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## ABSTRACT

Phase II of the research on the teaching and learning of higher order thinking and conceptual understanding in the elementary subjects, undertaken by the Center for the Learning and Teaching of Elementary Subjects, includes descriptions and analyses of commonly used and distinctive published curricula in each of the five subject areas. This report includes findings of the analyses of one commonly-used elementary mathematics textbook ("Addison-Wesley Mathematics") and three distinctive elementary mathematics curricula ("Real Math", "Comprehensive School Mathematics Program" and "Math in Stride"). A set of framing questions, developed by a team of researchers undertaking the curriculum studies, served to guide the analyses in each subject area and draw attention to evidence of instruction oriented toward developing critical thinking and understanding. While "Addison-Wesley Mathematics" purports a view of mathematics common to schools today, placing greatest emphasis on the development of computational skills in isolation of meaningful applications, the three distinctive curricula propose alternatives to this perspective and, in differing ways, have attempted to suggest that mathematics involves problem solving, sense making, complex thinking, and reasoning. They also have endeavored to propose different strategies for teaching and learning mathematics based on alternative pedagogies and perspectives on learning. This study examines the published or printed curriculum available to teachers. Study of how teachers use these alternative curricula is a logical next step. (Author)

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**IS THERE AN ALTERNATIVE? AN ANALYSIS OF  
COMMONLY USED AND DISTINCTIVE  
ELEMENTARY MATHEMATICS CURRICULA**

Janine Remillard



**Center for the  
Learning and Teaching  
of Elementary Subjects**

**Institute for  
Research on Teaching  
College of Education  
Michigan State University**

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ELEMENTARY MATHEMATICS CURRICULA

Janine Remillard

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## **Center for the Learning and Teaching of Elementary Subjects**

The Center for the Learning and Teaching of Elementary Subjects was awarded to Michigan State University in 1987 after a nationwide competition. Funded by the Office of Educational Research and Improvement, U.S. Department of Education, the Elementary Subjects Center is a major project housed in the Institute for Research on Teaching (IRT). The program focuses on conceptual understanding, higher order thinking, and problem solving in elementary school teaching of mathematics, science, social studies, literature, and the arts. Center researchers are identifying exemplary curriculum, instruction, and evaluation practices in the teaching of these school subjects; studying these practices to build new hypotheses about how the effectiveness of elementary schools can be improved; testing these hypotheses through school-based research; and making specific recommendations for the improvement of school policies, instructional materials, assessment procedures, and teaching practices. Research questions include, What content should be taught when teaching these subjects for understanding and use of knowledge? How do teachers concentrate their teaching to use their limited resources best? and In what ways is good teaching subject matter- specific?

The work is designed to unfold in three phases, beginning with literature review and interview studies designed to elicit and synthesize the points of view of various stakeholders (representatives of the underlying academic disciplines, intellectual leaders and organizations concerned with curriculum and instruction in school subjects, classroom teachers, state- and district-level policymakers) concerning ideal curriculum, instruction, and evaluation practices in these five content areas at the elementary level. Phase II involves interview and observation methods designed to describe current practice, and in particular, best practice as observed in the classrooms of teachers believed to be outstanding. Phase II also involves analysis of curricula (both widely used curriculum series and distinctive curricula developed with special emphasis on conceptual understanding and higher order applications), as another approach to gathering information about current practices. In Phase III, models of ideal practice will be developed, based on what has been learned and synthesized from the first two phases, and will be tested through classroom intervention studies.

The findings of Center research are published by the IRT in the Elementary Subjects Center Series. Information about the Center is included in the IRT Communication Quarterly (a newsletter for practitioners) and in lists and catalogs of IRT publications. For more information, to receive a list or catalog, or to be placed on the IRT mailing list to receive the newsletter, please write to the Editor, Institute for Research on Teaching, 252 Erickson Hall, Michigan State University, East Lansing, Michigan 48824-1034.

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### Abstract

Phase II of the research on the teaching and learning of higher order thinking and conceptual understanding in the elementary subjects, undertaken by the Center for the Learning and Teaching of Elementary Subjects, includes description and analysis of commonly used and distinctive published curricula in each of the five subject areas. This report includes findings of the analyses of one commonly-used elementary mathematics textbook (*Addison-Wesley Mathematics*) and three distinctive elementary mathematics curricula (*Real Math*, *Comprehensive School Mathematics Program*, and *Math in Stride*). A set of framing questions, developed by a team of researchers undertaking the curriculum studies, served to guide the analyses in each subject area and draw attention to evidence of instruction oriented toward developing critical thinking and understanding. While *Addison-Wesley Mathematics* purports a view of mathematics common to schools today, placing greatest emphasis on the development of computational skills in isolation of meaningful applications, the three distinctive curricula propose alternatives to this perspective and, in differing ways, have attempted to suggest that mathematics involves problem solving, sense making, complex thinking, and reasoning. They also have endeavored to propose different strategies for teaching and learning mathematics based on alternative pedagogies and perspectives on learning. This study examines the published or printed curriculum available to teachers. Study of how teachers use these alternative curricula is a logical next step.

IS THERE AN ALTERNATIVE?  
AN ANALYSIS OF COMMONLY USED AND DISTINCTIVE ELEMENTARY  
MATHEMATICS CURRICULA

Janine Remillard<sup>1</sup>

This report is one of the Phase II studies in the research on the teaching of higher order thinking and conceptual understanding in the elementary subjects currently being undertaken by the Center for the Learning and Teaching of Elementary Subjects. The studies in this phase are intended to describe current teaching practice, particularly those viewed as exemplary with regard to the teaching of critical thinking and application of the elementary subjects, mathematics, science, social studies, literature, and the arts. Included in these descriptions are reports and analyses of state- and district-level policies and curriculum guides that potentially impact the teaching of elementary subjects.

Textbooks and other published curricula are also considered to play a significant role in influencing classroom instruction (Freeman & Porter, 1989) and are often used as vehicles for creating change in teaching (Peterson et al., 1990). This report presents descriptions and analyses of four elementary mathematics textbooks or curricula. The interest in exemplary practice, particularly that which goes beyond traditional or commonly accepted approaches to incorporate or focus on developing the critical thinking abilities and understandings of students--what Graybeal and Stodolsky (1987) term "the good stuff"-- led us to examine alternative or distinctive curricula in addition to those most commonly used. Graybeal and Stodolsky examined the pedagogical visions and assumptions of several commonly used mathematics and social studies textbooks and found them to embody limited conceptions of learning and subject matter that involve recitation, drill, and solitary tasks. Finding these textbooks devoid of any "good stuff," they suggest further study of texts and their uses that could lead to the improvements in or alternatives to what is currently offered.

The distinctive curricula in this study, though not in common use, present alternatives to common practice. They can be viewed as aiming to provide epistemological or pedagogical alternatives to the approaches most frequently suggested by typical elementary mathematics curricula. Thus, a central purpose of these reports is to examine and assess alternatives for

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mathematics curricula provided by authors and publishers of distinctive curricula. To what degree and in what ways do their offerings break from the patterns found in commonly used curricula, as reported by Graybeal and Stodolsky (1987)? What are the epistemological and pedagogical foundations on which these alternatives are built? Do the conceptions of mathematics teaching and learning that they provide correspond with the visions held within the mathematics education community (National Council of Teachers of Mathematics, 1991)?

*Addison-Wesley Mathematics*, published in 1987 (Menlo Park, CA), is the most widely used of the four analyzed curricula and is typical of many elementary text series. The three distinctive curricula, *Real Math*, published by Open Court in 1987 (Open Court Mathematics and Science Curriculum Development Center, LaSalle, IL), *Comprehensive School Mathematics Program* (CSMP), published by CEMREL in 1978 (Mid-Continent Regional Educational Laboratory, Kansas City, MO), and *Math in Stride*, published by Addison-Wesley's Innovative Division in 1988 (Menlo Park, CA), propose views of mathematics and mathematics teaching and learning that break from that traditionally found in American schools. Their offerings seem to approach the type of self-regulated thinking and reasoning commonly referred to as higher order thinking (National Assessment of Educational Progress, 1983) or high literacy (Bereiter & Scardamalia, 1987; Resnick, 1987). The emphases on mathematical thinking, reasoning, and problem solving found in these distinctive curricula can also be linked to demands within the mathematics education community to place greater emphasis on the development of conceptual understanding and sense making and less on rote memorization of isolated skills (Dossey, Mullis, Lindquist, & Chambers, 1988; National Council of Teachers of Mathematics, 1989; National Research Council, 1989). All four programs are designed to be used in typical elementary classrooms from first to sixth grade. Together they present four varying portraits of the nature of mathematical knowledge, knowing, and learning.

### Methodology

The team of researchers analyzing curricula from the five elementary subjects developed a common set of framing questions (see appendix) to use in structuring and guiding the analyses. Establishing common dimensions to examine the different curricula, these framing questions were intended to facilitate comparisons of curricula within and across subject areas. They also served to ensure a consistent and useful degree of breadth and depth of each analysis, taking into account questions of overall goals, as well as specific details about the texts and how they were intended to be used in the classroom. The questions in the goals category inquire into the explicit and implicit goals of the curriculum. The next several categories focus on the mathematical content selected, its organization and sequencing, and its explication in the text.

The questions in these categories look specifically at the content found in the text and how it is treated. Considering that the curriculum experienced by students and teachers is not limited to the content found in the text but includes activities and discourse developed around that content, questions were developed to examine the types and nature of student activities and teacher-student interactions recommended by the authors. Recognizing the significance of the teacher's role in interpreting and implementing any curricula, one category of questions focuses on the assistance provided the teacher in using the curriculum.

To correspond with other work being done by Center researchers, the fifth- and second-grade texts of each curriculum were selected as particular foci of the analyses, although each curriculum was also examined as a whole. The following reports are the analyses of each of the four curricula. Contrasts between the three distinctive curricula and Addison-Wesley, the commonly used program, are taken up in the conclusion.

### *Addison-Wesley Mathematics*

#### Goals

Goals inferred from program rhetoric. The *Addison-Wesley Mathematics* teacher's guide does not provide a statement of program goals. Nevertheless, the flashy advertisement-like pages proceeding the actual content and the information provided to assist the teacher in using the text suggest certain goals, desired student outcomes, or activities that the authors deem important and significant to the teaching and learning of mathematics. The following are explicitly stated, although the lack of specific details of each maintains enough ambiguity to leave the desired goals open for interpretation. Popular terms, such as "skill development" and "problem solving" seem to be used to earn the approval of the potential reviewer, yet little explanation of these terms is provided. The statements of these expected and desired outcomes, however, imply a hierarchical view of mathematics and learning that attaches a specific remedy to each problem:

-Solid skill development with measurable results (The skills referred to include algorithmic procedures that can be measured by paper and pencil tests. The authors claim to address this through the use of "clear, consistent" introductions to and development of skills with examples provided; opportunities for independent practice; skill application and extension.)

-Problem solving (Addressed through several problem-solving pages at the end of each chapter, some of which include locating and using information from the three-page data bank of charts and graphs; a chapter opener problem; and word problems throughout the chapter. The types of problems presented are primarily routine story problems from situations created by the authors.)

-Individualization (which refers to the provision of reteaching and enrichment pages used according to the student's performance on the chapter tests.)

-Experience with technology (Two pages of Logo computer programming instruction and two pages of calculator activities are at the end of the second-grade text and six pages of BASIC programming and one of Logo and one page of calculator-assisted problems are in the fifth-grade text. The use of technology, particularly computers, is not integrated into the rest of the curriculum.)

-Estimation skills (Students are taught procedures for estimating and are often encouraged to estimate a reasonable answer before solving the problem algorithmically.)

-Mental math skills (Lessons are provided that develop mental mathematics skills--such as adding two numbers by beginning with the larger addend and "counting on" the smaller, breaking numbers into component parts that facilitate mental computation--and recognizing patterns.)

Several more implicit goals can be inferred from the question and answer section provided for the teacher:

-Memorization of number facts, which includes moving away from using fingers or other manipulatives.

-Solving problems, which includes collecting data and selecting the correct operation. There is emphasis on integrating problem solving into the daily skills lessons so the students will "understand why computational skills are needed."

Knowledge goals: The relationship among conceptual, procedural, and conditional knowledge. The underlying assumption about the relationship between conceptual, procedural and conditional knowledge seems to suggest a linear view of knowledge development: Students learn computational procedures first and in isolation and then use this knowledge in conditional (application) situations. Although it is not clearly articulated, in many cases, conceptual understanding often is used to refer to understanding the procedure, evidenced by the emphasis on algorithmic mastery, rather than the underlying concept.

Conceptual development is often used to provide a foundation for the introduction of the corresponding skill. For example, trading ten 1s for a 10 when putting groups together is practiced before students are taught to regroup using the algorithm. This passing attention to conceptual development before proceeding to the development of procedure, may reflect the assumption that mastery of the algorithm is a valid indicator of conceptual understanding. There is some emphasis on strategy development, that is, steps in problem solving, strategies for memorizing facts or determining the correct operation to use in a problem. In each case, these strategies are taught prescriptively and in isolation from one another and the other mathematical content of the chapter.

The types of attitude or dispositional goals implied in the teacher's manual include making math meaningful, alive, and enjoyable. Much of these are said to be accomplished through the layout of the book, that is, colorful graphics, clear examples, or the opportunity to solve problems. There does not seem to be any attempt to meet these dispositional goals through the actual tasks students engage in; the contexts of these tasks--lifelike situations or eye-catching illustrations--are used instead.

Opportunities for group work are scattered throughout each chapter. The authors also provide information for the teacher on establishing groups and helping them work successfully together. However, cooperative or collaborative work does not seem to be a central focus of the curriculum or of mathematical development. Instead, group work is only treated as a way of structuring classroom activities and is not seen as a fundamental part of doing mathematics. The development of mathematical content or application skills, regardless of the organization of the classroom, is focal. The frequency of the use of small groups seems to be more a function of current education trends than being grounded in the purpose or goals of the curriculum.

Connectedness of knowledge goals. There are few connections made between the knowledge goals. Rather, they are treated incrementally and in isolation from one another, although there are some surface attempts to connect the mathematical content across the curriculum. In some cases, certain skills are introduced in relation to skills already developed. Inverse operations, for example, are introduced in a way that highlights the inverse relationship between addition and subtraction. The relationship, however, is not fully extended as the topic is developed. The practice with each of the skills is very definitely divided into isolated segments.

Conception of mathematics driving the curriculum. The above goals present mathematics as a subject consisting of rules and procedures, which can be directly applied to word problems, once they have been memorized. Indeed, this perspective drives the skill-focused curriculum at each grade level. Aspects of mathematics commonly associated with conceptual understanding and application are not absent. There is an emphasis on problem solving, estimation, group work, and using manipulatives, which is undermined by the prescriptive approach. No emphasis is placed on constructing knowledge, justifying answers, encouraging divergent solution strategies, or helping students to experience mathematics as something other than a fixed set of useful rules.

#### Content Selection, Organization, and Sequencing

The substances and nature of mathematics. The content selection and organization represent the substance of mathematics as discrete pieces of content that should be taught in isolation from one another. The content is broken into the following topics, taken from the scope and sequence chart: addition, subtraction, multiplication, and division of whole numbers;

decimals; fractions; numbers and numeration; estimation and mental math; number properties and theories; integers and rational numbers; ratio proportions and percent; graphing, probability, and statistics; geometry; algebra; time and money; measurement; technology; problem solving and application. The majority of these topics are treated incrementally--broken into pieces and taught in isolation. Topics that are more common--the basic operations of whole numbers, geometry, decimals, fractions, and time and money--have an entire chapter or more devoted to them. Others are found on specific pages in other chapters.

This may be an attempt to integrate topics such as problem solving, applications, and technology with the rest of the content, but in actuality, they are added as separate exercises, rather than integrated into the mathematics of each chapter. The fact that fractions and decimals are found in isolated chapters suggests that operations with whole numbers have no relationship to operations with decimals and fractions. Similarly, two-digit subtraction without regrouping is taught separately from subtraction with regrouping. The implication seems to be that these are unconnected operations or that the student's understanding of them would not be enhanced by seeing them as connected.

The segments in which the content is grouped are organized hierarchically. Two-digit subtraction without regrouping is a precursor to two-digit subtraction with regrouping; finding common denominators precedes adding and subtracting fractions with unlike denominators. Because students are introduced to one procedure at a time, a disconnected view of the subject is communicated. It is likely that students who learn mathematics in this way would not make connections between the various pieces of the subject and would fail to see meanings behind procedures. For example, students are not likely to make connections between multiplying whole numbers and fractions because they are treated as separate entities in the text. Such a perspective ignores the inherent connections between ideas which are fundamental in mathematics.

Moderately evident in the authors' content selection and sequencing is attention to mathematical thinking. Reasoning activities are scattered throughout each chapter. These include the "Think Math" problems found at the bottom of every few pages. In the fifth-grade text, every other page ends with a Think Math problem which is usually challenging and thought provoking. The Think Math problems in the second-grade text appear much less frequently (every 5-10 pages) and are much more straightforward and less challenging. Otherwise, there is not evidence that ways of thinking and communicating are seen as part of the content.

Relationship between conceptual, procedural, and conditional knowledges. The relationship between these three types of knowledge that is represented in the way the content is organized and sequenced seems to be linear. Many of the skill introductions include activities

or work with manipulatives which tend to highlight the underlying concept of the procedure. However, the students are taught the corresponding procedures and are given ample, repeated practice. The application problems at the end of the section are contrived word problems that require the application of the particular mathematical procedure being taught and are designed to demonstrate practical uses of the skill. No attempts seem to be made to integrate the development of skills, their conceptual foundations, and possible applications, or to provide authentic application problems.

Applications. Although applications of mathematics do not seem to be a criteria for content selection, it is clear that the authors have heeded calls to make mathematics relevant to the lives of children by including a great deal of "application." The conception of application is limited, for the most part, to single-step, single-answer word problems in life-like contexts. Frequently the mathematical task is tangential to the problem situation, reducing the problem to a dressed-up procedural exercise. Although the Addison-Wesley curriculum is replete with word problems, this limited structure fails to represent authentic applications of mathematics in the world. Instead, their purpose seems to be motivational. The authors explain that problem solving demonstrates to students the importance of mastering computational algorithms and story problems which are used to introduce and follow up the introduction and teaching of computational skills. Each skill introduction is begun with a "motivational problem" and concludes with application problems. The skill section in the middle is predominantly procedural, the implication being that the procedures are prerequisite to the application, rather than the applications providing a vehicle for developing and using the procedures.

Student interests and dispositions to learn. An attempt to consider student interests and dispositions to learn is reflected in the use of many application situations that may seem life-like to students, suggesting that mathematics is useful in one's life. The message seems to be that math is useful and relevant to one's life. There are also attempts to make the content attractive to students. Colorful, busy pictures, both realistic and caricatures, pervade the curriculum. The intent seems to be to perk up, or even glitz up, a subject that has traditionally come to be known as boring and irrelevant to one's life. Engaging the students in challenging and interesting mathematics does not seem to be a priority.

Prior knowledge and assumptions about student ability. No mention is made of informal or practical knowledge of mathematics that students bring to the formal mathematics class. References to and assumptions about prior knowledge refer to the skills previously presented in the text. These are frequently reviewed and practiced through "Skillkeeper" problems which provide regular practice with previously presented procedures.

The contrast between the presence of reasoning activities in the fifth-grade text and their absence at the second-grade level implies that second-grade students are not able to engage in much analytical or complex reasoning. The fifth-grade text includes many problems which require students to use logical reasoning to solve problems. The problems in the second-grade book, including the "Think Math" problems, require more straightforward solution strategies. The difference in the types of problems presented between the two levels can not be completely accounted for by the ability differences between the students. Nonetheless, the difference in reading ability does contribute to this disparity. The amount of reading required for the second-grade text is very minimal. There is considerably more reading in the third-grade text; and the projects, problems, and reading assignments are significantly more integrated with other elements of a child's experience and the world. The Think Math problems are also more complex in the third-grade text. The implication seems to be that young students need solid exposure to and practice in basic skills before proceeding on to "higher order" thinking skills.

Provisions for student diversity. Provisions for an ethnically diverse range of students can only be found in illustrations where an ethnic and gender balance of children is pictured. The dress and activities of these children, however, are culturally and socioeconomically homogenous. Going to movies, giving parties, participating in typically American sporting activities and making purchases are the norm. Activities originating from diverse cultures or that do not include consumption are seldom found.

### Content Explication in the Text

Topic treatment. The text is divided into chapters which include the various strands considered appropriate for the particular grade:

#### Second grade

Addition/subtraction facts  
 Place value and counting  
 Sums to 18  
 Differences to 18  
 Time and money  
 Addition: 2-digit numbers  
 Geometry and graphing  
 Subtraction: 2-digit numbers  
 Measurement—metric units  
 3-digit place value  
 Add/subtraction: 3-digit numbers  
 Multiplication  
 Fraction and customary measures

#### Fifth grade

Basic facts  
 Numbers and place value  
 Addition and subtraction  
 Decimal add/subtraction  
 Multiplication  
 Division: 1-digit divisors  
 Division: 2-digit divisors  
 Measurement  
 Fractions: add/subtract  
 Larger fractions  
 Geometry  
 Fractions: multiply/divide  
 Decimals: multiply/divide  
 Graphing and data  
 Ratio and percent  
 Measurement: customary

As stated earlier, the use of many chapters tends to segment the content into isolated concepts and operations rather than structuring it around key ideas such as additive or multiplicative structures, spatial concepts, and so forth. Within each chapter a mastery approach is followed: The skill is introduced, illustrations are used to represent the concept, and opportunities for practice through repeated problems and story problems are provided. The chapter ends with a chapter review test, as well as a reteaching page and an enrichment page. Cumulative review tests are also scattered throughout the curriculum. Each chapter in the fifth-grade text begins with a factual or informative story which involves the chapter topic in some way (e.g., the story at the beginning of the division chapter is about the monarch butterfly. It explains that "In just 8 days it travels 319 km"). This and brief informative paragraphs at the beginning of many lessons seem to be an attempt to demonstrate the relevance of mathematics to the world. The fact that the fifth-grade text emphasizes more complex thinking and makes more connections to the child's experience than the skill-dominated second-grade text suggests a hierarchical view of learning. The implication is that young children must master basic skills before they can engage in complex thinking and applications. The result is that the mathematics has lost some validity at the primary level.

Common to the entire series is the lack of emphasis on the construction of knowledge and divergent thinking strategies. There are problems which could potentially facilitate strategy development through problem solving, yet these possibilities are somewhat undermined by their presentation in the text. An example from the fifth-grade text poses a multiple step problem in which the student must decide what time to leave home to get to the movie theater for a 1:00 show and to pick up Sandy. The textbook provides a grid map of the city, information about bus schedules, the time it takes to travel a city block on foot and by bus, and the location of Sandy's house. This situation presents a potentially problematic task in which the student must develop a strategy to approach this problem in addition to using the data to locate points on the map and calculate the amount of time to travel distances between them. The text, however, breaks the problem into pieces for the students and walks them through a convergent, step-by-step process to determine an answer. "How long will it take you to walk to Sandy's house?" asks the first question (Grade 5, p. 213). Exact answers to these subquestions are provided in the teacher's guide; yet, it is expected that answers to the question of what time to leave home "will vary." Expecting that these answers are reasonable is not explicitly encouraged. In fact, the teacher's guide suggests that there is "no right answer." In this problem, and others like it, mathematics is presented as filling in the blank, even in cases which require complex thinking.

Promotion of conceptual understanding. There is an attempt, through the use of pictorial and concrete models, examples, and application problems, to promote conceptual understanding of the ideas being taught. As the lessons progress, however, the stress is taken from these conceptual ideas and placed on repeated practice of the procedure. In the end, the underlying concepts are not emphasized to the degree that computational mastery is. This could indicate the implicit assumption that computation mastery is an indicator of conceptual understanding.

Other activities which take the student beyond computational mastery include Think Math (patterns and logical reasoning), Data Bank (gathering data and posing questions), calculator activities (exploration of number concepts using calculators), and other enrichment problems found every few pages in the fifth-grade text and about every 10 pages in the second-grade text. Interestingly, it is recommended that only "extended level" students do these problems. It is recommended that the "average level" do occasional Think Math problems and that the "minimum level" do only guided and independent practice.

Use of representations. In the student text and teacher's guide, illustrations, stories, and pictorial representations are used to represent the mathematical content. Pictorial models are used most frequently, and the second-grade teacher's guide often recommends that the students use manipulative models of some sort. The role of these models is usually to represent the algorithm. They are temporary aides to assist the student in performing the operation. Once algorithmic mastery has been achieved, which seems to be the central purpose, the aide is no longer necessary and is omitted. The illustration and stories are used to represent applications of skills.

Adjunct questions. The adjunct questions inserted in the text include the Think Math and the Skillkeeper questions. They are found in a box at the page of the student page. Not only are these exercises physically isolated from the others on the page, but they rarely are related to the content of the lesson that contains them. The Skillkeepers are designed to promote memorization and review of procedural skills. The Think Math questions are designed to promote application, logical thinking, and problem solving. Sometimes they may stimulate more questions or direct a student's interest to new ideas. For example, a Think Math problem in the fifth-grade text introduces Egyptian hieroglyphic numerals and directs students to use them to write several numbers. This may lead to an appreciation of various number systems or an interest in hieroglyphics or codes (Grade 5, p. 39).

#### Teacher-Student Relationships and Classroom Discourse

Opportunities for discourse. Few opportunities for discourse are provided in the second-grade curriculum. Teacher instruction and individual practice predominate. Some of the "problems of the day" activities in the second-grade text, found in the teacher's guide,

recommend that the class solve a problem and then "discuss why" the particular answer is correct. The fifth-grade curriculum provides more opportunities for discourse, although the main thrust of each unit is mastery of the computational skill being introduced. The small groups and discussion activities seem to be tacked on. There are no objectives in the teacher's guide that include discourse, and the chapter summary written for the teacher emphasizes the procedural skills being taught. The lesson components that include discourse are found in the teacher's guide, not the student text.

These opportunities for discourse, restricted to chapter openers, are separate from the rest of the content of the chapter. Discussing why an answer is correct, explaining, clarifying, or justifying a solution or ideas is not suggested during skill development portions of the lessons. Neither is developing and sharing various strategies used to find an answer. In short, talking about one's thinking and listening to the thinking of others is not considered part of elementary mathematics.

Forms of discourse. Much of the classroom discourse suggested is teacher-student, rather than student-student. This is characterized by the teacher asking a question and the student answering it. What opportunities there are for student-student discourse are found under the category of cooperative, small-group activities. A recommended activity at the beginning of each chapter suggests that the students work in cooperative groups. The fifth-grade teacher's guide recommends a small-group activity with each chapter opener. These do not have objectives attached to them and are given one class day in the unit calendar. The teacher is to allow the students to work on the project (often data collection) at the end of the period each day. Suggestions as to how the projects might be presented or displayed are also provided. One such project poses this situation: "Your group of students is investing \$1,000 in one or more stocks. Disregarding dividends and broker fees, what is your profit or loss after a three-week period?" It is recommended that the students work in groups of four to formulate questions and problems, collect data from the newspaper, formulate and implement a plan, and present the results.

The second-grade curriculum recommends the teacher begin each chapter with a motivational lesson read from the teacher's manual. These are stories during which the teacher pauses to ask both comprehension and mathematically related questions. A follow-up activity is also recommended with each opener which has the students work in small groups and present the results to the class. Unlike the fifth grade plan, there is no time provided for this activity in the daily plans and there are no suggestions on setting up small group activities provided for the teacher.

Authority for knowing. The teacher and text stand out as the authority for knowing. The text proposes a complete curriculum, designed for teachers to follow. Answers are given for all problems and exercises and guidance is provided in organizing and teaching each lesson.

There are few lessons which suggest that the teacher take the students outside of the text in teaching mathematics, although the above project, as do other chapter openers, has students draw on other forms of information, such as the newspaper.

### Activities and Assignments

Range of activities and assignments. The activities and assignments vary from computational exercises to application word problems to enrichment exercises. The bulk of the activities consist of computation exercises and word problems. The additional activities, Think Math, Skillkeeper, problem solving, chapter openers (fifth-grade only), data bank activities, calculator and computer activities, are found consistently, but less often, throughout the curriculum. The teacher's guide also suggests activities not in the student text. The information about each chapter includes a math lab and game suggestion. These are usually one or two game-like activities the teacher can make for the students to use in class. They tend to be games that students can play in small groups oriented at skill practice. The purpose of each math lab or game, as the text described it, is to practice, in a "fun" way, the specific skills being taught in the chapter.

Examples of interesting activities. The fifth-grade chapter opener activities, previously described, are strong in developing data collection and application abilities through peer interaction. They require student independence, cooperation, thinking and reasoning. In some cases they could lead to the student's development of conceptual understanding of the particular topic, but, more often, they seem to be a way to engage the student in the relevance of the procedure. How they connect to the rest of the chapter is not always clear.

The Think Math activities are also quite challenging and require logical problem solving abilities. This is much more the case in the fifth-grade text. A second-grade Think Math example provides the first inning score of a baseball game: Birds--2, Eagles--4. The final score is given as 5 to 3. The questions are "Which team won? How do you know?" (Grade 2, p. 158). The student must reason that "The eagles cannot have a final score of 3 since they already have 4 in the 1st inning." The following fifth-grade example is representative of the degree of abstract thinking required for many of the fifth-grade Think Math problems: A sheet is folded as shown and cut along the red line. (A diagram shows a square page being folded in half twice and one corner is cut off.) Draw a picture of what the sheet will look like when it is unfolded (p. 143).

Integration with other subjects. Subject integration is minimal, but occurs more frequently and more in depth in the fifth-grade curriculum than in the second-grade curriculum. The fifth-grade teacher's guide lists the topics in the upcoming chapter which are related to other subjects. Although no activities are recommended to facilitate the integration, the

teacher is given the opportunity to pursue these topics. The chapter openers in fifth grade and the motivational stories in the second grade, especially if they are followed through to the extent that the manual recommends, certainly involve integration of "other than math" topics, skills, and concepts into the mathematics in ways that would integrate the subjects, rather than replace one with the other. The data collection activities at the beginning of the chapter are an example of this. They take a mathematical skill and have the students use it in a somewhat realistic situation. Furthermore, because there is an attempt to make math fit with real-life experiences in both texts (although it is more evident in the fifth grade) many of the word problems and examples are set in contexts which usually are associated with other subjects, particularly science and social studies.

The advantages of this are that there is a great attempt to keep mathematics skills in their context when introducing them, and there is emphasis on application of skills to real-life problems. The disadvantages in the degree to which integration occurs is that there is really no attempt to facilitate the integration. Various subjects are touched upon, but the teacher is not given assistance in expanding those opportunities. There is also no mention of integration (having children see and experience the connections between the various subjects) in the objectives. It seems to be another tacked-on activity.

Types of questions and answers. The format of the questions in the text requires fill-in-the-blank, convergent answers, which are given in the teacher's guide. Those with multiple possible answers say "answers will vary." No encouragement is given to the teacher to assess the reasonableness of the varying answers. The fifth-grade chapter openers seem to be the only questions which require more than a short, specific answer or solution.

### Assessment and Evaluation

Forms of assessment. The formal assessment provided in the text consists of chapter review and cumulative review tests. After each chapter review test there is a reteaching page for "those students who experienced difficulty with the chapter review test" and an enrichment page for those students who successfully completed the chapter review test." In addition, follow-up activities are suggested frequently within the chapter, which provide strategies for reteaching and suggest activities for enrichment. This suggests the necessity of regular formal assessment, followed by enrichment or reteaching when necessary. Informal assessment is also encouraged. When new skills are introduced, a chart of frequently made errors and recommended remediation strategies is included in the teacher's guide. It is suggested that the teacher make note of the nature of student computational errors and then use the chart to diagnose and correct them.

What is being assessed. The assessment items call primarily for computational competency. A small number of word problems are also included; however, the suggestion seems

to be that correctly performing algorithmic procedures constitutes mastery. The problem-solving strategies are assessed to some degree in that there are story problems on the test, but the right answer is emphasized over the strategy. Questions requiring the type of strategies used in the Think Math questions are not found on the tests. Self assessment is not emphasized in the curriculum.

The forms of assessment used in *Addison-Wesley Mathematics* are limited because they are relatively uninformative about what the child understands about the procedures and their uses, how the child calls upon her own knowledge to solve problems, the child's ability to articulate and reason about mathematical ideas, or engage in forms of mathematical discourse.

#### Directions to the Teacher

Model of teaching represented. The model for teaching and learning from which the teacher suggestions grow seems to be one that includes motivating students through activities involving application or a real-life situation, introducing the skills, providing opportunities for students to practice the skill and use it in application problems, and test for mastery of the skill. The Madeline Hunter model of instruction is emphasized explicitly in the teacher's manual. In addition to the basic model described above, the teacher is presented with many enriching activities that can be added to the basic curriculum in the text. Because all of the ideas presented to the teacher are not manageable in the amount of time suggested for each chapter, the teacher must choose which activities to include. The choices include activities that involve problem solving, cooperative groups, and integration and are likely to take the student beyond basic computation. They are given secondary importance, however, by their place in the curriculum.

Information and explanations provided. The curriculum comes with clearly stated scope and sequence charts, introductory sections that provide clear, sufficiently detailed information about what the program is designed to accomplish and how it is designed to do so. Each chapter is preceded by a four-page introduction for the teacher summarizing the chapter, stating the objectives, explaining the mathematics, suggesting ideas to use in introducing the content and extending it, and providing a daily calendar which includes pages and problems to assign to each of the three levels of students: basic, average, and extended. There seems to be little emphasis placed on rationale, although there are various questions answered for the teacher explaining why certain activities are included or are introduced in the way they are. Some of the answers provided in this section seem unsubstantiated by the text. An example is the claim that problem-solving skills are taught as an integral part of any computational skills lesson. The authors claim, "This is why every skill lesson begins with a motivational problem and ends with additional application." While problem-solving skills are taught

alongside computational skills or in the same chapter, the two are rarely integrated in such a way that the student is expected to call upon both in solving computational and story problems.

Content and pedagogical knowledge required. The view of mathematics and how it is learned implied by the Addison-Wesley text fits closely with a fixed view of the school subject--that it is primarily a fixed body of computational procedures with single answers and that teachers should explain the rules to students who should practice using them. The assumption is that teachers must know these rules and understand the reasons behind them in order to explain them to the students. This is instantiated by the explanation of the procedures provided in the beginning of each chapter. The pedagogical knowledge required by the teacher in order to use the curriculum includes the ability to assess assignments and activities and make decisions about their appropriateness to her particular students. This ability, together with an understanding of mathematics, and knowledge of her students' understanding, would allow the teacher to adjust and adapt the assignments to foster their understanding of the content presented.

### *Real Math*

#### Goals

Program Goals. The authors of *Real Math* identify a set of mathematical goals "that tomorrow's citizens must have" and comment on the "overall purposes and effects of a mathematics education" (p. xx). A central purpose emphasized throughout the text is to facilitate the student's understanding of technological advances and their benefits, enabling the child to become the master of, rather than slave to, such advances. The authors also emphasize the "value to the individual of sound mathematic education," which can "develop the ability to *think clearly*." (pp. xx)" Both this ability and the confidence it generates are considered important to the critical thinking and decision making necessary in a "complex" and "changing" world. Related to this, the authors assert that "a good mathematics education may instill a strong respect for truth and an appreciation of beauty, symmetry, and pattern" (p. xx).

The authors list 21 goals which represent the skills "a child should acquire through a well-designed mathematics education in the elementary school" (p. xx). Going beyond observable skills that students are expected to obtain, these goals attend to the development of conceptual understanding and dispositional stances toward mathematics:

- Firm understanding of the significance and use of numbers in counting, measuring, comparing and ordering
- Mastery of the basic operations with whole numbers
- Sufficient familiarity with the number system to avoid applying tedious algorithms in special cases
- Ability to use the appropriate mathematical operation or operations to solve realistic problems

- Understanding of when an appropriate calculation is appropriate and the ability to make such an approximate calculation
- Appreciation of the role of estimation in intelligent behavior and the capacity to make reasonable estimates
- Ability to use a calculator effectively
- Familiarity with the nature and purpose of computers
- Firm understanding of magnitude with respect to measurements and of the role of units in assigning numerical magnitudes to physical quantities
- Ability to organize and arrange data for greater intelligibility
- Ability to extrapolate and interpolate from data and from graphic representations
- Understanding the role of functions in modeling the real world
- Understanding of rational numbers and of the relationship of fractions to decimals
- Understanding of the meaning of rates and of their relationship to the arithmetic concept of ratio
- The ability to use probabilistic ideas in ordinary, elementary applications
- The ability to solve problems involving money
- An understanding of, and ability to use, the geometric concepts of perimeter, area, volume, and congruency
- An understanding of, and ability to add and subtract, signed numbers
- The ability to make, read and use a map
- The ability to think intelligently, using numbers
- A positive attitude toward mathematics (p. xx)

**Knowledge goals.** The authors' goal statement implies that knowledge of procedures and their underlying meaning and use should not be treated separately, as most of the goals link the ability to perform procedures with understanding their underlying concepts and the ability to use them. For example, goals that aim to develop the ability to organize and arrange data for greater intelligibility; understanding of and ability to use the geometric concepts of perimeter, area, volume, and congruency; or the ability to use the appropriate mathematical operations to solve realistic problems suggest an interweaving of skills, concepts, and their applications.

**Dispositional goals.** The program also attends to the dispositions and attitudes students develop toward mathematics. The final goal listed, that students should acquire a "positive attitude toward mathematics" and the suggestion that this includes having the conviction that mathematics does "solve real, interesting problems and . . . is a tool that the children can use confidently" (p. xxi), suggest that the authors expect that students should come to enjoy mathematics through experiencing it as an inviting domain which makes sense and is useful, rather than a subject that has been made artificially appealing through pictures and games.

**The conception of mathematics driving the curriculum.** The program goals and the authors' purpose statements grow out of a conception of mathematics as a logical, sensible, and organized way of looking at and understanding the world and daily experiences. This does not

imply that mathematics is limited to its application within the physical environment. Human experience includes ideas and abstractions that have both aesthetic and logical appeal. Through understanding mathematics, the authors feel, children will gain a greater appreciation for the beauty and symmetry in the world around them.

### Content Selection

The strands used to group the content are numbers, place value, and numeration; operations; applications; reasoning; geometry; measurement; probability and statistics; estimation; algebra, relations and functions; and rates. Each strand is addressed to some degree at each level. The second-grade text includes more number and numeration than rates, while the fifth-grade text places greater emphasis on rates than number and numeration. There are several topics treated in *Real Math*, some in great depth, which are not addressed in standard curricula. Functions, introduced in the third grade and thoroughly developed in fifth and sixth, is the most striking example of such a topic.

The substance and nature of mathematics. The content selection represents mathematics as more than rules and procedures, the concepts underlying them and their applications. The authors emphasize that mathematics is a real part of the child's life: "It is not enough for mathematics to deal with objects from the child's world. . . . [It] must fit into how the child perceives and understands these objects" (p. xvi). Mathematics, then, is a way of thinking about and making sense of the rules, procedures, and concepts that grow out of and are applied to the real world.

Evidence in *Real Math* for this view is found in the nature of mathematical activities designed and quality of thinking expected. Both the thinking stories and the lessons seem intended to engage the students' thinking, rather than merely being tasks to be completed. An example in the fifth-grade text is a Thinking Story about Mr. Sneeby who opens a T-shirt store. In part I of the story he is visited by the children who discover he is not making enough money to pay his bills, although he is selling lots of T-shirts for five dollars each. Their initial reaction is that "he must be making lots of money if he is selling lots of T-shirts." The follow up questions ask (1) "What are they forgetting about in thinking this?" and (2) "What are some other things you need to know in order to figure out whether Mr. Sneeby's store is making money or not?" (p. 121). In proceeding parts of the Thinking Story the students use various bills (some are monthly, others cover different periods of time) to estimate how much per month it costs Mr. Sneeby to run his store. When doing this they are encouraged to "think about which bills are almost the same each month and which bills will change from month to month" (p. 135). The students must deal with how Mr. Sneeby should pay himself and other considerations in running a business. Problems and activities such as these represent mathematics as being

applicable to realistic situations and requiring reasoning and sense making, as well as knowledge of specific skills.

**Applications.** Applications of mathematical skills, procedures, and ways of thinking are emphasized throughout the *Real Math* text. Concepts are introduced most frequently through the use of realistic contexts to which the concept applies. Application problems, which require the student to formulate and solve problems, are found in the majority of lessons. Real-life applications are used to provide a context in which new concepts are taught and applied.

**The role of prior student knowledge.** The authors emphasize building on students' prior and intuitive knowledge. They recognize that children bring this knowledge with them to school and explain that instruction should sharpen and clarify their concepts, relate them to one another, and develop the skills through which they can make effective use of them. This is evident by the opportunities for discussion and sharing knowledge, ideas, and intuitions provided in the lessons. This emphasis on discussion and student knowledge is unlike many commonly used curricula which tend to be centered around workbook pages. The student workbook pages in *Real Math* play a less central role in each lesson. Often they are used to extend a whole-class lesson. Other times they are intended to be part of the lesson. This is particularly true in the second-grade text. The fifth-grade text plays a more central role in the lessons, but class discussions are frequently suggested, as is having the students work on the text pages in groups.

**Student interests and dispositions.** Throughout the curriculum the authors attempt to make mathematics "real" for children. This includes setting it in contexts with which they are likely to be familiar, emphasizing the underlying meanings of the concepts, and involving students in interesting and challenging applications and discussions. The Thinking Stories problems used at each grade level are examples of these efforts. They are amusing, interesting, and entertaining stories that involve complex, nonroutine mathematical thinking. In the younger grades the stories are read aloud to the class. In the older grades they are part of the student text. In each case they are treated as a whole-class activity. A second-grade story tells of a group of children trying to weigh a dog, Bowser. As they struggle to keep Bowser on the scale they discover that "The harder you press to hold him down, the more he seems to weigh . . ." (p. 313). Through asking questions the teacher is encouraged to lead the students in a discussion about the many problems the children in the story encounter.

Many of the characters in the first-grade stories reappear in stories at other grade levels. In the fifth-grade stories, they are involved in using clues to solve a mystery. In one story the children must decipher a code of numbers on a piece of paper in a dictionary. It is discovered that each number in the code represents the page number in the dictionary on which

the word can be found. The code is partially cracked before the dictionary is stolen. Given page numbers, some with corresponding words, the children must use their knowledge of alphabetical order to figure out possible words for the remaining numbers. The story situations are likely to be interesting to students, engaging them intellectually and encouraging them to reason logically.

### Content Organization and Sequencing

The content organization differs between the primary and intermediate grades. The primary grade curriculum is not organized into units. Instead it consists of 140 lessons which are grouped such that a series of three or more are devoted to a particular idea, such as counting and writing numbers or congruency. Each strand is not addressed in an isolated section but is touched upon frequently through various topics. For example, the counting numbers are reviewed in lessons one and two. They are addressed again through the use of money, the calendar, and the number line in later lessons. There is also a great deal of natural overlap and interconnection between the strands. For example, the calendar is part of the following strands: numbers and numeration, applications, and measurement. There are four diagnostic tests spread throughout the lessons, for the purpose of diagnosing "difficulties the children may have with the material covered to date." These tests are primarily made up of computation items and routinely phrased story problems. Unlike in the daily lessons, most of the exercises on them require single operations. The implication seems to be that the bottom line of understanding includes being able to do the procedures correctly.

Beginning at the fourth-grade level, the content is organized into units. The fifth-grade curriculum is divided into six units which conclude with unit tests. The units do not have titles, but the topics in each unit fit together and build on one another. For example, Unit V consists of the following geometric topics: angles and their measurement, congruency and similarity, maps and scale drawings, properties of figures, and area of triangles and parallelograms. The unit tests in the fifth-grade text are also primarily made up of computation and routine story problems. One feature that distinguishes the *Real Math* tests and practice pages from those in *Addison-Wesley Mathematics* is that the problems presented are mixed, in that they do not all call for the same procedure or solution strategy. In other words, there are not separate sections devoted to a single operation or procedure. Subtraction problems that do not require regrouping are peppered throughout problems for which regrouping is necessary. Each set of story problems calls for a variety of operations or solutions procedures, although they all may be connected mathematically. The students are, thus, required to examine the problems carefully and select or determine the correct approach. Often, more than one operation is required.

Substance and nature of mathematics represented by the content organization. The organization represents mathematics as problem setting and sense making as well as problem solving and applying algorithms and procedures correctly. Doing mathematics includes deciding what the problem is, as well as being able to solve it. The integration and spiraling of topics and concepts reflects the interconnected nature of mathematical topics. The chapter and diagnostic tests, however, suggest a discrepancy in this perspective as they focus on algorithmic proficiency, although they also tend to present a heterogeneous mixture of problems.

#### Content Explication in the Text

Content presentation. Unlike traditional lesson formats, which usually begin with whole-class instruction or review of a mathematical procedure followed by individual student practice of the particular skill in the form of workbook or text pages, the main focus of each lesson in *Real Math* is usually a whole- or small-group activity. There are a good deal more textbook pages to be completed by fifth graders than second graders, but it is recommended that many of them be completed in small groups. The whole-class lessons begin with a mental math segment, during which the students are given problems to compute mentally, and then proceed into instruction on or an activity with the main topic of the lesson. Another distinction of *Real Math* is the type of problems presented. As mentioned earlier, it presents genuine rather than contrived problems that require careful thinking on the part of the students. The regular and integrated use of application is another prominent feature of *Real Math*. Complex and interesting mathematics problems grow out of situations presented to the students throughout the text, not just at the end of units.

Games are used frequently in *Real Math* lessons to provide enjoyable practice of the skills being developed and applications of these skills (i.e., making change, using addition table, using monthly calendar, using place value, comparing decimals, approximating quotients). Unlike the games recommended in the teacher's guide of *Addison-Wesley Mathematics*, which seem to have the purpose of repeated practice of basic facts, these games are challenging. A number game played during the second lesson in the second-grade text involves two players taking turns writing one, two, or three consecutive numbers. One student designates the beginning and ending numbers before the players start. The goal is to be the player to write the designated final number. The players must respond to the number of numerals the other writes and plan how many to write while attempting to end up with the last turn. Games such as this are likely to encourage the students to reason about the outcome and develop logical strategies to use when playing.

Topic treatment. To a large extent, *Real Math* focuses on developing conceptual understanding of mathematical ideas and logical approaches to setting and solving problems. Algorithmic procedures, such as regrouping to subtract or finding common denominators, are

taught after much work has been done to develop the underlying concepts. Furthermore, there are attempts to make explicit the connections between the procedures and their conceptual underpinnings.

Use of representations. Concrete and pictorial models and stories are used metaphorically throughout *Real Math* to represent meanings or contexts for applications of the content being taught. These representations range from frequently used models, such as bundled sticks to represent base-10 place value or cut up pizzas to represent part-whole fractions, to uncommon models. In the teacher's guide, the authors claim to have based their selection of representations on research findings of how children learn mathematics, although the particular research used is not cited.

One particularly unusual representation, introduced in the second-grade curriculum, is the paper calculator. Essentially a number line and markers, this model emphasizes the nature of the base-10 system and the sameness of operations involving digits in any position (units, tens, hundreds, etc.). The students' initial introduction to the paper calculator involves only single digit numbers (see figure below).

Using markers, two addends are represented. For example, a marker on 8 and 5 would represent the expression  $8 + 5$ . By moving the markers in opposite directions the same sum can be represented with different addends:  $8 + 5 = 9 + 4 = 10 + 3$ . When these moves are introduced, it is also recommended that the students represent these relationships with a fixed number of objects distributed between two circles. The circles, in this case, would begin having eight objects in one and five in the other. One object would be transferred from the second circle to the first to show that  $9 + 4$  also equals 13. As the paper calculator is developed, it is expanded to include tens and hundreds. The similar relationship is emphasized between positions:  $80 + 50 = 90 + 40 = 100 + 30$ .

More commonly used models for place value are also used in *Real Math*. In each case, their presentation seems to be thoughtfully considered to highlight essential conceptual ideas. For example, bundling sticks are used to emphasize the base-10 grouping of numbers. Rather than showing a fixed number of bundles and loose sticks, the picture shows some of the bundles coming unbundled. The focus seems to be on the fluidity between representing the quantity in bundled and loose sticks.

To represent the meaning of the multiplication of fractions in a fifth-grade lesson, rectangular cakes are used together with a story about Larry who shares the cake with three other friends. Each friend cuts his or her fourth into three equal pieces, eating one piece a day. The picture shows that one-third of one-fourth is one-twelfth of the whole cake. This is an example of the explicit and careful use of commonly used representations found in *Real Math*.

They focus on the underlying concept and are carefully explained in the student text (or the whole-class lesson in the primary level).

Multiple representations are used, highlighting various components of the same concept. For example, fractions are represented as pieces of pizza, pie, fruit, round and rectangular cake, quantities of liquid, and portions of a line segment (in the fifth-grade text), and parts of shapes, parts of a set of discrete objects, and amounts of money at different points (in the second-grade text).

**Skill development.** The development and use of mathematics skills is an explicitly stated goal in *Real Math*. Skills, however, are broadly construed. They include facility with algorithmic procedures, the ability to construct and use charts, problem solving, and mathematical reasoning.

**Provisions for student diversity.** Attention to ethnic, cultural, and gender diversity is reflected in the divergent range of students represented in pictures and names in the text. The culture represented is less socioeconomically homogeneous and more authentic than that in *Addison-Wesley Mathematics*. Although typically American activities are portrayed, there is a greater range of activities and less emphasis on consumerism. The characters in the stories have human characteristics, expressing depth and feeling. The stories and pictures represent children and adults engaging the activities of their lives thoughtfully and naturally. A range of activities are represented from running, shopping, and baking to constructing, solving mysteries, and collecting bottles to recycle.

With regard to intellectual diversity, exploration and discussion are encouraged, and there is some room for divergent ways of thinking evident in the margins of the teacher's manual, particularly for the discussion questions and Thinking Stories. Most of the problems presented have several possible answers or ways of expressing the solution, or it is recommended that the teacher lead the students to a certain conclusion or make sure they consider specific points. The implication is that specific purposes underly these questions, and it is these purposes that are of central importance in considering the problem. So, although there is often one right answer, there are multiple ways to express it. The exact form of the answer is less important than the mathematical issues the problem raises.

Two very interesting recommendations to teachers represent well the authors' views regarding divergent thinking and appreciating differences. These are intended to aid the teacher in using the Thinking Stories and are found in the fourth- through sixth-grade manuals:

1. Don't praise students only when they give the answers you happen to have in mind. Recognize that for most questions there are a number of possible answers and that even a clearly incorrect answer probably entailed some worthwhile thinking . . .

10
9
8
7
6
5
4
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2
1
0

Paper Calculator, *Real Math* (Grade 2, p. 79).

2. Don't encourage snobbery. The characters in these stories are well aware of one another's foibles. They are all warm-hearted people who support each other and treat each other with respect. The stories are written so as to convey this attitude to the class as well. A lapse in thinking is not necessarily more contemptible than a broken arm, although both are handicaps that one does well to recognize and avoid if possible. So don't ridicule the characters or make a point of the students' intellectual superiority over them. (p. 467)

### Teacher-Student Relationships and Classroom Discourse

Forms of discourse. The authors of *Real Math* describe mathematics as a collaborative activity done with other people. Consequently, a large portion of lesson time is devoted to class discussions and small-group work. There is, however, noticeably more emphasis on individual work in fifth grade than in second. The second-grade curriculum allows 10 to 20 minutes of the 45-minute lesson for independent workbook pages. The remaining 25 to 35 minutes are spent in whole-class work, which usually includes demonstrations by the teacher; seminar (teacher-guided discussion relating to demonstration); workshop, for the purpose of practicing and applying skills (often in the form of a game); and discussions of Thinking Stories. The fifth-grade curriculum devotes more time of each lesson to completing textbook pages, although about half of the time it is recommended they be completed in small groups. There is less teacher instruction time, but Thinking Story discussions are included about every five lessons. At all levels, a small amount of time in each lesson is devoted to an individual response activity, during which the students respond verbally or individually, using mechanisms such as cubes or spinners with numbers on them, to questions asked by the teacher.

The Thinking Story lessons suggest a class discussion format in which students and teacher interact around problem situations. They are designed to facilitate reflective, creative, and logical thinking within a group setting. They encourage the students to think about problems within contexts that maintain the complexity of real life. The stories and accompanying questions seem constructed to lead students to develop collective solution strategies and explore mathematically significant ideas as a class. The directions given to the teacher explain the central ideas that students should consider, provide ways the teacher might respond to the students' answers, and suggest possible answers to expect. Ways in which the teacher might shape the discourse around these Thinking Stories are not provided in the teacher's guide. Although it is likely that student-student discourse might develop around the story situations, it is also possible that the talk must be limited to student-teacher interactions.

Authority for knowing. A prominent feature of *Real Math* is the presentation of mathematics as logical and rational. Not only is emphasis placed on making sense of the problems presented, but the context in which the ideas must make sense is the real world. While maintaining complexity, the problems tend to be set in concrete contexts which the

students can actually visualize and reason about. Thus, it seems that the authority for knowing and for what makes sense is the child's experience. This is not to say that the teacher or the text has no authority in determining correctness. However, the text seems to refer to actual concrete experiences (in stories or within models) as the basis for what is reasonable and accurate. The authors have carefully attempted to demystify mathematical processes, emphasizing that they are reasonable and make sense, through the characters in the Thinking Stories. One particular character, Minolita, tends to explain events as occurring magically when there are quite reasonable explanations that are discovered and explored. The magic power that she ascribes to unexplained events is not unlike the mystery that often surrounds mathematical procedures that are not understood. Students working with the Thinking Stories have the opportunity to find reason beneath seemingly magical events.

### Activities and Assignments

The recommended student activities and assignments are varied in *Real Math*. The formats include whole-class instruction, teacher-led discussion, small-group work, games played in small groups, and individual work. In these various formats, it is intended that students develop, practice, and use mathematical procedures and ways of thinking using manipulatives or pictorial devices to represent the procedures and their underlying concepts; applying procedures in standard story problems and complex situations; and collecting, organizing, and analyzing data. The text provides for a mixture of critical and creative thinking, problem solving, decision-making activities, and work with basic skills, rules, and procedures.

Several or all of the following eight components make up each lesson:

**Demonstration**--When a new topic is being introduced, about 10 minutes of demonstration time are used to explain and demonstrate.

**Seminar**--teacher-guided discussion, often combined with or following the demonstration.

**Response Exercise**--a brief period during which the students respond individually to mental computation problems posed by the teacher.

**Workshop**--approximately 15 minutes during which the students practice and apply the skills taught in the lesson, often including game playing.

**Storybook**--Every several lessons the students read, or listen to, a brief story and discuss questions afterwards.

**Whole-Group Activity**--Activities designed to give the students a chance to apply mathematics to various events in cooperative settings.

*Student book*--Two pages from the student books are usually included as a supplement to the practice provided during the workshop and response exercise.

*Extra teaching and practice*--Suggested ways in which special help can be given to children who show evidence of having difficulties during other parts of the lessons.

Integration of other subjects. Because the *Real Math* authors have used daily situations as contexts for many of the problems, there is a natural integration of other subjects running throughout the text. The subject provides the background for the development of mathematical ideas. The intent does not seem to be to develop the other subjects along side the mathematics, but to provide a richer experience through recognizing their connections to mathematics in real life. One way in which literature and an appreciation of story is integrated into the *Real Math* is through the Thinking Stories. While not part of a canonical body of literature, these stories are like chapters in the lives of a group of friends. The same characters appear in all of the Thinking Stories from first to sixth grade, although they grow older each year. Each character has a well developed personality and unique character traits. The teacher's guide provides a brief introduction to each character and an explanation of their contributions to the mathematical thinking in the stories. It is likely that children will come to know these characters in the same way they might come to know a character in a novel.

There are also instances of activities that are explicitly integrated with other school subjects. In these the teacher's guide suggests activities to do alongside those designed to develop the mathematical content. Science is most frequently used as a means of applying data collection and organization. After exposing children to the concept of congruency, the second-grade text recommends that a class field trip to a local factory be arranged. "Factories will probably utilize techniques for producing congruent objects and it may have control techniques to reject objects that do not meet certain specifications" (p. 144). Suggestions for organizing the visit and possible types of factories are provided.

In the fifth-grade text, several of the Thinking Stories are placed in a historical context. "Land" examines the concept of area through a story set in the Western frontier; although, this is not an attempt to teach history through mathematics. Nevertheless, these stories provide interesting problem contexts and contribute to the richness and variety of situations found in the text. Throughout the curriculum, the degree and form of integration is such that connections are made to other subjects without distracting from the mathematics.

#### Assessment and Evaluation

Tests in *Real Math* are found less frequently than in many commonly used curricula. In the primary grades there are four diagnostic tests spread throughout the lessons. It is stated that these are for the purpose of diagnosing "difficulties the children may have with the material covered to date." These tests are primarily made up of computation items

and routinely phrased story problems. Most of the exercises on them require single operations. As explained earlier, the emphasis on computational skill seems to be inconsistent with the emphasis in the lessons on conceptual understanding. In the upper grades there is a test at the end of each of the six or seven units. The tests are also mainly computation and routine story problems. Unlike the primary-grade tests, which are somewhat cumulative, the content is specific to that of the completed unit. The purpose of the daily response exercise is to provide the opportunity for the teacher to informally assess her students' mental computation skills.

#### Directions to the Teacher

Information for the teacher is divided into three parts. At the beginning of the teacher's guide a brief overview states the philosophy and goals of *Real Math* and describes its development. A scope and sequence chart is also included in this section. The second section includes the daily lesson plans: Purposes, materials needed, and suggested guidelines for teaching are found with each lesson. The guidelines are not scripted, but give a summary of what should be done, pointing out particular things to note. The final section, provides the teacher with strategies, tips, and pedagogical information. This portion includes more thorough details about using the curriculum. It also includes letters that could be reproduced and sent to parents describing the *Real Math* program.

The information to the teacher about the program is straightforward and informative, not glitzy or appearing to be an advertisement, as is the case with the information at the beginning of the Addison-Wesley teacher's manual. The level of detail provided in the lesson plans is such that a clear lesson framework is constructed, allowing the teacher to follow the lesson. There is an effort to emphasize the importance of questions that have several possible answers, and the sorts of answers students might arrive at are suggested in the margins. Some more general strategies for developing and managing class discussions, helping the students to justify their answers, and encouraging the class to consider whether or not their answers make sense are briefly mentioned in the pedagogical section.

Completeness of program. *Real Math* presents a complete program which treats mathematics with greater breadth and depth than many commonly used curricula. The materials included in the curriculum and others available from Open Court comprise, for the most part, all the materials needed. When the teacher will need to secure other materials or prepare to use some that are included with the curriculum, this is mentioned two lessons in advance. There are also suggestions of other materials that can be made to use with *Real Math*.

Teacher's mathematical knowledge. *Real Math* aims to develop and foster students' mathematical reasoning and understanding. In order to fully use and appreciate this program the teacher must be able and disposed to reasoning mathematically and have an understanding of mathematical ideas that goes beyond being able to manipulate symbols. For example, it is

important that the teacher have a sense of the purpose of the questions asked in the text. Without this, many of the questions intended to encourage students to think mathematically and explore different possibilities, could be treated convergently. Furthermore, it is important that she understand that reasoning mathematically is both a process and goal in the mathematics curriculum; that is, while being a goal in and of itself, it is also a vehicle through which a variety of other mathematical understandings are developed.

Because *Real Math* is quite distinct in content and approach--although its appearance is much like many commonly used texts--a teacher who does not understand the reasoning and philosophy behind the particular strategies used could possibly use the materials the way standard texts are used. For example, the focus of each lesson is designed to be on the class activity. When textbook pages are assigned in fifth grade they are often intended to be used within a small-group or class discussion. A teacher must use these materials as independent practice, rather than as a whole-group lessons, in which the collaborative process through which the students find answers is an important part of the lesson.

Because class discussion and small-group activities are frequently used, the ability to manage an active classroom, in which children work collaboratively, or the willingness to develop these abilities seems necessary. The hope of the authors is that teachers using this program are able to foster mathematical thinking and exploring through the activities provided in the text. A teacher not accustomed to student talk, and listening mathematically to students' responses, or with a low tolerance for noise and activity, might restrict these activities and miss the potential richness they intend.

### *Comprehensive School Mathematics Program*

#### Goals

Program philosophy and conception of mathematics. The *Comprehensive School Mathematics Program* (CSMP) authors do not provide a list of program goals stated in terms of student outcomes. Nevertheless, their perspective on mathematics and mathematics pedagogy is clearly spelled out in the "Philosophy and Goals" section of the CSMP Final Evaluation Report (Herbert, 1984), which provides a rationale for the content and pedagogical selections made. The curriculum is based on the belief "that mathematics belongs to the part of the school curriculum which deals with ideas rather than that part which stresses techniques" and that learning mathematics should be an "interesting, challenging, intellectual and aesthetic experience for all children" (Grade 5, Semester 2, p. 1). The CSMP developers believe that drill and practice in arithmetic computation should not be unnecessarily stressed in elementary school mathematics. Instead, children should be introduced to "mathematically important

ideas" (Herbert, 1984, p. 3.) The authors list three basic principles that guide the CSMP curriculum:

Mathematics is a unified body of knowledge and should be organized and taught as such, so that, for example, the artificial separation of arithmetic, algebra and geometry should not be maintained.

Mathematics as a body of knowledge requires certain ways of thinking and cannot be done by the exclusive use of memory.

Mathematics is best learned by students when applications are presented which are appropriate to students' levels of understanding and to their natural interests. (p. 3)

These three emphases--the connectedness of mathematics, the importance of thinking and understanding, and the use of appropriate and challenging applications--are evident in the content selection, organization and pedagogy. Based on the above philosophy, CSMP has attempted to introduce a different perspective of school mathematics. Not only is the content new and unlike standard math curricula, but the way it is organized and taught is also unlike traditional elementary curricula. The rationale provided for this content and structure is based on assumptions about how children learn:

An underlying assumption of the CSMP curriculum is that children can learn and can enjoy learning much more math than they do now. Unlike most modern programs, the content is presented not as an artificial structure external to the experience of children, but rather as an extension of experiences children have encountered in their development, both at the real-life and fantasy levels. Using a "pedagogy of situations," children are led through sequences of problem-solving experiences presented in game-like and story settings. (Herbert, 1984, p. 3)

The term "pedagogy of situations" is used often to refer to the learning that takes place when children experience and react to interesting situations, whether they are real or fantasy. This is similar to the perspective of the *Real Math* authors that for young children fantasy is part of "real life," and they can learn from and find meaning in fantastic situations.

**Knowledge goals.** The authors of CSMP expect that students will learn to "mathematize" situations. This refers to the ability to interpret situations through a mathematical lens and solve them using mathematical skills. This requires the integration of conceptual, procedural, and conditional knowledge. Consequently, the authors claim that the development of skills and procedures should occur within conceptual and situational contexts. The General Introduction stresses that "training for rote skill should never precede understanding" (Grade 5, Semester 2). The importance placed on the "pedagogy of situations"

emphasizes applications that require complex thinking and are truly problematic to students. The authors emphasize the importance of seeing connections and understanding interrelationships among ideas. They see learning mathematics within the context of situations, real or imagined, as being a way to have students experience these connections as they really are, rather than learning isolated facts and then afterwards applying them. CSMP does encourage skill development, such as mental and written response to situations in which they are needed, but the student is not expected to learn skills in isolation of the application or a larger context of ideas.

CSMP emphasizes conceptual and conditional knowledge as being preliminary to the development of procedural knowledge. The procedures (such as the subtraction, multiplication and division algorithms) are introduced and developed once related concepts have been thoroughly developed through the representational languages, making their introduction later than many math texts. The representational languages are designed to develop conceptual understanding, rather than knowledge of computational procedures. CSMP students are involved in using mathematics in a variety of different conditions, both real-world and imaginary. Furthermore, connections are made between events in one's everyday experience and mathematical ideas, as students learn to think and reason mathematically. In this way, conditional knowledge is emphasized.

For example, lesson 11N (Grade 5, Semester 1) begins with the teacher showing a spinner divided into fifths, two of which are red and three of which are blue. The students use fractions to figure out the probability of the needle pointing to each of the colors. They then calculate the probability of a die coming up with a divisor of 6. They continue to work with other probabilities and begin to look at equivalent fractions and then the decimal names for some of the fractions. The lesson intends to help the students apply a mathematical concept to a situation that they can imagine and understand. It uses the familiarity of the situation to make connections between mathematical ideas that may seem difficult if taught in isolation.

### Content Selection and Organization

The CSMP curriculum is also divided into four levels: Kindergarten, Grade One (Parts I and II for first and second semester), Upper Primary Grades (Parts I and II for the first and second semester of second grade and Parts III and IV for third grade), and Intermediate Grades (Parts I to VI, two semesters for fourth grade through sixth grade). The classroom set of materials includes teachers' guides, student workbooks and worksheets, and all other materials suggested in the guide.

How the content represents the substance and nature of mathematics. The content selected is intended to match the discipline of mathematics more closely than commonly used elementary curricula. In their selection of content, the authors aimed to represent mathematics

as interesting and challenging, having intellectual and aesthetic value. The content includes integers, rational numbers, decimals and their operations, geometry and measurement. The following topics, which are generally not found in standard curricula, are also included in CSMP:

*Geometry*--Construction of figures with translators, angle templates, compass and straightedge; properties of shapes independent of distance; parallelism and parallel projections; reflections and symmetry; generalized distances other than Euclidian; tessellations; the "map" of a cube; the triangle inequality.

*Probability*--Prediction and comparing results of probabilistic experiments simulated by marbles, spinners and dice; probability concepts such as randomness, equally likely events, fairness, selection with and without replacement; combinatoric analysis of probabilities; the multiplication principle in multistage events; geometric solutions to multistage random experiments.

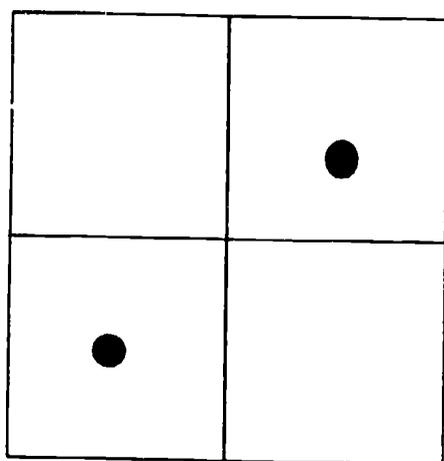
*Number and Number Theory*--Prime factoring modular arithmetic; various abaci and positional notations (binary, base 3, 4, 8, 2); codes and decoding in combinatorics; representation of fractions by infinite series; introduction to approximation and relative magnitudes; relations, functions, operations as functions, converses and compositions; negative numbers.

*Logic*--Negation of attributes; logic terminology; strategic thinking in special games.

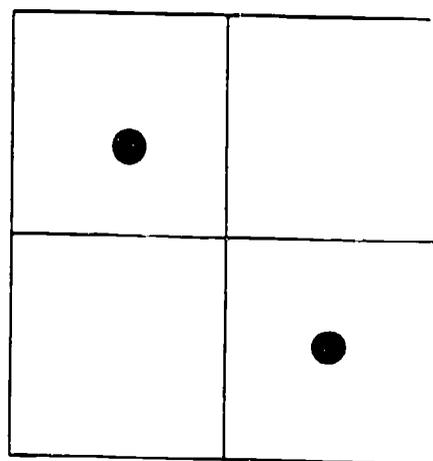
The authors of CSMP have employed three representational languages (pictorial devices) to facilitate young children's development of and communication about mathematical ideas and concepts: the minicomputer, arrow diagrams, and string pictures. The minicomputer is a paper abacus, invented by Georges Papy (Herbert, 1984), on which numbers can be represented using magnetic checkers placed in any of its four squares. Each number can be represented on the minicomputer using a variety of different arrangements, including negative checkers. Basic operations can be performed using this representation. Place value is visually demonstrated, as several cards can be put side by side, each representing one base-10 place. The basic minicomputer consists of four colored squares, each square having its own value:

8	4
2	1

Numbers are represented on the minicomputer by placing magnetic checkers on the square and summing their value. For example, nine can be represented in a variety of ways: A checker on the 8 square and a checker on the 1 square [ $8+1=9$ ], or three checkers on the 2 square and three on the 1 square [ $(3 \times 2) + (3 \times 1) = 9$  or  $2+2+2+1+1+1=9$  or  $3 \times 3=9$ ]. A second board placed on the left of the ones board represents the tens place. Each checker on this board is worth 10 times its value on the ones board. The minicomputer below shows 69: Sixty on the tens board and nine on the ones board.



Tens board



Ones board

Arrow diagrams are colored arrows used to represent relations among numbers or objects, which are represented by dots. These provide an early introduction to functions. String pictures are used to show classification of objects or numbers according to various attributes. Each of these represental languages is used throughout the elementary curriculum, increasing in degree of complexity with each level.

Topics not included or less developed in CSMP, which are common to many curricula are time calendars, common English measures, and coins and money. When standard measurement is part of a lesson, the metric system is used. Although CSMP gives little emphasis to teaching money, the students' knowledge of money is built on in working with decimals (i.e., 3.25 is often compared to 3 dollars and 25 cents.) In addition, little emphasis is given to canceling with fractions and multiplying and dividing mixed numbers. The long division algorithm is not as fully developed in CSMP as it is in many curricula.

The content selected suggests that learning mathematics is more than mastering computational skills and memorizing rules. The teachers' guides refer to the objects of mathematics as being mathematical ideas, from the simple to the complex. These ideas include concepts such as number properties, relationships and operations, geometric properties

and figures, relations and functions and operations of them (IG, Part IV, p.1). Also included are probability and logic.

Applications. The emphasis CSMP places on "pedagogy of situations" is illustrative of the authors' interest in encouraging students to apply their developing mathematical thinking to a variety of situations. What it means to apply mathematics, however, has a different meaning in the CSMP curriculum than in many commonly used curricula. The use of applications in many texts refers to the situations in which students apply their knowledge to solve problems. These situations tend to be contrived in that the mathematics is tangential to the situation rather than an inherent part of it. The situations in CSMP that require students to apply their mathematical knowledge are more than situational stories that "dress up" computation exercises. They present students with problematic situations that require the application, and consequentially the development, of mathematical thinking and reasoning, rather than mere algorithmic skills.

Prior student knowledge. CSMP seems to make two assumptions about students' prior knowledge and experiences in math. The first is clearly spelled out: "Children can learn and can enjoy learning much more math than they [presently] do" (Herbert, 1984, p. 3). The second is that most non-CSMP students have been exposed to mathematics consisting primarily of drill and practice computation presented in an isolated and meaningless context. These experiences engender misconceptions about what mathematics is and how it is learned. Thus, CSMP attempts to challenge by exposing students to a broad range of mathematical concepts and by involving them in complex thinking activities. Additionally, inherent in CSMP's pedagogy of situations--that through grappling with problem situations students will develop important mathematical strategies and skills--is the assumption that students come to the classroom situation with informal knowledge and abilities that they can call upon, develop, and formalize. Familiarity with the representational languages is also assumed beginning at the second-grade level. Assistance in introducing these languages to a class new to the program is provided at each level.

Student interests, attitudes, and dispositions to learn. Student dispositions to learn, interests, and attitudes seem to be considered in the CSMP content selection. The regular use of imaginary or realistic situations, the fact that the students as a group are intended to be actively involved in complex thinking during the lessons, the use of representational languages to provide students with multiple ways to represent concepts, and the use of games within lessons are aimed not at "making math fun" in an artificial way but at involving students in authentic and interesting mathematical thinking.

Provisions for student diversity. CSMP does not mention provisions for cultural, gender, or ethnic diversity. Neither is attention to diversity evident in the few pictures in the

student workbooks. Provisions are made for student diversity in ability and in ways of thinking. The lessons are designed to be taught to a whole class with students of varying abilities. Many of the activities the students do are open-ended, allowing for various levels of work. The spiral development of the curriculum is designed to bring students back to earlier concepts throughout the year. It is not expected that every student will develop a thorough understanding of each concept before the class moves on. The assumption is that there will be more opportunities for the concept to be developed, often in a new or different context, which might make more sense to the child. The worksheets, which accompany the lessons, are sequenced according to level of difficulty, beginning with "basic" and increasing to "very challenging." Each student is not expected to do each worksheet.

Topic treatment: Spiral organization and content strands. The organization of the content is unlike most mathematics curricula and reflects the authors' notion that mathematics is a "unified whole," rather than a collection of various topics. Four strands are identified: The World of Numbers (number, number theory, and work with the minicomputer), the Language of Strings and Arrows (logical and strategic thinking and reasoning and many elements of algebra such as functions, converses and compositions, and operations), Geometry and Measurement, and Probability and Statistics (begins as a strand in fourth grade, although it is included in the other strands as early as second grade). The substantive content of each strand necessarily overlaps with that of others. For example, fractions appear, in various forms, in each of the above strands. Within each strand, the lessons are sequenced into "blocks" dealing with the same idea, developing more fully as the lessons progress.

Each block, however, is not designed to be taught as an isolated group of lessons. The first semester of the fifth-grade curriculum includes in the World of Numbers strand a three-part block on multiples and divisors. They are placed in the curriculum as the 3rd, 23rd, and 26th lessons. Other complementing concept blocks are developed during lessons between these. Thus, ideas are introduced in a spiraling fashion throughout each strand. Additionally, the strands are designed to spiral as well. Except for Kindergarten, the curriculum is organized into daily, spiraling lessons. The Kindergarten curriculum is a linear sequence of 108 lessons spread over the school year (about three lessons per week). In grades 1-6, the World of Numbers lessons are taught two days a week, while the Language of Strings and Arrows and the Geometry and Measurement (and Probability and Statistics in fourth grade and above) are taught each on one other day. One day of the week is devoted to workbook lessons. These days begin with a brief lesson, followed by independent work in workbooks, which include problems from recent lessons from all the strands.

In the General Introduction of each teacher's guide the rationale behind the spiral sequencing is given. The spiraling, it is explained, "reflects that learning is not a linear

process" (Grade 5, Semester 2, p. 9). Ideas are touched upon recurrently at various points, giving students various opportunities to "catch on to an idea at each turn of the spiral" (p. 10).

Continually fostering student interest in mathematical ideas is one part of the rationale for the spiral development of the curriculum. This approach is contrasted with the mastery approach, in which a child may spend a great deal of time working to "master" low-level skills or concepts (often measured by computational accuracy) before moving on to ideas which develop and nurture complex thinking. The CSMP authors argue that all students, rather than a select group of what Addison-Wesley calls "extended" students, should be exposed to interesting and complex mathematical ideas. The authors admit that teaching in this way will require a great deal of faith on the part of the teacher who is accustomed to working with a particular topic until all, or almost all, of the students understand it or can show proficiency in it. The spiraling of concept blocks and strands, which overlap in content, and the consistent use of the representational languages throughout the program, create a curriculum which seems to be coherent and consistent across grade levels and within each grade.

How the organization of content represents mathematics. The CSMP authors hold to the proposition that mathematics is a "unified whole." The spiraling curriculum, as opposed to the typical segmented approach, continues to draw connections between different mathematical ideas and allows them to build on one another. The implication is that mathematical ideas are not discrete pieces, but are interconnected with other ideas that should be developed simultaneously. The fact that all students have the opportunity to work with all the ideas in the curriculum seems to reflect the idea that mathematics is not arranged hierarchically, dictating that certain concepts must be mastered before others are introduced.

#### Content Explication in the Text

CSMP does not have what is generally considered student texts. The essence of the CSMP curriculum is in the whole-class lessons, for which scripts are provided in the teacher's guide. There are student workbooks and worksheets to accompany some (in younger grades) and many (in upper grades) of the whole-class lessons. Additionally, there are 24 storybooks (twelve for ages 5-8, eight for ages 8-12, and four for ages 10-14) which use fictional stories to present students with mathematical ideas and problems. They can be read aloud to the class or students can read them individually. While a few of the stories are part of regular CSMP lessons, all are included with the class materials, and most are to be used by the teacher as seen appropriate.

Content presentation. CSMP presents the content primarily through whole-class lessons, which are often followed by individual student work. The representational languages contribute to the clarity of the presentation of the mathematical ideas. The languages are used to represent ideas and relationships visually. Each representation is developed more fully at

each grade level, building on the student's experience with it the previous year. Through the use of the representational languages, the authors are attempting to introduce students to authentic mathematical ideas and ways of thinking mathematically, but within formats that are appropriate to the child's age, ability, and interest. Once introduced to mathematical concepts through the representational languages, students have a good deal of experience with these ideas before conventional names and rules are attached to them.

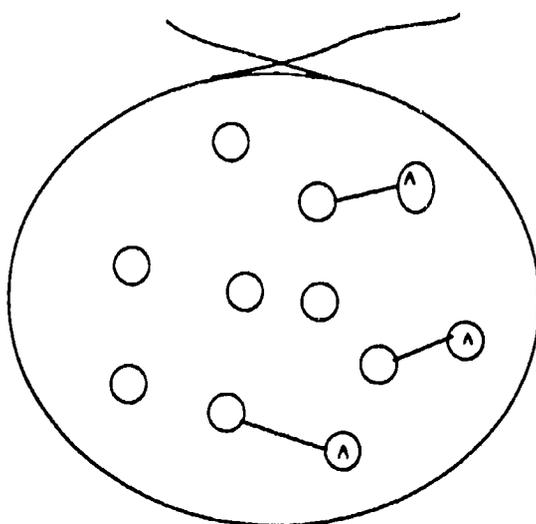
In contrast to the concept of functions which is introduced as such at the fifth-grade level in *Real Math*, CSMP uses arrow roads, in which each colored arrow indicates a function (+4, -3, 2x, etc.) to introduce students to the concept of functions as early as first grade. CSMP uses the representational languages to introduce the students to mathematical concepts at an earlier age than might be traditionally expected but tends to introduce the conventional symbols, terms, or algorithm later than commonly used curricula. For example, negative numbers are introduced as magic peanuts that cancel regular peanuts; and students begin using fractions such as  $\frac{1}{2}$ , as early as second grade, to divide numbers or objects in half. They are introduced, much later, to the terms "numerator and denominator," or to the relationship between dividing by 2 and taking half of.

Stories (both fantastic and realistic) are also employed to provide a context for the ideas presented, as well as stimulating student interest and activating prior knowledge. For example, taxi distance--distances between points along the lines of a grid--is introduced in the context of "Nora's neighborhood." Using a grid (representing the streets of the neighborhood), students work with the concept of distance. Decimals are first introduced in relation to dollars and cents (a context with which most students are likely to be familiar).

The worksheets are used to provide independent work after the lessons, and directly follow up the content of the lesson. It is not expected that every student will finish each sheet. When there is more than one worksheet, they are sequenced in order of difficulty. The instructions on each sheet are brief and not explicit. Problems could result if they were not used in conjunction with the lesson they follow, as the worksheets tend to follow up or extend the activity of the whole class lesson. The workbook strand, designed to be taught once a week, reserves the majority of the lesson time to be spent on independent work. Each lesson begins with a brief whole-class lesson and then devotes the rest of the math period to students working at an individual pace in workbooks. Like the worksheets, the workbook pages increase in difficulty and complexity for each topic. The first 10 pages are easiest. The next 10 to 12 are designed for students of average ability, and the last 10 pages are the most challenging in the book. It is recommended that two lessons are spent on each workbook, and consequently, it is not expected that all students will complete all 30 to 32 pages. The instructions are brief, and there seems to be an attempt to use the representational languages on the page to provide instructions.

This is quite different from *Real Math* and *Addison-Wesley Mathematics*, both of which require the students beginning at the fourth-grade level to do a significant amount of reading. While there is considerably less reading at the second-grade level than there is at the fifth-grade level in CSMP, the reading at fifth-grade required by the CSMP authors, is minimal.

The use of multiple representations. The following are several examples of how different representations are used to represent negative number concepts in multiple ways: In first grade the students are introduced to Eli the elephant and his magic peanuts. Each magic peanut combines with a regular peanut, and they both disappear. Using pictures of bags with both kinds of peanuts, students cancel regular peanuts with magic peanuts to figure out how many are left (e.g., 8 peanuts plus 3 magic peanuts equals 5 peanuts. 4 peanuts plus 6 magic peanuts equals 2 magic peanuts.)

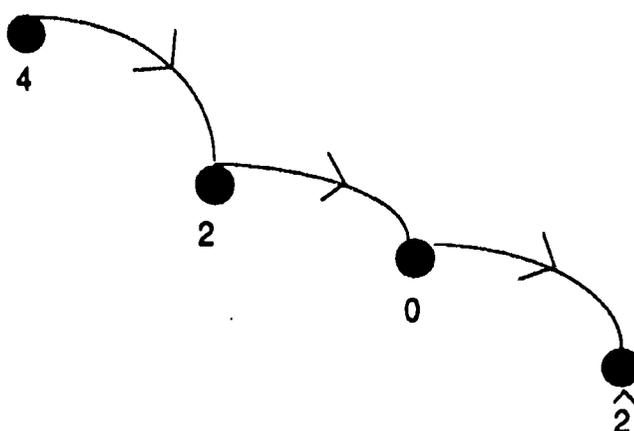


In second grade the students work with number sentences which represent what is happening in Eli's bag:

$$8 + \overset{\wedge}{3} = 5$$

The hat is used to signify that the number is negative. This notation is used to distinguish the negative value of the number from the operation of subtraction.

In the second semester of second grade, arrow pictures begin to include negative numbers. The arrow is a pictorial representation of a  $-2$  function.



Magic peanut chips are later used on the minicomputer and have the same characteristics as they do in Eli's bag.

A story about the various floors of the Empire State Building is part of a third-grade lesson. The floors are numbered. The ground floor is 0. The floors above the ground are labeled with positive numbers and those in the basement with negative numbers. An elevator moves people up or down to various floors. After figuring how many floors to the next stop, they pretend that the elevator only has certain buttons. For example:  $-5$ ,  $+5$ ,  $-1$ ,  $+10$ . The students strategize ways to get to certain floors within this limitation.

The particular stories used to represent negative number concepts (Eli the elephant and the Empire State Building) are used to help students visualize or concretize particular ideas, rather than emphasize direct applications. There are instances, throughout CSMP, in which stories are used to represent real-life applications of mathematical ideas. Comparing money to decimals and working with dividing quantities into equal parts are two examples. The stories and representational languages seems likely to be both interesting to the students and helpful to their making sense of the ideas, as they are represented visually and in multiple ways.

Promotion of conceptual understanding. The authors are explicit in their explanations that developing the student's conceptual understanding of mathematical ideas is a central purpose of the curriculum. In the General Introduction to teachers the authors explain, "CSMP believes that training for rote skill should never precede understanding" (Grade 2, Semester 1, p. 35). The use of the representational languages, which represent ideas and relationships, is one method employed to promote conceptual understanding. Additionally, the lesson scripts frequently encourage students to explain how they arrived at an answer or why it is so. Students are also asked about the different ways they think about a specific number (e.g., draw

a picture of 4 times 9 dots; what are some ways you think about 6 times 7?). What is being emphasized in these situations are the multiple and flexible ways one might think about a particular number or concept and the importance of different solution strategies.

### Teacher-Student Relationships and Classroom Discourse

Purposes and forms of recommended discourse. The CSMP program consists primarily of daily, whole-class lessons. The predominant format is one which emphasizes classroom discourse and student participation, although developing mathematical discourse is not an explicitly stated goal of the authors. In the teacher's guide, the authors stress the importance of group instruction to providing students with rich mathematical experiences. For this reason, grouping students according to ability is strongly discouraged. "The purpose of the full group experiences is to allow interaction among all the children in every situation posed in the lessons. Students learn quite a bit from each other's reactions" (Grade 2, Semester 1, p. 39).

In each lesson the teacher poses questions or problems to the students, many of which have multiple answers. Student contributions are an essential part of each lesson, not only in giving answers but in explaining their thinking or solution strategies. For example, a second-grade lesson begins by having the teacher ask all the students to raise one hand and then asks a student how many hands are raised. Then the students raise both hands and the teacher asks for the number of hands raised. The teacher then asks how the student knew.

In lesson N19 the teacher draws 4 rows of 7 dots on the board and asks the students what they see. The teacher's guide instructs, "Let the students express themselves. Many reactions are possible. If a student says there are 4 rows with 7 dots in each row, express this as:  $7+7+7+7$  and  $4 \times 7$ " (Grade 2, Semester 1, p. 133). The students' descriptions of what they see should be written down in number sentences. The teacher is encouraged to "help the students look at the picture in different ways by holding a ruler in different places to partition the array of dots" (p. 134). Thus, lessons are designed to be teacher-guided interactions between students and the teacher. The discourse emphasizes mathematizing situations; looking for patterns and relationships; verbalizing and explaining these processes; exploring new concepts and thinking critically, creatively, and reflectively. The types of questions asked and situations created require thoughtful and sometimes complex answers. The group discourse also facilitates problem solving. There is not, however, a great deal of emphasis on defending or justifying answers or solutions or challenging one another's solutions. It is evident that student-student discourse has the potential to grow out of the format of the lessons, but it is not suggested or necessarily encouraged.

Authority for knowing. Although the CSMP lessons are designed to be led by the teacher, the scripted lessons do not imply that the teacher stands for the authority for knowing. As mentioned earlier, the authors believe that students can do a lot more than

traditional mathematics curricula have expected of them, and the questions the students encounter require the students to use their knowledge and intuition. Student responses seem to be expected, valued, and necessary to the lesson. In this sense, the authority for knowing is shared by the teacher and the students, although this detail is not made explicit in the teacher's guide. How this is played out will be determined by the particular teacher using this material. It is conceivable that some teachers will assume more authority in determining what makes sense, while others will share it with the students.

The authority for knowing is more clearly the teacher's in relation to the student workbooks. The authors recommend that the teacher check the students' work and indicate which answers are correct and which are incorrect. It is then suggested that the teacher have the students redo the incorrect problems.

### Activities and Assignments

Completeness of activities as a set. The form, nature, and content of the activities in which the students engage are varied. There are whole-class lessons and individual work. The spiraling strands facilitate exposure to a variety of topics each week. The representational languages are consistent throughout the strands and are built upon at each grade level. The content of the group lessons varies from day to day and even within the lessons. The lessons are often divided into parts. The parts are usually conceptually related to one another but involve the students a different activities or representations. Some lessons begin with a few minutes of mental computation related to the lesson. The activities emphasize making connections, integrating ideas and thinking critically over recall of facts or definitions. The teacher's guide explicitly explains that CSMP believes that "drill" is not the most productive way to teach students skills of calculation. The problem solving and situational activities provide many opportunities for students to use and sharpen calculating skills. "Calculation is always considered as a tool for doing interesting things, rather than a chore done for its own sake" (Grade 2, Semester 1, p. 3).

Summaries of a second-grade and a fifth-grade lesson, serve as examples of the type of thinking and problem solving fostered in CSMP.

#### *Second Grade (first semester): L6 Detective Story with Coins*

##### Exercise 1

The teacher puts one dime, three nickels, and one penny into a container before the lesson. She opens the lesson by asking the students if they have piggy banks. Pretending that the container is a child's bank, the teacher tells the students that they can be detectives and discover how much money is in the bank. The teacher proceeds to give clues and works with the students, using pictures of coins and charts to narrow down the possibilities:

Clue 1--We are five coins and we are only dimes, nickles, and pennies.

Clue 2—At least one of us is a dime; at least one of us is a nickel; and at least one of us is a penny.

Clue 3—If you exchange each dime for two nickles, then we would be six coins.

Clue 4—If you put one more nickel in your bank, then there would be more than 30 cents. After giving the fourth clue it is recommended that the teacher go around the room and let the students whisper the amount in her ear.

### Exercise 2

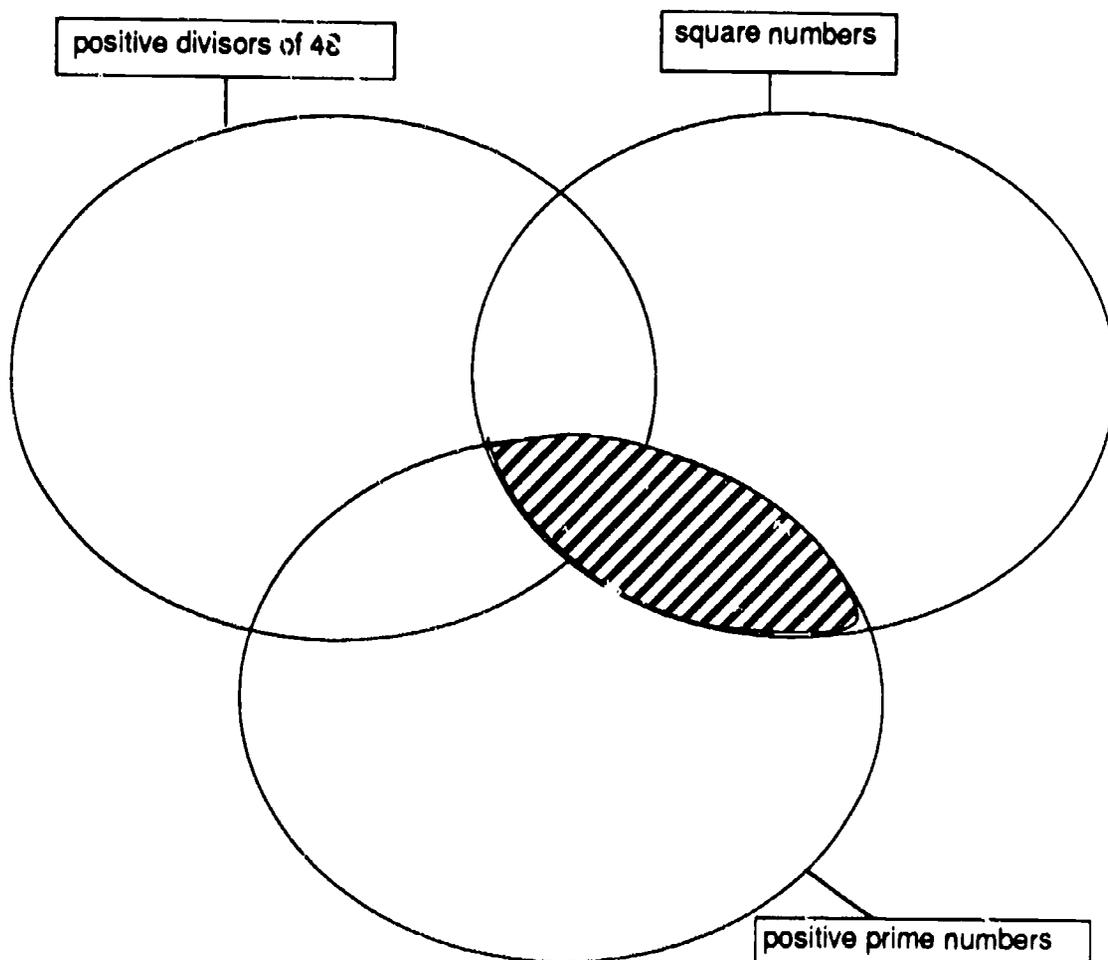
The teacher announces that they will go around the room counting by twos but first asks how many students are present. She then names the beginning student and ending student, and asks the class for their predictions about the ending student's number. The class counts and discovers which predictions were correct.

The teacher then has them pretend that they each have a dime and asks how they could find out how much money they have altogether. Before counting by tens, the students predict how much money the class would have. After counting, the teacher encourages the students to give the answer in cents and in dollars and cents (260¢ or \$2.60). She then draws a +10 arrow picture on the board and has students label one dot for each dime counted. If there is time, the teacher is encouraged to repeat the counting activity with nickels. (pp. 39-45)

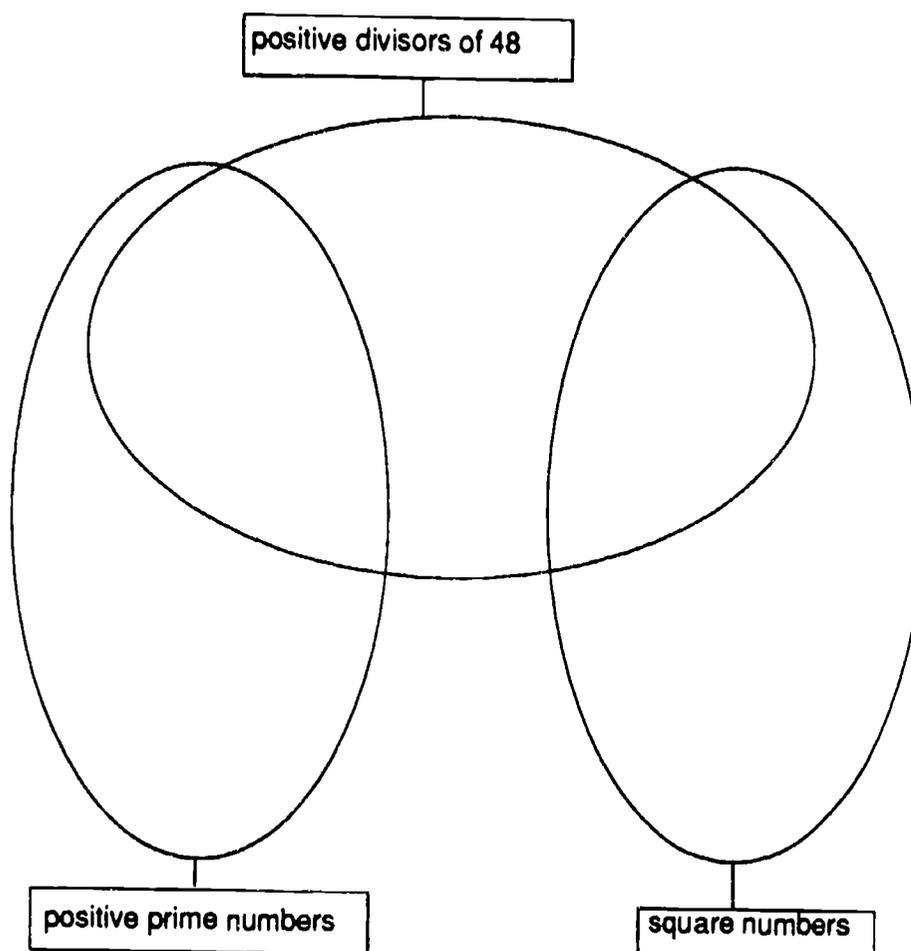
### *Fifth Grade (first semester) 2N String Pictures*

### Exercise 1

The teacher draws a Venn diagram of three intersecting strings on the board with colored chalk. The strings are labeled positive divisors of 48, square numbers, and positive prime numbers: The class works together to add numbers to the diagram. Through the process they should determine that there are no square numbers which are also positive primes, so a student hatches (marks off) the empty region.



If the student does not hatch the intersection of all three, the teacher is encouraged to ask the students if another region should be hatched. The teacher then asks the students if they can draw the diagram another way without hatch marks and allows them to work independently. A student draws her picture on the board and the class checks it against the original. The authors suggest that a student should draw a picture similar to the following:



### Exercise 2

The teacher draws another Venn diagram with labels: multiples of 3, multiples of 6, numbers that can be put on the minicomputer using exactly one of these checkers: ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨. The class follows the same procedure as in Exercise 1, filling in numbers, determining the empty regions and redrawing the diagram without any hatch marks. In the process of finding the solution, the students list and discuss multiples of 3 and 6 and determine that all multiples of 6 are also multiples of 3. They also use weighted checkers on the minicomputer. A ③ checker represents 3 times the value of the square it is placed in. Thus, the students discover that the ③ and ⑥ checkers will always yield multiples of 3 and 6. The lesson concludes with four possible worksheets, all of which involve placing numbers in Venn diagrams. (pp. 23-40)

In both the second-grade and fifth-grade exercises, the students are required to make sense of the information they are given, together with what they already know, to solve a problem. Rote recall of facts would not be applicable in these lessons. Furthermore, the stories used in the second-grade lesson are likely to be interesting to the students. One type of activity that is missing from the CSMP curricula is opportunities for students to work cooperatively in small groups, without being directly supervised by the teacher. There are many ways this type

of activity can be brought into the extant curriculum, but unlike *Addison-Wesley Mathematics* and *Real Math* such a format is not something that is addressed by the authors.

Linkages between activities and content. The activities engage the students in doing and thinking about mathematics. They are directly linked to understanding and becoming involved in mathematics and mathematical thinking. To some degree, there is less of an emphasis on real-world, direct applications as is the case with many texts. There is, however, a great deal of emphasis on using mathematics in situations, whether they are real or not. The sort of application emphasized is the ability to mathematize situations, reason about problems, and develop solutions.

#### Assessment and Evaluation

Recommended evaluation procedures. CSMP does not include any kind of formal testing or evaluation. The authors state their position that

learning often stops when the purpose of a lesson or sequence of lessons is the mere mastery of skills. Furthermore, we do not believe it is desirable or reasonable to attempt to assess student mastery of skills from day to day or even from week to week. (Grade 5, Semester 1, p. 12)

While acknowledging that reliable methods of measuring what or how much a child has been learned from any situation, the authors claim that it is possible to monitor student progress on the workbook and worksheet assignments. This would provide an ongoing measure of the student's ability to apply her knowledge to these tasks.

CSMP materials do not include "remedial" work. The regularly spiraling topics are expected to provide multiple opportunities for the student to develop an understanding of the topic. Not all students are expected to have the same understanding after each lesson. Although authors also suggest additional work (not remedial worksheets) with a small group of students who might be having extraordinary trouble with the lessons, they do not provide guidance for planning or organizing this work. It is further emphasized that being able to use standard algorithms is not the final goal, but perhaps a small piece, of learning math. "Students need plenty of time to explore the world of numbers," they argue, "mastering an algorithm should be the last step of this exploration, a sort of natural synthesis of previous experience" (Grade 4, Semester 1, p. 13).

There is no explicit mention of student engagement in the assessment of their own understanding or skills. Yet, the emphasis in the curriculum on applying one's knowledge to various situations implies that the students would increasingly become aware of their own knowledge and how they understand certain ideas.

While the stand that CSMP takes on testing seems to be well supported and very much in line with the entire program, it is likely that, given school emphasis on testing as a measure of student progress, the lack of concrete measures of student progress will inhibit the use of CSMP as the primary mathematics curriculum in a school or class. The authors recognize that students and teachers are often under a great deal of pressure to perform in certain ways. They recommend that a teacher to whom parents voice concern about the lack of emphasis on mastery of standard algorithms should reassure them that the students will have an advantage in applying mathematics to new situations, strategic thinking, organization ability, and understanding mathematical method. It is also suggested that, if a teacher feels enough pressure to do so, she might use supplementary materials that provide practice with standard algorithms.

### Directions to the Teacher

Format of directions. The directions to the teacher can be found in two places. Most apparent are the scripted lesson descriptions, with a summary and list of necessary materials at the beginning of each. In addition, each set of teacher's guides includes a General Introduction, which provides a detailed explanation of the CSMP philosophy, overview of the program, schedule, description of how to use the materials, a question-and-answer section, and a letter that can be sent to new CSMP parents. A teacher might be able to follow the scripted lessons without much preparation, but it is apparent that reading the General Introduction is likely to help the teacher understand the CSMP philosophy and be aware of why certain approaches are taken. An understanding of these purposes and knowing how the materials (such as workbooks and worksheets) are intended to be used might be a determining factor in the success of the use of the curriculum. Because this curriculum is so uncommon and is based on assumptions about mathematics and learning not commonly held by teachers, a teacher inservice has been designed and is taught by the CSMP coordinators and is likely to contribute to a teacher's success in implementing the program.

The highly scripted lessons are referred to, in the General Introduction, as a guide. It is recommended that the teacher use creativity and exercise professional judgement in making the lessons fit with the students' responses and deciding when to move on. The introduction does, however, give the teacher guidance concerning the discourse and possible responses to the student ideas in each lesson and examples of structured activities. There is also a suggested procedure for checking student workbooks. Giving grades is not a topic discussed by the authors.

Teacher content and pedagogical knowledge. CSMP is uncommon both pedagogically and in content, proposing alternative ways of thinking about mathematics, teaching, and learning. Through the whole-class lessons, during which student participation is essential, students are encouraged to think about and verbalize mathematical ideas and approaches.

Furthermore, the spiral approach of organizing the curriculum is antithetical to the "mastery" model, more commonly used in mathematics teaching. In order to successfully use CSMP, the teacher will need to be familiar with the representational languages and will need to see mathematics as more than procedures and arithmetic algorithms. The authors stress the importance of students exploring the world of numbers. A CSMP teacher must be familiar with this world of numbers herself and be willing to explore it with her students. This includes being able to accept more than one correct answer when appropriate and understanding mathematics well enough to know when answers or strategies are partially correct or are soundly based. The teacher must also understand the philosophy behind the spiral structuring enough to move on to the next lesson when it is apparent that not all students have mastered the topic, knowing that when the spiral returns to the topic again the increased exposure to other related topics may have prepared the student to make better sense of the initial topic.

The teacher's guide encourages every teacher to be trained by CSMP coordinators (18 hours for Kindergarten and First Grade, 32 hours for Upper Primary, and 56 hours for Intermediate Grades). The training introduces the teachers to the CSMP content, suggests ways to organize their classes, and takes the teachers through sample lessons. They are designed to use the spiral approach, aiming "not only to teach CSMP mathematics but to familiarize teachers with the spirit and pedagogy of the CSMP curriculum" (Grade 2, Semester 1, p. 46).

### *Math In Stride*

#### Goals

Program goals. The authors of *Math in Stride* do not provide a list of program goals. The introduction, however, includes an explanation of the program, the actual learning experiences provided for the child, and expected student outcomes, from which goals can be inferred. The authors describe the *Math in Stride* program as intending "to place the child and the child's responses to the broad field of mathematics at the center of the mathematics curriculum" (Grade 1, p. v). Rather than "imposing behavioristic techniques that force children to perform mathematical tasks in an arbitrary and rote fashion," the program "creates environments and opportunities for children to learn" (p. vii). It is an activity-based program which focuses on the child's development of mathematical understanding through interaction with his environment.

Each child, the authors state, goes through similar stages of intellectual development, but in very different ways; their pace, strength and aptitude, previous experience, level of interest, attraction to particular teaching styles and materials, and level of independence and initiative are factors cited by the authors as influencing the way children develop. The pedagogical goal of the program is to plan carefully a learning environment which will foster

the child's development of mathematical concepts from concrete to representational to abstract. An important goal for students is that they will progress toward more abstract understanding through concrete and representational activities. It is also expected that the students will develop correct uses of mathematical terms, ask clear and stimulating questions, and make concise observations and express reasoning well; it is suggested the teacher encourage these abilities.

Role of manipulatives in attaining knowledge goals. *Math in Stride* aims to give students "many opportunities to explore mathematical concepts through the use of manipulative materials" (Grade 1, p. viii). The authors stress that since conceptual understanding begins with a concrete experience, the manipulatives are central at all levels. Through concrete experiences the student will develop visual images which contribute to an enduring understanding. Although the relationship between the concrete experiences with manipulatives, their visual images, and conceptual understanding is not explicitly made clear, it seems from their use in the lessons, that the manipulatives are used in two ways: to represent or model mathematical concepts (place value, multiplying decimals, etc.) and to present challenging problems (involving patterning, permutations, attributes, etc.) which encourage and facilitate complex thinking and problem solving.

Conceptual, procedural, and conditional knowledge goals. *Math in Stride* places conceptual understanding as a prerequisite to learning computational procedures. The authors argue that without understanding the underlying concept the procedure is meaningless, inaccessible, and subject to being used incorrectly. They also emphasize that a student's ability to use a particular algorithmic procedure, such as regrouping to subtract, does not necessarily indicate understanding of its underlying concept. The activity-based program is an attempt to use problem situations to develop conceptual knowledge and then attach the procedures to their corresponding concepts. This would mean a great deal of preliminary work with place-value models before learning the addition and subtraction algorithms.

### Content Selection

According to the authors, the content is designed to provide problem-solving opportunities in three areas: data collection (classification, patterns, estimation, graphs, probability, statistics), spatial relationships (geometry, measurement, estimation), and number sense (addition and subtraction, multiplication and division, estimation, place value, negative numbers, fractions, decimals, percent and functions, and relations between numbers). These three areas are not treated as discrete topics but are interwoven throughout the curriculum. Because the program philosophy is based on the assumption that students learn by making sense of their physical environment, many experiences are provided for students to

collect, organize, and interact with data about spatial and numerical relationships around them.

The substance and nature of mathematics presented by the content selection. The content selection represents mathematics as a way to make sense of and organize one's physical and intellectual environment. Thus, it includes more than computational skills and work with numbers. The strong emphasis on spatial relationships and data organization indicates that mathematical thinking is a way of looking at and thinking about the world. That problem solving is the context through which students explore mathematical relationships suggests that doing mathematics involves solving problems. That mathematics is a way of thinking and organizing data is implied by the experiences students are intended to have and the abilities they are expected to develop. These include exploration, prediction, invention, experimentation, calculation, developing and testing strategies, and discovering relationships. Mathematical argument is not represented in *Math In Stride* as being a component part of mathematics.

Real-life applications. As do the authors of CSMP, the authors of *Math in Stride* take a different perspective on the meaning of application than that of conventional texts. The application of mathematical knowledge and abilities is found in the emphasis on applying mathematical ways of thinking to one's immediate experience. For *Math in Stride* this means manipulating, organizing, making sense of, and imposing logical structure on objects in one's physical environment. Such applications form the context of the lessons, which give rise to the development and use of mathematical ways of thinking. These include experiences collecting and problem solving using real data and objects, for example, comparing lengths of objects to a set length or surveying and graphing data collected. The implication is that application means applying abilities (such as graphing, computing, classifying) and understandings (of spatial relationships, patterns, probability, etc.) to real life situations, rather than rote skills to contrived situations. Consequently, *Math In Stride* emphasizes ways of thinking about the world, organizing and analyzing information, and developing and employing problem solving strategies. The authors see this activity-based program as giving students "opportunities to develop ways to think creatively and solve problems" (Grade 5, p. vii) that the students will extend and use in new problem situations.

There are some more conventional story problems at each grade level. These are referred to as "translation" problems--story problems that present real-life situations which can be translated into a symbolic statements and solved algorithmically. In some cases, students are given the symbolic statement and are expected to write illustrative stories. In other cases, the students are given a list of statements (e.g., "Snowy has 4 kittens, Peaches has 2 litters of 3 kittens each. Puff has twice as many kittens as Snowy." Grade 5, p. 218) and they

are to write questions and equations about these statements. The emphasis in these cases seems to be on making the translation and communicating information through mathematical symbols.

Student prior knowledge and experience. The program is written as if the students have had experience with *Math in Stride* in previous grades. Experience with the commonly used manipulatives and familiarity with terms and concepts previously introduced is assumed, as is the ability to work with partners or in groups. The teacher's manual often refers back to the grade level at which the students worked with a specific topic. It is not assumed, however, that students enter each grade level knowing specific facts or understanding certain concepts. The authors' perspective that students develop conceptual understandings through experience with and making sense of things around them implies that building and enriching one's understanding is an ongoing process. It is also emphasized that children develop at different rates, thus it is not safe to assume that all students at the same grade have the same knowledge.

Student interest, attitudes, and dispositions. It is assumed that all students are disposed to exploring their environment in an attempt to make sense of it. It is also assumed that, as a result of these varied experiences, students will synthesize ideas and understandings, develop strategies, and make connections between mathematical concepts. It is not suggested how the teacher should guide the students in making these connections. For example, the fifth-grade program begins with a variety of activities during which students estimate various quantities (the number of beans in a jar, cups of beans in the jar, value of stack of place-value blocks, fractions of quantities of objects). The students have many opportunities to estimate, analyze and revise their estimations, discuss them with the class, and check them against the actual amount. Simultaneously, concepts such as place value, fractions, percents, and arrays are being introduced as these provide the context for each estimation. The student is expected to develop and refine his estimation strategies through these experiences, although strategies are not explicitly addressed in the lesson and the teacher is not explicitly guided to attend to these developing abilities.

Provisions for student diversity. It is not apparent that special provisions have been made to accommodate student diversity with regard to culture, gender, race, or ethnic group. This is not to say that the program appears biased; rather, race, gender, and ethnicity are seldom portrayed. Instead, it focuses on the activities in which the students participate. The teachers' guides are peppered with photographs of students engaged in the activities, and there is attention to racial and gender diversity in these photographs. The student workbook does not include pictures of children although there are drawings of everyday objects. There are a small number of translation problems at each level which use "real-life" situations to pose problems. The authors have attempted to create situations to which many children can

relate without substantiating extreme social biases. Nevertheless, many of the situations and names mentioned reflect a somewhat homogeneous culture whose members go to movies, have pets, and eat salad, sandwiches, and ice cream. That students are expected to be disposed to using and manipulating concrete materials could be construed to reflect assumptions about children's experiences based on socioeconomic biases. In other words, children who have had experience playing with concrete objects are more likely to be accustomed to manipulative-rich environments and may be advantaged in functioning in them.

There is an attempt to make provisions for student diversity in ability and rate of development. One of the five foundational principles of *Math in Stride* emphasizes the individuality of children. Although children go through similar stages of intellectual development, the authors assert, they do so at various rates and in varying ways. They have different levels of interest and initiative, respond differently in each circumstance, and have different strengths. The program aims to maintain and respect this diversity. Each lesson includes a suggestion for extension activities, usually in the form of classroom stations, with the intention of providing students opportunities to explore more problem-solving activities. Additionally, students are encouraged to develop their own strategies and create their own problems, allowing for the activity to accommodate various ability levels.

#### Content Organization and Sequencing

Topic treatment. The *Math in Stride* authors attempt to depart from the traditional, topical organization of the curriculum "where one topic is taught in isolation and often does not occur in practice or application for as long as a year" (Grade 5, p. viii). They have attempted to integrate the development of skills and abilities such as computation, estimation, inductive and deductive reasoning into three areas: data organization, spatial relationships, and numerical relationships. The curriculum is made up of a number of foci (seven in second grade, five in fifth grade) under which a variety of topics fall. The topics recur under several or all of the foci and are thus further developed in different situations. In this way, the *Math in Stride* curriculum is clearly spiralled, but more subtly than CSMP, which spirals from week to week. Topics in the *Math in Stride* curriculum are readdressed in a different context every two months.

For example, in the second-grade curriculum, classification is a topic addressed in various ways throughout the year. During Focus I, Patterns, students classify various linear patterns of unifix cubes, they have created and identified (AB, ABB, ABC, AABC, ABBC, etc.) and graph the frequency of each. Under Focus II, Spatial Relationships, the students sort tangram pieces (a set of geometric shapes of various sizes from which other figures can be constructed) and then three-piece tangram designs. During Focus III, Place Value, students are introduced to Attribute Blocks (a set of objects each with a different combination of attributes. For example, an object can be red, blue, yellow, or green; a triangle, square or circle; and large or

small) which they sort and classify according to the various attributes (size, shape, color). They note differences and similarities and create matrices, Venn diagrams, and patterns using the blocks. Classification is again touched upon during Focus VI, Problem Solving, through work with classification problems using attribute blocks and Venn diagrams.

This example illustrates the spiraling of one topic throughout one year. They are also spiraled and built upon at each level. Fractions, for example are introduced in first grade through the classification of partially shaded shapes into halves, thirds, and fourths. In second grade, fractions are brought up again in several different contexts (e.g., 3 of 5 things, fractional parts of a shape, and symbolic expressions of fractions matched with pictorial representations). Throughout third and fourth grade, fractions are further developed through sorting, identifying and naming, recognizing equivalent fractions, addition and subtraction of fractions, writing tenths and hundredths as decimals, and multiplying fractions. These concepts are further developed and built upon in fifth and sixth grade. Classification also appears at each grade level.

How the organization of the content represents mathematics. The use of problem solving as a lesson context implies that mathematics is problem solving. The three areas through which problem solving is explored--data organization, spatial relationships, and number sense--communicate that mathematics includes more than computation. There is equal, if not greater, emphasis placed on identifying relationships, drawing connections, and mathematizing situations in order to set or solve problems and communicate mathematical ideas. The emphasis on manipulatives and pictorial representations through which students explore mathematical ideas and concepts, and the authors' discussion about allowing students to move from the concrete to the abstract, imply a perspective that mathematical ideas are abstract but can be represented concretely and be understood through interacting with these ideas through various models or representations.

The authors clearly intend to encourage the development of conceptual understanding before attaching symbolic expressions to each concept. Symbols are seen as one of many ways to represent mathematical ideas; facility with symbolic manipulation does not indicate understanding of the underlying concept. There seems to be a rigid notion about the sequence through which conceptual understanding is developed--from the concrete to the symbolic to the abstract. In other words, all students must move from the concrete to the abstract with each concept. Encouraging students to use and develop intuition, make conjectures, or argue mathematically is not explicitly mentioned.

### Content Explication

Content presentation. *Math in Stride* is designed to be an activity-based, rather than text-based, program. The lessons consist primarily of work with manipulatives that the

students do individually or in small groups. Whole-class lessons also have a similar format; the teacher poses a problem or gives directions and the students work in small groups or alone. It is suggested in quite a few cases, however, that the class discuss their results or conclusions during or after the activity. There is a student workbook, which contains supplementary pages to extend the work they have done with the manipulatives. There is also a set of black line masters, that may be copied, some of which are used during the activities, others intended for extension activities.

The content is presented to the students through the activities. Through the experiences provided in the program, guidance from the teacher, and interaction with other students, the students are expected to construct a meaningful and deep understanding of mathematical concepts, including their relationship to one another

Although the degree of complexity increases at each grade level, the emphasis on manipulatives and representations remains central. There is greater emphasis on free exploration and experimentation in the early grades. This seems to be built upon in the later grades which encourages deep and logical thinking to grow out of the different experiences. In second grade, a student might be given the problem of constructing various quadrilaterals on the geoboard (a square board containing equally spaced pegs around which rubberbands are stretched to construct various geometric figures). In fifth grade the students are asked to construct all the quadrilaterals possible on the geoboard. This is followed by a class discussion in which the students determine how they would know when all the possible quadrilaterals have been constructed.

The differences in the types of activities suggested for the lower and upper grades seem to parallel Piagetian theory of cognitive development. The activities in the lower grades tend to focus on manipulation of concrete objects and modeling concepts concretely or pictorially; the implicit intent being to foster movement from the preoperational to the concrete operational stage. In the upper grades, particularly in fifth and sixth grade, students begin to engage in more abstract and complex levels of thinking. For example, the logical problem-solving activities in second grade involve deducting and determining rules based on observations (e.g., probability patterns.) The logic problems in the fifth grade, reflecting a move toward formal operations, focus on deductive and inductive reasoning and deal with several variables at once (e.g., permutation experiments with as many as five different variables.)

Many of the activities are likely to pose genuine problems to the students, as well as being interesting and engaging. That manipulatives and pictorial representations are used well before students are introduced to symbolic expressions emphasizes that conceptual understanding is the central focus of the program. Because the activities are intended to be open-ended, it is expected that students who are able will take the activity further and work

at a more challenging level. It seems that the degree to which the authors' intentions are actually brought to fruition depends on the teacher's ability to facilitate such conceptual development, open-ended exploration, and problem setting. The degree to which the written curriculum provides assistance and guidance to teachers in taking on these roles is discussed in the section "Directions to the Teacher."

### Teacher-Student Relationships and Classroom Discourse

Purpose and forms of discourse. Teacher-student and student-student discourse seems to be assumed throughout the *Math in Stride* program, although it is not specifically addressed as such. The term used in the introduction is "appropriate conversation . . . quality rather than quantity of verbal exchange is important" (Grade 5, p. v). The authors go on to emphasize that using correct terms, asking clear questions, making concise observations, and giving well expressed reasons should be encouraged to promote efficient and effective concept development. Throughout the programs, collaborative or side-by-side work and discussions are expected. It is also recommended that a discussion conclude many of the whole-class lessons, particularly in the upper grades. Nevertheless, sufficient guidance in carrying out these expectations is lacking. Other than what is cited above, the authors have not delineated the forms of discourse expected or the role discourse plays in the development of critical thinking, problem solving, or conceptual development. It is not made clear whether students would have opportunities to justify ideas, explain concepts, or defend their thinking during the class discussions. Most often, the instructions say, "discuss findings." Explicit assistance in facilitating discussions appropriate to the particular mathematical ideas being developed is not provided.

Authority for knowing. The underlying emphasis on learning through discovery throughout *Math in Stride* places authority for knowing in the hands of the student. As students work through problems and synthesize their ideas and construct their own understandings they become the authority for their own knowing. The teacher's role is to facilitate this process by providing experiences through which the students explore their environment, eliciting student responses, and encouraging students to develop their understanding.

### Activities and Assessments

Because of the integrated nature of the *Math in Stride* curriculum, it is difficult to extract one or two lessons and examine them isolated from other lessons in which the same concept is being developed in different contexts. Several activity sequences provide a sample of *Math in Stride* lessons. The second-grade lessons on linear measurement illustrate the tendency to involve students in preliminary activities, aimed at laying a conceptual foundation, to foster understanding of the standard approach when it is introduced. The fifth-grade lessons involve

work with fraction and decimal concepts and are examples of the authors' parallel use of symbolic expressions and algorithms with concrete representations of the procedure.

The *linear measurement* topic, in the second-grade curriculum, is under Focus V, Multiplication/Division and Spatial Relationships. It consists of a sequence of five lessons. A brief introduction explains that it is important for the students to come to understand the necessity of using a uniform unit of measure and that a certain number of units--the total number of units equivalent to the length of the object--determines the length. Understanding these concepts provides a base on which students can build as they learn to use a ruler, which represents a series of uniform units, rather than mere "little lines."

In the first lesson, the students use a nonstandard unit, such as a tongue depressor, and a unit of measure. They identify objects in the room that are about the same length as their unit of measure, making a list of these things. It is suggested that these items be listed on a large sheet of paper with the tongue depressor glued to the top and hung in the room. The activity is repeated with other nonstandard units, such as a short chain of jumbo paper clips, a stack of unifix cubes, a straw, and a piece of paper. The lesson ends with a class discussion during which the students compare the various lists. The recommended extension activity is to have students find objects shorter and/or longer than the nonstandard unit.

The following day's lesson builds on the use of nonstandard unit measurement using five stations. Each station contains a box of 8 to 10 different items, one of which is identified as the unit of measure. The students work individually, moving from station to station, using a sorter sheet which has three categories: shorter, same, longer. At each station the students measure the objects in the box against the nonstandard unit and lay them on the sorting sheet in the correct category. The suggested extension activity is to have students compare the items in the boxes and order them from shortest to longest.

The next day each student uses a chain of six jumbo paper clips as the unit to measure a few objects, estimating how many more or fewer paper clips would make the chain equivalent to the object. They then estimate the number of paper clips needed in a chain to match the length of other objects (crayon, scissors, hand, etc.). A recording sheet is provided on which the students write their estimate and the actual measure. Discussion is encouraged throughout this activity. Several worksheets are available that continue the use of nonstandard measures as parallel activities. As an extension activity it is suggested that the students construct chains of straight macaroni to use as a nonstandard measure.

The following day's activity is similar to the previous day, but the students use trains of unifix cubes, each cube being one unit. It is suggested that the students measure fairly long items and then break the train into tens to count them. So a length of 37 cubes can be written  $30 + 7 = 37$  cubes long. The extension activity for this lesson involves the use of a "preruler," a string

of 50 to 100 beads of macaroni, with buttons dividing the smaller units into groups of ten. The students use this device to estimate and measure, counting by 10s as they total the number of units.

The final lesson in this sequence involves construction of a ruler out of two different colors of uniform oaktag strips. The strips are taped together in an AB pattern, reinforced, and then used to measure. Once the students have recounted each unit to measure several items, the teacher is to ask them how the measuring tape could be improved to make the recounting unnecessary. It is anticipated that a student will suggest numbering each unit. The students use their rulers to measure items in the classroom, recording, and then discussing their results. The extension activity involves more work with the paper measuring tapes.

The fifth-grade topic, fraction/decimal concepts, is part of Focus V, Extending Mathematical Connections. The introduction explains that, rather than providing students with rules used in performing the procedures, these activities are intended to "demonstrate the mathematical validity of these steps allowing students to discover the shortcuts on their own" (Grade 5, p. 168). In the first lesson the students use Place Value Blocks to model decimal numbers. They are told that the 100-square is worth 1. They then are to determine the value of a 10-rod (.1) and a unit-cube (.01). The students build constructions and estimate the total value of its block. Then they record the number of each type of block on a place value sheet, which has a space for ones, tenths, and hundredths, and figure the actual value of the construction. This is repeated several times.

The following day, the students use graph paper to shade in squares representing decimal numbers, which are then added together and recorded. The students continue this activity, intermixing fractions and decimals and recording their answers. The worksheet used with this lesson leads the students to rewrite the fractions and decimals in equivalent form. The shaded squares assist in determining common denominators. A decimal equivalent game is suggested as an extension activity.

The final lesson in this sequence involves rewriting two addends written as decimals as fractions, adding the fraction and the decimal forms, and comparing the answers. It is necessary to convert fractions with denominators of 10 into equivalent fractions with denominators of 100. As extension activities the students are encouraged to start with fractions with denominators of 10 or 100 and convert them to decimals and to make up problems of their own, using the calculator to aid in arriving at the answer.

### Assessment and Evaluation

Recommended assessment procedures. All recommended assessment is labeled as diagnostic, that is, the purpose is to assess the student's current understanding as a way of informing instruction, rather than evaluating the success of instruction. The authors strongly

recommend that the teacher make diagnostic observations of students as they work, and that these are essential to assessing students' understanding. Additionally, an appendix of diagnostic tasks, complete with explanations and interpretations of results, is provided in each Teacher Sourcebook and is intended to be used throughout the year, particularly at the beginning as a planning guide. The purposes of assessment appear to be somewhat different in the lower and upper elementary grades. At the lower elementary level the intent is to "assist the classroom teacher . . . in assessing the young child's level of operation" (Grade 2, p. 148). The focus is on determining the students' stages of development.

The purpose at the upper elementary level is to assess "the level of skills and concepts the students have achieved in order to provide a suitable mathematics program" (Grade 5, p. 189). Less emphasis is placed on developmental stages and more on skills and abilities that may be evidence of various stages at this level. In both cases, greater emphasis is placed on the student's ability to work with and demonstrate understanding of concepts represented with concrete objects. At the fifth-grade level, the ability to translate the models into their symbolic expressions is also assessed. Application is assessed at all levels, although it receives greater attention at the upper grades where there is a section devoted to organizing information and problem solving. The teacher is expected to use the knowledge gleaned from such assessment in planning lessons and activities appropriate to the student's abilities and understanding, as well as understanding and interpreting student errors and misconceptions. The diagnostic tasks are not to measure "attainment of specific knowledge and skills" as achievement tests might, but a student's performance with the tasks should "provide a way to evaluate and more carefully analyze a student's performance of achievement tests. For example, a student who does not conserve area would have difficulty interpreting a spatial model for fractions" (Grade 5, p. 190). The term "diagnostic," however, is often used in the context of determining what needs to be fixed. Such a perspective seems to focus primarily on what the student does not know or understand, rather than what she does understand about a particular concept.

In many cases, particularly at the lower grades, multiple approaches are used to assess understanding of a concept. For example, a student's number recognition is assessed through the following tasks: reading numbers, matching numbers to sets, building a set from a number, ordering numbers, and writing numbers. In the upper grades, some very complex problem-solving abilities are assessed: using the transitive property (given statements comparing quantities of marbles in boxes, the student must deduct which box holds the most), solving difference problems by inference (using Attribute Blocks), and solving analogy problems. Again, the differing degree of complexity in the lower and upper grade tasks seems to reflect the Piagetian notion of developmental stages, which assumes that lower grade students are limited in their

ability to think abstractly. There are no apparent attempts to assess attitudinal or dispositional goals, or to encourage student self-assessment.

### Directions to the Teacher

Format of directions. The Teacher Sourcebook, one for each grade level, provides the teacher with information about *Math in Stride* and directions and suggestion for its use. It does not provide step-by-step instructions or lesson plans to be followed. The introduction includes a description of the program and a rationale for its approach. It also provides suggestions for organizing the program and the classroom. Unlike the majority of mathematics curricula, *Math in Stride* does not present a lesson-by-lesson curriculum. Instead, each focus, intended to constitute about two months of mathematics activities, contains several topics. These topics are divided into activities described in the Sourcebook. There is not one particular activity for each school day; instead, the teacher will determine the amount of time to be spent on each activity according to the ability and interest of the students. Many of the activities, once introduced, are designed to be made into "stations" or learning center activities, potentially extending them over a longer period of time. Nevertheless, which activities to extend, how long to spend on a topic, how to organize and structure instruction must be determined by the teacher based on his or her knowledge of the students. The authors give suggestions for organization, provide lists of diagnostic tasks to aid in determining students' developmental ability, and provide rationales for each activity to assist the teacher in making these decisions. They also encourage the teacher to experiment with different approaches, suggesting that the *Math in Stride* philosophy of learning by doing is not limited to the young: Each year that you use the program you will build on your experiences and grow in implementing a manipulative approach just as the children are learning by doing" (Grade 2, p. xi).

The degree to which such a model of teaching and learning is manageable and appropriate would be determined by the teacher's understanding and view of mathematics and ways of knowing it, her ability to make and use diagnostic observations regarding her students' understandings and developmental stages in making curricular decisions, and her assumptions about how children learn. A potential strength of *Math in Stride* is its flexible approach to learning, allowing for student and teacher differences. Thus, how students will respond to the activities is not predetermined. Implicit in this format, however, are assumptions about the teacher's ability to use such open-ended guidance. Thus, a potential weakness is that some teachers (particularly those with limited mathematical understanding, knowledge of child development, or who are accustomed to determining student understanding through their computational ability) may have difficulty making informed decisions necessary to implement the curriculum. (This is an interesting contrast to CSMP, in which the authors seem to have sacrificed a degree of flexibility to ensure that teachers have explicit guidance in using the

curriculum. The authors of *Math in Stride* have given teachers the benefit of the doubt, risking teacher misinterpretation or even misuse to ensure flexibility.)

Rationale and introduction. In addition to the introductory section, which provides a program description, philosophy, and rationale, each focus begins with an "overview" and each topic begins with a "perspective." These sections provide rationales and explanations for the particular focus and topic according to developmental theory. Each topic perspective explains why the activity is included and what sort of development it is expected to foster. The focus overviews provide this information in addition to a rationale for the inclusion of the topics, and how they are connected, in the particular focus.

Scope and sequence. Information about scope and sequencing is contained in the "lesson plan chart" for each grade level. This chart divides the school year into segments and assigns a focus to each. The topics and topic activities are listed below each focus. As mentioned earlier, the teacher is encouraged to determine the amount of time to spend on each activity, topic, and focus. However, it is also stated that the sequencing of activities is carefully planned and should, thus, be preserved.

Completeness of curriculum materials. *Math in Stride* is a manipulative, activity-based program. Thus, the Teacher Sourcebook, student worksheets, and black line masters, available from Addison-Wesley, cannot be used without a classroom supply of materials. The Sourcebook lists commercial materials--such as multilinks, unifix Cubes, Pattern Blocks, geoboards, tangrams, and Attribute Blocks--and noncommercial materials--such as collections, tongue depressors, dry lima beans, yarn, number die, and containers--that the teacher will need to have in the classroom.

Completeness of information for teacher. The Teacher Sourcebook provides information about determining the student's level of development (through diagnostic observations) and planning appropriate activities and extending them. The Sourcebook, however, lacks thorough information that would guide the teacher in helping students make connections, leading discussions, and probing student thinking. The description of each lesson gives the teacher details about what the teacher and students should do and suggests an appropriate group size. Many of the lessons (particularly in the upper grades) suggest that the lesson be concluded with a discussion about the findings of the lesson. It is not made clear what "discuss" means in these situations or what key ideas (if any) should be discussed. The Sourcebook does not provide information about the role of these discussions in student learning. The assumption seems to be that the teacher has experience with student discourse and class discussion.

Another assumption the authors make is that the students are familiar with working in small, self-directed groups or independently and that the teacher has had experience fostering group and independent work. There are many activities in the *Math in Stride*

curriculum that are designed for a small group. Some of these require that the teacher work with the group of students while the remaining students work independently on assignments. Suggestions as to how the teacher might help students learn to work in these situations are not provided, as it is not recognized that these situations may be novel to the child. It is recognized that the students will need to learn to work with the manipulatives productively and that more time should be spent at the beginning of the year establishing rules and learning to get materials from and return them to specific areas: "Whenever a problem arises such as destructive use of the material, not sharing, or unreasonable noise level, stop the activities and discuss the situation with the children and ask them how the problem might be solved" (Grade 2, p. xi).

Teacher's content and pedagogical knowledge. In order to use *Math in Stride* effectively the teacher must have some knowledge of developmental theory and be able to recognize stages of development. She must also have conceptual understanding of mathematical ideas and their connections, in order to recognize the conceptual paths being constructed through the various activities, where students are on these paths, and to provide the appropriate next stepping stone for each student. The program authors assume that the teacher will make pedagogical decisions based on this knowledge and understanding. As mentioned earlier, the teacher should also have the ability, or be disposed toward developing the ability, to organize independent and small-group activities, observe students closely and ask questions that help them understand underlying concepts and make connections between them, and lead class discussions that further student understanding. It is possible that the authors' intentions could be misconstrued without these abilities.

### Conclusion

#### Breaking from the Mainstream

As these reports suggest, the three distinctive curricula provide alternative approaches to conceiving of and teaching elementary mathematics. They have broken from the confined conceptions of mathematics and the traditional teacher-centered practices embedded in mainstream curricula. The program goals of the three distinctive curricula suggest not only alternative approaches to teaching mathematics but alternative conceptions of what is being taught and what it means to know mathematics. For example, all three closely tie algorithmic proficiency with understanding of the underlying concepts, and attempt to avoid prescribing the practice of rote skills in isolation. The authors of CSMP and *Math in Stride* make explicitly clear their intent in developing the conceptual foundation well before introducing the related algorithm. The implication is that algorithmic mastery is not the epitome of mathematical knowledge or an indicator of it. Instead, these three alternative curricula rest on epistemological and pedagogical visions not found in commonly used textbooks,

such as *Addison-Wesley Mathematics*. Furthermore, the views of mathematics and the assumptions about how it is learned driving these curricula seem to be potentially closer to the vision proposed by the mathematics education community, such as detailed in the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards* (NCTM, 1989).

### Vision of Mathematics

Although the views of mathematics embedded in each of the three alternative curricula vary somewhat from one another, they are similar in that they construe mathematics as being more than a collection of rules and procedures. The authors have taken great strides to organize and present the content to reflect this view. The inherent beauty of mathematical ideas, the importance of the connections between mathematical ideas, and its natural connections to events, situations and phenomena in the world are characteristics of mathematics that the authors of these curricula claim children should come to appreciate. Additionally, to varying degrees, each of the alternative curricula recognizes and fosters forms of knowing mathematics other than that which can be demonstrated by a paper-and-pencil test. This vision of mathematics connects in a variety of ways with the vision presented by the NCTM *Curriculum and Evaluations Standards* (NCTM, 1989). In this document, the council defines mathematics as problem solving, reasoning, and making connections--all abilities that seem to play significant roles in the philosophies of the three distinctive curricula.

### Pedagogical Visions

The three curricula all veer from the traditional conception of teacher as the source of intellectual authority and knowledge, and of students as receptacles to be filled. Consequently, the teacher's role is more often to facilitate the construction of knowledge or to provide experiences that challenge students intellectually. This is particularly evident in the degree of complexity found in many of the situations in each of the three alternative curricula. The assumption is that students will develop mathematical understandings as they grapple with difficult and compelling problems. In other words, complex problem situations are used in the distinctive curricula as ways to motivate the relevance of particular ideas and as sites for instruction or development of these idea. *Addison-Wesley Mathematics*, in contrast, often uses situations to demonstrate uses of particular procedures or ideas but teaches the component skills in isolation before encouraging their use in application contexts.

Another characteristic common to the three distinctive curricula and not found consistently in *Addison-Wesley Mathematics* is the emphasis on student discourse or discussion. The focus on discourse complements the description of mathematics as communication in the *Curriculum and Evaluation Standards* (NCTM, 1989) and the emphasis on classroom discourse in the *Professional Standards for Teaching Mathematics* (NCTM, 1991). This implies that communication and discourse are more than pedagogical tools, that the

ability to communicate mathematically is one of the curriculum goals, rather than a strategy used only to achieve other goals. In each case, although to varying degrees and in different formats, student talk about mathematical ideas is a fundamental part of the intended instruction. This is quite different from the emphasis on solitary, paper-and-pencil tasks in *Addison-Wesley Mathematics* and in most commonly used curricula.

### Constraining Factors

Despite the fact that these three curricula provide compelling and desirable alternatives to what is most commonly used in elementary school--an alternative that seem intended to move instruction closer to the goals of higher order thinking or high literacy--these curricula are used minimally across the country. Teachers, district level administrators and specialists tend to avoid selecting them. Not only do their uncommon organizations, presentations, underlying philosophies, and goals seem foreign to many teachers and decision makers, but accepting them is likely to undermine the old philosophies and goals on which the traditional curriculum is based. They are often viewed as radical, ignoring important, basic skills (Peterson et al., 1990). Furthermore, there are multiple and conflicting agendas and influences confronting school officials. Decisions must be made to satisfy a wide range of purposes. Thus, widespread use of such alternative, and sometimes controversial, curricula is limited.

Alternative curricula are particularly subject to being reshaped and altered through their use. Teachers bring to any published curriculum their own knowledge and beliefs about learning and knowing mathematics, and this influences how they use these resources. Their uncommon representations of the content leads teachers to interpret and reconstruct them through their own understandings and perceptions of mathematics. In some cases in which these curricula have been selected and used, there is evidence of teacher resistance to their use; and in other cases, teachers have interpreted them in a variety of ways (Peterson et al., 1990; Remillard, 1991). In many of these cases the intent of the alternative curricula is to initiate changes in mathematics instruction, yet the alternative curricula seem to have minimal influence in initiating significant changes in mathematics instruction. It is this question of how teachers use and interpret alternative curricula which needs further study. We need to know more about what teachers bring to new and nonstandard mathematics curricula and how they make sense of and use them. This question needs to be explored across a variety of alternative curricula to begin to delineate aspects of curricula, in the form of organization, instructions, and pedagogical approaches, which tend to be usable and manageable for teachers, from those that tend to foster resistance or are likely to be grossly misinterpreted. There seems to be a great deal of potential in alternative curricula, but, because curricula cannot be separated from

practices, the potential these alternatives hold must also be examined within the context of practice.

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## Appendix

### Framing Questions<sup>2</sup>

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<sup>2</sup>The following framing questions were developed by a team of researchers representing five subject areas in an effort to establish some shared vantage points for examining curricula. While it was not anticipated that all questions would be relevant to all curricula, the construction of the instrument benefited from the breadth of perspectives of its contributors. These perspectives also contributed to the author's insight as a researcher. The analysis drew from the instrument questions relevant to mathematics and adapted others to be appropriate to each curriculum.

Phase II Study 2: Curriculum Materials Analysis  
Framing Questions

A. GOALS

1. Are selective, clear, specific goals stated in terms of student outcomes? Are any important goals omitted? As a set, are the goals appropriate to students' learning needs?
2. Do goals include fostering conceptual understanding and higher order applications of content?
3. To what extent does attainment of knowledge goals imply learning networks of knowledge structured around key ideas in addition to the learning of facts, concepts, and principles or generalizations?
4. What are the relationships between and among conceptual (propositional), procedural, and conditional knowledge goals?
5. To what extent do the knowledge goals address the strategic and metacognitive aspects of processing the knowledge for meaning, organizing it for remembering, and accessing it for application?
6. What attitude and dispositional goals are included?
7. Are cooperative learning goals part of the curriculum?
8. Do the stated goals clearly drive the curriculum (content, activities, assignments, evaluation)? Or does it appear that the goals are just lists of attractive features being claimed for the curriculum or post facto rationalizations for decisions made on some other basis?

B. CONTENT SELECTION

1. Given the goals of the curriculum, is the selection of the content coherent and appropriate? Is there coherence across units and grade levels? (Note: all questions in this section should be answered with the goals in mind.)
2. What is communicated about the nature of the discipline from which the school subject originated?
  - a. How does content selection represent the substance and nature of the discipline?
  - b. Is content selection faithful to the discipline from which the content is drawn?

c. What does the relationship among conceptual (propositional), conditional, and procedural knowledge communicate about the nature of the discipline?

3. To what extent were life applications used as a criterion for content selection and treatment? For example, in social studies, is learning how the world works and how it got to be that way emphasized?
4. What prior student knowledge is assumed? Are assumptions justified? Where appropriate, does the content selection address likely student misconceptions?
5. Does content selection reflect consideration for student interests, attitudes, dispositions to learn?
6. Are there any provisions for student diversity (culture, gender, race, ethnicity)?

C. CONTENT ORGANIZATION AND SEQUENCING

1. Given the goals of the curriculum, is the organization of the content coherent and appropriate? Is there coherence across units and grade levels? (Note: All questions in this section should be answered with goals kept in mind.)
2. To what extent is the content organized in networks of information structured in ways to explicate key ideas, major themes, principles, generalizations?
3. What is communicated about the nature of the discipline from which the school subject originates?
  - a. How does content organization represent the substance and nature of the discipline?
  - b. Is content organization faithful to the discipline from which the content is drawn?
  - c. What does the relationship among conceptual (propositional), conditional, and procedural knowledge communicate about the nature of the discipline?
4. How is content sequenced, and what is the rationale for sequencing? For example, is a linear or hierarchical sequence imposed on the content so that students move from isolated and lower level aspects toward more integrated and higher level aspects? What are the advantages and disadvantages of the chosen sequencing compared to other choices that might have been made?

5. If the content is spiralled, are strands treated in sufficient depth, and in a non-repetitious manner?

#### D. CONTENT REPLICATION IN THE TEXT

1. Is topic treatment appropriate?
  - a. Is content presentation clear?
  - b. If content is simplified for young students, does it retain validity?
  - c. How successfully is the content explicated in relation to students' prior knowledge, experience, and interest? Are assumptions accurate?
  - d. When appropriate, is there an emphasis on surfacing, challenging, and correcting student misconceptions?
2. Is the content treated with sufficient depth to promote conceptual understanding of key ideas?
3. Is the text structured around key ideas?
  - a. Is there alignment between themes/key ideas used to introduce the material, the content and organization of the main body of material, and the points focused on in summaries and review questions at the end?
  - b. Are text structuring devices and formatting used to call attention to key ideas?
  - c. Where relevant, are links between sections and units made explicit to students?
4. Are effective representations (e.g., examples, analogies, diagrams, pictures, overheads, photos, maps) used to help students relate content to current knowledge and experience?
  - a. When appropriate, are concepts represented in multiple ways?
  - b. Are representations likely to hold student interest or stimulate interest in the content?
  - c. Are representations likely to foster higher level thinking about the content?
  - d. Do representations provide for individual differences?
5. When pictures, diagrams, photos, etc. are used, are they likely to promote understanding of key ideas, or have they been inserted for other

reasons? Are they clear and helpful, or likely to be misleading or difficult to interpret?

6. Are adjunct questions inserted before, during, or after the text? Are they designed to promote: memorizing; recognition of key ideas; higher order thinking; diverse responses to materials; raising more questions; application?
7. When skills are included (e.g., map skills), are they used to extend understanding of the content or just added on? To what extent is skills instruction embedded within holistic application opportunities rather than isolated as practice of individual skills?
8. To what extent are skills taught as strategies, with emphasis not only on the skill itself but on developing relevant conditional knowledge (when and why the skill would be used) and on the metacognitive aspects of its strategic application?

#### E. TEACHER-STUDENT RELATIONSHIPS AND CLASSROOM DISCOURSE

1. What forms of teacher-student and student-student discourse are called for in the recommended activities, and by whom are they to be initiated? To what extent does the recommended discourse focus on a small number of topics, wide participation by many students, questions calling for higher order processing of the content?
2. What are the purposes of the recommended forms of discourse?
  - a. To what extent is clarification and justification of ideas, critical and creative thinking, reflective thinking, or problem solving promoted through discourse?
  - b. To what extent do students get opportunities to explore/explain new concepts and defend their thinking during classroom discourse? What is the nature of those opportunities?
3. Who or what stands out as the authority for knowing? Is the text to be taken as the authoritative and complete curriculum or as a starting place or outline for which the discourse is intended to elaborate and extend it? Are student explanations/ideas and everyday examples elicited?
4. Do recommended activities include opportunities for students to interact with each other (not just the teacher) in discussions, debates, cooperative learning activities, etc.?

## F. ACTIVITIES AND ASSIGNMENTS

1. As a set, do the activities and assignments provide students with a variety of activities and opportunities for exploring and communicating their understanding of the content?
  - a. Is there an appropriate mixture of forms and cognitive, effective, and/or aesthetic levels of activities?
  - b. To what extent do they call for students to integrate ideas or engage in critical and creative thinking, problem-solving, inquiry, decision making, or higher order applications vs. recall of facts & definitions or busy work?
2. As a set, do the activities and assignments amount to a sensible program of appropriately scaffolded progress toward stated goals?
3. What are examples of particularly good activities and assignments, and what makes them good (relevant to accomplishment of major goals, student interest, foster higher level thinking, feasibility and cost effectiveness, likelihood to promote integration and life application of key ideas, etc.)?
  - a. Are certain activities or assignments missing that would have added substantially to the value of the unit?
  - b. Are certain activities or assignments sound in conception but flawed in design (e.g., vagueness or confusing instruction, invalid assumptions about students' prior knowledge, infeasibility, etc.)?
  - c. Are certain activities or assignments fundamentally unsound in conception (e.g., lack relevance, pointless busy work)?
4. To what extent are assignments and activities linked to understanding and application of the content being taught?
  - a. Are these linkages to be made explicit to the students to encourage them to engage in the activities strategically (i.e., with metacognitive awareness of goals and strategies)? Are they framed with teacher or student questions that will promote development?
  - b. Where appropriate, do they elicit, challenge, and correct misconceptions?
  - c. Do students have adequate knowledge and skill to complete the activities and assignments?
5. When activities or assignments involve integration with other subject areas, what advantages and disadvantages does such integration entail?

6. To what extent do activities and assignments call for students to write beyond the level of a single phrase or sentence? To what extent do the chosen forms engage students in higher order thinking?

## G. ASSESSMENT AND EVALUATION

1. Do the recommended evaluation procedures constitute an ongoing attempt to determine what students are coming to know and to provide for diagnosis and remediation?
2. What do evaluation items suggest constitute mastery? To what extent do evaluation items call for application vs. recall?
  - a. To what extent are multiple approaches used to assess genuine understanding?
  - b. Are there attempts to assess accomplishment of attitudinal or dispositions' goals?
  - c. Are there attempts to assess metacognitive goals?
  - d. Where relevant, is conceptual change assessed?
  - e. Are students encouraged to engage in assessment of their own understanding/skill?
3. What are some particularly good assessment items, and what makes them good?
4. What are some flaws that limit the usefulness of certain assessment items (e.g., more than one answer is correct; extended production form, but still asking for factual recall, etc.)?

## H. DIRECTIONS TO THE TEACHER

1. Do suggestions to the teacher flow from a coherent and manageable model of teaching and learning the subject matter? If so, to what extent does the model foster higher order thinking?
2. To what extent does the curriculum come with adequate rationale, scope and sequence chart, introductory section that provide clear and sufficiently detailed information about what the program is designed to accomplish and how it has been designed to do so?
3. Does the combination of student text, advice and resources in teachers manual, and additional materials constitute a total package sufficient

to enable teachers to implement a reasonably good program? If not, what else is needed?

a. Do the materials provide the teacher with specific information about students' prior knowledge (or ways to determine prior knowledge) and likely responses to instruction, questions, activities, and assignments? Does the teachers manual provide guidance about ways to elaborate or follow up on text material to develop understanding?

b. To what extent does the teachers manual give guidance concerning kinds of sustained teacher-student discourse surrounding assignments and activities?

c. What guidance is given to teachers regarding how to structure activities and scaffold student progress during assignment completion, and how to provide feedback following completion?

d. What kind of guidance is given to the teacher about grading or giving credit to participating in classroom discourse, work on assignments, performance on tests, or other evaluation techniques?

e. Are suggested materials accessible to the teacher?

4. What content and pedagogical knowledge is required for the teacher to use this curriculum effectively?

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