activity 7	Investigating the Graphs of Quadratic Equations

Summary ►

Students use a graphing utility (graphing calculator or computer) to investigate the changes in the shape and location of the graphs of $y = Ax^2 + Bx + C$, $A = 0$ and $B = 0$, and $x = Ay^2 + By + C$, $A = 0$ and $B = 0$, by varying the parameters A and C.

Objective 27 ►

The student will explain the change to the graphs of $y = Ax^2 + Bx + C$ and $x = Ay^2 + By^2 + C$ as A and C vary.

Essential Element ► 4B

Materials ►

Any one or any combination of the following will be needed for this activity:

- graphing calculators
- overhead projector graphing calculator
- computers
- software package that will graph several functions and/or relations on the same screen

Procedures ►

The following procedures are based on the assumption that the students will have had prior experience with graphing utilities and will be familiar with terms such as complete graph and viewing rectangle.

Suggestions—The procedures should be hands-on exercises for each student. If there are not enough graphing utilities for each student to work independently, arrange students in pairs. Having more than two students per graphing calculator will be a waste of time, while two or three students per computer works well. Also, the procedures work quite well as controlled discovery activities in which the teacher uses an overhead projector graphing calculator or a computer for demonstration purposes. The screen of the computer monitor will need to be projected, using a TV or PC projector, for best results.

This activity will require more than one class period. Be patient and thorough, taking plenty of time to allow students to make the observations. Do not worry that the class is getting behind. Understanding developed here will save time later.

A. The role of A

Since we want to focus on the parameter A, we will let both B and C equal zero. Thus, the quadratic function $y = Ax^2 + Bx + C$ becomes $y = Ax^2$. Now we can attribute any changes in the graph to the values assigned to A. To make any comments about how a parameter affects the graph, some curve must be used as a basis for comparison. In this case, $y = x^2$ is that curve.

1. Graph $y = x^2$ in a suitable viewing rectangle. Have students experiment with various range settings as they try to get a larger graph. Select a viewing rectangle suggested by a student or give them one such as $[-5, 5, 1]$ by $[-1, 10, 1]$.

After changing the range setting and regraphing the function, have students make observations about the graph of $y = x^2$. If they are reluctant, prompt with questions such as these:

a. What is the value of A?

b. Do you think the graph is complete? Explain.

c. Where is the lowest point on the graph? What is the name given to this point?

d. Do you see any other distinguishing characteristics? (Students should notice that the graph is symmetric with respect to the y-axis. If not, point it out and have them explain why.)

(Do not clear the screen.) Graph $y = 2x^2$ and observe, but do not comment.

(Do not clear the screen.) Graph $y = 3x^2$ and observe, but do not comment.

Ask the student what would happen if A were 10? Accept all conjectures before checking by graphing $y = 10x^2$. (Do not clear the screen.)

Students will now have the graphs of $y = x^2$, $y = 2x^2$, $y = 3x^2$, and $y = 10x^2$ on the same set of axes. List the students' observations as they consider the graphs. The list should include the following:

a. The graphs have the same vertices; i.e., the same low point.

b. The graphs are symmetric with respect to the y-axis.

c. The graphs open in a positive direction; i.e., upward.

d. The graphs are not congruent to the graph of $y = x^2$.

e. The graphs become steeper (narrower) as the value of A increases. (Keep this list for use later in the activity.)

2. So far we have considered values for A such that $A \geq 1$.

   Ask the students what results if $A < 1$ but is still positive; i.e., $0 < A < 1$? What will happen to the graph of $y = Ax^2$?

   (Clear the screen.) On the same set of axes, graph $y = x^2$ and two or three other $y = Ax^2$ graphs, using values of A suggested by the class. Have students make observations as before. List them.

   Before continuing, have the students summarize their observations; i.e., compare and contrast the graphs of $y = Ax^2$ when $A \geq 1$ and when $0 < A < 1$.

3. ASK:

   a. Can you think of any values we have not assigned to A?

   b. What effect do you suppose values for $A < 0$ will have on the graph of $y = Ax^2$?

   (Clear the screen.) Graph $y = x^2$ and $y = -x^2$ on the same set of axes. Because students are still using the original viewing rectangle, they will not have a good view of both graphs. Ask how the students would remedy the situation. An appropriate viewing rectangle would be one such as [−5, 5, 1] by [−10, 10, 1].

   Change the range settings and regraph. Have students state the effect of $A < 0$ on the graph and add this to the list of observations.

4. Have students look at the list of observations again. Point out that not everything listed is a result of changing the values of A. Have students identify those that are.

   Ask the students how they think the graph of $y = Ax^2 + Bx + C$ would look if x and y were interchanged? In other words, what do you think the graph of $x = Ay^2 + By + C$ would look like?

   Have students tell how A affects the graph when it is "lying on its side," when the x-axis is the axis of symmetry.

NOTE ►

- Most graphing calculators and most software packages graph functions only. Consequently, you will have to show the students how to "trick the machine" before they can explore the graph of the quadratic relation $x = Ay^2 + By + C$. Students will need to manipulate the equation to put it into function form. If $x = y^2$, then $y = \pm\sqrt{x}$. Now the graphing utility will accept $y = +\sqrt{x}$ and $y = -\sqrt{x}$ as separate entries. (Of course, students could do the investigation by constructing a table and plotting points.)
- Students will need to graph only a few equations to draw correct conclusions; for example, $x = y^2$, $x = 2y^2$, and $x = 0.5y^2$. Ask the same kinds of questions you asked when students were investigating the quadratic function $y = Ax^2$. List them.
- Students will need time to discover how to reflect the graph of $x = y^2$ across the y-axis. (Graph $y = \sqrt{x}$; Graph $y = - -\sqrt{x}$)

B. The Role of C

We have thoroughly examined the effects of A on the graphs of $y = Ax^2$ and $x = Ay^2$. If we let B = 0 in the equation $y = Ax^2 + Bx + C$ and $x = Ay^2 + By + C$, the result is $y = Ax^2 + C$ and $x = Ay^2 + C$. Thus by using the basic equation $y = x^2$ or $x = Ay^2$ and changing only the value of C, we will be able to discover how C changes the graph. Encourage students to make predictions before graphing any equations.

1. Graph $y = x^2$ in the standard viewing rectangle; i.e., [−10, 10, 1] by [−10, 10, 1].

   (Do not clear the screen.) Graph $y = x^2 + 2$ and observe, but do not comment.

   (Do not clear the screen.) Graph $y = x^2 - 2$ and observe, but do not comment.

2. Ask:

   a. How does C change the graph of $y = x^2$?

   b. What else can you say about C even if it does not change the location of the graph? (Students seldom see that C is the y-intercept; you may have to coax them for this information.)

   c. What else can you say about these graphs?

3. Ask the student how they think C will affect the graph of $x = y^2$? Accept the students' predictions, then check by graphing.

4. Summarize: Have students tell three things about C: it controls vertical movement, it is the y-intercept, it does not change the shape of the graph.

SAMPLE PROBLEM:
The following problem illustrates the above concepts:

Mike wants to enclose a portion of his yard for his dog. He plans to use 60 m of fencing for three sides of the rectangular pen and the house as a fourth side. Let x represent the length of the sides perpendicular to the house.

a. Write an algebraic representation of the area of the pen as a function of x.

b. Draw a complete graph of the algebraic representation. Specify its domain and range.

c. Which portion of the graph represents the problem situation? Specify the domain and range of the problem situation.

d. Use the graph to find x so that the greatest possible area will be enclosed. What is that maximum area?

e. Use the graph to determine the values of x if Mike constructs a pen with a maximum area of 300 m^2.

SOLUTION:

a. Draw a picture of the problem situation.

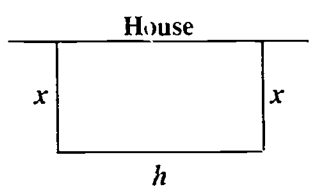


Figure 7

Let h represent the side of the pen parallel to the house. The perimeter can then be described by the equation $2x + h = 60$ and $h = 60 - 2x$. Thus, the area A as a function of x is given by $A(x) = x(60-2x)$. This equation is the algebraic representation of the problem situation.

b. Finding an appropriate viewing rectangle takes practice. To help students develop a feeling for where the graph lies, the teacher might ask some key questions: What one point do we know is on the graph? Will the graph have a maximum/minimum point? Do you think the vertex will lie to the right/left of or on the y-axis? These kinds of questions can guide students as they experiment with various viewing rectangles. Students can also build a table of values and observe some reasonable range settings for x and y.

$x(m)$	$A(m^2)$
−10	−800
0	0
10	400
20	400
30	0

Using this table, the student might guess that an appropriate viewing rectangle might be [−10, 40, 10] by [−100, 500, 100]. After graphing the equation, the student may need to adjust the range settings.

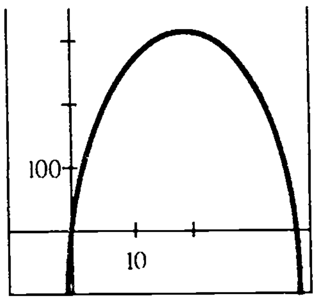


Figure 8

Domain: R

Range: $\{y : y \leq 450\}$

Use the trace key to find the domain and range.

c. The portion of the graph in the first quadrant represents the problem situation. (Measurements must be positive.)

Domain: $\{x : 0 < x < 30\}$

Range: $\{y : 0 < y \leq 450\}$

d. Use the trace key to locate the x- and y- values of the maximum point.

$x = 15$ m

$y = 450$ m^2, maximum area

e. Overlay the graph of $y = 300$. Use the trace key to find the points of intersection of $y = 2x^2 + 60x$ and $y = 300$.

Answer: $x = 5.96$ m and $x = 23.51$ m (no zoom-in)

5. Discuss accuracy with the students. Then teach them how to zoom-in to obtain the desired accuracy.

Suggestion: Divide the class into groups. Have each group write one or two problems with questions such as those asked in the sample problem. Have groups exchange problems.

Activity 8	**Constructing a Model: To Introduce the Concept of a Variable and Illustrate Multiplication of Binomials and Factoring**

Summary ►

Students construct a physical model that introduces the concept of a variable. The model also illustrates multiplication of binomials by use of an area model. Factoring is also included by using the same model and reversing the process used for multiplication.

Objective 34 ►
The student will add, subtract, multiply, and divide polynomials.

Objective 36 ►
The student will factor simple polynomials.

Essential Elements 5C, 5E

Materials ►
One packet per group of four students containing six to seven small squares (all one color), four to five large squares (all the same color but different from the small squares), and three extra 8½ × 11 sheets of a third color of construction paper; one pair of scissors per group; algebra tiles for overhead projection.

Procedures ►

A. Have the students explore combinations of rectangles.

1. Out of the large pieces of construction paper, each group of four cuts 12 rectangles whose widths are the same as the small squares and whose lengths are equal to the length of the large squares. After pieces are cut, have students try making as many different rectangles as they can, using:
   - 2 small squares
   - 2 large squares
   - 2 rectangles
   - 1 square, 1 rectangle
   - 3 rectangles
   - 4 rectangles

2. After this exploration, ask students how many different rectangles can be formed using at least one of every shape (1 small square, 1 large square, 1 rectangle) and no more than 12 pieces all together. Different rectangles are defined as rectangles of different dimensions, not different designs. Instruct students to find ways to record their creations so that they do not duplicate rectangles.

3. After a period of exploration, ask students, "Were any of your rectangles made with the same exact pieces?" "Is it possible to make different rectangles using the same pieces?"

4. Ask members of each group to recreate their favorite rectangle. Have students move around the room to view the rectangles made by other groups. Then ask, "Did one group make shapes that you don't think you can make?" Ask students to try to make the rectangle of the group next to them. Then have them try to make a group's rectangle that they do not think they can duplicate. Ask everyone to try to make that rectangle; then you construct it using algebra tiles or colored acetate squares and rectangles on the overhead projector.

5. Construct this rectangle on the overhead and ask all groups to duplicate it.

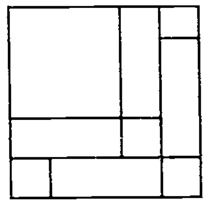


Figure 9

Be sure that everyone arrives at the conclusion that everyone can duplicate someone else's rectangle. Then ask if we can exchange the places of the small and large squares in Figure 10. That is, wherever you have a large square put a small one and wherever there are small squares put large ones. What happens?

(Some may notice the rectangles are turned a different way.)

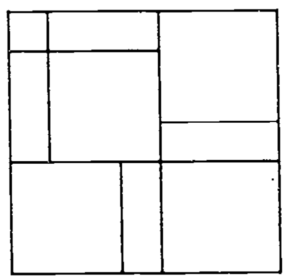


Figure 10

6. Ask the students if the above procedure will work with any rectangle? Create a rectangle using up to 12 pieces and the same rules as before. Try to exchange large and small squares. This cements the idea of a variable since each group has used different sized squares and rectangles, but they all arrive at the same conclusions. This will also help to validate the following activities even though you only use one set of squares and rectangles.

B. Have students use combinations of rectangles to illustrate factoring.

1. Ask students to make a rectangle for the equation, $3a^2 + 7ab + 2b^2 = (3a + b)(a + 2b)$, using three small squares, two large squares, and seven rectangles. Make this on the overhead while the students duplicate it at their desks.

2. Have the students rearrange small and large rectangles so that they fall into four quadrants.

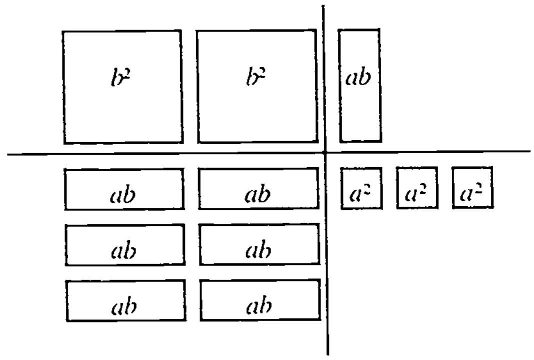


Figure 11

Tell students to attempt to make a rectangle by arranging large squares into Quadrant II (upper left) and small squares into Quadrant IV (lower right). Then they can try to fit the rectangles in (squares may be rearranged as long as at least one of the large squares and one of the small squares is touching the vertical axis). If this can't be done, the pieces cannot make a rectangle. Assign the worksheet "Possible/Impossible" on page 39. Ask: Which groups of pieces can be made into rectangles? Tell students to record their arrangements, if they are possible, and to write "impossible," if not, for numbers 1-10. After the groups of four students have completed their tasks, ask the following questions for feedback (check answers):

- Any discoveries?
- What is the area of this rectangle?

  Answer: $2b^2 + 7ab + 3a^2$
- What is the area of each piece?
- The large square has dimensions $b \times b$.

  Area $= b^2$.
- The small square has area $a \times a$.

  Area $= a^2$.
- Rectangles have dimensions $a \times b$ or area ab.
- The area of the large figure $= 2b^2 + 7ab + 3a^2$.

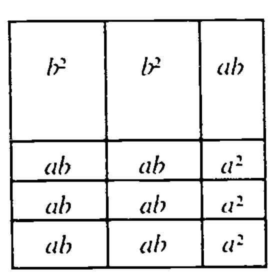


Figure 12

Have students find how each term is represented in the model.

What are the dimensions of the rectangle?

Answer: $(b + 3a) \times (2b + a)$

Connect the concrete representation to FOIL.

$(b + 3a)\ (2b + a)$

First $2b^2$ (two large squares)

Outside $\underline{ab}$ (rectangle above three small squares)

Inside $6ab$ (six rectangles below two large squares)

Last $\underline{3a^2}$ (three small squares)

3. Have students find the areas and dimensions of the rectangles in Numbers 1, 3, 4, 6, 7-10 of the "Possible/Impossible" worksheet. Ask students to compare answers with each other. Ask: Does anyone see any patterns or discoveries? Were any of the rectangles on the worksheet squares? Can you make a square using at least one of every piece? What would the dimensions be?

Example: $a^2 + 2ab + b^2\ (a+b)\ (a+b)$

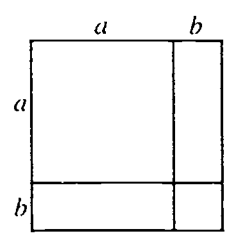


Figure 13

4. Relate the above procedures to the concept of perfect squares. Can you make other perfect squares?

Ask students if they could let the length of a side of a small square equal 1. Then the length of two small squares would equal 2. A rectangle of length x would then have the dimension $x \cdot 1$ (where x = large square's length or $1x$).

Example: $x^2 + 3x + 2$ = area

$(x + 2)(x + 1)$ = dimensions

Note the terms represented by the model in Figure 14:

$x^2 + 1x + 2x + 2$

$x^2 + 3x + 2$

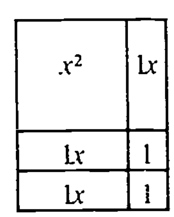


Figure 14

Possible/Impossible

A rectangle consisting of:

	Large Squares	Rectangles	Small Squares
1)	1	5	6
2)	2	5	6
3)	2	7	6
4)	3	5	2
5)	3	6	2

Complete the following to make rectangles:

	Large Squares	Rectangles	Small Squares
6)	3	7	2
7)	1	____	3
8)	1	____	4
9)	2	3	____
10)	2	5	____

Activity 9	Laws of Exponents

Summary ►
Students use calculators to discover the laws of exponents.

Objective 35 ►
The student will form equivalent expressions using laws of exponents (including all integral exponents).

Essential Element ► 5 D

Materials ►
Scientific calculator/Explorer calculator

Procedures ►

A. Have students calculate $8^3 \cdot 8^2$ and arrive at 32.768.

$\boxed{8}\ \boxed{y^x}\ \boxed{3}\ \boxed{\times}\ \boxed{8}\ \boxed{y^x}\ \boxed{2}\ \boxed{=}$ 32.768

Then have them divide 32.768 by 8 until the quotient is 1. They must keep up with how many times it took. Ask students how many times 8 must be used as a factor to yield a product of 32.768 and lead them to write the equivalent expressions of $8^5 = 32.768$.

Do other examples such as the following until students are able to generalize the rule

$x^n \cdot x^n = x^{n+n}$ $\qquad 9^2 \cdot 9^1 \qquad 2^3 \cdot 2^4 \qquad 7^1 \cdot 7^0$.

Ask students if this rule would apply to $6^{-3} \cdot 6^3$?

Have students enter $\boxed{6}\ \boxed{y^x}\ \boxed{3}\ \boxed{=}\ \boxed{\frac{1}{x}}\ \boxed{\times}\ \boxed{6}\ \boxed{y^x}\ \boxed{3}\ \boxed{=}\ \boxed{1}$

Lead students to see that $6^{-3} \cdot 6^3 = 6^0 = 1$.

B. Ask students what they think would happen if 6^{-3} were multiplied by 6^1.

Have students enter $\boxed{6}\ \boxed{y^x}\ \boxed{3}\ \boxed{=}\ \boxed{\frac{1}{x}}\ \boxed{\times}\ \boxed{6}\ \boxed{y^x}\ \boxed{1}\ \boxed{=}$ .027778

Have students discuss if they could discover ways to prove that $6^{-3} \cdot 6^1$ follows the rule of $x^n \cdot x^n = x^{n+n}$.

Two ways this could be done are the following:

- List the factors then use the calculator to arrive at the decimal equivalent.

$\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot 6 = \frac{1}{36} \qquad \frac{1}{36} = .02778$

- Assume that $6^{-3} \cdot 6^{1} = 6^{-2}$; using keystrokes

$\boxed{6}$ $\boxed{y^x}$ $\boxed{2}$ $\boxed{=}$ $\boxed{\frac{1}{x}}$ arrive at .027778.

Then have students prove that $5^{-3} \cdot 5^{-2}$ follows the rule also.

To arrive at the generalization $\frac{x^n}{x^n} = x^{n-n}$, follow the same procedure using the examples to arrive at the rule.

$\frac{6^5}{6^2}$ $\boxed{6}$ $\boxed{y^x}$ $\boxed{5}$ $\boxed{-}$ $\boxed{6}$ $\boxed{y^x}$ $\boxed{2}$ $\boxed{=}$ 216 $\quad$ $216 = 6^3$

$\frac{3^5}{3^3}$ $\quad$ $\frac{5^6}{5^1}$ $\quad$ $\frac{8^1}{8^0}$

After the generalization has been reached, investigate what happens if the exponents are negative. Again have students discover ways to prove that these also follow the rule they have generalized.

Activity 10 — Dimensional Analysis

Summary ►
Students work problems involving different units of measure by carrying the units throughout the computation. This helps students arrive at the appropriate units of measure in their answers.

Objective 42 ►
The student will determine appropriate units of measure in problem situations involving denominate numbers by carrying units throughout the computation.

Essential Element ► 6F

Materials ►
None

Procedures ►
Ask students what units of measure are used to measure area? (square units)

What kind of units are used to measure volume? (cubic units)

The correct units of measure can easily be found by working problems with the unit measure included in them. Demonstrate with some examples.

How much carpeting will be needed to cover the floor of a room 12 feet long and 9 feet wide?

$A = lw$

$A = (12 \text{ ft})(9 \text{ ft})$

$= (12 \cdot 9)(\text{ft} \quad \text{ft})$

$= 108 \text{ ft}^2$

How much soil can be placed in a dump truck that is 5 meters long, 3 meters wide, and 2 meters deep?

Solution ►

$V = lwh$

$V = 5\ m \cdot 3\ m \cdot 2\ m$

$V = 30\ m^3$

This technique can be used for units of measure that may not be so easily remembered. The formula $d = (\frac{b^2}{2})(s)(n)$ relates the displacement of an automobile engine, d, to the diameter of the engine's bore, b; the engine's stoke, s; and the number of cylinders in the engine, n. What is the displacement of an engine with a bore of 8 cm, a stoke of 9 cm, and 6 cylinders.

Solution ►

$d = (\frac{b^2}{2})(s)(n)$

$= (\frac{8\ cm^2}{2})(9\ cm\ (6)$

$= (4\ cm)^2\ (9\ cm)(6)$

$= (16)(9)(6)(cm^2)(cm)$

$= 864\ cm^3$

You can easily show that the displacement is measured in cubic centimeters by keeping the unit measures in this problem.

The formula $s = \frac{1}{2}gt^2$ relates the distance, s, that a falling object travels; the acceleration due to gravity, g; and the length of time, t, that the object falls. What would be the rate of acceleration of an object that has traveled 396 meters in 9 seconds?

Solution ►

$s = \frac{1}{2}gt^2$

$397\ m = \frac{1}{2}g(9\ sec)^2$

$794\ m = g(81\ sec^2)$

$\frac{794\ m}{81\ sec^2} = g$

$9.8\frac{m}{sec^2} = g$

This technique demonstrates that acceleration in this problem is measured in meters per second squared.

Emphasize to students that including the measurement units throughout the solution of a problem will help them arrive at the proper units of measure in their answers.

Activity 11	Using Scientific Notation

Summary ►

Students are introduced to the use of scientific notation and work problems that illustrate the need for scientific notation.

Objective 43 ►

The student will solve problems involving scientific notation.

Essential Element ► 6G

Materials ►

Scientific calculator

Procedures ►

Scientists often encounter very large and very small numbers which they express in scientific notation. In addition, large numbers are given in newspapers in scientific notation such as $1.2 billion. Thus students have studied and used scientific notation in other disciplines and in previous mathematics classes. One objective of Algebra I is to have students understand that the mathematics studied in algebra is the same as they use in science and other disciplines.

Computers and calculators are limited in the number of digits in a display or computation. To use these tools, students must be able to convert to scientific notation and operate with these types of numbers. For example, using a calculator to multiply 0.00000654 by 0.00000000567, the student must use scientific notation.

Assign the students the following problems:

I. Compute the following, using scientific notation and the laws of exponents:

1. $\dfrac{(8.76 \times 10^{25})}{(1.8 \times 10^{-6})} \; \dfrac{(1.2 \times 10^{-8})}{(7.3 \times 10^{-4})}$

   Write your answer in scientific notation and standard notation.

   Use a calculator for computation.

   Answer: 8.0×10^{26} or 800,000,000,000,000,000,000,000,000

2. Which is larger? By how much?

   5.4×10^{62} or 6.2×10^{61}

   To perform the operation of addition or subtraction without a calculator that uses scientific notation, use the distributive property to simplify the expression.

   $5.4 \times 10^{62} - 6.2 \times 10^{61} = 10^{61}\,(5.4 \times 10 - 6.2) = 10^{61}(47.8) = 4.78 \times 10^{62}$

II. Use a calculator when appropriate and scientific notation to solve the following:

1. A rectangular object 3 *cm* $\times$ 4 *cm* $\times$ 6 *cm* is made of tiny cubes, each 10^{-2} *cm* on a side. How many cubes does the object contain?
2. The speed of light is approximately 300,000 *km* per second or 186,000 miles per second.
   a. If a beam of light from earth reaches the sun in 8.3 minutes, how many kilometers is the earth from the sun?
   b. If a light year is the distance light travels in a year, approximately what is one light year expressed in miles?
3. A cube has a side of 9.1×10^{-7} *cm*. What is the volume of the cube?
4. How many oleic acid molecules occupy 0.5 cm^3 if the volume of one oleic acid molecule is 10^{-2} cm^3. Assume that the molecules are cubical in shape and that there is no empty space between them.
5. The volume of an atom equals the volume of the sample divided by the number of atoms in the sample. From an experiment, the volume of a sample of polonium is

   $$\frac{3.9 \times 10^{-4} g}{9.3\ g/cm^3}.$$

   The number of atoms in the sample is 1.3×10^{18}. Find the volume of one atom of polonium.

Activity 12	Applying the Pythagorean Theorem

Summary ►
Students work a problem that calls for an application of the Pythagorean Theorem. They consider what other factors might influence a final solution.

Objective 48 ►
The student will determine solutions to problem situations using the Pythagorean Theorem.

Essential Element ► 7E

Materials ►
None

Procedure ►
A student was sent to buy a piece of plywood but was not told exactly what size to buy. The directions were to buy the largest piece possible that could fit in the back of a van with the door closed. The van opens from the back and the opening is 44″ high by 60″ wide. The opening has a depth of 72″. What would be the dimensions of the piece that a student would buy, and what factors other than just calculations would enter into the choice? Write a short paper to justify the student's purchase.

ALGEBRA II

Essential Elements of Instruction, Objectives, and Activities

Essential Elements for Algebra II

Algebra II (1 unit). Algebra II shall include the following essential elements:

(1) Development of mathematical structure. The student shall be provided opportunities to:

(A) compare and contrast the real number system and its various subsystems in terms of structural characteristics;

(B) investigate examples and nonexamples of fields using the real number system and its various finite and infinite subsystems; and

(C) develop the complex number system and its operations.

(2) Quadratic functions. The student shall be provided opportunities to:

(A) solve quadratic equations by completing the square;

(B) develop and apply the quadratic formula;

(C) find a quadratic equation given its roots;

(D) explore the effects of simple parameter changes on the graph of a quadratic function, using computer graphing techniques where appropriate;

(E) use characteristics of a quadratic function to sketch the related curve;

(F) determine the equation of quadratic functions from their graphs; and

(G) use quadratic functions as models in real-world problem situations.

(3) Quadratic relations. The student shall be provided opportunities to:

(A) explore the graphs of algebraic representations of conic sections and make generalizations that allow classification of these algebraic representations as circles, ellipses, hyperbolas, or parabolas, using calculators or computers where appropriate;

(B) verify graphs of conic sections using computer graphing techniques where appropriate;

(C) use characteristics of conic sections to sketch the related curves;

(D) determine equations of conic sections from their graphs; and

(E) use quadratic relations as models in real-world problem situations.

(4) Systems of equations. The student shall be provided opportunities to:

(A) use the linear combination (addition-subtraction) method to solve systems of three linear equations in three variables;

(B) use augmented matrices by hand or by computer to solve two- or three-variable linear systems;

(C) apply linear programming techniques to model and solve real-world situations, using the computer or calculator, where appropriate; and

(D) solve quadratic-quadratic and quadratic-linear systems, and confirm the solution by computer graphing techniques.

(5) Numerical methods and higher degree polynomials. The student shall be provided opportunities to:

(A) use successive approximations on the calculator or computer to solve higher degree equations;

(B) apply synthetic substitution to find functional values of higher degree polynomials;

(C) use the Fundamental Theorem of Algebra and the Factor Theorem to factor higher degree polynomials;

(D) graph higher degree polynomial functions using computer graphing techniques;

(E) solve higher degree polynomial equations using computer graphing techniques; and

(F) use an iterative process (algebraic or geometric) to approximate irrational roots of higher degree functions.

(6) Exponential and logarithmic functions. The student shall be provided opportunities to:

(A) investigate the concept of nth root and convert between exponential and radical forms of an expression;

(B) extend the properties of exponents to include rational exponents;

(C) investigate exponential functions and their inverses to develop the definition of logarithm;

(D) explore the graphs of exponential and logarithmic functions using computer graphing techniques;

(E) convert between logarithmic and exponential forms of an equation;

(F) apply properties of logarithms to solve equations; and

(G) apply logarithmic and exponential functions in problem situations using the computer or calculator.

(7) Rational algebraic functions. The student shall be provided opportunities to:

(A) simplify complex fractions;

(B) graph rational algebraic functions (using computer graphing techniques where appropriate) to develop an intuitive understanding of the concept of limit; and

(C) use direct and inverse variation functions as models to make predictions in real-world situations.

(8) Sequences and series. The student shall be provided opportunities to:

(A) investigate patterns in given sequences and use the patterns or recursive or generator formulas to find additional terms;

(B) investigate and graph geometric and arithmetic sequences;

(C) find the nth partial sum of geometric or arithmetic series and find n given the nth term or partial sum;

(D) investigate convergent geometric series;

(E) use sequences and series as models in real-world problem situations;

(F) use the binomial theorem to expand powers of binomial expressions; and

(G) solve enumeration problems involving permutations and combinations.

(9) Data handling and analysis. The student shall be provided opportunities to:

(A) recognize the importance of unbiased sampling and valid reasoning in statistical arguments;

(B) select an appropriate sampling method for a given real-world problem situation;

(C) interpret probabilities relative to the normal distribution;

(D) design a simple statistical experiment to test a hypothesis generated by a real-world problem situation and interpret the results; and

(E) use computer simulation methods to represent and solve problem situations involving uncertainty.

Sample Objectives for Algebra II

District educators should develop or adapt objectives that will ensure that all the required essential elements of Algebra II are taught. The objectives should clarify the essential elements and describe expected student outcomes.

Forty-nine sample objectives for Algebra II are listed on pages 50-54. (These objectives are not mandated but are intended only to show examples of types of objectives teachers may use for each essential element.) District curriculum planners may use the sample objectives in the development of their own course objectives. The sample objectives are arranged in a logical sequence of instruction, and correlations to the essential elements are noted in brackets. To the left of many of the objectives is a star (☆), which indicates that an activity appropriate for that objective is included among the activities on pages 54-72. Although certain objectives are especially appropriate for using calculators or computers, the teaching and learning of virtually every objective can be enhanced through the use of technology. Finally, objectives marked [H] are appropriate for an honors curriculum.

Essential Element 1

Development of mathematical structure.

OBJECTIVES. The student will:

1. determine the field properties possessed by subsets of the real and complex numbers under addition and multiplication [EE 1A]

2. determine whether or not a given set of real numbers under two binary operations is a field [EE 1B]

3. add, subtract, multiply, and divide complex numbers [EE 1C]

[H] 4. graph complex numbers

[H] 5. find square roots of complex numbers

Essential Element 2

Quadratic functions.

OBJECTIVES. The student will:

6. solve quadratic equations by completing the square [EE 2A]

7. develop the quadratic formula and use it to solve quadratic equations [EE 2B]

8. find a quadratic equation given its roots [EE 2C]

☆ 9. explain the changes to the graph of $y = Ax^2 + Bx + C$ as A, B, and C vary [EE 2D]

☆ 10. sketch the graph of a quadratic function using
- axis of symmetry
- vertex
- symmetric points
- intercepts
- concavity (direction it opens)

[EE 2E]

11. find the equation of a quadratic function given
- its roots and one other point
- its vertex and one other point on the graph
- any three points on the graph

[EE 2F]

☆ 12. determine solutions to problem situations by using appropriate algebraic models (write equations) that involve quadratic equations [EE 2G]

[H] 13. solve equations that are reducible to quadratic form

Essential Element 3

Quadratic relations.

OBJECTIVES. The student will:

☆ 14. classify equations of the form $Ax^2 + Dx + Cy^2 + Ey = F$ whose graphs are circles, parabolas, hyperbolas, or ellipses for various values of A, C, D, E, and F [EE 3A, EE 3B]

☆ 15. sketch the graphs or determine the equations of conic sections using data such as

- centers
- foci
- axes
- asymptotes
- vertices

[EE 3B, EE 3C, EE 3D]

☆ 16. determine solutions to problem situations by using appropriate algebraic models (write equations) that involve quadratic relations [EE 3E]

Essential Element 4

Systems of equations.

OBJECTIVES. The student will:

☆ 17. solve systems of linear equations with two or three variables by:

- linear combinations
- augmented matrices

[EE 4A, EE 4B]

☆ 18. solve optimization problems using linear programming involving two variables [EE 4C]

☆ 19. solve quadratic-quadratic and quadratic-linear systems of equations and confirm the solution by graphing the system on a computer or graphing calculator [EE 4D]

[H] 20. calculate inverses of square matrices and solve systems of equations using the inverse of a square matrix

Essential Element 5

Numerical methods and higher degree polynomials.

OBJECTIVES. The student will:

21. find functional values of higher degree polynomials using synthetic substitution [EE 5B]

22. find the linear and quadratic factors of higher degree polynomials using the Factor Theorem and the Rational Root Theorem [EE 5C]

☆ 23. locate roots and maxima/minima points of higher degree polynomials by graphing on a computer or graphing calculator [EE 5D, EE 5E]

☆ 24. approximate roots of higher degree polynomials by successive approximations using a calculator or computer [EE 5A, EE 5F]

[H] 25. find the number of real roots of a polynomial using Descartes' Rule of Signs

Essential Element 6

Exponential and logarithmic functions.

OBJECTIVES. The student will:

26. write expressions containing rational exponents in radical form and vice versa [EE 6A]

27. form equivalent expressions using the laws of exponents (including rational exponents) [EE 6B]

28. express $y = b^x$ as $x = \log_b y$ and vice versa [EE 6E]

☆ 29. sketch the graphs of logarithmic and exponential functions on a computer or graphing calculator [EE 6D]

30. explain the relationship between $f(x) = \log_b x$ and $g(x) = b^x$ [EE 6C]

31. use the calculator to determine y in the following equations:
    - $y = b^x$
    - $y = e^x$

- $y = \log x$
- $y = \ln x$
- $x = b^y$

[EE 6G]

32. solve equations using the properties of logarithms and laws of exponents [EE 6F]

☆ 33. determine solutions to problems by using appropriate algebraic models (equations) that involve logarithmic or exponential expressions [EE 6G]

Essential Element 7

Rational algebraic functions.

OBJECTIVES. The student will:

34. simplify complex fractions [EE 7A]

35. sketch graphs of functions of the form $h(x) = \frac{P(x)}{Q(x)}$ where P and Q are polynomials of degree 0, 1, or 2 and identify:
    - intercepts
    - horizontal and vertical asymptotes as limits
    - discontinuities

    [EE 7B]

☆ 36. determine solutions to problem situations by using appropriate algebraic models (equations) that involve direct and inverse variation [EE 7C]

Essential Element 8

Sequences and series.

OBJECTIVES. The student will:

☆ 37. determine a possible recursive or general formula for a given sequence and find additional terms in the sequence [EE 8A]

☆ 38. identify arithmetic and geometric sequences and sketch their graphs [EE 8B]

39. find the nth partial sum of geometric and arithmetic series and find n given the nth term or partial sum [EE 8C]

40. identify geometric series that converge and find their sums [EE 8D]

41. determine solutions to problem situations by using appropriate algebraic models (equations) that involve sequences and series [EE 8E]

42. find powers of binomial expressions using the binomial theorem [EE 8F]

☆ 43. solve combinatorial problems involving the fundamental counting principles, permutations, and combinations [EE 8G]

[H] 44. prove theorems using mathematical induction

Essential Element 9

Data handling and analysis.

OBJECTIVES. The student will:

45. identify biased samples and invalid statistical arguments [EE 9A]

46. select an appropriate sampling method for a given real-world problem situation [EE 9B]

☆ 47. solve problems that involve probabilities associated with a set of normally distributed data [EE 9C]

☆ 48. design a simple statistical experiment to test a hypothesis generated by a real-world problem situation and interpret the results [EE 9D]

☆ 49. solve problem situations that involve uncertainty using computer simulation methods [EE 9E]

Sample Activities for Algebra II

The 10 activities on pages 54-72 illustrate how teachers may address different topic areas by having students perform the prescribed tasks. The activities are designed to illustrate methods of teaching and to help clarify what is meant by some of the essential elements. The compilation is not intended to be a comprehensive list of activities to teach the essential elements nor are these activities required to be used. However, the activities demonstrate the kind of work that students should be required to do in Algebra II. The activities relate to some of the sample objectives on pages 50-54.

Activity 1	Graphing Pictures

Summary ►
Students duplicate pictures by writing equations the graphs of which will create the pictures. They then graph the equations on a computer or graphing calculator to check for accuracy and to make adjustments.

Objective 14 ►
Classify equations of the form $Ax^2 + Dx + Cy^2 + Ey = F$ whose graphs are circles, parabolas, hyperbolas, or ellipses for various values of A, C, D, E, and F.

Objective 15 ►
Sketch the graphs or determine the equations of conic sections using data such as:

- centers
- foci
- axes
- asymptotes
- vertices

Essential Elements ► 3(A), 3(B), 3(C), 3(D)

Materials ►
Graphing calculators or a computer program that will graph second degree equations

Procedure ►
Have students work in pairs, with one calculator or computer for each pair. Give students pictures for duplication. Tell them to write equations that will duplicate the pictures you have given them.

Students may experiment and check their work by using the calculator or computer. However, the work is self-checking since they will be able to tell if their equations are correct by comparing their pictures to those given to them.

Answers ►

Snowman

$(x + 1)^2 + (y - 8)^2 = \frac{1}{4}$

$(x - 1)^2 + (y - 8)^2 = \frac{1}{4}$

$x^2 + (y - 7)^2 = \frac{1}{9}$

$x^2 + (y - 7)^2 = 4$

$x^2 + (y - 2)^2 = 9$

$x^2 + (y + 5)^2 = 16$

Clown

$y = 2 - x^2 + 5$

$(x + 3)^2 + (y - 2)^2 = \frac{1}{4}$

$(x - 3)^2 + (y - 2)^2 = \frac{1}{4}$

$x^2 + y^2 = 1$

$25x^2 + 49y^2 = 1225$

$x^2 + 9(y + 3)^2 = 9$

$64(x - 7.5)^2 + 4y^2 = 16$

$64(x + 7.5)^2 + 4y^2 = 16$

$y = -\frac{1}{5}x^2 - 5$

Answers ►

Spider Web with Spider

$x^2 + y^2 = 4$

$x^2 + y^2 = 9$

$x^2 + y^2 = 25$

$x^2 + y^2 = 49$

$x^2 + y^2 = 81$

$x^2 - y^2 = 9$

$x^2 - y^2 = 25$

$x^2 - y^2 = 49$

$x^2 - y^2 = 81$

$y^2 - x^2 = 9$

$y^2 - x^2 = 25$

$y^2 - x^2 = 49$

$y^2 - x^2 = 81$

Butterfly

$x^2 + (y - 8)^2 = 1$

$(\frac{1}{2}, 8\frac{1}{2})\ (-\frac{1}{2}, 8\frac{1}{2})$

$y = \frac{1}{2}x^2 + 7$

$y = x^2 + 9$

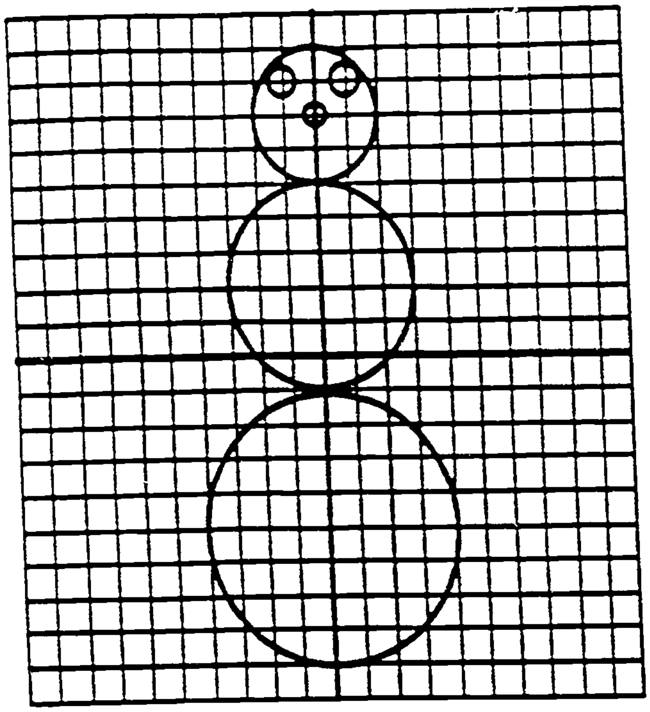
$49x^2 + (y - 1)^2 = 49$

$(1,10)\ (-1,10)$

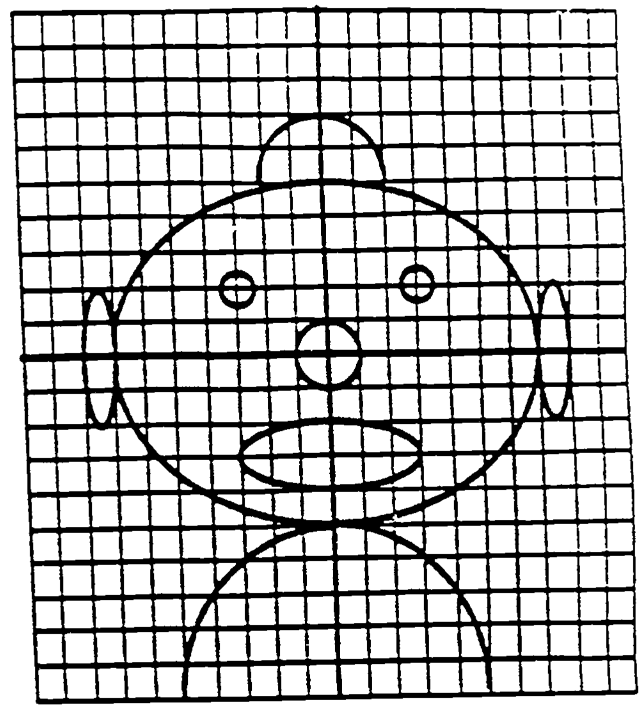
$x^2 - (y - 1)^2 = 1$

$x = -\frac{2}{81}(y - 1)^2 + 10$

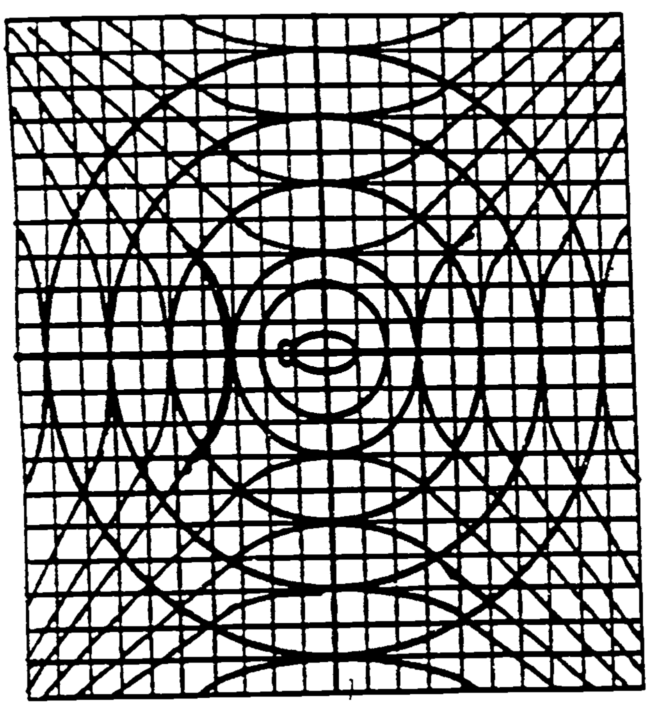
$x = \frac{2}{81}(y - 1)^2 - 10$



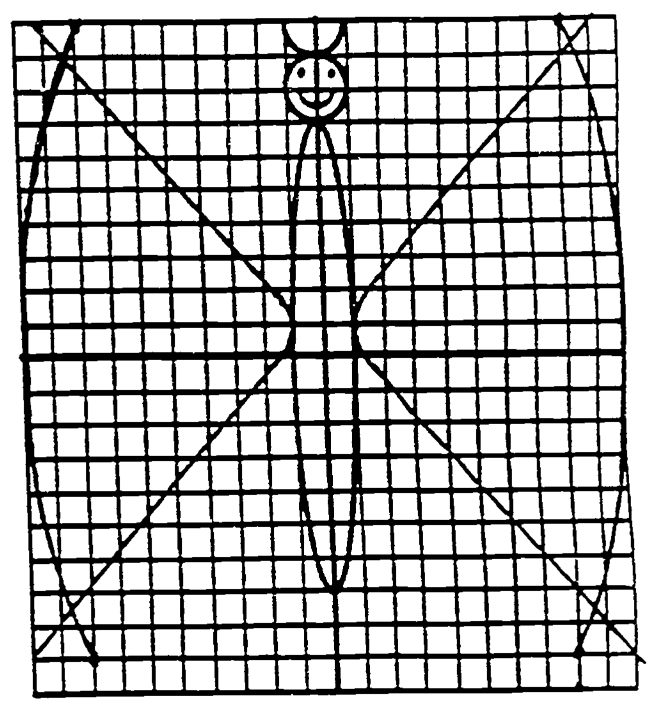
Snowman



Clown



Spider Web with Spider



Butterfly

Figure 15

Activity 2	**Linear Programming**

Summary ►

Linear programming is a technique for optimizing quantities (usually maximizing profit or minimizing costs) whose mathematical formulation involves only linear equations and inequalities. Usually there are thousands of variables, but students gain some appreciation for the technique by examining situations in two variables.

Objective 18 ►

Solve optimization problems using linear programming involving two variables.

Essential Element ► 4(C)

Materials ►

Student solution in small groups: coordinate graph paper, colored pencils, straightedge

Teacher demonstration: coordinate plane transparency, transparency with a long segment drawn, colored pens

Procedure ►

Here is a typical problem with a sequence of questions and answers that could be used for a teacher demonstration or a student worksheet with small groups.

Problem ►

You are the owner of Ponderosa Plywood which produces plywood boards by using a press to glue together thin panels of lumber. The panels come in two types—pine and oak. Two kinds of plywood boards are made—exterior and interior. One board of exterior plywood requires 2 panels of pine, 2 panels of oak, and 10 minutes of press time. One board of interior plywood requires 4 panels of oak (none of pine) and 5 minutes of press time. On Monday there are 1000 panels of pine available and 3000 panels of oak. There are 12 presses, each of which can press 4 panels into 1 plywood board. Each press can be run for 500 minutes a day. Your company must make a minimum of 600 plywood boards (interior and exterior) per day to make expenses.

In a typical linear programming problem, you are producing a product to minimize cost or maximize profit. You can solve a linear programming problem having two variables by graphing lines and finding corner points.

- In this problem, what is your company producing?

  Answer: Exterior and interior panels

- The number of each of these two products that are sold will be the two variables: x = number of exterior panels, y = number of interior panels.

• What restrictions or constraints are there in producing these panels?

Answer: In this case, there are constraints about materials, time, and number needed to make expenses.

(Materials)	Pine	Oak	(Time)
Exterior (x)	2	2	10
Interior (y)		4	5
Total available	1000	3000	500(12) = 6000

• What are the inequalities (constraints) for each condition that determine the feasible region?

(Make expenses)	$x + y \geq 600$
Materials:	$2x \leq 1000$
Materials:	$2x + 4y \leq 3000$
Time:	$10x + 5y \leq 6000$

(On the overhead, draw the graph of each inequality and shade the feasible region.)

• What is the minimum feasible number of interior plywood panels?

Answer: 100

• Suppose you can make a profit of $6 for each exterior plywood panel and $5 for each interior plywood panel. What is the objective function that gives the total profit?

Answer: $P(x) = 6x + 5y$

• Shade the portion of the feasible region in which the profit would be at least $3600 on Monday.

Answer: Shade $\{(x,y) \mid 6x + 5y \geq 3600\}$

(On the overhead, overlay the transparency with the segment representing the profit line $P = 6x + 5y$ and move it across the indicated region above. This also shows the corner point at which the maximum profit occurs.)

• What is the maximum feasible profit you could make on Monday, and how many of each type of plywood panel would need to be manufactured to achieve it?

Solution: Check $P(x) = 6x + 5y$ at the five corner points:

Corner point:	(0,600)	(0,750)	(300,600)	(500,200)	(500,100)
Profit:	3000	3750	4800	4000	3500

• How much more profit do you make on Monday by operating at the maximum profit than by operating at the minimum profit in the feasible region?

Answer: $4800 - 3000 = 1800$

• Suppose you can make a profit of $10 for each exterior plywood panel and $5 for each interior plywood panel. What is the objective function now?

Answer: $P(x) = 10x + 5y$ (same as the time constraint)

- What is the maximum feasible profit you could make on Monday, and how many of each type of plywood panel would need to be manufactured to achieve it?

  Solution: Check $P(x) = 10x + 5y$ at the five corner points: (or just use the constraint $10x + 5y = 6000$)

Corner point:	(0,600)	(0,750)	(300,600)	(500,200)	(500,100)
Profit:	3000	3750	6000	6000	5500

  (On the overhead, overlay the transparency with the segment representing the profit line $P(x) = 10x + 5y$ and move it to show it coincides with one of the sides of the feasible region, thereby indicating the 201 solutions.)

- The profit is maximized along the segment joining the points (300,600) and (500,200). How many points on this segment have whole number coordinates?

  Answer: 201

Note ►
This discussion indicates the major points involved in a linear programming program with one extension that shows more than one solution is possible in such a problem. A feature not illustrated by this problem is a case where the solution to the problem requires whole numbers, but the coordinates of the appropriate corner point(s) are not whole numbers. In that case, we must look near the corner point to determine the solution(s).

Activity 3 — Investigating the Graphs of Polynomial Functions (Locating Real Zeros and Maxima/Minima Points)

Summary ►
Students use a graphing utility to investigate the behavior of higher degree polynomial functions. Specifically, students find real zeros and maxima/minima points without using calculus. Students examine graphs of equations in the form $y = x^n$ before they investigate the behavior of more complex polynomials. At the end of the investigation, students should be able to make general statements about the graphs of polynomial functions, given their equations.

Objective 23 ►
Locate roots and maxima/minima points of higher degree polynomials by graphing on a computer or graphing calculator.

Essential Elements ► 5(D), 5(E)

Materials ►
Any one or any combination of the following will be needed for this investigation:

- graphing calculators
- overhead projector graphing calculator
- computers
- software package such as Master Grapher or IBM Mathematics Exploration Toolkit

Procedure ►

When using a graphing utility to answer specific questions about higher degree polynomials, students need to have some general knowledge about the behavior of these functions. Because the complete graph is not always in the viewing rectangle, students will need to have some idea of what to look for and when to quit searching. They will need to know, for example, that the graph of a third-degree polynomial will always have one of the following shapes:

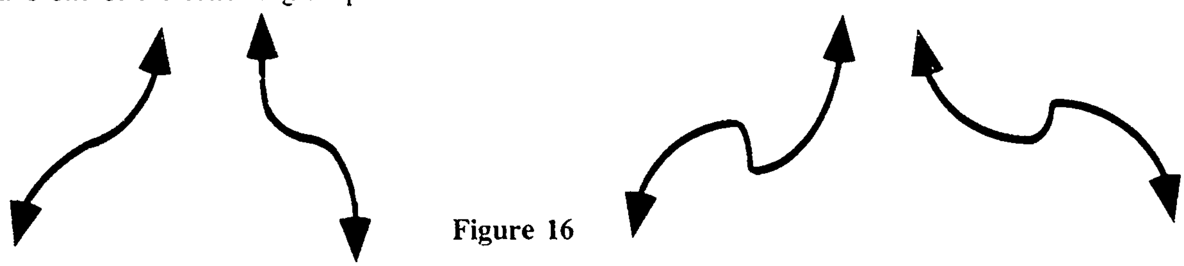


Figure 16

Also, they will need to know that the graph of a fourth-degree polynomial will look like one of the following:

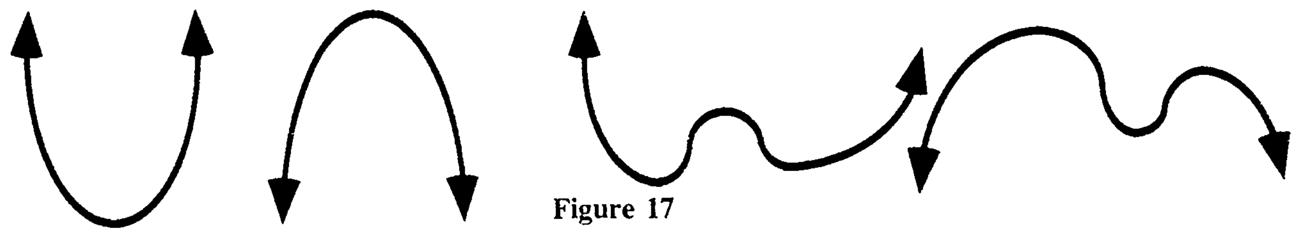


Figure 17

If students have not had an opportunity to investigate the graphs of $y = x^n$, they should do that before examining the graphs of higher degree polynomials.

The Graphs of $y = x^n$

STEP 1

Graph $y = x^2$ in a suitable viewing rectangle (e.g., $[-7,7,1]$ by $[-2,10,1]$). Students should recognize this as the graph of a quadratic function. Conduct a brief review of "things we know about the graphs of quadratic functions." For this investigation, however, the focus should be on the roots (zeros) of the function and the maximum or minimum points.

Ask

1. How many zeroes does this equation have?

   Answer: 2

2. Why, then, does the graph indicate only one zero?

   Answer: double root

3. Where is the zero?

   Answer: (0,0), where the curve is tangent to the x-axis

4. Is there a maximum or minimum point? Where is it? *Answer:* min., (0,0)

   Locate this point using the trace key. *Answer:* $(-3.1\text{E} - 12, 9.61\text{E} - 24)$

   Students may not recognize these values as (0,0).

(Clear the screen.)

STEP 2
Graph $y = x^2 + 3$.

Ask

1. How many real zeros are there?

   Answer: 0

2. But we were expecting two. Where are they?

   Answer: no real zeros, but complex ones

3. Is there a maximum or minimum point? Where is it? *Answer:* min., (0,3)

   Locate this point using the trace key. *Answer:* $(-3.1E - 12, 3)$

   As the pixel moves from top to left toward the minimum value, have students trace the y-values so they can see that the function is decreasing. Allow tracing to continue to the right of the minimum value to allow students to see that the function is increasing. This would be an appropriate time to discuss intervals on which the function is decreasing or increasing.

(Clear the screen.)

STEP 3
Graph $y = x^2 - 3$ (Students will need to reset the range.)

Ask

1. How many real zeros are there?

   Answer: 2

2. What name is given to the points?

   Answer: x-intercepts

3. What are the coordinates of the zeros?

   Answer: algebraically $x = \pm\sqrt{3}$;
   calculator $x = \pm 1.75$

   Discuss the exact versus the approximate solution and the desired degree of accuracy. In an Algebra II class, students do not need to get bogged down in error analysis.

4. Locate the maximum or minimum points by using the trace key.

5. Identify intervals on which the function is decreasing or increasing.

(Clear the screen.)

STEP 4
Graph $y = x^2$, $y = x^4$, $y = x^6$ on the same set of axes. To get a good picture, students may need to experiment with the range setting.

Ask

1. What similarities/differences do you observe about the graphs? What, if any, points do they share?
2. What would you expect the graph of $y = x^{14}$ to look like? Verify with the calculator.
3. Can you draw any conclusions about the graph of $y = x^n$ if n is an even natural number (i.e., as the value of n increases, the graph of $y = x^n$ . . .)? Students may need some guidance. (1) What happens at $x = 0$? At $x = \pm 1$? (2) For what values of x is the graph of $y = x^2$ greater than (above) or less than (below) the graph of $y = x^4$, etc.? (3) Are the graphs symmetric with respect to an axis? A point? (4) On which intervals is the function increasing? Decreasing?

Repeat
Repeat Steps 1-3 of this procedure by graphing $y = x^3$, $y = x^3 + 3$, $y = x^3 - 3$, asking the same questions. Then repeat Step 4 having students graph $y = x^3$, $y = x^5$, $y = x^7$ on the same set of axes, asking the same questions.

Ask
What would happen to the graphs if the first term were negated? What, if anything, changes? Stays the same? Verify your predictions by using a calculator.

Investigating the Graphs of Higher Degree Polynomials ►
Give the following example to students: Graph $f(x) = x^3 - x^2 - 9x + 9$. Locate the real zeros and the relative maximum/minimum points to the nearest 0.1 or 0.01 (or to whatever degree of accuracy desired).

Do not suggest a viewing rectangle. Students may use a range setting that will result in an incomplete graph. This would necessitate students having to search for the rest of the curve. Ask questions such as the following to facilitate the exploration:

1. What do you expect the graph to look like?
2. How many real zeros are guaranteed? How many are possible?

   Answer: 1, 3

3. How can you locate the real zeros algebraically?

   Answer: factor by grouping

   What are the real zeros?

   Answer: (1,0), (−3,0), (3,0)

4. What is one point that we know is on the graph?

Answer: y-intercept: (0,9)

5. How would knowing the answers to Questions 1-4 help you select a viewing rectangle for a complete graph?

Allow students time to experiment with various range settings. Once they each have a complete graph, have them locate the real zeros by using the trace and zoom-in features. Compare these calculator values with the calculated values. Discuss the differences, if any.

Now have students use the trace and zoom-in features to locate the relative maximum/minimum points. After locating these points, have students identify the intervals on which the function is increasing/decreasing.

Give the following example to students: Graph $f(x) = x^3 + x^2 - 8x + 4$. Locate the real zeros and the relative maximum or minimum points to the nearest 0.1 or 0.01, etc.

Ask the same questions as in the first example. This time, however, students will not be able to locate zeros by factoring. Before graphing on the calculator, have students:

- plot points. (Use negative values, zero, and positive values of x.) Have students observe changes in the sign of $f(x)$ to locate real zeros between consecutive integers and continue to use the calculator to find increasingly better approximations for $f(x) = 0$.
- list and check possible rational roots, using synthetic division (Rational Zero Theorem, Descartes Rule of Signs, Upper and Lower Bounds Theorem). Discuss how these theorems can reduce the amount of work and guide the student in locating zeros. Having students graph one or two polynomials without the aid of a graphing utility should increase their appreciation for the powerful tool they hold in their hands.

Suggested Practice ►

(1) Have students write 5 to 10 third-degree polynomials. Using a graphing utility, students should locate the real zeros and the relative maximum/minimum points. Students can then exchange their problems for more practice.

(2) Divide students into groups (or pairs). Have a contest to see how quickly students can (a) find a complete graph and (b) locate the zeros and extrema.

(3) Have students write equations to satisfy given conditions such as coming in from the top, having one real zero (or three), having one negative and two positive zeros, etc. After experimenting with the equations to get the appropriate graphs, students should find the zeros and extrema.

Give the following example to students: Graph $f(x) = 2x^4 + 5x^3 - 3x^2 - 9x + 1$. Locate the real zeros and the relative maxima/minima points to the nearest 0.1 or 0.01, etc.

Ask the same kinds of questions as in the first example. Use a graphing utility to draw the graph and to find the required points. Have students graph several fourth-degree polynomial functions. Students should select equations to illustrate various possibilities for real zeros, including one with complex roots only.

Suggested Practice ►
After students have graphed a number of third- and fourth-degree polynomial functions, have them draw some conclusions about the behavior of these functions. Ask students if they can state a relationship between the number of peaks and valleys (turning points, bumps) and the degree of the polynomial. Is this generalization always true? How can we tell whether a polynomial will come in from the top (or bottom), or can we?

The use of graphing utilities allows students to solve advanced problems early in their mathematical careers because they do not need calculus.

Give the following example to students: A mathematics teacher plans to treat the students to popcorn and a movie but does not have enough containers. The teacher gives each student a piece of construction paper measuring 8.5 in. by 11 in. and instructs them to remove a square from each corner. They will then fold up the sides and have an open box for the popcorn. Students who create a box of maximum volume will also receive a can of soda. What are the dimensions and maximum volume of such a box?

Solution: Draw and label a diagram, letting x represent the length of each side of the square to be removed.

Dimensions

length (l) = $(11 - 2x)$ in.

width (w) = $(8.5 - 2x)$ in.

height (h) = x in.

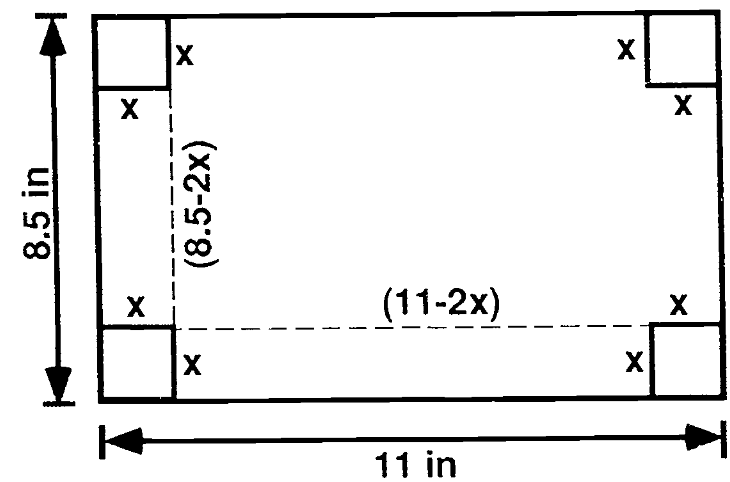


Figure 18

Volume = lwh

$V(x) = ((11 - 2x) \text{ in.})((8.5 - 2x) \text{ in.})(x \text{ in.})$

Graph $V(x)$. After getting a complete graph, adjust the viewing rectangle to get a large but complete graph.

Ask

1. What part of the graph represents the problem situation? Explain.
2. What is the domain of the problem situation?
3. For what value of x is the volume at a maximum? What is this maximum volume?
4. What are the dimensions of the box with a maximum volume?
5. What happens to the volume as x increases from 0 to about 4? What is the volume when $x = 5$? Is this reasonable? Explain. What if $x = 6$? Is this reasonable? Explain.

Activity 4	Approximating Roots of Higher Degree Polynomials

Summary ►
As a practical matter, the only equations that can reasonably be solved through the use of paper and pencil are linear and quadratic polynomials and simple examples of other equations involving radicals, exponentials, and logarithms. Calculators and computers and/or numerical methods should be used for other types of equations. Here, students solve an equation graphically, using a graphing calculator or computer graphing program and two basic root-finding algorithms.

Objective 24 ►
Approximate roots of higher degree polynomials by successive approximations using a calculator or computer.

Essential Elements ► 5(A), 5(F)

Materials ►
Programmable graphing calculator and/or a computer with a graphing program

Procedure ►
The first step in using either of the following methods is to write the equation in the form $f(x) = 0$.

METHOD 1
Produce a complete graph of the equation. Create a nested sequence of viewing rectangles that zoom in on each root to any desired accuracy possible with the calculator or software.

METHOD 2
Ask students to write and use a program based on one of the following algorithms. Both algorithms assume that students have done some preliminary work to locate the roots on some interval $a \le x \le b$, hereafter denoted $[a,b]$.

A. 1. Find an interval $[a,b]$ for which $f(a) < 0$ and $f(b) > 0$.

2. Start with $x = a$ and $h = 1$. Choose an error-bound E (desired accuracy)

3. Move to the right in steps of h until $f(x) > 0$.

4. Go back one step.

5. Replace h by $\frac{h}{10}$. If $h \ge$ E, then go to Step 2. Otherwise, stop with x as the root.

Questions
1. Why does the algorithm work?

2. How can it be adapted to the case $f(a) > 0$ and $f(b) < 0$?

3. Why might such a program miss roots that are very close together?

B. 1. Choose an interval $[a,b]$ where $f(a)f(b) < 0$.
The general idea is to halve this interval at each stage until it is smaller than some error-bound E.

2. Set $x = \frac{a + b}{2}$

3. If $f(x) = 0$, then stop.

Note: This is highly unlikely to happen.

4. If $f(x)f(a) > 0$, then set $a = x$. Otherwise, set $b = x$.

5. If $|a - b| > E$, then go to Step 2. Otherwise, stop with x as the root.

Questions

1. Why does the algorithm work?

2. Might such a program miss roots that are very close together? If so, why?
A classical example is to solve $x^3 - 3x + 1 = 0$

Answer: .347, 1.532, −1.879

Activity 5	Introduction to Sequences

Summary ►
Students look for patterns in strings of numbers. The intent is to show students many kinds of patterns before they concentrate on arithmetic and geometric sequences.

Objective 37 ►
The student will determine a possible recursive or explicit formula for a given sequence and find additional terms in the sequence.

Essential Element ► 8(A)

Materials ►
Chalkboard or overhead projector

Procedure ►
A classroom example would be: 2, 6, 12, 30, 42, 56, ...

To convey the function idea, call out a number such as "3" or "5" and see if students can think of an appropriate response (you want students to think of the value of the number in the third or the fifth place). Then see if students can think of the value of the eighth term and ninth term by observing a pattern. Finally, challenge students to tell you the 1000th term; for this purpose they will need a formula linking the term number with the term value. By writing the numbers below the terms, some students will notice that the term numbers are factors of the term values. For example, 7 is a factor of 56, and $56 = 7 \cdot 8$. Writing these factors under the term numbers reveals a pattern from which a formula is obvious.

2	6	12	20	30	42	56	—	$t(n)$
1	2	3	4	5	6	7	—	n
$1\cdot 2$	$2\cdot 3$	$3\cdot 4$	$4\cdot 5$	$5\cdot 6$	$6\cdot 7$	$7\cdot 8$	—	pattern for $t(n)$

The term equals the term number times one more than the term number.
That is, $t(n) = n(n+1)$.
From this formula, $t(1000) = 1000(1001) = 1001000$.

Activity 6 — Arithmetic and Geometric Sequences

Summary ►
The most important points to stress in an introductory lesson on arithmetic and geometric sequences are the two definitions. An arithmetic sequence is formed by adding a constant, and a geometric sequence is formed by multiplying by a constant. The formulas used simply say to start with the first item and add $(n-1)$ differences or multiply by $(n-1)$ ratios.

Objective 38 ►
Identify arithmetic and geometric sequences and sketch their graphs.

Essential Element ► 8(B)

Materials ►
Chalkboard or overhead projector

Procedure ►
For classroom presentations, first show examples of arithmetic and geometric sequences before stating their definitions. For example, you could have students find patterns in

a. 3, 12, 21, . . .
b. 3, 12, 48, . . .

In the first sequence, you add the constant 9 to get the next term. In the second, you multiply by the constant 4 to get the next term. From these examples, the definitions arise naturally and can even be worded by the students themselves. A third example could be:

c. 3, 12, 72, . . .

The pattern is, "Multiply by 4, multiply by 6, multiply 8, . . . ," so the sequence is neither arithmetic nor geometric.

Students should become familiar enough with the formulas to be able to use them backwards and find n when $t(n)$ is given. Good classroom examples are:

Arithmetic: $t(n) = 305$, $t(1) = 17$, $d = 8$ (*Answer:* $n = 17$)
Geometric: $t(n) = 34816$, $t(1) = 17$, $r = 2$ (*Answer:* $n = 12$)

Activity 7	Combinatorial Problems

Summary ►
Students learn the factorial formulas for both permutations and combinations, and they learn the difference between the two concepts.

Objective 43 ►
Solve combinatorial problems involving the fundamental counting principles, permutations, and combinations.

Essential Element ► 8(G)

Materials ►
Chalkboard or overhead projector

Procedure ►
Have students think of deriving the combinations formula in two sequential steps.

A. Select a combination of 3 of the 4 letters (x ways).

B. Arrange this group of 3 letters (6 ways).

Since both steps are taken in succession, the total number of ways is found by multiplying $x \cdot 6$ or $6x$.

But this is equal to the total number of permutations of 4 letters taken 3 at a time. This number is known to be 24. Therefore, $6x = 24$.
Dividing by 6 gives $x = 24/6 = 4$.
This could also be thought of as:

$$\text{The number of combinations of 4 things taken 3 at a time} = \frac{\text{Number of permutations of 4 things taken 3 at a time}}{\text{Number of permutations of one of these combinations}}$$

Make sure that students write the symbols ${}_nP_r$ and ${}_nC_r$ with the P and C on the line and the n and r as subscripts.

Activity 8	Weather or Not

Summary ►
Students gather data, find the mean and standard deviation, and then determine the probability of an event.

Objective 47 ►
The students will solve problems that involve probabilities associated with a set of normally distributed data.

Essential Element ► 9(C)

Materials ►
Resources that provide daily high and low temperatures for your city over the last 10 years.

Procedure ►
Have students research the high or low temperature for their city over the last 10 years for the day on which the senior class is to graduate.

1. Have them find the mean and standard deviation.

2. Have them sketch a graph or use a computer to produce one.

3. Then assuming the temperatures are normally distributed, have students determine what the probability is that the temperature on graduation day will be within one standard deviation of the mean.

Suggestion ►
Invite a weather forecaster as a guest speaker and have him or her bring the data.

Activity 9 — Data Analysis of Tomorrow's Adults

Summary ►
Students design an experiment to test a hypothesis then interpret the results obtained. The activity is given in a time-line format.

Objective 48 ►
Design a simple statistical experiment to test a hypothesis generated by a real-world problem situation and interpret the results.

Essential Element ► 9(D)

Materials ►
Optional—computer program to analyze data and print graphs.

Procedure ►
DAY 1
Hold class discussion to form hypothesis. Possible subjects might include:

- amount of time students spend studying
- cheating
- student involvement in extracurricular activities
- working students
- spending habits of students

DAY 2
Design association questionnaire or other means of collecting data.

DAY 3-5
Collect data.

DAY 6
Compute data and do printouts of results, using a computer if available.

DAY 7
Prepare graphic representations of data results, using a computer if available.

DAY 8
Prepare a written paper on whether or not the collected data and its analysis support the hypothesis.

Activity 10	Random Draw

Summary ►
Students determine a theoretical probability and then test it with a computer simulation.

Objective 49 ►
Solve problem situations that involve uncertainty using computer simulation methods.

Essential Element ► 9(E)

Material ►
Computer, computer program which is given

Procedure ►
The following computer program simulates drawing a card at random from a deck of 52 cards. First, find the theoretical probability of picking each card (replacing each time); then run the program 50 times and determine how close the actual results are to the theoretical probability. What other theoretical probabilities could be tested using this program? Determine at least one and then test it.

```
  4 RANDOMIZE TIMER
  5 FOR X=1 TO 100
 10 LET Y= INT (13*RND(N+1)) + 1
 20 ON Y GOTO 30, 50, 70, 90, 110, 130, 150, 170, 190, 210, 230, 250, 270
 30 PRINT "ACE OF";
 40 GOTO 280
 50 PRINT "DEDUCE OF";
 60 GOTO 280
 70 PRINT "THREE OF";
 80 GOTO 280
 90 PRINT "FOUR OF";
100 GOTO 280
110 PRINT "FIVE OF";
120 GOTO 280
130 PRINT "SIX OF";
140 GOTO 280
150 PRINT "SEVEN OF";
160 GOTO 280
```

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170 PRINT "EIGHT OF";
180 GOTO 280
190 PRINT "NINE OF";
200 GOTO 280
210 PRINT "TEN OF";
220 GOTO 280
230 PRINT "JACK OF";
240 GOTO 280
250 PRINT "QUEEN OF";
260 GOTO 280
270 PRINT "KING OF";
280 LET Z=INT (4*RND (N+1))+1
290 ON Z GOTO 300, 320, 340, 360
300 PRINT "DIAMONDS"
310 GOTO 361
320 PRINT "HEARTS"
330 GOTO 361
340 PRINT "SPADES"
350 GOTO 361
360 PRINT "CLUBS"
361 PRINT "++++++++++++++++++++++"
362 PRINT
363 PRINT "PRESS ANY KEY TO CONTINUE"
364 A$=INKEY$: IF A$="  "THEN 364
365 PRINT: PRINT: PRINT: PRINT: PRINT: PRINT: PRINT
366 PRINT: PRINT: PRINT: PRINT: PRINT: PRINT: PRINT
379 NEXT X
380 END
```

Appendices

Appendix A: Evaluation

Local Evaluation

The following material is adapted from the National Council of Teachers of Mathematics document, *Curriculum ai Evaluation Standards for School Mathematics.*

The main purpose of evaluation is to help teachers better understand what students know. Teachers can then make meaningful decisions about instruction. Evaluation includes tests, observations of students as they work, and assessments of products that are more substantial than the answer to a single question; for example, proofs, reports, and cooperative projects. Successful evaluation in Algebra will guarantee that:

- student assessment is integral to instruction
- multiple means of assessment are used
- all aspects of algebra knowledge and its connections to other content are assessed
- both classroom instruction and curriculum materials are considered in judging the quality of geometry instruction

Assessment must be properly aligned to the curriculum. That is, the content, processes, and skills assessed must reflect the goals, objectives, and breadth of topics specified in the curriculum.

Student performance will reflect the amount of opportunity students had to learn particular information; students cannot be expected to perform well on material for which they had not received adequate instruction. But beyond this, the emphases of the assessment should reflect the emphases of instruction; questions that deal with small, relatively insignificant parts of the algebra curriculum are not appropriate. Rather, teachers need to structure assessment items around the central ideas of algebra, and they need to provide opportunities for students to demonstrate their understanding of the connections among the major concepts presented in the courses. Too, assessment must reflect the emphasis that instruction places on technology; to the extent that calculators and computers are important during instruction, they should also be available during assessment.

Assessment in algebra should include more than just tests. Students should also be required to write convincing arguments, both formal and informal; to construct models through drawing and the use of other manipulatives; and to solve significant problems. Such tasks are intrinsically motivating, allowing students to integrate what they know; however, they also have extrinsic value to teachers by helping construct a more complete picture of the extent of a students' understanding. Because algebra is a content area that is dense with concepts and principles, teachers should monitor the development of students' understanding throughout the course. When teachers notice discrepancies in student understanding early on, they can quickly remediate any significant misunderstandings indicated, so that students' future learning is not jeopardized.

Problem solving is the focus of Algebra I and II, as in all of school mathematics. Hence, problem solving must also be the focus of assessment. Students' progress in problem solving should be assessed systematically, deliberately, and continually to influence effectively students' confidence and ability to solve problems in various contexts. Giving students feedback about the results of this assessment, on both process and results, is critical to their development as problem solvers. Assessments should determine students' ability to perform all aspects of problem solving. Evidence about their ability to ask questions, use given information, and make conjectures is essential to determine if they can formulate problems. Assessments also should yield evidence on students' use of strategies and problem-solving techniques and on their ability to verify and interpret results. Students should also be assessed on their ability to generalize.

Writing and discussing content help students to develop their reasoning skills and to integrate the concepts they have learned. For example, organizing and writing a report of data generated by use of exploratory software can enable students to make conjectures through a reasoning process. In addition, oral discussions and written work help students learn that one counter example is enough to discount the truth of a statement, while any finite number of examples is never enough to verify a general statement (at least one that applies to infinitely many cases). Students likely will not learn this by being told; rather, they must gradually construct and integrate the meaning of this fact.

Algebra content requires students to learn facts, concepts, and procedures and then to organize this knowledge into communications, typically in the form of informal and formal arguments or as solutions to problems. Algebra teachers have the opportunity to show students how to integrate knowledge as they learn it. The objectives suggested on pages 14-18 and pages 50-54 support a broad view of Algebra I and II. Teachers can encourage this approach through the nature of the assessment tasks they develop for students.

State Evaluation: Texas Assessment of Academic Skills

The Texas Education Agency is implementing a new statewide testing program for the period 1990-95. This assessment program—the Texas Assessment of Academic Skills (TAAS)—has as its primary purpose to provide Texas schools with an accurate measure of student achievement. The new program differs significantly from the previous assessment program entitled Texas Educational Assessment of Minimum Skills (TEAMS), both in content and in format. The scope of content eligible for testing has been broadened to include more of the instructional targets delineated in the essential elements, which should encourage greater emphasis on a larger set of essential elements rather than on the relatively limited set of TEAMS objectives. Every section of TAAS contains a certain number of broad objectives. These objectives remain consistent from grade to grade, representing the core concepts that form the basis for a sound instructional progression form Grade 1 through Grade 12. Instructional targets, however, differ from grade to grade. A portion of the instructional targets will be selected for assessment each year; not every instructional target will be tested every year. Thus teachers should focus instruction on the essential elements, which reflect these targets, instead of on a narrow body of assessment objectives.

The broadened scope of the new assessment program will allow for a different focus, which will better address the academic requirements of the 1990's. Skill areas that demand little more than rote memorization will be de-emphasized, while areas that improve a student's ability to think independently and to solve problems logically will receive increased emphasis. The new assessment emphasis corresponds well to the current emphases in mathematics education, which stress the importance of teaching students higher order thinking skills.

Texas educators are required to administer TAAS to students in certain grades. The tests assess students on objectives/instructional targets that they should have mastered in previous grades. Although both the Algebra I and Algebra II courses go beyond the content of the mathematics exit-level test, students who take Algebra I and II will study many topics that are directly related to the objectives and instructional targets of TAAS. The domains, objectives, and instructional targets of the TAAS Mathematics Exit-Level Test are the following:

DOMAIN: Concepts

Objective 1	The student will demonstrate an understanding of number concepts.

Items for this objective assess the following instructional targets:

- Use scientific notation.
- Compare and order real numbers.
- Round whole numbers and decimals.
- Determine relationships between and among fractions, decimals, and percents.
- Find squares and square roots.

Objective 2	The student will demonstrate an understanding of mathematical relations, functions, and other algebraic concepts.

Items for this objective assess the following instructional targets:

- Use real number properties and inverse operations.
- Determine missing elements in patterns.
- Identify ordered pairs and solution sets in one and two dimensions.
- Apply ratio and proportion.
- Evaluate variables and expressions (formulas).
- Solve simple equations involving integers, decimals, and fractions.

Objective 3	The student will demonstrate an understanding of geometric properties and relationships.

Items for this objective assess the following instructional targets:

- Use the basic elements of geometry (point, line, segment, ray, angle).
- Use geometric figures and their characteristics.
- Use right-triangle properties.
- Use indirect measurement with similar triangles.
- Apply geometric properties.

Objective 4	The student will demonstrate an understanding of measurement concepts using metric and customary units.

Items for this objective assess the following instructional targets:

- Use metric and customary units.
- Solve problems involving measures.
- Find distance, perimeter, circumference, area, surface area, and volume.
- Recognize precision.

Objective 5	The student will demonstrate an understanding of probability and statistics.

Items for this objective assess the following instructional targets:

- Use counting procedures (tree diagrams, multiplication).
- Find probability of simple and compound events.
- Determine the mean, median, and the mode.

DOMAIN: Operations

Objective 6	The student will use the operation of addition to solve problems.

Items for this objective assess the following instructional target:

Use the operation of addition with real numbers in practical situations.

Objective 7	The student will use the operation of subtraction to solve problems.

Items for this objective assess the following instructional target:

Use the operation of subtraction with real numbers to solve problems.

Objective 8	The student will use the operation of multiplication to solve problems.

Items for this objective assess the following instructional target:

Use the operation of multiplication with real numbers in practical situations.

Objective 9	The student will use the operation of division to solve problems.

Items for this objective assess the following instructional target:

Use the operation of division with real numbers in practical situations.

DOMAIN: Problem Solving

Objective 10	The student will estimate solutions to a problem situation.

Items for this objective assess the following instructional target:

Estimate solutions.

Objective 11	The student will determine solution strategies and will analyze or solve problems.

Items for this objective assess the following instructional targets:

- Identify strategies for solving or solve proportion problems.
- Determine methods for finding or find percent and percentage.
- Determine methods for solving or solve measurement problems.
- Formulate or solve problems using geometric concepts.
- Analyze or solve probability and statistics problems.
- Make predictions.

Objective 12	The student will express or solve problems using mathematical representation.

Items for this objective assess the following instructional targets:

- Formulate equations/inequalities.
- Analyze or interpret graphs, charts, tables, maps, or diagrams, and use the information derived to solve problems.

Objective 13	The student will evaluate the reasonableness of a solution to a problem situation.

Items for this objective assess the following instructional targets:

- Determine the validity of conclusions drawn from statistical data.
- Evaluate reasonableness.

The study of Algebra I and II should help students in all the content areas of the TAAS Mathematics Exit-Level Test. Algebra teachers should be aware of the TAAS objectives, but they should not emphasize these objectives to the neglect of essential elements. If the Algebra I and II essential elements are taught as intended, students should have a solid background for the TAAS objectives.

Appendix B: Resources for Algebra Instruction

Books and Articles

Algebra for Everyone. National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 22091.

Dalton, Leroy. *Algebra in the Real World.* Dale Seymour Publications, P.O. Box 10888, Palo Alto, CA 94303.

Driscoll, Mark. "The Learning and Teaching of Algebra," *Research Within Reach: Secondary School Mathematics* (1983). National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 22091.

Gadanidis, George. "Problem Solving: The Third Dimension in Mathematics Teaching," *Mathematics Teacher*, Vol. 81, No. 1 (Jan. 1988), pp. 16 ff.

Green Globs Student Activities [to go with *Green Globs* graphing software]. Center for Research in Mathematics and Science Education. Dr. Sarah Berenson. 315 Poe Hall. P.O. Box 7801. North Carolina State University. Raleigh. NC 27695. Phone: 919/737-2013.

Howden. Hilde. *Algebra Tiles for the Overhead Projector* (1985). Cuisenaire Company of America. 12 Church Street. New Rochelle. NY 10805.

Instructional Aids in Mathematics, 1973 Yearbook of National Council of Teachers of Mathematics. 1906 Association Drive. Reston. VA 22091.

Kinach. Barbara. "Solving Linear Equations Physically." *Mathematics Teacher*, Vol. 78. No 3 (Sept. 1985). pp. 437-447.

Laycock. Mary. "Discovering the Sum of a Power Sequence." *CMC ComMuniCator*. Journal of California Mathematics Council. Vol. 13. No. 3 (Dec. 1988). Bob Schultz. CMC Editor. 25271 Alpha Street, Moreno Valley. CA 92388.

———. and Smart. Margaret. *Solid Sense in Mathematics 7-9* (1981). Activity Resources Co., Inc., P.O. Box 4875. Hayward. CA 94541.

———. and Schadler. Reuben. Vol. 82. No. 3 *Algebra in the Concrete* (1987). Activity Resources Co., Inc., P.O. Box 4875. Hayward. CA 94541.

McFadden. Scott. *Algebra Warmups*. Dale Seymour Publications. P.O. Box 10888. Palo Alto. CA 94303.

Secondary Activities for Graphing Calculators. Monograph of Michigan Council of Teachers of Mathematics. Dr. Elizabeth Phillips. Monograph Editor. Dept. of Mathematics. Michigan State University. East Lansing. MI 48824.

Smart. Margaret. *Focus on Pre-Algebra* (1983). Activity Resources Co., Inc., P.O. Box 4875. Hayward. CA 94541.

Sourcebook of Applications (1980). National Council of Teachers of Mathematics. 1906 Association Drive, Reston VA 22091.

Stimpson. Virginia C. "Using Diagrams to Solve Problems." *Mathematics Teacher* Vol. 82. No. 3 (March 1989). pp. 194-199.

The Ideas of Algebra, K-12. 1988 Yearbook of National Council of Teachers of Mathematics, 1906 Association Drive. Reston VA 22091.

Thompson, Frances. *Five-Minute Challenges for Secondary School* (1988). Activity Resources Co., Inc., P.O. Box 4875, Hayward, CA 94541.

Waits, Bert and Demana, Franklin. "A Computer-Graphing-Based Approach to Solving Inequalities." *Mathematics Teacher*, Vol. 82, No. 5 (May 1989), pp. 327-331.

———. "An Application of Programming and Mathematics: Writing a Computer Graphing Program." *The Journal of Computers in Mathematics and Science Teaching*, Vol. 7, No. 4 (Summer 1988).

———. "Solving Problems Graphically Using Microcomputers." *The UMAP Journal* (Spring 1987). UMAP Journal, 60 Lowell Street, Arlington, MA 02174.

State-Adopted Textbooks (1990-1996)

Algebra I

Brown, Richard; Dolciani, Mary; Sorgenfrey, Robert; and Cole, William. *Algebra.* Houghton Mifflin Company, 13400 Midway Road, Dallas, TX 75244.

Coxford, Arthur and Payne, Joseph. *HBJ Algebra 1.* Harcourt Brace Jovanovich, Publishers, P.O. Box 612267, Dallas, TX 75261-2267.

Dilley, Clyde; Meiring, Steven; Tarr, John; and Taylor, Ross. *Heath Algebra 1.* D.C. Heath and Company, 1815 Monetary Lane, Carrollton, TX 75006.

Fair, Jan and Bragg, Sadie. *Algebra I.* Prentice Hall, Inc., 641 W. Mockingbird Lane, Dallas, TX 75247.

Foerster, Paul. *Algebra I.* Addison-Wesley Publishing Company, Inc., 1815 Monetary Lane, Carrollton, TX 75006.

Foster, Alan; Rath, James; and Winters, Leslie. *Merrill Algebra One.* Merrill Publishing Co., % Glencoe Publishing Company, 320 Westway Place, Suite 550, Arlington, TX 76018.

McConnell, John; Brown, Susan; Eddins, Susan; Hackworth, Margaret; Sachs, Leroy; Woodward, Ernest; Flanders, James; Hirschhorn, Daniel; Hynes, Cathy; Polonsky, Lydia; and Usiskin, Zalman. *UCSMP Algebra.* Scott, Foresman and Company, 2105 McDaniel Drive, Carrollton, TX 75006.

Smith, Stanley; Charles, Randall; Dossey, John; Keedy, Mervin; and Bittinger, Marvin. *Addison-Wesley Algebra.* Addison-Wesley Publishing Company, Inc., 1815 Monetary Lane, Carrollton, TX 75006.

Algebra II

Brown, Richard; Dolciani, Mary; Sorgenfrey, Robert; and Cole, William. *Algebra & Trigonometry.* Houghton Mifflin Company, 13400 Midway Road, Dallas, TX 75244.

Coxford, Arthur and Payne, Joseph. *HBJ Algebra 2 with Trigonometry.* Harcourt Brace Jovanovich, Publishers, P.O. Box 612267, Dallas, TX 75261-2267.

Dilley, Clyde; Meiring, Steven; Tarr, John; and Taylor, Ross. *Heath Algebra 2.* D.C. Heath and Company, 1815 Monetary Lane, Carrollton, TX 75006.

Foerster, Paul. *Algebra and Trigonometry.* Addison-Wesley Publishing Company, Inc., 1815 Monetary Lane, Carrollton, TX 75006.

Foster, Alan; Rath, James; and Winters, Leslie. *Merrill Algebra Two with Trigonometry.* Merrill Publishing Co., % Glencoe Publishing Company, 320 Westway Place, Suite 550, Arlington, TX 76018.

Hall, Bettye and Fabricant, Mona. *Algebra 2 with Trigonometry.* Prentice Hall, Inc., 641 W. Mockingbird Lane, Dallas, TX 75247.

Senk, Sharon; Thompson, Denisse; Viktora, Steven; Rubenstein, Rheta; Halvorson, Judy; Flanders, James; Hynes, Cathy; Jakucyn, Natalie; Pillsbury, Gerald; Usiskin, Zalman. *UCSMP Advanced Algebra.* Scott, Foresman and Company, 2015 McDaniel Drive, Carrollton, TX 75006.

Smith, Stanley; Charles, Randall; Dossey, John; Keedy, Mervin; and Bittinger, Marvin. *Addison-Wesley Algebra and Trigonometry.* Addison-Wesley Publishing Company, Inc., 1815 Monetary Lane, Carrollton, TX 75006.

Computer Software

Algebra I and II. IBM Corp., P.O. Box 1328-W, Boca Raton, FL 33429-1328.

Function Plotter [Graphing program for Apple]. Microcomputer Curriculum Project, Price Laboratory School, University of Northern Iowa, Cedar Falls, IA 50613-3593.

Master Grapher [Function graphing program developed at Ohio State University for IBM, Macintosh, or Apple II]. Addison-Wesley Publishing Co., 5851 Guion Road, Indianapolis, IN 46254.

Math Exploration Toolkit (MET). IBM Corp., P.O. Box 1328-W, Boca Raton, FL 33429-1328.

Omnifarious Plotter by Walter R. Dodge, New Trier High School, 385 Winnetka Avenue, Winnetka, IL 60093 [Public domain graphing software].

Superplot [Function graphing program for Apple II with 64K]. EduSoft, P.O. Box 2560-OJ, Berkeley, CA 94702.

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(2) operation of school bus routes or runs on a non-segregated basis;

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(4) nondiscriminatory practices in the hiring, assigning, promoting, paying, demoting, reassigning, or dismissing of faculty and staff members who work with children;

(5) enrollment and assignment of students without discrimination on the basis of race, color, or national origin;

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