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AUTHOR Mittag, Kathleen Cage

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ABSTRACT

A pivotal theorem which is of critical importance to statistical inference in probability and statistics is the Central Limit Theorem (CLT). The theorem concerns the sampling distribution of random samples taken from a population, including population distributions that do not have to be normal distributions. This paper contains a brief history of the CLT; several forms of the CLT--Kreyszig, Groeneveld, Rahman, Harnett, Dubewicz, and Marzillier; and a proof of the CLT. Examples, both using a small population with hand calculations and using large populations with computer programs, are included to illustrate concepts of the CLT. A HyperCard stack and the program "Resampling Stats" are also demonstrated in the paper. The appendixes contain a proof of Form 5 CLT, a computer worksheet on CLT, three examples of the CLT HyperCard, and a computer program. (Contains 14 references.) (Author/ALF)

Using Computers to Teach the Concepts of the Central Limit Theorem

Kathleen Cage Mittag
Texas A&M University 77843-4232

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ABSTRACT

A pivotal theorem which is of critical importance to statistical inference in probability and statistics is the Central Limit Theorem (CLT). The theorem concerns the sampling distribution of random samples taken from a population, including population distributions that do not have to be normal distributions. This paper contains a brief history of the CLT, several forms of the CLT, and a proof of the CLT. Examples, both using a small population with hand calculations and using large populations with computer programs will be included to illustrate concepts of the CLT. A HyperCard stack and the program *Resampling Stats* will be demonstrated in the paper.

A pivotal theorem of critical importance to statistical inference in probability and statistics is the Central Limit Theorem (CLT). The theorem concerns the sampling distribution of random samples taken from a population, including population distributions that do not have to be normal distributions. According to Brightman(1986, p. 107), the importance of the CLT is "if the central limit theorem didn't exist, statistics would have little practical use." Emphasizing this point, Brightman (1986, p. 141) wrote, "Without it, estimating population parameters from sample statistics -- inductive inference -- would be practically impossible." This paper contains a brief history of the CLT, several forms of the CLT, and a proof of the CLT. Examples, both using a small population with hand calculations and using large populations with computer programs, will be included to illustrate concepts of the CLT.

HISTORY

The CLT first appeared in print in 1733 when A. de Moivre derived a special case of the theorem. In 1812, P. S. LaPlace gave a general form of the theorem. C. F. Gauss also worked with normal distributions using the theorem. In 1901, A. Liapounoff was the first to offer a proof of the CLT (Hald, 1962).

NORMAL PROBABILITY DISTRIBUTION

A critical concept that must be understood before properties of the CLT can be presented is the normal probability distribution. Brightman (1986, p. 107) presents three features of the normal probability distribution. These features are:

1. The normal probability histogram is symmetric. The highest point of the curve is the mean. The part of the curve to the right of the mean or expected value is a mirror image of the part to the left.

2. As with all continuous probability histograms, the total area underneath the curve is equal to 100 percent.

3. The curve appears to hit the x-axis but it never does. The chance of events very far above and below the mean or expected value is, however, very small.

DIFFERENT FORMS OF THE CLT

Several forms of the CLT have been presented by various authors. Six forms will now be discussed with their headings being the name of the authors describing the forms.

1. **Kreyszig** (1970, p. 191).

Central Limit Theorem. Let $X_1, X_2, X_3, \ldots$ be independent random variables that have the same distribution function and therefore the same mean μ and the same variance σ^2. Let $Y_n = X_1 + \ldots + X_n$. Then the random variable

$$Z_n = (Y_n - n\mu)/(\sigma n^{0.5})$$

is "asymptotically normal" with mean 0 and variance 1, that is, the distribution function $F_n(x)$ of Z_n satisfies the relation

$$\lim_{n \to \infty} F_n(x) = \phi(x) = 1/(2\pi)^{0.5} \int_{-\infty}^{x} e^{-u^2/2}\,du.$$

2. **Groeneveld** (1979, p. 185)

The Central Limit Theorem. If $X_1, X_2, \ldots, X_n$ are independently and identically distributed random variables, with common distribution given by the random variable X, for which $E(X) = \mu$ and $Var(X) = \sigma^2$, then

$$\lim_{n \to \infty} P\,[(\overline{X} - \mu)/(\sigma/\sqrt{n}) \leq t] = F_Z(t)$$

for all t, where $F_Z(t)$ is the cumulative distribution function for the standard normal distribution. We say $\overline{X}$ is asymptotically normally distributed, with mean μ and variance σ^2/n.

3. **Rahman** (1968, p. 364)

If the non-normal population sampled has a finite variance σ^2 and n is large, then,

$$Z = (M - \mu)/(\sigma/\sqrt{n})$$

is approximately a unit normal variable. This is one form of a remarkable result in statistical theory, which is known generally as the *Central Limit Theorem*; and it is this theorem which, indeed, gives to the normal distribution its unique position in statistics. An extension of this result also shows that if σ is replaced by s then, again for large samples

$$t = (M - \mu)/(s/\sqrt{n})$$

is approximately a unit normal variable. Hence the tests of significance used in the case of sampling from a normal population can also be used approximately when the population sampled is non-normal but with finite variance.

4. **Harnett** (1970, p. 165 & 167).

The Central Limit Theorem has two forms:

a. The distribution of the means of random samples taken from a population having mean μ and finite variance σ^2 approaches the normal distribution with mean μ and variance σ^2/n as n goes to infinity.

b. $\lim\limits_{n \to \infty} (M - \mu)/(\sigma/\sqrt{n}) = N(0,1)$.

5. **Dubewicz** (1976, p. 149)

Central Limit Theorem. Let $X_1, X_2, \ldots$ be independent and identically distributed r.v.'s with $EX_1 = \mu$ and $Var(X_1) = \sigma^2 > 0$ (both finite). Then (for all z, $-\infty < z < \infty$), as $n \to \infty$

$$P\{[(X_1 - \mu) + \ldots + (X_n - \mu)]/(\sqrt{n}\sigma) < z\} \to [1/\sqrt{(2\pi)}] \int_{-\infty}^{z} e^{-.5y^2}\, dy$$

The proof of this theorem is included in Appendix A.

6. **Marzillier** (1990, p. 179)

The Central Limit Theorem

If all possible samples of size n are drawn from a population with mean = μ and standard deviation = σ, and M, the sample mean, is calculated for each sample, then the frequency distribution of M, thus obtained, has the following three properties:

a. Its mean = μ, the mean of the population.

b. Its standard deviation = $\sigma/\sqrt{n}$, the standard deviation of the population divided by the square root of the size of each of the samples.

c. It will tend to have a normal distribution, regardless of the shape of the population.

DISCUSSION OF FORM 6

Form (6) provides a different perspective of the CLT, which would be applicable for use in an introductory statistics course. Different notation used in the explanation of form (6) will involve the mean of the frequency distribution of M, μ_M, and the standard deviation of the frequency distribution of M, SD_M. Considering part (a), it states that the mean of all the sample means equals the population mean. This is reasonable if it is considered that some of the sample means will be greater than the population mean and some of the sample means will be less than the population mean.

Part (b) states that the standard deviation of M is equal the population standard deviation, σ, divided by the square root of the sample size, n.

$$SD_M = \sigma/\sqrt{n} \qquad 1$$

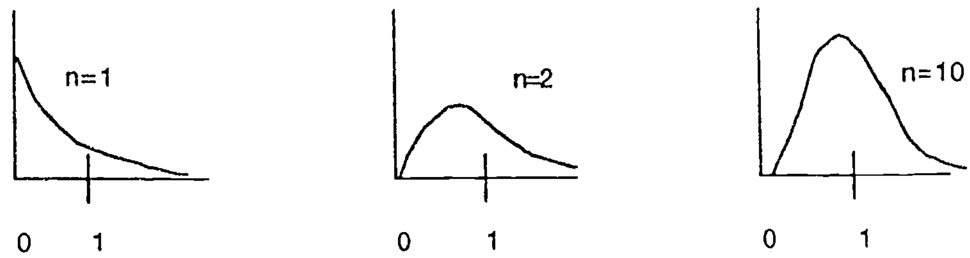
This means that the sample means are less spreadout than the population mean and the spreadoutness decreases as sample size increases. For example, if n=9, SD_M = 1/3 of σ and if n=36, SD_M = 1/6 of σ. There is a finite correction formula for finite population of size N. The formula is

$$SD_M = (\sigma/\sqrt{n})\ \sqrt{[(N-n)/(N-1)]} \qquad 2$$

If $\sqrt{[(N-n)/(N-1)]}$ is close to 1, then formula 1 and formula 2 are equivalent.

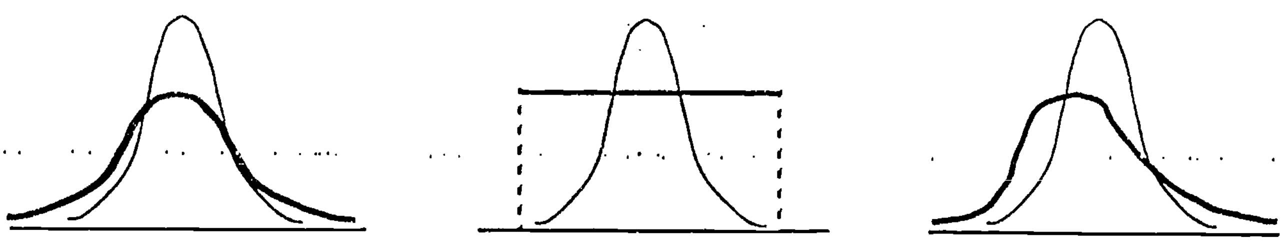
Moore (1989, p. 417) illustrated the concept that the larger the sample size,n, the standard deviations decrease by the value of $\sigma/\sqrt{n}$. The population is an exponential distribution with μ=1 and σ=1.

FIGURE 1
Density Curves



If n=1, then σ=1; if n=2, then $\sigma=1/\sqrt{2}$; and if n=10, then $\sigma=1/\sqrt{10}$. It can be seen in Figure 1 that the variability of the x-values actually decreases as n increases.

Part (c) is the section of the CLT which is referred to most often. Marzillier (1990, p. 180) illustrated part (c) as follows:



The thin lines are graphs of the frequency distributions of M. No matter what the distribution of the population is, the frequency distribution of M tends to be normal. Intuitively, if many samples are taken from a population, then many more of these samples will have means close to μ. This forms the "bell-shaped" normal curve.

EXAMPLE

A very small population will first be used to illustrate concepts of the CLT. Let the population be the numbers 2, 3, 4, 7, 9, and 11, then μ=6 and σ=3.27. Take all the possible samples of size n=3 from the population, then calculate M for each sample. There will be 20 possible samples since C(6,3) = (6!)/(3!3!) =20.

Sample	Mean	Sample	Mean
2,3,4	3	3,4,7	4.7
2,3,7	4	3,4,9	5.3
2,3,9	4.7	3,4,11	6
2,3,11	5.3	3,7,9	4.3
2,4,7	4.3	3,7,11	7
2,4,9	5	3,9,11	7.7
2,4,11	5.7	4,7,9	6.7
2,7,9	6	4,7,11	7.3
2,7,11	6.7	4,9,11	8
2,9,11	7.3	7,9,11	9

Using the means of each sample as the data set, then μ_M=6. This agrees with part (a) of form (6), which stated that $\mu_M = \mu$.

Again using the means as the data set, then SD_M = 1.46.

Using formula 1 from form (6),

$$SD_M = \sigma/\sqrt{n}$$

$$= 3.27/\sqrt{6}$$

$$= 1.34$$

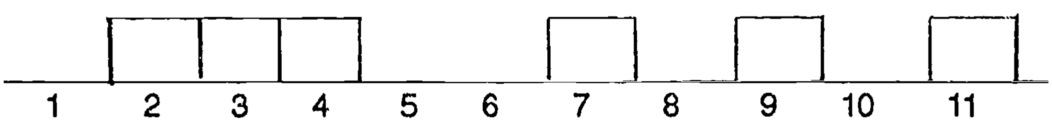
These two values for SD_M are very close and differ because of round-off errors in M calculations.

To illustrate part (c) from form (6):

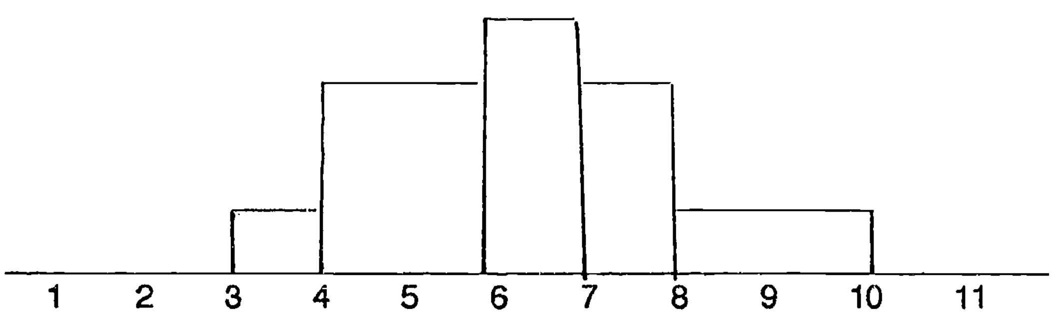
Frequency Distribution of M

Class	f
3.0-3.9	1
4.0-4.9	4
5.0-5.9	4
6.0-6.9	5
7.0-7.0	4
8.0-8.9	1
9.0-9.9	1

Histogram of the Population



Histogram of Sampling Distribution of M



The histogram of the population definitely is not a normal distribution, but the histogram of the sampling distribution of M is approaching a normal distribution.

COMPUTER EXAMPLES

Since the introduction of computers into the classroom, it has become much easier to teach statistical concepts. The first computer example will use a HyperCard Stack developed by James Lang. HyperCard version 2.0 or later is needed to run this program on a Macintosh computer. A sample worksheet (Lang, 1991) is included in Appendix B.

Sample printouts, using HyperCard, are included in Appendix C. In Example 1, there were 150 samples taken of size n=5 resulting in μ=4.5 and μ_M=4.4 and SD=2.87 and SD_M=1.29. Since SD_M should equal $2.87/\sqrt{5}$, which is 1.28, the results are consistent with the CLT. In Example 2, there were 150 samples taken of size n=10 resulting in μ=4.5 and μ_M=4.5 and SD=2.87 and SD_M=0.869. Since SD_M should equal $2.87/\sqrt{10}$, which is 0.906, the results are also consistent with the CLT. Example 3 shows the relationship of mean and standard deviation of the sample means generated by a computer to the mean and standard deviation of the population. As sample size gets larger the mean of the sample means gets closer to the population mean, as is expected.

Another computer program which can be used to demonstrate the CLT is *Resampling Stats.* Monte Carlo simulations, bootstrapping, and randomization procedures are used in this language. Both IBM and Macintosh versions are available. Simon and Bruce (1991, p. 3) define resampling as "use of the data, or a data-generating mechanism such as a coin or set of cards, to randomly generate additional samples, the results of which can be examined." A program, included in Appendix D and adapted by Mittag, was used to demonstrate the concepts of the CLT. A data set was entered, 100 trials of size 5(10) were taken, the means were calculated, then a histogram of the means was graphed. Also, the mean of the sample means was calculated and recorded as D. The results of the histograms and the μ_M calculations agree with the concepts of the CLT when the program is run.

CONCLUSION

The CLT should be taught in a statistics course because of its vital importance to statistical inference. Four primary topics can be emphasized when teaching the CLT. These topics are: a. the variability of M; b. the distribution is centered around the mean; c. the variance of the distribution gets smaller as sample size gets larger; and d. the distribution of M is a normal probability distribution. These points can be taught both by hand calculations with small samples and by using computer programs for large samples. The PC is such a powerful tool that it should definitely be used to teach statistical concepts. Students can see and actively participate in the sampling procedure when using HyperCard, and seeing is believing. The computer can simulate sample repetitions rapidly, graphically present the results, quickly perform calculations, and allows students to think more about the concepts (Groeneveld, 1979). The CLT is just one important statistical concept in which computers can be utilized in the explanation.

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APPENDIX A

The following proof of Form 5 CLT was offered by Dudewicz (1976, p. 149).

Proof:

$$\phi\frac{(x_1-\mu)+\ldots+(x_1-\mu)}{\sqrt{n}\sigma}(t) = \phi\frac{(x_1-\mu)}{\sqrt{n}\sigma}+\ldots+\frac{(x_1-\mu)}{\sqrt{n}\sigma}(t)$$

$$= \left[\phi\frac{x_1-\mu}{\sqrt{n}\sigma}(t)\right]^n \quad \text{, By Theorem 6.1.2}$$

$$= \left[\phi x_1-\mu \; \{t/\sqrt{n}\sigma\}\right]^n \text{, By Theorem 5.4.12}$$

$$= \left[1+\frac{iE(X_1-\mu)}{1!}\frac{t}{\sqrt{n}\sigma}+\frac{i^2E(X_1-\mu)}{2!}\frac{t^2}{n\sigma^2}+0\left(\frac{t^2}{n\sigma^2}\right)\right]^n$$

$$= \left[1-\frac{1}{2}\frac{t^2}{n}+0\left(\frac{t^2}{n\sigma 2}\right)\right]^n$$

$$= \left[1+\frac{-.5t^2 \quad + \quad no\,(t^2/n\sigma^2)}{n}\right]^n$$

$$\rightarrow e^{-0.5t^2}, \quad \text{as } n\rightarrow\infty$$

WORKSHEET
CENTRAL LIMIT THEOREM

Part I.

1. Once you have loaded the program, click on **Clear** so you are beginning with a new screen.

2. The population you will be working with is in the box at the lower left corner of the screen. This is a uniform distribution containing one of each value between 10 and 29. Calculate the population mean to verify that the mean of this population is 19.5. **Show work.**

3. -Set speed to **slow**, **n** = 5, and **sample count** = 150.

4. Click **take sample** and watch the computer select 5 items at random from the population to be in the sample. Record the sample, its mean, and the sampling error below. Recall that the sampling error is the difference between the sample mean and the population mean. Notice how the mean is added to the stem and leaf diagram. Repeat this two more times (click **take sample** to repeat).

Sample 1	Sample 2	Sample 3
1/	1/	1/
2/	2/	2/
3/	3/	3/
4/	4/	4/
5/	5/	5/
Mean = _______	Mean = _______	Mean = _______
Sampling error = ____	Sampling error = ____	Sampling error = ____

5. Now change the speed to **medium** and take three more samples (click **take sample** to sample). Notice how the items chosen from the population are no longer highlighted but the sample is still displayed. Also notice how the means are continually added to the stem and leaf display. Record the three sample means below.

Sample 4	Sample 5	Sample 6
Mean = _______	Mean = _______	Mean = _______
Sampling error = ____	Sampling error = ____	Sampling error = ____

6. Now change the speed to **fast** and click **take sample** to finish the sampling. Notice at this speed the samples aren't displayed (but are being taken as before) and the means are entered into the stem and leaf display. (Watch the **sample count**, it will countdown to 0 when sampling is complete).

   Describe the shape of your sample mean distribution. (Is it symmetrical? mounded? What is the range? What group has the most data ? etc.)

From James Lang with permission

7. Click on **Stats**. This shows the mean of the sample means, the standard deviation of the sample means, and the percentage of points within 1, 2 and 3 standard deviations of the mean. Copy the information for you data set here.

mean of xBars = ______
st.dev. of xBars = ______

_____% of the xBars are within 1 standard deviation of the mean.
_____% of the xBars are within 2 standard deviations of the mean.
_____% of the xBars are within 3 standard deviations of the mean.

What percent of the data in a normal distribution can be expected to be found within 1 standard deviation of the mean? ______%, within 2 standard deviations of the mean? _______%, within 3 standard deviations of the mean? ______%

How do the percentages from the computer simulation compare with those expected in a normal distribution?

Part II. Now we will see what effect a larger sample size has. Click on **Clear** to get a clean screen. Set speed to **medium, n** = 25, and **sample count** = 150.

8. Click **take sample** and watch the computer select 25 items at random from the population to be in the sample. Record the sample mean below. Notice how the mean is added to the stem and leaf diagram after each sample is selected. Repeat five times (click **take sample** to repeat).

Sample 1	Sample 2	Sample 3
Mean = _______	Mean = _______	Mean = _______
Sampling error = ___	Sampling error = ___	Sampling error = ___

Sample 4	Sample 5	Sample 6
Mean = _______	Mean = _______	Mean = _______
Sampling error = ___	Sampling error = ___	Sampling error = ___

9. How does the sampling error compare with those in question 4 ?

10. Now change the speed to **fast** and click **take sample** to finish the sampling. Watch as the means are entered into the stem and leaf display. (The **sample count** will countdown to 0 when sampling is complete).

Describe the shape of this sample mean distribution. (Is it symmetrical? mounded? What is the range? etc.)

How is the shape of this sample mean distribution different than the one for n = 5?

What can you conclude about the effect that larger sample size has on the variation of the sample means ?

11. Click on **Stats**. This shows the mean of the sample means, the standard deviation of the sample means, and the percentage of points within 1, 2 and 3 standard deviations of the mean. Copy the information for you data set here.

mean of xBars = ______
st.dev. of xBars = ______

______% of the xBars are within 1 standard deviation of the mean.
______% of the xBars are within 2 standard deviations of the mean.
______% of the xBars are within 3 standard deviations of the mean.

- What percent of the points in a normal distribution can be expected to be found within 1 standard deviation of the mean? ______%, within 2 standard deviations of the mean? ______%, within 3 standard deviations of the mean? ______%

How do the percentages from the computer demonstration compare with those expected in a normal distribution? (Is it possible this is a normal distribution?)

Part III. Now we will compare the results of the computer simulation to the theoretical calculations presented in the textbook.

12. Click on Clear to clear the screen. Set **n** = 10, the **sample count** = 150, and choose the **fast** speed. Click **take sample** to sample and then examine the stem and leaf display. Count the actual number of means that are less than 17.
How many? ______

What percent of the samples is this? ______

13. Note that for the population given here $\mu = 19.5$ and $\sigma = 5.916$. Suppose n = 10. Calculate (as in example 7.9, page 286) the theoretical percentage of samples that have a mean less than 17, i.e. $P(\bar{x} < 17)$. Show your work including the sketch.

$P(\bar{x} < 17) =$ __________

How close is this percent to the actual value calculated above? ______

Part IV. Now we will look at the relationship of the mean and standard deviation of the sample means generated by computer to the mean and standard deviation of the population.

14. Click on right arrow (→) to go to the next page of the display. Here you are asked to choose a sample size and click on **calculate** to find the mean and standard deviation of the sample means. Note the computer is doing the sampling as before but not showing each sample. Complete the following table:

μ = ________

σ = ________

$\bar{n}$	mean of set of sample means	standard deviation of set of sample means
4		
9		
16		
25		

15. How does the mean of the set of sample means appear to be related to the population mean ?

16. How does the standard deviation of the sample means change as n increases ?

17. According to the Central Limit Theorem, the standard deviation of the sample means is equal to $\frac{\sigma}{\sqrt{n}}$. Calculate $\frac{\sigma}{\sqrt{n}}$ for n = 4, 9, 16 and 25 and compare to the standard deviation of the set of sample means found in the table above. Does the computer experiment support the theoretical formula ?

Click on the left arrow to return to page 1. Before leaving the program, feel free to experiment with different values for **n** and the **sample count.**

Part V. Reflecting on the experiment. You may answer the following questions at home.

18. What is a sampling distribution? How is the sampling distribution generated in this exercise?

19. How does this computer simulation demonstrate the Central Limit Theorem as discussed in Chapter 7 of your text?

CENTRAL LIMIT THEOREM HYPERCARD

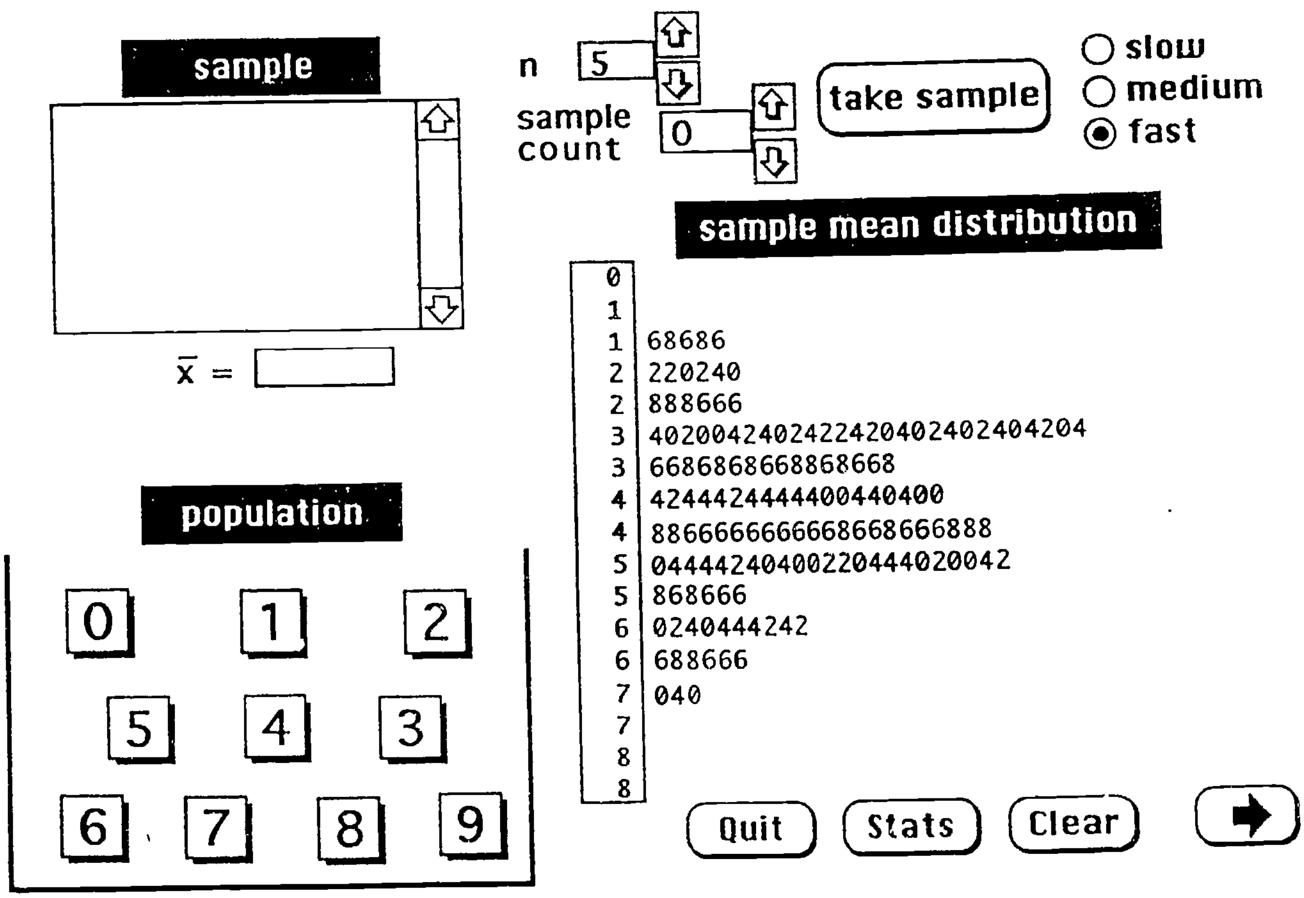
The purpose of this program is to let you watch the sampling process that leads to the sampling distribution for the sample mean. You may also compare the results of the program to the theoretical statement called the Central Limit Theorem. Click on the population below that you want to sample.

Population: 10,11,...29

Population: 0,1,2, ... ,9

EXAMPLE 1

CENTRAL LIMIT THEOREM HYPERCARD



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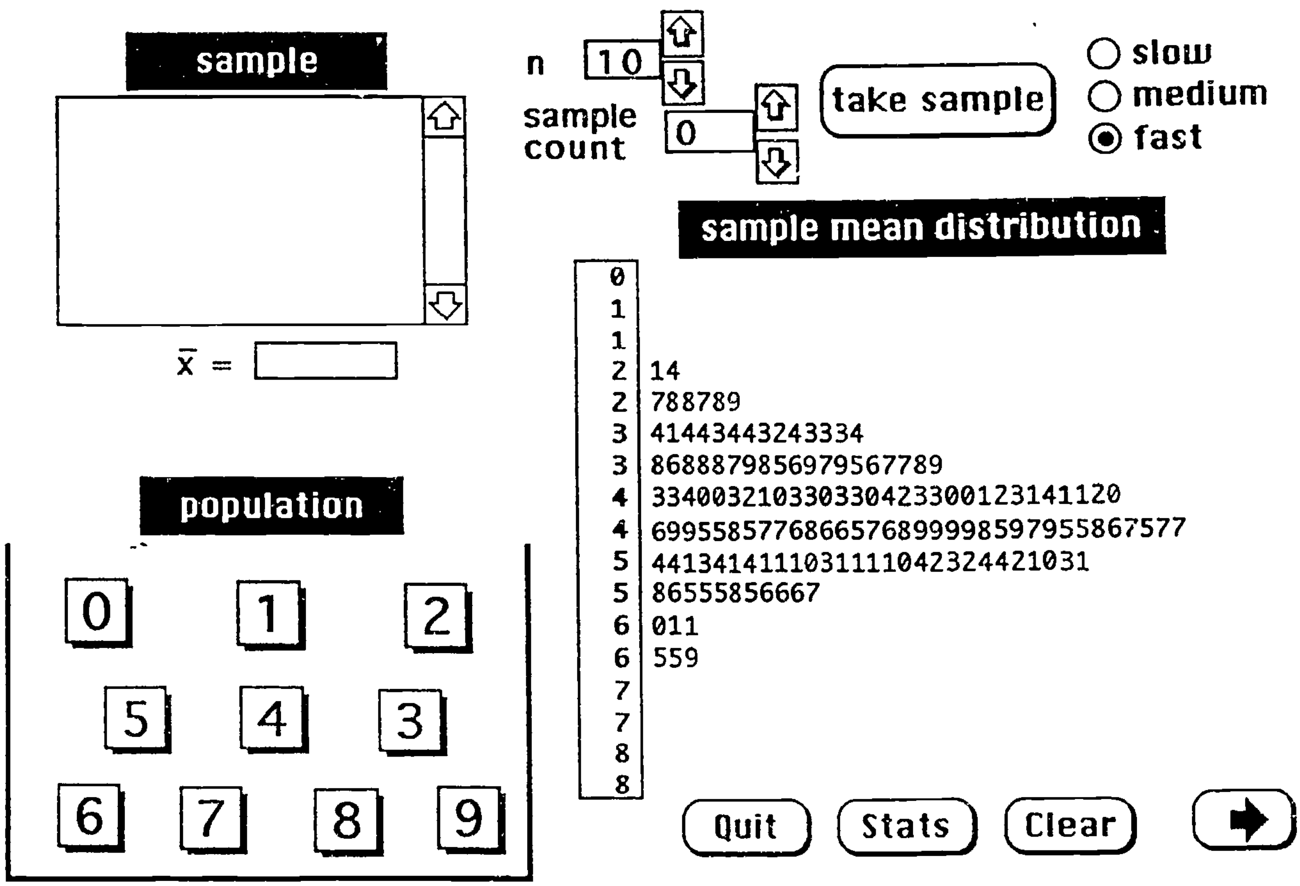

EXAMPLE 3

CENTRAL LIMIT THEOREM HYPERCARD

Enter a sample size,n, below and click calculate. The computer will then take 500 samples each of size n from the population below. For each sample the mean is then calculated. Then the mean and standard deviation of this set of 500 sample means is calculated. These values approximate the mean and standard deviation of the sampling distribution of the sample mean.

$\mu = 4.5$

$\sigma = 2.87228$

population

0 1 2

5 4 3

6 7 8 9

n	Mean of set of sample means	SD of set of sample means
2	4.432	2.003598
3	4.537333	1.615771
4	4.511	1.47121
5	4.5072	1.320665
6	4.531667	1.136551
7	4.496857	1.056398
8	4.4745	0.977823
9	4.481111	0.974203
10	4.5044	0.906943

n = 10

calculate clear

```
COPY (11 12 13  14 15 16 17 18 19 20) A
REPEAT 100
  SAMPLE 10 A AA
  SUM AA AAA
  DIVIDE AAA 10 B
  SCORE B Z
  SUM Z ZZZ
  DIVIDE ZZZ 100 C
  SCORE C D
PRINT D
END
GRAPH Z
SORT Z ZZ
PRINT ZZ
```