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ABSTRACT

The School Science and Mathematics Association seeks to improve the teaching and learning of mathematics and science and to promote the integration and interrelationships among these disciplines. This monograph presents 20 articles that have appeared in the association's journal, "School Science and Mathematics," between the years of 1905 and 1988 that addressed that goal. After an introduction that explains the rationale and purpose of this monograph, the articles are divided into six sections. The sections present the following: (1) three articles that discuss the interdependence of science and mathematics; (2) four articles that discuss integrating science and mathematics in the school curriculum; (3) three articles that discuss science and mathematics in secondary education; (4) four articles that discuss science and mathematics in elementary education; (5) three articles that present unifying themes in science and mathematics; and (6) three articles that discuss science and mathematics in a technological age. (MDH)

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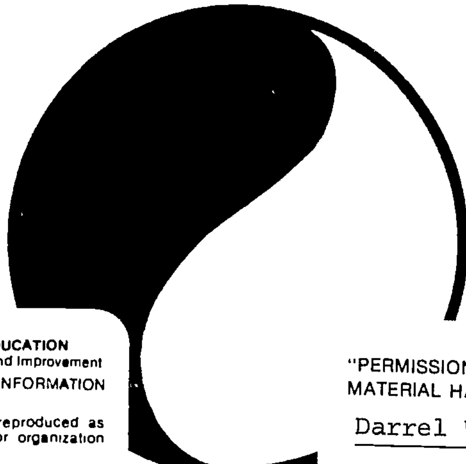
Topics for Teachers Series Number 5

Science and Mathematics:
Partners Then . . . Partners Now



Readings from *School Science and Mathematics* on the Integration of Science and Mathematics

Edited by Peggy A. House

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Topics for Teachers Series Number 5

Science and Mathematics:
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FOREWORD

The integration of mathematics and science can provide a broad base of relevant experience to promote meaningful learning. Such learning, coupled with opportunities for application of mathematics and science, facilitates generalization of concepts and processes as they are encountered in the environment.

The School Science and Mathematics Association seeks to improve the teaching and learning of mathematics and science and to promote the integration and interrelationships among these disciplines. SSMA, therefore, has as its goals:

1. To identify and explicate interrelationships between mathematics and science.
2. To facilitate the teaching of science and mathematics in formal and informal settings in a manner that reflects the interrelatedness of the disciplines.
3. To inform mathematics and science teachers of current and future trends, innovations and teaching techniques pertinent to their fields.
4. To encourage curriculum developers and publishers to create products that focus on the interrelationships of science and mathematics.
5. To promote standards of excellence in the preparation and professional development of teachers of mathematics and science.
6. To provide a forum for critical examination of the issues and trends in teaching and learning science and mathematics.
7. To stimulate research on issues related to mathematics and science education.
8. To support and cooperate with other professional organizations sharing mutual concerns for the improvement of science and/or mathematics instruction.
9. To influence policy makers at all levels on issues related to mathematics and science education.
10. To build public awareness and support for the importance of science and mathematics education for all citizens.

From the Purpose Statement of the
School Science and Mathematics Association.
1986

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Introduction

Peggy A. House

As an undergraduate physics major, I was once startled to find a question of the final exam that asked us to give the title of the book by X, an author whose name I have long since forgotten. Having no idea of the correct answer, but knowing that I could not fake so direct a question as that, I left the item blank. To my great surprise, I later learned that I had received +2 points for *not* knowing. The book, destined to become a standing joke around our laboratory, was titled *Physics Without Mathematics*, and we had been admonished never to check it out or use it. The exam question was comic relief from a good-natured professor, but it also served as a stimulus to serious discussion about the absurdity of trying to estrange science from mathematics.

Although that author's name quickly faded into oblivion, the lesson I learned from it did not, and as I completed majors in physics and mathematics, I came to believe that *everyone* assumed, as I did, that science and mathematics were inseparable partners.

My belief might well have been shattered quite abruptly had I begun my teaching career in a larger high school than I did, but instead I selected a small rural high school in which I was the whole mathematics department and half of the science department. Having the same students in science and mathematics courses, together with the benefit of a flexible schedule, afforded us the opportunity to study mathematics and physics in a much more integrated manner than is normally possible. I soon came to realize that I was a better mathematics teacher because I taught science, and I was a better science teacher because I taught mathematics. It is a belief I would cling to all my life.

So it came as something of a surprise when I eventually realized the degree to which science and mathematics have become separated in our educational system. Indeed, separation has become a characteristic of the curriculum in general, a trait that deepens as one progresses up the grades to high school and beyond. In his report on secondary education in America, Ernest Boyer, former United States Commissioner of Education and President of the Carnegie Foundation for the Advancement of Teaching, commented on the problem as follows:

> *The current instructional program reflects the compartmentalized view of curriculum. Students study world history at 10 a.m., economics at 1 p.m., biology at 9, health at 2. They are taught literature in one room, civics in another; fine arts on the second floor; French on the third. While we recognize the integrity of the disciplines, we also believe their current state of splendid isolation gives students a narrow and even skewed vision of both knowledge and the realities of the world.*[1]

Recently there has been increased effort on the part of educators to address the problems caused by the "splendid isolation" which Boyer described. Jacobs[2] reported that in 1988 the Association for Supervision and Curriculum Development (ASCD) conducted a poll that sampled ASCD members, chief state school officers, deans of schools of education, and others. The need for curriculum integration was identified as the number one issue among the respondents.

The same spirit of integration is reflected by the identification of *Mathematical Connections* as one of the four strands upon which the National Council of Teachers of Mathematics (NCTM) has woven its curriculum standards. The underlying assumption for that standard is the belief that, "The curriculum should include deliberate attempts, through specific instructional activities, to connect ideas and procedures both among different mathematical topics and with other content areas."[3]

Such a spirit of interrelatedness between mathematics and science is the foundation upon which the School Science and Mathematics Association (SSMA) was conceived. A history of the Association, published in 1950 in honor of its fiftieth anniversary, recounted the following developments:[4]

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[1]Boyer, Ernest L. High School. New York: Harper and Row, 1983. Page 114.

[2]Jacobs, Heidi Hayes. Interdisciplinary Curriculum: Design and Implementation. Alexandria, VA: Association for Supervision and Curriculum Development, 1989. Page 3.

[3]National Council of Teachers of Mathematics. Curriculum and Evaluation Standards for School Mathematics. Reston, VA: NCTM, 1989. Page 11.

[4]A Half Century of Science and Mathematics Teaching. Oak Park, IL: Central Association of Science and Mathematics Teachers, 1950.

By the turn of the century, science had set its roots deep in the society of America. Scientific discoveries became topics of everyday conversation. The experimental method of science rapidly gained popular approval and acceptance. A golden age of science--a phenomenon of western civilization--was at hand.

This rapid growth and public acceptance of science, coupled with a rapid increase in the enrollment of secondary schools, stimulated new development in the teaching of mathematics and the sciences.

On the crest of this wave of scientific interest and emphasis, a group of physics teachers from schools in the Central States met in Chicago in the spring of 1902 to consider the organization of an association of physics teachers. A committee of three was appointed which, after further consideration of the matter, called a meeting to be held in Chicago, June 7. Twenty-five schools were represented at this general meeting. It was a business meeting with no program of papers, and here it was that an organization entitled the Central Association of Physics Teachers *was formed. A constitution was adopted and . . . officers (were) elected. . . . Preparations for the next meeting were at once begun . . . .*

The fact that the scientific impetus which stimulated the organization of physics teachers influenced other areas is evidenced in the action taken by the Mathematics Section of the Educational Conference of Academies and High Schools in November of the same year. These teachers were concerned with the improvement of instruction in mathematics by introducing the laboratory method and by bringing about a closer correlation of mathematics with other subject matter of the curriculum, especially physics.

To this end, they set in motion plans which resulted in a petition, signed by the teachers of mathematics, being presented at the Thanksgiving session of the Physics Association. This petition embodied the request that a larger association to include all the sciences and mathematics be considered. During the winter, plans were formulated to include mathematics and the other science fields in the April meeting of the Physics Association. This meeting . . . was the culmination of the

> *unification movement. The larger organization was named the* Central Association of Science and Mathematics Teachers. . . .
>
> *The third meeting, held November 27-28, 1903, . . . was the first program meeting of the larger organization. By now, the Association was well established and the general pattern of its meetings was set up much in the manner that it remains today. Dr. John Dewey, then Professor of Philosophy and Education and Director of the School of Education at the University of Chicago, made the leading address. His subject was* The Disciplinary Value of Science Teaching.
>
> *Following the third meeting, the close correlation of science and mathematics was evidenced in the papers presented. This correlation has remained a major theme in the meetings of the Association to the present time.*

One year after the expansion into the Central Association of Science and Mathematics Teachers (CASMT), in November 1904, the treasurer reported an Association balance of $35.52. A yearbook published that year listed 272 members in Illinois, Ohio, Wisconsin, Indiana, Michigan, Iowa, Minnesota, Missouri, Colorado, Nebraska, and North Dakota.

In the years that followed, growth of the CASMT was steady, if somewhat slow. In 1928 the Association was incorporated under the laws of the State of Illinois. As the membership continued to expand beyond the central states from whence the CASMT had begun, the Board of Directors sought an identity that better reflected the national character of the Association. Accordingly, in November 1970 the bylaws of the Association were amended to change the name of the organization to the *School Science and Mathematics Association*.

The name *School Science and Mathematics Association* (SSMA) was chosen very deliberately, for it represented the activity for which SSMA and its predecessor, CASMT, were perhaps best known: the publication of the journal *School Science and Mathematics*.

Like the Association itself, the journal that was to mature into *School Science and Mathematics* began with science. The first issue of *School Science* was published in March, 1901, one year before the birth of the infant Association. In 1903 the *Mathematical Supplement of School Science* appeared; in 1904 it became a separate journal, *School Mathematics*. By the end of that year, the two publications had been wedded into one: School *Science and Mathematics*.

Although in the early years the journal and the CASMT were, in fact, two separate educational agencies, they were always closely associated and *School Science and Mathematics* was always considered to be the official journal of the CASMT. Following the legal incorporation of the CASMT, the Association purchased the journal which it has owned, managed, and published ever since.

The early years of this Association were, without a doubt, exciting years to be teaching science and mathematics, although teachers of today can easily lose sight of the revolutionary developments taking place at the time. In 1900, the number of elements on the periodic table was only 83. It had been only three years since Thomson's discovery of the electron, and only two years since his measurement of the charge-to-mass ratio. Millikan would not measure the electronic charge for more than a decade, and Einstein was still five years away from publishing his theories of the photoelectric effect and of special relativity. It would be a quarter of a century until the development of quantum mechanics, and high-speed computers were still many decades in the future.

For ninety years, *School Science and Mathematics* has reflected developments in science and mathematics teaching in the United States. During those same years, SSMA has remained true to its original goal of promoting the partnership of science and mathematics as well as improving the teaching of each. This volume commemorates ninety years of dedicated professional activity.

The papers that follow, all reprinted from *School Science and Mathematics*, span more than eighty years. They reflect the continuity of our disciplines and of our problems. Many of the issues that confronted educators decades ago are as contemporary as any we can name. Thus, for example, Kinney (in 1930) recognized the mutual interdependence of science and mathematics but bemoaned students' inability in making transfers and applications. Karpinski (1929) contended that progress in mathematics necessarily precedes progress in science and that a mathematically prepared mind is essential for great scientific insights to occur. Ingraham (1945) portrayed science and mathematics as basic components of a liberal education and insisted that they must be taught along with their historical and cultural underpinnings.

In 1905, Bishop described attempts to unify physics and mathematics instruction, and the laboratory approaches used in the process. Sixty years later, Kullman (1966) traced the history of attempts to integrate science and mathematics in the schools. Questions of how to structure the learning process were addressed by Ost (1975) and by Cooney and Henderson (1972).

Breslich (1936) saw problems of science as the vehicle for increasing students' mathematical power, and he offered examples of strategies for furthering such integration. Carpenter (1962) examined the relationships between school mathematics and physics, while Schaaf (1965) proposed a sample of scientific concepts that can and should be taught in conjunction with junior high mathematics.

The partnership between science and mathematics in the elementary school curriculum was not overlooked. Brogan (1939) posed the question of whether science (including arithmetic) is a body of subject matter to be acquired or a set of procedures students need to develop in order to secure increasing control over their lives. Nelson (1962) pointed out that opportunities for normal integration of science and mathematics occur daily when working with young children, and she urged teachers to take advantage of such opportunities. Brown and Wall (1976) discussed content common to mathematics and science that can and should be dealt with in a laboratory situation with children, and Benham, et.al. (1982), described activities that encourage very young children to develop necessary observation skills.

Other writers examined specific aspects of the curriculum. Georges (1926) saw *functional thinking*, i.e. the investigation of relationships between associated quantities, as central to both mathematics and science, and he noted that mathematics and physics are inseparable in their dependence upon relationships. Goodrich (1935) presented algebra as a medium for interpreting and controlling nature; Bubb (1937) saw geometry as guiding the development of natural science.

Contemporary issues were addressed by Dean (1975) and by Ost (1987) who discussed the role of modelling and simulation in contemporary science and mathematics. House (1988) reflected on the dispositions that contemporary students and teachers of science and mathematics will require for success in the technological society that will characterize SSMA's second century.

In some ways, there is evidence of progress over the century. For example, the obvious references to virtually all mathematics and science students and teachers as male, a characteristic of all of the older papers, has vanished from contemporary writing. On the other hand, it is hard not to be depressed by the realization that we are still, after all these years, trying to solve so many of the same problems that plagued our predecessors.

So as we near the close of this century, it behooves us to reflect on the themes that are interwoven in this sampling of writing taken from throughout the past 90 years. The motivation that gave birth to the infant CASMT ninety years ago and the challenges that drove the members over the decades remain the central focus and the urgent needs of the mature SSMA:

Science and Mathematics–partners then, partners still.

Although mathematics has developed, and is developing, within itself as a system of thought, its development has been, and is now, spurred on by the demands of the sciences. The sciences, in turn, owe their development, in large measure, to mathematical methods. Moreover, the more mathematics contributes to their development, the more do they become dependent upon it.

J. M. Kinney, 1930

I. The Interdependence of Science and Mathematics

The fundamental mathematical theory has always developed independently of the physical phenomena which it explains. Again and again it would almost seem that the experimental scientists waited to allow the mathematician to go ahead and pave the way. The physicists and the scientists who have made progress have approached the science, in general, with the mathematical tools prepared, with adequate mathematical equipment.

L. C. Karpinski, 1929

May I say quite frankly that I do not think a man who has narrowly specialized in mathematics or who has narrowly specialized in one of the sciences has gained, thereby, enough to make up for what he has lost.

M. H. Ingraham, 1945

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Cooperation in the Teaching of Science and Mathematics

J. M. Kinney
(Vol. XXX No. 3 March, 1930)

This Journal bears the name *School Science and Mathematics* because of the recognition of the fact that science and mathematics are closely related systems of thought. The interdependence of these two fields comes about in large part because they are both interested in variables and functionality. Thus, to carry out a scientific investigation of a phenomenon is to note variables associated with it, to collect quantitative data relative to these variables, to arrange the data in some sort of order for the purpose of displaying a relationship among them, and finally, if the investigation has a successful outcome, to find the precise character of this relationship; that is, to find the mathematical law governing the phenomenon and express it in the symbolism of mathematics.

We find a good example of scientific procedure in the work of Tycho Brahe, Kepler, and Newton relative to the behavior of the planets. The great mass of data collected from observations on the planets made over a long period of time by Tycho and Kepler led the latter to the discovery of three relationships between certain variables and known as the Laws of Kepler. From these laws as a basis of a mathematical investigation, Newton discovered a law of gravitation which held for the planets, a law which he later assumed to be the Universal Law of Gravitation.

The laws of science give rise to functions which become objects of investigation in the field of mathematics. Many times it happens that these functions arising from widely different fields of science are "concrete" instances of an abstract mathematical function. Thus, the law of gravitation, $F=k/d^2$, the law of the intensity of illumination, $I=k/d^2$, and the resistance of a wire to the flow of electricity,

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$R=k/r^2$, may all be expressed in the form, $y=k/x^2$, having no reference to a concrete situation. This function may be studied in the abstract and the results of the study may be applied to any concrete situation giving rise to a function of this form.

We have been saying that science and mathematics are related. This relationship is put quite vividly in evidence in the highly developed physical sciences. Some of the more recently developed sciences, such as chemistry, biology, medicine, psychology, education, sociology, economics, anthropology, meteorology, and many others, are assuming a mathematical form. Although mathematics has developed, and is developing, within itself as a system of thought, its development has been, and is now, spurred on by the demands of the sciences. The sciences, in turn, owe their development, in large measure, to mathematical methods. Moreover, the more mathematics contributes to their development, the more do they become dependent upon it.

It follows, therefore, that people working in the scientific field are acutely conscious of the need of an extended mathematical training. Those who have not made an adequate mathematical preparation find that they are greatly handicapped in reading modern scientific literature. On the other hand, many people working in the field of mathematics feel the need of keeping in touch with science. They get pleasure from seeing the applications of mathematics to concrete situations. In recent years teachers of mathematics have been pleased to see numerous articles in this Journal and elsewhere bearing on the applications of mathematics to widely separated fields of scientific investigation.

On account of this increasing reciprocal interest on the part of teachers of science and mathematics, there will no doubt be a marked increase in the enrollment of students in these fields. The percentage of increase should be greater in the department of mathematics. The large majority of students enrolling in this department will expect to get therefrom ability to do the quantitative thinking of science.

The problem of making the mathematical training of the student function in the field of science presents itself for solution more urgently than it has in the past. The solution of the problems calls for a sympathetic spirit of cooperation on the part of teachers in both fields. In the past this spirit has not been sufficiently in evidence. Teachers of science have complained that their students could not satisfactorily apply their mathematics; that not only was there but slight transfer but that also there was but little evidence of mastery of mathematical principles. This statement holds for arithmetic, algebra, and geometry. Many teachers of mathematics have replied that it was their business to teach mathematics as such, and that it was the business of the teacher of science to see that the transfer was made.

We feel that both of these attitudes are wrong and should be replaced by a sincere desire for cooperation. The proper spirit of cooperation can be brought about if teachers of science and mathematics recognize the fact that they have very much in common. They are both dealing with quantitative problems. They are both dealing with variables and their relationships. It is this notion of functionality, especially, that is of such fundamental importance in both science and mathematics. They are both interested in a symbolic language for the expression of their quantitative thoughts.

The learning product turned out in both fields will be much more satisfactory if their respective teachers can agree as to the emphasis that should be placed by each on the various phases of the quantitative problem. We feel that such an agreement can be attained. We suggest that teachers of mathematics teach the symbolic language. They should not confine their use of letters to x and y. but should use freely the letters that are used in the formulas of science. They should teach the fundamental processes and see that they are applied not only to abstract but also to concrete situations. They should stress functional thinking from the beginning to the end. They should lead students to see that an abstract functional form may have many different concrete applications. They should give students abundant practice not only in passing from the concrete to the abstract but also in passing from the abstract to the concrete.

Teachers of science should recognize the fact that their quantitative problems deal for the most part with technical situations. They should note the fact that students may be required to recall a mathematical notion or process from arithmetic, algebra, geometry, or, possibly, trigonometry and bring it to bear on their problem. If the problem involves a functional relation, as it quite probably does, they should perhaps help the students recall the abstract form of the relationship. Above all teachers of science should be conscious of the fact that transfer for the average student takes place with difficulty even in such closely related fields as science and mathematics.

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Mathematics and the Progress of Science

Louis C. Karpinski
University of Michigan
(Vol. XXIX No. 2 February, 1929)

The other day I read an advertisement of a new book on the practical applications of the fairly old subject of probability. The practical author asserts that probability is "no longer the plaything for the entertainment of the erudite mind, but is a powerful instrument of practical science." This statement reflects an absolute misconception of the nature and purpose of mathematical reasoning. Fortunately it is only a few shallow scientists to whom the powerful tools placed in their hands by pure mathematical research will appear as playthings devised for entertainment. Mathematics needs no justification! The theory of probability has been studied by intelligent men as a part of fundamental truth. It happens, then, that the theory applies to many situations in the work-a-day world; it happens that in manufacturing the theory applies to the distribution of a large group of objects, like electric light bulbs, into groups having given characteristics; it happens that in insurance and in other industrial operations probability has its applications. But when these light bulbs are replaced by some more effective method of illumination, and when these industrial devices have long been placed in the discard, the theory of probability will continue to illuminate, will continue worthy of serious study by intellectual beings.

The world of mathematics is a reality more enduring than the animals and plants, the changing objects and situations among which we live. To this reality the mathematician devotes his attention because he must. It is an inner compulsion.

Mathematics is closely akin to art in its methods and in its inspiration. The poet feels within himself an impulse to write in verse; the painter has before his mind's eye a beautiful picture; the architect

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sees a majestic building; the great musician hears a beautiful melody. And in each case the artist creates primarily for himself, to satisfy that inner demand. So it is with the mathematician; the theories which he elaborates are ends in themselves, sufficient unto themselves.

These numbers, these algebraical symbols, these geometrical elaborations, these extended speculations do correspond, it frequently happens, to diverse actual situations in the world of commerce and of science. By means of these tools the things which happen in the laboratory, in the community, and in the universe of the stars or the atoms may often be measured, studied and understood.

The fundamental mathematical theory has always developed independently of the physical phenomena which it explains. Again and again it would almost seem that the experimental scientists waited to allow the mathematician to go ahead and pave the way. The physicists and the scientists who have made progress have approached the science, in general, with the mathematical tools prepared, with adequate mathematical equipment.

Take such simple curves as the conic sections associated through the analytical geometry with quadratic equations.

These curves, first as cut from cones with varying vertex angles, the ancient Greeks studied simply and solely because these curves seemed to them logically to follow upon the circle. A whole succession of Greek mathematicians studied the properties of these curves; they learned how to draw tangents and normals and even tangents from an external point. Finally Archimedes learned how to compute the area of segments and the volumes of related solids. All of this the mathematicians did without happening to find the focus of the parabola which Pappus placed upon the map some four hundred years later. No one of these men knew or cared about any practical application of these curves.

Nearly two thousand years later Kepler, a student of mathematics and astronomy, found that the orbit of the earth was not circular. Kepler turned naturally first to the ellipse with whose properties, as a serious student of the mathematics of his day, he was familiar. So he was able to find that the paths of the earth and other planets are ellipses with the sun as one focus. Kepler was able, through his familiarity with the conic sections, to enunciate the three theorems of planetary motion. This knowledge of the properties of the conic sections Kepler obtained as a student of mathematics. Neither the Greeks who began these studies nor Kepler when he studied the Greek developments had any notion of their application to astronomical problems. Had Kepler not had this background of mathematical information and training the world would have waited considerably longer before the astronomical facts would have received their correct interpretation.

Not long afterwards Galileo Galilei wished to investigate by experiment the motion of a falling body. Here again a simple mathematical law was found to explain the motion. Galilei was able to show that the space passed over in successive seconds was proportional to the successive odd numbers. As a result, the total space passed over by a freely falling body is given by the quadratic equation $s=16t^2$. This of course is connected with the beautiful fact that the sum of first n successive odd numbers is n^2.

Upon these foundations of the conics and their properties, and of the planets and their motions, Isaac Newton built his universe. Essential to his formulation and explanation was not only the mathematical work on the conics, the work on the planets and on falling bodies as given by Kepler and Galilei, but equally the new tool of analytical geometry which had just been placed in his hand by Descartes. The three laws of planetary motion, as enunciated by Kepler, became special instances under a more comprehensive law, as given by Newton, and because Newton's equations are more powerful and more comprehensive than Kepler's, for that reason Newton is a greater genius than Kepler.

Kepler and Galilei and Newton are universally regarded as great scientists, not because of their remarkable powers of observation but primarily because back of the observed facts these men saw mathematical formulas. Not only are the observed facts subjected to the mathematical formulae, but out of the mathematical formulation comes new light upon the observed facts, with new facts and new aspects revealed only by observation again in the light of the formula. The real value of a formula is revealed, not in its explanation and interpretation of what has happened, but rather in its power to guide the intelligent observer to new truths.

Today the automobile engineer uses the parabola to fashion the automobile headlight; the architect uses the parabola to build the auditorium and to build his finest bridges; the student of projectiles begins with the parabola. Progress is made by the use of these geometric curves with simple algebraical properties. While these curves were originally studied as simple geometrical exercises, the teacher today who does not point out their many uses in practical affairs misses a great opportunity to impress upon the pupil the part of mathematics in the progress of science and to impress upon the pupil the universe as ruled by mathematical law.

As I have said, the outstanding characteristic of a great physicist or astronomer is the ability to interpret and extend his observations by means of mathematical formulas. Only rarely is there an exception to this rule. Such an exception was doubtless Faraday whose contributions to the science of electricity press in upon us wherever we live. However, Faraday himself knew that lack of familiarity with mathematics was a serious drawback. Upon Faraday's observations Clark Maxwell reflected and evolved the famous Maxwellian equations. As an almost immediate magnificent product of the mathematical consideration came the electro-magnetic theory of light. Again twenty years later, upon the basis of the mathematical formulation, the German Hertz regarded the phenomena anew, under the light of the mathematical formulas, and the marvels of wireless electricity began to appear.

These mathematical speculations and their mathematical consequences created a new heaven and a new earth, for the old physics is passed away. The new heaven is mathematically determined to be finite, while the new earth consists of mathematical combinations of a hydrogen nucleus with attendant electrons. However, while these mathematical speculations so intimately connected with the relativity theory are beyond the ordinary scientist, yet everyone who listens to the radio should recognize also the voice of the mathematical formula.

By the recent development in the new physics, the work of Maxwell takes a place alongside Newton's *Principia*. In these works we have the universe subjected to that mathematical formulation. A great body of physicists have contributed to the mathematical extension of Newton's universe as expressed in the Einstein emendations. In this new universe the great groups of phenomena such as those connected with light and electricity and magnetism are treated simultaneously as different aspects of similar phenomena. The same equations govern different phenomena and often give remarkable analogies. Chemistry and astronomy and even medicine cannot remain aloof from these mathematical considerations of the electrons. No one can see or prophecy the end, but one can say with confidence further progress will be reflectcd in further mathematical formulation.

Doubtless the Maxwell equations represent the highest contribution of mathematics to the understanding of the universe in which we live. Yet the same type of illumination is afforded by diverse other fields of pure mathematical speculation. For many centuries the students of mathematics labored with the problems arising out of the representation of numbers. When it was found that negative numbers could be represented by points on a line symmetrically placed with respect to an origin, representing 0, a great step was made for advance in the theory of equations. At the same time the notion of a vector was suggested, so that by consideration of the line segment as having magnitude and direction a powerful tool was given to the physicist. It was comparatively easy for the mathematician to see that the $\sqrt{2}$, $\sqrt{3}$ and other quadratic irrationalities would be represented by absolutely definite points upon this line of real numbers. The fact, too, that to each real number corresponded a definite point upon this line also offered little difficulty.

In this way the negative and irrational solutions of equations came to be accepted as on a par with solutions in rational numbers. However, the complete acceptance of the complex number waited upon some graphical representation. To this problem no less than three mathematicians, the Norwegian Kaspar Wessell, the great German scholar Gauss, and the Frenchman Argand, found independently the solution, the complex number diagram which is known today even to our high school students.

For the theory of algebraical equations this conception gave to that great genius, Gauss, the suggestion for the proof of the theorem that every polynomial in x with real and complex coefficients has a root, the so-called fundamental theorem of algebra. But it was not long before the physicist discovered that this new speculation of the mathematician afforded elegance and simplicity in the consideration of diverse problems of nature. Thus the theoretical treatment of alternating currents is made easily intelligible by a Steinmetz, employing the symbolism and the methods of operations with complex numbers.

Progress in science has been preceded by progress in mathematics. Certainly so far as the science of physics and astronomy is concerned it is true that progress has been expressed in mathematical formulas. The laws of physics and the laws of planetary motion are mathematical statements concerning the physical phenomena. Today even the biologist and the student of medicine, as well as the chemist, are looking to the physicist for an attempt at a mathematical formulation of the phenomena of the test tube, of the growing plant and the human body.

For the secondary school teacher the question is vital: Can the student study physics and other science first and learn the necessary mathematical formulas and methods as the need arises? The answer of history is absolutely and emphatically, *No*. Had Kepler not been familiar with the mathematical theory of the conic sections he would not have been able to formulate the laws of planetary motion. Had Newton not been acquainted with the mathematical advances of his day, and the geometry of the Greeks, his conception of the universe could not have been formed or formulated.

Helmholtz made enduring contributions to the theory of sound because as a young student at Berlin he occupied himself with mathematics and physics. When his work in medicine brought to his mind the problems of sound, Helmholtz had the indispensable preparation, and he was able to solve problems which would have been for him forever unsolvable if he had been required to go back to build up the theoretical foundation.

Maxwell had that thorough mathematical preparation which enabled him to see the formulas back of the electro-magnetic phenomena. A whole host of recent physicists like Michelson and Einstein have had such a profound grasp of mathematics that the new physics, a mathematical creation, has emerged. The mathematical training of these men began in their early student days and it has continued ever since.

For the boys in your high school the lack of the instruction in elementary algebra, in geometry, in elementary trigonometry will absolutely bar their way to the sciences; it is as serious for them as lack of mathematical training is for the physicist today. For 99 out of 100 in college it is too late to turn back, to pick up that elementary instruction in the mathematics which today, more than ever, is essential for progress in science. "Time never turns back" and the student who waits to learn his mathematics and the physics, the theory, as he needs it, that student will never find it necessary. Progress will not wait for him.

It is not possible in the limited time to touch upon many of the contributions of mathematics to progress. In economics, in many phases of biology, in insurance, in almost all phases of engineering, mathematics plays an important role. Newton would be surprised and pleased that his binomial expansion proves so usual in modern business. Leibniz and Newton and Descartes would congratulate themselves upon the wide uses found for the methods of analytical geometry and the calculus. The mathematicians of Greece and India and Arabia and Europe who for pure love of science evolved the trigonometry would be surprised to find their tables and their methods in daily use in the great factories and in the workshops.

But no one of these thinkers of past ages would feel that his search for mathematical truth needed the practical application to justify his study. Even in those ancient days the circle and the parabola and the ellipse were realities, and the numbers of arithmetic and the symbols of algebra and geometry and trigonometry and calculus created a world whose reality cannot now be disputed.

Today more than ever number rules the universe; and appreciation of the universe in which we live as ruled by number and by form is cultivated by the proper study of mathematics. Number and form are guiding principles in all reasoning involving quantitative relationships. Even in the fields of ethics and philosophy and religion the serious student is frequently compelled to become precise and definite by mathematical illustration. Thus it is related that when Plato lectured on beauty some of the listeners protested because he began with geometry. And today for any intelligent comprehension of the economic foundations of our society and for any slight understanding of the universe of the stars or the atoms, for appreciation of many of the most fundamental developments which distinguish the reasoning creature, man, from the dumb brutes, the study of mathematics is essential. Show to your pupils that mathematics opens to their minds many doors which will in all likelihood forever remain closed if in the high school the opportunity to begin this study is neglected.

3

Mathematics and Science in a Liberal Education[1]

Mark H. Ingraham
University of Wisconsin
(Vol. XLV No. 2 February, 1945)

Ladies and gentlemen, as I understand it, when the Romans used the phrase "Liberal Arts," *Artes Liberales*, they referred to those branches of learning which only free men were permitted to pursue. Although I do not, myself, believe in always strictly following the original meaning of the word or phrase, today I want to explore the contribution that the study of mathematics and science can make to the life of a freeman, and to say something about how this contribution can be enhanced.

I wish to discuss this subject in relation to three aspects of the education of a free man. First, we should seek the enrichment of the life of the student, and I do not believe it is selfish on his part to seek this enrichment. It is his birth right, left to him by generations of scholars. Second, the early education of a free man should leave freedom for later specialization and later choices of occupation. Third, the education of a free man should equip him for his duty as a citizen of the state.

When I say that the education of a free man should enrich his own life, I mean this in a very broad sense, spreading all the way from the mere relief of boredom to the deepest intellectual and esthetic satisfactions. Even the relief from boredom is no mean accomplishment. Americans pay billions of dollars a year for this and, until rationing, untold quantities of gasoline. But, of course, the enrichment of life goes far beyond any escape mechanism. To lead a

[1]Presented before the Junior College Group of the Central Association of Science and Mathematics Teachers at Chicago, November 25, 1944. The author was Dean of the College of Letters and Science, University of Wisconsin

rich life a man must be the kind who will draw out the best in the companionship of others. No man is the same to two different persons, and another person may give little to me where he has much to give to another, not because of the fault of the giver, but of the receiver. Enrichment of life includes a sense of the unity of one's life with that of the race and with one's environment. Enrichment of life includes a satisfaction at the level of the appreciation of poetry, or art, and of the understanding of knowledge. It also includes just plain fun.

A man is not a free man if his past has tied his future to such an extent that he is constantly not making choices of volition but by the compulsion of inadequacy. It is the minority of cases, and I believe the unfortunate ones, where a boy in high school really knows what he wants to be when he is a man. I had the good fortune of not deciding on my profession, although I thought I had, until I was through college and through two years in the service. One of the ablest scientists I know, a member of the National Academy, majored in the classics and lives without regret that he did so. One's freedom should be maintained as long as possible. One of the greatest accomplishments that I know is to come up to the emeritus status with the flexibility to still make a decision as to what one should do. How would this do for a definition of liberal education? A man is liberally educated if, when he becomes emeritus, he genuinely has a large freedom of choice of occupation. I think with pleasure of three scientists, one of whom spent his years after retirement in the intensive study of Japanese prints; the second in writing delightful essays of noble inspiration on the lives of great scientists; and the third, who had had his scientific career interrupted by about forty years of college administration, has made an international reputation in biology since reaching seventy-five.

But no freedom is complete. One man's activity limits his neighbor, and the free man must also share in the government that determines the controls of his and his fellows' freedom. The education for freedom must include an education for this participation. Freedom without such education is dangerous, and I am not sure but that the Romans were right in also considering as dangerous such education without freedom.

I turn now to consider the contributions that science and mathematics make, or can make, in the education of a free man, especially with respect to the three forms of freedom I have mentioned. Because I put certain objectives first and the means last, do not think that I have fallen into the currently fashionable error of believing that is how we build sane educational practice. Some day it may be possible to do this, but as yet we know far better what portion of our education is good than why it is. The real logic of this paper is that 1) experience, subjective and objective, has amply demonstrated the liberalizing value of mathematics and science and some inadequacies in its teaching; 2) the knowledge derived from this experience may be organized in many ways, some of them illuminating. The present description is derived from what to the author is such an illuminating organization. But before that attempt to fit these contributions into a pattern, I would like to say a few words as to what those contributions themselves are.

First of all, the study of science and mathematics introduces us to a large body of organized and valuable information. I think we too often, in saying what scientific education can do in the way of introducing us to methods of thinking, forget the value of an informed mind. It seems to me that we should mark as fatuous any scientific education that isn't very largely informative. Secondly, a study of science and mathematics introduces us to great examples of constructive imagination. Of course, not the only examples. On one occasion, I risked my reputation by listing as, in my opinion, the greatest book ever written by a single individual, Dante's *Divine Comedy*, and further risked it by placing Newton's *Principia* second. In the company I then was, the second statement was the greater risk. I presume that is not the case now. It is not just the facts, but the organization of the facts that marks science. The arrangement of the Periodic Table, the Mendelian Hypothesis, the tracing of the history of the glacial epoch, and the quadratic reciprocity theory are not examples of mere noting of facts, but of a type of imagination that can conceive those facts in their relationships. Thirdly, though trite, we must not forget that the study of science should give insight into the scientific method including such aspects as the method of forming deductive patterns, most clearly exemplified in mathematics; the method of forming hypothetical patterns to be exploited deductively and checked inductively; the method of the laboratory, which both checks these hypothetical patterns and leads one to form new patterns; and throughout the attaining of certain standards of excellence, as for example the standards of mathematical rigor and mathematical elegance, and the standards of scientific verification. We should also recognize that both mathematics and the mathematical sciences develop the ideas of quantitative thinking to a marked degree.

With this introduction I would like to point out how the study of mathematics and science can serve the education of the free man as detailed above, with some comments as to how this service can be maximized.

First of all, I think the study of mathematics and science is for a great many students fun, and can be made so for many others. The last time I looked up the figures there were a higher percentage of F's and a higher percentage of A's in mathematics than in any other subject of large election in the University of Wisconsin. Correlative to that, I believe there are more students who hate mathematics and more students who love it than any other subject. I just cannot understand how a person would not enjoy Euclid's proof that there are an infinite number of primes or the fact that every integer can be expressed as the sum of four squares. Incidentally, in my opinion there is more fun in number theory than in most branches of mathematics, and there are parts of number theory, for instance the sieve of Eratosthenes, which may be shown to the students far before algebra is usually introduced. Other people find a great deal of pleasure in the coordination of brain and hand, which is developed in the demonstration of natural phenomena by technical laboratory means, and it would be pretty difficult to be bored in the country side if one knew a little of botany and ecology.

The man who has a knowledge of natural phenomena is far less lonely than he who has not. It not only opens for him the companionship of many of the best minds around him, but also the sense that he is a part of an orderly yet wonderfully diverse universe. In the teaching of science, in particular, it seems to me to be well to stress this wholeness and this diversity even at the expense, at times, of time spent on acquiring technical knowledge.

May I say quite frankly that I do not think a man who has narrowly specialized in mathematics or who has narrowly specialized in one of the sciences has gained, thereby, enough to make up for what he has lost. The man who has shut out from himself a continuing interest in literature and in history is blind to a whole band in our intellectual spectrum. I am, therefore, not pleading for the life of a scientist to the exclusion of all else but for the enrichment which science can bring to a life which is also to be enriched by the study of the humanities. I think, therefore, one should consciously consider in teaching either mathematics or science that broad relationships of man and his environment should be revealed to the student and the plain pleasure that even a non-scientific student can get from the study, as well as any technical efficiency that may be attained.

In planning a curriculum I believe one of the prime considerations is to keep, as long as possible, as many doors open for the student's future as can be. Mathematics is a good illustration. To a mathematician, mathematics is a joy in itself, but for everyone it can be a useful tool. The degree of usefulness depends upon the extent of the knowledge, but to even a greater degree upon the mastery of the portion of the subject covered. At the University of Wisconsin a student may enter the university without any high school mathematics. However, unless he has had a year of geometry and a year and a half of algebra, he may not have full admission to the College of Engineering nor the course in Agricultural Engineering; and unless he has had a year of geometry and a year of algebra, or unless he has made these up (which is seldom done) he may take no work in astronomy, mathematics or physics; he may not enter the course in Chemistry, Medical Technology, Nursing or Pharmacy; and he may not major or specialize in American Institutions, Bacteriology, Biology, Chemistry, Commerce, Economics, Geology, Humanities, International Relations, Mathematics, Physics, Pre-Medicine, Political Science, Psychology, Sociology or Zoology. He may not become a junior in the College of Agriculture. These decisions were not made upon the urgings of the Department of Mathematics, but upon the advice of the individual departments concerned as to whether or not the student was qualified to do the work unless he has had this small amount of mathematical preparation. These are too many doors for a boy of fifteen or sixteen to slam in his own face.

No one of the sciences could probably furnish a similar list, and yet whole areas of understanding are closed if at the proper time a student does not become acquainted with the basic scientific facts in several fields, and with the scientific method. To keep one's choices open as to both vocations and avocations, it is clear that a person should have a fundamental knowledge of history, science, and the social studies, an appreciation of literature, ability to read and write accurately (the study of science is one of the media for attaining this ability and should be consciously used), the ability to think quantitatively and at least to be accurate in elementary calculations. I would grant immediately that a person may not meet all these requirements and may still have a rich life, and by no means be a failure. I am asserting that such a person is not as free as he should be.

When we deal with the duties of the citizen, it must be granted that the first subjects we must insist upon would be history and the analytical social studies. However, the problems with which a community must deal and deal intelligently require not only the advice of the expert scientist, but I believe also the judgment of a citizenry, not too ignorant of the basic ideas of science and technology. One of the men I know who has the broadest conception of the proper controls of radio is an electrical engineer. The decisions as to the issuance of bonds for a new sewer system certainly involve problems of bacteriology, and the person who gets scared every time he sees a statistical table is, in my opinion, unqualified to vote.

I wish to end with a few more or less dogmatic statements concerning the teaching of science and of mathematics which I believe are the corollaries of the foregoing development. The more dogmatic statements will be concerning mathematics; the less, concerning the teaching of science. First, throughout the mathematical training should be placed more emphasis on the mastery of arithmetic, including the ability to carry out long arithmetic processes accurately with suitable checks. This facility with arithmetic gives the insight that makes algebra easy, opens up many applications to the individual, makes it easier for him to catch the buncombe of a grocery clerk or a politician, and next, perhaps, to concise and grammatical speech and understanding reading, is the most used of our intellectual tools. Secondly, I would make mathematics more amusing by including some topics just because of the fun involved. Illustrations are items connected with prime numbers, and I would say a reintroduction of continued fractions. Thirdly, I would place more emphasis on proofs than is currently the fashion. Please do not misunderstand me in this regard. I think there is little value in memorizing a proof in the step-by-step detail fashion that some students resort to, and I do not know that we need to cover more proofs than at present. I do feel that greater emphasis on the nature of proof and the understanding of proofs is needed. We do not ask students to go through proofs chiefly to convince themselves that a statement is true. Generations of scrutiny of Euclid will be a better guarantee of the validity of the reasoning than the student's own examination. But proofs of theorems indicate the interrelationship between one portion of a theory and its general background. It is only through the proofs that the structural pattern is seen. It is only through the proofs that the constructive imagination of a mathematician is understood. It is chiefly through the proofs that the beauty of mathematics is revealed to the student. I would suggest that frequently, in place of proving five theorems, it would be wise to get five proofs of the same theorem. Take a theorem like that of Pythagoras. I have used that theorem constantly and have known how to prove that theorem occasionally for the last three decades, but it was not until recently when I noted a proof new to me that I understood certain aspects of its relationships to other parts of mathematics that I had never seen before, and that the theorem became a "natural" instead of a sport.

Where, you may ask, will the time come for more drill in arithmetic, for more fun in subject matter, and for more attention to proof? My answer is a clear-cut compromise: You will have to get some of the time by omitting topics, and you will have to get a great deal of the time by demanding more of the student than is usually done, and in some cases you will have a chance to do what I indicate only for the better students who can brush off the ordinary amount of mathematics in a trivial amount of time. I may add, however, that I am so firmly convinced that much of the time and energy that a student puts on mathematics could be saved if the arithmetic were more soundly founded, that I believe that aspect will be time saving rather than time using.

I am very much more tentative in my conclusions as to the teaching of science. My first conclusion, however, is the negative one that it is unwise to give up laboratory work in science to teaching exclusively by lecture and by demonstration. However, I believe that for a very large number of students, both those who are not going to be scientists and those whose scientific endeavor is going to be in a different field, the laboratory work could be modified so as to emphasize to a lesser degree modern technology and to a greater degree scientific imagination. Certainly a student should have the opportunity to have a question posed and find out for himself the answer by planning and carrying out the experiment with, if necessary, quite crude laboratory techniques. In large institutions, and perhaps even in large high schools, there should be the choice for the introductory courses between one that is especially planned to prepare the student for the more advanced phases of the same subject and those that are the background of the culture intelligent citizen. I do not like survey courses which merely attempt to cover a larger area by making the coating thinner. I do believe, however, that for the student who is only going to devote a strictly limited time to the physical sciences, a more satisfying selection of topics and of laboratory experiments can be chosen from the fields of physics and chemistry than from either one alone. I do believe that for the student who is not going to be a biologist, a more useful selection of topics can be chosen from the fields of zoology, botany, human physiology, and bacteriology than from zoology or botany alone, and that the general citizen who does not complement his geography with geology or vice versa has paid highly for his dab of specialization by only dipping into one of these subjects. Courses which spread much beyond the ranges just indicated lack so much in unity that I doubt the wisdom of giving them.

These interdepartmental programs lead to some difficult educational problems: problems of advising, problems of the student who changes his mind (a privilege I wish to maintain), problems of selection of laboratory material that will not be too spotty, and at the same time problems of maintaining standards which have been established for the technical introductory courses and have not been a tradition of the survey courses. Some of these things I have said tentatively even if they sounded dogmatic. May I be dogmatic on this: that unless these broader courses are planned by able scientists rather than as the escape activity of a man who has not done any research, the breath of life will not be in them. And for both mathematics and science I would like to make one further remark. The subjects should be taught in such a way that social and economic significance of modern technology and modern scientific knowledge is brought to the attention of the future citizen. Moreover, both of these subjects should be taught with much greater emphasis on their history, on their development as human activities, and on their place along with literature and art as the flowering of the human spirit, than has been done in the past. The fundamental theorem of algebra without a knowledge of Gauss, Newton's interpolation theory with no idea of who Newton was, the Mendelian law with no picture of the priest, Mendel, and his garden, does not represent the knowledge of science as a living organism that we should have.

Mathematics, it is affirmed, has made physics unpopular. Language without words would be about as sensible as physics without mathematics.
F. L. Bishop, 1905

The giants of mathematical thought often have been scientists of the first magnitude, and a certain degree of reciprocity exists between even the purest mathematics and the most practical science.
D. E. Kullman, 1966

II. Integrating Science and Mathematics in the School Curriculum

Teacher education, whether it be pre-service or in-service, is a major factor that must be faced realistically. Discipline-oriented individuals must be re-oriented if they are to successfully work with unified curricula. Discipline-oriented college curricula certainly are major obstacles.
D. H. Ost, 1975

One can conjecture that, should mathematics and science teachers help students structure their knowledge, the student himself may develop strategies for organizing his knowledge in other subjects. It is a researchable conjecture.
T. J. Cooney and K. B. Henderson, 1972

4

Progress in the Correlation of Physics and Mathematics[1]

F. L. Bishop
Bradley Institute
(Vol. V No.3 March, 1905)

The progress that is being made in the correlation of physics and mathematics is so extensive as regards amount of territory covered, and the methods employed differ so greatly that it is almost impossible to rightly estimate its absolute value.

Professor E. H. Moore of the University of Chicago writes "that distinct advances in this direction are being made in England, Germany and France, at least to the extent that first, the work in mathematics is treated as a single whole; second, it is done simultaneously with physics; third, it is done as far as possible by the same instructors. This facilitates actual and continuous correlation."

At a meeting of the Mathematical Club of the University of Chicago held during the last summer quarter, reports were made showing progress on the Pacific coast, in the South and in the Middle West. A factor in this country tending toward correlation is the organization of such clubs as the Association of Teachers of Mathematics in the Middle States and Maryland. Its first meeting was held at Teachers College, New York City, on November 28, 1903. Among the papers read were "The Laboratory Method of Teaching Mathematics," "Geometry in the Grammar School," and "Has Algebra Any Genuine Application?"

By such organizations as the Eastern Association of Physics Teachers this subject has received more or less consideration, the

[1] Address given before the Mathematics and Physics Sections, Central Association of Science and Mathematics Teachers, November 26, 1904.

following quotation being taken from an address by Vice-president George A. Cowen of Simmons College: "Fifteen years ago at Phillips Academy, Andover, Professor Graves performed the experiments while the boys looked and wondered. Now they do and know. With the change came the necessary demand for accurate measurement, but measurements of length and weight and force are of no use unless properly correlated. This is mathematics. Mathematics, it is affirmed, has made physics unpopular. Language without words would be about as sensible as physics without mathematics."

The methods that are being employed are illustrated by the following: Professor G. W. Greenwood, of McKendree College, writes: "We are trying here to bring mathematics and physics into closer relationship by showing that algebra is a means of expressing precise relations among magnitudes which may be measured and of expressing relations deduced from given relations. I am using entirely new definitions with these ends in view, and so far as I know they differ widely from the text books. We take any verbal statement from physics or arithmetic and then state it in the form of an equation. From equations we state verbal equivalents or rules. We leave entirely in the physics department the experiments from which the laws are deduced, and so far we are making good progress."

From Professor Newhall of the Shattuck School we have the following: "The courses are conducted entirely separate, but the department of mathematics aims to teach such subjects as the metric system, ratio and proportion, variation, graph, a knowledge of the trigonometric functions, etc., before they shall be needed in the study of the sciences. The instructor in physics has written out for me a full list of the formulas, equations and geometrical proofs which occur in a year's work in physics, and I see that these identical equations and proofs are studied in the algebra and geometry. In return he emphasizes such subjects as the parallelogram of forces, direct and inverse variation, etc." Mr. Newhall also writes that they have a close correlation between the mechanical drawing and mathematics. Further he states: "I do not think I like the idea of correlating the two subjects to such an extent that either loses its identity."

Professor H. E. Cobb of Lewis Institute, in outlining the interesting work he is doing, says: "My experience is no doubt of value in this, that I am laboring under the difficulties that most teachers encounter when they try to get out of the beaten path. While I am free to use any method I choose in my classroom, at the close of each quarter and often during the quarter, students are transferred to or from my section. Hence I must go over the ground with the new ones, and always have my students in such shape that they can do work with other sections. In the first year algebra I use the balances and levers to illustrate the equation and the operations with positive and negative numbers. Squared paper is used in the solution of problems and graphical work. During the last two quarters one day a week is given to concrete geometry, measurements being made in the metric and English systems, and constructions with compasses and ruler. In first year geometry great emphasis is laid on doing and on numerical computation. During the last quarter elements of trigonometry are introduced and triangles are solved, and computations of heights and distances made with the squared paper instead of tables of natural functions. In geometry various blocks are measured with both ruler and calipers, which are weighed and the volume and specific gravity computed."

Professor Donecker of the Richard T. Crane Manual Training High School of Chicago has devised a balance which he calls the "Algebraic Equation Balance," which serves the purpose of making it possible to present equations in concrete form. The primary purpose is to give a concrete idea of negative numbers. It can also be used as a basis for problems on levers. I suggest that every mathematical teacher investigate this apparatus.

Professor Risley of the Mathematical Department of Armour Institute of Technology outlines their work as follows: "We have gotten outside problems from the work in Physics and used them in our class work. We have not discarded a text, as some have advocated. In those subjects requiring mathematical statements and calculations, our instructors agree that the difficulty is fundamentally one of arithmetic. The mistakes are made in adding, etc., and in getting the decimal point in the right place; in handling ordinary common fractions and in placing the work in a clear logical order. In our plane geometry practically all the work is original. This is somewhat slow at first, and usually a little discouraging to the student for about six weeks. After that, if the instructor has been sufficiently strenuous in exacting the niceties of geometrical logic and clearness, there is a most pleasant outlook ahead. In our geometrical notebook we have a great deal of construction work illustrated by the following problems: Draw 5 lines of different lengths, measure them in centimeters and inches, find the ratio of the length of an inch to that of a centimeter in each case; also the ratio of the length of a centimeter to that of an inch in each case. Obtain the mean ratios. Why do your values differ from the true ratios? As another example: Draw 5 different angles, acute, obtuse and reflex. Measure each three times and obtain the mean. Describe the process of measurement carefully. The student will not study construction as such from his text this term, the idea being to have him become thoroughly familiar with his inch and centimeter rules, his compasses and dividers, protractor, etc. Later he will consider the parallelogram of forces from data obtained in actual experiments. One primary object in our work is the development of the initiative, and we hold that analysis bears an important place in this development."

Dean Raymond of the Department of Physics writes as follows: "We have found at Armour Institute of Technology that in attempting to do specified physics experiments with our mathematics classes, we sacrificed the formal drill in the manipulation of algebra expressions. This is too important a part of the training of an engineer to be studied in any but a rigid manner. In place of the 'booky' problems that are found in almost every text, we have supplied a long list of problems from physics, especially mechanics, etc. The law is stated and the student has problems to solve that arise from this law. The interest of the student is assured at the outset, knowing that he will later meet with the principle in his engineering work. Besides the interest of the student, he is being drilled to manipulate those forms which will make the study of the subject of mechanics or physics very much easier when taken up. Complaints were frequent from those teaching the applied mathematics that the men could not handle the mathematics of the subject after having made the application. The list of problems was written after scanning the books used in the engineering classes, in hopes that the men might be better trained to handle the work. From results attained thus far, we feel that the work is proving very beneficial."

It would be much easier for Professor Comstock or Professor Plant, who have so successfully and untiringly pushed the work of correlation at Bradley Institute, to outline their work for you and explain exactly how and why it was begun. From the point of view of the physicist, it commenced some six years ago when the mathematics department discarded the algebras then in use and made out in outline one which used extensively the graph and introduced a large number of physical problems. These problems were selected in what appears to me now an almost ideal manner. The physics department furnished a list of all the typical equations used in elementary physics and later a series of problems which covered every type of equation. The mathematical department then selected from these and added many others which appeared especially well adapted to students in elementary algebra. From physical problems to simple apparatus, such as balance, lever, thermometer, etc., was only a step, and the logical outcome of the introduction of such work. This work has been developed along this line until it seems to me, as I come in contact with the students taking the course, to be very successful.

In geometry the students were first given a few plane figures to find their dimensions in both the English and metric systems. Various geometric problems that had a more or less direct bearing on the physics were introduced.

Two years ago the mathematical laboratory for work in concrete geometry was established. This simply meant the better systematizing of the experiments already given and the addition of many more. Some of these experiments were taken directly from the physics, while others were original and not ordinarily given at the present time in the elementary physics. Great care was taken in the selection of these experiments to include only in general those that have a geometrical proof, thus enabling the student to obtain a very clear comprehension of the practical applications of his geometry. The second object that was aimed at was the selection of experiments which required only the simplest form of apparatus. Those which do not fulfill this latter condition are, in my opinion, absolutely worthless for elementary mathematics. As soon as the apparatus becomes sufficiently complicated to need explanation by the instructor, the student has his mind turned from the essentials of the experiment to the apparatus. It is not the aim of this laboratory course to teach the student manipulation of complicated apparatus.

It was early recognized that it would be impossible to have a close correlation between the physics and mathematics unless the instructors were familiar with the work of both departments. For this reason one mathematical instructor taught three-fourths of his time in mathematics and one-fourth in physics, while one of the physics instructors gave three-fourths of his time to physics and one-fourth to mathematics, i.e., the physics instructor had one class in geometry or algebra and the rest of his time in physics. This also furnished a bond of interest between the two departments, in that each knew the aims and objects of the other.

The results obtained from this arrangement cannot be overestimated. It seems to me doubtful if as much could have been accomplished in any other way; at least it would have required a much longer time. Another feature which contributed materially to the success of this correlation was the introduction three years ago of a course in physiography, which is taken by all students during the first quarter of the first year. This course, given under the direction of the physics department, is made an introductory course in science, so that the student is more or less familiar with the words and phrases that he will be required to use in his problems in algebra and geometry.[2]

Thus we have outlined briefly the work of correlation as it is being carried on in some schools. There are many others where the work is probably as far advanced as in the cases noted, but I was unable to obtain accurate and specific information concerning them. A pertinent question would be: What assistance has this been to physics? The student is familiar with many of the words like *velocity, acceleration, force, centigrade*, etc., the metric system in detail, and the graph. He can solve all algebraic equations occurring in physics with numerical problems under each. The working of examples in the composition and resolution of forces with the trigonometric functions--sine, cosine and tangent--is but a continuation of his work in geometry. The laws of the lever, reflection and refraction of light, of the inclined plane, and the relation between the Centigrade and Fahrenheit thermometers come as easy as the simplest equation in algebra. From his laboratory work he is familiar with the method of doing accurate laboratory work. He knows the *degree of accuracy* which he may expect to obtain, i.e., a clear relation between the theory and practice. He is also familiar with the source of error and form of laboratory reports, and he knows the best methods of computation.

There are, of course, many others; but beside all these which we can state more or less definitely, he possesses the power to use his mathematics to a degree that is almost unknown to students who have not had this work.

Another question which is often asked and which is certainly much to the point is: Do these students know their pure mathematics as well as students who have had only the abstract mathematics? This is, of course, a very difficult question to answer. I have made an attempt to find an answer in this way. I have in my classes not only students who have taken this work, but also students who have come to us from first-class high schools where I know that the preparation in pure mathematics is very good. I have sometimes asked for the proof of some geometrical theorem that we have been using, as for instance the Pythagorean theorem. I have never yet found a student with only the abstract preparation who would attempt to demonstrate one of these off-hand, while I have found that

[2] These experiments were published in full in a report of the committee on the correlation of mathematics and physics in secondary schools made to this association in 1903.

a large number of the students who have had the concrete geometry were able to give the demonstration.

While this cannot be considered in any sense a proof, it certainly indicates to me that the student has lost none of his reasoning powers by taking up this applied work. That I am not the only teacher who believes that the concrete work has added materially to the student's power to deal with mathematics is easily seen from the following: Professor Cobb of the mathematics department of Lewis Institute, who has made decided progress in the correlation of physics and mathematics, says: "I am thoroughly convinced that my students are getting hold of mathematics in a way that is not possible under the old formal method of teaching." Again, to quote from Professor P. B. Woodworth: "I take great pleasure in reporting progress in the correlation work in mathematics and physics at Lewis Institute. I have been more than pleased with the results as they develop this year. The students who had the mathematical work based upon actual measurements are much better prepared than those who have had the same amount of abstract work in mathematics. The work seems in some way to have developed a thinking mathematical method which largely prevents those mathematical blunders which have been so exasperating to physics teachers. I also think the attempt at precision measurement has increased the students' reverence for mathematics. The only ill effect I have observed is that those who have not had the course given by Professor Cobb are having a hard time to get in line." Professor R. A. Milliken of the University of Chicago quotes Mr. Lynde, instructor in physics at the School of Education, as saying that the students who took the concrete mathematics and have now entered physics take to the graph like ducks to water; beyond this he is not prepared to make a report at present, as this is only the second year that the physics courses at the School of Education have been in operation.

From the statements of various persons quoted in this paper, and from many others which I have obtained, it seems that correlation does not mean that either physics or algebra or geometry is to be eliminated, but, as Dr. Milliken very aptly expresses it, "you can teach all the physics you want in algebra and geometry and then there will be plenty left for us."

It appears that the correlation is an accomplished fact in so far that problems from physics are made the basis of the original work in algebra and that wherever this work is carried on we find both the mathematics and physics teachers enthusiastic concerning the progress of the student in his power not only to use his mathematics, but he seems to possess a clearer and a more logical and a more correct idea of the abstract mathematics. It would seem that to attain the highest degree of efficiency in this work, not only must the teacher of mathematics be a student of physics, but if possible he should teach physics for a time in order that he may better comprehend the needs of the physics teacher and at the same time study carefully in all ways the effect that the correlation is having on the student's work.

5

Correlation of Mathematics and Science Teaching

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Mathematics has been called "the queen and servant of science," and it is true that, both historically and in our contemporary situation, mathematics and the natural sciences are very closely related. The giants of mathematical thought often have been scientists of the first magnitude, and a certain degree of reciprocity exists between even the purest mathematics and the most practical science.

Mathematics is already the language of the physical scientist, and it is becoming rapidly an important language for the biological and social scientist as well. But if science depends on mathematics for the formulation and solution of many of its problems, mathematics also must acknowledge that it acquires meaning as it is used to describe the physical world. Moreover, many significant advances in mathematics have come as the result of attempts to solve complex physical problems.

Since mathematics and natural science are related intrinsically to each other, it would seem that the relationship should be spelled out clearly in the teaching of these subjects. Indeed it often has been pointed out by mathematicians, scientists, and educators that we should review our present curricula in mathematics and science, with a view toward better coordination of instruction in both areas. This type of coordination has been advocated in the United States for more than a half-century. Yet it is still not unusual to hear complaints from science teachers that their students cannot apply mathematics in the solution of physical problems. At the same time, the mathematics teachers reply that they cannot teach "scientific applications" in their mathematics classes because their students do not understand the basic principles of science.

Perhaps a review of the historical aspects of this problem and attempts to overcome it will shed some light on the path toward a solution in the future.

The need for greater correlation of mathematics and science teaching was expressed as early as 1901 by John Perry, a professor of mathematics at the Royal College of Science, London. In an address on "Teaching of Mathematics," given at the Glasgow meeting of the British Association for the Advancement of Science, Perry attacked what he felt was a "system of teaching boys elementary mathematics as if they were all going to be pure mathematicians." He also called for a laboratory approach to the teaching of mathematics.[1]

This latter suggestion was taken up by E. H. Moore, then a professor of mathematics at the University of Chicago and president of the American Mathematical Society. In his presidential address, delivered in December, 1902, he again called for a laboratory system of instruction in both mathematics and physics. He further asked whether it would be possible "to arrange the algebra, geometry, and physics of the secondary school into a thoroughly coherent four years' course."[2]

Other educators in the United States were interested also in relating science and mathematics instruction to each other. In April, 1903, the Central Association of Science and Mathematics Teachers was founded, with one of its objectives as set forth in the constitution being "to obtain a better correlation of [mathematics and science] to each other and to the other subjects of the curriculum."[3] Many of the articles in early issues of *School Science and Mathematics* were concerned with the correlation of mathematics and the sciences.

Some other indications of the widespread interest on this problem were the beginnings of experiments involving various degrees of correlation between mathematics and science. Some authors of textbooks also began to include problems from the physical sciences.

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[1]Mock, Gordon D. "The Perry Movement." *The Mathematics Teacher*, 56: 130-133 (March, 1963).

[2]Moore, E. H. "On the Foundations of Mathematics." *Science*, 17: 401-416 (March 13, 1903)

[3]Central Association of Science and Mathematics Teachers. Report of the Committee on the Correlation of Mathematics and Physics in the Secondary Schools. 1903

It seemed as though real progress would be made toward the integration of science and mathematics. However, one unfavorable reaction to Professor Moore's view was noted by David Eugene Smith of Teacher's College in New York City. He reported that the eastern part of the United States was content to let the Central States continue their experiments, but that the East did not support any efforts to introduce physical experiments into mathematics classes; for "the consensus of opinion is that the number of applications of algebra to physics, for example, is exceedingly small." Smith further stated that the East would make every effort to "get the pupils to walk in the domain of pure mathematics."[4]

In 1923 the National Committee on Mathematical Requirements of the Mathematical Association of America, recommended several plans for a Mathematical Curriculum.[5] It is significant, however, that none of these plans included suggestions for the correlation of mathematics and science. By 1934, E. R. Breslich, of the University of Chicago, could say, "It is unfortunate that the correlation of science and mathematics, which had such a promising beginning, did not continue to make the progress expected by those who advocated the plan. Indeed, the ground made has been almost lost."[6]

After 1935 interest in the correlation of mathematics and science teaching was renewed. When the Curriculum Committee of the Central Association published a "paper panel" on "Desirable Curriculum Adjustments in Science and Mathematics" in 1941, the writers on this panel expressed a concern for the greater use of mathematics in science courses and the use of problems from science in the mathematics classroom.[7] The following year the National Council of Teachers of Mathematics published its 17th Yearbook, *A Source Book of Mathematical Applications* Significantly, a large percentage of these applications were drawn from the physical sciences.[8]

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[4]Smith, D. E. "Movements in Mathematical Teaching." School Science and Mathematics, 5: 135-139 (March, 1905.)

[5]National Committee on Mathematical Requirements. The Reorganization of Mathematics in Secondary Education. Boston, Houghton Mifflin Co., 1923.

[6]Breslich, E. R. "Coordinating the Activities of the Departments of Science and Mathematics in Secondary Schools." School Science and Mathematics, 34: 144-157 (February, 1934).

[7]Carnahan, Walter H. "Some Desirable Curriculum Adjustments in Science and Mathematics." School Science and Mathematics, 41: 103-114 (February, 1941).

[8]National Council of Teachers of Mathematics. Seventeenth Yearbook: A Sourcebook of Mathematical Applications. Washington, The Council, 1942.

In recent years many articles have appeared in educational journals which emphasize the need for physical applications to promote better understanding of mathematics. A typical viewpoint is expressed by the following quotation: "Mathematics as it originates in the curriculum is taught too frequently as symbolism without reinforcing by experience. Historically we have become accustomed to teaching mathematics without a laboratory where some of the badly needed experience in this subject could be acquired.... Unless a definite effort is made to assist [the student] in applying mathematics to science, he will continue to fail to solve equations and formulas which arise in the science classroom when he has had little trouble in solving the same basic problems in the regular mathematics classroom."[9]

One problem which always arises in attempts to correlate mathematics and science instruction is that of the order in which topics are taught. Traditionally, the topics which might best be taught together have been taught at quite different times in the instructional sequence. For example, scientific notation is used very early in the physics or chemistry course, but it is usually not taught in a mathematics class until fairly late in the second year of algebra. To overcome this incompatibility will require some major changes in the order of teaching topics, beginning at the elementary school level.

One such attempt is now being made by the Minnesota Mathematics and Science Teaching Project (MINNEMAST). The Minnemath center at the University of Minnesota is working on a combined math-science curriculum for grades K-9. They have already prepared some units for the primary grades in which children perform experiments leading to scientific concepts and then learn the mathematics needed to express these concepts.

On the junior high school level, the School Mathematics Study Group has published three units entitled *Mathematics Through Science*, and a fourth unit entitled *Mathematics and Living Things*. These units are intended for use in the mathematics classroom, and although they use physical and biological experiments to motivate mathematical ideas, their primary purpose is to teach mathematics.

A survey of recent volumes of journals in science and mathematics education shows many articles concerned with the relationship between science and mathematics teaching, both generally and in specific applications of mathematics to science. A few such references are listed at the end of this article.

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[9]Hall, Arthur J. "Relations between Science and Mathematics in the Secondary School." National Association of Secondary School Principals Bulletin, 37: 191: 92-95 (January, 1953).

The above programs may be the beginning of a more unified math-science curriculum, but there is still much room for experimentation at all levels. Much of this experimentation could be done by classroom teachers who are willing to break away from the compartmentalized curriculum which is presently found in so many of our schools. When educators stop thinking of algebra, geometry, trigonometry, biology, chemistry, and physics as separate entities and begin to see natural science and mathematics as interwoven disciplines, they may improve the "transfer" of concepts between the mathematics and science classrooms.

There may be some dissention on the part of "pure" mathematicians who will rightly claim that mathematics can be learned apart from its applications, and that these applications should never be allowed to obscure the mathematical structure itself. One answer to this objection is that, although mathematics *can* exist as a completely isolated science, it is not in the best interests of elementary and secondary education to present it in this way. A large number of students will be using their mathematics primarily as a tool in some other field. The responsibility of both science teachers and mathematics teachers is to help them to see the possibilities of using mathematics in describing the "real" world. This can best be done when the interrelationships between mathematics and science are brought out in both classrooms.

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6

Changing Curriculum Patterns in Science, Mathematics and Social Studies

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Introduction:

Due to the nature of cultural evolution, curriculum development must be an ongoing function of the educator. There are always individuals who see the need for educational reconstruction. John Dewey wrote some 50 years ago "if there is a special need of educational reconstruction at the present time . . . it is because of the thorough-going change in social life of accompanying the advance of science, the industrial revolution, and the development of democracy."

Any change is at least in part a reflection of the needs of society. The current move towards the fusing of disciplines, conceptual schemes, or instructional procedures is such a response. For several years it has been recognized that instruction in disciplines for the sake of increasing content competencies is insufficient. The national curricular movement of the 1960's, with their stated objectives to increase the scientific manpower, is no longer a vital force in our curriculum movement. There is no longer the pressing national security need, whether real or manufactured, for more scientists and engineers. The pressure is ever-increasing to prepare a public which is literate in the interactions of the science-technology-society complex. This is in spite of reactionary activities pressing for skill development, short-range limited objectives, and job orientation.

There are a variety of individual projects and group efforts put forth in an attempt to modify curricula as a response to the changing society needs. Not unexpectedly, each group has its favorite terms which are in need of operational definitions. Rubrics such as *interdisciplinary, unified, integrated, correlated, coordinated* and *comprehensive problem solving* need to be clarified. Operational definitions will facilitate communication. No doubt there are other labels currently in favor by the current generation of curriculum developers, but these terms seem to be most prevalent in the literature.

Interdisciplinary:

Interdisciplinary is generally applied to courses of study which merge for purposes of instructional expediency two or more bodies of knowledge. The results are courses with titles such as "Organic Chemistry, Pesticide Use and Residue Control," "An Interdisciplinary Approach to Environment Improvement," or "The Chemistry of Drug Use and Abuse." Such courses are frequently in response to student demand and thus reflect the contemporary societal concern. Examples of such courses are cited in *Science for Society* (1). It is not unusual for such courses to be "team taught." This is an indication that the instructors have not conceptually integrated the content from their respective disciplines and consequently "team." Unfortunately, the frequent result is that the instructors concentrate on their separate areas of expertise, producing two or more minicourses with the burden placed upon the student to interrelate the information. A theoretical basis for the interdisciplinary curriculum, instruction, and/or program is generally lacking.

Unified:

The term *unified* is usually applied only to courses or programs within which unifying themes or concepts can be developed. Most frequently this approach is applied to the various disciplines of the natural and physical sciences. Unified science takes the major themes and builds conceptually upon or through them. It has been stated that all the unified science projects are constructed around concepts, principles and processes that permeate all science (2). An example might be the concept of energy, which would be studied from the biological, physical, chemical and geological dimensions.

The unified approach appears to have a firm theoretical basis for its development. Science is treated as a discipline in and of itself and is therefore not conceptually divided into a variety of disciplines. Instructional materials are patterned in a manner which reduces redundancies and thereby increases efficiency. In addition, the total content of science is perhaps better sequenced in a manner more logical and consonant with the student's ability to understand. Persons committed to this approach have formed a semi-structured organization, the Federation for Unified Science Education (FUSE).

Integrated:

The term *integrated* most often is used in connection with mathematics and science, although in recent years other disciplines are being included under the umbrella of integration. Integrated programs are perhaps really interdisciplinary, but at a more sophisticated level. For example, programs which are classified as integrated science and mathematics usually involve the teaching of applied mathematics for the solving of scientific problems. It is frequently found that one or two disciplines, usually chemistry and physics, employ mathematics to raise to the level of cognition certain concepts in science. In such programs, mathematics is relegated in a large part to a service function. The mathematics may be taught separately, but conceptually it is integrated into the curricula of the sciences, which may or may not be unified.

Correlated:

Correlated is a term used to describe attempts made to relate skills or concepts from one discipline to the other. Most of the disciplines retain a separate identity. The strategy is sometimes to relate skills or concepts from one discipline to the other, sometimes to relate the basic structure of the disciplines. For example, at the college level a course in statistics may utilize problems for biology, physics or chemistry. Or at the high school level the mathematics teacher may accommodate to the needs of the physics students. The usual rationale behind such a program is to increase the relevancy to the student or to simply provide appropriate skills when needed.

The Report of the Cambridge Conference on the Correlation of Science and Mathematics in the Schools (3) clearly suggests that mathematics is taught in an irrelevant manner. The report suggests that the teaching of mathematics could retain its integrity and be made more meaningful by being correlated with other disciplines. In correlated programs, mathematics makes use of everyday problems, modeling conditional probability, and other factors related to the students' environment. Effort is made to apply the skills or concepts developed in one discipline to another discipline. The theoretical basis is one of correlating skills and applied problems. The sequencing of content is not of prime concern.

Coordinated:

Coordinated programs are probably attempts to remove redundancies. In part, these types of programs may be the result of attempts to unify or are transitory between traditional and unified programs. The coordinated programs may also focus on conceptual schemes set in a traditional spiral curriculum with different courses treating the materials with differing degrees of sophistication. Textbook series for elementary school are frequently written endeavoring to coordinate the content conceptually as well as to coordinate the material with intellectual development of the student.

At a less sophisticated level, individual instructors may have gotten together to coordinate the content on two or more courses. Usually such efforts are done for the purposes of efficiency and to remove what are thought to be redundancies. "Core courses" in college curricula are examples of what might result.

The basic premise of coordination is to increase the efficiency of instruction. An unfortunate side effect is the increased interdependency of the courses.

Comprehensive Problem Solving:

Large scale programs dealing with comprehensive problems are just beginning. There are multitudes of materials which might be classified as being congruent with this approach which are usually based on the "unit concept." Here the student defines a comprehensive problem and is asked to apply various skills and knowledges from the sciences, mathematics, and social sciences in an attempt to optimize some solution. Problems are frequently drawn from current social interests such as consumer research, community recreation, and transportation. In the process of studying such a problem, the student might consider costs, governmental involvement, water usage of the area, projected needs, and other such related topics. Data collected and the optimized solution might, in fact, be transmitted to the proper authorities for action.

The strengths of this approach lie in the ability of the student to enter completely into the problem. He is able to enter and exit at his own level of competency. Specifically, the student requires no predetermined knowledge, but rather can either apply what knowledge he has available to him or develop the new knowledge necessary to solve the problem or to work towards a tentative solution of the problem. Proponents of this approach suggest that it serves an additional function of helping the student make moral and ethical sense out of his environment and the happenings in his surroundings (4).

Some Implications:

Teacher education, whether it be pre-service or in-service, is a major factor that must be faced realistically. Discipline-oriented individuals must be re-oriented if they are to successfully work with unified curricula. Discipline-oriented college curricula certainly are major obstacles. There is evidence to suggest that colleges are also moving to the types of programs described above. Perhaps the current interdisciplinary movement called "structuralism" may unify, at least methodologically, the various research disciplines. Such a change would certainly have ramifications for college teaching and teacher education.

It is really not significant what a program or an approach is named. What is important is what schools communicate to students. It is obvious that today's schools are in need of a great revitalization. Any and all of the above approaches can be important factors for the revitalization. As with any change in education, it can only be as good as the changers: output is always a function of input.

Let us hope that those individuals who see the need for educational reconstruction push forward so that education can be in step with the needs of society.

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7

Structuring Knowledge in Mathematics and Science

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One of the goals of instruction is to help students formulate a conceptual network which will render their knowledge of specifics more useful. Such a structure should assist the student in recognizing the interrelationships of concepts and principles and also in assimilating newly acquired concepts and principles into his cognitive structure.

While this goal may be regarded as obvious, it is not immediately apparent that teachers usually teach with the idea in mind of helping students develop some kind of structure. Observations of teachers, especially beginning teachers, reveal that teachers often neglect the interrelationships between various concepts and principles. It is the purpose here to suggest a method by which teachers can explicitly direct their students' attention to meaningful relationships in mathematics and science. It is assumed that if a teacher is consciously aware of ways of organizing material into a desired structure, he is better able to assist students in developing such a structure.

CLASSIFYING

One of the fundamental abilities in learning mathematics and science is the ability to classify according to some criteria. It is through classification that we obtain much of our power in understanding what we learn. Classification allows us to form concepts, transfer knowledge from one set of objects to another set of objects, and, in general, allows us to subsume much of our knowledge into broader more inclusive categories. The power of classification lies in the fact that *whatever* item is classified that item possesses those attributes associated with the set to which it belongs.

The act of classifying occurs whenever an object is stated to be an element of a particular set or whenever a set is declared to be a superset of another set. The first situation will be called *set membership* and the second *set inclusion*. For example, classifying $\sqrt{2}$ as an irrational number or NaOH as a base are instances of set membership. This activity plays an important role in the classroom, for now the student can associate properties of irrational numbers and bases in these two objects respectively. In particular, the student should realize that $\sqrt{2}$ can not be expressed as the ratio of two integers as can 3 1/7. Since NaOH is a base and bases are generally caustic, feel slippery, neutralize acids, and cause litmus paper to turn from red to blue, he can associate these same properties with sodium hydroxide.

Set inclusion occurs when squares are noted to be rectangles, the set of differentiable functions is a subset of the set of continuous functions, sugars are carbohydrates, or in general when a superset of a given set is stated. The importance of this type of classification is essentially the same as that given for set membership. It provides an organization to concepts that are learned and allows one to associate characteristics of the superset with the more restrictive subset. For example, since carbohydrates are composed of carbon, hydrogen, and oxygen we can deduce the composition of sugars. Hence, the student minimizes the amount of specific knowledge he must memorize through a structure of the knowledge he has acquired.

The fact that the solutions to the equation $x^4-1=0$ form a cyclic group of order four under the operation of multiplication provides us with information on how these four elements behave. If a student is aware that the trigonometric functions are periodic functions this facilitates his graphing of these functions.

ANALYSIS

Another important strategy that teachers can use in helping students organize their thoughts is to assist them in analyzing a particular concept or set by breaking it down into its constituent parts--be they subsets of the set or elements of the set. Giving subsets of the referent set of a concept will be referred to as *analysis*, whereas the listing of elements of a particular set will be called *specifying*.

A chemistry teacher might use an analysis move by categorizing the four general types of chemical reactions or by having students realize that hydrocarbons consist of alkenes, alkynes, alkadienes, and the aromatic hydrocarbons. The latter realization enables students to associate the properties of hydrocarbons to the particular kinds of hydrocarbons. An analysis can also help students consider special cases of a generalization. Consider, for example, the generalization that the altitudes of a triangle are concurrent. If one partitions the set of triangles into acute, right, and obtuse triangles, the generalization still holds for each of these types of triangles but the point of concurrency lies inside, on, or outside the triangles respectively.

An analogous situation occurs in physics classes when a student considers the generalization:

The sum of the forces acting in one direction equals the sum of the forces acting in the opposite direction.

The student can then investigate the application of this principle to the three types of levers. The structure provided here is that the student need not learn three separate principles, one for each class of lever, but rather one principle made applicable to three different situations by the use of analysis.

The same sort of structure can be provided by specifying, i.e., giving elements of a set. The listing of the elements may or may not exhaust the set, but in any case a structure is provided by grouping items of knowledge into a single set. Consider the set of ways of proving that a quadrilateral is a parallelogram. Two of the elements of this set are:

1. If the opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

2. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Specifying can help students organize techniques for performing given tasks. For example, students can catalog various ways of solving quadratic equations or the chemical reactions which produce salts. Chemistry teachers can utilize this move by having students learn some of the metallic sulfides, or a biology teacher might have the students identify the organs of the body and their functions.

CHARACTERIZATION

A logical move similar to analysis and specifying is the eliciting of characteristics of an object. Mathematics teachers can have students associate various properties of parallelograms and rectangles--such as opposite sides parallel and diagonals bisect each other, etc. Also those properties which differentiate rectangles from other parallelograms can be given, viz., congruent diagonals and congruent angles. This type of move could follow an analysis move. Once the hydrocarbons have been analyzed into their different subgroupings--alkines, alkynes, etc., the characteristics both common and unique to these compounds can be discussed. For example, each of the substances is composed of the same two elements, hydrogen and carbon, however the groups differ with respect to their molecular structure.

EXPLAINING

There are two different but related classroom activities which can help students gain an understanding of the interrelationships of principles in mathematics and science. The first, called *justifying*, involves challenging an assertion whereupon the person challenged is expected to provide facts and generalizations from which the assertion follows. The assertion may be a singular statement or a universal generalization. For example, a student may be asked to explain why $\sqrt{2}$ is irrational, or to justify the claim that the arctic is a desert. The justification for both of these claims would be predicated in part, on definitional statements, e.g., what do we mean by $\sqrt{2}$, *irrational*, and *desert*? However, while the mathematical assertion could be justified solely in terms of postulated statements, the second assertion requires some observations to establish its truth claim.

A mathematics teacher may wish to have a student explain why the diagonals of a rectangle bisect each other in terms of the principle that the diagonals of a parallelogram bisect each other and the relationship between rectangles and parallelograms. A physics teacher may wish to have a student validate the claim that the downward motion of an object fired horizontally from a gun is the same as the motion of a freely falling object. The student's explanation may entail Newton's second law of motion and hence provide a link between this law and the asserted claim.

Sometimes a teacher may ask for a justification of a certain procedure, such as why constructing congruent corresponding angles with two lines and a transversal produces parallel lines. One way to justify such a procedure is to show that the prescriptions involved are predicated on established generalizations. In this case the procedure is based upon the theorem:

If two lines form congruent corresponding angles with a transversal, then the lines are parallel.

IMPLICATING

The second activity, called *implicating*, utilizes this same sort of structure by giving students certain conditions and asking them to formulate the conclusion following from these conditions. For example, from the fact that a parallelogram is inscribed in a circle one can conclude that the parallelogram is a rectangle. Another instance of this activity might occur in a science class in the following discussion:

T: If a warm moist mass of air meets a polar mass of air so as to form an indentation in the cold front, what weather condition would we expect to result?

S: It would be the beginning of a cyclonic storm.

The conditions that are set forth are the meeting of the two air masses and the indentation formed by the warm air pushing into the cold air mass. The conclusion which follows is so stated by the student and thus provides an organization between the given weather conditions and the anticipated resulting climatic conditions.

ABSTRACTING AND GENERALIZING

Discovery lessons also provide students with the opportunity to internalize and structure their knowledge. The essence of such lessons is abstracting and generalizing which are the upshots of the students' probing, questioning and exploring hypotheses. If teachers are to be effective in designing and using discovery techniques, then it is essential that they understand the activities of abstracting and generalizing. Quite simply, abstracting is the realization of similarities amid differences. That is, when considering a set of exemplars, abstracting involves the recognition of those properties which are common to all of the exemplars. Consider, for example, the concept of prime numbers. A possible strategy in teaching this concept is to present to the students a sequence of examples and nonexamples of this concept, e.g., 12, 8, 7, 10, 2, and 11, and ask the students to form rectangular arrays of dots representing the given numbers. The student realizes, it is hoped, that the only rectangular arrays that can be made to represent 7, 2, and 11 are 1x7, 1x2, and 1x11. That is, the student abstracts that property which is common to prime numbers, viz., that the only positive factors of a prime number are 1 and the number itself.

Generalizing involves two cognitions. The first is a statement about a particular set of objects and the second is a statement about a superset of the first set. For example, students could be given three objects of different composition and do an experiment with these objects involving Archimedes' principle. The students then discover that when each of the three objects was submerged in water individually, the apparent loss of weight of each object was equivalent to the weight of the water displaced. Now the students might generalize and form the conjecture that this phenomenon is true for any object submerged in water, not just the three specific objects examined. The students may also wish, further, to generalize from water to any liquid. While the latter conjecture would be correct and does represent a broader extensive of Archimedes' Principle than was originally formed by the students, care must be taken not to have students generalize (in the sense of proclaiming a truth not just the utterance of a conjecture) in the absence of data. Mathematics students may find that 1^2+1+41, 2^2+2+41, 3^2+3+41 all represent prime numbers and hence generalize that all numbers of the form n^2+n+41 are prime when in fact they are not (n=multiples of 41 yields numbers that are not prime). Teachers can help students avoid making incorrect generalizations by carefully selecting the sample to be examined, making sure the sample is truly representative of the more inclusive set about which the students are to make their discovery, and encouraging the students to seek counterexamples.

CONCLUSIONS

Discussed above are activities which teachers can utilize in their classroom to assist students in structuring their knowledge. Many teachers are already aware of these strategies, at least on a nonverbal basis. Since one of the emphases of curriculum movements in mathematics and science in the last decade has been to produce materials which help students in seeing the overall structure of the subject matter rather than viewing the particular items of knowledge within the subject as ends in themselves, it behooves teachers to utilize teaching strategies which maximize the understandings of the interrelationships present in knowledge which are exhibited in these curriculum materials. One can conjecture that, should mathematics and science teachers help students structure their knowledge, the student himself may develop strategies for organizing his knowledge in other subjects. It is a researchable conjecture.

Indeed, one drastic recommendation for the solution of the problem of mathematical difficulties in high school science has been to demathematize the science courses by making them essentially informational. Most science teachers, however, feel that the study of science is as much one of mathematical relationships as of accumulating a body of facts and information, and that high school science courses freed from mathematics are not the best science preparation for pupils who expect to continue the study of science.

E. R. Breslich, 1936

III. Science and Mathematics in Secondary Education

Any physics course, be it academic or practical, must be based on nature's physical laws and phenomena--and these laws and phenomena are mathematical. We physics teachers have the responsibility of showing our students the importance of quantitative reasoning, experimentation, and observation in trying to understand nature's physical phenomena. If we only give students an appreciation for the physical world as it 'mathematically and quantitatively' exists, we will at least have helped to prepare them for their science-centered world.

R. E. Carpenter, 1962

Due to the contemporary emphasis in secondary mathematics on logical foundations and structure, there appears to be less likelihood than ever of introducing a significant amount of science material into mathematics courses. If so, this would be most unfortunate for both subjects, inasmuch as on the highest levels, modern science leans heavily on "pure" mathematics of the most abstract nature.

W. L. Schaaf, 1965

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Integration of Secondary School Mathematics and Science[1]

E. R. Breslich
University of Chicago
(Vol. XXXVI No. 1 January, 1936)

Historical statement of the problem: The problem of integrating mathematics and science is not new. As far back as thirty-five years ago plans have been suggested for establishing close relationships between the two fields. At that time it was hoped thereby to decrease or eliminate many of the difficulties encountered by the pupils in the study of mathematics and to attain more satisfactory results. Perry of England and Moore of the University of Chicago expressed the conviction that this could be brought about by placing greater emphasis in teaching on the practical application of mathematics, particularly by teaching mathematics in continual relation to problems of physics, chemistry and engineering.

The suggestions of these leaders were enthusiastically received by teachers of high school mathematics and also by teachers of the high school sciences. Historically the development of a great deal of mathematics grew out of the needs of the sciences. This fact made it seem logical that if some of the experiments usually performed in the science laboratory were performed in mathematics classes in such a way as to lead to discussions and formulations of the underlying mathematical problems and principles, the teaching of mathematics could thereby be greatly improved.

A striking indication of the widespread interest of the importance of the problem is the formation of the Central Association of Science and Mathematics Teachers with *School Science and Mathematics* as its official publication. One of the major purposes of the association

[1] An address given at the Conference of Administrative Officers of Public and Private Schools, The University of Chicago, July, 1935.

was to find and establish legitimate contacts between the mathematical subjects and the sciences. It was hoped that the constant training which the pupil derives from applying mathematics to problems in science would increase his mathematical power and that his interest in mathematics would grow with the opportunities of using it in other school subjects. Indeed, some of the leaders of the movement were advocating that algebra, geometry and physics be organized into one coherent course. If possible this course was to be taught by the same teacher or at least by two teachers who were in sympathy with the ideas of correlation.

It must be admitted that these expectations have not been realized. The record shows that several committees have reported on ways of correlating science and mathematics, that the topic was discussed in the yearly meetings of the Central Association of Science and Mathematics Teachers, and that some progressive teachers and schools developed integrated courses which have been reported in *School Science and Mathematics*. Furthermore, writers of textbooks were quick to include among the verbal problems in algebra applications taken from the fields of physics and chemistry. Nevertheless, the movement did not gain widespread endorsement. It is significant that in 1923 the National Committee on Mathematical Requirements, which recommended several plans for a mathematical curriculum, failed to include in any of them suggestions as to the correlation of mathematics and science.

The foregoing historical sketch might give the impression that integration of science and mathematics is a closed issue and that it is no longer to be taken seriously as a plan of organization of materials. It is not intended to discourage interest in the plan. It does show, however, that great difficulties have to be overcome before success in the solution of the problem may be assured. The problem is much more difficult than that of the correlation of the various mathematical subjects. The soundness of that movement has never been questioned. Yet its progress has been very slow. Indeed, the movement might have failed had it not received new impetus from the junior high school movement, from the growing tendency toward integrated courses in the colleges, from the general educational movement toward integration of high school subjects and comprehensive examinations, and recently from the College Entrance Examination Board.

It is possible that the causes contributing to the failures of the early attempts to integrate mathematics and science do not operate at the present time. The attempts were made during a period of high specialization and departmental organization when few teachers of mathematics were qualified to teach another subject. Furthermore, the plan of including among the verbal problems of algebra a number of exercises on applications to science failed because detailed explanations of the situations and of the new science concepts were usually lacking. Thus, the science problems made little or no contribution to the teaching of mathematics. On the other hand, they actually added to the difficulty of algebra. The mathematical training derived from such problems was necessarily very small.

The mathematical needs of pupils taking courses in high school science: The extent to which high school mathematics and science offer possibilities for integration may be seen from a number of studies that have been made during the last fifteen years. These studies were usually made by science teachers who undertook to determine the mathematics actually used in high school physics and chemistry. The detailed findings of most of the studies have been reported in *School Science and Mathematics*. As a rule the investigators employed the method of analyzing textbooks in physics and chemistry. They listed the mathematical processes and principles which they found. Other investigators went further and worked out all the problems that involved mathematical manipulation. Records were thus obtained of the specific operations and skills required of the pupils who were to solve the problems. A third method of obtaining the required information was to analyze the notebooks and other written work of the pupils.

The studies disclose two interesting facts. The mathematics involved in high school physics and chemistry are of a much simpler type than most of the mathematics presented in courses in algebra and geometry, and all of the mathematics required in science are ordinarily taught in these courses. Anyone interested in the detailed findings may obtain them from the published reports of the investigators. For the present it is sufficient to summarize them briefly as arithmetical, algebraic, geometric, and trigonometric. Thus, a thorough knowledge of *arithmetic* is required. Since high schools, as a rule, do not and probably should not offer courses in arithmetical computation, responsibility for this type of training should rest with the science teachers fully as much as with the teacher of mathematics. Indeed, better opportunities for teaching arithmetic are offered in science courses than in the regular mathematics courses. The training in arithmetic should aim to develop: proficiency in the fundamental operations with integers, common fractions, and decimal fractions; knowledge of percentage; ability to use ratios and proportions; knowledge of the metric system and of other standard units of measure; ability to use and to interpret numerical tables; ability to employ the type of reasoning used in solving verbal problems; and skill in solving such problems.

The *algebra* used in high school science comprises ability to choose and employ good symbolic notation; ability to make substitutions in formulas, to evaluate formulas, and to solve formulas for specific required literal numbers; a knowledge of the laws of ratio and proportion; direct and inverse variation; logarithmic computation; signed numbers, and positive and negative exponents; ability to solve linear equations in one or two unknowns with integral or with fractional coefficients; ability to solve simple quadratic equations; ability to interpret relationships in formulas and equations; and ability to solve verbal problems by use of equations and formulas.

The methods of investigation make it impossible to discover the informational type. The pupil should be able to make simple diagrams and geometric drawings, including scale drawings; to read and interpret drawings; and to make and interpret graphs. A knowledge of the basic geometric concepts also is required. Acquaintance with the fundamental geometric constructions and with about a dozen geometric theorems is all the investigations disclosed.

The *trigonometry* of high school science seems to consist of a knowledge of the meaning of the trigonometric ratios and of the fundamental identities by which they are related to each other.

Determination of the mathematics required in the science course in a particular school: The foregoing summary of the studies aiming to determine the mathematics used in high school science gives only an idea of what is *generally* required. Before an attempt is made to integrate the two subjects in a particular school, a careful survey should be made of the mathematical needs in the specific science courses offered in the school. The method used by the department of mathematics of the University High School illustrates how this may be done. Mr. G. E. Hawkins, one of the instructors, has examined the materials of the physical science and chemistry courses and the problems assigned to the pupils. Each unit of the course was carefully analyzed and the required mathematical skills were recorded in the order in which they occur in the course and in the classroom. He listed typical problems and gave in each case the complete method of solution which was expected by the science teacher. The next step taken by the department was to study his report to determine when and where contacts should be made in the two fields. Three examples taken from the study will indicate the type of analysis that is being made. The examples are taken from the unit on matter and energy.

1. **Problem:** If one allows 1/2% for slippage on account of ice and snow, how many revolutions would the 80 cm. wheels of a car make in running a kilometer?

Solution:

$$\frac{100{,}000}{80\pi} + .005\ \frac{100{,}000}{80\pi} = 399.9 = 400$$

It is evident that the solution of the problem involves many abilities, any one of which may decide success or failure. The pupil must be able to read the problem understandingly. He must grasp the situation described. He must know how to express measures. He must know how to express the number of revolutions in terms of circumference and distance traveled, i.e., the distance has to be divided by the circumference (80)(3.14). He must be able to multiply and divide decimal fractions. One-half per cent of the number of revolutions is to be found, and finally the two resulting decimal fractions have to be added. Thus, the problem involves arithmetical computation with whole numbers and decimal fractions; a knowledge of metric units, of the circumference formula, and of the relation between distance traveled and circumference. Moreover, ability in arithmetical reasoning is required to determine which arithmetic processes are to be performed and in which order. In this case it will be advantageous to add before dividing. Finally, the pupil must know how many figures to use in the value of π, how far to carry the multiplication by 80 and the division by 80π. Finally, his answer must be a reasonable one. Thus, an answer of 399.88 revolutions would pretend greater accuracy than is in keeping with the nature of the problem. The best he can say is that the wheels make about 400 revolutions.

Few teachers of science will take the trouble to identify the mathematical difficulties in this problem which at first thought may seem to be merely a simple arithmetical exercise. The chances are that if the pupils fail the teacher will dispose of the whole matter with the sweeping statement that the "pupils do not know how to use their mathematics." Problems of this type make integration highly desirable. Somebody has to see to it that the required abilities are developed. The problems supply the mathematics teacher with interesting applications and if properly taught they offer excellent training in mathematical computation and thinking.

2. **Problem:** Find the resultant of two forces of 200 grams each acting at an angle of 60 degrees with each other.

Solution: Draw to scale or construct a parallelogram with two adjacent sides equal to 200 units each forming an angle of 60 degrees. Draw the diagonal and measure it in the unit used for the sides.

The solution of this problem requires a knowledge of some of the properties of the parallelogram; of drawing an angle of 60 degrees either by use of protractor or by geometric construction; and of the use of ruler and squared paper in making scale drawings. The problem could be assigned to a class in plane geometry.

3. **Problem:** A captive balloon whose lifting force is a ton was blown during a storm until its anchor rope made an angle of 60 degrees with the surface of the earth. Find the force of the wind.

Solution: The problem may be solved in three ways: by a scale drawing, by a proportion, or by trigonometry. The trigonometric method which is the most convenient is as follows:

A right triangle is drawn with the acute angle equal to 60°. The side opposite is 2000 and the side adjacent is the unknown, f.

$$\tan 60^\circ = \frac{2000}{f}$$

$$\therefore f = \frac{2000}{\sqrt{3}} = 1154.7$$

The force is 1155 pounds.

The solution involves the ability to make simple geometric diagrams; a knowledge of the tangent function; to find $\sqrt{3}$; to solve a simple equation; and to divide by a decimal fraction. The pupil must know how to express different measures of weight in the same unit. He must know to how many figures he should carry the arithmetical computations.

It will be noted that all of the skills and abilities which the three problems presuppose are contained in the general list derived from the studies mentioned earlier in this paper. The advantage of an analysis of the courses offered in a particular school is that it makes it possible to determine where the various processes and facts are needed in the science courses and where they may be taught in the mathematics courses. When that has been done the first step toward integration has been taken.

Attempts to solve the problem of mathematical difficulties in study of the sciences. It has been shown that mathematical problems which occur in science courses and which seem simple at first thought involve serious mathematical difficulties. A detailed analysis of the abilities required to solve the problems would enable the teachers of both subjects to help the pupils overcome them. A detailed list of such problems would provide the teachers of mathematics with vital applications for the abstract facts and processes of mathematics. They offer excellent opportunities for bringing about a closer relationship between the two departments.

Ignorance in these matters has been the cause of criticisms of the teaching of mathematics and sometimes of friction between two departments that should be in the closest possible agreement. Indeed, one drastic recommendation for the solution of the problem of mathematical difficulties in high school science has been to demathematize the science courses by making them essentially informational. Most science teachers, however, feel that the study of science is as much one of mathematical relationships as of accumulating a body of facts and information, and that high school science courses freed from mathematics are not the best science preparation for pupils who expect to continue the study of science.

A second method of solving the problem of mathematical difficulties in science is to leave the science and mathematics courses undisturbed and to form two groups of students corresponding to their mathematical ability. Those who are able to make easy and rapid progress in mathematics are permitted to advance rapidly and with a considerable saving of time. The group of students less capable in mathematics is given some mathematical instruction. They finish the science courses at a slower rate of progress.

A third plan that has been tried in some schools is to administer to all students who enroll for courses in physics and chemistry an examination covering the mathematics needed in science courses. Usually the examination is given during the first week of the science course. The students who fail to pass the examination either do outside assigned work in mathematics along with the science course or they may postpone the science course until they are able to give evidence of possessing the required mathematical knowledge.

The three plans do not attempt a reorganization of courses and involve only a small amount of cooperation between the two departments.

A fourth plan aims to reach an agreement between the departments as to which one is to assume responsibility for removing mathematical deficiencies. Either the students are referred to the mathematics department for correction of deficiencies when they appear, or the science teacher assumes the responsibility of teaching mathematics whenever such needs become apparent.

A fifth plan tries to establish complete departmental cooperation. The science situations requiring mathematical knowledge are determined. Each is analyzed as to the particular mathematical skills and processes involved. The mathematics department is informed as to the time when these skills and processes will be needed and undertakes to provide them.

The last plan has been modified in some schools by actually transferring certain science experiments of a mathematical nature to the mathematics department. This requires also the transfer of a certain amount of laboratory equipment. A few typical experiments which may be performed in the mathematical laboratory are: the relation between the metric system and other systems of measurement of length, area, volume and weight; specific gravity; laws of leverage; law of vibration of the pendulum; center of gravity; images in the plane mirror; refraction of light; parallelogram of forces, composition and resolution of forces; the inclined plane, pulleys; development of certain formulas such as the relation of Centigrade and Fahrenheit, of distance and time in the law of falling bodies, and their graphical representation; curvature of a lens.

Experiments like the foregoing are based on mathematical principles and may be performed in the mathematical laboratory whenever these principles appear in the science courses. The plan suggests a type of integration which may be introduced in schools with little disturbance and difficulty.

Finally, a sixth plan has been advocated which recommends the merging of two departments into one. The same teacher gives instruction in science and in mathematics. The plan has been successfully used in European schools. Its introduction in American schools requires rather radical departmental changes which seem to make the plan prohibitive.

A modification of the plan was tried in the University High School during the past year. Since the mathematics and general science courses alternate each semester in the seventh grade, one general science class was taught the first semester by one of the science instructors and continued with mathematics the second semester with the same instructor. Another group took mathematics the first semester and continued with general science the second, being taught the entire year by one of the mathematics instructors. Since not a great deal of mathematics is used in the general science course, there were few opportunities for integration. The major advantage was that the two instructors became thoroughly familiar with both fields and that each developed an acquaintance with certain problems peculiar to the other's field. One of the first steps toward closer cooperation between the two departments has thus been taken.

Summary: It has been shown that the problem of integration of mathematics and science is not new. Professor Moore's recommendation made in 1903 "that algebra, geometry and physics of the secondary school should be organized into a thoroughly coherent course" sounds perfectly modern today, although it was made more than thirty years ago. For a number of years much interest was shown in the problem. A number of plans were advocated. However, these plans, after being tried in various schools, were abandoned and the problem received practically no more attention until recently.

The modern trend toward integration of high school subjects has revived interest in the problem, and again certain progressive schools are experimenting. It is as yet too early to expect any measured results, and so far no results have been published that may be of assistance to other schools. However, the chances for success are more favorable, especially since most students now preparing to teach mathematics are qualified to teach courses in a second field. For mathematics this second field is usually in the sciences. Furthermore, the administrators of the secondary schools have become interested in the problem and are ready to encourage experimentation.

For schools planning to experiment with an integrated program in science and mathematics the following steps are recommended:

1. Analyze the science syllabus, texts, manuals, notebooks of pupils, guide sheets, and the specific units for situations which are mathematical in nature and require a knowledge of mathematical concepts, principles and processes.
2. Make a complete list of the types of mathematics which pupils need to solve the science problems and to read understandingly the assigned literature.
3. Analyze the problems and reading matter to discover the specific abilities necessary for understanding.
4. Collect specific criticisms of the science department as to the mathematical deficiencies of the pupils.
5. Determine the place and time in which certain knowledge and skills are expected to have been acquired.
6. Check the findings against the mathematics courses to see if the requirements have been adequately provided for.

7. Hold joint conferences of the two departments to determine the mathematical responsibilities to be assumed by the science department and the science responsibilities to be assumed by the mathematics teachers. No progress can be expected unless both departments have a real interest in the plan.

8. Make arrangements for transfer of subject matter and experiments of science to mathematics, and of mathematics to be taught by the science teachers.

9. Encourage teachers of either department to offer courses in the other.

Advantages to be derived from the plan: It is to be expected that the work of both departments will improve if science and mathematics are integrated because of the sympathetic attitude of all teachers toward the problems in both fields. The methods of the two departments will be in agreement and the pupil will not be confused by conflicting procedures employed by different departments. The plan should result in a saving of the pupils' time and effort. The chances are that the great scientific ideas and principles common to the two fields will really be learned by the pupil when he is given a broad outlook of both. The methods of science will find a way into mathematics teaching. Mathematics will tend to become more experimental and therefore less formal and mechanical--a study of relationships, a mode of thinking.

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How Much Mathematics Should Be Required and Used in High School Physics?[1]

Robert E. Carpenter
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(Vol. LXII No. 5 May, 1962)

The answer to the question, "How much mathematics should be required and used in high school physics?" is one that has been of great importance in the past few years because it is the *use of mathematics* in physics which takes the blame for the drop in enrollment and the failures in high school physics classes. Some authors and educators have suggested that mathematics should be removed from these courses and that physics should be taught only as "cultural" courses.

> *". . . we may teach it [physics] as a cultural subject--as an appreciation and understanding of the concrete objects of our scientific environment; or we may teach it as preparation for becoming a physicist or an engineer. For the majority of high school students only the cultural course is meaningful. . . . if we teach physics as a cultural understanding of the concrete objects of our physical environment, mathematical problems are not necessary." (3)*

This writer wishes to present the opposite point of view. I believe that the high school physics course should require all the mathematics that the student can get before enrolling in the subject. I agree fully with the belief that ". . . for permanent value, the teaching of physics must still center in a real understanding of its principles and relations and of the units of measurement employed.

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[1]A paper presented at the convention of the Central Association of Science and Mathematics Teachers, Chicago, Illinois, November 24, 1961.

Otherwise, the student will merely learn about physics instead of learning the subject itself." (1)

Before we can arrive at any solution to the problem of how much mathematics should be used, we must define the types of physics courses taught. Generally, these courses are divided into three major categories based on the type of student enrolled. These are:

(1) The College Preparatory or "Academic" course. This includes the advanced placement or honors students.

(2) The General Course. This is probably the course found in most schools and includes all types of students.

(3) Practical Physics. An examination of the outlines for this course indicates that it probably should not be called physics at all.

Whichever type of course is taught, the most important aim should be to introduce the student to the processes of exact, quantitative thinking, along with careful observation and honest reporting and analysis of what he observes. Such an aim means that the student must know how to apply his arithmetic, algebra, geometry, and trigonometry. He must be taught that these are useful tools. He must be taught the proper use of these tools in his physics courses. The physics teacher cannot expect the mathematics teacher to do this for him. The mathematics instructor can give the pupil the concepts and principles of mathematics, but it is the responsibility of the physics instructor to show him how to apply these concepts in a physics class.

Certain minimum mathematical requirements should be set as prerequisites for *any* high school physics course if quantitative reasoning is to be a part of the course. These are two years of algebra and one year of geometry with a grade of "C" or better. For the college preparatory course, one semester of trigonometry should also be required. In some cases, the trigonometry and physics may be taken concurrently.

I realize these requirements may create some scheduling problems in schools in which a minimum curriculum is offered. Any school, however, should be able to offer two years of algebra and one of geometry by the student's senior year. The trigonometry needed for the physics course may have to be included as a part of the course. Another major problem created by these requirements involves the administrators and counsellors, who might have to tell a student which subjects he can take instead of allowing him the usual freedom of choice. Perhaps the physics teachers can help to show parents and students that school is a place to learn and that pupils should be placed where they belong and not where they wish to be.

How these mathematical prerequisites will be used in a course will be determined by the type of course taught and the ability of the students enrolled. In the general and practical courses, it is not necessary for all students to derive the formulae for the laws of motion in order for them to understand the meaning of such laws. All students, however, do need to work with these laws in a quantitative way. How can one fully understand that it will require at least four times the distance to stop a car at sixty miles per hour as it does at thirty miles per hour if he has no opportunity to prove this mathematically?

If the practical course is to be taught, and if the majority of students in the course are non-college preparatory, one might ask, "Why should these students be required to have mathematics prerequisites of algebra and geometry?" The answer to this question can be found in the outline for the course. Such a course should deal with the applications of the physical laws and concepts, applications which have been developed or discovered because of the mathematical and quantitative nature of the laws. The best way to give a student a real understanding of the practical uses of a physical concept is to provide him with opportunities to set up and solve applied problems.

Granted, a student can be taught the proper way to put a plug on the end of an extension cord. He can also find out whether the appliance on the other end of the cord will blow a fuse by the simple experiment of plugging it in. Is this physics? Would he not have a better understanding of why a fuse might blow if he has had the experience of actually calculating the power consumed in an a.c. circuit by several appliances? Such experiences might prove to be less expensive and, in a few cases, less disastrous.

So far I have tried to present a few arguments for the need for mathematics pre-requisites as a requirement for students enrolled in high school physics classes. How much and what kind of mathematics should be used in the classes should also be considered.

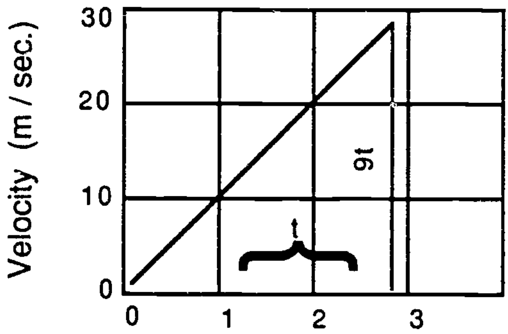
A student should be able to read a problem, analyze it, and apply the proper arithmetic, algebra, geometry, or trigonometry in solving it. He should be able to state the problem in a mathematical way, using properly defined symbols and equations. The equations must show all relationships that are stated or implied in the problem. It becomes necessary for the physics teacher to make the student understand that he must clearly define the terms and symbols which he plans to use in solving a problem. An example illustrating the use of analysis and equations can be found in the problem of finding the resultant of two forces acting at right angles to each other from the same point. The student should be able to translate such a problem in terms of the Pythagorean theorem and set up a mathematical expression to show this. Have you ever asked your students to state and explain this theorem? Most of them will probably say "$c^2=a^2+b^2$." If you now ask them what c^2, a^2 and b^2 are, few can tell you even with the usual *ABC* right triangle drawn on the chalk board. Incidentally, it should not be too great a task to relate the use of basic trigonometric functions to such problems.

Since the quantitative work in physics involves measurement, the use of units becomes an important concept. Teachers should insist that students include the proper units throughout the solution of every problem, experimental or otherwise. Students should be allowed to develop their own units so long as they have a logical reason for doing so. Timing an event in terms of heart beats should be as thrilling and useful today as it was in Galileo's time.

Measurement immediately calls to mind the importance of error. Students need to know how to account for errors, and how to analyze measurements in terms of uncertainty. They also need to know the meaning of per cent of error. An understanding of measurement and errors due to measuring can be related to significant numbers.

The use of exponential notation for very large or very small numbers should be considered also. Such numbers as Avogadro's number (6.02×10^{23}) or Angstrom units (1×10^{-10} meters) are much easier to comprehend than if they are written out the "long" way.

The construction and interpretation of graphs is another phase which should be included in every physics course. Direct and inverse proportion can be shown quite readily by graphs. A plot of velocity-time relationship for a freely falling body can be used to show the distance. The distance is equal to the area under the curve or the area of the triangle with base t and altitude gt. This area is equal to $1/2(gt^2)$. Is this use of graphs an introduction to calculus?



Many examples, other than the ones discussed, could be listed, each one illustrating the need for the use of mathematics in physics. Any physics course, be it academic or practical, must be based on nature's physical laws and phenomena--and these laws and phenomena are mathematical.

We physics teachers have the responsibility of showing our students the importance of quantitative reasoning, experimentation, and observation in trying to understand nature's physical phenomena. If we only give students an appreciation for the physical world as it "mathematically and quantitatively" exists, we will at least have helped to prepare them for their science-centered world.

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Scientific Concepts in the Junior High School Mathematics Curriculum[1]

William L. Schaaf
(Vol. LXV No. 7 October, 1965)

Theoretically, a curriculum in secondary mathematics can be constructed with little or no reference to the physical sciences. Indeed, most mathematics courses are of this nature. It is hardly conceivable, on the other hand, that a substantial curriculum in the physical sciences can be designed without a modicum of references to mathematics, and yet this is commonly done. To be sure, mathematics at the secondary level and even below can be taught as a self-consistent deductive system without any allusions whatever to natural phenomena. But it is difficult to see how science--particularly physics, chemistry, and portions of general science--can be taught meaningfully without introducing certain mathematical concepts.

From time to time, during the past fifty or sixty years, educators have advocated some form of "integration" of science and mathematics to the end that each discipline should benefit from the other. Thus from John Perry, E. H. Moore and Felix Klein at the turn of the century, down to the contemporary champion of such a program in the person of Morris Kline, many a voice has been raised for greater fusion of these two disciplines. Such proposals, however, have never really taken hold in the United States, although they are common enough in Europe. In the past, the alleged explanation for the failure to teach mathematics and science together was the fact that many mathematics teachers didn't know enough science, and, per-

[1]A paper presented at the 64th annual convention of the Central Association of Science and Mathematics Teachers, November 28, 1964.

haps less frequently, many science teachers didn't know enough mathematics. Due to the contemporary emphasis in secondary mathematics on logical foundations and structure, there appears to be less likelihood than ever of introducing a significant amount of science material into mathematics courses. If so, this would be most unfortunate for both subjects, inasmuch as on the highest levels, modern science leans heavily on "pure" mathematics of the most abstract nature. I am convinced that junior high school pupils can learn a number of scientific concepts in conjunction with their mathematics.

PSYCHOLOGY OF CONCEPT LEARNING

Let us begin with a brief consideration of the psychological aspects of learning concepts, particularly concepts such as those encountered in science and mathematics, which are characterized by generality, preciseness, and abstractness. Conventional psychology texts, frankly, do not offer much help. The researches of Piaget, Bruner and others, however, throw considerable light on the matter, and much of what follows is based directly on the findings of Professor Bruner and his associates at Harvard University.[2] Their findings furnish a useful background for our discussion. In general, the learning of concepts involves two major factors: (1) discovery and intuition, and (2) communication. To be sure, Bruner breaks this down into four factors, but from my point of view it is more convenient to think of discovery and intuition together, while "translation" and "readiness" are so inextricably associated with language and semantics that probably the translating of intuitive ideas into generalizations and abstractions is part and parcel of communication between teacher and learner.

According to Bruner, the procedure or mode of thinking by which a discovery is arrived at is more important than the discovery itself. He points out, further, that at one end of the spectrum the learner can accommodate by accepting what he encounters or is presented with and changing his behavior accordingly; at the other end, he can assimilate by converting current experience into what are for him already existing concepts and meanings. Probably neither of these extremes is ideal, and a middle course is more realistic. In any event, the discovery itself--the end product--is a built-in reward. At this point, I should like to make a few observations from the standpoint of the classroom teacher. For some pupils this is a wonderful

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[2]Cf. Jerome Bruner, On Learning Mathematics, *The Mathematics Teacher*, 53:610-619 (December, 1960).

experiment. But at best it is rather time-consuming, and at worst, very uneconomical. Let's be honest: how many of our pupils are potential Newtons or Gausses or Einsteins? Even with skillful guidance and expert heuristic teaching, it is unrealistic to expect the majority of pupils to discover (or rediscover) many of the major significant concepts in any area of science or mathematics.

We should note that an intuitive discovery is in essence a conjecture, an inner realization of the meaning or nature of a concept or relation without recourse to logical analysis or formal proof. Suffice it to say that there are good reasons for encouraging pupils--especially in the lower grades and in the junior high school--to use intuition freely. Under no conditions should the disposition to "guess" and discover be inhibited or penalized, despite the possible drawback mentioned above.

Having made a discovery, or on the threshold of grasping an idea intuitively, how shall it be transformed into a general principle, or an abstract concept, or an analytical tool? This is largely a matter of language and communication. As a general rule, technical and semitechnical terms should not be introduced until the learner has already grasped the meaning intuitively. It may also be helpful to distinguish (loosely perhaps) between concepts somewhat as follows:

Reality	Evidence	Examples
(1) Obvious "real" existence	Direct observation; sensory impression	Volume; weight; motion; illumination
(2) Quasi-real existence	Indirect observation; surrogate sensory impressions; measurements	Capacity; force; pressure; temperature
(3) Intellectual existence	Abstract concept; symbolism	Center of gravity; moment of inertia; chemical bond; true value of a measurement

In addition to considering the nature of the concept as just suggested, there is also the matter of sequence. Some concepts are clearly prerequisite for understanding other concepts. For example, before a pupil can understand "angle of elevation" and "angle of depression," he must understand the meaning of *horizontal, vertical, perpendicular, altitude, parallel, alternate interior angles between parallels*. Or, again, before he can grasp the meaning of the precision of an instrument or a measurement, he must know the meaning of *scale, unit, subdivision, midpoint, estimate, absolute error*.

Finally, will the learner grasp the meaning which we want him to? This is far from a simple matter. It goes beyond mere "readiness." It is a function of maturation, to be sure, but there are probably other more significant considerations, such as both the learner's and the teacher's purpose and objectives; the learner's individual personal satisfactions; recognizable utility of the thing being learned, immediate or ultimate; intrinsic interest or appeal in the context of the learner's milieu. Not the least important factor is the question of whether the desired goal is to achieve wide coverage rather than depth and continuity of content, that is, appreciation rather than clear understanding and insight. But this is a matter of educational philosophy which I do not propose to enter upon here.

SCIENCE CONCEPTS IN MATHEMATICS

Let me say at once that the following selection of topics for possible inclusion in the mathematics curriculum for Grades 7-9 is entirely arbitrary, and merely suggestive; it represents a personal conviction, or, if you will, a hunch as to what might be feasible and desirable. You may feel that some of the topics suggested are too difficult for 12-14 year olds, or you may believe that the topics are too unrelated to each other. No doubt other objections may suggest themselves. Nevertheless I am reasonably certain that these concepts are (1) easily motivated; (2) intrinsically significant; and (3) readily related to and illustrative of important mathematical concepts currently being taught at these grade levels.

GRADE 7

The Lever. The lever as a simple machine is easily motivated. Many of the tools, gadgets and other artifacts with which modern man is surrounded and which illustrate the lever are perfectly familiar to boys and girls. Sports and Scout activities furnish other examples of levers. The principle of moments is a significant physical concept. The intuitive grasp of the role played by a force and its arm flows readily out of children's experiences on the playground. The final analytical formulation

$$W_1d_1 = W_2d_2$$

is easily enough reached. True, the concept of a *turning moment* is an "intellectual concept," or an abstract concept; but the vivid experiences on a physical teeter-board, both as to the effect of changing a W or a d, should lend plausibility to the product Wd. The symbolic expression of the law of the lever exemplifies the concept of equality and furnishes an opportunity for solving simple equations.

Temperature. The average adult layman is often confused about heat and temperature and their measurement. And yet many everyday experiences involve these notions. Frozen foods openly displayed in the supermarket; the deep freezer in the home; questions of temperature in outer space, in the earth's interior, or on the sun: could a topic be more easily motivated? We note that the concepts involved in measuring temperature are not readily arrived at intuitively, since direct sensory observations (excluding reading a thermometer) are notoriously misleading. Among the prerequisite concepts involved are the phenomenon of expansion, the stability of melting points and boiling points, and the arbitrary selection of the scale to be used. The ultimate analytical expressions

$$F = \frac{9}{5}C + 32 \text{ and } C = \frac{5}{9}(F - 32)$$

are not particularly difficult to arrive at, although there is the danger that the net achievement may simmer down to skill in numerical substitution and conversion from one scale to another. However, it seems likely that this can be avoided if attention is given to the notion of comparison by means of a ratio rather than by difference, stressing the relation of the ratio of 180:100 or 9:5.

Density. The volume of a physical object, the room that it takes up or the space that it occupies, is presumably a simple concept to grasp. Its mass, often confused with its weight, is not quite so simple, although intuitively quickly understood. Density, however, is a concept that probably belongs to the abstract construct category. Although the symbolism $D=M/V$ is simple enough, the fractional form could be deceptive, suggesting a ratio rather than a rate. Yet this is a rate form that occurs fairly frequently in the sciences: "so much of some property per unit of some other property."

Liquid Pressure. Here a number of prerequisite concepts are indicated, namely, the concepts of *force, pressure, the virtual incompressibility of liquids, the relation of pressure to depth*, and *Pascal's principle of transmission of liquid pressure.* All these concepts are "slippery"; certainly they are of the quasi-real existence type. The intuitive discovery of these concepts may not be particularly difficult to come by, especially with "structured" experimentation and skillful heuristic guidance. But to make the transition to the generalizations, grasping their full significance, is another story. It is not easy to "visualize" a force acting although apparently "nothing is happening". *Force per unit of area* (i.e., *pressure*) is less difficult to conceive, although the learner may have to supply his own hypothetical unit surface as a category of the mind. That internal pressure exists upon every particle of a liquid, due to molecular motion and the mass of the particles, is by no means a simple idea, but probably within the comprehension of pupils; the notion that pressure at any point within the liquid is exerted equally in all directions is one that may have to be accepted on faith, for a while at least, despite the fact that the indirect sensory evidence is easy enough to come by. Once the last two concepts have become meaningful, the idea that pressure varies directly as the depth and as the density is relatively easy. The symbolic forms for these generalizations are simple algebraic expressions:

Pressure (in general):

$$P = F/A, \text{ or } F = PA$$

Liquids:

$$P = HD$$

(where H = height and D = density).

for horizontal surface:

$$F = AHD$$

for vertical surface:

$$F = AHD/2$$

As for buoyancy, this is a concept which has easy motivation that is rooted in many common experiences: it is easier to lift a stone under water than when it is out of the water; it is easier to swim and float in salt water than in fresh water; push a floating object under the surface and when released it bobs up immediately; and so on. Yet the principle of Archimedes, that an object immersed in a liquid apparently loses weight to the extent of the weight of the liquid it has displaced, is for most learners a particularly difficult concept to understand. That there should be *some* apparent loss of weight most learners will agree to intuitively without much difficulty; but to understand exactly *how much* weight is lost, and why, is something else again. Yet, with a simple mathematical analysis of a hypothetical cube immersed successively at different depths, it is possible to develop the concept that the buoyant force equals $A(h_2-h_1)d = Vd$, or the weight of the liquid displaced by the cube.

Boyle's Law. The tyro finds it somewhat more difficult to understand the properties and behavior of gases than those of solids and liquids. Here is a case of "quasi-real existence." Most gases can't even be seen or felt; that the familiar air which constantly surrounds us should have weight and be able to exert "real" pressure is not easy to believe. Yet the phenomenon of atmospheric pressure, the barometer, and Boyle's law are significant concepts of science. These topics are easily motivated: the boy's bicycle tire, effortlessly inflated at the neighborhood filling station; the role of the barometer in weather forecasting; variations in atmospheric pressure at different altitudes; pressurized cabins in aircraft; the complete lack of air pressure in outer space; all these are of interest to junior high school pupils. As for the mathematical concept involved, Boyle's law provides a simple dramatic instance of inverse variation, and the alternative symbolic expressions, $V_1/V_2=P_2/P_1$ and $P_1V_1=P_2V_2$, can be extremely illuminating.

GRADE 8

Suggested concepts for this grade level might well include a discussion of the speed of sound; linear and cubical expansion caused by heat; the inverse-square law of illumination; further discussion of the gas laws; uniformly accelerated motion and freely falling bodies. To be sure, the several topics mentioned are unrelated, but each one is more or less self-contained. Moreover, each of these topics is readily motivated, and the mathematical concepts involved are simple. The scientific concepts vary in difficulty and subtlety and there are admitted pitfalls.

Sound and Heat. Perhaps the simplest concept among this group is that the velocity of sound varies with the temperature and with the nature of the transmitting medium. When trying to understand and use a linear coefficient of expansion, the pupil may run into difficulty of a skill nature, namely, in handling 5- and 6-place decimals. He may also find the symbolism a bit formidable: $L_2-L_1=L_1k(t_2-t_1)$, although it would seem that neither the concept of the difference between two lengths or between two temperatures, nor the use of subscripts to designate similar quantities, is intrinsically as difficult as some of the rather subtle mathematical concepts that contemporary eighth-graders are expected to learn, such as the distinction between a subset and a proper subset, or the concept that between any two given rational numbers there exists an infinitude of other rational numbers.

Light. The inverse-square law may present one or two unique difficulties. It may be the pupil's first encounter with inverse variation; the additional consideration that the variation is of the form $y=k/x^2$ rather than $y=k/x$ may also be confusing. Then there is the concept of intensity of illumination, involving the use of a compound unit, the *foot-candle*. If it be pointed out that the idea of a compound unit such as this is too sophisticated, it could be said that a few years ago, when "social and economic arithmetic" was in favor, eighth-grade textbooks did not hesitate to introduce such terms as man-hours, ton-miles and passenger-miles.

Gas Laws. Extension of the gas laws to include the relation between volume and temperature when the pressure remains constant, and the relation between pressure and temperature when the volume remains constant, are conceded to be a bit sophisticated. For one thing, the concept of absolute temperature must be developed first. Then there is the danger of becoming involved in a discussion of isothermal expansion and adiabatic expansion, and I am not advocating teaching those ideas at this grade level. However, it is quite feasible to emphasize the contrast between Boyle's law and Charles' law--one being an instance of inverse variation, the other, direct variation. These are the mathematical concepts that we should seek to clarify.

Motion. When we come to problems of uniformly accelerated motion and freely falling bodies, we are dealing with a subtle concept--the rate of change of a rate of change. For most pupils, the notion of a simple time rate of change--a car driven at 55 miles per hour, or a jet plane flying at 650 miles per hour--is readily comprehended; so is the relation $D=RxT$, or $R=D/T$. From this it is an easy step to the analytic formulation of

$$s = vt, \qquad v = \frac{s}{t}, \qquad t = \frac{s}{v}.$$

If the motion is uniformly accelerated, we will have to introduce the concept of change of velocity per unit of time, or

$$\frac{v_2 - v_1}{t};$$

this is where the rate of change of a rate of change enters the picture. The learner must grasp the idea that if the velocity of an object changes from 10 ft./sec. to 50 ft./sec. during an interval of 5 sec., its acceleration is at the rate of (50-10)/5=8 ft./sec./sec. This unit of "ft./sec./sec." is a novel situation and may cause difficulty. In order to arrive at the distance relation for uniformly accelerated motion, we must first develop the idea that $v=at$; then we must get across the idea of average velocity, and why average velocity$=(0+at)/2 = 1/2at$. Once this is understood, the relation $s=1/2at^2$ falls into place easily enough. Lest it be thought that the idea of an average velocity is too recondite, we might remember that youngsters are realistic: They know that if you ride a bicycle or drive a car any distance, the velocity is likely to fluctuate during the journey, and so if it took 3 hours to go 120 miles, the *average velocity* during that 3-hour period was 40 mi./hr. Discovering from the earlier relations that $v^2=2as$, or $v = \sqrt{2as}$, is a good example of the power of analytic expressions.

If average eighth-grade pupils can master these concepts (and it is quite possible that only the brightest among them can do so), then the idea of freely falling bodies should not be difficult. The chief hurdle here is to make the constant acceleration of gravity a meaningful concept--and I don't mean just memorizing the fact that g=32.2 ft./sec./sec. The analogous relations

$$v=gt, \quad s=1/2gt^2, \quad v = \sqrt{2gs}$$

should then become meaningful.

GRADE 9

As likely topics for inclusion at this level one might suggest: Hooke's law of elasticity; concurrent forces; horizontal and vertical components; parallelogram of forces; quantity of heat and specific heat; Ohm's law; parallel and series circuits; electrical resistance; electric energy and power; heat effect of an electric current. Although some of these topics may not be quite obviously motivated, they do touch upon fairly familiar everyday experiences: the strength of materials used in constructing bridges and sky-scrapers; the effect of winds on aircraft travel, the angle of drift, etc.; the effect of large masses of water upon surrounding climate; method of wiring strings of Christmas tree lights, or city street lights, so that if one lamp goes out, it does not affect the others; do we purchase electric energy or electric power from the utility company? Why are electric toasters, electric irons and electric heaters more expensive to operate than electric fans and electric motors?

Elasticity. The concept of elasticity is not always clearly understood by the layman. Prerequisite concepts are *stress (F/A); strain (Δx/x); deformation (Δx)*; and *elastic limit.* Within the elastic limit, the ratio of stress to strain is constant, exemplifying direct variation:

$$\frac{stress}{strain} = K;\ or\ \frac{F/A}{\frac{\Delta x}{x}} = k = Young's\ Modulus$$

Forces. The concepts of horizontal and vertical components of forces and of the parallelogram of forces afford an interesting application of numerical trigonometry, and should not cause great difficulty. The skillful teacher will present the sine, cosine and tangent functions in the role of multipliers as well as ratios.

Heat. Heat exchange may present a slight problem. Again we are in the area of concepts having quasi-real existence. Associated and prerequisite concepts include temperature gradient, specific heat, familiarity with calories, and the principle of the conservation of energy. Once the concept of equivalence of total heat lost and total heat gained has been grasped, and the significance of specific heat is appreciated, the analytic statement

$$W_1 = (t_1 - t_f)(s_1) = W_2 (t_f - t_2) s_2$$

may not be as frightful as it might seem.

Electricity. When we come to topics in electricity, we have the relation $I = E/R$, showing that current varies directly as the electromotive force (e.m.f) and inversely as the resistance. What the learner should come to understand is that

$$(1)\ I = E/k \quad \text{and} \quad (2)\ I = K/R$$

where k and K are constants. It must be realized, of course, that here we are almost in the realm of "intellectual existence" insofar as the concepts of current intensity, e.m.f., and resistance are concerned. There is a danger that, with insufficient experimental and other science background, these terms may become mere verbal symbols unassociated with any clear conceptual understanding. (This danger exists, I suppose, for the science pupil as well as for the pupil in the mathematics class.)

Series-connected circuits present no difficulty. A parallel circuit, however, presents a slight conceptual difficulty, viz., that the voltage across each branch remains the same. There is also a possible difficulty on the skill side: the handling of fractional equations of the form

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

This matter of electrical resistance is a bit "sticky." The electrical resistance offered by a conductor is a function of (1) the nature of the conductor; (2) the length of the conductor; (3) the temperature of the conductor; (4) the cross-sectional area of the conductor. Possibly the two significant mathematical concepts here are that resistance varies :

(1) directly as the length, and

(2) inversely as the area, or inversely as the square of the diameter, the latter exemplifying a property of the areas of similar figures.

To understand the relation of electrical power to electrical energy, a number of preliminary concepts must have been grasped:

(1) Power (watts) = volts x amperes, or $W=IE$
(2) Since $E = IR$, we say $W = I(IR) = I^2R$
(3) Kilowatts = volts x amperes/1000
(4) 1 kw.= 1 1/3 horsepower
(5) Power is the rate of doing work
(6) Energy = Power x Time
(7) Kilowatt-hours (energy) = *IxExt/1000*

I am willing to concede that there are considerably more science concepts in this group of ideas than mathematical ideas; yet somehow it seems to be something worth considering in a mathematics class nevertheless. Finally, there are a few additional topics that might well be worked into any one (or all three) of these years, but for lack of time they can only be indicated here. In connection with measurement, these are the concepts: *unit of measurement; standard measure; scale; absolute error; relative error; precision; accuracy; tolerance; deviation; scientific notation.*

SUMMARY AND CONCLUSION

We have arbitrarily singled out some fifteen or twenty pertinent topics of physical science, involving possibly half a hundred concepts concerning natural phenomena. We have indicated that these concepts stand in close relation to a small number of highly significant mathematical concepts which apparently are "built into" the contemporary junior high school mathematics curriculum along with the present day emphasis on structures and logic: these concepts include *ratio; variable and constant; function and graph; direct, inverse and joint variation; linear function; quadratic function; orthogonal projection; Pythagorean relation; measurement and error; precision and accuracy; computation with approximate numbers; exponential notation.*

It is the writer's considered opinion that if these science concepts were appropriately introduced into the mathematics course in Grades 7-9, they would dramatize and reinforce some of the mathematical concepts which we hope will be learned in these grades, and that the use of the mathematical concepts would in turn contribute to greater insight, now as well as later, into the meaning of the science concepts. In the light of the psychological considerations presented in the beginning, it is my further conviction that these science concepts can be learned effectively by the majority of junior high school pupils. Most of them can be drawn from familiar experiences and sensory observation, and nearly all of them are of real interest to children of this level. To be sure, it means adding something to a rather full curriculum as it is, and it means increasing the responsibility of the mathematics teacher. But we have hardly given it a real trial, and it would appear to be eminently worth trying. We have been disposed to underestimate our pupils in the past; perhaps if we gave them the opportunity they might surprise us agreeably.

Very early in this paper I hinted at the relation of science to mathematics. Although it does not bear directly on the question under discussion, I cannot refrain from a few observations. We all know that the precise meaning of the terms "pure mathematics" and "applied mathematics" is somewhat of a controversial matter, if, indeed, there is any valid distinction today. We are all too familiar with the cliche that mathematics is the language of science. As a recent writer[3] has said: "Mathematics is the only language we have by which statements about nature can be combined according to logical rules; a language which not only permits us to describe the order in

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[3]Cf. Morris Shamos, "The Language of Science," in the *New Jersey Mathematics Teacher*, 22:5-11 (October, 1964).

nature, but by providing the logical tools for dealing with these descriptions, leads us to a better understanding of that order. It is an essential part of the structure of science, not simply an accessory." This same writer feels that his colleague, Morris Kline, exaggerated somewhat when he asserted that, "Science has become a collection of mathematical theories adorned with a few physical facts." I am disposed to say that the former statement is too naive and not nearly strong enough; while the latter, although much nearer to the truth, is unfortunately a little too cryptic for comfort.

Nearly forty years ago the case was stated very well by Heisenberg, when he wrote:

> *"On the one hand, mathematics is a study of certain aspects of the human thinking process; on the other hand, when we make ourselves master of a physical situation, we so arrange the data as to conform to the demands of our thinking process. It would seem probable, therefore, that merely in arranging the subject in a form suitable for discussion we have already introduced the mathematics--the mathematics is unavoidably introduced by our treatment, and it is inevitable that mathematical principles appear to rule nature."*

Personally, I feel that one of the most perceptive observations ever made in this connection was that of the late J. W. N. Sullivan. Discussing mathematics as an art, he wrote:

> *"The significance of mathematics resides precisely in the fact that it is an art; by informing us of the nature of our own minds it informs us of much that depends on our minds. It does not enable us to explore some remote region of the externally existent; it helps to show us how far what exists depends upon the way in which we exist. We are the law-givers of the universe; it is even possible that we can experience nothing but what we have created, and that the greatest of our mathematical creations is the material universe itself."*

We all realize that science can be kept perfectly safe for elementary school children. We can give them little electric outfits, cute little steam engines that whistle, and houses with glass fronts so we can peep in on the private lives of ants. We can go further and have a workshop where youngsters can repair simple machinery brought from home and learn how to work safely with electricity, leaves, and lizards. We can do all of this, and never let it penetrate the child's mind that science has any significance beyond a mechanical one.

W. Brogan, 1939

One of the fundamental purposes for teaching science to little children is the same as the one offered for the teaching of arithmetic: to help the child understand his environment, the many quantitative and phenomenal aspects of it, and to help him develop in sequence the concepts and skills which will allow him to pursue at a later date these studies as they appear in specialized areas and fields.

P. A. Nelson, 1962

IV. Science and Mathematics in Elementary Education

It should be apparent that science and mathematics may be related in several ways in our elementary schools. There is a continuum from mathematics for the sake of mathematics to science for the sake of science. Between these two extremes lie at least three points of importance: (1) mathematics for the sake of science, (2) mathematics and science in concert, and (3) science for the sake of mathematics. Any program that does not include all five points on this continuum is not representative of both disciplines and the related aspects of both fields of study.

W. R. Brown and C. E. Wall, 1976

The teacher needs to provide an interactive environment for children and help them focus on what they see and do. This environment must be constructed with care so that authoritarian, erroneous views and theories are not formed through inadequate feedback. The teacher should help focus, not dictate, the learner's attempt to give structure to what is observed.

N. B. Benham, et. al., 1982

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Science and Arithmetic in the Elementary School Curriculum

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(Vol. XXXIX No.2 February, 1939)

I am assuming the privilege of all speakers who are assigned a topic; namely, that of interpreting the title in such a way as to permit me to have my say unmolested by the programming committee. Hence you will hear very little about arithmetic as such because that subject has always seemed the basic science to me. It has lost its vitality because we have destroyed that position and made it a series of numerical abstractions.

My discussion today will be on science, but do not forget that arithmetic is included in that term. The last reference to arithmetic will be to call your attention to the excellent statement of R. L. Morton and his committee in the November issue of the *Curriculum Journal.*

A healthy, curious small boy, until he has been squelched by school procedure, can tell us more about the place of science in an elementary school than can pedagogues. Instead of listening to me this afternoon you could get a better answer to the question implied in the title of this talk by listening to that boy pour forth a constant stream of "How?", "Why?", "When?", or "What makes it go?"; "How do you work that?" *et cetera.* Science, insofar as possible, should help him answer his questions, and that is its place in the elementary school curriculum.

Of course, that is a lovely generalization which will probably give rise to some hows, whens, and why of its own. Nevertheless, I'll stick to that generalization as an adequate description of the place of science in an elementary school.

Science is a newcomer in the elementary curriculum family. Moreover this babe was born during a combination of a family quarrel and spring house cleaning in our pedagogical household. Father, mother, aunts, uncles and distant relatives all have plans for the correct up-bringing of this newcomer, and the plans do not harmonize. In addition, this babe seems to be the only male child in a somewhat effeminate group.

Actually, and seriously, the problem in science is exactly the same as it is for all other school subjects. Is science in the elementary school a body of material and a method with which children answer their questions, or something that teachers teach, regardless of immediate questions but directed toward future needs? To put it as specifically as possible, should a teacher show a pupil how to repair household electric equipment, or have him go through standard experiments on fixed apparatus to teach laws in physics? Does Johnny bring his broken bicycle to the laboratory or does the teacher buy expensive machinery to demonstrate wheels, pulleys, leverage, et cetera? Or, to reduce it to the standard phraseology, is science a body of subject matter to be acquired in a systematic fashion, or is it a procedure whereby children secure an increasing ability to answer questions which bewilder them, repair machines they use and need, to secure an increasing control over their ordinary lives?

"Neither one exclusively, but some of both" is the easiest answer, but I doubt if it will work for the busy and hurried classroom teacher who already has her hands full. She will teach science as she teaches other material, be it by textbook, project, experience, or what have you. She will take care of this curriculum baby the same as she cares for the older members, no doubt wondering all the while when birth control will be exercised in the educational family.

So far I've merely presented the controversial issue, perhaps over elaborately, but because I want to give an opinion concerning science it seemed necessary. Maybe it is just the inevitable pedagogical prologue. Now for the opinions.

First, science is a method of inquiry, not a subject. Science, as is true of any method of inquiry, does not exist in a vacuum but in relationship to the material subjected to inquiry. Please, or as the children say, pretty please with sugar on it, do not take that one sentence to mean that I recommend an abstract, unrelated "method." There is no such animal. Scientific inquiry in various areas has given to us a tremendous body of subject matter of indubitable value. It is our duty as educators to help children acquire much of this information because they need it to live today. Yet it is a possible and frequent happening to see the results of scientific inquiry taught in such a manner as to kill any spirit of investigation on the part of the learner. And this in the name of science. We must realize that a pupil can acquire a large amount of knowledge given him by science, yet remain totally without scientific ability to deal with any problems on his own environment.

Just to avoid quibbling, it may be necessary to add that a child who practices investigating, experimenting with, correcting and controlling the ordinary features of his life will study, of necessity, the organized knowledge of science. But the above statement is not reversible. Repeat that sentence three times daily.

Second, we must decide whether or not our primary concern in the elementary school is to be with the content or the method of science. This is not an "either or" question, but merely one of emphasis. It is proposed here mainly as a forerunner of the next point, but a few words of elaboration may help. This elaboration is going to take the form of a specific classroom situation.

Two second grade children ran in from a recess indulging in a heated "It does," "It doesn't," flurry. The "does" and "doesn'ts" were over iron, does it sink or float in water? Teacher would know, hence must settle the question. She did very simply. There was an empty tin can on her desk, and after convincing the skeptical young minds that tin and iron were much alike she went with them to a wash basin, filled it with water and let the children see the can float. She then took a hammer, pounded the can into a lump of metal, and let them see it sink. She took another can, punched a hole in the bottom and let them watch it fill, then sink. She then told them about boats and why a steel ship floats. In other words, she gave them a complete and satisfactory answer to their questions, verbally and visually, without bothering to guide them through the process of discovering the answer for themselves. Was she right or wrong? I do not know, but will always feel it was one of the best jobs of incidental teaching that I've ever seen if one considers only the subject matter or knowledge side of the question.

Third, do we want children to develop inquiring minds, together with an emotional capacity to cope with the situations arising therefrom? Do not answer this one too fast.

Let us return to the first proposition, that science is a method of inquiry, and add a corollary that all phases of human life, mechanical, moral, political, religious, are legitimate fields of investigation. Elementary school children do not come into direct conflict with established patterns very often, but are helpless and submissive. Nevertheless, circumstance is such that small children are becoming, increasingly, the emotional victims of a bewildered adult society.

The above sounds highbrow and theoretical when one is supposed to be considering science for elementary school children, but it isn't. Almost any day now some obstreperous young soul is going to discover that not only are the habits of birds and bees and flowers and trees natural science, but that human beings come under the same category. It has always seemed a strange paradox to me that we humans consider ourselves so much superior to animals, but when we want to teach our young natural science we have them study beasts lest they become contaminated by knowledge of the conduct of humans. Hence, I believe the average teacher, which is all of us, should understand rather clearly the consequences of any thoroughgoing science program that is concerned with inculcating the inquiring attitude of science into the pattern of childhood behavior. Our answers will be questioned; our right to give answers will be questioned; the basis of our assumed authority will be questioned. We will have to face the task of dealing with children emotionally and intellectually as opposed to the authoritarian basis of school control which now predominates and will be faced with the need of a teaching method based upon reasonable explanation instead of flat statement. Of course, all of this is extreme, but not at all improbable, and should represent the goal of teaching which is concerned with the method as well as the content of science.

However, there is another important matter we must consider which grows directly out of the points suggested in the preceding paragraphs, especially one question. If we guide children into the emotional and mental set of questioning and inquiring, how much of life is it "safe" to let them question? If the experimental technique is sound as a learning method, how far should the experimenting be permitted to go?

Children haven't acquired our knowledge of taboos and *a priori* truths. There are a number of "correct" answers which we know are correct "just because." For example, respectable teachers do not inquire into validity of our structure of property rights, nor doubt the soundness of our moral precepts in sex relationships. Young minds trained to inquire and not accept might not be so acquiescent. Do we want that? What about religion, the nature of patriotism, et cetera? Do not forget that all Socrates did was to ask questions about things respectable people were not supposed to question.

We all realize that science can be kept perfectly safe for elementary school children. We can give them little electric outfits, cute little steam engines that whistle, and houses with glass fronts so we can peep in on the private lives of ants. We can go further and have a workshop where youngsters can repair simple machinery brought from home and learn how to work safely with electricity, leaves, and lizards. We can do all of this, and never let it penetrate the child's mind that science has any significance beyond a mechanical one. However, let's not deceive ourselves as to what we are doing. We are not teaching science, a method of inquiry and control, except as applied to mechanical objects. Perhaps that is enough. Probably we do not want humanity's actions exposed to the light of scientific inquiry or rational judgment. Obviously we do not want children to develop an emotional mind-set which leads them to believe custom is open to question. And I think we are wrong.

Suppose we became interested in helping children develop experimental attitudes and techniques in social matters. Then how would we use science in the elementary curriculum? Briefly, we must let them experiment with the curriculum. Insofar as the school is concerned, the curriculum and its regulatory discipline is society for the child. Through the construction of that curriculum, by the nature of its attendant disciplines, we can contribute somewhat to teaching the child to analyze, evaluate, and control his social environment; or we can teach him to submit as graciously as possible to imposed learnings and autocratic control. As we are learning from world affairs, we must acknowledge that imposition and autocracy are the antithesis of science in human life, yet we continue these practices even in the teaching of science subject matter.

Why not guide children, even little ones, into establishing objectives of learning? Why not aid them as they conduct experiments with learning materials directed toward achieving these ends? Why not assist them in evaluating their objectives, the experiments directed towards achieving them, the results, and the construction of new and better objectives? Quite simply, it can be stated in one more question. Why not apply the rational, scientific method of action to all of the learning in the public schools? Or are we afraid of it?

The few preceding paragraphs are all questions which we must answer truthfully if we are concerned with real science in the curriculum; however, I want to give the closing moments to some suggestions for inquiry in an area which is sometimes overlooked in a discussion of curriculum-- namely, the knowledge and the attitudes of teachers. These two items are, in my opinion, the heart of any curriculum, despite the voluminous tomes turned out by curriculum experts.

This discussion is centered around science in the curriculum. Supposedly we are all interested in it as a source of real value in learning. What does science mean to us as adults in society, as teachers in society? What is our concept of science as a method of inquiry, as a guide to social action?

What we teach children in science is entirely dependent upon what we believe. An obvious truism, but a neglected one. Yet, I'm becoming worried about the attitude which says we human beings can conquer the physical world scientifically but must depend upon dogma, superstition, and a reverence based on repetition to direct our usage of mechanical power.

The last election is a good illustration of this point, and I shall try to elaborate it. (By the way, do not take this as a plea either for or against Roosevelt.) The present administration has done something which had never been done before in this country. It has furnished us with information out of which we can frame intelligent questions concerning the direction of our national future. The National Resources Board, in its reports, is outlining clearly our potentialities and our problems. This material is startling in its revelations and, to anyone who will think, a source of enlightenment. Despite that fact, this last election showed a distinct trend back toward the mumbo-jumbo which produces starvation in the midst of such plenty that one of our chief concerns is limitation of food production. A bunch of monkeys in a coconut tree would have more sense than to starve because there were too many coconuts.

Someone may ask, "What business is this of a science teacher? We are interested in chemistry, physics, zoology, natural science, et cetera. You are talking about sociology and things like that." No, I'm talking about science, a method of intellectual inquiry in the field where it is most needed, human relationships. I'm also talking about the responsibility of science teachers for training children to become accustomed to such methods of thought and action. Scientists, above all persons, must realize that increased mechanical efficiency directed toward achieving barbaric goals is not progress, but merely animal cunning raised to a higher level. On the other hand, the use of old tools to more humane ends is progress.

Germany, today, is a perfect example of mechanical science dominated by brutish superstition. However, we do not need to cross the ocean to find our examples. On Monday evening, November 14th, Dorothy Thompson gave a brilliantly bitter speech concerning the persecution of 500,000 Jews under the control of Naziism. She did not mention the 2,000,000 sharecroppers living under very near the same conditions here because of our superstitious faith in capitalism. Water Lippmann once observed, in a bitter attack on Fascism, "that if Democracy loses one election, there will be no more elections," but his columns are just as bitter against workers in this country who presume to fight for a right to vote on conditions of employment and wages.

What has all this to do with science? Do you suppose a person accustomed to scientific thinking would be guilty of the above inconsistencies? Perhaps, but let us hope not. We cannot go on much longer creating twentieth century engines to appease the appetites and sooth the gods of cavemen.

Lancelot Hogben, the noted English scientist, has stated simply and clearly the function of science as a social instrument. After discussing many of the beliefs which hindered progress he wrote:[1]

"In their place modern science now offers us a new social contract. The social contract of scientific humanism is the recognition that the sufficient basis for rational cooperation between citizens is scientific investigation of the common needs of mankind, a scientific inventory of resources available for satisfying them, and a realistic survey of how modern social institutions contribute to or militate against the use of such resources for the satisfaction of fundamental human needs. The new social contract demands a new orientation in educational values and new qualifications for civic responsibilities–the power to shape the future course of events so as to extend the benefits of advancing scientific knowledge for the satisfaction of common human needs, guided by an understanding of the impact of science on human society."

In this statement is the true charter for science teachers, a description of their responsibility for liberating us from superstition into the freedom to enjoy the fruits of our achievements.

[1]Hogben, Lancelot, Scientific Humanism, *The Nation*, November 12, 1938.

12

Precision in Science and Arithmetic in the Elementary School

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STATUS OF ELEMENTARY SCIENCE AND MATHEMATICS

The importance of both elementary school science and mathematics has long been established, although it is perhaps true that today's mathematical studies and research groups at the elementary school levels have been given more prominence than those in the science areas. Study groups investigating the learning development of arithmetic concepts are making steady progress in many sections of the nation, but although there is some research in elementary school science relative to how children think scientifically, much research is still in the formative stage.

Although the advocates of problem-solving methodology have long recommended inductive approaches to science learning, most elementary science text books still retain a preponderance of descriptive science, and much lip-service is offered to teaching by problem-solving.

It is true that, in the past, it was necessary that science at the lower level be "bootlegged" under the term *social studies.* During the past decade, however, elementary science, per se, has emerged in dignity, with no apparent necessity for forced correlation with history and geography. This is true of pseudo integration with other subjects in the school curriculum, also, particularly language-arts and reading. It is recognized that for long periods of time, science was often a weak but concomitant part of the language-arts program, and children learned to write, perhaps not too creatively, while reading directions on how to construct something or perform an "experiment." Learning how to outline paragraphs involving descriptive science was a particularly offensive gimmick. In the reading program a story on the conservation of wild flowers, or an episode from the life of Louis Pasteur, often constituted the science program for the day or even for the week! It is an interesting observation that this forced integration has not occurred to such an extent in the arithmetic program. Indeed, quite the opposite has taken place. Science and arithmetic have normal integration, although teachers have often failed to take advantage of the many varied opportunities that exist, and have always existed, in integrating the science and arithmetic programs.

SIMILAR PURPOSES OF SCIENCE AND ARITHMETIC

One of the fundamental purposes for teaching science to little children is the same as the one offered for the teaching of arithmetic: to help the child understand his environment, the many quantitative and phenomenal aspects of it, and to help him develop in sequence the concepts and skills which will allow him to pursue at a later date these studies as they appear in specialized areas and fields.

The encouragement of rigorous thinking occurs in both science and mathematics, although it is a matter of history that mathematics pioneered first in the fields of problem-solving. It has become increasingly accepted that before a child can attempt problem-solving in science, he must be taught a *method* of approaching and solving the problem. In the field of number, this teaching of procedure before attempting the solving of a problem has long been used.

There are many so-called "scientific methods," and the current investigations and studies are endeavoring to determine which of the many methods is best suited for the elementary child of today. It has been determined that once, by good inductive teaching, the child has learned *how* to approach a problem and to work it through, he is on the way to thinking and conceptualizing. As in arithmetic, however, the student may go through the motions of writing down an answer without actually knowing the *meaning* of either the question or the answer. For decades, the meaning theory has been accepted by educators in the field of arithmetic as the most effective means for understanding and learning. Educators have also long recognized the importance of *readiness* in arithmetic. Pathetically enough, this has not been the case in the field of elementary science. It is true that some investigators have provided certain materials for the pre-school and kindergarten child, usually for the purpose of encouraging introductory scientific *discussions*, but little groundwork has been offered for recognizing science readiness at the primary level. Currently, some excellent materials are being produced relative to the training of the senses, with undertones of acceptance of the theories and philosophies of the sense-realists of the past.

SOCIAL UTILITY OF NUMBER AND SCIENCE

Because of the priority of skill subjects in the primary grades, increased time or emphasis on science cannot be advocated to the detrimental effect on the reading program. Since communication skills are so heavily emphasized in the lower grades, arithmetic as well as science often takes a secondary or even tertiary place for a valuable two-year period. This does not need to be. The social utility of number as well as science may be emphasized as a felt need of the child in understanding his own environment. The young person is awakened by the *sound of an alarm clock* in the morning; he reads the numerals to tell the time that he has arisen; he may watch his mother prepare his breakfast by *timing a 3-minute egg* or stirring cereal for *four minutes*. As he eats, he may read the numbers in *percent* on the back of the cereal box which tells him the *amounts of* vitamins and minerals his body requires in a day. From morning until evening the child's day is normally replete with numbers and science. Is this enough, then, for the young boy or girl who today talks in terms of space travel, of the *number of G's* an astronaut experiences, and the virus epidemic which may be engulfing his community? Secondary school educators have long accepted the fact that science and mathematics can be integrated quite normally for effectual learning. Teachers of elementary grade children, and particularly those in the lower grades, need to be alerted to the normal opportunities which arise daily to integrate arithmetic and science. Precision in working with and developing *measurement and time* concepts offers excellent means of combining the two disciplines. A good scientist is *accurate*; he measures amounts *carefully*; he *follows* directions with great respect. When large numbers are used, the child is trained to make comparisons with other amounts, to determine quantitatively *how much, how much larger, wider, smaller*, etc. He makes analogies naturally. "This is like a cumulus cloud only the bottom of it is straighter. It is closer to the earth. How far is *close*? Is it as far as *near*?"

NORMAL INTEGRATION

It is recommended that there be *no forced attempts* to integrate science and arithmetic in the elementary school. Rather, it is suggested that the integration come about naturally, and that the teacher be alert during the teaching of science to include accuracy of measurement and time concepts in all teaching situations.

AVOIDANCE OF VERBALISMS

It is of great importance that the science and arithmetic teacher be alert constantly to verbalisms, those parroted words and phrases which have not real meaning and understanding to the learner. When large numbers are used with little children, they must be *written* on the chalk board and made as meaningful as possible by comparisons, analogies, and even by use of kinesthetic methods. The following represent suggestions in the field of science teaching, where integration of arithmetic and science may occur with normalcy. There has been no attempt here to provide a plan or plans for the teacher to teach the following material. In fact, because of the close normal integration of the two subjects, this listing may serve only as a stimulus to the teacher who is not aware of this close association.

Research in elementary science has not revealed any definite subject-areas for grade placement. Because of this, after each statement a suggested grade or grades has been indicated where numerical as well as scientific understanding may occur.

1. The angle of incidence is equal to the angle of refraction. Grades 5-6.
2. The speed of light is approximately 186,000 miles per second. Grades 5-6.
3. Sound travels slowly, moving only one mile in five seconds. Grades 3-6.
4. The speed of sound in water is about 4 times its speed in air. Grades 4-6.
5. Echoes are heard as distinct, separate sounds only if they reach the listeners' ears one tenth of a second after the original sound. Grades 3-6.
6. The middle C of a piano vibrates 256 times per second if the piano is in tune. Grades 4-6.
7. Image persistence (a movie projector is constructed to make each image persist for one twenty-fourth of a second on the screen; the retina of the eye retains a picture for as long as one fifteenth of a second after it has disappeared). Grades 5-6.

8. The human ear is able to pick up vibrations ranging from 20 to 20,000 vibrations per second. Grades 4-6.
9. Destructive vibrations. Thousands of pages of calculations are necessary to insure the safety of plane passengers. High velocity affects wing vibrations. Grades 3-6.
10. During the launching of an ICBM or satellite vehicle, its position is carefully monitored and calculations made by large computers on the ground in regard to its flight path. Grade 6.
11. Digital computers have been developed for use in aircraft to determine the course required for target interception. Grade 6.
12. Combat planes used to fly at speeds of 250-400 miles per hour. Today, their speed is 3 times as great. Grades 5-6.
13. The core of the new atomic energy plant will be about the size of a 55 gallon drum. Grades 4-6.
14. A new dry cell has an electrical push (volts) of about 1$^1/_2$ volts; a storage battery in an auto, 6 volts; house circuits have 110-120 volts.
15. The length of time an element remains radio-active is stated in seconds, minutes, days and years. Grade 6.
16. Metals and their melting points. Gallium, an unusual metal, melts at body temperature. It will not boil until heated to a temperature of about 3,000 degrees Fahrenheit. Grades 5-6.
17. About 10 million atoms would stretch across the head of a pin. Grade 6.
18. One cubic centimeter would equal a cube 1/2 inch on all sides. Grade 6.
19. Plant researchers and farmers have in store for us within the next 10 years: grapes as big as plums; midget watermelons that won't crowd the refrigerator; blueberries the size of large marbles. Grades 3-6.
20. The poppy seed became a means of precise measure in the 1700s. One inch was divided into 3 barley corns, and each barley corn equalled 4 poppy seeds. Grades 4-6.
21. Today, we have approximately 478 million acres of good cropland. We shall need an added 140 million acres of cropland to produce what we will consume in 1975. Grade 6.
22. The keeping of a daily weather calendar. In the lower grades, simple symbols for rain (umbrellas) and clouds (cotton) placed at the date-space; at the upper level, actual weather symbols, degrees of barometer and thermometer readings may be taught; weather instruments constructed and calibrated. Grades 1-6.
23. The troposphere varies in height from 10 miles at the equator to about 5 miles at the poles. Grades 4-6.

24. Carbon dioxide makes up only 3-hundredths of one percent of air. Grade 6.
25. Moist air is lighter than dry air because a large percentage of it is water vapor. Grades 4-6.
26. The jet stream is about 300 miles wide, but varies in its form and position. It occurs in cycles of from about 4 to 6 weeks. Grades 4-6.
27. Tornadoes are much smaller in area than hurricanes, averaging about one fifth of a mile in diameter. Grades 5-6.
28. Lunar explorers would need to avoid extremes of temperature from 260 degrees Fahrenheit during the two-week lunar day to -270 degrees Fahrenheit during the lunar night. Grade 5-6.
29. One space vehicle carried animals, weighed 5 tons and circled the earth 17 times at an altitude of 200 miles. When it returned to earth it was only 6.25 miles off target. Grade 6.
30. The United States Air Force snagged the capsule of Discoverer XIV at a height of 8,500 feet in mid air. Grades 5-6.
31. Satellites have revealed to us that the earth has a 50 foot depression around the South Pole, a 25 foot bulge around the North pole, and a 25 foot depression around the North Mid Latitudes. Grades 4-6.
32. The sun is 93,000,000 miles away from the earth. Grades 4-6.
33. The crust of our earth is from 10-25 miles thick. Grades 3-6.
34. Evidence suggests that the innermost core of the earth is a very hot liquid. It is probably about 2,160 miles in radius. Grade 6.
35. American scientists use the metric system in their scientific work. Grades 5-6.

SUGGESTED RESTATEMENTS AS PROBLEMS

The above sampling of evidence relative to the simple and normal integration of science and arithmetic may well lead the way to further brainstorming in the area. Certainly, to insure thinking and meaning, it will be necessary to re-state these declarative statements in the form of questions or problems. For example, Number 35 might be presented: "Why do we learn one number system in school and then have to learn another way when we get to high school, or read science books?"

SUGGESTION TO TEACHERS

If the above compilation of samples of the normal integration of science and arithmetic has quickened an interest in furthering the cross-fertilization in this area, it is suggested that certain *areas in* *science* be explored as an initial approach. It may be as simple as a unit or study on *weather* which is taught at any elementary school level. How *many tablespoons* of baking soda did we add to the cup of vinegar? How *many inches* of water were shown on the rain-gauge? *What number* was the mercury of our thermometer on when we placed it in the shade? *How much* is 15 pounds per square inch: indeed, what *is* a square inch?

PRECISION A NEED TODAY

Measurements must be very accurate in our world today. In the past, a yardstick passed the test often as a precision instrument, but in this age, dimensions of all kinds must be within a thousandth of an inch. Things must fit together just right. Specifications must be followed accurately so that assembly lines can run smoothly. The day of the elementary teacher pouring "some" baking soda into "about two tablespoons" of vinegar is outmoded. In fact, this type of introduction to scientific learning and thinking is as dangerous and as inaccurate as learning multiplication "tables" without meaning or drawing the design of a hot-air furnace into the science notebook directly from the text with no understanding and concomitant learning.

Once the relationship between measurement and scientific accuracy has been established, the elementary teacher should have no problem in accepting the interchangeability of the two subject areas. Accuracy is indeed one of the keystones to excellence in science whether at the industrial, professional, secondary or elementary level. As accuracy permits mass production in industry, so will precision in science measurement at the elementary level foster a healthy respect for number and arithmetic, which, in turn permits a mass production of intellectually alert children--the hope of the nation.

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A Look at the Integration of Science and Mathematics in the Elementary School-1976

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The elementary teacher in 1976 is faced with several dilemmas. This specialist with young children is expected to teach content from several disciplines such as language arts, mathematics, science, social studies, art, music, and physical education. Children are to be prepared to progress through an educational system that, for the most part, has given little thought to the interrelated aspects of disciplines; that does not functionalize the articulation of grade levels so that a child really continuously progresses from kindergarten to grade twelve; and that is currently placing a major emphasis on the reading component of language arts and the computational aspect of mathematics.

How can an elementary teacher cope with the numerous demands of society? How can mathematics be designed so that adults will be able to balance a checkbook (assuming that a cash-less society has not evolved by the time current elementary-age children become adults) and at the same time develop an understanding of the structure of that discipline called mathematics? Can science curricula deal with major concepts and with process in a K-12 program? What will children read about and talk about in language arts?

A way of treating the dilemmas of a massive amount of material, interrelatedness of disciplines, articulation of grade levels, "practical" application of content, concept and process emphases, and the structure of disciplines is to integrate those facets of traditionally-defined disciplines where commonalities interface.

There have been renewed calls for efforts to integrate science and mathematics. The Third Cambridge Conference[1] is a notable example. However, the idea that we should *NOW* integrate science and mathematics is fallacious since the two have never been conceptually separated. One discipline cannot advance indefinitely without the other, as many mathematicians are rediscovering to their dismay. At times the impetus for new lines of research in mathematics comes from science. On the other hand, mathematics developed as "pure mathematics" years ago is currently being applied by scientists to explain present advances in science.

It should be apparent that science and mathematics may be related in several ways in our elementary schools. There is a continuum from mathematics for the sake of mathematics to science for the sake of science. Between these two extremes lie at least three points of importance: (1) mathematics for the sake of science, (2) mathematics and science in concert, and (3) science for the sake of mathematics. Any program that does not include all five points on this continuum is not representative of both disciplines and the related aspects of both fields of study. Unfortunately, only the two categories at the ends of the continuum have a long history in U.S. educational programs. This exclusion, among other reasons, has led to the poor teaching of mathematics and science in our schools today. Mathematics and science are typically unrelated subjects taught as two different "worlds." Numerous "new" mathematics and science programs have evolved in the past twenty years to attempt to change the styles of teaching from a "take-it-here-it-is" style to an involvement, hands-on approach. The learning theories of Jean Piaget, Zoltan Dienes and others shed doubt on the abstract, non-involvement approach of some versions of "new math" and many science series. Children learn mathematics and science by doing, as exemplified by the old Chinese proverb:

I hear and I forget
I see and I remember
I do and I understand.

The doing emphasis has given birth to mathematics laboratories and hands-on science projects. Six areas of learning that are common to mathematics and science can be dealt with in a hands-on laboratory setting: (1) sorting and classifying, (2) measuring,

[1]Goals for the Correlation of Elementary Science and Mathematics, Houghton Mifflin Co., 1969.

(3) using spatial and time relationships, (4) interpreting data, (5) communicating, and (6) formulating and interpreting models.

The remainder of this paper deals with these six categories with an emphasis on how mathematics and science are related. Examples of available materials are cited in each category. No attempt has been made to dictate or even suggest a program of study.

I. Sorting and Classifying--

Teachers must begin with what the child knows. What does a child know when he enters school? He knows about the concept of a set. He has heard about sets of dishes, herds of animals, families, church groups or club members. However, this concept must be refined. Sorting and classifying are activities that increase precision in the child's thinking. A child begins by sorting a group of objects into two sets, e.g. rough vs. smooth. Then he sorts his objects again using a different criterion. A child should always try to sort, classify, compare and contrast in more than one way using a different criterion. In this manner he begins to characterize sets of objects by their attributes and values of these attributes, e.g. the attribute is color and values are red, green, blue. Sorting and classifying also lead to more sophisticated operations with sets. If a child sorts a group of objects using values from different attributes, intersecting sets result. For example, suppose the child chooses the values *metal material* and *red color*. The following sets would result: (1) objects neither metal nor red, (2) metal objects but not red, (3) red objects but not metal, and (4) objects both metal and red. This could be represented by the following Venn diagram (Figure 1).

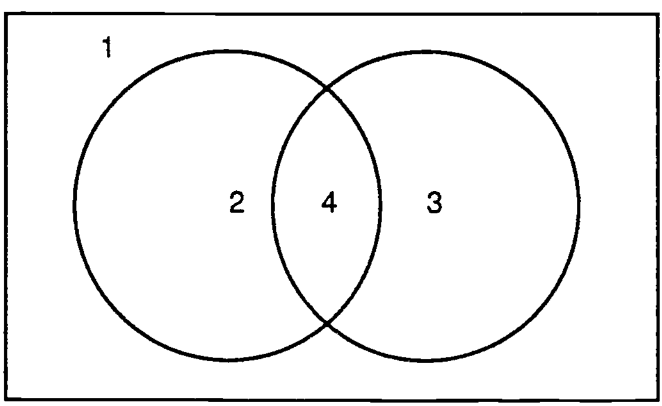

Figure 1 This is an excellent illustration of intersecting sets.

Scientists and children who do science typically collect data. The term "raw data" may be used to infer the state of unorganized observations. How can these raw data be organized? One way is to examine attributes of the phenomenon under study and to assign values to these attributes. The Periodic Table of the Elements is an example of organization based on attributes and properties. A dichotomous key to woody plants is another example. The two examples cited have been devised to give functional order on what would be a chaotic mass of data.

Children can construct their own organizational schemes such as the Venn diagram mentioned earlier or by a dichotomous key (Figure 2.) Subset D corresponds to Set 1 on the Venn diagram; subset C corresponds to 2; subset B corresponds to 3; and subset A corresponds to 4.

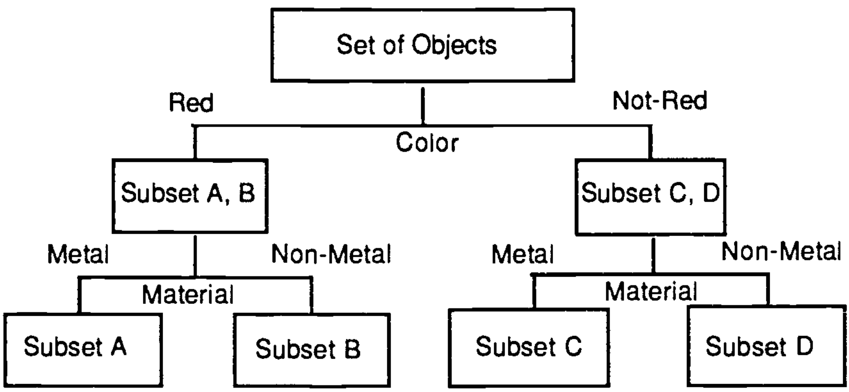

Figure 2

Several examples of commercial materials are appropriate for developing, sorting, classifying, and working with sets. *The Elementary Science Study* (*ESS*) unit called *Attribute Games and Problems* has four components useful in sorting and classifying[2]. Color Cubes, A-blocks, People Pieces, and Creature Cards all provide varied but redundant experiences for children grades K-8. The *Material Objects* unit of the *Science Curriculum Improvement Study* (*SCIS*) aids children in grade-level one to identify properties of ob-

[2]Elementary Science Study, McGraw-Hill Book Co., 1969.

jects and to organize these properties into useful schemes[3]. Children manipulate buttons, geometric shapes, pieces of wood, metals, rocks, shells, liquids, gases and other familiar objects.

Classification is a formal topic in the *Science--A Process Approach II* program (*SAPA*)[4]. Module Four uses leaves, nuts, and seashells to introduce classification. Animals, familiar things, terrarium organisms and mixtures are used in modules 14, 16, 32 and 42. All these modules deal with the identification of similarities and differences that can be assigned values. Module 56 introduces a punch card system where two values of an attribute may be coded as a hole or a notch. Cards are coded and separated by values of attributes determined by children.

Of course a teacher does not have to buy commercial materials. A shoebox or coffee can full of assorted objects obtained locally can be used in sorting, classifying, working with sets, and for counting. The use of locally-obtained materials also has the advantage of "personalizing" learning materials.

The repetitive experiences of sorting and classifying can be used as a basis for building a number system and its operations. Raw data can be organized in a functional manner as a result of hands-on manipulative experiences with familiar concrete objects.

II. Measuring--

Reference is made to *SAPA*, *SCIS*, and *ESS* as these programs are representative of the distinct approaches to science education. *SAPA* emphasizes process; *SCIS* is concept oriented; and phenomena are the basis for *ESS*. Several other commercial programs are available that deal with the six major topics in this paper.

Several of the *SAPA* modules formally deal with measuring. One sixth of the modules for K-3 emphasize measuring. Topics such as length, volume, metric, use of a balance, distances, forces, temperature, and moving objects are the content areas of these modules.

The *Match and Measure* unit of *ESS* provides several activities for primary children in comparing objects, in using arbitrary units, in using standard units, and in experiencing many modes of measuring such as meter sticks and long strips of paper. *ESS* Pattern Blocks are also useful in measuring.

It would seem that good pedagogy as well as the integration of science and mathematics dictates a certain order for the teaching of

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[3]Science Curriculum Improvement Study, Rand McNally and Co., 1970.

[4]Science--A Process Approach II, Ginn and Co., 1974.

measurement concepts. Children should begin by simply comparing objects, e.g. object A is heavier, longer, or larger (area or capacity) than object B. Then the student proceeds to order several objects. Next, objects can be compared by the use of nonstandard units of measurement. Through the use of nonstandard units, several new concepts can be introduced without confusing the child. First, students should learn the need for standard units since they will be unable to compare results such as their height or the length of a table. Second, the concept of a unit of measurement is emphasized without the confusion of parts of units. In order to learn this, children fill out a table as illustrated in Figure 3[5].

Unknown Object	Unit Object	Greater Than	Less Than	Closest To
Side of Desk	Straw	5	6	6

Figure 3

Here the student is forced into the answer of *6* rather than an answer like *5 3/4* straws. Third, accuracy and precision are best learned with nonstandard units, since it is much clearer that the smaller the unit the more precise the measurement.

Accuracy, the agreement of an observed measurement with the actual or true value, is important in quantifying science. Students typically do not have trouble with this idea as they are willing to accept standard methods and units of measure.

Precision presents a problem to many students and to many teachers. The agreement among observed values in repeated measurements lends support to reported results. For example, a student should be able to rank order the precision of the following measuring devices by taking repeated measures and by determining the mean and the range of obtained values. If a graduated cylinder calibrated in milliliters is used to measure the capacity of a non-calibrated container; if a meterstick calibrated in mm. is used to measure the length of a table; and if a spring scale in newtons is used to measure the force required to move an object, the results should indicate the relative precision of these instruments. If the table length is 26 $\pm$ 1 mm., the capacity is 35 $\pm$ 3 ml., and the force is 20 $\pm$ 10 newtons,

[5]Hart, Alice. University of Illinois, Chicago Circle Campus.

the instruments are ranked meterstick, graduate, spring scale in order of precision.

After having learned the concept of measurement, the child is ready to proceed to measurement with standard units. The metric system should be taught before the English since it is used in science. Also, the student will become more proficient in the system taught first. Only after the student has mastered the preceding steps, ratio, proportion, and decimals should he be concerned with conversion from one system to another.

The metric system is here! Children can establish personal reference points that will aid them in "getting a feel" for metric units. For example, if a child knows he is 120 cm. tall, he can judge that something 10 cm. long is a lot shorter than he is and that something 1,000 cm. long is a lot longer than he is. Similar reference points of mass and capacity help students to interpret metric measures.

Many non-commercial materials can be used in measuring. For linear measurement a teacher could use straws or popsicle sticks for non-standard units and assorted scrap pieces of wood or objects in the room as unknowns. For mass, one can use paper clips, marbles or poker chips as standard masses, and any object as an unknown. Plasticene can be used to make up standard masses. When measuring capacity, the children and teacher can collect different sized jars, cups, and cans. Rice, corn, beans or sand can be used to compare capacity.

Area presents a special problem because of its relationship to linear measurement, i.e. area of an object equals the linear unit squared. For example, the English system measures in square inches. Consequently, in filling out a table the child may obtain results as shown in Figure 4.

Unknown	Unit	Greater Than	Less Than	Closest To
Desk Top	Triangle	30	35	33

Figure 4

This diminishes the accuracy of area measurements. The *ESS* Pattern Blocks, or ceramic or plastic tiles, make good unit measures for area, although a teacher may use homemade shapes. Unknowns may be irregularly shaped pieces of paper or wood. Another *ESS* unit that is useful for the study of area is the *Tangram* unit (Figure

5). For example, let the area of $a = 1$. What is the area of *b, c, d, e, f, g* or some combination of these shapes? Tangrams can also be used for the study of congruence and similarity.

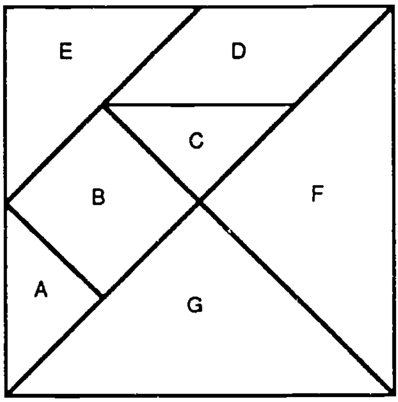

Figure 5

III. Using Spatial and Time Relationships--

Essential to the study of any aspect of science is the description of the physical environment. Topics included are spatial relationships and their change with time, geometric shapes, symmetry, motion, and rate of change.

Recognizing and using two-dimensional shapes; direction and movement; spacing arrangement; three-dimensional shapes; shadows; symmetry; lines; curves and surfaces; rate of change; and relative motion are the topics of nine modules in *SAPA K-3* that are directly oriented to using space-time relationships.

In the *SCIS* programs, level four deals with the topic of relative position and motion. Children use a stick figure, Mr. O, to relate to positions such as *above* Mr. O, *below* Mr. O, to Mr. O's *right*, *behind* Mr. O, and *close to* Mr. O. Mr. O is placed in situations where objects are moving and position is relative to a particular point of reference. The topic of coordinates is introduced as the need arises to locate points in space.

The *ESS* series has several units related to space-time relationships. Mirror cards are excellent devices to practice with symmetry. Geo blocks can be used to teach geometric shapes and the relationships of various shapes. Units such as *Shadows* and *Daytime*

Astronomy deal with abstract ideas in concrete situations. Relative positions of the earth and the sun are difficult for youngsters to comprehend unless they have the types of hands-on experiences exemplified as *ESS*.

Topics such as linear speed, circumference and diameter of a circle, and the relationships of circumference and the diameter of a circle can be illustrated by easily obtainable items. Take three or four different sized jar or can lids and measure the circumference of each one by carefully surrounding it with a length of string. Determine the length of the circumference by holding the taut string along a meterstick. Measure the diameter of each lid using a string and a meterstick. Divide the circumference by the diameter. Students will have calculated the value pi (π). This activity also reinforces the principles of accuracy and precision since it is unlikely that 3.14 will be obtained with each operation. Students can now relate to the formula $C = \pi D$. The relationship can now be used to solve such problems as calculating the linear speed of a man standing on the geographic equator of the earth if you know the diameter of the earth at the equator.[6]

Using space and time relationships is essential in science and employs many types of activities traditionally associated with mathematics.

IV. Interpreting Data--

In science and mathematics, process should be considered as important as cognitive learning. It is the process that develops new science or mathematics, and the process is the same for both disciplines. A beginning point is to ask questions and to be curious about the environment, whether it be "why does an apple fall to the ground?" or "how does our place value system work?" Data pertinent to the question must then be interpreted, and the data are collected as a result of some experiment or activity that will provide information. The data must then be interpreted, and the data are most frequently in some mathematical form. This implies that they must be interpreted mathematically. The next step is to form some hypothesis and to determine its validity. Here science and mathematics part company since the truth or falsity of a statement can be deductively determined in mathematics with the exception of open questions where the hypothesis is in doubt. In science a hypothesis may be rejected or supported, but never proven in an absolute sense.

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[6] Brown, W. R., Handbook of Science Process Activities, Education Associates, Inc. 1974.

Data are usually presented in the form of tables or graphs so that students search for patterns in order to formulate interpretations and/or new hypotheses. There are many science programs based on these principles; *SAPA* is one. Several modules in the series 61-105 for grades 4-6 place an emphasis on data interpretation.

In both *SCIS* and *ESS* the interpretation of data is a built-in on-going activity. Students interpret data they have collected by observing living communities and by shooting stopper-poppers in *SCIS*. *Clay Boats*, *Behavior of Mealworms*, *Growing Seeds*, and *Optics* are examples of *ESS* units where students are involved in collecting and interpreting various forms of data.

The *Madison Project* is a collection of lessons dealing with modern mathematics topics with a hands-on or mathematics laboratory approach. Graphing, interpreting data, and making inductive hypotheses are integral aspects of each *Madison Project* unit (although this was not the original intent of the program). These units are commercially available as sets of boxes[7]. Each box contains task cards and appropriate materials to complete the tasks. Units dealing with masses and springs and with circles are appropriate to science.

V. Communicating--

Communicating is a process not only of science and mathematics but of all human endeavors. Clear, precise, unambiguous communication is desirable in any activity and is fundamental to all scientific work. Communication can be by oral and written words; by diagrams and pictures; by maps; by two and three dimensional graphs; by table, by expressions of central tendency such as the mean, median, mode and range; by mathematical equations; and by various kinds of visual demonstrations.

Seven of the K-3 *SAPA* modules, starting with module 22, place a formal emphasis on communicating. Several of the topics listed in the preceding paragraph are the content of these modules.

The communicating process is important in all *ESS*, *SCIS*, and *SAPA* activities. One of the common features of these three programs is the lack of a student text. Students do not read about what someone else found out but are instead involved in generating data of various types. In these three programs, students *MUST* be provided with opportunities to represent, pool, and discuss their observations.

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[7]Madison Shoe Boxes, Math. Media Inc., 1960.

A major emphasis on reading is a phenomena of the U.S. schools of the 1970s. This emphasis may be important, but perhaps youngsters should also have in-depth experiences in oral communication. The content of science can provide something for children to talk about, something for them to read about, and something for them to write about. One hopes that the "cookbook fill-in-the-blanks" laboratory guides are a historical facet of elementary science education. The hands-on activities under the banner of science and mathematics may serve as vehicles to make communication, in all forms, a functional skill.

VI. Formulating and Interpreting Models--

An adult understanding of scientific concepts is highly abstract. A useful way to represent the physical world is to create mental models of systems or phenomena that have mathematical equivalents. An atom is a mental model created by man to explain his physical environment. This model is constantly subject to modification as man extends his observational ability by the use of technological devices.

A characteristic of models is that they have predictive components. If an atom of hydrogen has been modeled, the model can be tested in the physical world. If oxygen does indeed combine with two atoms of hydrogen to form water, then the model is supported. An additional component of a water molecule model is the expression of water as a dipole. This model can be useful in predicting the properties of water.

The *SCIS* program formalizes the idea of models in the sixth grade unit called *Models: Electric and Magnetic Interaction.* Electricity and magnetism are abstract phenomena that may be mathematically modeled. The sixth-grade life science unit in *SCIS* is called *Ecosystems.* In reality an ecosystem is a model of the biotic/abiotic world. Cycles such as the water cycle, the food-mineral cycle, and the oxygen-carbon dioxide cycle are models of phenomena that help to build a model of cycles in general. The recycling of beer cans can be related to a general model of cycles in the physical world.

ESS units such as *Bulbs and Batteries*, *Heating and Cooling*, *Kitchen Physics*, *Mapping*, and *Pendulums* deal, to some degree, with formulating and interpreting models.

Several *ESS* units designed for science also facilitate the development of mathematical concepts. The pattern blocks have already been mentioned as being useful in measurement. However, with the addition of mirrors they can be useful in the teaching of transformational geometry. They can be used to teach the relationship of different shapes (triangles form a hexagon).

Another *ESS* unit already mentioned is *Tangrams*. They are useful for teaching congruence and similarity. For instance *a, b,* and *c* form a shape similar to *d* (see Figure 5). Also, shapes *a, b, c, d* and *e* form a shape congruent to *f* and *g*. There are an almost unlimited supply of such questions that children can answer through the manipulation of these objects.

The *ESS* unit *Mirror Cards* is particularly useful in the study of symmetry. The cards increase in difficulty as one proceeds through the box. The first levels could be used by kindergarten or first graders with no difficulty.

Geo Blocks, another *ESS* unit, is designed to increase a student's perceptual abilities in the three-dimensional world. However, the shapes are quite useful in the teaching of solid geometry and the relationship of three-dimensional objects. The similarity of three dimensional objects is also quite apparent when the different sized cubes are stacked one on top of the other from largest to smallest. Also, by using wax paper or tin foil the student can "look" inside the objects to study their structure.

Two More Programs--

In addition to the sources listed previously, two programs are commercially available that integrate science and mathematics. The *Minnesota Mathematics and Science Project* (*Minnemast*) is a program of twenty-nine units in mathematics and science for early childhood[8]. The units are intertwined and sequenced spirally although some deal only with mathematics or science. It is an active-learning, laboratory situation where the mathematics is coordinated with the science units. The units emphasize the contribution of both disciplines. The aim of the units is the development of the child's logical processes. The following concepts are emphasized: real numbers, geometry, system, change, interaction, reversibility, invariance, space, time, matter, force, and field. Processes emphasized are observations, experimentation, and generalization. The originators of *Minnemast* feel that their program is based on the psychology of Bruner and Piaget. This is important since the project was begun in the early 1960s when mathematics educators had not yet "discovered" and applied the work of Bruner and Piaget.

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[8]Minnesota Mathematics and Science Project, University of Minnesota, 1970.

A more recent project (early 1970s) is *USMES* (*Unified Science and Mathematics for Elementary Schools*)[9]. The originators of *USMES* feel that their program is based on the goals of the Third Cambridge Conference. *USMES* is composed of twelve independent teaching units that integrate mathematics, science, language arts, and social studies. Real-life problems called challenges are presented for the class to solve. The work of every class is different since circumstances differ depending on the needs and the interests of each school environment. *USMES* units are not designed to replace the regular mathematics, science, language arts, and social studies curricula. This is an enrichment, problem-solving series that may be used in addition to more traditional discipline-oriented lessons. Examples of *USMES* units are *Lunch Lines*, *Consumer Research-Product Testing*, *Advertising*, *Burglar Alarm Design*, *Traffic Flow*, *Weather Prediction*, and *Soft Drink Design*. The names of the units imply the problem-solving, interdisciplinary nature of the program.

It should be evident that there is a wealth of materials available at the elementary school level that can be used to integrate at least six aspects of science and mathematics. Besides the abundance of materials, an additional factor that favors the integration of mathematics and science is that these disciplines complement and motivate each other. A basic premise of this paper is that a hands-on laboratory approach will be most suitable in the implementation of the six areas.

I hear and I forget
I see and I remember
I do and I understand.

Now that you have seen an argument for the integration of science and mathematics, please try some of the ideas presented and you will understand!

[9]Unified Science and Mathematics for Elementary Schools, Education Development Center, Inc., 1973.

14

Making Concepts in Science and Mathematics Visible and Viable in the Early Childhood Curriculum

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Education during early childhood is generally based on an informal, experience-centered curriculum philosophy. If it is safe to conclude that humans live "in a world of concepts rather than a world of objects, events and situations," then it is safe also to conclude that the child must become oriented and begin to live in that world (Ausubel, 1968). How the orientation process takes place and what pedagogical intervention best encourages that process is still hypothetical. From experience and training, the teacher acquires the disposition to justify, give reasons for, setting up a particular classroom environment from the perspective of child growth and development. This perspective claims that through experience children acquire concepts naturally as they interact with the environment. The concepts to be acquired are not explicitly presented but hidden within the objects and materials, activities, games and social simulations that are offered to the child.

The authors believe that complete dependence on concepts to present themselves by the mere existence of materials and activities in the classroom is in error, especially science and mathematics concepts. The teacher needs to plan and assess for instructional effects as well as nurturant effects (Joyce & Weil, 1980). The following assumptions are delineated from this position:

1. To assure conceptual learning will take place, structured interactions between child, teacher, and material will be designed by the teacher highlighting science and mathematical concepts which may be hidden in the informal curriculum of early childhood education.

2. To assure clear communication of concepts, the teacher will consciously plan classroom activities with language enforcement which invites the child to rationally attend to these concepts.

3. To assure a viable, open attitude toward science and mathematics via concept development, the teacher will plan classroom socialization and structured activities in order to enhance self-correction, child deference, and tolerance for learning from incorrect responses as well as correct responses.

The purpose of this article is to deal with these philosophical assumptions in the following manner. First, we will discuss our view of the cognitive processes characteristic of the early childhood learner as well as the potential benefits of learning through "incorrect responses." Second, we will delineate the science and mathematical concepts that we believe are within the early childhood learner's cognitive capabilities. Finally, we will illustrate how science and mathematical concepts, which are presently in traditional classroom activities, can become more visible and viable in an early childhood setting.

Learning Concepts in Young Children

In order to design instructional activities where the concepts are communicated with clarity, the teacher should consider the developmental theory of Jean Piaget (Copeland, 1979). According to this theory, the early childhood learner's conceptual thought structures are preoperational. The following description briefly characterizes the nature of concept development in the young child: preoperational children are egocentric, seeing themselves as the center of their immediate environment; they are pre-logical in their thought structures, causing them to be unable to reverse those thought processes when relationships between physical objects are varied; they are unable to conserve invariant properties of physical objects, and they cannot see the "part" in relation to the "whole"; the thought processes of the child who is in transition from preoperational to operational thought are in a state of disequilibrium.

Since we are subscribing to Piaget's theory, which is developmental in nature, we believe that the learners are necessarily going to be revising and refining their view of the world. This process of revision and refinement of knowledge is what Henry Perkinson (1979) refers to as "learning from our mistakes."

"Learning from mistakes," Perkinson believes, is a common theory shared by leading twentieth century educational theorists such as John Dewey, Maria Montessori, A. S. Neill, Jean Piaget, Carl Rogers, and B. F. Skinner (Perkinson, 1979). For example, Piaget's theory confirms that children are active, not passive, learners of concepts, seeking order in their world. Perkinson had developed a non-justification theory of teaching, applying Sir Karl Popper's philosophy of fallibilism (Perkinson, 1969). Briefly, fallibilism is a position about rationalism and truth based upon such notions as: human beings are fallible and certainty is denied them; secondly, coherence and consistency are not criteria for truth, but incoherence and inconsistency do establish falsity (Perkinson, 1969); finally, though absolute truths do exist, obtaining absolute truths about the working of the universe is impossible, particularly by inductive reasoning. However, by eliminating error in our theories we can get closer to the unattainable goal of absolute truth (Swartz, et.al., 1980).

Perkinson sees the fallibilist teacher as one who creates a free, but responsive, environment where the children are fallible creators of knowledge (Perkinson, 1980). The teacher will treat knowledge, called *subject matter*, as conjectural. The teacher, through dialogue with the learner, will focus on critical feedback aimed at guiding the learner in the identification of errors in concept formation (Perkinson uses the term *theory* instead of *concept* because theories can logically be true or false.) The teacher must see to it that children receive this critical feedback. In this role as a critic in a free and responsive environment, children will learn the critical attitude and, as John Dewey suggests, become problem solvers (Perkinson, 1969).

Teachers who subscribe to this position will note that their instructional goals are to teach for understanding, and that the emphasis in the classroom is on exposure to ideas and materials, not on mastering a body of factual information.

The content which is appropriate for the preoperational learner in an environment which emphasizes revision of knowledge is that content which is within the realm of a child's ability to understand. Since our position has been that the early childhood learner is in the preoperational state, we feel that the content or activities which are appropriate are those which move the child in the direction of operational thought. To encourage this facilitation, we must take into consideration that this child is one who is not yet exhibiting, but is developing, the Piagetian cognitive structures: 1) Conservation of number, 2) Seriation and an understanding of ordinal numbers,

3) Multiplicative classification, 4) Multiple seriation, 5) Conservation and measurement of length, 6) Hierarchical classification, 7) Class inclusion (Copeland, 1974). For the child who is developing these cognitive structures, the curricular content for mathematics and science should deal with: scientific observation and limited interpretation of the observed data; classification; patterning; introduction of measurement; seriation; one-to-one correspondence; equalities and inequalities; and discrete quantification.

Visible and Viable Activities

The following activities for which we have developed modifications are representative of those commonly found in the early childhood classroom. In critiquing these activities, we find two types of problems frequently occurring. The first problem occurs when the teacher does not do enough to facilitate the observation process which is vital to the child's development of knowledge. The second type of problem is an activity in which the situation presents erroneous information and allows the child, through observation, to draw incorrect theories without providing the critical feedback necessary for revising "incorrect" to "correct."

ACTIVITIES TO ENCOURAGE OBSERVATION SKILLS

I. A. *Content*: Seed Germination-- Every Spring there seems to be proliferation of seed germination activities in the early childhood classrooms. The activity ordinarily consists of the children planting various seeds in some growth medium: dirt, sponge, paper towel, or other easily obtained, non-toxic materials.

B. *Problem*: Each child usually plants individual seeds, labeling the container with his or her name, and then places it in a light source. The next child contact is often when the sprout reaches the surface and is visible. When the activity proceeds in this way, the benefits of observation during the germination process are lost. Since we are dealing with the preoperational child who is unable to visualize the intermediate phases, the concept of germination as a gradual process is not emphasized.

C. *Modifications*: The most important modification might be the careful selection of materials for planting. The use of transparent containers and the arrangement of a growth medium that would force the seed against the side of the container would facilitate observation. Once the child has planted the seeds using these materials, the teacher might ask the child to draw a picture of the seeds in the container. This step in the activity assumes that the child's art development is at the representational stage. If the child is not at the representational stage, the teacher might encourage a discussion about the newly planted seeds, audio-tape recording it for future comparisons. As an extension of this activity, the teacher should lead the children in an audio-tape recorded brainstorming session during which the children make predictions about what will happen to the seeds.

Each child should be responsible for the daily care of his or her seeds, perhaps using rebus picture directions demonstrating what the child is to do and in what order. While the children are watering their seeds, the teacher can encourage them to observe any changes occurring, calling attention to both the roots and the sprouts.

When the seeds have produced an observable sprout, the children might draw an updated picture and/or tape a description of the plant. Comparisons should be noted between the earlier picture/description and the most recent depiction. The children should be encouraged to review their predictions made during the brainstorming session and compare them with the actual outcome. One suggestion for a culmination to this activity would be to eat the sprouts as part of a salad or sandwich.

II. A. *Content*: Pets in the Classroom-- Many early childhood classrooms contain small pets such as gerbils, mice, rats, hamsters, guinea pigs, birds, fish, or other easily-housed animals. These pets are usually displayed on a daily basis in an area accessible to the children.

B. *Problem*: Quite often, the teacher neglects to facilitate the environment to enhance observational skills of pets in the classroom. Frequently, the teacher assumes the responsibility for the care and feeding of the pets and neglects to plan for observation and discussion of the animals by the children. Consequently, the children do not become involved with the pets, and may view their presence in the classroom as no different than any other classroom equipment.

C. *Modifications*: The teacher can plan for the children to be involved in the daily care of the pet by designating this a "daily helper" task. Rebus picture directions for the necessary care may be displayed next to the pet so that non-readers can perform this task independently. The teacher can plan for daily observation of the pets, directing the children's attention to changes, growth, feeding habits, odors, skin texture, general activity or movement, or the lack of activity. All of these items are observable characteristics upon which inferences can be made. The children should be encouraged to verbalize their observations, thus becoming acquainted with new vocabulary.

The children should also be encouraged to question their findings, hypothesize about what they have seen, and test and re-test their hypotheses, in their own neophyte manner. The teacher may want to follow up the children's observations by graphing or charting what the children have verbalized. It is important that the teacher encourage children to use observable data to support incorrect as well as correct statements.

III. A. *Content*: One-to-One Correspondence--Setting the table, passing out treats, playing musical chairs, matching games, and teacher management procedures are some of the many places one-to-one correspondence occurs in the early childhood learning situation.

B. *Problem*: Overlooked opportunities for building concepts and comparative language are the source of the problem in dealing with one-to-one correspondence. When a teacher says, "Joan will you pass out the cookies so that each child has one?", "Here are the napkins, put one at each place,", "There is only room for five people at the block center-- who wants to go? O.K.-- one, two, three, four, five-- you five go.", then proceeds with the activity, an opportunity for developing *same number, greater than, more, less than, fewer* concepts and language is missed.

C. *Modifications*: When doing these types of activities, the teacher should take care to provide situations where the exact amount of an item needed is not present, i.e., too few napkins, too many cookies. By having extra or insufficient amounts, the teacher provides the situations where "more than" and "less than" conclusions can be made by the child through observation of the activity's outcome. The teacher can then provide the language to talk about what was observed.

In distributing treats the following language reinforcement should occur: "Are there enough cookies? Do we have enough cookies for each child?"

In management situations, the teacher could have a set of five clothespins and ask each child who works in the block area to wear a clothespin. As children leave the center they should place their clothespins in a specified location so that other children, seeing a free clothespin, can go to the block center. The teacher interacting with children who want to play with the blocks can ask, "Are there more clothespins than children playing blocks? All those who want to play with the blocks raise your hand. Are there more hands than clothespins? Which group has less-- clothespins or blocks?"

In this way, instead of counting out children to go to the center, the teacher matches children to clothespins. Beginning prediction and estimation skills can also be developed by asking before matching, "Do you think we have more (cookies) or (children)?" or "Which group is smaller?"

ACTIVITIES THAT CORRECT ERRONEOUS INFORMATION

I. A. *Content*: Water Play-- Many early childhood classrooms contain some apparatus to allow for playing in the water. The equipment may vary in complexity from a sink or dishpan to specially designed water tables.

B. *Problem*: We might preface by saying that water play, in and of itself, with or without accessories, has therapeutic benefits for the child. However, it is also a versatile medium for presenting both science and mathematical concepts. If we look at the mathematics and science concepts to be gleaned during water play, the problem with commonly used water activities seems to lie in the choice of accessories to accompany the water table. When all of the tall containers hold more water than the low, flat containers, then it is possible for the child to draw erroneous conclusions concerning conservation of volume. Therefore, the facilitators are at fault if their choice of materials leads children to false conclusions without feedback to assist them in correcting their errors.

One water-related science activity traditionally done in early childhood classrooms is a sink/float experiment. The child is asked to explore a set of objects and determine the buoyancy of each. The facilitator may choose large, dense objects which sink, and smaller, less dense objects which float, pairing up characteristics that do not necessarily have any relationship. Again, this may lead the child to draw false conclusions regarding the characteristics of sinkable items.

C. *Modifications*: The primary modification would be to carefully select accessories and materials that would offer appropriate feedback by which the child can create accurate knowledge. Using tall containers, and wide, flat containers which hold equal quantities of water, or a variety of different shaped containers which have equal capacity, would facilitate exposure to the concept of conservation of volume. The child's experimentation with these materials, paired with the teacher's appropriate questioning, changes a random water play activity into one focusing upon mathematical concepts. The use of an assortment of materials, suggested above, sets up an experience in discrepant events, forcing the child to adapt to these discrepancies.

In the sink/float experiment, the choice of objects is equally important. The placement of large, lightweight materials (e.g., large pieces of balsa wood) and small, dense materials (e.g., lead sinkers) next to a tub of water would encourage the child to test and observe buoyancy. The teacher might then assist the child in verbally drawing conclusions about the discrepancies they have observed. An extension of this activity for children who are able to approximate the concepts presented would be to use a quantity of clay, shaping and reshaping it while testing for buoyancy.

II. A. *Content*: Classification of Fruits and Vegetables--Fruits and vegetables are often the first food group classification introduced to the early childhood learner because of the child's familiarity with these foods in his/her diet.

B. *Problem*: The classification of foods into categories seems to be the primary focus of these early childhood activities. The children are often encouraged to categorize these food objects according to erroneous characteristics. For example, classification of objects as fruits solely on the basis of being sweet or because they are desserts is an incorrect classification. Categorizing vegetables as "what we eat with our meat at supper" is also incorrect. Neither of these examples takes into account the exceptions such as tomatoes in our salad (fruit) or sweet potato pie (tuberous vegetable). It is our contention that these errors are made universally, not just by early childhood educators. Furthermore, if it is our intention to be scientifically accurate, the category of *vegetables* is not a *bona fide* botanical classification. Rather, it is a general description of herbaceous, food-producing organisms. Fruits, nuts, seeds, flowers, stems, roots, and tubers are all types of vegetables.

C. *Modifications*: As a preface to all modifications for activities which offer erroneous information, we would like to emphasize that it is the responsibility of educators at all levels to be sure of the legitimacy of their content. Just by the nature of the school setting, there is an aura of efficacy about the information presented.

The teacher should first become familiar with botanical or scientific criteria used to classify these foods. For the most part, these criteria are somewhat obtuse for the preoperational learner. As a result, it may be inappropriate for the teacher to expect the child to adopt these criteria as the touchstone for categorization. The teacher, then, needs to choose concrete criteria for classifying foods. For instance, grouping by taste (sweet/sour) or grouping objects that have many seeds rather than just one seed would be within the cognitive capabilities of the early childhood learner. The teacher should encourage children to focus upon the properties of the real object (not inferences) such as size, shape, color, and texture. Emphasizing the development of new vocabulary in formulating accurate descriptions is a necessary part of this activity. As a result of a series of observations, it would be possible to lead the children into making predictions about unfamiliar objects based on their past observations. One very positive by-product of this modification could be willingness of children to taste unfamiliar foods, relying on their predictive skills. The emphasis in classification activities should be placed on observable characteristics.

NOTE: These same modifications apply to the errors made in geometric classifications where squares are classified as a separate group from rectangles rather than as a subgroup of rectangles (all squares are rectangles).

III. A. *Content*: Cardinal/Ordinal Numbers--Storybooks, labels on cabinets, chairs, naming of groups, and sequencing of activities are examples of the early childhood learner's contact with numbers.

There are three common ways numbers are used. First, as *cardinal numbers*--those that tell *how many* or *quantity*; i.e.,. there are 5 birds on the fence. Second, as *ordinal numbers*--those that tell the *position* of an element in a designated sequence, i.e. I live in the second house from the corner. And third, as *nominals*--those that merely *name*, i.e. my room is 24.

B. *Problem*: In the presentation of numbers to children, the cardinals, ordinals and nominals are often used in incorrect or confusing manners, thus creating confusion as to what the numbers actually mean. Many children when asked to count a group of five apples will count them by matching a number name to an object in the set and report that "there are five apples." However, when asked to show the five apples, they point to the fifth apple, thus pairing the number word with the apple in an ordinal sense rather than as a cardinal number which represents the entire group of apples. This problem is often compounded by storybooks. One such book is Sendak's *Seven Little Monsters* (1977). In this book he gives each monster a name that is a cardinal number and refers to the monster using this number, i.e. "Six sleeps late but not in bed." Meeks makes a similar error in *One is the Engine* (1956) by pairing "Three is a flat car" with a picture of a car with a "3" on the side. He further confuses the concept of cardinality by showing at the bottom of the page an entire train with the cars discussed to this point (1 is the engine, 2 is the box car) shaded in blue. One would expect to see three cars shaded. Four cars are actually shaded, the engine is represented by a two car unit (1956).

Further confusion arises when numbers are used as nominals. "The paper is in cabinet four."

C. *Modifications*: The teacher should always be careful when using ordinals to show position to use the terms, *first, second, third*, . . . and never the numerals *1, 2, 3*,. . .

When discussing *four, five, six,* as numbers that show quantity, the entire set or group should be emphasized. Activities which match cardinal numbers one-to-one with a set of objects in order to find the quantity of the set reinforce the misconception that the object matched with the numeral *4* is seen as representing the number *four* rather than being part of the set of four. An illustration that encompasses all of the four objects with a border would reinforce the concept of cardinality.

When using number books, two courses of action are possible. The first would be to eliminate all books and situations that use the numbers incorrectly such as *Seven Little Monsters* (1977) or *One is the Engine* (1956), and only use books such as Sazer's *What Do You Think I Saw?* (1976) or Kulas' *Puppy's 1-2-3 Book* (1978) where the numbers refer to sets, i.e. "There were three jungle mice eating brown rice," and "Puppy sniffs 'round the barn door. Inside are four ducklings." (Kulas, p 4). The second approach is to revise the books so that cardinals are used appropriately: "The sixth monster sleeps late but not in bed," and "the third car is a flat car."

Nominal numbers should probably be eliminated in the classroom situation. Their explanation may serve only to further confuse the issue.

Long Range Implications

Although scientists and science educators do not always agree on what science is, they do agree on what scientists do. These methods of investigation are referred to as the *science processes* and are divided into the *basic processes* and the *integrated processes*. Since the preoperational learner is not developmentally ready to grasp many of the abstract concepts of the science content, the emphasis should be on the basic science process skills. These skills include observation, communication, inference, measurement, classification and prediction and are appropriate for the preoperational learner (Funk et al., 1979).

The proposed modifications in the previously cited activities should have many long range effects, the most immediate effect being the student's ability to find errors and to approach new problems or situations using the basic science process skills as emphasized in these activities.

Upon close examination of these activities, it is to be noted that the process of observation is incorporated within each. In the early childhood years most of the observations will be qualitative in nature (using one of the five senses), but as the students move up to the elementary grades they will be able to make many quantitative observations (making reference to some standard unit of measure). The students are asked to verbalize a description or draw a diagram of their observations. This is the first step in collecting data and communicating these observations to others. Communicating observations is basic to everything else in science and may be helpful in promoting good communication skills for the learner when dealing with many types of information. This repetition in using the same process skills in the activities described should help reinforce their use by the learner. In some instances, such as in the activity *Pets in the Classroom*, the children will be able to make some simple inferences and predictions from the data they have collected.

In the fruits and vegetables classification activity, the learner begins to realize through observation that similarities and differences exist among objects and that these characteristics may be used for classification. This grouping should in turn lead to the ability to impose some type of order to the many varieties of objects encountered in the world. Again, observation is emphasized by classifying the objects using observable characteristics. This type of classification encourages the learner to realize that some objects may be subsets of other objects.

Although early preoperational learners may not be able to make accurate measurements, they will be capable of making some crude measurements. This is demonstrated in the "Water Play" activity. With the modifications suggested in this activity, the learner will begin to realize that only with some form of measurement can volume (amount of space within) of the containers be determined.

The long-range implications for mathematics may not be as obvious as one may think. Most children do eventually sort out the differences in cardinal, ordinal and nominal numbers, and use correct comparison words. Along the way, they have accumulated many mathematical behaviors that are built on erroneous or incomplete foundations. Many adults have problems with the inclusion idea that is found in "all squares are rectangles but not all rectangles are squares."

Later, when teachers attempt to correct erroneous information, the learner fails to perceive the logic and consistency of mathematical system. Rather, mathematics seems as a variable, illogical and capricious. Without the understanding of the content, many people learn mathematics in a rote manner, where the reason for a procedure is the authority of the instructor--not the logic of the content structure. Research has indicated that this type of perception can lead to mental blocks against learning mathematics, anxieties, and avoidance behaviors which often limit career and life-style choices.

These are only a few of the science and mathematics processes hidden within the early childhood curriculum and the long range effects of emphasizing or uncovering them from the learners.

Summary

Conceptual learning can be enhanced in mathematics and science at the early childhood level. It is important that the teacher carefully structure the activities to use materials and language enforcement to focus on concepts and that the teacher provide a classroom environment which encourages the children to create knowledge, recognize errors, and learn from mistakes.

Our position has been that the early childhood learners are most likely to be in a stage of cognitive development that Piaget refers to as "preoperational." These children need to learn by making mistakes, seeing the consequences of these mistakes, and revising their views and theories of the world to include the new information gleaned from mistakes. In order to facilitate this learning process, the teacher needs to provide an interactive environment for children and help them focus on what they see and do. This environment must be constructed with care so that authoritarian, erroneous views and theories are not formed through inadequate feedback. The teacher should help focus, not dictate the learner's attempt to give structure to what is observed.

This view suggests that what is formally presented to the learners must be within the realm of their ability to understand, and not just for the exhibition of a non-meaningful behavior. Teaching at this level should aim at the exposure to content, not to its mastery.

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Because of the universality of quantitative relationships, the habit of functional thinking is of utmost importance to the individual. Its acquisition should be the emphasized goal of every course dealing directly or indirectly with relations between things or processes.

J. S. Georges, 1926

V. Unifying Themes in Science and Mathematics

Not only does algebra increase our happiness by unfolding before us a larger meaning to nature than could otherwise be found, but the study of algebra leads directly to the control of nature and the pleasures that come from the possession of these powers. In brief, algebra may be considered as a medium for the interpretation and control of nature.

M. T. Goodrich, 1935

Thus I say that Geometry, the first and noblest of sciences, has played in the past and will probably continue to play the dominant role in science. It certainly dictates the form and to some extent the content of all scientific concepts. It is the stage upon which those abstract actors, the causes and effects, play their scientific roles. It is the picture frame wherein that greatest of all artists, the creative scientist, paints his pictures.

F. W. Bubb, 1937

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Functional Relations and Mathematical Training[1]

J. S. Georges
University High School, Chicago
(Vol. XXVI No. 7 October, 1926)

"It is not things themselves," says Henri Poincare, "that science can reach, as the naive dogmatists think, but only the relations of things." The outstanding problems of modern science in things quantitative are indeed problems of mutual dependence and relationships. In his groping after the truth the scientist begins with measurement, when measurement is possible, then with the compilation of data, and finally the study of the relationship between the associated quantities. Thus the study and investigation of the quantitative relationships form the underlying principle of understanding the laws of nature. They enable man to know the material world in which he lives. They enter into every phase of the life of every thinking man. In fact, life is made up of relationships: relationships which unite the individual in a definite manner to the society, or the group of individuals, of which he is a part; to the inorganic world and the processes of nature upon which he depends for subsistence; and to the other organisms with whom he inhabits the earth.

Furthermore, the whole process of thinking is based upon, and in terms of, certain acquired relationships which form the background or the apperceptive mass of the individual. His ability to investigate, interpret, and comprehend, or even appreciate new relationships, depends in a large extent on the knowledge of quantitative relationships already in his possession. These constitute his mental

[1]Read at the Educational Conference of the Academies and High Schools in cooperation with the University of Chicago, May 8, 1926.

capacity. Now the true purpose of general education is to endow the individual with methods of thinking. These methods of thinking are acquired only after passing through a long series of experiences, and after undergoing a long period of training. Similarly, the habit of relationship or functional thinking is acquired only through a long and slow process of experiencing with simple relations, and with specific instances, each instance shedding some light on the exact nature of the more general relationship.

Because of the universality of quantitative relationships, the habit of functional thinking is of utmost importance to the individual. Its acquisition should be the emphasized goal of every course dealing directly or indirectly with relations between things or processes. In its preliminary report on "the Reorganization of the First Courses in Secondary School Mathematics," the National Committee on Mathematical Requirements specified that "the primary purpose of the teaching of mathematics should be to develop those powers of understanding and analyzing relations of quantity and of space which are necessary to a better appreciation of the progress of civilization and a better understanding of life and of the universe about us, and to develop those habits of thinking which will make these powers effective in the life of the individual." Thus in the realization of the value of the formation of the functional thinking habit and its inculcation in the individual lies the true purpose or usefulness of mathematics. But as Professor Hedrick[2] points out, the danger lies in the fact that those who have acquired the habit may underestimate the value of training in its formation, and overlook to emphasize the essential significance of relations, because they seem to be obvious and self-explanatory. How often, in working with algebraic processes, the pupil is left alone to see the "obvious" relations, and as should be expected, the obvious is too obvious for his untrained mind to see. It will be pointed out later on in this paper that generally the text books in the secondary school mathematics fail to emphasize or bring out the significance of the relationships involved between the quantities with which they deal. "Our courses of study have failed, generally, to the present time, to give our high school pupils a grasp of functionality. Thus both the basic mathematical purpose of the course and the foundational thinking purpose have not been fulfilled."[3]

Among the multitude of ideas and endless variety of concepts that constitute any course of study, there are a few which form its general framework. These fundamental concepts with their great or-

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[2]*Mathematics Teacher*, April, 1922.

[3]Rugg-Clark, *Fundamentals of High School Mathematics*, p. VIII.

ganizing powers form what Professor Morrison calls the true learning products or units of the course. They are to the course what the spinal column is to a vertebrate animal, giving the whole structure its character, its stability, and its coherence. They are the unifying principles about which, as nuclei, the important materials of the course are grouped. The cellular organism in biology and the electron theory in physics are examples of such supreme and important concepts. In mathematics the one great idea which is sufficient in its scope to form the basis of unification is the function concept, "a concept which since the seventeenth century has dominated advanced mathematics, a concept which in the twentieth century, according to the auspices, will play a fundamental role in the reorganization of elementary mathematics."[4] Because of the inter-relations of the equation, the formula, the graph, and the geometric relations inductively acquired, the function concept, as Professor Felix Klein pointed out in a paper read before the International Congress of Mathematicians which met in Chicago in 1893, should be the unifying principle around which mathematical material should be organized and correlated.

"Functional relations will occur on every page of every book of mathematics, unless we suppress them," says Professor Hedrick, whose statement, because of his special investigation of and interest in the reorganization of secondary school mathematics, may be accepted as authentic. And the significance of this statement becomes at once evident if we realize that the science of mathematics is primarily engaged in the study of quantitative relationships, both the spatial and numerical, which constitute its direct field of study, and those which are investigated and recorded by other sciences. Perhaps it is for this reason that the mathematician has claimed this fundamental and universal notion as his very own, even though its application is of utmost importance in all fields of thinking.

In its mathematical setting the term *function* has gone through a long process of modifications of meaning, from its original meaning as any power of a number to its present true meaning as the notion of relationships between quantities and the manner in which changes in one of the two or more related quantities produces a change in the others. The relationship is represented in terms of dependence of one variable upon another in such a way that, when one variable, the independent, is assigned any value from a set of values, called the *range of the variable*, the corresponding value or values of the other, the dependent variable, may be determined. The variable, such as x, in terms of which the function is defined, is simply a mathematical

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[4]E. H. Moore, *The School Review*, May, 1906.

symbol denoting in any given discussion any one of a set of objects. In elementary mathematics this set of objects which constitutes the range of the variable consists of numerical facts. Thus any mathematical expression involving one or more variables embodies the functional relationship.

Perhaps no notion is as common and familiar to all as the dependence of one variable quantity upon another. The notion of mutual dependence and reciprocal evaluation is exemplified in every turn and feature of life and the world. For instance, the perimeter of a square depends upon the length of one side; the area of a circle upon the square of its radius; the distance traveled upon the rate and time of going; the time of vibration of a pendulum upon its length; the volume of a gas upon temperature and pressure; the cost of a railroad ticket upon the number of miles travelled; the parcel post rate upon the weight of the parcel and the distance it is sent; the amount of work a man does upon the number of hours he works; the cost of a suit of clothes upon the supply of cloth, labor, and style; the rent one pays upon the size of the house, improvements, location, and the conscience of the owner; the size of the crops upon the acreage, heat, moisture, fertility of the soil, and the industry of the farmer; the rate at which potatoes cook upon the amount of gas burned under the cooker; the sweetness of a thing upon the amount of sugar in it; the amount of light coming through a window upon the size of the window; the prosperity of a throat specialist upon the moisture of the climate; the rate of chemical change upon the amount or the mass of the substance involved; the interest on a sum of money upon the rate and time; and so on without end.

The notion of dependence forms the basic study of all physical sciences. As soon as a science reaches the stage, in its groping after the truth, where measurement is possible, the observed data are set down and studied as special instances of some general law which it seeks to discover. Whenever possible the mathematical method of representing dependence and showing relationships are utilized to advantage. This is especially true in physics. Every law of physics may be expressed as an equation. Hence the necessity of cooperation between physics and algebra. It is here that we find an indispensable need for the grasp of the principle of functionality as expressed in algebraic shorthand.

"An equation is the most serious and important thing in mathematics," says Sir Oliver Lodge[5]: The concept is perhaps the oldest in mathematics, for when the rational mind of man began to count, he used the idea of equality of the things counted. The elementary alge-

[5]*Easy Mathematics*, 1906, p. 127.

braic equations and their solutions have been studied with various degrees of success by the ancients. The Ahmes papyrus is the oldest deciphered work treating the solution of equations in one unknown. The unknown quantity is called *hau* or *heap*[6]: Thus, "heap, its 1/7, its whole, it makes 19": i.e.
$x/7 + x = 19$. On fragments of papyri which have been deciphered more recently, but are probably older than the work of Ahmes, statements equivalent to the system of two simultaneous equations

$$x^2 + y^2 = 1000, \quad y = \frac{3}{4}x$$

have been found.[7] The Greek mathematicians, as well as the Hindu, Arabian, and the European mathematicians of the Middle Ages, sought numerical solutions of particular quadratic and cubic equations.

The algebraic equation interpreted rightly is a convenient method of representing the functional relationship. For example, the equation $y = 2x+3$ determines a unique value for y corresponding to every value of x, the relation of dependence being stated in the polynomial in the right member. Furthermore the concept is not only obvious in the equation involving two variables, but is also inherent in the equation with only one variable, such as $2x+4 = 10$. For in the latter case, as Professor Young explains[8], the problem is to find how the expression $2x+4$ varies as x varies. Among the different values of $2x+4$, that of 10 can be found, and it is but one of the many values. When the curve of the equation is drawn, the variation is at once recognized.

Mathematics had its origin in trying to solve practical problems, and mathematical knowledge has grown because it has been useful. In view of the usefulness of equations in expressing physical relations compactly, the emphasis should be placed on interpreting the relations and not on the manipulation of the symbolism. There seems to be a perverted attitude on the part of the text book writers and the teachers of mathematics in neglecting the numerical relationships and in making the manipulation of the equation their primary objective. The pupil is taught how to solve a given equation, and he acquires a great deal of skill in his performance, but he has no idea what the equation stands for. He solves the equation for the value of

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[6]Cajori, *A History of Mathematics*, p.13.

[7]*Monographs on Modern Mathematics*, p. 213.

[8]*Teaching of Mathematics*, p. 387.

x, and when that is done he is through, even though the equation may be a representation of relationship between two or three associated quantities, expressed in terms of a single variable x. And when he comes to verbal problems, he is told to study the relations between the parts of the problem and express them in algebraic form, and then solve the equation. But he sees no relations, for his attention has not been called to any relations in an equation. Give him the equation, he will solve it; but to set up the equation, that is another story.

An analysis, by the writer, of the text books of elementary algebra, and of the general mathematics for the junior high schools, in connection with this problem, has shown that the equation is invariably defined as a statement of equality between two numbers, and to emphasize the equality, the equation is thought of as a balance. What is done to one side must be done to the other side, if the balance is to be preserved. It is true that this statement inherently means, "a change produced in one member of the equation necessitates a corresponding change in the other;" but this latter meaning should be made clear because it brings out the idea of dependence and relationship. Now a formula, because of its nature, would be expected to state explicitly the relationship between the quantities which it symbolically represents. But a formula is commonly defined as "an abbreviated rule." Where is the relationship? The pupil is expected to get it from the nature of the problem. He does not. He tries to remember the rule. There is no relation specified, and if his attention is not called to it, he does not see it. Many writers will warn the pupil that a formula is always an equation, but an equation is not always a formula. If defined on the basis of relationship and dependence of quantities there may be some justification for this attitude, since the relations expressed in most equations are artificial and not real and practical as those expressed by the formula. But they do not always define an equation in terms of any relationship. Then, why try to confuse the mind of the child? Since his attention is not called to any relations involved, and he cannot be expected to distinguish between an equation and a formula on the basis of any relationship, how is he to know when an equation is a formula and when it is not a formula? Is it a wonder, then, that he can solve equations involving x, y, z, a, b, c, but gets stuck on those having m, d, v, t, f, a? If an equation is defined and interpreted correctly, this apparent confusion disappears. Many writers[9] use the two terms synonymously, and they define an equation correctly as a method of representing rela-

[9]For example see Breslich, *Junior Mathematics, Book I.*

tions between numerical facts. $X=4y$ and $p=4s$ both show that the perimeter of the square is directly proportional to the side.

When, in connection with the solution and the application of the equation, the idea of the functional relation which it represents is duly emphasized and illustrated with numerous concrete examples, algebra will become a homogeneous subject grouped about the equation as the central notion and will not consist, as at present, as Professor Bliss expresses it[10] of "topics, related perhaps inherently, but with no indicated relationship, following each other in a confusion of radicals, exponents, progressions, imaginaries, probabilities, and other algebraic conceptions, in a way which must tend to develop a very disjointed understanding on the part of the beginner." Some text books of general and unified mathematics in the secondary schools, especially in the junior high school mathematics, have introduced the algebraic materials as a homogeneous unit centered about the equation. The closer the connection between the equation and the function concept, the more uniform and coherent is the unit.

The concept of relationship can be acquired by the pupil in proportion as the teacher calls his attention to it in every case where relation exists between the quantities with which he works. But it cannot be taught or learned if the teacher fails to interpret the equation as representing relations, and waits until nearly the end of the course when he devotes a few days to variation and proportion. Furthermore, the pupil's acquaintance with function should not consist of drawing a few perfunctory functional graphs. The acquisition of the concept must start from the very beginning, when the pupil is introduced to the simple linear equation in one variable, and continue as long as he works with algebraic expressions; with special emphasis and amplification on dependence and variation as expressed by proportion; substitution, which means the calculation of one quantity in terms of another; tabulation, which, actually states in full the value of one quantity in terms of another; and graphic representation of related facts.

The functional relation is often more explicitly stated in formulas expressing physical or geometrical laws. For the most part a clear understanding of these laws is reached through the study of special instances of the variation between the variables, in the form of equal ratios of the measured magnitudes. The ratios determined, the law is stated by indicating how one quantity varies with the other, directly or inversely. Proportion is thus conceived of as a special instance of variation and of the functional relationship of the form $y=kx$. The

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[10] *Monographs on Modern Mathematics*, p. 264.

dependence or variation may be direct, that is, both variables increase or decrease in value together, as is the case when the expression has the form $y=kx$; or one quantity may vary inversely as the other, that is, one variable increases in value while the other decreases, which case has the form $xy=k$. The two special instances of the former case give rise to the proportion

$$\frac{y_1}{x_1}=\frac{y_2}{x_2}$$

while those of the latter case give the proportion

$$\frac{y_1}{x_2}=\frac{y_2}{x_1}$$

Thus many problems in physics, chemistry, general science, domestic science, astronomy, as well as in mathematics, may be solved by either variation or proportion. And since the whole theory of proportion is involved in variation, this fact should be made obvious to the pupil. He should clearly recognize the relation between variation and proportion. Proportion should mean more to him than "the product of the means equals the product of the extremes." An understanding of the true nature of proportion will enable him to interpret and appreciate the various possibilities which occur in the relations expressed by proportion. The untrained mind cannot think of such relations readily, as shown by Sir Oliver Lodge in his book, *Easy Mathematics, Principally Arithmetic*. He points out that most persons will attempt to use proportion in cases to which it is not adapted, and when the relations are impossible and ridiculous, as in the following: "If a camel can go without water for ten days after drinking fifteen gallons, how long could he go if he drank one hundred gallons?" "If a boy can slide eighteen feet on the ice with a running start of twenty feet, how far could he slide after running half a mile?"

As intimated above, the study of functional relations and functional thinking are useful even in branches of thought where precise mathematical formulation is impossible. In this case the observed facts are tabulated and the data may be studied to interpret the relations between the quantities they represent. Tabulation and interpretation of tabular relations is of significant importance, not only for its own direct and practical application, but also because of its direct bearing on formulation, when formulation is possible, and graphic representation. A table is strictly functional in character. It states the value of one of two related quantities when the other is given. For example, consider the accompanying table, in which the numbers in the first row represent hours of the day, and the numbers in the second, the corresponding temperatures.

Hours	8	9	10	11	12	1	2	3	4	5	6	7	8
Temperatures	62	63	67	75	80	82	83	81	79	73	74	67	59

The table sets up a definite correspondence between the numbers representing hours and the numbers representing temperature, such that whenever a certain hour is selected a corresponding temperature is uniquely determined. It should be noted that the functional relation here is not expressible as an algebraic equation.

Tables of statistics furnish useful information, collected through a great number of observations, about the weather, the growth of population, crop productions and prices, the cost of living, the death rate, etc. The facts found in most statistical tables are not expressible in exact mathematical laws, or formulas, though such a formula would be of immense value. Such, for example, if a formula could be deduced by means of which the rainfall or the temperature for any specified time could be computed in advance. Nevertheless, the study of statistical tables warrants fairly safe and useful conclusions.

The functional character of the tabular representation should be emphasized, because the ability acquired by the pupil is an essential tool which may be used to advantage in other courses and in life, and also because of its importance in connection with the graphical representation of relationships.

The graphical representation of numerical facts and relationships by means of geometric line segments and curves is of utmost importance in mathematics, and at the same time is very interesting to the child. The notion is not altogether new to him for he has become familiar with it in the newspapers and magazines. The principles underlying this representation are easily understood and are tacitly assumed without much difficulty. As a method of representing statistical facts the graphic method has many advantages over the tabular method: in comparison of different tables of facts, in bringing out more clearly the meaning of the numerical facts and the relation between them at a glance, in showing the maxima and minima, the range of change, etc. Furthermore, as in the case of the continuous graph, this method furnishes additional facts not stated in the table.

A common method of representing statistical data is by means of the bar graph. This is supplementary to the tabular. The facts in the accompanying table are interpreted more readily by means of the diagram using line segments for numerical facts.

Months	Jan.	Feb.	Mar.	Apr.	May	Jun.	July	Aug.	Sept.	Oct.	Nov.	Dec.
Average daily hours	62	63	67	75	80	82	83	81	79	73	74	67

The table states the number of electric light units used in the average residence in a certain city for each month of the year. To represent these facts graphically, squared (centimeter) paper is used. A horizontal line is used for the base line on which the months of the year are marked off; the number of corresponding hours are represented by means of vertical line segments. The selection of the unit for the length of the line segments, the distance between two successive bars, and their width are optional and depend on the nature of the numerical facts in the table. It should be noted that this type of work is really interesting to the pupil and develops initiative and independent thinking, for example in the selection of the baseline, the unit, etc.

After the pupil has learned to make and read the bar graph, he is naturally led to the next step in the graphic representation, namely, the continuous line graph, by joining the end points of the bars. The bars are then entirely omitted, leaving only the end points. The line graph is used to represent related facts, and it not only illustrates given facts and enables comparison, but also provides additional information not contained in the table. It also paves the way for the representation of algebraic expressions by means of the geometric lines and curves, for he becomes familiar with the relation between points and pairs of values which is the underlying principle of the method of analytic geometry. The continuous line graph is best used for representing changing prices of a commodity, temperature, stock fluctuations, etc.

The graph is a powerful instrument in representing relationships admitting mathematical expressions both in their interpretations and their solutions. It is here that we see a true fusion of the geometric and the algebraic methods. Though the properties of the simple geometric curves have been known from the earliest times, little use was made of them in interpreting algebraic expressions. In fact these two branches of mathematics were studied separately following two parallel lines of development, and they were jealously kept separated until in the seventeenth century the great philosopher and mathematician Descartes prepared the way for the union of the algebraic function concept with the geometric curve notion. The method consists of putting the points in the plane into a one-to-one correspondence with the pairs of real numbers. The position of any point may be uniquely determined by its distances from two fixed lines called the *coordinate axes*. The geometric curve is then thought of as generated by a moving point and its study reduces to the study of the variation of the coordinates of the points. This variation of the coordinates for most curves may be represented in an algebraic form, that is the ordinate y is expressed in terms of the abscissa x by means of a formula or equation. The graph becomes the picture of the algebraic formula $y=f(x)$, and the functional relation expressed by the equation is made obvious by the graph.

In the secondary school mathematics the formulas and equations studied are for the most part linear and quadratic functions; consequently the straight line and the parabolic graphs are of special interest. In the case of the linear equations, the straight line graph always intersects the x-axis in one and only one point, except in the special case when the graph is parallel to the x-axis, and the point of intersection gives the real solution of the equation. The graph of the quadratic equations, on the other hand, intersects the x-axis in two, one, or no points, giving two real distinct, two real equal, or two imaginary roots of the quadratic equation. We thus see that in connection with the representation of dependence the graph may also be used in solving equations.

Thus, interpreted correctly the real work of algebra consists of the study of relationships between numerical facts. The real criterion, then, should not be the number of symbols it contains, but whether there is any question of variation and dependence involved. The emphasis should not be placed on the symbols or their manipulation, for symbols are only the shorthand for representation of relations. The shorthand is essential but it is not algebra. Algebra has been and may be studied without the shorthand. To spend all the time on the shorthand, without any realization of the relationships for which the shorthand is to be the handmaid, is "to spread a feast without any guests."

Turning to geometry we find the relations involved are those existing between geometric or spatial quantities, as distinguished from the algebraic relationships between numerical quantities. The points, lines, planes, and solids which constitute the material of geometric study are related in certain definite ways. The relations existing between the parts of a triangle have been completely developed and their functional character recognized by the terminology applied to them, namely, trigonometric functions. The parts of all geometric figures are inter-related. Certain definite relations exist between the angles and the sides of polygons. For example, in the triangle, the fundamental relation $x^{\circ}+y^{\circ}+z^{\circ}=180^{\circ}$, existing between the three interior angles, gives rise to the variations: $x=180-y-z$, $y=180-z-x$, and $z=180-x-y$. Besides the trigonometric ratios, there is the relation between the squares of the side opposite an acute, right, or obtuse angle, and of the other two sides. If the functional character of this relation is recognized, it would not be divided into three separate and unrelated theorems. Functional thinking in geometry necessitates reasoning by the Principle of Continuity to discover the general relation, and applying it to the special cases. For example, the four separate theorems concerning the measurement of the angle included between two intersecting lines, in terms of the intercepted arcs of the circle, become but one general theorem, each special case depending on the location of the point of intersection of the lines with respect to the circle. Similar conclusions apply to the relations between similarity, congruence, and equivalence of plane figures.

As in algebra, so in geometry the proper emphasis is often placed not on the quantitative relationships which constitute the real geometry but on the formal logic which is an instrument used in the study of those relationships and in discovering other relationships from the existing ones. Our traditional courses in geometry have become so absorbed in the formal logic in the proofs of the theorems, that the functional relation is neglected and often not recognized. Studied this way geometry fails to be of practical value in the interpretation of spatial relationships. It is open to question whether geometry as it is taught at the present time enables the pupil to see, for example, the changes produced in the shape of a parallelogram with the change of the angles while the sides remain fixed, or the change in the sides while the area remains fixed. To study changes produced in figures by the variation of one or more of its parts is to study real and practical geometry, and it is of utmost consequence toward a real mastery of geometric notions. Ability to think such reasonable relationships quickly and accurately should be a part of the reasonable mental equipment of all educated men and women.

In view of the importance of functional thinking in all fields of thought and investigation, and the important role it plays in mathematics, permeating all of the branches, and being represented by different forms of varying degrees of applications, we are led to believe that the practical value of mathematics to a non-mathematician lies in the development of the functional thinking habit. It is generally felt that mathematical training is indispensable in the study of other sciences, and by its devotees mathematics is designated as "the language of all sciences." Of course no one will deny the absolute importance of the number concept and the four fundamental arithmetical operations to the individual in carrying on intelligently and economically his duties of life. But how much of algebra, geometry, and trigonometry he needs in his further studies and in life is another question. Since he continues to deal with quantitative relationships throughout life, it is to be expected that the habit of functional thinking acquired in his courses in mathematics will abide through its constant use, and that of the mathematical processes learned only those which are directly associated with this habit will be retained and utilized.

The problem of finding the exact amount of mathematical training needed in special types of study and work is indeed a difficult one. Various investigations have been made by students of education in this line, but most of them consist in analyzing text books, magazines and newspapers to determine what mathematical terms are used and the minimum vocabulary of mathematical terms needed for a comprehensive reading of the subject matter.[11] These investigations, of course, throw but little light on the actual applications and practicabilities of mathematical concepts and processes.

Of the algebraic work found in the use of mathematics in the industrial occupations, Florence Morgan[12] shows only ratio and proportion used in carpentry. Of the mathematics used in shop problems, equations comprise 11.1 percent, literal equations and formulas 22 percent, ratio and proportion 66.9 percent. Thus it seems that there is a predominance of formulas and proportion; the fundamental operations used are only in connection with the manipulation of the equation.

Mary O. McClusky[13] concludes that only a few simple exercises of an algebraic nature, requiring an acquaintance with equations and the use of formulas, are needed in home economics.

R. C. Scarf[14] points out in his thesis, *Mathematics Necessary for Reading Popular Science,* that of the algebraic processes only formulas, bar, linear and circular graphs are used. He concludes that "the evidence presented shows clearly that one of the important functions of mathematics is furnishing a vocabulary for describing spatial and quantitative relationships." One may reasonably doubt if the furnishing the individual with a mathematical vocabulary alone is

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[11] See Bobo's Thesis, The School of Education, The University of Chicago.

[12] Thesis: The School of Education, The University of Chicago.

[13] Thesis: The School of Education, The University of Chicago.

[14] Thesis: The School of Education, The University of Chicago.

sufficient to enable him to interpret and appreciate spatial and quantitative relationships.

Similarly, the functional character of the practical mathematics is seen in the use of mathematics in agricultural studies where, according to H. B. Roe[15], perimeter and area formulas of polygons and circles, volume formulas, and formulas of the relationship of a right triangle are needed mostly. Problem analysis is considered of special significance.

An analysis, by the writer, of some typical high school physics text books has disclosed the following concepts and processes used: (1) formulas, (2) graphs, (3) ratio, (4) direct and inverse variation, (5) proportion, (6) geometrical constructions needed in the composition of forces, velocities, and optics, (7) trigonometric functions. With the exception of number 6, the others are the processes which we have shown to be directly involved in the function concept. The chemistry text books, on the other hand, use very little algebra. The chemical formula, such as $Fe_2+0_2 = Fe_20_2$, is not a true algebraic equation. Some proportion is used in connection with determination of atomic weights. However, the ability of the pupil to interpret variously stated real relations, and to represent them symbolically, is of paramount importance in chemistry as well as in physics.

The outstanding finding of these and other studies confirms our convictions in the importance attached to a full understanding of functional relations and to the acquisition of the habit of functional thinking. Herein lies the primary function of a high school course in mathematics, and herein lies the true solution of the problem of reorganization of secondary school mathematics. The School of Education of the University of Chicago has been a center for educational experimentation for a number of years, and here the solution to the problem of mathematical reorganization has been along the lines of fusion and correlation of material. Following in Mr. Breslich's wake, many text books of general mathematics have recently appeared which indicate a favorable and concerted effort toward a recasting of the subject matter. In many of them this reorganization consists merely of mixing algebraic and geometric material. True fusion, however, should be in the form of a "chemical compound," coherent and based upon the inter-relations of concepts. Reorganization of material is taking place even in the traditional courses in algebra, where more practical problems are brought in and more use is made of the formula and the graphic representation.

But, after all, the text books are but tools in the hands of the teachers. When the teachers of mathematics themselves realize the

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[15]*Mathematics Teacher*, January, 1922.

importance of the function concept, and are willing to lay stress on it as the primary and underlying principle of the course, and have constantly in mind the pupil's training in the formation of the habit of functional thinking, they will utilize this fundamental concept in making mathematics of direct value in the development of more intelligent citizenship. They will present the supreme ideas, of which mathematics is the science, as never before in their more obvious aspects to be understood and utilized by the individual for his personal use, and by the society and the state for the advancement of scientific knowledge. They will present them with such earnestness that the science which Plato called "divine"; which Goethe called "an organ of the inner higher sense"; which Novalis called "the life of the gods"; and which Sylvester called "the Music of Reason" shall be the very essence of reality, penetrating life in all its dimensions. It is this "new mathematics" which H. G. Wells describes in his "Mankind in the Making," as "a sort of supplement to language, affording a means of thought about form and quantity, and a means of expression more exact, compact, and ready than ordinary language. The great body of physical sciences, a great deal of the essential facts of financial science, and the endless social and political problems, are only accessible and only thinkable to those who have had a sound training in mathematical analysis."

16

Algebra as a Medium for the Interpretation and Control of Nature

Merton Taylor Goodrich
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(Vol. XXXV No.3 January, 1935)

Not only does it give delight to find algebraic laws in nature, but the discovery and application of these laws make the world a happier and better place in which to live. It is a joy to perceive that there is a relation between speed, time, and distance which can be represented by a simple formula. But is an even greater pleasure to find that by means of algebraic formulas, machines can be constructed for the production and control of motion. Not only does algebra increase our happiness by unfolding before us a larger meaning to nature than could otherwise be found, but the study of algebra leads directly to the control of nature and the pleasures that come from the possession of these powers. In brief, algebra may be considered as a medium for the interpretation and control of nature.

The world in general does not use algebra to solve puzzle problems, to remove complicated nests of parentheses, or to perform other valueless tedious tasks. Society uses algebra to shape the wings of the airplane, to build the headlights of an automobile, to arrange the parts of a radio tube. Our present civilization could not exist without algebra. Society makes use of algebra to obtain new meanings to the universe and to control the forces which it discovers.

It is impossible to control the force of exploding gasoline without the use of formulas for the construction of cylinders and valves. It is impossible to control the force of electricity without formulas which determine how the wires shall be wound on the generator. At the present time, the natural and the social sciences are finding it desirable to use symbols and formulas to clarify their terminology and to establish their laws more definitely. To an increasing extent, all sciences are making use of simple algebra. Society is finding algebra useful not only in the study of numbers, forms, and forces, but in such studies of real life as are illustrated by the Mendelian law. When a law is determined with sufficient certainty to be represented by a formula, it becomes algebraic. But not until a law has become algebraic can it be used effectively or accurately for the control of nature.

Let us, then, refrain from teaching algebra as a tool subject for the solution of obsolete and absurd problems, such as how long it will take a dog to catch a rabbit or how long it will take an automobile to overtake a bicycle. Let us not make algebra a monotonous series of tedious drills.

When algebra is regarded merely as a tool subject, when it is taught in a mechanistic way, many pupils hate it, and there is justification for their attitude.

Let us teach algebra as a real, living, power-giving subject. Then, teaching algebra will become an enjoyable occupation, and studying it will be a thrilling, happy experience.

It is natural for human beings to use symbols even in childhood. The toy automobile is a symbol for a fire truck or a five passenger car. The doll is a symbol for a darling baby, a little boy, or a little girl. Some see a symbolism in all nature and, as an illustration, think of a flower as a symbol for a divine thought. For these reasons, algebraic symbols are not to be taught as meaningless characters to be juggled at will. Algebraic symbols are to be taught as the normal development of the natural instinct to symbolize. Leading pupils to see that the symbols of algebra may be used to represent elements and factors in real life just as toys are used, leading pupils to see that by the use of symbols formulas may be written which represent real relationships, leading pupils to see that the symbols and formulas of algebra show the order that pervades the universe, leading them to see how the laws of nature may be interpreted and controlled by the aid of algebra is leading them ultimately to appreciate more fully the laws of God.

Specifically, this means the teaching of all things concretely as far as possible, so that negative numbers actually can be observed as common in the daily life of the people, so that pupils in the classroom can derive simple formulas experimentally or demonstrate dramatically the existence of complex numbers.

When pupils realize that algebra is a subject shining with beautiful symbols, that the formulas most commonly used are the simplest ones, and that algebra when taught concretely may be easily understood by the normal student, then pupils like to study it and teachers like to teach it. The study of algebra begins to give real pleasure just as soon as pupils see that it gives a broader and fuller meaning to life, and this pleasure increases as to this power of interpretation is added the power to control nature.

17

On the Role of Geometry in Science[1]

Frank W. Bubb
Washington University
(Vol. XXXVII No. 1 January, 1937)

I wish to explain how geometry guides—one can even say dictates—the development of natural science.

In presenting this thesis, I must call attention to certain aspects of geometry not commonly emphasized. Geometry is usually regarded as a measure of external space. In popular conception this body of knowledge is supposed to have been put into permanent form by a Greek gentleman named Euclid. Both of these beliefs are entirely untrue.

Our knowledge of space is obtained through the senses. The sensory apparatus may be likened unto a lens system which throws upon the mind an image of external space. Geometry is our knowledge of this mental image, and this internal picture may or may not be a faithful reproduction of external space. I shall show from historical evidence that geometry has, during the last two thousand years, undergone some startling changes. Now surely it is easier to believe that these changes have occurred in our conceptions of space rather than in external space itself.

I think I had better try to impress upon you more clearly the distinction between external space and these several internal or mental images thereof. I can make my meaning clear if you will perform an experiment with me. Close your eyes (after a moment) and imagine or visualize yourself standing, say, before your own home. Enter your house, in imagination, and walk through the various rooms, noting their arrangement as well as that of the furniture. You may

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[1]Address before the Physics Section of the Central Association of Science and Mathematics Teachers in St. Louis, Missouri, November 27, 1936.

indulge your fancy a bit by rearranging the furniture. You will quite miss the point of our experiment unless you stop long enough . . . to exercise thus your space intuition. You may picture internally in this manner any familiar scenes, persons, and events. You may dramatize your past and attempt to visualize your future. It is this power of the mind to construct pictures (of the lovely heroine, for example) which enables you to enjoy reading books. Without this power, you could not betake yourself from your work to your home in the evening; it would be unsafe for you to go anywhere beyond your immediate range of eyesight. You can (and if you performed the experiment proposed above, you did) visualize a distant scene so vividly that you are bewildered for a moment upon coming out of such trance-like state. Then is the time to ask yourself where is that place in which you seemed to be but a moment ago. You were not in that distant place itself. If you have an astral double, it may have gone there, but certainly your body did not. Neither is that place inside your head; your head is not large enough to hold that house. But there certainly is something inside your head corresponding to external space. This is what I call the mental picture frame. This is the internal filing cabinet wherein we store our perceptions. This is the seat of our visual memory. This is the laboratory wherein we make our mental constructs, develop and fit together those abstractions called the concepts of science.

Is the internal picture frame, which a person may have, a faithful representation of external space? I shall let you judge of this by actual example.

The ancients, before the time of Euclid, had a 2-and-1 geometry. They conceived all horizontal directions to be alike, thus forming a homogeneous 2-dimensional manifold. But the one vertical direction was conceived to be unique and different in nature from any of the horizontal directions. Gravitation was so universal a phenomena that apparently it was not thought necessary to explain it. Gravitation was simply part of the uniqueness of the vertical.

Children seem naturally to form this 2-and-1 conception of space, a matter which may be verified by conversation with almost any little person who has competence in self-expression. I had a most interesting talk with one little chap who was hearing for the first time that the Earth is spherical. He objected at once, pointing out to me the fact that the vertical *is* different than the horizontal. He wanted to know how people on the opposite side of the earth cling on to its surface, why they do not start sliding when they journey too far, why the oceans have not run off, and demanded the why-nots of certain other phenomena which his imagination led him to expect. In short, by the simple device of extrapolating the vertical parallel to itself, he proceeded in masterly fashion to demolish my absurd notions on the sphericity of the Earth. And to clinch the argument, he challenged me to step off the roof and test the essential correctness of his views. I am having a slight measure of success in persuading this small Newton that his assumption that all verticals are parallel does not agree with experience.

The ancients extended this local horizontal-vertical partition of space to the uttermost limits, and consequently supposed the Earth to be flat, as children do. Upon the seas, mariners feared to sail too far lest they go over the edge into the bottomless abyss of space. Sailors who failed to return were commonly supposed to have perished thus.

Such a flat world had to be supported somehow. To perform this function, the orientals posted an elephant at each one of the four corners. But this is not enough. Some scientifically minded fellow, lacking in respect for his ancestors, eventually raised the question as to what the elephants stand upon. The question cried as logically for answer as the original question concerning what supported the Earth. The question was answered by asserting that the four elephants stand upon the back of a turtle, a sturdy turtle. But the question recurs and the suite of answers caused the foundations of the Earth to take on resemblance to a totem pole.

This question as to the foundations of the Earth is important because it provides a perfect example of how growing science disposes of some quite logically formulated questions. Science has never answered the question as to what supports the Earth; it quietly murdered the question, made it nonsense. The question as to how empty space transmits the enormous gravitational force between Earth and Moon is of this category. This gravitational problem, which has produced since the time of Newton some 200 theories (at least 199 of which must be wrong) as to means of transmission, is not answered by relativity. It is simply removed; the force does not exist. And there are doubtless numerous questions now tormenting us poor humans which have intrinsically no meaning, no matter how logically they may now appear to demand answers.

The common device for explaining natural phenomena in the ancient 2-and-1 cosmogony was the nature-myth. Rivers flowed because local river gods pushed the waters along. The Sun rolled across the heavens because it was dragged by the chariot of a mythological being. The winds were controlled by angels who stood at the four corners of the Earth. Heaven above, hell below, and the Earth between were filled, each with a diversity of gods and demons: Zeus with his thunderbolts, Pluto with his fires, and Poseidon with his storms. Greece is still dotted with the ruins of temples dedicated to their propitiation. Natural phenomena occurred or did not occur according to the caprice of gods and demons. The nature-myths peopled the Earth, the skies, and the seas with living beings, as senseless and savage as those who framed them.

What was this 2- and-1 geometry? We now regard it as a distorted mental image of space. This affords an example of the point I made above that geometry is our knowledge, not directly of external space, but of the internal space image. For, surely no one except children now regards external space as actually shaped according to this ancient model.

There can be little doubt that our conception of space is intimately bound up with the particular sensory equipment which humans happen to possess. Now the problem of perception is given little attention by the scientist in spite of the emphasis placed upon it by the philosopher and psychologist.

Let us examine, if only for a brief moment, the sensory equipment of the natural philosopher. He is endowed with five senses which enable him to partake, slightly, in the profound actions of natural forces. He can not participate intimately in chemical reactions; he can not be dissolved, go colloidal, or explode. Perhaps he might do one of these things but his knowledge of what had happened to him would not be available to the rest of us. He can not indulge with too much abandon any taste which he may have for thermodynamics; he might be gassified one way or solidified the other. He can not personally commune with lighting flashes. In short, he is careful to stand well away from and intrude himself only with circumspection into the affairs of nature. Most of what he knows has not come to him through his sensory channels; he has invented most of it. His senses are like the pinhole of a pinhole camera. Through this pinhole he peers out at the world. And just how tiny this pinhole is is rather frightening. It makes one feel like a child lost in the dark when one realizes the vast extent of the reality which one seems barred from ever perceiving directly.

How, in view of this, can one believe that the external world is shaped the way we now conceive it to be? Or to put the same question differently, how can we expect the mind to construct within the convolutions of the brain a framework which shall be a faithful representation of the external world? Isn't it possible that space-time is just our mode of thinking about external phenomena? Do we not externalize this way of arranging our perceptions and conceptions and superimpose this framework upon the external world? The historical evidence that geometry has changed, although, alas! with evolutionary slowness, is positive evidence for this view.

I shall anticipate here a question which someone is sure to ask. If one should train oneself to think with facility in four dimensions when one is young and before one begins to believe the external world is three dimensional, namely, if one were to develop systematically a mental four-dimensional framework, would one then externalize this mode of thought and regard the external world as four dimensional? I think so. Unfortunately, I studied the properties of four-dimensional figures after I was taught to regard the external world as three dimensional. But in spite of this handicap, I occasionally glimpse relations which can not be expressed in three dimensions.

But let us return for a moment to the ancient 2-and-1 geometry. In this framework, gods and demons were regarded as the efficient causes of things. Mythologies, which are now regarded as nothing more than poetic fancies, were then both the science and religion of their times. Nature-myths were the explanations of natural forces. They were attempts at rationalizing natural phenomena. And that is precisely the purpose of any of the accepted theories of modern physics.

Such was the grotesque scientific picture which our ancient brethren painted in their 2-and-1 picture frame. But within such a distorted, poorly shaped frame, how could anyone paint other than a fantastic picture?

The next frame for our mental picture of the universe was constructed by Thales, Pythagorus, Euclid and others. We shall call this the homogeneous 3-frame because all directions in space were conceived to be alike in geometric properties. This homogeneous 3-geometry makes room for a spherical Earth. In this enlarged frame, the 2-and-1 geometry appears as an unwarranted extension of the local horizontal-vertical partition, which we make of space. The 3-frame permits individuals at different points upon the Earth to make their own local horizontal-vertical partition, as each individual will if he wishes to keep head-end up.

In this picture frame, which goes by the name of Euclidean Geometry, many artists have wrought. Isaac Newton nearly filled the picture with his cosmic theory, a stupendous structure. Magnificent designs such as Maxwell's electromagnetic theory endow the picture with life. The radiant patterns of Fresnel and Michelson, of incredible delicacy, transcendent beauty, adorn this wonderful picture. This is no dead thing wrought in metal, painted on canvas, or cut in stone to catch a fleeting mood. This is a representation of nature herself. Its beauty consists in the play of intelligence upon matter, spirit upon energy, and the scientist is an artist vastly more subtle than the greasy fellow who smears paint upon canvas. His creations are so convincing that he frequently fools himself into believing that he has grasped reality itself instead of an image thereof.

In order to show what I mean by the assertion that geometry dictates the form and content of scientific concepts, I shall become a bit technical. I make the point by discussing a bit critically the simplest of the physical sciences, statics. This branch of mechanics consists of four basic items.

(1) The Concept of Force. In our recurring groups of muscular sensations, we find the thing we call *force* and in it the properties of direction, magnitude, and position. To construct a geometric image of force, we draw a line through the point of application of the force, in the direction of the force, make its length equal to the magnitude of the force to some arbitrary scale of length, and place an arrow on the line to indicate the sense of the force one way or the other. So far as statics is concerned, we assert that such a directed line or vector is a perfect symbol of force.

As a matter of fact, there are many properties of force which this geometric symbol is incapable of depicting. For example, a force never acts at a point but is always distributed over a finite area. There is nothing about the vector to indicate for how long a time the force may be applied, whether it be moving or at rest, whether it be applied with kindly or malicious intent; yet a force which is moving and doing work is quite different than a force which does no work. However, we end this quibbling by defining statics as that branch of mechanics which concerns itself only with those properties of force which are represented by the vector symbol.

(2) The Moment of a Force. The *moment* of a force about a point is represented by and thought about in terms of precisely the same geometric symbol we use to represent force, the vector or directed straight line segment. The moment vector is constructed as follows. Pass a plane through the point in question and the line of action of the force. Through the point erect a line perpendicular to this plane: this line is called the moment axis. Lay off along this moment axis a length equal, on any arbitrary scale, to the product of the force by its moment arm or perpendicular distance from the point to the line of the force. Place an arrow on this moment vector to indicate the sense of the turning moment, clockwise or counterclockwise.

Moment is a derived concept. The fundamental concept, formulated from experience and intelligible only in terms of experience, is force.

(3) The First Law of Statics. If a body be in equilibrium under the action of a number of forces, the vector sum of these forces is

zero; namely, if the vectors be put by parallel translation into a concurrent chain, the end of the last vector in the chain coincides with the origin of the first. This law is a generalization from experience. It is sometimes called the parallelogram law.

(4) The Second Law of Statics. If a body be in equilibrium under the action of a number of forces, the vector sum of the moment vectors constructed through any point in space is zero. This law is a generalization from experiment. It is also called the *principle of moments.*

These four items plus Euclidean geometry constitute the science of statics. These four items are obviously geometrical: they fit the Euclidean frame. There is no problem, theorem, or method in statics, however it may be disguised analytically, which is not geometrical or which may not be made so.

Other classical sciences are likewise geometrical in structure. Most of them are more complicated than statics. For example, the Lorentzian formulation of electricity and magnetism employs six scalar or one-component quantities, eight vector or three-component quantities, and two dyadic or nine-component quantities. These are connected by some fifteen equations in the nature of defining relations or experimental laws. They geometrize electricity and magnetism quite as completely as the four fundamentals of statics geometrize that science.

In fact, we may summarize the structure of classical science quite briefly. The concepts, the things we call causes and effects, of classical science fall into a regular hierarchy of geometric figures called *tensors*. The simplest of these is the tensor of rank zero or the scalar having one component such as mass, temperature, energy, action, entropy. The next is the tensor of rank one or the vector having three components such as force, moment, velocity, acceleration. Then comes the second order tensor or dyadic having nine components such as the inertia dyadic used in rigid dynamics and consisting of the moments and products of inertia of a rigid body. The nine stresses at a point in an elastic body make a dyadic and so do the nine strains at that point. The hierarchy continues with the numbers of components mounting in powers of three. Thus the 81 elastic constants required to express Hooke's linear relation between stresses and strains make a fourth order tensor (although the number of independent components is reduced to about one fourth that number by symmetrical properties of the stress and strain dyadics.) All these conceptual forms are represented by characteristic geometric figures. Thus the first order tensor is represented by a directed line segment. The second order tensor is represented by a second degree surface, such as stress, strain, and momental ellipsoids. This explains what I mean by saying that the form or structure of the concepts of classical science are made to fit into the Euclidean picture frame.

The manner in which the fundamental concepts of a science connect with our experience is the vital part of science. The marvelous accuracy with which our abstractions, our thought symbols, fit upon and describe experience; the exactness, completeness, and artistry with which we frame the basic concepts of a science; this part which is not a matter of reason alone is the part which grips our imagination, satisfies and awakens the creative instinct. And it is in just these things, these roots which lie below the reach of reason, which are grasped only by the intuition or artistic appreciation, that a science has its most intimate contact with reality. And we seek, as Alexis Carrel puts it in his book *Man the Unknown*, in the external world only those things which may be geometrized; and geometry is, in all probability, a thing not of the external world but a thing within ourselves.

The pretensions of the ancient 2-and-1 mythology stand discredited: no one believes any longer in Zeus. The homogeneous 3-dimensional picture called classical physics is now in its turn becoming discredited under the attacks of the quantum and relativity theories. It was, no doubt, difficult for our forebears to abandon their faith in nature-myths and accept the classical picture of nature. The effect of mass faith is not only to inhibit original thinking but to punish quickly such a breach of manners. Socrates and Aristotle both poisoned themselves to escape the consequences of having denied the nature-myths of ancient Greece. It will be difficult for many scientists of the present day to abandon their faith in the Euclidean-Newtonian myths.

Is it necessary to put one's faith in any science? Is it even desirable? Surely the history of science shows it to be a man-made and continually changing scheme for thinking about nature. If one scheme works well, let us praise the artist who conceives the scheme. But it is unsafe and unfair to bind the next generation in the bonds of faith and prevent the construction of a better picture. Why should we not be taught to regard science as a game, leaving out, as the mathematician does, the element of faith.

I shall cite one, among many possible, series of modern scientific nature-myths.

Not many years ago, perhaps not beyond the memory of some of you, electricity was regarded as a sort of "fluidic virtue." For more than two hundred years a controversy raged as to whether there were two electrical fluids or only one. Then came the discovery of the electron by J. J. Thomson. He conceived it to be a tiny sphere of negative electricity. The objections of some of his contemporaries that this conception was impossible because such an electron ought to explode under its own inverse square law of repulsion were gradually silenced by the success of the conception, although the question, as a matter of fact, was never met. Some ten years or so ago the electron, in the hands of Schroedinger, expanded in the most astounding fashion from a mere concentrated speck until each individual electron filled the whole universe. If one may still apply the definite article to such a conception, *the* electron operated through a mysterious function called y the square of whose absolute value was vaguely reminiscent of charge density, but whose principal duty was to remain finite, single-valued, to vanish at infinity, and be quadratically integrable. This view of the electron as a mass of waves filling the whole universe again underwent modification. The newer view, proposed by Max Born and Norbert Weiner, associates y with a probability. Thus, one is free to form practically any picture one may fancy of the electron itself, but one must agree that y times its conjugate complex $\overline{\psi}$ is the local probability that the electron, whatever the thing may be, is somewhere about. In these y spaces, one part keeps in touch with another by means of little messengers whose essence was described by the term *probability packets*. These packets of *perhaps* or *maybe* travelled hither and yon on the business of the atom. Now I submit that there is nothing in the ancient mythologies or even in Saint John's *Revelations* to match these ideas for sheer fantasy. Those conceptions quite surpass that transcendental operation whereby Lewis Carrol detached the grin from the Cheshire Cat. My practical point here in citing this example is that all these conflicting conceptions can not be right, yet some or all of us have believed them. I think we could have played the game just as well without believing in anything more than the convenience of these conceptions.

The most effective means of showing the scientific apprentice that his science is a man-made scheme for thinking about nature is to make him read sympathetically the history of science. He will discover that the causes which man has found or thinks he has found operative in nature are pure romance. He will find that certain facts and mathematical formulae stand out like rocks but that those things he calls causes, those things which for the moment make the picture intelligible, are naught but shifting sands formed by his fancy into all sort of fantastic figures. Thus he will find out for himself that the Ptolemaic theory of the heavens was a beautiful theory and that it worked. He finds that Carnot derived the second law of thermodynamics from the old phlogiston theory of heat, and that this law not only worked then, but continues in good standing. He finds that river-gods pushed water along in the rivers until Isaac Newton invented the gravitational force. He now finds that rivers flow because of the curvature of space-time. There is reason to believe in the efficiency of Einstein's space-time curvature; there was reason to believe in the efficiency of Newton's gravitational force; and, in spite of the fact that a river-god does not fit nicely into an equation, he was, at least, sufficient cause for the water's motion. Why should one believe in the reality of any of these "causes" just because they fit into the picture? Surely the only reasonable requirement of a picture is that it possess realism, likeness to reality.

Before passing on to a discussion of the latest conceptions of space-time, I must point out certain limitations of the time concept as it is used in the classical theories.

The role which time plays in science developed slowly. The recurrence of night and day and the regular procession of the seasons must have impressed themselves upon man very early. But the first effective use of time in the present mechanical (and not astronomical) sense appears to have been made by Galileo who, in 1581, timed with his pulse the swing of a lamp in the Cathedral of Pisa and discovered that its period was independent of its amplitude of swing. Certainly he made effective use of his discovery, for by employing the principle of the pendulum he constructed an accurate clock and by its use discovered the laws of falling bodies. Upon this reasonably firm foundation, the science of mechanics developed rapidly, particularly in the hands of Newton. And with the basic science of mechanics established, the classical picture was rapidly sketched in as to its broad outlines. Incidentally, the homogeneous 3-geometry of Euclid passed over gradually into a 3-and-1 manifold, 3-space and 1-time.

According to Newton, "Absolute, true, and mathematical time flows in itself and in virtue of its nature uniformly and without reference to any external object whatsoever." All motions may be accelerated or retarded, but the flow of time can not be changed. The same measure of persistence and duration applies to all things whether their motions be rapid or slow. If the progress of everything in the universe were exactly reversed, the flow of time would be unaffected. In other words, throughout its vast extent, all events in the universe click with our little whirligigs; they always have marched to the tune of our clocks, are now so marching, and will always do so. The little whirligig is the pulse of and runs the universe. One is reminded of Rostand's chanticleer whose crowing is the Sun's signal every morning to arise.

Is it not possible that this unique, absolute and universal time coextensive throughout the universe may be merely an extension, purely imaginary as to its reality, of our local time, comparable to the extension throughout all space of the local vertical made by the ancients? The answer to this question gives us our next picture of the universe.

Hermann Minkowski, in a notable address entitled *Raum und Zeit* delivered to the Cologne meeting of the Scientific Congress in 1908, sketched in broad outline the new picture frame. He seized upon a peculiar interpretation which Einstein had made of certain experimental results due to Michelson, Fizeau, Trouton and Noble, Rayleigh and Brace, H. A. Wilson, and others, all of which exhibited a certain family failing in refusing to live up to the requirements of classical science. Without going into details of this family failing, we may say briefly that Minkowski advanced cogent reasons for suggesting that a homogeneous 4-geometry of space-time might work better than the 3-and-1 space-time of the classical theory. He suggested that the local classical partition, space-time, had been extended too far. Time is undoubtedly experimentally distinguishable from space locally, just as the vertical is locally distinguishable from the horizontal. But just as the vertical at a point, say, on Mars is not the same as the local vertical, so may the time at that point on Mars be not the same as the local time. I quote from Minkowski's lecture: "In nature all is given; for her the past and future do not exist; she is the eternal present; she has no limits, either of space or of time. Changes are proceeding in individuals and correspond to their displacements upon worldways in a 4-dimensional eternal and limitless manifold. These concepts in the region of philosophical thought will produce a revolution considerably greater than that caused by the displacement of the Earth from the center of the universe by Copernicus."

The essential features underlying both Einstein's special and his general relativity theories was pointed out nearly a century ago, in 1854, by Bernard Riemann in his doctor's thesis, *On the Hypotheses which Lie at the Foundation of Geometry*. Riemann noted that the propositions of Euclid may be divided into two classes: those having to do with numbers, ratios, measurements, and on the other hand those having nothing to do with these matters. All propositions having to do with measurements are called *metrical propositions*, and are, one and all, dependent upon the theorem of Pythagorus to the effect that the sum of the squares of the legs of a right triangle equals the square of its hypotenuse. That part of geometry exclusive of metrical propositions is called *affine geometry*. This part, affine geometry, is common to all geometries, and the various geometries differ only in the metrical propositions. Before proceeding to lay down his various types of geometries, Riemann interchanged the Theorem of Pythagorus and the Parallel Postulate, that is, he elevated the 47th proposition, called the *Bridge of Asses* by the British, to the rank of axiom and relegated the Parallel Postulate to the status of a theorem in Euclidean geometry. The new axiom is called the *metrical axiom*. Hence, according to Riemann, geometries differ only in their metrical axioms. Inasmuch as the metrical axiom is logically independent of, namely, can not be deduced from, the nonmetrical axioms, he pointed out that we might assume any other metrical axiom and obtain a geometry as logically consistent as that of Euclid. The three types of geometries, *elliptic*, *Euclidean*, and *hyperbolic*, are characterized by the assertions that the sum of the squares on the legs of a right triangle are greater than, equal to, and less than the square on the hypotenuse.

We usually characterize Euclidean geometry by the Cartesian quadratic form, $dx^2+dy^2=ds^2$, where dx, dy are the legs and ds the hypotenuse of a right triangle; occasionally by the plane polar quadratic form $ds^2=dr^2+r^2d\theta^2$; and on rare occasions by more complicated coordinates. All these forms characterize the same flat Euclidean geometry, for anyone of the forms may, by a continuous coordinate transformation, be carried over into another. However, the rather simple quadratic form $ds^2=a^2d\theta^2+a^2\sin^2\theta d\psi^2$, which belongs to a sphere of radius a, can not by any continuous transformation be expressed in any of the above forms. This, incidentally, is the reason why a sphere cannot be rolled out upon a plane without altering lengths, angles, and areas. The last quadratic form is the axiom of 2-dimensional spherical geometry. In this geometry, the sum of the interior angles of a right triangle exceeds two right angles; it is impossible to pass through a given point a line parallel to a given line; two straight lines (straight in the precise sense in which *straight line* is defined in Euclid, namely, as being the shortest line connecting two points on the sphere) intersect in two points; together with many other propositions which are false in Euclidean plane geometry. Geometries of this kind are called *curved* or *non-Euclidean* geometries.

I shall discuss briefly the special relativity theory. The fundamental element in Euclidean geometry is, of course, the point. In the new space-time 4-dimensional geometry, the fundamental element is the event. An event occurs when something happens at a particular point (of Cartesian coordinates x, y, z, say) at a particular time, t. Thus the event has four coordinates, x, y, z, t. The interval between two events (x_1, y_1, z_1, t_1) and (x_2, y_2, z_2, t_2) may be written as $dx=x_1-x_2$, $dy=y_1-y_2$, $dz=z_1-z_2$, $dt=t_1-t_2$. The new metrical axiom adopted by Einstein asserts that two different observers (ourselves and our friend on Mars) agree on the quantity

$ds^2=dx^2+dy^2+dz^2-c^2dt^2 = dx'^2+dy'^2+dz'^2-c^2dt'^2$, where c is the velocity of light. Either one of the sides of this equation is called ds^2, and ds is called the space-time interval. The appropriateness of the name is apparent since the quantity is obviously a combination of length and time.

Since the two observers previously agreed both on the space part and the time part of the interval, this generalization seems mild enough. However, it commits plain murder upon the time-honored notions of Euclid, Galileo, and Newton, for it now permits them to disagree on both the time and the space parts and only requires them to agree on the space-time interval. Furthermore, it requires that all the concepts of natural science be reshaped to fit the new picture frame. One or two examples will suffice to show this effect.

Obviously a vector can no longer have only three components; it must have four if it is to be made in imitation of the new four-component space-time interval. If not, it will not fit the new picture frame. Thus, our old friend, the 3-vector momentum (mv_x, mv_y, mv_z), takes on a fourth or time-component, mc, where m is the mass and c is the velocity of light.[2] Two different observers must agree on the quantity

$$(mv_x)^2+(mv_y)^2+(mv_z)^2-(mc)^2=(m'v'_x)^2+(m'v'_y)^2+(m'v'_z)^2-(m'c)^2.$$

In particular, suppose the primed observer to be moving with the mass m': he gets zero for his estimate of the velocity, and measures the mass-momentum vector as simply $m'c$. This particular mass m' measured by one relative to whom the mass is at rest is called the *rest mass*. Solving the above equation for m in terms of m', we have

$$m = m' / \sqrt{1-v^2/c^2}$$

This means that the mass is a function of the velocity: the greater the velocity, the greater the mass as measured by him relative to whom the mass is moving. Similar generalizations are made for other three-vectors. The electric 3-vector *current* takes on a fourth or time component, pc, where p is the charge density. The old 3-vector *force* of classical physics adds a time component, the activity or *power*, namely, the rate of doing work. On the contrary, the classical 3-vectors, *electric and magnetic force*, do not take on classical

[2]To be strictly accurate, this is not a 4-vector. In the mass-momentum 4-vector, m is the mass density

scalars to make 4-vectors; the two are combined into a single concept, a second order tensor whose 16 components are reduced by skew symmetry to six independent components, for which reason the electromagnetic field is called a six-vector, although this terminology is poor.

The theory of relativity is, properly speaking, not simply a theory; it is a program for the generalization of classical science. The new forms are not to be deduced from the classical concepts; they are to be made over, reshaped. The new concepts are analogous, if you please, to the classical concepts, but this is quite a different matter than supposing them to be logically dependent upon classical forms. The new science is a separate structure, being erected not upon the foundations of classical science, but as an entirely new structure. Just as the causes and effects of those mythological animals of classical physics were dragged from the land of imagination over the Pythagorean Bridge of Asses, so are the newer, more multi-headed myths to be brought into science over the Einsteinian Bridge, his newer metrical axiom.

But perhaps this is enough of the special relativity theory to convey what is relevant here. And besides, the theory has already been found to be inadequate. Whilst it works better than classical science, it offers no explanation of gravitation one whit more rational than Newton's.

And now comes Einstein again. Having pondered for a number of years after stating his special theory, it occurred to him that he had been, in effect, doing exactly as the classicists had done. In accepting his special relativity metric, he had fixed the frame of his picture, just as the Pythagorean metric had fixed the classical picture frame. Possibly phenomena refused to fit into this rigid frame because of its form. The special relativity frame was better than the Euclidean, but why take a fixed frame? This brings us to the next attempt to find a suitable picture frame.

In his general relativity theory, Einstein still clings to a homogeneous 4-geometry. But he assumes an elastic, not a fixed, frame. He takes for his new metrical axiom the most general conceivable quadratic form, $ds^2 = \Sigma g_{ab} dx_a dx_b$, where the 16 (10 independent) coefficients g_{ab} may be functions of the four coordinates, and where he does not say to begin with which one of the four coordinates is the time, leaving the question open, in fact, so that the time may be found to be a function of all four of the coordinates. Leaving the g functions undetermined, he took over Riemann's general propositions and let physical phenomena themselves decide upon the form of the space-time frame. That is, he took phenomena as it appears from careful measurement and attempted to fit the picture frame to the picture. The analytical details of this picture frame construction are out of place here. Suffice it to say that Einstein did succeed in masterly fashion in geometrizing the gravitational phenomena.

Unfortunately, this is not yet the end of the matter. The present metric of relativity does not permit of the simultaneous geometrization of both mechanics and electromagnetism, which also exhibits "forces" acting through empty space. The theory which shall accomplish this miracle is already named the *Unified Field Theory*, but it is not yet formulated. There have been several attempts, notably by Einstein, Weyl, and Eddington, to construct a unified field theory; but upon comparison of the facts of measurement with the predictions of these theories, none have stood the test.

The most valuable contribution of the relativity program appears then to be one of method. Instead of accepting a geometry fixed in form and seeking to force phenomena to fit it by the invention of divers fantastic "causes," we set ourselves the task now of seeking that frame for our picture which best fits upon nature, letting nature herself be judge of the excellence of the fit.

There is now coming over the horizon a new possibility which may have an important effect upon our notions of space and time. In Dirac's one of the various formulations of the quantum mechanics, a new type of concept appears called the *spinor*. The simplest spinor is a two-component entity, a sort of square root of a special relativity 4-vector. The components of a spinor do not fit nicely into our picture frame. We can not lay off one component along x, one along y, one along z, and one along t. Yet it is possible to show by the theory of groups that any special relativity equation between tensors may be translated into an equivalent equation between spinors. What does this do to all the imagery, the mythology, the causes and effects, behind our equations? Tensors have certain intuitive properties; they fit into the frame. But spinors are so queerly shaped—they even involve the hitherto little used imaginary numbers—that so far only certain combinations of them can be made to fit the picture frame. It is an interesting speculation (I offer it as nothing more) that these new "causes" and "effects," namely, the spinors, may later on produce rather violent changes in our notions of space and time, particularly if some mathematician succeeds in formulating some similar concept covariant to general relativity transformations.

Thus I say that Geometry, the first and noblest of sciences, has played in the past and will probably continue to play the dominant role in science. It certainly dictates the form and to some extent the content of all scientific concepts. It is the stage upon which those abstract actors, the causes and effects, play their scientific roles. It is the picture frame wherein that greatest of all artists, the creative scientist, paints his pictures.

So far the natural philosopher has painted in his space-time frame only the primeval 2-and-1 picture, the Euclidean 3-frame picture, the Galilean-Newtonian 3-and-1 picture, and the several Riemannian-Einsteinian 4-dimensional pictures. But there are possible many such frames, and but these few are without form and void, and darkness lies over their depths; but the creative spirit of man will move over the face of their waters and people them with his creations.

Now we try to make pupils realise that from studying real phenomena one can get data which provide a quantification of the behaviour of a phenomenon at the time studied. When there are sufficient data (an important condition both mathematically and scientifically), we can select from it and propose a model. This must be satisfied 'closely' by the data; it does not have to explain the interactions within the phenomenon, and it may be useful for finding extrapolated results.

P. G. Dean, 1975

Models and modeling are part of the fabric of science; instruction in science and mathematics as well as the curriculum should so reflect them.

D. H. Ost, 1987

VI. Science and Mathematics in a Technological Age

We are told that scientific and technological knowledge currently increases by 13 percent per year, thus doubling every 5.5 years, and that the rate is expected soon to jump to 40 percent per year--a doubling time of 20 months. Clearly an education built on the transmission of information is impossible. What is urgently needed is an education focused on the *use* of knowledge in new and yet unanticipated situations, and use, not by a select few, but by everybody.

P. A. House, 1988

18

A View of Computing, Mathematics and Science in British Education

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Introduction

Modelling and simulation could be a link between science, mathematics, and computing courses for the more able British secondary pupil. The teacher is prepared to include new methods if he thinks that they will help him to teach his subject. We must encourage the development and use of computer programs which can be integrated into a teaching scheme.

The Development of Computing for Education

Only since the early 1960's have computers been available *in education*, and then you needed experience and understanding of the machine before you could get information in or out. In the few British schools which had access to a computer it was normally used by the mathematics teachers. They taught a programming language to some of the mathematics pupils and then encouraged them to write useful programs. Some of these useful programs were a stimulus in the next stage of development, for the teacher kept them as 'library programs' and provided copies of them for other schools. They now began to be valued as a teaching resource rather than as a programming exercise, and we saw the beginning of computing *for education*. Even at the end of the decade (1969) the position was well described by Bryan's introductory remark[1], "It is difficult to report on 'Computers and Education,' because the topic is so new and what it includes is so fragmentary that it almost defies concise description."

It is gratifying to see how the situation has developed just four years later. There are now many schools with telephone access *(on-line mode)* or courier access *(batch mode)* to a computer whose control is within the capabilities of most teachers and secondary-level pupils. Education authorities appoint advisers in computing, who organise courses for teachers and who encourage working and interest groups. There are thus many mathematics teachers, but far fewer science teachers, who are on the 'inside' of computing for education. They develop various programs and texts and make use of similar materials obtained from educational or commercial sources.

Our big problem is to prove to teachers on the 'outside' of computing for education that it may contain something which will help them to teach better.

The Teacher and Interdisciplinary Studies

"Changing the curriculum is a matter first of changing people and attitudes."[2] Most mathematics and science teachers have been trained in one subject, have evolved satisfactory ways of teaching this, and are not dissatisfied with the work they are doing. This should be considered as the base from which interdisciplinary studies must grow if we wish to involve many teachers.

A realistic expectation is that we will retain the structure of a school system which recognises subject departments and teachers, and that by crossfertilisation each subject will in part relate, and be seen to relate, to other subjects. It is interesting to look for a moment at actual developments within the School Mathematics Project[3] which use computing. Their library programs include *FOXRAB* (foxes and rabbits; growth and decay of two populations), *LIFE* (cellular growth and decay), and *LUNAR* (a lunar module to the moon) which all encourage explorations in the field of science. However these are presumably being used by teachers and pupils of mathematics in the expectation of giving "new insights into mathe-

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[1]Bryan, G. L. "Computers and Education." *Computers and Automation*, 18.3 (March 1969), 16-19.

[2]Rogers, G. "Curriculum Reform and Teacher Reform in the Secondary School." *London Educational Review*, 1.2 (Summer 1972), 39-44.

[3]*SMP*. Westfield College, Kiddepore Avenue, London W3.

matics" and a "better understanding of mathematics."[4] Here we see the genesis of an 'infiltration method,' where the teachers can use computing, biology, and physics alongside a syllabus with which they feel secure. This method of crossfertilisation has begun naturally and may be the one most likely to succeed.

In Britain, in-service courses in Modern Mathematics or Nuffield Science have attracted large numbers of teachers of the relevant discipline, but teachers have been reluctant to enroll for courses such as *Computing for Science Teachers*, let alone for *Links Between Science and Mathematics*. Thus it seems sensible to meet teachers in their subject groups. The other week I, a mathematician, led one session of a course for chemistry teachers and was amazed at their interest and enthusiasm when they saw a direct relevance with their subject. It is expected that all teachers will become entitled to regular release for in-service training within a few years. Perhaps we should plan to use this 'infiltration method' much more widely in the subject courses or other training schemes which must become available.

There is also the possibility of teachers developing an interdisciplinary approach within a school, using the science pupils who study computing. When writing or using programs, these pupils may need advice from two or more teachers. In return the pupils can give support to the mathematics teacher who lacks confidence with his science, or vice versa. Teachers are often so busy with their own subjects that they need this interest of a pupil to show the relevance of some aspect of another subject.

The Development of Mathematics and Science Teaching

"The mathematics and science reformers have been working with very similar philosophies. Both parties have been concerned with (1) the search for big ideas, and (2) the search for the ways in which children learn to handle them and apply them to the solution of problems."[5] The interesting interdisciplinary development has been the application to a sequence of different curriculums. The first British reformers were dissatisfied (a very necessary catalyst) and therefore set out to discover a better way of teaching their subject. Typical outcomes were the *School Mathematics Project*[3] and

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[4]Webb, N. G. G. "SMP Computing in Mathematics." SMP duplicated sheet, (May 1973).

[5]Matthews, G., and Seed, M. "The Co-existence in Schools of Mathematics and Science." *International Journal of Mathematical Education in Science and Technology*, 1 (1970), 21-26.

Nuffield Biology.[6] Only after intensive work within their subjects did the scientists begin interdisciplinary approaches such as the *Schools Council Integrated Science Project*.[6] Similarly the mathematicians began to integrate computing into their curriculum.

There is now evidence of further developments which may encourage co-operation between science and mathematics teachers, and two examples will suffice: (1) "Modern views in chemistry teaching--certainly in schools--stand on two main premises: the method of pupil investigation, and the empirical approach to generalisation . . . In both these processes, mathematics is increasingly involved-."[7] and (2) "They (mathematics teachers) knew all the old stuff about momentum and moments of inertia but . . . the real trouble was with some of the 'new-look' examination questions. These tended to be unorthodox, omitting to specify the assumptions to be made."[8] As with others I have experienced, these two are examples which I would classify as infiltration towards interdisciplinary studies between mathematics, computing and science. Maybe infiltration is all that we should expect in the majority of schools during the 1970's.

Modelling in Mathematics and Science

". . . I would assert that the meaning of the concept of *model* is the same in mathematics and the empirical sciences."[9] "There is nothing wrong with Newtonian mechanics as a description of the physical world as most of us see it. The fault lies in the simplified models which we have to construct in order to make particular problems manageable."[10] Many teachers have realised that they no longer need to use very simple models to make particular problems manageable when they have a computer to do the calculations. This is certainly one way to make the computer serve the teacher. But, joy oh joy, this release of a restriction so far borne can now lead us

[6]Centre for Science Education, Chelsea College, Bridges Place, London SW6.

[7]"Mathematics in Chemistry." Paper written by a working party for the British Committee on Chemistry Education, Duplicated sheets, (1970).

[8]Brown, M. "'Real' Problems for Mathematics Teacher." *International Journal of Mathematical Education in Science and Technology*, 3 (1972), 223-226.

[9]Suppes, P. "A Comparison of the Meaning and Uses of Models in Mathematics and the Empirical Sciences." In *The Concept and Role of the Model in Mathematics and the Natural Sciences*, International Union of History and Philosophy of Sciences: D. Reidel, Dordrecht, Holland. (1961), 165.

[10]Flisher, F. W. "Main mathematics in Colleges of Education." *The Mathematical Gazette*, LVII, 402 (December 1973), 321-333.

on to using models in different, and perhaps more enlightening, ways.

A teacher who plans to work to a yearly syllabus cannot aim to use a free discovery method, and he therefore provides guided discovery. The same restriction applies when using computer modelling, for it is unrealistic to expect a pupil to search almost at random for his own model. It would also be difficult, if not impossible, to program the computer to accept any model. The three likely alternatives are: (1) to provide one fixed model, and allow the pupil to investigate different variables within it; (2) to provide several models, and allow the pupil to test each and decide which is the 'best;' and (3) to provide a generalised model which the pupil can particularise. Which alternative we choose will depend on the phenomenon being studied and on our educational aim.

Probably one aim is to understand the place of mathematical relationships in everyday science. When I was a pupil at school, applied mathematics, physics, and chemistry appeared to me to include many laws which stated how physical objects should behave; my calculations gave the correct answer and the real objects did not--because of natural disturbances. Now we try to make pupils realise that from studying real phenomena one can get data which provide a quantification of the behaviour of a phenomenon at the time studied. When there are sufficient data (an important condition both mathematically and scientifically), we can select from them and propose a model. This must be satisfied 'closely' by the data; it does not have to explain the interactions within the phenomenon, and it may be useful for finding extrapolated results. As illustration I take the topic of chemical reaction kinetics, on which we have developed and used a computer program and supporting texts. When used with a class, only one pupil said he had not gained further understanding of the need for the mathematical model

$$[\text{Concentration}] = [\text{Initial concentration}]\ e^{-kt}$$

which is part of the computer program.

The chemistry pupils had taken readings for at least one reaction in a laboratory experiment, had plotted the readings, and had calculated the first-order rate constant. They were asked to supply their values for the rate constant and initial concentration to the computer model above. The computer calculated the concentrations at each time interval, and the pupil was asked to decide whether it gave results 'close' to his laboratory readings. If he was satisfied with this model he could go on to use it to investigate the calculated results for other reactions, where the data were unknown to him but were provided from a table of experimental results. These included reactions which were too fast, or too slow, for study in the school laboratory.

When used as described, the computer program is designed to help chemistry pupils to learn about reaction kinetics. The study of a chemical reaction put into a different teaching sequence could provide an illustration of the exponential function. This is a 'real' application of a function studied by many mathematics pupils. They are familiar with Figure 1 and can be led to see it as a model for Figure 2. Interesting discussion can come from the absence of points in the second quadrant of Figure 2, and as to whether both curves are asymptotic to some horizontal line. With several 'real' illustrations from science the young mathematician should also get a better understanding of the relation *dy/dx ay*.

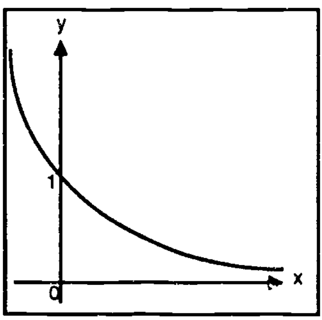

Figure 1. $y = e^{-ex}$

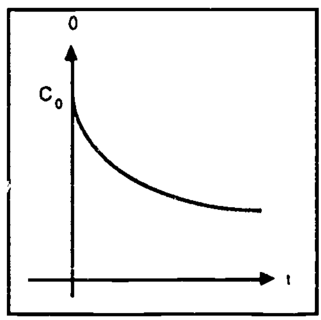

Figure 2. $C = C_0e^{-kt}$

A second aim may be to emphasise the strategy for solving a problem with more than one variable. The method of controlled change in one variable while trying to keep the others fixed is as applicable in mathematics (e.g., combinations of reflections, rotations and translations) as in science. In biology, where the interrelationships are some of the most difficult to express, it is possible to utilise models published in books[11,12] or in journals, where computer modelling has been receiving increasing attention over the last decade. Some programs are available in Britain, and phenomena

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[11] Smith, J. Maynard. *Mathematical Ideas in Biology.* Cambridge University Press, England, (1964).

[12] Slobodkin, L. B. *Growth and Regulation of Animal Populations.* Holt Rinehart and Winston, New York, (1961).

studied include the simple food chain,[13] the behavior of chromosomes at crossover,[14] and the co-existence of two animal species.[15] The teaching scheme for this last computer model illustrates the method of controlled change in one variable.

After studying populations, e.g., in a jam jar, the program is used in on-line mode and the biology pupil is first asked to choose several values for the number of offspring per generation and then for the generation time. From the calculated results he can see the corresponding changes in growth pattern for a typical population of a single species. He now has a background from which he can discuss two of the scientific aspects of control of population in a social environment. Returning to the computer program, he can investigate the model to find the predicted effect of starting with different numbers of animals. There is then a sequence of investigations of the interactions between two species. It is expected that the teacher will relate the results to recorded examples of co-existence or extinction, and that the pupil will be able to see that a realistic study of a habitat can be made by synthesis.

As well as the strategy for problem solving, there is a lot of explicit mathematics in this model which could be brought out by a different teaching scheme. Four which might be used with mathematics pupils are: (1) linear or non-linear relations between variables; (2) convergence of a series, and its dependence on terms in the series; (3) difference and differential equations; and (4) perturbations (after co-existence one species is depleted by human interference, what then happens?).

Pupils are often prepared to accept that the laws of nature contain integral powers of the variables. The computer allows them to decide whether the integer is there to make calculations simple or to provide the most accurate model. For example, in a program containing the generalised model $F \alpha \frac{1}{r^n}$, the pupil can be allowed to choose $n = 2$ (the familiar inverse square law) or other values. Bennett et al.,[16] consider this as a model for Geiger and Marsden's

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[13]Regional Centre for Computer Education, Dundee College of Education, Park Place, Dundee, Scotland.

[14]Advisory Unit for Computer Based Education, 19 St. Albans Road, Hatfield, Hertfordshire.

[15]*Computers in the Curriculum.* Schools Council Project, Chelsea College, Bridges Place, London SW6.

[16]Bennett, J. W.; Bignall, B. G.; and Bradley, R. A. "Some Uses for a Digital Computer in an A Level Physics Course." *Physics Education*, 7.3 (May 1972), 155-157.

alpha particle scattering experiment. They show that small variations in n produce significantly different results.

One change brought in with new syllabuses is the increased awareness and acceptance of different ways of solving problems. The teacher or pupil who writes computer programs can hardly fail to realise that there are alternative strategies, for his companions will have said "Why not do it this way?" The crunch comes when one or more programs have been written (i.e., models have been constructed), for we cannot find out whether any provide a solution to the problem until we clarify what we mean by a solution. I suggest there are three classes of problem: (1) If the problem has an analytical solution, as for example with certain integrals, then we may define a solution as 'for every example tested, it has provided the numerical answer which is equal to the analytical answer when the latter is rounded to six significant figures;' (2) if the problem is based on experimental observations, some statistical analysis (see next section) is required within the definition of a solution, but it may be valuable to compare parts of two models. Bennett et al.,[16] provide thought for both mathematicians and physicists by comparing

$$tan^{-1}\left(\frac{i}{\mu}\right) \text{and } sin^{-1}\left(\frac{sin\ i}{\mu}\right)$$

as alternative models to predict the angle of refraction; and (3) if the problem can only produce a prediction, possibly using a stochastic model, the decision as to whether it is a solution may be as uncertain as with the Club of Rome/MIT predicament of mankind.

Experimental Results, Mathematics and Computing

"If in a mathematics lesson the pupil is dealing, for example, with the function $x \rightarrow 1/x$, he will be dealing with this precisely, with no 'noise' in the system. . . . In science, by contrast, the results of school experiments are noisy, sometimes even deafening; the data contain errors, which make the inference of relations between variables much harder."[17] The gap exemplified by this quotation is being narrowed by developments in each discipline, mainly

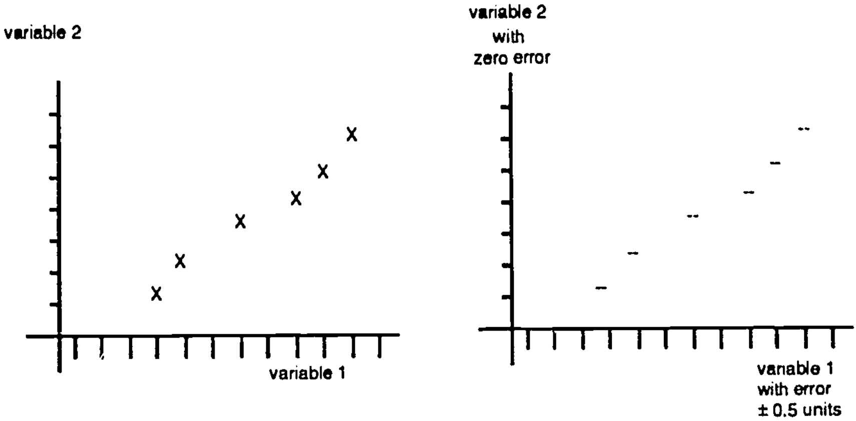
[17] Malpas, A. J. "Mathematical Interpretation of Experimental Data in Schools." *International Journal of Mathematical Education in Science and Technology*, 4 (1973), 263-270.

independently of the others. Teachers need to come close enough to communicate, and maybe share a computer.

The now-common slide rule, with its logarithmic scale, makes it easier to use logarithmic graph paper for analysing the results of experiments. Let mathematics and science pupils plot familiar functions using different graph papers or scales. When they try to draw smooth, continuous curves they will appreciate the apparent distortions. Then provide them with sets of results of experiments where the relations are unknown and guide them in finding the relations. In mathematics classes they may have seen the advantage of combining all their sets of data, and if this was done with the results of a science experiment the pupils might find it easier to "draw a straight line through" the non-linear points. The statistics taught to school pupils has often omitted explicit mention of fitting a curve to a set of points by the least-squares method, but it is a standard computer library program and the principle can easily be understood. The science pupil may have one or two rogue points within his results; do they significantly alter the curve or line calculated by the computer?

In the language of numerical methods, the rogue points mentioned above are 'mistakes' which the pupil may be able to identify and subsequently correct by repeating part of the experiment. Numerical methods also quantify 'errors,' and here mathematics and science are drawn closer together. In an experiment data may have zero error, such as the number of objects (i.e., an integer); data may have a known non-zero maximum error (e.g., 0.5 units); or data may have an estimated maximum error. This last is obtained by combining the errors of the dependent variables. If a mathematician plotted the results making allowance for the errors he might get Figure 4 instead of Figure 3. Instead of drawing a curve to pass near the points in Figure 3, he would draw a curve to intersect the line segments in Figure 4 (or the areas if variable 2 also had non-zero error).

A difficulty with sampling methods on a biology field course is that time spent on the mathematics might otherwise be spent on biology, and there is one London school where data from a field course were analysed using a computer. Another scheme in which I have taken part is for mathematicians and scientists to combine for a field course, each contributing to, and learning from, the rest of the group, but it can be difficult to plan this to meet the needs of each discipline.



Conclusion

The work done in Britain has highlighted the following difficulties in making computer library programs into a resource for mathematics and science teachers:

1. Programs and accompanying texts: These must be devised, produced, tested and distributed. It is too big a job for the average teacher, but national developments have now begun.

2. The teaching scheme: There is no accepted way of integrating computing to benefit one or several subjects.

3. Teacher training: Is the 'infiltration method' going to be successful and is there a better method?

4. Machine compatibility: The design and writing of programs should enable them to be transferred easily to other educational computers.[18]

[18]Lewis, R., and Dean, P. G. "The Design and Writing of Programs." Project Paper 4, *Computers in the Curriculum,* Chelsea College, Bridges Place, London SW6 (November 1973).

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Models, Modeling and the Teaching of Science and Mathematics

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One of the most prominent features of modern science is the model. Modern developments in mathematics and in computer science have made available new analytical models which have resulted in a revolution of modeling as part of the scientific method.

A successful model assumes attributes which last far beyond the problem-definition stage or its use in specific research. The model may become the dominating paradigm in the field, not only in the sense of directing scientific inquiry but in the way the student and the public think about the issue. The Watson-Crick model of DNA has been the research paradigm for molecular biology for nearly three decades. And the students of biology during the same period have come to think of DNA as stick-balls and paper cut-outs of nucleotide bases.

Because of the nature of models, the teacher of science or mathematics must be alert, since students are easily seduced. Students need constant reinforcement of the fact that it is reality, the phenomenon, which is the focus of study. The model is the tool and modeling a process. It is important in science and mathematics teaching to clearly differentiate between a given model and the reality it depicts.

There are four basic kinds of models found in contemporary science which have direct applications in the teaching of science and mathematics. Some are more useful than others.

Representative Models

Representative models, as the name implies, are partial facsimiles of nature. Examples range from plastic representations of cells and tissues to stick-balls to assemble molecules to simulation exercises using computers. They can be extremely valuable for communicating descriptive information as well as for constructing hypotheses.

The most common feature of representative models is that they are done to scale or include some aspect of scaling. Maps, whether mathematical projections or not, are so closely allied to experience and to photographs that this type of model is frequently underestimated. The fact that the map *represents* reality is readily apparent to anyone who can "read" maps. In a sense the map model has become a sophisticated language all its own. Witness, if you will, satellite weather tracking or the analysis of features on a distant planet.

Scale models are generally constructed on reduced and/or proportional scale. A scale model of a bridge might be built to obtain numerous kinds of data such as stress, tension, approach roads, etc. Precisely developed, the representative scale model is a useful tool for solving real problems without the enormous expense and investment of trial and error. Natural processes which may not be fully understood can be somewhat duplicated or replicated through models.

Just as a wind tunnel can be used to model a host of environmental conditions on a scale smaller than nature, so too can scale models be used for teaching. Models of river channels, air or fluid movements, and other natural conditions can be used to determine the effects of modifications and human interventions such as dams, changes in humidity, pollution, etc. Variables can be manipulated and tests which produce useful data can be run. Unfortunately, in teaching this type of model is used primarily to *demonstrate how* (e.g. how sedimentation accumulates or how nucleotide bases pair in DNA) or to *communicate* ideas (e.g.,. proportionality, construction of time lines, distance in space, or the special relationship of the particles of the atom.) It would seem that representative models are not being used to their full potential.

Analogue Models

Analogue models compare two things which are in some sense similar and is some sense different. The flow of energy in an ecosystem is sometimes modeled as a hydraulic flow system. The two systems have various attributes in common. Through careful study the researcher or the student may realize that ecosystems have a property in common with the hydraulic system which was never anticipated. In this way, the analogue model can lead students as well as scientists to a *discovery.*

Note that this use of the model demands little of the model itself. In this sense, the analogue model has little experimental value. For instance, the chemist Kekule is reported to have discovered the nature of the benzene ring from dreaming that a snake swallowed its tail. Clearly no herpetologist would accept the truth of the tail-swallowing notion, and no one would accept this type of model as evidence for fact. Nevertheless, it was an analogue model which provided Kekule the direction to search for his discovery.

Although some data may be gleaned from using the analogue model, the primary purpose of the analogue is to identify and relate variables in the beginning stages of problem formulation. Perhaps one of the best classic examples of this was the work of Charles Darwin. In his support of the theory of natural selection, he brought forth artificial selection. His analogue model went something like this: in the protected environment of the farm, breeders select adult forms to reproduce; in a natural environment, the selection of which adult forms reproduce is determined by nature.

Darwin used the analogue model as a *justification.* The two systems were different but similar enough to form the basis of a logical premise. His contemporary critics attacked this premise on the basis that the analogy was inappropriate. This argument is still voiced today, illustrating the limitation of the analogue model in supporting scientific concepts.

Logical Models

In a sense, the logical model describes reality as it "ought" to be, *not* as it "is." In teaching, the logical model may be used to explore the interrelationships among science, technology and society (S-T-S). Discussions of the problem of acid rain require consideration of economics and political science. *Ought* issues emerge just as they might in exploration of knowledge of human genetics and abortion decisions. The *ought to* problem is easily seen in computer simulations of the Three Mile Island nuclear power situation or in energy consumption in general. It is in this way that the teacher can relate technology or the advance of scientific knowledge to cultural ethics and belief systems.

The focus of the logical model is on particular sets of ideas which have been generated to satisfy pre-established axioms and theorems. There is no way scientific data can be gathered or hypotheses tested by using these models since the situation being modeled is based on philosophical positions or on hypothetical situations.

Theoretical Models

The most useful types of models in science and mathematics are mental constructions invented to account for observed phenomena. Although theoretical models are analogical in origin (e.g.,. simulation), they are formal in their use. They have a correspondence with reality. The implicit or explicit attributes of the model are closely tied to ideas, facts, or theories which are well documented or familiar.

In theoretical models the physical systems are represented mathematically. In the classic model of gas, interrelationships of mass, velocity, energy, etc., are illustrated by equations. Although none of these attributes is observable, calculations originally derived from an analogue model (the billiard-balls model) provide evidence that the primary system has the suspected properties. The model can be the basis for expanding theory or for developing prediction. In the case of the analogue billiard-balls model, it was extended to develop the van der Waal's equation.

A theoretical model may be the first step in the development of a representative model. In teaching, for example, mathematical formulas or theoretical models of electrical resistance can be readily transformed into physical (representative) models of circuits. If students trace the development of a model, insight will not only be gained into the concept but into the ways knowledge is generated.

Theoretical models are rich sources of information and ideas which have allowed science to go far beyond the simple cause and effect mechanisms implicit in representative models. The concepts of *feed-back* and *adaptation*, when worked into a model, allow the modeling of change, particularly change requiring large amounts of time.

Models and Methodology in Science and Mathematics

There is an increasing number of modeling skills which are not directly tied to science but which could be incorporated as part of the methodology of scientific inquiry. The fields of statistics and computer science are rapidly developing new tools. The skills, tools, and models which result could provide new insight into sampling schemes, classificatory devices, multivariate analysis, and the design of factorial experiments.

Although it is possible that the majority of these new methodologies may remain outside of science or mathematics as technique, they could just as well become integrated components of the inquiry structure. Generations ago a lens grinder provided biology and astronomy with major tools of research. Microscopy has become part of biology and the health sciences, the telescope part of astronomy.

The formulation of hypotheses, the design of investigations and analyses are each determined by the available methodologies. As biology went through a transition with the advent of the microscope, the student of biology developed new skills and techniques. These skills and techniques are now part of biology. We can expect models and modeling to impact on science and mathematics in a similar manner. The skills, corollary to modeling, will become part of curriculum and instruction and enrich the student's education. Lack of skills in the new modeling methodologies will relegate students to limited understanding of modern science and technology and thus limit career opportunities.

Implications for Science and Mathematics Education

Models are not new to the classroom. Plastic representative models of planetary systems, cells, reproductive systems, etc., are commonplace in the life and earth science classrooms. The physical sciences have long used analogue and theoretical models to *explain*. But even in these sciences the full potential of models has not been developed in the classroom. The models are used as instructional strategies. *Models and modeling are part of the fabric of science;* instruction in science and mathematics as well as the curriculum should so reflect them.

Theoretical models are ideal for integrating science and mathematics. A mathematical model of population growth can be readily transformed into a model of population dynamics. Populations and social systems can be modeled as equations. The variables which affect population growth or gene changes can be illustrated mathematically. The effects of emigration, immigration, death and birth can be quantified. Models and modeling can be used to develop skills of explanation, interpretation, prediction and analysis. In general, theoretical models serve all the function of theory. They are a constant source of plausible hypotheses and, as such, are basic tools in inquiry as well as the teaching of science and mathematics.

Theoretical models and modeling are inexpensive to use in the classroom or laboratory. Perhaps one of the greatest values of the computer is its availability as a sophisticated tool to work with data and to develop important skills of modern scientific inquiry. The relationships among variables can be readily observed. Modeling is an effective method to provide students with experiences in hypothesis formulation and the design of investigations. Of course, the analysis of the data can be readily accomplished through the use of the computer if the study is properly designed. However, a major barrier is the lack of needed quantitative skills on the part of the teacher as well as the student.

Modeling is an ideal way to introduce decision-making as it is used in S-T-S issues. An introduction to risk or uncertainty calculations drawn from probabilistic models is a natural part of decision-making. Further, computer-based mathematical models are the main theoretical model used for prediction. Students must learn that the predictive value as well as decision-making is a direct function of the quality of the data put into the model. Human values are *not* easily introduced into any prediction equation.

As students develop the appropriate skills and expertise in modeling, the importance of sensitivity analysis to validate models will become clear. Individuals trained in the use of models in science will understand their limitations. Individuals who receive no such training will be overwhelmed by models in whatever field (science, economics, management, etc.) they enter.

In addition to developing important skills for use in science, the student trained in modeling will have gained an important *general* education. Society is asking the student to develop *technological literacy*. Understanding models, knowing how to model, and recognizing the use of the model in society are certainly part of such literacy. Furthermore, models and modeling can be effective and efficient ways to relate science and technology to society.

Some Thoughts on Drawbacks

Since the days of Faraday, the warning that there are problems and dangers associated with models is recurrent in the scientific literature. This warning holds, too, for the teaching of science. In general, models should be taken seriously but not literally. It must be remembered that models are partial and inadequate ways of representing reality. Unless constantly reminded, students will lose sight of the phenomena being modeled and focus solely on the model. For some, the model becomes an intellectual barrier, a limiting factor in their learning. An oversimplified representative model cannot accommodate sophisticated information. Students must then unlearn ideas tied to simple models before they can progress.

The result is that many individuals still picture the gene/chromosome relationship as snap-together beads. No matter how hard they try some students continue to think of an atom as a miniature planetary system. The mental construction developed from the model. The students did not *synthesize* the information; they *learned* the model.

Many times it is the teacher who forgets that the model is a mental construction and not a picture of reality and neglects to point out there is no direct correspondence of the model with reality. Textbooks neglect to illustrate that a model is neither true nor false but only more or less useful to picture whatever is being studied. The power of the model is in the *conceptual* simplification. As a tool of science it can foster the creative process. If modeling is considered as only a methodology, it can fossilize the intellect.

As with theory, the model evolves to be consistent with newly-found phenomena. Personal commitment to specific models can become intense, and literalism subsequently creeps in. As teachers develop expertise in an area of science, they are subjected to a subversive indoctrination in the models of the time. Although the model continues to evolve in the science, the teacher's knowledge of it very likely will not. The teacher may unconsciously select materials that are in harmony with a given model or magnify particular phenomena to highlight conformity of the research with a specific paradigm. Opposing scientific models, even though they may not accommodate all data, must be considered if science and the student of science are to maintain objectivity.

Furthermore, the teacher must be aware when reading popularized literature that the author may be stressing data which support a *belief* in a particular model. When a new model is brought forth to explain data or phenomena, many researchers focus their work to gather evidence in *support* of the model. The importance of falsifying hypotheses (*a la* K. Popper) is frequently overlooked in the eagerness to develop support for the model. At times it is even forgotten that it is the deductions from the theory, to which models lead, which must be tested and retested, not the model itself. If this is true of professionals, it can be assumed that the student of science is equally seduced.

Summary

Modeling is one of the most basic tools in the emerging information society. Modern decision-making, at all levels, tends to be based on modeling. The better the model, the better the decision. The future citizen, whether a scientist or an office worker, should understand not only the strengths and mechanisms of models but the limitations and pitfalls as well. The modeling process may be the best way to develop this understanding.

The increasing use of the computer in all facets of society will result in greater dependence on models. The student who has worked with models, has developed modeling skills (particularly computer related), and understands their use as well as their limitations will have an edge.

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Components of Success in Mathematics and Science[1]

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It is indeed fitting that we should be gathered together at the beginning of this school year to address the questions facing mathematics and science teachers in the late twentieth century. It is particularly fitting this year, 1987, because we are at a special vantage point for looking both back at where we have been and forward to where we must go.

Only a short time ago we observed the 200th anniversary of the signing of the United States Constitution. We were given a renewed sense of history that reminded us of the age and heritage of our nation. Yet at the same time that organizers in Philadelphia and Washington were making last-minute preparations for parades and fireworks to celebrate the constitution, mothers in Billings and Helena and Great Falls and throughout the country were unceremoniously sending eager, or perhaps apprehensive, young children off to their first day of kindergarten. The connection between these two events may seem remote, but those kindergarteners represent the high school graduation class of the year 2000. It is these children, already in our classrooms, who will lead us not only into a new century but into a new millennium and toward the tricentennial. It is for their sake that we consider the question of how to prepare them to be effective citizens of a technological world about which most of us can only dream today.

It has long been the case, whether we like it or not, that the best metaphor for education has been the pendulum. If you close your

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[1]Keynote address to the School Science and Mathematics Association and the Montana Council of Teachers of Mathematics, Billings, MT, October 1987.

eyes and visualize a pendulum in motion, you see that it swings back and forth between two extremes. It moves fastest through the middle part of its path, slowing as it approaches one end where it must momentarily come to a stop before gaining momentum in the opposite direction. As it oscillates in one direction, forces are constantly at work exerting pressure to change the direction and speed of the swing. From time to time an outside event will occur that suddenly accelerates the swing in a new arc or a new direction, sometimes with the result that the motion runs wild and uncontrolled to an extreme.

On October 4, 1987, we observed the thirtieth anniversary of such an event: the launching of the Russian *Sputnik*. That occurrence shattered our national ego and created an outbreak of doubt and criticism of our technological strength. It was further accompanied by widespread public awareness of, and concern for, the educational process, especially in science and mathematics as these related to our competitive position in world affairs. It also was followed by a major infusion of public and private dollars in support of the reform efforts.

What resulted then were the movements we came to call "new math" and "new science." That was a case where educators, concerned with the perceived lack of ability of large numbers of students and graduates, attempted to find a solution through curriculum reform. The movement took life in the "alphabet soup" curricula of the 1950's and 60's which gave us *SMSG* and *PSSC* and *BSCS* and *UICSM* and a litany of other projects.

For a little over a decade, those activities flourished. Curricula were written; new textbooks were purchased; and large numbers of teachers spent their summers at NSF institutes on university campuses across the country. Americans landed successfully on the moon, and the country seemed to heave a collective sigh of relief that the race had been won and we could relax.

At the same time, national attention was being diverted away from the Soviets and the space race and toward two new targets: the war in Viet Nam and internal troubles at home, especially in our cities. Technology often was associated with the unpopular war effort, and it began to lose its mystique and allure. There was growing concern for minorities and the disadvantaged who were not well-served by the college-preparatory curricula of the new math and new science. Events like Watergate contributed to a growing demand for accountability which, in education, took the form of minimum essentials or basic skills testing. The educational pendulum had begun its swing to the opposite extreme in what we would come to know as "back to the basics."

In both mathematics and science, the emphasis during the back-to-the-basics era of the 1970s was on the acquisition of information and the development of algorithmic skill. Scores on standardized tests were taken as the measure of success, and every school system in the country set for itself the impossible goal of becoming like Lake Woebegone where "all the children are above average." At the same time that developments in technology were making the hand-held calculator and the microcomputer commonplace and inexpensive, teachers were directing most of their energies to making sure that children could behave like a calculator, producing rote answers to low-level questions with little regard for thinking or conceptual understanding.

What eventually brought an end to the back-to-the-basics cycle were results such as those of the National Assessment of Educational Progress that showed that children at all of the tested age levels could calculate with a reasonably high level of proficiency. Unfortunately, the same could not be said for the higher level thinking skills needed for problem solving, or even for what we might call the "middle level" skills of knowing when to perform computations and with what numbers. The assessment also revealed that students spend a large portion of their time watching and listening to the teacher, and working alone, probably doing textbook assignments. Other studies showed that, across the country and almost without exception, the routine in classes, day after day after day, was to correct the homework, do a few examples on the board, then start the new homework. Mathematics and science classes were found to embody little of the spirit of inquiry, exploration, laboratory investigation or individualization required for problem solving, and the adjectives most frequently applied, especially to mathematics classes, were *dull* and *boring*.

Then, as has happened before, certain events, this time the publication of several national reports beginning in the early 1980s, gave a push to the school pendulum and hurled education back into national attention, putting us once again under public scrutiny for our teaching practices and outcomes. This time, however, the arena is not only technological, it is economic as well. Americans have begun to recognize that we are being edged out of world markets by nations we once took for granted, notably Japan, and recently-published international comparisons of educational achievement have shown the United States to be mediocre at best when compared to other countries.

On this last point, an explanation is in order because it touches all of us in our mathematics and science classrooms. It should be noted that our very best American students compete favorably with the best students worldwide. The troublesome fact is that the vast majority of students in Japan or the Soviet Union far outperform their counterparts here. It is this discrepancy in the performance of the majority of students that should alarm us.

Futurists tell us that the twenty-first century world will be characterized, above all else, by exponential growth and ever-accelerating change; by a society built on information technology rather than on industrial systems; by connectedness within the world community; and by a myriad of new realities and new problems only vaguely imagined today. The children we educate for life in that world will need new coping skills if they are to live as productive citizens. The writers of an important 1983 report by the National Science Board entitled *Educating Americans for the 21st Century* (1983, p. v) saw the task as unequivocal. They wrote:

> *We must return to basics, but the "basics" of the 21st century are not only reading, writing and arithmetic. They include communication and higher order problem solving skills, and scientific and technological literacy -- the thinking tools that allow us to understand the technological world around us.*
>
> *These new basics are needed by all students -- not only tomorrow's scientists -- not only the talented and fortunate -- not only the few for whom excellence is a social and economic tradition. All students need a firm grounding in mathematics, science and technology.*

But a truth that frequently eludes those who make a profession within either science or mathematics is that both disciplines have come to be viewed as the province, not of the masses, but of an intellectual elite. For most people, science and mathematics beyond the most rudimentary levels are the pursuits of a gifted minority. Both disciplines also have been accused of focusing primarily on the transmission of factual information or on what Alfred North Whitehead (1929, p. 13) termed "inert ideas" -- i.e.,. ideas that are received without being utilized, tested or thrown into fresh combination. Such learning results in the *possession* of knowledge, but rarely in the *use* of knowledge.

At the same time, we are told that scientific and technological knowledge currently increases by 13 percent per year, thus doubling every 5.5 years, and that the rate is expected soon to jump to 40 percent per year -- a doubling time of 20 months (Naisbitt, 1984, p. 16). Clearly an education built on the transmission of information is impossible. What is urgently needed is an education focused on the *use* of knowledge in new and yet unanticipated situations, and use, not by a select few, but by everybody. Mathematicians call it *problem solving*. Scientists call it *inquiry*. The message is the same. Indeed, Whitehead (1929, p. 16) gave as the very definition of education that education is "the acquisition of the art of the utilization of knowledge."

What, then, shall we expect of an education designed to enable our students -- all of them -- to cope successfully with the twenty-first century? To attempt to answer that question, I want to suggest for your consideration what I believe are important components of success both for pupils and for teachers. I hope you will give some time throughout the year to exploring these ideas further and to developing strategies that will help you transform your classrooms into learning environments filled with successes for your students and for yourselves. Let's begin by spelling out success on the part of students.

Components of SUCCESS for Students

S elf-confidence
U nderstanding
C ompetence
C ommunication
E nthusiasm
S ignificance
S ticktoitiveness

Self-Confidence

The first component is, I believe, self-confidence. This is an important starting point because without self-confidence and a positive self-concept our best efforts to educate students can still meet with failure and frustration. Students must believe they can learn mathematics and science or they never will.

Sometimes it is hard for those of us who chose mathematics and science as our major fields to grasp the sense of helplessness that many people experience in these areas. But a negative self-concept can be so serious an impediment that it can totally prevent learning. Each of us can undoubtedly identify some area where we feel incompetent. Perhaps it is art or music or athletics. For me it has always been music. I have been the victim of a long string of teachers, from early elementary grades through college, who have done to me all the things that I am sure music teachers are taught *never* to do. But they long ago succeeded in convincing me that I cannot sing, and, sure enough, I can't. Although, when I am safely alone in my car with the windows up, I will turn on the radio or tape recorder and sing my heart out, I cannot and will not attempt to do so in public. My self-concept in this matter has totally prevented me from ever attaining success. Such is also the case for many students when it comes, not to music, but to mathematics and science.

The starting point for all of us who teach mathematics and science will be to convince *all* students that, to some appropriate degree, these subjects are *for them*. (I say "some appropriate degree" because I do not expect all students to specialize in science or mathematics, but I do expect all students to attain a functional level of mathematical and scientific competence.)

A special comment is in order here. All of us who teach mathematics, and many who teach science, especially physics, know what it is like to meet a total stranger who, on finding out what we do, immediately lets us know that "I can't do math!" or "Yuk! Math! My worst subject" or some equivalent exclamation. This is symptomatic of a serious problem related to self-concept, namely the acceptance that it is okay to be mathematically incompetent. It is NOT okay, and we must make a conscious effort to combat that idea.

Understanding

My next component is understanding. The ability to "do" mathematics or science, as we traditionally expect students to do them in school, is not a measure of understanding. When students truly understand our subjects they internalize the concepts and principles on which the subject is based; they *think* in the discipline; they differentiate between the reasonable and the unreasonable, the relevant and the irrelevant; they take ideas learned in one way and view them from different perspectives or combine them in new ways. They are able to *use* knowledge, not just repeat it.

Let us be sure we are clear on one important point here: facility with computation or algebraic manipulation, or quick recall of facts and formulas, is neither a necessary predictor nor a sufficient measure of understanding.

This discussion always reminds me of two chemistry teachers with whom I worked on separate occasions in the past. The first insisted that a student simply did not know chemistry unless he had committed to memory the names, symbols, atomic numbers, valences and atomic weights of all the elements in the periodic table. The second teacher, by contrast, ran a very active laboratory classroom in which he had permanently mounted on each of the four walls large posters of the periodic table that could easily be read from anywhere in the room. These two men personified the difference between transmitters of information and stimulators of useful knowledge. Only the latter will be able to fulfill the challenges of the future.

Competence

Successful students must be competent in all of the basic skills of the technological society. Ten years ago, the National Council of Supervisors of Mathematics gave us an expanded definition of "basic skills" by insisting that these included not just computation or arithmetic but also problem solving and alertness to the reasonableness of results and estimation and the ability to interpret tables, charts and graphs, and using mathematics to predict, and more. The new basics of our age also include facility in analyzing and interpreting quantitative situations, in generating models, in designing algorithms, in planning and carrying out solutions, in evaluating and verifying results. These are the components of problem solving and higher-order thinking which mathematics and science educators espouse; they are not easily measured with paper and pencil tests. But if we are not stressing these outcomes in our classrooms, we are not preparing students with the competence they will need for future success.

Communication

Communication refers to the ability to read, write, speak, think, demonstrate, and persuade with meaning and understanding. Students must learn to communicate ideas clearly and concisely both with the specialist and with the layman. Mathematics is itself a language, precise and specialized, with a unique symbol system that condenses vast ideas to a dense, concise representation. It is also the language in which most great scientific ideas are encoded. Students must learn to communicate in that language as well as in correct, fluent, effective English. We should expect no less of them because we teach science or mathematics. This is a point on which I am very intolerant, and I have pounded on many a podium insisting that mathematics is not a license for illiteracy.

During the basic skills years we heard frequent laments that "Johnny can't read," and publishers responded, especially in mathematics texts, by draining them of language, leaving little more than facts and rules stated in the simplest of terms. Such textbooks have not yet disappeared from the market, so we must be vigilant in guarding against them. Our students will never appreciate the richness of our subjects unless they can communicate with us about great ideas.

Enthusiasm

If science and mathematics are to become anything more than required drudgery, students must be enthusiastic about them. They must feel a sense of ownership, a curiosity and drive to learn more. Enthusiasm includes respect for science, mathematics and technology; it involves a sense of potency with respect to one's place in the world; it means willingness to think creatively, to take risks, to subject one's ideas to scrutiny by others; to make decisions and accept their consequences. Enthusiasm leads us to anticipate new and unimagined futures and to accept that we can help shape that future. Enthusiasm is excitement about learning the subject, yet, as we saw earlier, students find little to be excited about in typical classes. The continuing challenge to each of us will be to kindle and then nurture enthusiasm.

Significance

Perhaps the reason students are not more enthusiastic about mathematics and science is that they do not perceive our subjects as having significance *for them.* Too many students who have asked, "Why do we need to learn this? What's it good for, anyway?" have received the common but unimaginative answer that, "You'll need it next year in trigonometry or physics," or "You'll need it in college." Pages of drill or formulas have very little significance to a learner.

But we have today new tools that we never had before. Technology in the form of calculators and microcomputers and video disks allow us to present applications of science and mathematics that are relevant, interesting and nontrivial. We must seriously examine the way we use these tools, as well as the problems we present, to assure that the significance of our subjects for our students is not lost. We must, every day, help our students to see the applications of mathematics and science all around them in their own lives. Students need to learn how to view the world through math-and-science-colored glasses. When they do they will begin to transfer school learning to life beyond the classroom. They will begin to develop an awareness of the interrelatedness of science, mathematics and technology with wider social issues, and they will learn to understand and cope with the world in which we live.

Sticktoitiveness

Finally, students will need perseverance or sticktoitivness, and they will need it on more than one level.

On the immediate small-scale level, this means that students must learn to stay on task and to grapple with important ideas or hard problems. I see this as a difficulty for many students, especially our so-called "better students." They, in particular, have come to believe that what we expect of them is quick performance on low-level tasks, and, indeed, they are very good at that. They are used to getting the answer, to finishing the assignment quickly, and to knowing that their answers are correct. But they come apart when presented with problems that require them to think and struggle and trust their own judgements and even risk being wrong. So they give up on the problem or are easily distracted by other amusements.

The other level of sticktoitiveness is the long-range one: More students will need more years of mathematics and science regardless of their plans beyond high school. The world they live in will demand that. But it also is the case that we must seriously examine the kinds of courses we offer them.

In mathematics, for example, the traditional offerings beyond algebra and geometry have always been designed to prepare students for the calculus. But those who plan to study in fields like business, economics, sociology, political science, psychology and the like need, not calculus, but probability, statistics, matrix algebra, finite mathematics, graph theory, and related topics. Likewise, not all students need engineering physics, but all do need a solid grounding in the principles of physical and life science.

The bottom line is that we cannot allow students to end their study of mathematics and science in the ninth or tenth grade. But, at the same time, we must have more to offer them than bonehead arithmetic and Sandbox II.

And this leads me quite naturally to teachers. Education is a partnership, and we won't develop successful students without also developing successful teachers. So let us ask ourselves how we will recognize success in teachers.

Components of SUCCESS for Teachers

S tudent-centeredness
U rgency
C ommitment
C reativity
E xpectation
S urprise
S elf-direction

Student-Centeredness

We will know them first because they are student-centered. This is a dimension on which mathematics and science teachers have not always received the highest marks. Many of us intimidate our students because they see us as "real smart" and as talking a language they can't possibly understand.

But student-centered teachers understand those apprehensions. They have empathy and patience; they find ways to make the subject relevant to students. This means more than being an expert in the subject yourself; it means knowing it so well that you can make it understandable to a child.

Urgency

The successful teacher also exhibits a sense of *urgency* or *mission* that convinces students that what we teach is vitally important for them. Out of our urgency is born their sense of excitement about learning. Teachers who have lost, or who failed to develop, a sense of urgency about teaching are the ones who have begun on-the-job retirement. Uninspired teaching lulls students into complacency or alienates them altogether. Teachers without a mission are unhappy people who can do little to benefit either themselves or their students.

Teachers can do much to encourage and support one another and thereby to stimulate their sense of mission or urgency. In this regard I give you the same advice I give my students who are preparing to become teachers: align yourselves with colleagues who are professionally alive and excited about their teaching -- whatever the subject -- because they are the ones most likely to keep you alive and excited.

Commitment

Urgency leads to commitment, and commitment is the hallmark of successful teaches. It is also a quality that seems to have eroded seriously in recent years.

As a teacher educator, absolutely the most difficult part of my job is seeing my eager undergraduate students return from the schools where they have their practicum and student teaching field experiences and listening to their horror stories about the things teachers tell them about teaching: "Don't do it! You're making a big mistake!" "Get out while you can before it's too late . . ."

How will we replace the present cadre of teachers, a majority of whom will retire or leave teaching in the near future, when teachers themselves do not take pride in and are not committed to what they do?

Each summer I teach in a program for gifted high school mathematics and science students from throughout the state of Michigan. During the institute I have an opportunity to talk to them about teaching. I try to impress upon these extremely talented young people that teaching is a good and honorable profession, that when the alarm rings in the morning I *like* to get up and go to work, and that we hope many of them will join us in this vocation. But I am always left to wonder what their teachers back home say and do to give credibility to the things I tell them.

Creativity

Creativity goes hand-in-hand with all the other characteristics. There is nothing creative about reading answers to yesterday's homework, doing a few examples on the board, then assigning more homework. But it takes considerable creativity to use time and space effectively, to assure that every minute in your classroom is used productively to engage the students in your subject. It takes creativity to find interesting problems and relevant applications; to prepare effective demonstrations and hands-on experiences; to ask stimulating, thought-provoking questions; to unfetter yourself from the textbook and do what you believe will be best for your students.

Creative people are curious. They are risk takers. They are sometimes unconventional. Creative teachers capture the attention of their students; they conduct stimulating and productive classes that students *want* to attend and are afraid to miss.

Expectations

Successful teachers also have high expectations for themselves and for their students. Students resonate to the self-fulfilling prophecy of teachers' expectations. When we expect students to be stupid or bored or rowdy, they seldom let us down. But when we expect them to be competent, to accept responsibility, to perform according to their true abilities, they usually come through for us if they recognize that our expectations are grounded in respect and trust. Obviously, I do not expect a six-year-old to master trigonometry or chemistry, but I do expect students to perform tasks appropriate to their age and grade. I also expect them to do more than they think they can, and I expect to provide them with support and guidance and help to make sure that they succeed.

Surprise

Like a young child discovering the world, I believe teachers must never lose a sense of wonder and surprise about their subjects. For example, when I read the mathematical writings of Martin Gardner, I come away with renewed awe and wonder and surprise about mathematics. I have a similar experience when I study the history of science.

One way to cultivate this wonder is through problem solving or laboratory research of your own. I don't mean working out the answer key for your class or running through the experiment before the students do it; I mean *real* investigation of your own discipline in an area that is new to you.

In this regard, mathematics and science teachers often differ sharply from teachers of other subjects. When I think about art teachers that I know, I find that they *do art* for their own pleasure and development. They paint, do sculpture, make pottery, take photographs, and the like. Similarly, the music teachers I know participate in music outside of school. They sing in choirs, play instruments, attend concerts. But I know precious few mathematics and science teachers who *do* mathematics and science on their own just for the enjoyment of it. Yet it is this personal involvement in the subject that is the source of our deepest sense of surprise.

Self-Direction

Finally, the successful teacher is a self-directed professional. He or she keeps abreast of the discipline and of issues in teaching that discipline, not because further study will lead to a lane change and a salary increment, but because the knowledge will make him or her a better teacher. Self-directed teachers regularly read the professional journals, attend professional conferences, serve on committees, take leadership roles, give talks and workshops, share teaching ideas, hold office in state and local professional organizations, participate in curriculum development, mentor student teachers, and more. On the national level, there are strong movements to restructure teaching to recognize differences in teachers' involvement in these activities. Those who wish to attain the status of advanced career professionals will have to become highly self-motivated and self-directed.

Conclusion

In a world where we must educate students to solve problems as yet undreamed of, we can only hope to impart to them the germs of the attitudes and abilities that may someday enable them to find breakthrough solutions. Our mathematics and science classrooms can become the laboratories for this learning.

So to science and mathematics educators of the present falls the challenge to begin to identify and explore the most powerful ideas of the future. We shall face the grave task of assuring that education in science and mathematics begins early and develops in a consistent, coherent and purposeful manner under the stimulation of a community of educators who model the knowledge, abilities, attitudes and commitment of persons in touch with their world and convinced of their ability to make a difference in that world.

These are imperatives that must be taken seriously now. We can, in truth, invent our future, but the time for invention is before the future arrives. We know all too well that the future is approaching with accelerating speed. Once it arrives, the best we shall be able to do is react to it.

So my sincere wish for all of you is a wish for success. Your students are depending on you. We all are depending on you. Please do not let us down.

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