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ABSTRACT

Classrooms in our nation's schools reflect an "assembly-line model" of work in which the educational product is a set of students' performance and abilities learned in school. Contemporary educators are critical of the old factory model and seek to replace it by classrooms whose goals are to learn and to develop knowledge. Studied is the case of third-grade teacher Keisha Coleman, who, during the 1989-1990 school year, revised her views of the learning and teaching of mathematics and significantly changed her teaching practice. Beginning in October, the researcher spent 1 day a week in Ms. Coleman's mathematics classroom. Her lessons were observed, classroom discourse was audiotaped, and Keisha was interviewed after each lesson about how and what she was trying to teach, why she was trying to teach it, and what she hoped that the students would get out of the lesson. Reported here is a mathematics lesson taught by Keisha in November 1989, focusing on several major revisions that occurred in her mathematics teaching. She moved away from "teaching as telling" and moved towards guiding discussions as students figured out solutions to mathematical problems for themselves. Rather than focusing on covering mathematical content, Ms. Coleman focused on solving mathematical problems and discussing students' solutions and explanations. The ways that Keisha revised her thinking about teaching are discussed and possibilities for further revisions in her thinking. (KR)

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Center for the Learning and Teaching of Elementary Subjects

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Abstract

During the 1989-90 school year, third-grade teacher Keisha Coleman revised her views of the learning and teaching of mathematics and significantly changed her teaching practice. Beginning in October a classroom researcher, Penelope Peterson, began spending a day a week in Ms. Coleman's mathematics classroom. She observed lessons, audiotaped the classroom discourse, and interviewed Keisha after each lesson about how and what she was trying to teach, why she was trying to teach it, and what she hoped that the students would get out of the lesson. In this report Peterson examines a mathematics lesson that Keisha taught in November 1989, focusing on several major revisions that occurred in Keisha's mathematics teaching after she observed the teaching of her long-time colleague and peer Deborah Ball, a university professor/researcher and third-grade teacher in the same school. As Peterson explores the thinking that underlies Keisha's teaching, she shows how Keisha's thinking has changed and speculates on the reasons why. She concludes by considering the possibilities for further revisions in Keisha's thinking and in her mathematics teaching.

REVISING THEIR THINKING: KEISHA COLEMAN AND HER THIRD-GRADE MATHEMATICS CLASS[1]

Penelope L. Peterson[2]

Classrooms in our nation's schools reflect an "assembly-line model" of work reminiscent of the era of Henry Ford. Yet contemporary experts in business and industry argue that the nature of work has changed even in manufacturing and that the quality of the educational "product"--students' performance and abilities learned in school--needs to be changed and improved to keep the United States competitive in a global economy (see National Academy of Engineering, 1985; Ross, 1988; Zuboff, 1984). Business and industry need workers who are literate and numerate, think for themselves, work and learn collaboratively, solve problems, access and use knowledge as needed, and revise and transform the information given.

Contemporary educators, too, are critical of the old factory model as a way of conceptualizing what should be going on in classrooms. For example, in a recent article, Marshall (1988) pointed out the limitations of the "workplace metaphor" as a way of thinking about classrooms. In its place, she suggested the metaphor of "learning-oriented classrooms." She argued that, in contrast to the old factory model where the goal was to

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[2]Penelope L. Peterson, professor of educational psychology and teacher education at Michigan State University, is codirector of the Center for the Learning and Teaching of Elementary Subjects. The author wishes to thank four people who read and commented on earlier versions of this paper--Sarah McCarthey, Jere Brophy, Deborah Ball, and Keisha Coleman. Ms. Coleman requested that her real name not be used so the author has honored her request. The author also thanks Nancy F. Knapp, Janine Remillard, and James Reineke who interviewed children in Ms. Coleman's class. Finally, the author expresses her appreciation to Ms. Coleman and the students in her third-grade class who allowed her to visit their mathematics class for a year and to look, listen, and learn along with them.

produce a product, in classrooms the goals should be to learn and to develop knowledge. Further, in contrast to these work settings where authority relationships were based on the status and expertise of the manager, in learning settings authority relationships should be "based on expertise and knowledge *to be shared or developed* rather than held by the authority and on the *desire to help individuals acquire or construct knowledge"* (Marshall, 1988, p. 14).

Underlying much of the rhetoric of the current reform both in industry and in education is a new view of knowledge. According to this view, knowledge is seen not as fixed and static but rather as continuously undergoing revision and transformation. Manufacturing is no longer seen as the application of fixed knowledge (e.g., as instantiated in the assembly line) to produce a static, unchanging product (e.g., your father's Oldsmobile). Rather, manufacturing is seen as "a process which transforms information into a product. The information includes design data, quantities required, and delivery dates. The transformation involves developing tools and processes, obtaining material, processing material, assembly, testing, and delivery" (Shea, 1985, p. 12). The information or knowledge itself is not viewed as static but rather as in need of ongoing revision and transformation because "the companies that gain nearly unassailable positions in the world market" will be those who are able "to produce quality products tailored to special customer requirements on a very short lead time" (Shea, 1985, p. 12).

Similarly, education reformers are creating new visions of the knowledge base for teaching and are offering new views of what it means to "know" and understand academic subjects. Shulman (1987) proposed that "a knowledge base for teaching is not fixed and finite," and he argued for building teaching reform on a view of teaching that emphasizes comprehension, reasoning, transformation, and reflection. The Mathematical Sciences Education Board of the National Research Council (MSEB, 1990) has asserted the need to change two popular and outdated assumptions--that mathematics is a fixed and unchanging body of facts and procedures and that to do mathematics is to calculate answers to set problems using a specific catalogue of rehearsed techniques. The Board proposes instead that "mathematics is a creative, active process;" that in mathematics, "reasoning is the test of truth;" and that "mathematics is a language--the language through which nature speaks . . . and an apt language for business and commerce" (MSEB, 1990, pp. 10-12).

What do these revised views of teaching and what it means to know mathematics imply for how mathematics should be taught and learned in elementary classrooms? This question has generated and is generating substantial debate and discussion among mathematics education researchers and education reformers as well as among teachers as they struggle to enact these new visions of mathematical knowing and teaching in their classrooms. Two compelling portraits of such attempts are provided by Lampert (1990) and Ball (1990).

In her teaching of elementary mathematics to fifth-grade students, Lampert tries to "bring the practice of knowing mathematics closer to what it means to know mathematics within a discipline by deliberately altering the roles and responsibilities of teachers and students in classroom discourse" (Lampert, 1990, p. 29). To do so, she has developed new forms of classroom discourse and teacher-student interaction where content and discourse are intertwined and words take on new meanings. Some of the words that take on new meanings are *knowing, thinking, explaining,* and *revising*. Lampert begins by posing a problem to her students. As students volunteer solutions to the problem, she writes them on the board for consideration. These solutions are up for discussion and revision. If students want to disagree with a solution, they say that they want to "question so-and-so's hypothesis" and then give the reasons for disagreeing. The student who gave the solution is free to respond or not to respond with a "revision." When a student says that he wants to "revise his thinking," he is using words that Lampert has encouraged her students to use, and he means that he wants to change his mind about an assertion that he made earlier in their class discussion. Lampert views this as important because "when a student is in charge of revising his own thinking, and expected to do so publicly, the authority for determining what is valid knowledge is shifted from the teacher to the student and the community in which the revision is asserted" (Lampert, 1990, p. 52).

Like Lampert, with whom she collaborates in a National Science Foundation project to document their teaching, Deborah Ball aims at developing a "practice that respects both the integrity of mathematics as a discipline *and* of children as mathematical thinkers" (Ball, 1990, p. 3). She strives to create a classroom environment in which the norms of discourse are informed by patterns of discourse in the mathematics community as well as by the culture of the classroom. Further, she strives to shift authority for mathematical knowledge from the teacher and the "text" to the community of knowers and learners of mathematics in her classroom.

While Ball and her students engage in extensive discourse in the whole-class setting, they also work in small groups. She tries to select and create mathematics tasks that engage students in learning the content of mathematics as they learn the ways of knowing. Ball (1990) provides an example of discourse from her third-grade mathematics class in which students discussed the problem 6 + (-6). Ball and her students spent over 30 minutes discussing solutions for that problem. At one point, a student gave the correct answer, but the student's explanation was problematic. Students gave two other solutions that received "equal air time." Ball states that at no time did she "tell or lead the students to conclude that 6 + (-6) equals zero--by pointing them at the commutativity of addition or at the need for the system of operations on integers to be sensibly consistent. At the end of class only half the students knew the right answer" (Ball, 1990, p. 26). However, Ball was not uncomfortable with this situation because she thinks that the time that students spend "unpacking ideas" is time well spent. Too often she has seen evidence of students who fail to understand even though they have been "taught" the mathematical procedure. Ball noted that when they "moved on from negative numbers a week or so later almost every student was able to add and subtract integers accurately if the negative number was in the first position, for example, -5 + 4, or -3 - 8" (Ball, 1990, p. 25).

The portraits provided by Lampert (1990) and Ball (1990) provide two examples of how teachers might enact revised views of mathematics and what it means to know mathematics in their elementary classrooms. Such case analyses are important because they provide insights into the dilemmas that elementary teachers face as they attempt to enact reformers' visions of desired changes in mathematics instruction--less emphasis on practice of isolated computational skills, more emphasis on understanding, problem solving, and flexible, mathematical reasoning (e.g., National Council of Teachers of Mathematics (NCTM), 1989; National Research Council, 1989). Such case analyses are needed both to advance researchers' understandings of what it means to learn and teach mathematics with understanding in light of calls for reform and also to inform teachers, teacher educators, and others as they attempt to effect fundamental changes in mathematics teaching and learning in our elementary schools (Hiebert & Carpenter, in press). The purpose of the present study was to attempt to understand another teacher's attempt to enact these revised views in her classroom.

During the 1989-90 school year, I studied an elementary teacher, Keisha Coleman, as she revised her teaching of third-grade mathematics. In this paper I relate a bit of the story of Keisha Coleman's mathematics teaching--a story that is, of course, still unfolding. I focus on the changes in her thinking and her mathematics teaching during the fall of that year and how they came about. I conclude with some tentative ideas about what I have learned and some questions for further thought.

A Brief Note on Method

In October 1989 I began spending at least one day a week in Keisha's third-grade classroom observing her teach mathematics. During my observations, I wrote narrative descriptions of what occurred, focusing particularly on the discourse and the mathematics that was taught. I recorded what was written on the board and any written work that the students did. The teacher and student discourse in each lesson was audiotaped and later transcribed. During the post-observation interview, my conversation with Keisha focused on what she was trying to teach, why she was trying to teach it, how she was trying to teach the mathematics, what she hoped that the students would get out of the mathematics lesson, and what she thought the students actually got out of the lesson. My post-observation interview questions and techniques were adapted from those we have used in the California Study of Elementary Mathematics (see Peterson, 1990) which were adapted from interviews developed by the National Center for Research on Teacher Education (1989). In conducting the interviews, I relied not so much on a structured interview format but, rather, on my own knowledge and experience gained from interviewing elementary teachers. Thus, I asked Keisha questions that would help me understand how she was thinking, how she construed the mathematics lesson, and how she thought about mathematics teaching and learning in her classroom.

In October, January, and June each student in the class was interviewed individually for one to two hours about their solutions for some mathematics problems and their thinking about these problems. The intent of the interview was to probe in depth the student's knowledge and understanding of key mathematical ideas and the student's attitudes and beliefs about mathematics and the learning and teaching of mathematics.

This chapter focuses on Keisha's mathematics teaching and her thinking about her mathematics teaching during the Fall of 1989. Peterson and Knapp (in preparation) provide further analyses of the teaching and learning of mathematics in Keisha's classroom during the Winter and Spring of the 1989-90 school year.

The "Learning" Context of Keisha Coleman's Classroom

Ms. Coleman teaches in a school that has a high percent of ethnically and linguistically diverse children, some of whom are eligible for and receive free or reduced lunch. Most of the children's parents are undergraduate or graduate students who are attending Michigan State University. Children in the school speak 20 different languages. Although some children attend ESL (English as a Second Language) classes, all the regular classroom teachers teach their lessons in English. Ms. Coleman, the principal, and the teachers in the school themselves represent the ethnic diversity characteristic of the school, and Ms. Coleman often focuses on issues of ethnic identity and culture in her teaching. She grew up in nearby Detroit, Michigan, and received her teaching degree from Michigan State University. She began teaching at her school 15 years ago, a year after her colleague Deborah Ball also began teaching there and the current principal started at the school.

During the previous year, Keisha's school had become a "professional development school" affiliated with Michigan State University in efforts "to develop and put in place new forms of teaching for genuine conceptual understanding in core subject areas, for problem solving and thinking skills, for higher order literacy, for the skills of learning to learn autonomously, for teamwork skills, and for other aspects of the education required for success in the emerging knowledge age society and economy" (College of Education, Michigan State University, 1989). A major focus of professional development schools is facilitating teachers' learning (see The Holmes Group, 1990).

At the beginning of the year, Keisha stated that one of her intentions for the year was to work on learning more and on changing and improving her mathematics teaching. This is how she defined her work for the year as an elementary teacher of mathematics. She agreed to be the focus of this study and to participate in the research project because she saw it as a way to reflect on, learn about, and work on improving her mathematics teaching. During the course of this case study year, Keisha engaged in several other influential learning activities including participating in long lunch-time meetings on Friday with the whole staff of the school and serving as a member of the East Lansing school district's mathematics committee. In addition, she observed two of her peers teach--Deborah Ball, a colleague in the same school, and Elaine Hugo. As the district's mathematics support teacher for the Comprehensive School Mathematics Program (CSMP) (CEMREL, 1985), Elaine Hugo came into Keisha's classroom and taught during the 1989-90 school year. (See Putnam & Reineke, 1991, for a case study of Elaine Hugo.)

CSMP is an innovative mathematics curriculum that focuses on mathematical problem solving and thinking and on providing students with distinctive mathematical tools (e.g., Venn diagrams or "string pictures," a kind of abacus called the "minicomputer," and "arrow roads" showing mathematical functions) for representing their mathematical ideas and thinking (see Remillard, 1991). As Hugo sees it, "CSMP has helped a lot of teachers do more verbalizing in math whether it's just by asking more questions or by getting the kids to talk more about mathematics" (Putnam & Reineke, 1991, p. 7). Based on their observations of Hugo's practice, Putnam and Reineke (1991) noted that as is consistent with her beliefs about the importance of giving students opportunities to talk about mathematics, Hugo structures her lessons to allow students to express their thinking. However, these researchers noted that in her classroom, Hugo gives the most attention to students' *correct* understandings. She is "fairly convergent about where she is going; she wants the students to say a particular thing" (Putnam & Reineke, 1991, p. 14).

Consistent with Hugo's idea that CSMP has helped teachers change, Keisha Coleman sees her mathematics teaching as having evolved over the last four years as she has been influenced profoundly by using CSMP. She reported that when she first started teaching, the district had an individualized mathematics program, and she hated it. "Everybody was just everywhere in the book," she said, and she felt that all her instruction "was just hit and miss." One of Keisha's colleagues in the school, a fifth-grade teacher who had also taught at the school for 15 years, referred to teaching mathematics during those years as "a paper chase." Then the district adopted CSMP, and Keisha felt that teaching CSMP really changed her feelings about mathematics and about teaching mathematics. As Keisha put it:

> With CSMP, I felt like I was *teaching* because of the questions that you're constantly asking children, trying to get them to rethink or to think about their responses rather than just giving an answer. I'm not real sure I was comfortable with that in the beginning, but I think that is a thing that helped me. I actually felt like I was *teaching* math, and that was a feeling that I wanted. There was all sorts of information, workshops that you could go to. We always had a reading consultant here in our district--somebody who could always assist you with any kind of problems in reading. We didn't have that in math so we began to reevaluate what we were doing. I think Debbie [Ball] sort of felt a lot of that too--trying to look for ways the teachers could actually teach math and feel comfortable with it. Before I had always told her [Deborah Ball], "Don't bring anything math-like my way because I'm not good at it."

Keisha's Classroom in November 1988: Following the "Text"

When I first met Keisha in November 1988 she had been teaching mathematics using CSMP for three years, and she taught like a teacher who was committed to using CSMP to teach mathematics. She held the CSMP teacher's guide in her hand while she was teaching, and she seemed to be reading from the guide much of the time. The picture (an "arrow road") that Ms. Coleman drew to represent mathematical ideas seemed to come from the text rather than from her own head or from the thinking of the students. She taught the entire mathematics lesson using lecture/recitation in teacher-led whole-group format. The CSMP teacher's guide is scripted with the kinds of questions the teacher should ask and the kinds of student responses the teacher should expect to get. Although Keisha's fourth-grade students verbalized some strategies that they used to solve mathematical problems that she posed, Ms. Coleman acknowledged or encouraged only students' contributions that seemed to fit with her "script." The classroom discourse was mostly convergent, focusing on coming up with a solution. For example, in one place in the mathematics lesson, Ms. Coleman asked the students to tell a story for the number sentence 20 - 14 = 6. One child told the following story: "Garner has 20 houses. A giant stepped on 14. How many are left?" Ms. Coleman responded by asking the students, "What is my question? What would my question be?" Although several children gave plausible responses to her query, Ms. Coleman ignored them because they were not the one she was looking for. Finally, she wrote on the board the response she had been looking for--the question "How many houses were left?"

The classroom dialogue was primarily teacher-student rather than student-student. Further, where Ms. Coleman had opportunities to explore students' thinking or "unpack" mathematical ideas, she often did not follow up on them. When Ms. Coleman asked her students to tell a story for the number sentence as described above, one fourth-grade child proposed, "There were 20 punks; 14 had mohawks. How many didn't have mohawks?" Another child suggested, "Steve has 20 cents. He bought gum with 14 cents. How many does he have left?" A third child pointed out that these two problems are different--one is comparing and one is subtracting--but he added "We still had to minus it." Although Ms. Coleman acknowledged this student's thinking as good, she did not build on it or ask other students what they thought of this student's idea.

A final noteworthy aspect of Keisha's mathematics teaching on that day was the extensiveness, yet disconnectedness, of the mathematical content that she covered and the classroom discourse that occurred. In a one-hour mathematics lesson Ms. Coleman covered four different mathematical topics, and she made no explicit connections between them. The first topic dealt with different ways to compute 6 x 7. The second topic involved asking the students to make up word problems for the number sentence 20 - 14 = 6. The third topic involved Ms. Coleman asking the students to make up a word problem like the one she gave: I have five envelopes. Each envelope has six picture postcards in it. How many picture postcards are there altogether? Finally, Ms. Coleman posed the following word problem from the CSMP teacher's guide:

> Andrew wants to buy a ticket good for one admission to Cedar Point. A ticket costs 9 dollars. Grandfather says he'll double whatever amount Andrew has. Grandma will give $1 more to Andrew. How much money does Andrew need before visiting his grandparents? Which should Andrew go to first (Grandmother or Grandfather)?

This problem proved to be quite challenging for the fourth-grade students in her class, so Ms. Coleman led the class through the solution by introducing a pictorial representation to help them solve it. Throughout the discussion of the problem solution, the talk was teacher-student-teacher-student with no student-student discourse about the solution to the mathematics problem.

To what extent did Keisha's mathematics teaching on that day reflect important elements of teaching mathematics for understanding? Certainly, her lesson did focus on students' mathematical thinking and mathematical strategies in addition to correct mathematical solutions. Students verbalized solution strategies to mathematical problems, and some of students' thinking was made visible to others in the class--at least that part of students' thinking that fit with the script. Recent experimental research in elementary mathematics classrooms has shown that teachers who spend more time having students verbalize their different solution strategies for solving word problems have students who do better on tests of word problem solving and as well or better on computation problems as students of teachers who spend more time on computation and less time on having students verbalize their different solution strategies for solving word problems (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). Further, reformers and researchers alike argue that having students verbalize different solution strategies for solving word problems is key to learning mathematics with understanding (see California State Department of Education, 1985; NCTM 1989; Fennema, Carpenter, & Peterson, 1989).

For this reason and because Keisha's mathematics teaching and her focus on students' thinking and problem solving stands out in stark contrast to most other elementary mathematics teaching where the focus is solely on mathematical computations and procedures, Keisha's mathematics lesson on this day was *exceptional.* Clearly, Keisha's purpose in teaching the CSMP lesson was for her students to think about and learn the mathematics she was teaching. Thus, if I had asked her last year and asked her again this year, Keisha probably would have agreed that, of course, the purpose of school and assignments should be *learning.* However, as we shall see, Keisha views herself as continuously learning how to teach mathematics, and she sees her mathematics teaching as undergoing substantial revision.

Learning From Her Peer and Colleague

Another major revision in Keisha's thinking and her mathematics teaching occurred during the Fall of 1989 when Keisha began observing the teaching of her long-time colleague and peer, Deborah Ball. Deborah Ball and Magdalene Lampert are Michigan State University professors and researchers, as well as experienced elementary teachers, who teach mathematics one period each day in the school in which Keisha teaches. During the 1989-90 school year, Keisha taught third grade in a classroom next tc the room where Ball daily taught third-grade mathematics and across the hall from the fifth-grade classroom where Lampert daily taught mathematics.

Keisha observed Deborah Ball's classroom for a week in November 1989. I also observed Deborah teach while Keisha was there. On November 17th, I returned to observe Keisha's mathematics classroom, and I was amazed by the discourse about mathematics that took place. Keisha posed questions and orchestrated the classroom discourse in ways similar to those of Deborah. These changes in Keisha's classroom behavior were unexpected, and I speculate that Keisha's easy facility and ability to perform these new behaviors might be related to her expertise as an aerobic dance instructor--a role that required her weekly to learn new steps and new verbal directions and to perform these with smoothness, drama, precision, and enthusiasm. Before I give you a glimpse into what I saw and heard in Keisha's classroom that day, I want to tell you about my conversation with Keisha before I observed her teach on that day.

Our conversation revolved both around what Keisha was going to do in mathematics class that day and what she had learned from watching Deborah. These were interconnected because her plans were based on ideas that she had developed from watching Deborah and talking to her about her mathematics teaching.

Thinking About Unpacking Mathematical Ideas

One thing that surprised Keisha was that Deborah and her students had spent an entire hour in thinking about and discussing only four mathematics problems. Keisha remarked that she was just amazed that the children were really involved in what they were doing, and it "was not tedious or busy-work." In one particular part of the lesson, Keisha recalled that two girls, Betsy and Leann, came up to the overhead projector. They had beans and sticks, and they were trying to "prove" why the answers to two problems, 92 - 65 and 93 - 66, were the same. Keisha retold what she had witnessed in an amazed voice:

> And you could just see the wheels turning in Betsy's head, just turning as she was trying to explain. Then she became frustrated because she knew that her solution or what she was trying to say was not clear to everybody. . . . Leann began to try to say what Betsy was trying to explain, but Betsy wasn't satisfied with that. . . . But Deborah let those two girls go on, and even some of the other children became involved with that exchange. . . . Never once did the kids turn to Debbie and say, "What is the answer? Why aren't you telling us?"

Keisha said that ordinarily she would never have let an exchange like that between Betsy and Leann go on in her class. However, now she is reconsidering that. She also was surprised that by the end of the class the students never came to any conclusion as to why those two numbers equal the same. That bothered her at first, and it still bothers her a bit, but then she thought about it and said to Deborah, "You know, in my opinion, that might even stimulate interest in the other children--to try to go home and work that out. And they might come back and say, 'Well, you know, I finally figured out why those two numbers equal the same.' "

Thinking About New Mathematical Language

Keisha also noted that not only in this episode but throughout Deborah's lessons a lot of talk went on about proving. She thought that the children obviously understood what Deborah meant when she talked about "proving" because the children themselves were using the word "proving." Keisha said that she planned to bring up the idea of "proving" with her children today by asking, "How do you think you might be able to prove an answer is right or correct?" She said that she was not sure that they would be able to give her very much but that she planned to write down what they said on the board and "just kind of go from there."

Keisha noted another important thing about Deborah's class:

> When a child comes up with a method for solving something, Debbie writes that down, and she posts it in the classroom. And the kids agree whether or not it works or it doesn't work. But then they are challenged to find out whether or not this method would work for other things or not or for other situations. And it may be two days or two weeks down the road where a child will come up and challenge the method and say that it needs to be revised. . . . Deborah also has these children write these methods down in their notebooks, and they are also still posted in the classroom, so they can challenge them at any point. What better way for an idea to stay with a child?

Then Keisha embellished upon why she thought it might be important to write down students' ideas about mathematical methods. She said that last week her children were working with a string picture (Venn Diagram), and one circle in the string picture stood for even numbers and the other circle stood for multiples of five. She and the students began talking about what an even number was. She commented that although this was very new to her, she listened to what the children had to say and then wrote it down. But she didn't write it down on construction paper and post it in the room. The next Monday when they came back to it and talked about it, she asked the children, "Can you remember what we said about that?" Nobody could remember. She said that she could just "kick herself for that" because she knows it is going to come up again. She would like to be able discuss with them the questions "Is this true for all even numbers?" and "Can you find a situation where it doesn't apply to an even number?"

Thinking About Deborah's Knowledge of Her Students' Mathematical Knowledge

A theme that Keisha brought up throughout the interview dealt with what she viewed as the impressive knowledge that Deborah had of her children's mathematical knowledge and thinking. She reported that she had seen the report cards that Deborah had done on her students. Deborah had written narrative descriptions of each student's mathematical knowledge and thinking.

Keisha wondered, "How does Deborah get that kind of in-depth knowledge of her children's mathematical thinking and understanding?" Keisha commented that all too often she was frustrated because although she had taught a mathematics lesson or given the children practice sheets, she was never able to sit down with the children individually and talk with them about how they figured something out. Thus, she felt like she never knew if the child had "grasped onto" what she had taught. Now she set a goal of being able to know more *in depth* what her children know and understand in mathematics.

Thinking About Tools to Assess Students' Understanding

Keisha talked about three tools that she noticed Deborah uses to get this kind of in-depth knowledge and understanding of her children's knowledge and thinking. She speculated on how she might incorporate some of these techniques in her own teaching. One thing she noted is that Deborah gains extensive knowledge and understanding of her children's mathematical thinking and knowledge from the kind of discourse that occurs during her mathematics class, in which the thinking and understanding of individual children become more visible because they are the focus of discussion. On the other hand, when she reflected on what

Deborah had written in her narrative reports of children, Keisha thought,"Geez, how is she getting all this? She can't be getting it just from the discussion period."

Keisha speculated that another way Deborah learns about her students' mathematics thinking is by having the children write their mathematics work in a notebook. Not only do they have to solve the problems in their notebooks but they also have to prove their answers, either by using manipulatives or writing out explanations. Further, Deborah has her students write in ink rather than pencil, so that by not being allowed to erase, she can, "Just kind of see their thinking so that she can see the children's thinking, through their mistakes, or as they cross things out, or as they try to rework the problems." Keisha noted that Deborah also has the children use manipulatives such as popsicle sticks to prove their answers so that she and other members of the class can see how the students are thinking about the problem.

Keisha told me that her goal is to get information about how her children are thinking about things. She feels that she does that for reading, but she wants to be able to do the same thing for mathematics, social studies, and science. Although she was not sure yet how she would use this information, she had a clear goal for what she thought she wanted the children to be able to do: She wanted her class to be able to say, "This is how I'm thinking about this," and then for students to say, "I think that your idea is good, but I think we need to look at this too." Keisha emphasized that she wanted to be "able to see the wheels turn in terms of their own thinking, and I don't have that right now."

A third tool that Deborah uses that Keisha planned to incorporate in her own teaching is one that she had just tried. Keisha gave her students a homework sheet that she had gotten from Deborah. On the top of the homework sheet was a number line. Then there were some problems like "$\hat{10}$ plus 10 equals . . . " and "0 minus 2 equals . . . " (In the CSMP textbook, a ^ above a number represents a negative number.) Under each of these problems was the question "How do you know?" Keisha thought that this is one way that Deborah is able to come to know and understand her students' mathematical thinking--by looking at her students' answers to such questions.

Keisha gave the worksheet to her children the night before, and she reported that although the children wrote down answers to the problems, only one or two of the children actually gave reasons why their answers came out the way they did. She said that she made comments on each one of the children's papers in terms of how they responded. Looking later at the kinds of comments that Keisha had written on the students' worksheets, I found they were open-ended queries that seemed to be intended to get the student to think about his or her answer and how he or she got it. For example, on Titon's paper, Keisha had written, "Are you sure about your answers?" On Ben's paper, she had written, "How do y know?" Tara had written "0" for the answer to "$\hat{10}$ + 10," and the reason that Tara had given was, "Because we give tacks away things." On Tara's paper, Keisha had written, "What does this mean?"

Keisha assessed her students' responses to the homework by saying, "You know, my kids have never had to do that before--write down a reason why their answer is correct or how they got the answer." She stressed that that was one of the things she intended to talk about today--proving your answer.

With that we concluded our conversation, and Keisha moved to begin her mathematics teaching for the day. What did Keisha's mathematics lesson look like that day and how did it reflect the thinking that she was doing about her teaching? Let us take a look.

<u>Looking in Keisha's Classroom in November 1989</u>

The mathematics lesson in Keisha's third-grade classroom on that day revolved around one problem from the homework assignment that Keisha had given her students the night before. The homework sheet was the one that Keisha had gotten from Deborah Ball. At the top of the worksheet was this number line:

<---. ---. ---. ---. ---. --- . --- 0 --- . --- . --- . --- . --- . --- . --->

Under the number line was the following direction: "Put the other numbers on this number line."

Below the number line were twelve different problems, some of which involved adding and subtracting positive and negative one-digit numbers. Ms. Coleman and the students spent the entire half-hour mathematics lesson "unpacking" students' mathematical ideas about the first problem on the page:

^
10 + 10 = ?   How do you know?

Ms. Coleman began by asking if the students remembered the homework sheet that she gave them the night before. After she handed back the homework sheets, she told the students that she wanted to find out what they thought she meant by "proving" their answers. She said that everyone was able to fill in the number line, and that was fine. She had the class look at the first problem on the page, "negative ten plus ten." Ms. Coleman reminded the students that they were supposed to give an answer and then tell why they knew their answer was the correct one. Ms.

Coleman then asked for a volunteer who would like to share the answer she got. Maria volunteered and said, "zero." Ms. Coleman wrote, "10 + 10 = 0" on the board and said, "Negative ten plus ten equals zero" aloud as she wrote. Ms. Coleman followed up by asking Maria if she could tell them *why* she knew that. Maria replied with an affirmative and then the following whole-class dialogue ensued:

> Ms. Coleman (C): What did you say? *I would like for the rest of you to listen very carefully because I want you to be able to tell us, or tell Maria, if you agree with what she says or perhaps you disagree with what she says.* Maria?
>
> Maria: You have to count ten numbers to the right. . . .
>
> [Here Ms. Coleman asked Maria to "say that again" and *Ms. Coleman wrote Maria's exact words on the board* as Maria said them.]
>
> C: All right. Maria says that negative ten plus ten equals zero so you have to count ten numbers to the right. What do you people think about that? Hilliard?
>
> Hilliard: I think it's easy, but I don't understand how she explained it.
>
> C: O.K. Does anybody else have a comment or a response to that? Tara?
>
> Tara: I think it's zero 'cause negative ten plus ten equals zero.
>
> C: O.K. And? Right now, I'm asking about what Maria said. A comment? Agreements? Disagreements? Tara.
>
> Tara: There's not. . . . I don't agree.
>
> C: You don't?
>
> Tara: I mean I do agree.
>
> C: What do you agree with?
>
> Tara: That negative ten plus ten equals zero.
>
> C: But that's not all Maria said.

Tara: I disagree with that.

C: What do you disagree with?

Tara: You have to count numbers to the right. If you count numbers to the right, then you couldn't get to zero. You'd have to count to the left.

C: Could you explain a little bit more about what you mean by that? I'm not quite sure I follow you. And the rest of *you need to listen very closely so you can make comments about what she's saying* or say whether or not you agree or disagree. Tara?

Tara: Because if you went that way [points to the right] then it would have to be a higher number.

In the above exchange, the thinking of both Tara and Maria became visible to Ms. Coleman and their peers. Tara's responses led me to suspect that her understanding of negative numbers was not the same as Maria's. Indeed, my earlier interviews with these two girls in October also suggested that this was the case. In the interview, the interviewer showed Tara two numbers, $\hat{79}$ and 2, and asked her which one was smaller. Tara pointed to the two and said that it was smaller because it was two and the other was 79. When asked what the numbers would add up to, Tara said, "81, because the next one [after 79] is 80 and then 81." In contrast, Maria showed greater understanding of negative numbers in her interview. In response to the same questions, Maria pointed to $\hat{79}$ when the interviewer asked which number was smaller. Further, Maria explained correctly it was because "when they have the hats like that, they are smaller--they're not as much as zero."

In this first part of the classroom dialogue, Ms. Coleman set the scene for the students to construct a new orientation to mathematics learning in her classroom by indicating, first, that students would need to explain why they got the answer that they did, and second, they would need to listen and think about their peers' explanations so that they can *decide whether or not they agree or disagree*. Further, they would need to know *with what* they agree or disagree and then be prepared to explain *why* to their classmates. Thus far in the dialogue, Maria and Tara had both gotten their thinking out on the table, and they indicated that they disagreed. However, up to this point, the discourse had still been teacher-student-teacher-student, and the conversation between Maria and Tara had been mediated by Ms. Coleman. In the next part of the dialogue, Ms. Coleman helped to change the pattern of discourse by suggesting that Ben, who disagreed with Tara, talk directly to Tara so that they could try to understand each other:

> Ms. Coleman (C): Any comments about what Tara's trying to say? Ben?
>
> Ben: I disagree with what she's trying to say.
>
> C: O.K. Your disagreement is?
>
> Ben: Tara says if you're counting right, then the number is--I don't really understand. She said, "If you count right, then the number has to go smaller." I don't know what she's talking about. Negative ten plus ten is zero.
>
> C: You said that you don't understand what she's trying to say?
>
> Ben: No.
>
> C: Do you want to ask her?
>
> Ben [Turns to Tara and asks]: What do you mean by counting to the right?
>
> Tara: If you count from ten up, you can't get zero. If you count from ten left, you can get zero.
>
> Ben [to Tara]: Well, negative ten is a negative number--smaller than zero.

Tara: I know.

Ben: Then why do you say you can't get to zero when you're adding to negative ten, which is smaller than zero?

Tara: OHHHH! NOW I GET IT! This is positive.

C: Excuse me?

Tara: You have to count right.

C: You're saying in order to get to zero, you have to count to the right? From where, Tara?

Tara: Negative 10.

From the October interview with Tara, I knew that Tara thought that "$\hat{79}$" was the same as "79." During the above interaction, I suspected that Tara was thinking of "$\hat{10}$" as the same as "10" so she said that when you count up from "$\hat{10}$" (actually negative ten but what she thought was positive ten), then you couldn't get to zero. In response to Ben, Tara said she knew that a negative number was smaller than zero. And it dawned on Tara that "10" is positive, and "$\hat{10}$" is a negative number.

The class went on to discuss what it means "to count ten numbers to the right"--the words in Maria's original explanation that Ms. Coleman had put on the board. However, at this point in the lesson, a concrete referent still had not been given for what was meant by counting ten numbers to the right. Ms. Coleman asked the students if anyone would like to *revise* what Maria had said (here on the board) "so that it will say exactly what you feel in terms of *proving* your response."

Ben suggested revising Maria's explanation to say "If you're on negative ten and add ten, it equals zero." Ben was a student who, in the October interview, demonstrated some understanding of negative numbers by correctly identifying "$\hat{79}$" as smaller than "2" because "negative 79 is smaller than zero. Zero is smaller than two." However, Ben thought that adding negative 79 and 2 would give you "negative 97" even though he then correctly told the interviewer spontaneously that, "If you had one and you had negative two, and you added them you would get negative one."

Later in the classroom discourse, Ms. Coleman became more directive than she had been thus far. She gave Ben the pointer and asked him to "show us on the number line." The number line (i.e., -10 -9 -8 -7 - 6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10) was posted above the chalkboard. Ben said:

> Negative ten [Ben pointed to negative ten on the number line] is ten times smaller than zero [Ben pointed to zero]. And the regular ten [Ben pointed to positive ten on the number line] is ten times bigger than zero so if you add negative plus the regular ten, you're going upwards. You're going the adding way--if you plus negative ten plus the regular ten, it would equal zero.

Then Ms. Coleman asked,"Is that clear to everybody what Ben is saying?" All the students chorused, "NO!" resoundingly.

In this part of the classroom discourse Ben's thinking became visible to others, reflecting the fragility of his mathematical understanding even though he had previously given the correct answer to the problem in question (negative ten plus ten equals zero). Not only was Ben unable to use the number line to explain or prove why his answer was correct but he also introduced some misinformation into the discussion by suggesting that "regular ten is ten times bigger than zero" and "negative ten is ten times smaller than zero." What was striking was the way that Ms. Coleman then responded to Ben's explanation and introduction of what might be considered incorrect mathematics. Rather than correcting him, as she might have done previously, Ms. Coleman asked the class whether Ben's explanation was clear to them. The students responded with an overwhelming "NO!" Then Ms. Coleman continued by requesting that the students ask Ben a question that might help make clearer what Ben was trying to say, but no one was able to do so.

Finally, Ms. Coleman asked again if anyone could "show us in some way that negative ten plus ten equals zero?" At that point, Tara, who had been struggling to understand throughout the class, volunteered. She picked up the pointer, walked to the board, and pointed to numbers on the number line as she said:

> You start at negative ten [she correctly pointed to this number]. Then you add, one [pointed to negative nine], two [pointed to negative eight], three [pointed to negative seven], four [pointed to negative six], five [pointed to negative five], six [pointed to negative four], seven [pointed to negative three], eight [pointed to negative two], nine [pointed to negative one], ten [pointed to zero]. That equals zero.

Rather than affirming Tara's explanation as correct, Ms. Coleman turned to the class and asked them to evaluate it by asking if there were any questions or comments about Tara's method or what she did. Ben said that he didn't quite see it, so Tara went to the board and explained again in the same way, using the number line and the pointer. At that point, Ben stated that he agreed with Tara, and he described what he thought she was doing:

> I sort of know what she's doing because she's counting by ten plus the other ten, she's counting by ten, but she started at negative ten and counted up ten times.

Ms. Coleman then asked for another volunteer to explain what Tara was doing. The incomplete understandings of two other students, Hilliard and Juleah, were then revealed. Juleah suggested that Tara "was going backwards." When Ms. Coleman asked what she meant, Juleah said, "going smaller." Hilliard then added, "When you're using negative numbers, it goes the same way as the regular ones, but the numbers go lower--go over zero, not higher than it." Ms. Coleman wrote Hilliard's words on the board exactly as he said them. Then Ms. Coleman turned to the class and asked the students if it was clear to everyone what Hilliard said. Ben and several other students said they didn't understand Hilliard's explanation; Ms. Coleman admitted that she didn't understand either.

Ms. Coleman then concluded the lesson by saying that on their homework for tonight they were going to have some more of the same kinds of things--thinking about ways that they can prove their answers. She finished by adding, "*What we need to start doing is thinking about how to formulate our words so they say exactly what we mean.*"

As students began to get ready to go home, Ms. Coleman got her notebook and wrote down what was written on the board, including the words and ideas that each student had come up with and the name of the student. I noted in my fieldnotes that the class "did not converge on a solution or answer although at one point it looked like Keisha might be trying to get some convergence."[3]

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[3]During this mathematics lesson, 16 students were present. At the end of the year, 14 of these students were still in the class. Two students had returned to their foreign countries because their parents had completed their education at Michigan State University. These 14 students were interviewed in a one and a half hour clinical interview at the end of the year by either Penelope Peterson or Nancy Knapp, a graduate assistant. Thirteen of the fourteen students were able to identify 79 as smaller than 2, to provide an explanation of why this was the case, and to correctly give a number that was smaller than 79. These students included Ben, Maria, Hilliard, Charles, Juleah, Wayne, Afsoneh, Amherstia, Berny, Titon, Siti, Andy, and Roy. The 14th student, Tara, showed partial understanding. While she identified correctly 79 as smaller than 2 and referred to it as "negative 79," she explained that it was because "negatives are a lot smaller than twos." Although she seemed to understand that any negative number was smaller than zero, when asked to give a number smaller than negative 79, Tara incorrectly gave "negative one." We also interviewed two additional students, Michelle and Freddi, who joined the class during second semester. Although Michelle had not been present for the November discussion, she identified correctly 79 as smaller than 2, gave a correct explanation, and said that "negative 100" was a number that was smaller than negative seventy-nine. Freddi incorrectly identified 79 as bigger than 2 and was unable to articulate why he thought so.

In What Ways Did Keisha Revise Her Mathematics Teaching?

This lesson captures some of the important ways in which Keisha's mathematics teaching changed during the fall of 1990. These changes began as tentative revisions and then became more stable elements of Keisha's mathematics teaching in the months that followed.

In her mathematics teaching, Ms. Coleman had definitively moved away from "teaching as telling" because she had found that she could "teach it one day, and two weeks down the road, the kids didn't even remember one iota of what we dealt with." As Keisha put it, you can't assume that the students "know" just because you, the teacher, told them.

Thus, Ms. Coleman no longer "told" students the answer or the mathematical procedure, nor did she indicate whether an answer was correct or incorrect because she had begun to believe that students learn mathematics better if they hear it from their peers. *With skillful guidance and questioning from the teacher, students had to figure out solutions to mathematical problems for themselves.* An important result was that Ms. Coleman was no longer the sole source of mathematical knowledge in the classroom. Students did the explaining, talked about how they solved the problem, and clarified the meaning of their explanations. As a result, students' thinking became visible, and students learned from each other.

Keisha began to think differently about how she assessed her students' mathematical knowledge. She planned to pay less attention to students' mathematics scores on the CTBS test as measures of students' knowledge and to rely instead on what students said during classroom discussion, what students wrote in their explanations for mathematical solutions on their written work, and what students said and did during small-group clinical problem-solving interviews that she began to conduct every couple of months.

Rather than focusing on "covering mathematical content," Ms. Coleman focused on solving mathematical problems and discussing students' solutions and explanations. Students worked on one or two mathematics problems for the whole period. The mathematics lesson focused on discourse about how to think about and solve the mathematics problem. In focusing her whole mathematics lesson on solving one or two mathematics problems, Keisha was able to focus on "unpacking" mathematical ideas in greater depth and in a more coherent way than she had previously (see Ball, 1990; Lampert, 1990; Stigler & Perry, 1988).

Keisha ceased following the CSMP script and, finally, ceased using a textbook altogether. Rather, she followed the chain of students' thinking about and sense making of the mathematics problem. Such an approach is characteristic of teachers who take a constructivist view of children's learning (see, for example, Lampert, 1988; Peterson, Fennema, & Carpenter, in press; Wilson, in press).

In all classrooms, part of the academic work for students is making sense of the task and what the teacher wants (see, for example, Doyle, 1983). *In Ms. Coleman's class, students had to work on making sense of the mathematics and how their peers were thinking about the mathematics.* Classroom work in Ms. Coleman's classroom focused on the construction of mathematical knowledge and the negotiation of shared mathematical meaning (Wood, Cobb, & Yackel, in press). However, Keisha did not use these words, and it was not clear that she saw or thought about the changes in her mathematics teaching in the same way as I saw them and thought about them. So how did Keisha think about the changes she was making in her mathematics teaching?

In What Ways Did Keisha Revise Her Thinking?

The major ways in which Keisha revised her thinking during the Fall of 1989 revolved around changes in her view of how students learn mathematics. In an interview in January, I asked Keisha how she thought she had changed. She replied:

> Basically building on what the children already know. . . . Before everything was more or less programmed [or scripted in CSMP], and it's not that programmed right now, if that makes sense. Because when I teach the lesson, I have an idea what I want them [the children] to understand. And however they arrive at that is okay. I don't know why it's different, but it is different. I'm just not getting the responses [from students] that I had in the CSMP book, because I'm not anticipating or saying to myself, "Well, this is what they should say." Rather, I'm taking what they're giving me and building upon that. I know a lot of times in CSMP, when you [the teacher] have the anticipated response there, if you don't get that, then you would somehow rephrase it and tell it to them. With this [the way I am teaching now] I don't do that; everything comes from them [the students]. And we can build upon what they bring to the lesson, and I think that's really exciting.

Why did Keisha believe that it was important to build on what children know and follow the chain of their thinking and ideas rather than the script in the textbook? A major reason was that Keisha was developing and trying out the idea that children learn mathematics better when they hear it from one of their classmates rather than from the teacher, even though what the children hear from their peers might be explanations of the same mathematical knowledge or procedures that she, as the "teacher," would have given. One might say that Keisha had this idea as a "working hypothesis." As she put it in the January interview:

> I think this [mathematical knowledge] will stick with more of the children than me trying to stand there and force feed them that [knowledge] which I think is really interesting. But I think it's good also because they [the children] hang onto what their classmates say a lot more than any information that I could give them. And I always think that I'm trying to explain it so that they understand it and then giving them examples to prove it. But rather than *me* doing that, give *them* that task.

By giving the students the task of learning, Keisha saw that students began to change their orientation toward the task so that they no longer viewed it as "just work to be done." Then in the interview Keisha gave the example of having taught concepts in CSMP last year and giving students a page from the CSMP textbook to work on. If they were asked to draw an arrow road, for example, the children would just *draw it*. However, at the time of this interview, Keisha saw the students as "thinking a little bit more" and asking themselves, "How should I do this? What is the best way for me to do this? Rather than just sitting down and going through it, students are thinking lots more." She then described an example of a problem students had worked on the day before: "See if you can figure out a way that you can show how you get from 7 to 135 on the mini-computer." The students had had the problem with arrow roads, but now they had to do it with their paper abacus--the mini-computer. Keisha noted with amazement:

> Students "were discussing it with one another. They were talking about it. . . . And I mean, they were *really* working with it, rather than sitting there. There's just a difference, I think. Because it's okay for them to come up with different strategies.

One reason Keisha believes that students are learning more is that they are more "involved." She sees students as more "involved" because as the teacher she uses "their input" and builds on what they say, and she believes that the more she, as the teacher, does that the more that students are "going to retain it."

Thus, through her own mathematics teaching and observations of her students' mathematics learning as a result of the revisions she has made in her mathematics teaching, Keisha seems to have discovered the power of what Flanders (1970) referred to as "use of student ideas." In his Interaction Analysis Scheme, Flanders included in this category acknowledging a student's idea, modifying the idea, applying the idea, comparing the idea, or summarizing what was said by an individual student or group of students. As Rosenshine (1971) noted, using students' ideas seems to be related to two of the greatest tributes and motivators in the academic world--being published and being cited. Rosenshine also noted in his review that in 8 of 9 studies where researchers observed teacher's use of student ideas, they found a positive relationship between the frequency with which teachers used students' ideas and student achievement.

However, an important difference exists between what Rosenshine had in mind and what seems to be happening in Keisha's classroom. The discourse in Keisha's classroom revolves around not just the *teacher's* use of students' ideas but also the *students'* use of other students' ideas. Not only are students using other students' ideas, but they are evaluating them, piggybacking on them, and building on them to construct new understandings.

The research on teacher behavior conducted and reviewed by Flanders, Rosenshine, and others formed the research base for models of direct instruction and effective instruction that were promulgated in the late 1970's and early 1980's. Indeed, Keisha views herself as teaching within what she refers to as an "effective instruction" frame.

Keisha says that she teaches everything "as part of her effective instruction" that she learned in a district workshop the previous year. One of the important elements of effective instruction is the use of "sponge activities." The mathematics problem or problems that she gives students to work on as they come into mathematics class is a "sponge activity" according to Keisha. Keisha gives her students the mathematics sponge activity because she likes her students to be "on task" as soon as they come into the classroom, and she wants them to be "ready for learning." She says that her sponge activity is basically her "anticipatory set." When asked why it was called a "sponge," Keisha speculated that it had to do with the "soaking in of information." When Madeline Hunter (1983) proposed the idea of a "sponge," she presented it as a way of soaking up loose time that otherwise might be wasted.

Conclusion

In her thinking as of January 1990 Keisha Coleman had a developing view of children's mathematics learning and her own teaching that reflected some elements of both constructivism and behaviorism. In this way Keisha is like others in mathematics education who are struggling to move from a behavioral view of mathematics learning and teaching, which has dominated American classrooms, toward a practice that takes seriously the question of what it means to know and do mathematics. It should come as no surprise that even within the same elementary school mathematics teachers such as Keisha Coleman, Deborah Ball, and Magdalene Lampert, like researchers within the mathematics education community, do not agree on this epistemological point. For example, in their summary of the research agenda-setting conferences held by the National Council of Teachers of Mathematics, Sowder and her colleagues

(1989) distinguished five contemporary scholarly views of what mathematics is and how one comes to know mathematics.

According to the first view, mathematics is external to the knower, static, and bounded. Learning and teaching mathematics involve the acquisition of information. In the second view, mathematics is also external to the knower, but it is a growing unbounded discipline that changes over time. Learning and teaching mathematics focus on how students acquire meaning for what is to be learned. The remaining three views involve some variation on "constructivist" ideas that knowledge is personal or social. According to the first constructivist perspective, to know mathematics means to do mathematics by "abstracting, inventing, proving, and applying" (Sowder, 1989, p. 22). The second constructivist position assumes an epistemology of mathematical knowledge that is consistent with the contents of individual minds. Finally, the last constructivist perspective regards mathematical knowledge as the product of social and cultural processes. This latter perspective seems to best describe the views of Lampert (1990) and Ball (1990), while the view of Keisha Coleman in 1990 seems to reflect elements of the first two perspectives. Keisha regards the learning and teaching of mathematics as the acquisition of information, but she also endorses the importance of her students acquiring meaning for what is to be learned.

How will Keisha's thinking develop and how will she revise her thinking and her mathematics teaching in the future? Although we cannot predict what changes will occur, we predict that changes will occur. At the point we leave Keisha Coleman in January 1990, she remains committed to learning and to improving her own mathematics teaching. She continues to feel excited by the challenge of her movement away from following the CSMP text. She continues to define her work as a teacher to include her own learning and reflection on her mathematics teaching. She engages constantly in thinking about her mathematics teaching and also in evaluating what she is learning, for example, from Deborah Ball. She views her principal and colleagues in her school as playing important roles in her own learning and development as a mathematics teacher over the last seven years, and she believes they will continue to do so. While the "professional development" context of the school is important to Keisha Coleman, she believes that the principal and the teachers had established such a context through their own efforts at supporting and learning from one another well before last year when the school officially became a "professional development school" associated with Michigan State University.

Revisions are likely to continue to occur in Keisha's thinking and in her mathematics teaching. For Keisha, her knowledge of the teaching and learning of elementary mathematics is all at once unbounded, dynamic, and changing and very much the result of personal and social processes occurring within her, within her classroom, and within the school context.

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