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ABSTRACT

Calls for elementary teachers to reform their classroom practice in mathematics are coming from all sides. They are being asked to shift their teaching from an approach based on "transmission of knowledge" to student-centered practice. In order to assess elementary teachers' current goals and activities in teaching mathematics, a survey of 493 teachers in California, Florida, and Michigan was conducted. Six schools recognized as particularly effective in helping students develop conceptual understanding were selected from one large urban district and one moderate-sized district in each state. Cluster analysis of survey responses to a six-page questionnaire yielded five clusters of teachers: (1) primary teachers who had students use manipulatives extensively (N=14); (2) teachers, most of whom used "Math Their Way", who had students use manipulatives and discuss problem solving extensively (N=20); (3) modal teachers whose profile reflected a softened version of drill-and-practice teachers (N=353); (4) drill-and-practice teachers (N=10); and (5) teachers in a cluster with three expert teachers, whose profile represented a more moderate version of the manipulative using and problem solving-oriented teachers (N=56). Profiles including demographic information and grade levels taught of each cluster are reported. The sheer numbers of teachers falling into the modal cluster may discourage mathematics education reformers, but if this picture is viewed as a brief picture on a changing scene and reformers focus on the profiles and portraits of teachers in the expert cluster, they can take heart that practice can be changed. (MDH)

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Penelope L. Peterson, Ralph T. Putnam,
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Abstract

In light of changes called for in elementary mathematics teachers' practices, we surveyed 493 elementary teachers in California, Florida, and Michigan about their current goals and activities in teaching mathematics. Cluster analysis of survey responses yielded five clusters of teachers: (a) primary teachers who had students use manipulatives extensively; (b) teachers, most of whom used *Math Their Way*, who had students use manipulatives and discuss problem solving extensively; (c) modal teachers whose profile reflected a softened version of drill-and-practice teachers; (d) drill-and-practice teachers; and (e) teachers in a cluster with three expert teachers, whose profile represented a more moderate version of the manipulative using and problem solving-oriented teachers.

PROFILES OF PRACTICE: ELEMENTARY SCHOOL TEACHERS' VIEWS OF THEIR MATHEMATICS TEACHING[1]

Penelope L. Peterson, Ralph T. Putnam, Jan Vredevoogd, and James W. Reineke[2]

From all sides calls are coming for teachers to reform their classroom practice in mathematics. For elementary teachers, the calls are strong, the voices are many, and the demands are great (Mathematics Sciences Education Board and National Research Council, 1990; National Council of Teachers of Mathematics, 1989; 1991; National Research Council, 1989a; 1989b). Elementary teachers are being asked to change not only their goals for teaching mathematics but also the ways they teach mathematics in their classrooms. They are being asked to expand their goals from "inculcating routine skills to developing broad-based mathematical power," which requires "that students be able to discern relationships, reason logically, and use a range of mathematical methods to solve a wide variety of nonroutine problems" (National Research Council, 1989a). They are being encouraged to shift their teaching from an approach based on "transmission of knowledge" to a student-centered practice featuring "stimulation of learning" (National Research Council, 1989a). As summarized by the *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989), the changes needed in instructional practices in elementary mathematics include *decreased emphasis on* rote practice, one answer and one method, written practice, and teaching by telling and *increased emphasis on* use of manipulative materials, discussion of mathematics, justification of thinking, a problem-solving approach to instruction, and writing about mathematics.

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[1]This article will appear as part of a special issue of the *International Journal of Educational Research*, edited by Walter G. Secada, entitled Researching Educational Reform: The Case of School Mathematics in the United States.

[2]Penelope L. Peterson, professor of educational psychology and teacher education at Michigan State University, is co-director of the Center for the Learning and Teaching of Elementary Subjects. Ralph T. Putnam, assistant professor of counseling, educational psychology, and special education, is a senior researcher with the Center. Jan Vredevoogd, a doctoral candidate in educational psychology, is a former research assistant with the Center. James W. Reineke, a doctoral candidate in educational psychology, is a research assistant with the Center.

How might we think about this profile of needed changes in light of elementary teachers' current classroom practice in teaching mathematics? To answer this question, we need to consider the goals elementary teachers hold and the current classroom practices they employ in the teaching of mathematics. Although researchers have documented and lamented the uniformity of teachers' practice in elementary mathematics, they have also noted some differences among teachers (Romberg & Carpenter, 1986). Differences may exist because some teachers have developed new knowledge and expertise, perhaps related to using new mathematics curriculum or texts or as a result of learning with others or alone. Differences may exist because some teachers have been working in curriculum policy contexts that promote reform, such as in California (California State Department of Education, 1985; 1987). Differences may exist because some teachers teach mathematics differently to younger learners than to older learners and because some of the new practices, such as use of manipulatives, fit more easily with existing knowledge, beliefs, and practices of primary teachers than with upper elementary teachers. What do these different profiles of elementary mathematics teachers look like? And how might we use these profiles of practice to inform our thinking in light of the need for ongoing reform of mathematics education in our nation's elementary schools? We addressed these questions through a large-scale survey of elementary teachers' content-related teaching practices.

Methodology of the Survey Study

We conducted the survey during 1988-89 as part of the work of the Center for the Learning and Teaching of Elementary Subjects at Michigan State University. The purpose of the survey was to get a picture of elementary teachers' current practice in six subject areas--mathematics, science, social studies, literature, art, and music. A six-page questionnaire addressed: (a) teachers' professed goals; (b) the proportion of time and emphases teachers report giving to specific activities in each subject area; (c) teachers' reported use of specific curricula and materials designed to promote greater student understanding, problem solving, or thinking in each subject area; and (d) teachers' judgments of how knowledgeable and effective they are in each subject compared to other elementary teachers.

We chose to survey elementary teachers in three states--California, Michigan, and Florida--because these states represent distinctly different policy contexts, and they differ significantly both substantively and procedurally in their approaches to subject area curriculum guidelines and policies at the state level (Freeman, 1989). Within each state we selected one large urban district and one moderate-sized district. Within each district we asked administrators to nominate four schools in their district that were recognized as particularly effective in helping students develop conceptual understanding, problem solving, and higher level thinking--two schools that served students of relatively high socioeconomic status (15% or less of the students qualified for free or reduced lunch) and two schools that served students of relatively low socioeconomic status (50-80% of the students qualified for free or reduced lunch). With the help of the state-level directors of either the Catholic schools or the private schools in each state, we also obtained nominations of two private Catholic elementary schools in each district.[3] We then contacted the principals of these six schools in each district and confirmed their willingness to participate in the survey study.

The principal of each of the 36 selected schools was asked to request that each elementary teacher in the school complete the survey. In a letter to the teacher, on the front page of the questionnaire, we indicated that the questionnaire was part of research being conducted by the Center for the Learning and Teaching of Elementary Subjects at Michigan State University and that we would use what we learned from the study to help improve education in our nation's schools. We stated that the questionnaire was being administered to teachers in several states and that it was important to get information from a large number of teachers on their goals and classroom practice in teaching elementary subjects. Packets of questionnaires were sent to principals at each school during August or September 1988. We indicated that teachers'

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[3]The private schools selected were Catholic schools near selected public school districts with a student composition comparable to one of the two participating public schools. It was felt best to identify private schools comparable as possible to public schools studied. Catholic schools are the largest subgroup and share common elements that make them comparable to one another as well as different from public schools.

participation was voluntary and that we would guarantee confidentiality of their questionnaire responses. Teachers signed the informed consent form on the bottom of the front page indicating their understanding of the purposes of the study, the guarantee of confidentiality of their responses, and their willingness to participate. Although the principals distributed the questionnaires to the teachers and returned the packet of questionnaires in a self-addressed express mail envelope, they did not see the completed questionnaires of individual teachers. To ensure confidentiality, each teacher placed the completed questionnaire in a sealed envelope before returning it to the principal. By the end of November, we received signed consent forms and questionnaires from 678 teachers in 35 of the 36 elementary schools we had selected. The 36th elementary school (that served students of low socioeconomic status [SES] in the large urban district in California) eventually declined to participate due to unforeseen difficulties that arose during the fall of the school year. Of the 678 teachers who responded, 493 reported teaching mathematics and responded to all 12 mathematics questions.

We also administered the questionnaire to three elementary teachers whom we had identified as experts as part of another study (Putnam, Prawat, & Reineke, 1990). To identify these expert teachers, we called scholarly leaders in mathematics education at universities around the United States; described the kinds of elementary teachers we were seeking; and asked for nominations of elementary teachers who were outstanding at promoting mathematics understanding, thinking, and problem solving in their students. We then contacted the nominated teachers by phone and interviewed them about their educational backgrounds, teaching experience, and ideas about goals and methods for teaching elementary mathematics. After stratifying to ensure balance between teachers with expertise and experience in the early and later elementary grades, we invited three teachers, who had seemed most impressive during their phone interviews, to participate as experts. Two of the experts had been involved in significant mathematics education reform efforts: Elaine Rosenfield served on the advisory committee that developed the *Mathematics Model Curriculum Guide* (1987) for the state of California; Yolanda

Rodriguez served on the Mathematical Sciences Education Board that has authored, *Reshaping School Mathematics: A Philosophy and Framework for Curriculum* (1990).

In this paper, we focus on our analyses of elementary teachers' responses to the survey questions regarding their goals and classroom practice in teaching mathematics. Figure 1 presents the questions to which teachers responded. The first four questions dealt with teachers' goals for mathematics teaching. Teachers' responses were scored from 1 to 6 depending on the extent of their agreement or disagreement with each goal statement. The next eight questions dealt with instructional activities in which the teacher or students might engage. Activity 4 was stated as being performed by the teacher ("You explain concepts or computational procedures"); the other seven activities were stated as performed by students. Teachers' responses were scored from 1 to 7 for each activity depending on the proportion of mathematics time the teacher estimated spending on each activity. If the goals and activities are considered in light of instructional changes suggested by the NCTM *Standards* (1989), then one would hope to see teachers place greater emphasis on goal 2 than on goal 1, on goal 4 than on goal 3, and on activities 2, 3, 5, 6, and 7 than on activities 1, 4, and 8. It would be impossible, however, to derive a specific profile of activities from the *Standards*, for that document does not specify a precise instructional model and because individual activities are only rough indicators of a coherent instructional approach.

Cluster Analyses of Teachers' Questionnaire Responses

Teachers' responses to the mathematics activities and goals questions were highly intercorrelated. We decided that each teacher's views of his or her mathematics teaching would be better represented by a profile of mathematics goals and activities taken together than by considering goals and activities separately. Given these many individual teachers' profiles on eight mathematics activities and four goals, we wondered what we might learn by grouping together teachers with similar profiles and whether our analysis might reveal important similarities and differences in the classroom practice of teachers in different groups. Accordingly, we used cluster analysis to group profiles of the individual teachers' practice in teaching elementary mathematics (see, for example, Aiken, Anderson, & Hinde, 1981). By

Please indicate how strongly you agree or disagree with each of the following statements about mathematics:

	Strongly Disagree					Strongly Agree
1) In teaching mathematics, my primary goal is to help students master basic computational skills.	①	②	③	④	⑤	⑥
2) In teaching mathematics, my primary goal is to help students develop the ability to solve problems and think mathematically.	①	②	③	④	⑤	⑥
3) Students need to master basic computational facts and skills before they can engage effectively in mathematical problem solving.	①	②	③	④	⑤	⑥
4) Students should learn computational skills within the context of solving problems.	①	②	③	④	⑤	⑥

Estimate the **PROPORTION OF MATH** time that is spent in the following activities:

	None	1 - 5%	6 - 10%	11 - 25%	26 - 50%	51 - 75%	over 75%
1) Students practice or drill on computational skills.	①	②	③	④	⑤	⑥	⑦
2) Students solve story problems or other problems that don't have obvious solutions.	①	②	③	④	⑤	⑥	⑦
3) Students discuss different ways that they solve particular problems.	①	②	③	④	⑤	⑥	⑦
4) You explain concepts or computational procedures.	①	②	③	④	⑤	⑥	⑦
5) Students use manipulative materials or drawings to solve problems.	①	②	③	④	⑤	⑥	⑦
6) Students discuss mathematical ideas, as a class or in small groups.	①	②	③	④	⑤	⑥	⑦
7) Students respond to questions or assignments that require them to write text at least a paragraph long.	①	②	③	④	⑤	⑥	⑦
8) Students take written tests.	①	②	③	④	⑤	⑥	⑦

Figure 1. Questionnaire items on mathematics activities and goals.

analyzing the clusters, we hoped to develop some picture of the profiles and range of teachers' current mathematics practice. Such a picture would provide some persp-tive on the changes in classroom practice that are hoped for by mathematics education reformers.

Questions

Our cluster analysis was guided by a series of questions:

Would the three expert teachers, whom we had chosen independently of our large three-state sample of teachers, join together in the same cluster?

How many other teachers would fall into the cluster with the experts?

What other clusters would appear, and how many teachers would fall into these other clusters?

Would the clusters reveal interpretable profiles of teachers' mathematics goals and activities, and if so, what would they look like?

Before performing the cluster analysis, we examined teachers' responses and found that different teachers had used different reference points in estimating the proportion of mathematics time that they spent in each of the eight activities. Because we were more interested in the relative emphases that teachers gave to each of these activities than in the absolute proportion of mathematics time they gave to each activity, we used deviation scores from the mean score for each teacher. We subtracted the teacher's own average response for the eight activities from his or her response for each activity. This resulted in eight deviated activity scores for each teacher. Similarly, we were interested in teachers' relative emphases on pairs of goals. Thus, we created two difference scores for each teacher by subtracting a teacher's response to goal 1 from his or her response to goal 2 and his or her response to goal 3 from goal 4. Using the between-groups average linking method (SPSS, 1988), we performed cluster analyses on teachers' deviated responses to the activities questions and the goals questions. Given eight activities and two goal pairs, a point was plotted for each teacher in 10-dimensional Euclidian space. Cluster analysis finds the two points that are closest to each other and then calculates the midpoints of those two points to form a cluster, resulting in one less point in space. The analysis continues to reduce the number of points in space until only one point remains. The researcher can stop the analysis at any point and choose the number of clusters that she wants to examine.

Five Clusters of Elementary Mathematics Teachers

The solution that we found most meaningful was a 15-cluster solution, of which 5 clusters included 10 or more teachers. Figure 2 shows the mean deviated responses to each math activity for the teachers in each of these five clusters. Figure 3 shows teachers' relative emphases on mathematics goals by cluster.

To begin to construct a picture of the teachers and teaching within each of these clusters, we did two kinds of analyses. First, we examined similarities and differences among clusters in teachers' relative emphases on different mathematics activities and goals. Second, we did cross-tabulations on other descriptive data from the questionnaire to explore the kinds of teachers that fell within each cluster. Before examining in detail the profile of mathematics practice for each cluster, we provide thumbnail sketches of the clusters. In our sketches we highlight the differences among the clusters that appeared as significant from the cross-tabulations.

Thumbnail Sketches of the Clusters

Cluster A teachers ($N=17$) were almost all primary teachers (94% taught K-1st grade) who had their students use manipulatives extensively. Although many Cluster A teachers (43%) had previously used an activity-based mathematics program called *Math Their Way* (Baratta-Lorton, 1976), only 29% of them were currently using it. Cluster B ($N=20$ teachers) was also predominantly composed of primary teachers (80%), most of whom (over 80%) currently used *Math Their Way*. Like the A teachers, the B teachers were high on having students use manipulatives, but whereas Cluster A teachers were high on teacher explanation, Cluster B teachers were high on having students discuss problem solving. Compared to teachers in other clusters, Cluster B teachers, along with Cluster E teachers, rated themselves as most effective in teaching mathematics and also most knowledgeable about teaching mathematics. Within the B Cluster, four times as many of the teachers came from California as from Michigan or Florida.

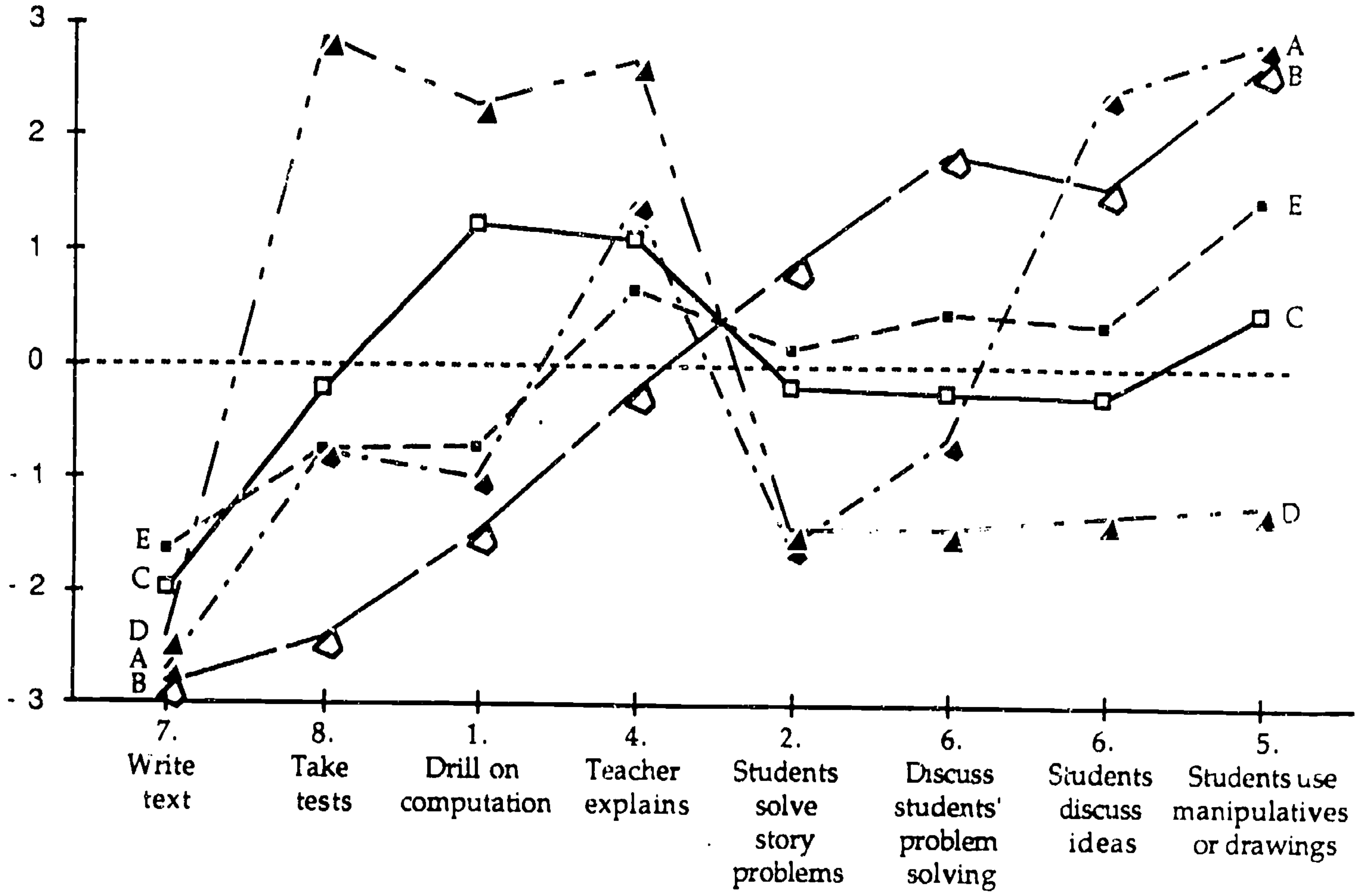

Figure 2. Deviated means of mathematics activities.

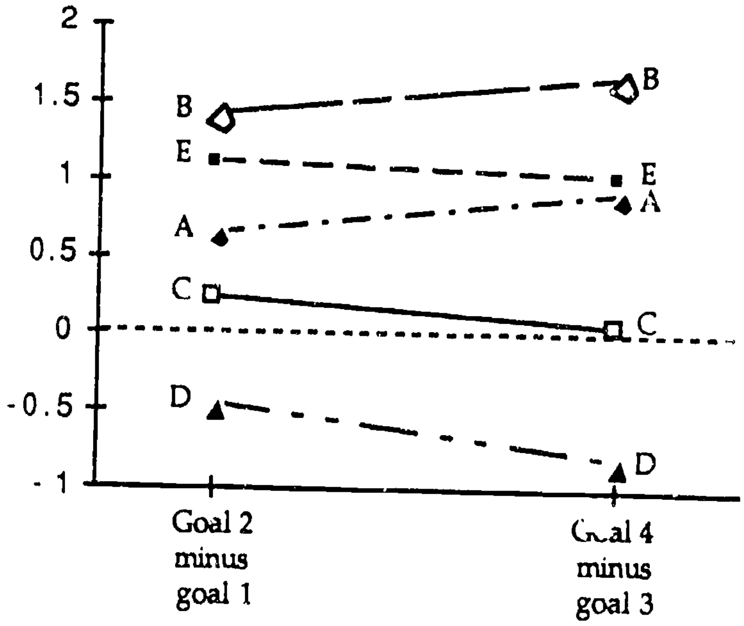

Figure 3. Deviated means of mathematics goals.

Cluster C (*N*= 353) represented the modal cluster where most of the teachers fell. Only 7% of teachers in the modal cluster were currently using *Math Their Way*, although an additional 16% had used it previously. Cluster C teachers were distributed across the three states equally, and across all grades with slightly fewer teaching K-1st grade (25%) than teaching grades 2-3 (34%) or grades 4-6 (34%). Cluster D (*N*= 10) consisted of teachers who might be characterized as strong drill-and-practice teachers. Compared to teachers in other clusters, teachers in the drill cluster rated themselves as least effective in teaching mathematics and also least knowledgeable in mathematics. Within the D Cluster, a larger proportion of teachers were teaching in private Catholic schools than in either high SES or low SES public schools. Cluster E (*N*=56) included the three teacher experts we had identified previously (Putnam et al., 1990). Cluster E teachers were distributed about equally across the grades and across the kinds of schools (low SES public, high SES public, and private Catholic).

Compared to teachers in the modal and drill-and-practice clusters, proportionally more teachers in the expert cluster had either used or were using one of a variety of "distinctive" mathematics programs including *Math Their Way* (40%), *Real Math* (Willoughby, Bereiter, Hilton, & Rubenstein, 1987) (37%), or *Comprehensive School Mathematics Program* (CEMREL, 1985) (17%). Almost an equal number of teachers in this expert group were from California and Michigan, but only half as many were from Florida. This latter finding may be related to differences in the mathematics curriculum policy contexts within which teachers work. Florida state and local policies have reflected more press toward basic skills and less press toward high-level mathematics thinking and goals than state and local policies in California or Michigan (Freeman, 1989).

Why did so few teachers fall into what we came to call the drill-and-practice cluster even though it is argued that this sort of teaching predominates in elementary school mathematics instruction (Romberg & Carpenter, 1986)? At least two explanations possible. First, the teachers surveyed in this study came from elementary schools that were nominated for their effectiveness so these schools might have had fewer traditional drill-and-practice teachers than the typical population. Alternatively, as we shall see, teachers in this cluster represented a rather extreme pattern of mathematics goals and activities that few teachers might have been willing to admit to, endorse, and report on their questionnaire.

Teachers' Profiles of Mathematics Activities and Goals by Cluster

How were teachers' profiles of mathematics activities and goals similar and different according to the cluster in which they fell? Did teachers in the expert cluster differ from those in the other clusters, and if so, how? We begin by discussing the drill-and-practice teachers (Cluster D) and then move to the *Math Their Way* teachers (Cluster B) because these two groups seemed most different.

Cluster D: Drill-and-practice teachers. Compared to teachers in other clusters, *drill-and-practice* teachers endorsed most strongly the primary goal of helping students master basic computational skills over developing the ability to solve problems and think mathematically. Further, they believed most strongly that students need to master basic computational facts and skills *before* rather than *in the context of* doing problem solving. The drill-and-practice teachers reported spending more of their time than teachers in other clusters on explaining concepts or computational procedures to students and on having students practice or drill on computational skills or take written tests. They reported spending the *least* proportion of their time on having students use manipulative materials or drawings to solve problems, discuss mathematical ideas as a class or in small groups, or discuss different ways that they solve problems. In addition, drill teachers spent little time on having students solve story problems or other problems that do not have obvious solutions.

Cluster B: *Math Their Way* teachers. The profiles of *Math Their Way* teachers were the mirror reversal of the drill-and-practice teachers.[4] Among all teachers, *Math Their Way* teachers endorsed most strongly the primary goal of helping students develop problem solving and mathematical thinking abilities over a primary goal of helping students to master computational

[4]Recall that only 80% of the teachers in this cluster were using *Math Their Way*.

skills. Compared to teachers in all other clusters, *Math Their Way* teachers reported spending the *least* proportion of their time on practice or drill of computational skills, giving students written tests, or explaining concepts or computational procedures. In contrast to all other teachers, *Math Their Way* teachers spent the *greatest* proportion of their time having students solve problems, discuss different ways that they solved problems, use manipulatives or drawings to solve problems, and discuss mathematical ideas. These activities are consistent with the *Math Their Way* program, which places a heavy emphasis on working with concrete materials and solving problems with manipulatives before learning the written symbols of mathematics.

Cluster A: Manipulative-using primary teachers. Like *Math Their Way* teachers (Cluster B), Cluster A teachers spent as great a proportion of their time having students use manipulatives as Cluster D teachers did on drilling and testing. Yet Cluster A teachers were the only group that spent less time on story and nonroutine problems than drill teachers. And, like the drill teachers, Cluster A teachers spent relatively greater time explaining concepts or computations and relatively less time having their students engage in or discuss problem solving. However, unlike drill teachers, whose lack of emphasis on problem solving and discussion of problem solutions seemed to fit with their overall profile, Cluster A teachers' lack of emphases on these problem-solving activities seemed not to fit with their strong endorsement of a problem-solving goal over a computational goal and their overall profile, which included as much use of manipulatives and discussion of mathematical ideas as the *Math Their Way* teachers. But the primary teachers' profile makes sense if one considers the possible perspectives and orientations of these teachers of young children. Cluster A teachers may have avoided involving their students in problem solving and discussing problem solutions because they believed that primary-age children were not ready or able to solve and discuss story problems and other problems because they lacked reading skills, mathematical knowledge, or verbalization abilities (see for example, Peterson, Fennema, Carpenter, & Loef, 1989). Further, they may have believed that their young students needed more teacher explanations to teach them concepts and procedures and that, as young children, they also needed to manipulate concrete materials and objects to understand the mathematics.

Cluster C: Teachers in the modal cluster. The profiles of both the modal group of teachers (Cluster C) and the experts (Cluster E) occupy the middle of Figure 2 and show flatter profiles compared to the other clusters. Although less extreme in profiles than the *drill-and-practice* teachers and the *Math Their Way* teachers, the modal teachers' profile might be viewed as a softened version of the drill-and-practice profile while the *experts'* profile seems to represent a balanced version of the *Math Their Way* profile. Compared to teachers in the expert cluster, teachers in the modal cluster endorsed less strongly a primary goal of mathematics problem solving and thinking mathematically over mastering basic computational skills, and they believed less strongly that students should learn computational skills within the context of problem solving rather than needing to master basic skills before solving problems. Teachers in the *modal* group spent a greater proportion of their time on mathematics drill and teacher explanations than they did on having students solve problems, discuss mathematical ideas and ways to solve problems, or having students use manipulatives. While modal teachers devoted a significantly smaller proportion of their time to having students take written tests than did drill teachers, nonetheless, their proportion of written test time was greater than teachers in the other three clusters and equal to the proportion of time spent discussing problem solving and mathematical ideas.

Cluster E: Teachers in the expert cluster. Like *Math Their Way* teachers and primary teachers, teachers in the *expert* group spent the greatest proportion of their time having students use manipulatives or drawings to solve problems although the proportion of time they spent on this was considerably less than the *Math Their Way* or primary teachers. They spent a roughly equal proportion of their time on having students solve problems, discuss ways to solve problems, discuss mathematical ideas, and on giving explanations of concepts and procedures. Teachers in the expert group spent a considerably smaller proportion of their time on drill and practice and on having students take written tests. Like teachers in all the other groups, teachers in the *expert* group reported spending the smallest proportion of their time on having students respond to questions or assignments that require students to write text at least a paragraph long. Indeed, in all clusters the average proportion of time that teachers checked for this category was 1 to 5%. However, although the proportion of time that expert teachers had students spend in writing about mathematics was low, it was greater than that for teachers in any of the other clusters.

In sum, our examination of teachers' profiles from the results of the cluster analysis provided some interesting insights into how teachers in the five clusters were similar and different in their thinking about mathematics goals and activities. Yet we wondered if there were another way of capturing more parsimoniously the salient dimensions of mathematics teaching along which the clusters of teachers varied. To address this question we performed a discriminant function analysis.

Discriminant Function Analysis

Given x number of groups and y variables, a discriminant function analysis finds the best linear combination of the variables that discriminates among these groups. We found two significant functions that represented the best linear combination of our 10 variables (8 deviated mathematics activities and 2 goal differences) that discriminated among the five clusters of elementary teachers. The first function accounted for 76% of the variance between the groups and the second function accounted for an additional 15% of the variance. Drill-and-practice activity correlated negatively with the first discriminant function (-.77). The variables that correlated positively were emphasis on a problem-solving goal over computation (.51), a belief that students should learn computational skills within the context of problem solving rather than master basic skills first (.37), and emphasizing students' discussion of ways of solving problems (.35). The variables that correlated positively with the second discriminant function were having students use manipulative materials and drawings (.68) and having students discuss mathematical ideas as a class or in small groups (.57), whereas the variable that correlated negatively was having students take written tests (-.50). Thus, the first function seems to represent a problem solving versus computational practice orientation; the second function represents an emphasis on understanding and exploration versus standard assessment.

Figure 4 shows a two-dimensional plot of the territories for each cluster of teachers on the two discriminant functions. The drill-and-practice teachers (Cluster D) occupy the territory in the lower left-hand corner of the plot, representing the lowest scores on both discriminant functions and suggesting both a computational practice rather than problem-solving orientation (function 1) and an orientation toward written testing rather than using manipulatives or discussion to determine students' understanding (function 2). Upward and to the right of the drill teachers, the modal teachers (Cluster C) occupy the largest territory, which covers the four quadrants of the plot of the two functions, suggesting that the modal teacher cluster has the greatest within-group variance although the group centroid is the closest of all the clusters to the origin of both functions. The territory of teachers in the expert cluster (Cluster E) comes next, with most teachers falling in the positive range on both discriminant functions.

This mapping suggests that teachers in the expert cluster had both a greater problem-solving orientation (function 1) and a greater orientation toward using manipulatives and discussion to develop students' mathematical understanding (function 2) than did teachers in the modal cluster. Such was also the case for primary teachers (Cluster A) and *Math Their Way* teachers (Cluster B). However, *Math Their Way* teachers were more extreme than experts in both their problem solving versus computational practice orientation (function 1) and in their manipulatives and discussion-based versus testing orientation toward teaching and assessing understanding (function 2). By comparison, primary teachers (Cluster A) were like the experts in their problem solving versus computational practice orientation (function 1), but more extreme than both the experts and the *Math Their Way* teachers in their emphasis on use of manipulatives and discussion rather than written tests to develop students' conceptual understanding (function 2).

In occupying the area between the modal cluster and the *Math Their Way* cluster, the territory of teachers in the expert cluster overlaps these clusters so that some teachers in the expert

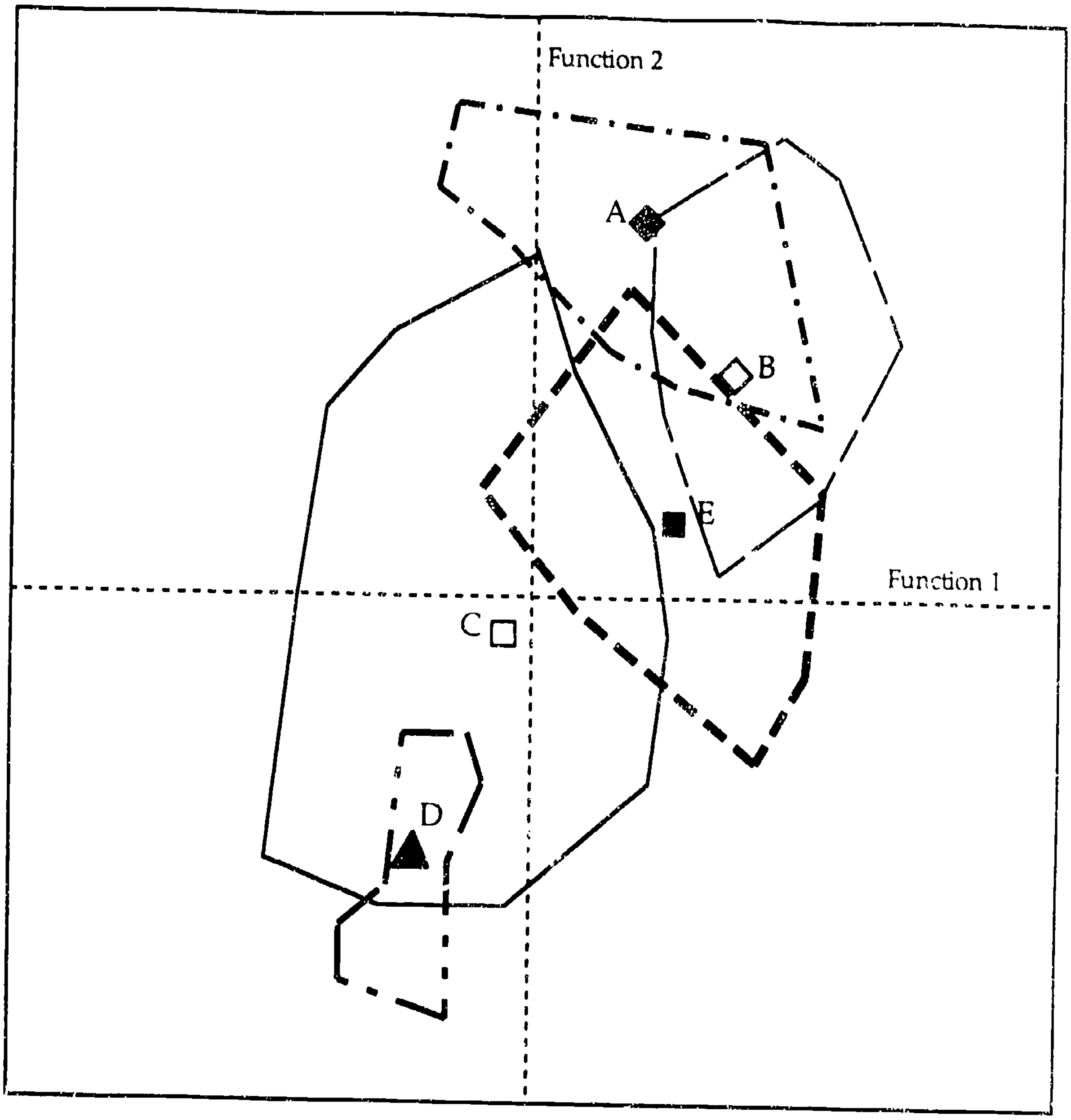

Figure 4. Territorial plot of the five clusters (A through E) of teachers on the two discriminant functions.

cluster have discriminant function scores like some *Math Their Way* teachers, and some have scores similar to some modal teachers. Similarly, about half the territory of *Math Their Way* teachers overlaps that of the primary teachers. These overlaps are partly a function of reducing the 10-dimensional space of the cluster analysis to 2 dimensions.

What Were the Expert Teachers Like?

We found our questionnaire data useful in providing us with a broad perspective on the different profiles of the current mathematics teaching practice of a large sample elementary teachers in three diverse states. The data helped to place teachers' practice on some of the broad dimensions of mathematics teaching and goals that are the focus of change in the current mathematics education reform movement. But the questionnaire data did not afford any in-depth insight into how teachers think about their mathematics practice or what teachers' mathematics teaching might actually look like in the classroom. This was the focus of another study that we conducted with teachers from four schools in the two California districts in this study (Cohen et al., 1990). In that study, in-depth on-site case analyses of teachers in the modal cluster revealed important similarities and differences in these teachers' thinking and classroom practice in elementary mathematics (See, Ball, 1990; Heaton, in press; Peterson, 1990; Prawat, in press; Putnam, in press; Remillard, in press; Wilson, 1990).

The Expert Teachers

Although we had no observational data on teachers in the expert cluster, we did have interviews that we conducted over the course of a one-day visit with each of the three elementary teachers whom we had identified as experts as part of another study (Putnam et al., 1990). Each teacher was asked to prepare a written document in which they addressed three representative but important mathematics goals, listed important understandings related to each goal, and developed a scenario for teaching one of the understandings at each of two grade levels--second and fifth. Then each teacher came to the Michigan State University campus where she participated in a five- to six-hour interview with us over the course of one day in the Summer of 1988. From the teachers' interviews and written documents we were able to construct three self-portraits of these expert elementary mathematics teachers.

Dolly Davis: Teaching math for relevance and understanding. Dolly Davis teaches third grade in Athens, Georgia, but much of her background and experiences are at the early childhood and primary levels. From Davis's point of view, schooling should be interdisciplinary, relevant, and appealing. She wants her students to view mathematics as fun and easy and to realize that mathematics is useful and relevant to their out-of-school lives now and in the future. She believes that children learn best through familiar and meaningful activities. Thus, when she is introducing mathematical ideas, she tries to use a variety of familiar objects and settings as embodiments of the mathematical idea she is presenting. For example, Davis uses counters and containers for sets, and she uses money (dollars, dimes, and pennies) rather than base-10 blocks to introduce regrouping in subtraction because students "can understand it with the money where they can't with the blocks. The blocks have no relevance for them, but money has relevance for them."

Davis designs her classroom activities to capitalize on familiar contexts, and she creates activities through which students can see the applicability and interelatedness of all they are learning. She has her third-grade students create a small town in their classroom that they design, build, and participate in throughout the school year. She uses this town situation to introduce mathematical problems. For example, students punch a time clock when they begin and leave work at their jobs in the town. Students are paid three to five cents per minute. Each student then has to figure out the amount of money they have earned at their job during a given week. Davis also uses the making of scale drawings prior to the actual building of the city as an opportunity to teach a number of measurement concepts. She views her ongoing town situation as offering multiple possibilities for students to encounter new mathematical ideas and to apply what they have learned in their mathematics lessons. For Davis, the town and the problem situations that grow out of it become a site for connecting important learning experiences in mathematics, social studies, art, science, and language.

In discussing the ideas within whole-number arithmetic, Davis emphasizes the importance of students understanding the meaning of various operations (i.e., addition, subtraction, multiplication, and division) as well as knowing how to perform them (i.e., knowing basic fact combinations and being able to carry out the steps of computational algorithms). Although Davis emphasizes the importance of students' understanding of mathematical operations and having the opportunity to use and apply math matical ideas (especially within the context of the town), she also believes that students need to master their basic facts. She does drill on facts throughout the year, and she insists that students learn to recall facts without relying on counting.

Elaine Rosenfield: Teaching the powerful tools of mathematics. Elaine Rosenfield wants students to become fluent users of the powerful tools of mathematics. She views mathematics as "a tool to organize information and to make decisions about real problems." She believes that thinking, problem solving, and sense making should permeate the whole elementary school curriculum including mathematics. Rosenfield is opposed to thinking of mathematical knowledge as hierarchical or as decomposable into many discrete learning objectives. Rather than viewing mathematical problem solving as a matter of applying basic skills and knowledge, she thinks that children learn various mathematical operations and numerical relationship through the process of solving problems. Rosenfield emphasizes that they should not learn simply the arithmetic skills and concepts that have previously dominated the elementary mathematics curriculum, but rather students should learn concepts and skills from all "strands" of mathematics (including geometry, measurement, probability, patterns, and algebra) with "key understandings" being interwoven within rich mathematical activities rather than being taught as discrete concepts or subskills.

In talking with us and in her writing, Rosenfield focused on the strands and the key understandings, and she drew heavily on the *California Mathematics Framework* (1985) and the *California Model Curriculum Guide* (1987), which she was involved in writing. She emphasized that key understandings should be dealt with as interconnected with one another and as interactive with what the child is thinking about; they should not be taught as separate skills or concepts in a particular sequence. The idea of patterns emerging from a variety of settings was a central one that pervaded Rosenfield's thinking about teaching multiplication and other topics. She thought that students should deal with multiplication in a variety of problem settings and with a variety of perspectives, gradually coming to make general abstractions--"developing a cognitive structure about multiplication."

Rosenfield sees it as of utmost importance that the mathematics students are learning makes sense to them. The mathematics students are learning must connect in meaningful ways with what they already know. Rosenfield never wants a child to carry out a symbolic procedure, such as the addition algorithm without first having established meanings for the symbols and procedures. Thus, when working to help students learn a particular mathematical topic or set of ideas, she always starts with concrete activities. As students solve problems within this concrete environment, they begin to see what Rosenfield describes as the connecting level of activity. Finally, students begin to use and learn about the traditional mathematical symbols and procedures for manipulating them. Rosenfield argues that learning virtually always proceeds best by going through the concrete, connecting, and symbolic levels. She thinks this is true whether the learner is a child or an adult; it is almost always helpful to begin to understand a mathematical idea by working with some kind of concrete representation of it. While asserting the importance of these levels, Rosenfield also emphasizes that she does not view the levels in a lock-step or linear way. Rather, expert learners of mathematics move productively back and forth among the concrete, the connecting, and the symbolic.

Yolanda Rodriguez: Empowering students to use mathematics. Yolanda Rodriguez has three major goals for her students. First, she wants to empower students to think mathematically so that mathematics becomes "a pump, not a filter." She sees this as especially important for the low SES and minority students whom she teaches in a school outside Boston. Second, she wants her students to see mathematics as useful--that they know when and how to apply mathematical skills in a variety of situations and that they can make rich connections between their mathematics knowledge and real-world and scientific contexts. Finally, Ms. Rodriguez wants to communicate to her students a sense of wondering and sense making through her own attempts to learn and understand mathematics because she believes that if she is curious and wonders about mathematics, then her students will also.

Rodriguez's view about teaching and learning revolve around the importance of sense making. Students need to work to make sense of the mathematics they are learning so that they can develop conceptual understanding as foundational for their mathematical knowledge. Rodriguez's biggest concern is that students arrive in her sixth-grade mathematics class having learned algorithms, but having no sense of when to use them and why they work. She sees connections as necessary to conceptual understanding in mathematics, and she strives particularly to help students make connections to the various contexts of use of mathematical concepts and procedures. Important sites for developing these connections are rich interdisciplinary problem contexts in which students can explore a variety of mathematical ideas. Two examples of such sites that Rodriquez has used are the "Voyage of the Mimi" and a unit on the solar system that she developed with a collaborative group of teachers (Regional Math Network, 1987).

Over a two-month period, Rodriquez involved her students as "crews" who had to solve real-world problems on the voyage. She sees conversation in mathematics as central to her teaching approach. She encourages conversation by encouraging students to share their solution strategies for solving a math problem, generating many different ways to solve a problem, and evaluating the strengths and weakness of these different ways. For Rodriguez, problem contexts serve as a source for representations of important mathematical concepts that students should learn in mathematics (e.g., the Red-Line subway in Boston as a representation for thinking about positive and negative integers[5]). Rodriguez spends a lot of time thinking about what makes a good

[5]Rodriguez attributed this representation to curriculum materials developed as part of the Algebra Project, directed by Bob Moses.

problem, and she is always on the look-out for problem settings that will lend themselves to exploring a variety of mathematical ideas with her students.

Summary and Conclusions

The portraits of these experts put some flesh on the profiles of mathematics activities and goals of three teachers who fell in the same cluster based on their survey responses. The portraits show how each expert in her own way endorses the goal of students being able to solve mathematics problems and think mathematically rather than merely demonstrating mastery of computational skills. The portraits illustrate how each expert develops and spends significant time with her students engaged in mathematics activities that involve solving problems, discussing mathematical ideas and ways of solving problems, and using manipulative and concrete materials. However, the portraits also reveal intriguing variations in perspectives and practices depending on the expert. Both the similarities across experts in the major themes, and the variations across experts in the playing out of these themes lead us to suspect that much could be learned by further exploration of such cases (Wilson, Shulman, & Richert, 1987). Exploration of such portraits of expertise as cases of mathematics teaching might inform mathematics education reform and help other teachers who are struggling to change their teaching practice.

The three teachers who came highly recommended as experts fell into a cluster of teachers with a moderate profile on the goals and activities. The *Math Their Way* cluster fit more neatly into the pattern of goals and activities we suggested at the beginning of this article as consistent with the recommendations of the NCTM's *Curriculum and Evaluation Standards:* emphasizing goal 2 over goal 1, goal 4 over goal 3, and activities 2, 3, 4, 5, 6, and 7 over activities 1, 4, and 8. A number of different interpretations might be advanced for this finding--all tentative because we have only questionnaire data on the teachers. Because the fairly rich descriptions of three of the expert teachers suggest their practice is consistent in many ways with the recommendations in the *Standards*, we could argue that the *Math Their Way* teachers appear to be overly extreme in the espousal of manipulatives and student discussion.

On the one hand, we might argue that the *Math Their Way* teachers have simply picked up more extreme language about what they do--that they argue more vehemently for making students' discussion and understanding more prominent in instruction. On the other hand, we might assert that teaching mathematics in rich and meaningful ways does not mean eliminating teacher explanations and having *all* instruction involve manipulative materials. Indeed, the *Standards* authors recommend "decreased emphasis" on such things as rote practice and teaching as telling, not necessarily the elimination of these activities or approaches. Because we have only teachers' reports of what they do in their classrooms and no evidence on the effect of different patterns of goals and activities on student learning, we can make no strong claims about which of these profiles is "better" or "more effective." Indeed, we think that trying to come up with a "best" profile is probably counterproductive. We take the fact that our three recommended experts fit into the more moderate cluster as evidence suggesting that good teachers may hold more ecclectic and balanced views rather than views that reflect adherence to a narrow ideology or taking an extreme position.

What have we learned from this study of profiles and portraits of the current practice of elementary mathematics teachers? If mathematics education reformers view this picture as a static one, and they focus on the sheer numbers of teachers who fell in the modal cluster on our survey and who reported a continuing emphasis on written tests, drill and practice, and teacher explanations, then reformers might be discouraged. But if reformers view the picture as an ever-changing video that was stopped for a brief moment in time in September 1988, and they focus as well on the profiles and portraits of teachers in the expert cluster, then they might take heart. Teachers' reports, descriptions, perspectives, and insights on their practice offer important information from which all of us can learn, whether we are reformers, researchers, administrators, policymakers, or teachers ourselves. We need only be willing to ask, look, and listen to teachers.

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