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ABSTRACT

This booklet is the second in a series of nine from the Teacher Training Institute at Hofstra University (New York), and describes the capstone course on calculus in secondary school mathematics, which was planned in response to numerous requests from the participants of the previous cycles. Included in this booklet are: (1) an introduction; (2) the preliminary considerations for course set-up and planning; (3) the general course framework; (4) an outline of the course particulars; (5) a discussion of testing and grading procedures; (6) results in terms of student and instructor course evaluations; and (7) appendices which contain the preliminary student questionnaire form, homework assignments, the final examination, and the course and instructor evaluation form. (JJK)

HOFSTRA UNIVERSITY

TEACHER TRAINING INSTITUTE

Department of Mathematics and School of Secondary Education
Hofstra University
Hempstead, NY 11550

DISSEMINATION PACKET - SUMMER 1989

Booklet #2

BARBARA BOHANNON

CALCULUS IN SECONDARY MATHEMATICS

NSF Grant # TEI8550088, 8741127

This booklet is the second in a series of nine booklets which constitute the Hofstra University Teacher Training Institute (TTI) packet. The Institute was a National Science Foundation supported three-year program for exemplary secondary school mathematics teachers. Its purpose was to broaden and update the backgrounds of its participants with courses and special events and to train and support them in preparing and delivering dissemination activities to their peers so that the Institute's effects would be multiplied.

This packet of booklets describes the goals, development, structure, content, successes and failures of the Institute. We expect it to be of interest and use to mathematics educators preparing their own teacher training programs and to teachers and students of mathematics exploring the many content areas described.

"Calculus in Secondary Mathematics" was planned as a 'coda' course in response to TTI participants' requests. The coda was an added Institute component, run during five weeks of the summer of 1988, and created to round off the program after two year-long cycles had been offered (June 1986 through May 1988). The other two courses of the coda were "Problem Solving via Pascal Data Structures" and "Discrete Mathematical Models".

This booklet describes the Calculus course - its creation, course outline, tests, course evaluation and it concludes with a discussion of possible improvements for future offerings.

TEACHER-TRAINING INSTITUTE REPORT #2:

CALCULUS IN SECONDARY MATHEMATICS

by

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1. INTRODUCTION. Calculus in Secondary Mathematics was planned in response to a request by Teacher-Training Institute students for a calculus course. First requests focused on a need to review calculus, a desire to upgrade calculus skills and teaching techniques, and a wish to see applications and alternate proof techniques. The course, with the undergraduate calculus sequence as prerequisite, was scheduled to be taught intensively in two consecutive weeks from July 25 to August 4,1988. Class would meet on Monday, Tuesday, and Thursday of each week, the first week for nine hours (from 9 AM to noon each day) and the second week for fifteen hours (afternoon sessions from 1 to 3 PM in addition to the morning hours). When I was asked to teach this course in May, only these broad parameters had been set, although the TTI structure of intensive and cooperative learning was already firmly established.

The following sections describe the planning, teaching, and evaluation of Calculus in Secondary Mathematics. Section 2 describes the preliminary work of setting up and planning, section 3 presents the general course framework, and section 4 gives a brief course outline. Some details of the outline appear in section 5 and a discussion of grading procedures and results makes up section 6. Sections 7, 8, and 9 deal with student evaluations of the course and my evaluation of the experience.

The reader should keep in mind that the course evolved in response to student-perceived need and that a different group of students would probably have elicited a different sort of calculus course.

2. PRELIMINARIES. An orientation meeting of TTI students and faculty was held in early June to discuss plans for the summer session and I had a chance to speak with students both formally and informally. A quote from a hand-out from this session (see Appendix A) states my goal:

> I'd like to have a unifying theme for each session, beginning with a review of relevant ideas, but progressing by the end of the session to more challenging concepts and problems. This seems to be the only practical way to tame the huge calculus beast and fit it into our relatively meager time frame.

A questionnaire (Appendix B) filled out by twenty prospective students confirmed what I had begun to suspect from informal talks with students: these students had widely varying calculus backgrounds.

Answers to "When is the last time you took a calculus course?" spanned nearly thirty years. Typical answers were: "ancient history," 29 years ago, 1967, all the way up to 1985.

A self-rating of current calculus knowledge and skill was revealing. Choices were *excellent, very good, good, so-so,* and *poor.* The responses were: excellent, 0 (this may have been modesty); very good, 3; good, 7; so-so, 4; and poor, 6.

Of the group surveyed, 5 had taught Advanced Placement Calculus, the other 15 had not.

Typical goals were revealed in answers to "What do you expect to gain from this course?" Many students wanted a calculus refresher; they felt rusty and unsure. (One student had been away from teaching for twelve years; another recently certified teacher felt underprepared to teach mathematics and welcomed any opportunity to upgrade skills.) Others wanted to learn alternative teaching techniques and new calculus applications, with an emphasis on an ability to do AP type problems. Others saw the course as a way to prepare to teach AP Calculus, tutor calculus, or help returning students with their questions. Some wanted to polish problem solving skills and proof methods.

Answers to "What topics would you like to see stressed in this course?" ran the gamut: epsilon-delta proofs, curve sketching, related rates problems, integration techniques, differentials, the fundamental theorm of calculus, sequences and series, differential equations.

Students were even more revealing in informal conversation: many felt enormous anxiety about their calculus rustiness and fear that the course would begin at a level beyond their ability. It was these voices I kept hearing as I planned the syllabus. I was trying to find a way to respond to this level of need, while still serving the better prepared and more confident students. And all of this in twenty-four hours spread over two weeks.

About three weeks before class began, students were asked in a memo (Appendix C) to do some preparatory work for the class. They were asked to review functions, limits, and the derivative, with a focus on the definition of the derivative and differentiation formulas (product rule, quotient rule, etc.). Because of this preparation, I felt able to assume that these formulas were familiar when the class actually began.

3. FRAMEWORK. The first class began with a short pre-test, meant to determine the entering level of the twenty-three registered students. The pre-test (Appendix D) consisted of ten routine questions on limits, derivatives, antidifferentiation, and integration. Each question was worth one point (no partial credit) and scores on this test ranged from 2 to 10 with a mean of almost 6 and the mode of 5 occurring five times. Once again the wide range of preparation and skill in this group was displayed.

A predominant student request was that we cover a lot of ground; in essence, we wanted to review the material of two semesters of undergraduate calculus in two weeks, and do it in as sophisticated a way as possible. We wanted to understand the connections between various topics, examine the hypotheses of theorems, look for the motives behind proofs, and escape from rote as much as possible. I decided that formal lecture was the most efficient way to use our time. If the group had been more homogeneous in its background, I would not have chosen this route, and would instead have focused on group projects with much more student self-direction.

So class consisted of lecture with a great deal of give and take. Homework problems were discussed, alternative techniques were offered, counter-examples were enthusiastically produced, different philosophies of teaching were espoused. The atmosphere was relaxed and fun, but we covered a lot of ground together.

There was homework each day, some of it taken from Barron's How to

Prepare for Advanced Placement Examinations: Mathematics, the rest on dittoed handouts (Appendix E). These handouts consisted of routine problems taken from several undergraduate calculus texts. The homework was self-corrected and not collected or graded. It often served to facilitate classroom discussion. At the end of the first week, a take-home "mid-term" (Appendix F) was distributed, to be returned on Monday. Students were permitted to consult any books for help with this exam, but other people could not be consulted. An in-class final (Appendix G), which was more routine than the mid-term, took up two hours of the last class meeting. About thirty percent of this final consisted, by agreement, of the same questions covered on the pre-test. The average of these two test grades determined a final average, which, together with my subjective evaluation of class participation, determined the final course grade.

Three members of the class, all strong calculus students, agreed to serve as coaches. They made themselves available outside class hours to the other students, helping enormously with drill and clarification. The cooperative spirit was further encouraged by the daily group lunch traditionally shared by TTI students and teachers.

Like most college mathematics departments, Hofstra is ordinarily awash with sample calculus texts, sent for examination by publishers. We had enough of these to provide a good reference library for the students in the course, and at the end of the course nearly everyone received one of these books in a raffle.

4. COURSE OUTLINE.

Week One (nine hours): Limits, continuity, the derivative, differentials.

Week Two (fifteen hours): Related-rates problems, curve sketching, antiderivatives, the integral, the exponential and natural log functions.

These topics had each been mentioned by several students as important to them. But because of limited time, many topics that students wanted to cover were not included. In particular, sequences and series and differential equations had garnered a lot of interest but had to be excluded, because our time was so limited.

5. DETAILS OF COURSE OUTLINE. This is not meant to be an exhaustive

discussion of the syllabus. Instead it will focus on some of the things that worked particularly well in the class.

Limits: We focused on intuitive results, but did do some simple epsilon-delta proofs, to demystify this technique. We spent a lot of time looking at the geometric figure usually used to prove $(\sin\theta)/\theta \to 1$, as $\theta \to 0$, focusing on the fact that the length of an arc and the length of a side of a triangle are being compared in this ratio. There was a lot of discussion as to why radian measure needs to be used to get this result, leading up to a homework question which asked what happens if θ is measured in degrees instead of radians. In the coverage of indeterminate forms, great emphasis was placed on what term dominates in a particular expression and the class was able to come up with many examples to clarify this subject.

Continuity: The focus was on removable singularities and essential singularities. A discussion of the Intermediate Value Theorem gave the class a chance to test the necessity of each of the hypotheses in the theorem. Why is continuity required? Why must the function be defined on a closed interval? The class quickly produced examples to show the need for these conditions. This analysis encouraged us to be more careful in examining the hypotheses of each theorem we were to encounter. Instead of looking for where the hypothesis is needed in a proof (a worthy goal) we asked ourselves what would happen if a hypothesis were dropped.

Differentiation: Of course we defined the derivative as the limit of the usual difference quotient, and discussed its significance as tangent slope and instantaneous rate of change. We noted that the tangent at a point is the best linear approximation to the graph at that point and briefly mentioned Taylor polynomials as an extension of this idea to polynomial approximations of functions. In addition we carefully examined the relationship between differentiability and continuity. The function

$$f(x) = \begin{cases} 1, & x \le 0 \\ 2, & x > 0 \end{cases}$$

helped in this discussion, since a careless look at this function gives the erroneous conclusion that $f'(0) = 0$. This gives new respect to actually taking the right and left-hand limits of the difference quotient and shows how important continuity is. We also spent some time sketching the graph of f' when f is known, and talking about how a knowledge of what f' looks like helps in sketching the graph of f.

The Extreme Value Theorem gave another opportunity to closely examine the necessity of each hypothesis in a well-constructed theorem, and Rolle's Theorem and the Mean Value Theorem were good exercises in proof techniques.

Instead of proving L'Hospital's Rule, we motivated the 0/0 case with the following argument. If $f(a) = g(a) = 0$ and $f'(a)$ and $g'(a)$ both exist with $g'(a) \neq 0$, then for $x \neq a$

$$\frac{f(x)}{g(x)} = \frac{f(x) - f(a)}{g(x) - g(a)} = \frac{[f(x) - f(a)]/(x - a)}{[g(x) - g(a)]/(x - a)}$$

Taking the limit as $x \to a$, this can be seen as a special case of L'Hospital's Rule.

We also noted that L'Hospital's Rule does not always work, sometimes cycling unproductively, as in question 2 on the take-home mid-term. Everyone felt this was a good lesson to pass along to students who become too enamored of L'Hospital's Rule.

Differentials: Many students wanted to cover this topic because it turns up with regularity on the AP Calculus exam and it really did turn out to be a way to once again emphasize the difference between the secant and the tangent and to view mathematical analysis as a process of approximation. It isn't that we did anything earth-shakingly new; it just sharpened insight.

Related Rates: Nothing new and exciting here. Just an emphasis on old-fashioned problem solving techniques.

Curve Sketching: Again, we used the tried and true methods of calculus, but also spent a lot of time talking about translations, behavior at ∞, symmetry, and asymptotes (some people were surprised to see that the graph of a function can cross an asymptote).

Antiderivatives: This is the point where the pressure of time was really felt. We had time for only the simplest methods of substitution and the tabular method of integration by parts. (A brief discussion of this integration by parts algorithm, which tends to be spread by word-of-mouth rather than in text books, can be found in the 7th edition of Calculus and Analytic Geometry by Thomas and Finney published by Addison-Wesley. The technique can also be glimpsed in the film "Stand

and Deliver.") One simple u-substitution that some students had not seen before occurs in an integral like $\int x(3-x)^{\frac{1}{2}}\,dx$, where letting $u = 3 - x$ shifts the power from the messy term to the simple term. Most people viewed this as an improvement over integration by parts for this type of problem.

Integration: Here we definitely emphasized concept and theory over technique. Most students felt that given time they cou master techniques on their own, while an exploration of theory together proved very valuable. Integral results about even and odd functions were easily deduced by looking at sketches. People were interested to learn that their students who go into engineering will find these kinds of results useful when dealing with Fourier series. We spent a lot of time discussing the implications of the Fundamental Theorem of Calculus, in particular the version that says any function which is continuous on a closed interval has an antiderivative, even if it cannot be found in closed form.

Proving the Mean Value Theorem for integrals, obvious from a sketch for a non-negative function, gave the class an opportunity to pull together results learned from the Extreme Value Theorem and the Intermediate Value Theorem. (Recall that this proof hinges on knowing that a continuous function on a closed interval attains both its maximum and minimum values and all the values in between.)

The exponential and natural log functions: We had a good time with this, first naively and intuitively constructing $y = a^x$, $a > 0$, by assuming continuity and using known results about rational exponents. We then noted that if this function is differentiable, then

$$f'(x) = \lim_{h\to 0} \frac{a^{x+h} - a^x}{h} = a^x \lim_{h\to 0} \frac{a^h - 1}{h}$$

Then we noticed that this function would be its own derivative (a very useful type of function) if there were a base that makes the limit on the right equal to one. By looking at graphs and arguing (again naively) from continuity, we concluded that such a number must exist, and that it lies between 2 and 3. We called it e.

Then from $\lim_{h\to 0} \frac{e^h - 1}{h} = 1$ it can be seen that $\frac{e^h - 1}{h} \approx 1$ when h is small, so $e^h \approx 1 + h$, and $e \approx (1 + h)^{1/h}$.

So we have $e = \lim_{h \to 0} (1 + h)^{1/h}$ and the substitution $x = 1/h$ gives one of the usual definitions of e.

Once e^x is defined it is easy to define the natural log of x as the inverse of e^x.

Once we had used this exponential to natural log approach, we did the development in the other direction, first defining the natural log as an integral in the ususal rigorous way and then obtaining e^x as its inverse.

There was interesting class discussion about which of these styles of presentation was better for teaching calculus to high school students, most people coming down on the side of the more intuitive "exponential first, log second" approach.

6. GRADES. Grading seemed to me to be the least important feature of this course. This was a highly motivated group of students, with very good basic mathematics skills, who wanted to upgrade those skills even further. They had been chosen to participate in ITI because of these attributes and they fitted the ideal very well. In addition, because students began with such disparate backgrounds, I was interested mainly in seeing individual improvement rather than measuring performance against some ideal. The take-home mid-term exam was somewhat challenging, but use of references was permitted. Grades ranged from 75 to 100. Grades on the fairly routine in-class final ranged from 73 to 105 (there was an extra-credit problem). Final course grades ranged from B- to A.

The pre-test became a post-test as the first part of the final. Post-test scores ranged from 8 to 10, with a mean of 9.3 and the mode of 10 occurring seventeen times. (Recall that pre-test scores ranged from 2 to10, with a mean of 6 and the mode of 5 occurring five times.)

7. STUDENT EVALUATIONS OF COURSE. An anonymous written evaluation form (Appendix H) was filled out by students at the last session. Most evaluations were very favorable. Apparently the course met the expectations of most members of the class. People felt their rusty skills were less rusty and were glad to have compared teaching techniques with other members of the class. Nearly everyone was surprised at how much

we had been able to accomplish in only two weeks. There was some concern about a lack of feedback on performance. (The first graded assignment wasn't due until the Monday of the second week and could not be returned until the next day.)

The one negative evaluation was extremely negative. The writer did not like the syllabus, the style of presentation, or me. This person would have preferred a much less structured presentation, with more independent and group work among the students.

Many students felt pleased to have been part of such a marathon and several asked when we could do it again to cover the topics that we didn't get to this time.

8. THINGS I WOULD CHANGE. The two week marathon schedule was exhausting for teacher and students. Life would have been much more pleasant if the class had run for three weeks, nine hours a week. This would have given time for more thorough absorption of material. Sometimes our pace was dizzying. A three week schedule would also have given time for more feedback to students on their performance and perhaps even provided a little leeway for cooperative projects among students.

Choosing a review book as a text was a mistake. It simply did not contain enough problems for practice or good enough documentation of key results. We did have a very good library of standard calculus books for students to use for reference, but it would have been better to have each student have a copy of the same text, in addition to having access to the library.

7. THINGS I LIKED. Teaching this eager and curious group of talented high school teachers was a pleasure. Even though the skill range was extremely wide, nearly everyone contributed thoughts, ideas, and opinions to the group. The sharing of teaching techniques and philosophies was particularly useful. It was a delight to have a classroom filled with bright, curious, and cooperative students.

APPENDIX A

MATH 299B

DATE: July 25-August 4
INSTRUCTOR: Barbara Bohannon
OFFICE: 203 South Hall
PHONE: 560-5569
TEXT: Barron's How to Prepare for Advanced Placement Examinations: Mathematics. third edition.

I'd like to have a unifying theme for each session. beginning with a review of relevant ideas, but progressing by the end of the session to more challenging concepts and problems. This seems to be the only practical way to tame the huge calculus beast and fit it into our relatively meager time frame. Themes that have occurred to me are:

1. Functions, limits, and continuity.
2. The derivative and differentiability.
3. Curve sketching.
4. Other applications of the derivative--related rates, applied max.-min. problems, rectilinear motion.
5. Techniques of integration.
6. Integration (the definite integral)--the fundamental theorem.
7. Applications of integration--area, volume, arc length, average value.
8. Sequences and series.
9. Elementary differential equations.

COMMENTS

1. This list is not meant to be exhaustive and some topics deserve more of our time than others.
2. It is probably best to restrict ourselves to topics of single variable calculus. After all, they take up two semesters in the ordinary college curriculum and we have only two weeks.
3. There should be a balance between theory and problem solving. Some proof methods are highly instructive and deserve our time.
4. Grade will depend on problem sets to be done for homework, two one hour in-class exams, and class participation.
5. I'm eager to hear your ideas and suggestions. Please fill-out the accompanying questionnaire.

1. When is the last time you took a calculus course?

2. Rate your current calculus knowledge and skill. (Please circle one but feel free to add comments as well.)

   excellent   very good   good   so-so   poor

   Comments:

3. Have you taught AP calculus?

4. What do you expect to gain from this course? Be as global or specific as you want. (For example, if gaining skill in epsilon-delta proofs is important to you, say so.)

5. What topics would you like to see stressed in this course? Use my list of nine unifying themes to get started, but feel free to depart from this list. Are there any items on my list you feel are not central to your purpose in taking this course? Is there anything missing?

6. Do you have any other comments or suggestions? Please use the other side of this sheet if necessary.

TO: Students in Math 299B
FROM: Barbara Bohannon
DATE: July 5, 1988

The questionnaires you returned show that there is a very wide range of current calculus knowledge and facility in the class. I'd like to try to make that range less wide by having you review some elementary material before class begins.

Please read chapters 1 & 2 of the text and do the associated exercises. Also read chapter 3 and do as many of the exercises as you can. In particular, you should know the definition of the derivative on p.28 and formulas 1-14 on p.29. If we all have these ideas in common, we'll be able to get to more interesting and challenging ideas sooner.

I'm looking forward to seeing you on July 25.

Math 299B Pretest

① Find the limit, if it exists.

ⓐ $\lim_{x\to 2} \dfrac{x^2+x-6}{x^2-4}$ ⓑ $\lim_{x\to +\infty} \dfrac{\sqrt{x^2+1}}{3x-2}$

② Find $f'(x)$.

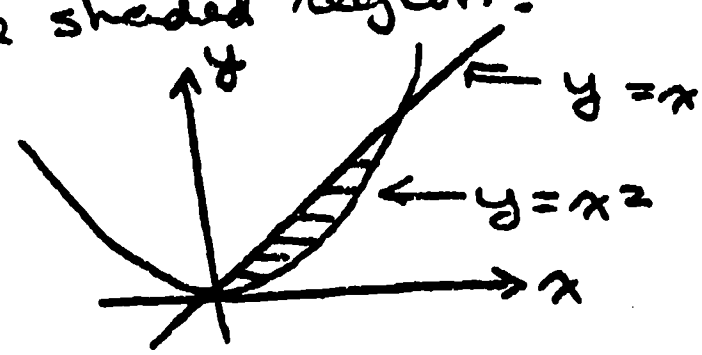
ⓐ $f(x) = \sqrt{3x-5}$ ⓑ $f(x) = \sin^2(5x+2)$

③ Find the equation of the tangent to $f(x) = 3x^3 - 1$ when $x=1$.

④ Suppose $f(3)=1$, $f'(3)=-1$, $g(1)=3$, $g'(1)=2$.

Find $\dfrac{d}{dx}\left[f(g(x^2))\right]\Big|_{x=1}$.

⑤ Find the area of the shaded region.



⑥ Find the following antiderivatives.

ⓐ $\int x\sqrt{4-3x^2}\,dx$ ⓑ $\int x\sqrt{3-x}\,dx$

299B Homework 1

① Text: Exercise set 2 (p. 24–27)

② What happens to $\lim_{\theta \to 0} \frac{\sin \theta}{\theta}$ if θ measured in degrees?

③ Use ε-δ proofs to show

ⓐ $\lim_{x \to c} k = k$ ⓑ $\lim_{x \to c} x = c$ ⓒ $\lim_{x \to c} (ax+b) = ac+b$

④ Give an example to show that the existence of $\lim_{x \to c} [f(x)+g(x)]$ does not imply the existence of $\lim_{x \to c} f(x)$ or $\lim_{x \to c} g(x)$.

⑤ Suppose $f(x)g(x) = 1$ for every x and $\lim_{x \to c} g(x) = 0$.

Prove $\lim_{x \to c} f(x)$ does not exist.

⑥ Find all points of discontinuity and classify as removable or essential.

ⓐ $f(x) = \frac{x^2 - 3x}{x-3}$ ⓑ $f(x) = \frac{x-2}{|x|-2}$

⑦ Let f and g be continuous on $[a,b]$ with $f(a) > g(a)$, $f(b) < g(b)$. Show there is at least one solution of the equation $f(x) = g(x)$ in (a,b).

[Hint: look at $f(x) - g(x)$].

⑧ Let $p(x)$ be a polynomial of odd degree. Show $p(x) = 0$ has at least one real root.

Math 299 B Homework 2

① Text: Exercise Set 3 p 43/1–10, 18, 19, 25, 32–35, 37, 43, 47, 63.

② Show $\frac{d}{dx}\tan x = \sec^2 x$

③ Given

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	1	3	−2	−1
−2	−2	−5	1	7

Find

ⓐ $\frac{d}{dx}\left[f^2(x) - 3g(x^2)\right]\Big|_{x=1}$

ⓑ $\frac{d}{dx}\left[f(x)g(x)\right]\Big|_{x=1}$

ⓒ $\frac{d}{dx}\left[f(g(x))\right]_{x=1}$

ⓓ $\frac{d}{dx}\left[g(g(x))\right]\Big|_{x=-2}$

ⓔ $\frac{d}{dx}\left[f(-\tfrac{1}{2}x)\right]\Big|_{x=-2}$

ⓕ $\frac{d}{dx}\left[f(g(4-6x))\right]\Big|_{x=1}$

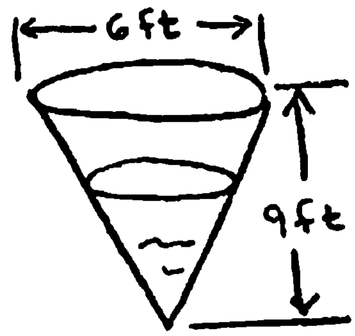
ⓖ $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]\Big|_{x=-2}$

MATH 299B SUMMER 1988 BOHANNON
DITTO 3

1. My house is ten miles from Hofstra. This morning it took me twenty minutes to drive to work. I didn't stop along the way. Prove that my speedometer said 30 miles per hour at some time on my trip.

2. Air is being pumped into a spherical balloon so that its volume increases at a rate of 100 cm^3/sec. How fast is the radius of the balloon increasing when the diameter is 50 cm?

3. Water leaks out the bottom of the conical tank shown in the figure at a constant rate of 1 ft^3/min.
   (a) At what rate is the level of the water changing when the water is 6 ft deep?
   (b) At what rate is the radius of the water changing at this instant?



4. A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/sec, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?

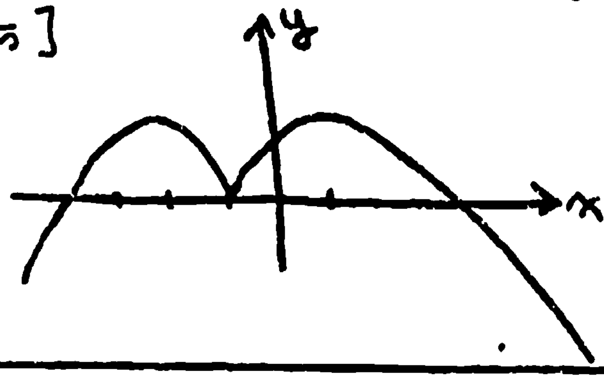
5. Sketch f(x) for each of the following.
   (a) $f(-1) = 4, f(1) = 0.$
   $f'(-1) = f'(1) = 0, f'(x) < 0$ if $|x| < 1, f'(x) > 0$ if $|x| > 1.$
   $f''(x) < 0$ if $x < 0, f''(x) > 0$ if $x > 0.$

   (b) $f(-1) = 4, f(1) = 0.$
   $f'(-1) = 0, f'(1)$ does not exist, $f'(x) < 0$ if $|x| < 1, f'(x) > 0$ if $|x| > 1.$
   $f''(x) < 0$ if $x \neq 1.$

Math 299 B Take Home Quiz Due Monday Aug. 1

All work should be your own. Consulting books is O.K. Consulting people is not o.k.

(1) Sketch the graph of $f'(x)$ if $f(x)$ looks like:
[7 points]



(2) Consider $\lim_{x\to+\infty} \frac{\sqrt{x^2+1}}{x}$. Try L'Hopital's rule
[6]
to find this limit. What happens? Find limit some other way.

(3) If $f(x)=x^2$, $h(x)=f(1+g(x))$, $g'(1)=1$
[6]
and $h'(1)=1$, Find $g(1)$.

(4) For what value of x is $f'(x)$ not defined for
[6]
$f(x)=|x+3|$?

Find $f'(x)$.

[5] (5) Find the equation of the tangent line to the graph of $2x^2+2xy-y^2=2$ at the point $(1,2)$.

[5] (6) Use differentials to approximate $\cos 31°$.
(Be careful. Remember derivative formulas are in terms of radians.)

[10] (7) Find a non-zero k so that $f(x)$ is continuous at $x=0$.

$$f(x)=\begin{cases}\frac{\tan kx}{x}, & x<0\\ 3x+2k^2, & x\geq 0\end{cases}$$

⑧ Let $g(x)$ be given by
[15]

$$g(x) = \begin{cases} -1-2x, & x < -1 \\ x^2, & -1 \le x \le 1 \\ x, & x > 1 \end{cases}$$

ⓐ At what values is $g(x)$ differentiable?

ⓑ Give a formula for $g'(x)$.

ⓒ Sketch the graphs of g and g'.

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[10] ⑨ Let $f(x) = x^{2/3}$, $a = -1$, $b = 8$.

ⓐ Show there is no point c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

ⓑ Explain why this result does not violate the Mean Value Theorem.

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⑩ Use the Mean Value Theorem to prove
[10]

$|\sin x - \sin y| \le |x - y|$ for all real values of x and y.

MATH 299B FINAL EXAM SUMMER 1988 BOHANNON

PART A: PRETEST REVISITED

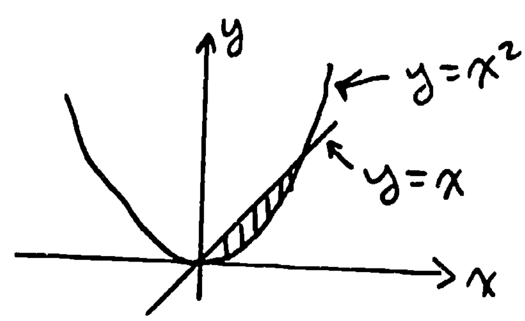
1. [2 points] Find the limit, if it exists.
   (a) $\lim_{x \to 2} \dfrac{x^2 + x - 6}{x^2 - 4}$ (b) $\lim_{x \to \infty} \dfrac{(x^2 + 1)^{\frac{1}{2}}}{3x - 2}$

2. [6] Find $f'(x)$.
   (a) $f(x) = \sqrt{3x - 5}$ (b) $f(x) = \sin^2(5x + 2)$

3. [4] Find the equation of the tangent to $f(x) = 3x^3 - 1$ when $x = 1$.

4. [2] Suppose $f(3) = 1$, $f'(3) = -1$, $g(1) = 3$, $g'(1) = 2$. Find
   $\dfrac{d}{dx}\left[f(g(x^2))\right]\Big|_{x=1}$

5. [5] Find the area of the shaded region.



6. [12] Find the following antiderivatives.

   (a) $\int x\sqrt{4 - 3x^2}\,dx$ (b) $\int x\sqrt{3 - x}\,dx$

PART B

1. [12] Prove (a) or (b), but not both.
   (a) If $f(x)$ is continuous on a closed interval I and $f'(x) = 0$ for every x ... the interior of I, then $f(x)$ is constant on I.
   (b) If f and g are differentiable and $k(x) = f(x)\,g(x)$, then $k'(x) = f(x)g'(x) + g(x)f'(x)$.

2. [25] Let $f(x) = \dfrac{2(x^2 - x)}{(x+1)^2}$.

Sketch the graph of the function, being sure to consider symmetry, intercepts, asymptotes, relative extrema, and inflection points. You may assume

$$f'(x) = \frac{2(3x - 1)}{(x+1)^3} \qquad f''(x) = \frac{12(1 - x)}{(x+1)^4}$$

3. [12] Two cars start from the same point. One travels north at 60 mi/hr and the other travels east at 25 mi/hr. At what rate is the distance between them changing two hours later?

4. [6] At what value or values of x does the graph of $y = xe^{-x}$ have a horizontal tangent?

5. [7] Find the antiderivative.

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$$

6. [7] Integrate.

$$\int_{-1}^{0} \frac{x^2}{x^3 - 1}\,dx$$

EXTRA CREDIT: Find the derivative of the function

$$y = F(x) = \int_{1}^{x^2} \frac{e^t}{t}\,dt \quad \text{, when } x > 0.$$

Hint: Make a change of variables.

HOFSTRA UNIVERSITY
Teacher Training Institute
SUMMER CODA 1988
August 1988

COURSE AND TEACHER EVALUATION
Questionnaire

The Institute would appreciate your considered and honest response to this questionnaire.

Below is a list of questions concerning your impression of the course and the instructor. Answer each question by circling the letter which corresponds most closely to your judgement. The extremes are defined as indicated.

The course is Math 299B

The instructor is Barbara Bohannon

		A	B	C	D	E	
1.	The instructor's daily presentation is (well organized)						(disorganized)
2.	The ability of instructor to transmit information is (excellent)						(poor)
3.	The instructor's knowledge of the subject matter is (outstanding)						(inadequate)
4.	Does the instructor see to it that the classroom situation is conducive to questioning? (very much)						(not at all)
5.	The answers the instructor given to student's questions are (excellent)						(poor)
6.	If a student asks a question, is the instructor concerned with whether or not the student understands the explanation the instructor gives (very concerned)						(unconcerned)
7.	Outside of class the instructor is (available)						(unavailable)
8.	Does the instructor stimulate interest in the subject matter? (very much)						(not at all)
9.	Would you recommend that a friend take a mathematics course from this instructor? (definitely)						(never)
10.	What is your perception of the amount of mathematics you are learning in this course? (a lot)						(little)

11. The tests are (too difficult) (too easy)

12. From taking the tests I learn (a lot) (nothing)

13. The grading is (too harsh) (very lenient)

14. Are the homework problems assigned helpful in learning the course material? (very helpful) (not helpful)

15. The amount of homework assigned is (too much) (too little)

16. Does the instructor provide you with a means to judge your progress in the course (for example, corrected homework, frequent quizzes or tests, etc.)? (yes) (no)

17. With respect to other college level instructors, I would grade this instructor

18. Including the homework, how much studying do you do for this course per week outside the classroom

19. Class attendance during semester (always came) (never came)

20. What is your grade so far in this course?

21. Usefulness of course

22. The following strengths impress me most about this instructor:

23. The following weaknesses are evident to me about this instructor:

4. Please make any other comments about this course, the instructor, the Institute or this evaluation questionnaire:

APPENDIX I

Student Commentary on the Teacher Training Institute
Math 299B: Calculus in the Secondary School

After reading Dr. Barbara Bohannon's report and commentary on the calculus course she presented, I was reluctant to offer any further discussion. Her report is all inclusive and certainly encompasses all of the thoughts I could have suggested.

I was and I stress the word was, one of the students who initially indicated that my calculus background was 20+ years old and by virtue of non-use very, very rusty. The careful preparation by Dr. Bohannon that went into this course has since changed my abilities. In other words, I feel that the course met the main objective I had when entering the course: a refresher that would enable me to feel confident in front of a classroom of first level calculus students.

By assigning an initial "get yourself ready" preparatory assignment, the students should all have a relative starting point. If the student did not take this assignment seriously, he/she would have been at a great disadvantage. Once the actual classroom instruction began, the tempo quickened each day. The pace was incredible,but dispite the fact that I was simultaneously taking a computer Pascal (data structures) course that demanded many hours of outside work, I was able (barely) to keep up with the assignments and course demands. During the second week of class, when the Pascal course was over, the work load seemed more acceptable. (Many outside hours were necessary for someone like me to keep up.)

The topics included in the course seemed appropriate and of interest to the varied backgrounds of the students. One must not forget that many were like me, while others were already teachers of the advanced placement high school course in calculus. All of us seemed to be challenged, just some more than others!

As a result of Dr. Bohannon's course, I accepted a surprise teaching assignment at Hofstra, namely a four credit undergraduate course in calculus with applications. Prior to taking Math 299B this would not have been possible as I am the type of person who will only do those things that I feel confident and comfortable with. Thus the National Science Foundation concept of sharing that which you have learned certainly is applicable in my case!

Respectfully submitted by
Irene Ober

April 13, 1989