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ABSTRACT

The interactions of three high school juniors (two females and one male) working together on a series of contextual mathematics problems using a multirepresentational software tool were studied. Focus was on determining how a constructivist model of learning, based on an individual problematic-action-reflection model, can be extended to offer explanatory power for small-group collaborative learning. This extension is constructed by adopting several concepts from the socio-historic or Vygotskian school, including the zone of proximal development, cultural tools, proleptic talk, and appropriation. The subjects worked together during a 10-week secondary mathematics course that focused on problem solving with Function Probe. Although constructivist and socio-historic approaches to cognition have, at times, been interpreted as offering opposing viewpoints, it is suggested that there is a potential complementarity, particularly in the area of collaborative peer learning, since researchers in neither area have as yet offered a strong explanatory model for how students jointly construct mathematical knowledge. Four figures and a 24-item list of references are included. (SLD)

Understanding Collaborative Learning: Small Group Work on Contextual Problems Using a Multi-Representational Software Tool

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"...the allocation of power affects how people take part in the formulating of knowledge. The effect of placing control of relevance in the hands of one person is to emphasize his content frame, and this will affect profoundly the basis upon which others participate. If on the other hand alternative frames are open to negotiation this will influence not only who takes part but also the knowledge which is celebrated. Thus, what is learnt by discussion in a group of peers will be different in kind as well as content from what is learnt from teachers."

(Barnes and Todd, 1977, p. 127)

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Abstract

In this paper we investigate the interactions of three students working together on a series of contextual math problems using a multi-representational software tool. The goal of the paper is to look at how a constructivist model of learning, based on an individual problematic-action-reflection model, can be extended to offer explanatory power for small-group collaborative learning. We construct this extension by adopting several concepts from the socio-historic or Vygotskian school, including the zone of proximal development, cultural tools, proleptic talk and appropriation. Although the constructivist and socio-historic approaches to cognition have, at times, been interpreted as offering opposing viewpoints, we suggest that there is a potential complementarity, particularly when looking at collaborative peer learning as researchers in neither area have, as yet, offered a strong explanatory model for how students jointly construct mathematical knowledge.

INTRODUCTION

Of the reforms which were called for by the National Council of Teachers of Mathematics in their recei. 'y revised "Standards" (NCTM, 1989), increased use of small groups, contextual problems, and computers received high priority. Although the desirability for changes in these directions may be widespread, there are obvious questions relating to these reforms which have not yet been fully answered. Of primary importance is the question: "What is the nature of the mathematics that students learn?" Closely tied to this is another issue: 'How do students come to learn mathematics?"

In terms of educational goals, the most important question regards the issue of what mathematics is learned. This is in no way a trivial question, nor one that can be answered through traditional pre and post testing, for it is not a question of how much is learned, but of the descriptive qualities of what is learned. Although in the past, we have often treated mathematical knowledge as if it were something that could be measured on an appropriate scale, there is growing evidence of the qualitative nature of mathematical knowledge which challenges us to find new ways of assessment. In summarizing the work of a number of educational researchers, Forman concludes "that the form of instruction influences the nature of the knowledge learned." (1989, p. 56). Guba and Lincoln extend this claim beyond formal instructional situations, claiming that knowledge "emerges as a product of interaction between humans or between humans and nonhuman objects" and is "created by that interaction." (1989, p. 67). In addition Pea (1987) argues that computers themselves do not simply "amplify" our ability to learn but alter the ways in which we learn and, thus, the nature of what is learned. This suggests that the question of 'what' is learned may be very different from that of 'how much' is learned and that the questions of 'what' is learned and 'how' that learning takes place may be inseparable. Thus the focus of our work in this paper is to develop models for the question: How do these students learn mathematics? We intend to concentrate specifically on how students learn mathematics through small-group peer collaboration. However, since the study was carried out in a situation where students were using a software tool and working on contextual problems, the interactions of problem and tool in this process will be significant and must be taken into account.

Peer collaboration seems to fall into a niche somewhere between the constructivist emphasis on the individual problem and resolution (Confrey, 1990a; Steffe, 1990a; von Glasersfeld, 1984) and a socio-historic or Vygotskian emphasis on thought as internalization of social interaction with more knowledgeable others (Wertsch, 1981; Davydov, 1989; Newman, Griffin, and Cole, 1989). In this paper, we will argue that the stereotypes of both sides, of constructivists requiring each child to reinvent the world and of Vygotskians ignoring individual differences by placing all thought in the social realm, grossly simplifies the work of researchers in both areas. Instead we see the differences as rising primarily from different, but potentially complementary, starting points in the investigation of cognition. Since the research methodology in the constructivist model has emphasized the individual learner and research on the Vygotskian model has emphasized interaction between student and expert, we believe that peer collaboration provides a fruitful area in which to explore this potential complementarity.

But a resolution at the theoretical level is not sufficient. We need to develop specific examples for which we can ask: Does this make sensed in relation to our model? Just as the model provides a lens for looking at interactions, we also need well developed instances of interactions which provide a way to focus and adjust our model. The model we will propose is partially a result of preliminary recursive viewings of this kind. In the last section of this paper we continue this process by describing how one set of interactions is seen through the lens of the model we propose.

Development of a Theoretical Perspective

The goal in this section is to develop a set of theoretical constructs that will provide explanatory power in understanding collaborative learning. This will be undertaking by first describing a constructivist perspective on learning, then building on the constructivist approach by incorporating concepts from the Vygotskian school and from the research on cooperative and collaborative learning.

Constructivism and Peer Collaboration

There are two fundamental concepts necessary to understanding a constructivist approach to cognition. One is the idea that cognitive activity is based on goals or intentions of the individual:

> Constructivism necessarily begins with the (intuitively confirmed) assumption that all cognitive activity takes place within the experiential world of a goal directed consciousness." (von Glasersfeld, 1984, p.32)

The second is that knowledge is constructed in reference to actions which are successful or unsuccessful in achieving those goals:

> ... *knowledge* refers to conceptual structures that epistemic agents, given the range of present experience within their tradition of thought and language, consider *viable*. (von Glasersfeld, 1989, p. 124).

The constructivist, in effect, makes the claim that it is the actions of the active epistemic agent that allow him to construct a world that is viable in the sense that it provides structure to experience and allows one to have some control over what is experienced.

Referring more specifically to mathematics, Steffe describes knowledge as "coordinated schemes (in a Piagetian sense) of action and operation" which "is based on coordinations of (mental) actions into organized action patterns to achieve some goal."(1989, p.1). His basic claim is that the mathematics of the student (or of anybody, for that matter) is not the mathematics of the traditional curriculum, and that by relying on textbook mathematics we prevent students from having the opportunity to coordinate their construction of mathematics with their construction of social and physical reality. Mathematics is an activity of solving problems (resolving perturbances) and, thus, must take place within the context of schemes already existing for the individual. As a researcher, teacher, or observer of another individual we can never know what those cognitive schemes are (communication is never perfect), but can only produce what seem to be viable models of their mathematics that allow us to communicate with them mathematically and thus possibly provide "possible mathematical environments" which the student can use to construct more useful schemes of action.

Confrey (1989) describes the activity of learning mathematics in terms of a cycle of problematic-action reflection. She emphasizes several points:

1) For the active problem-solver the problem that appears on a piece of paper is not the same as the "problematic". "A problem(atic) is only defined in relation to the solver. A problem is only a problem(atic) to the extent to which and in the manner in which it feels problematic to the solver. When defined this way, as a roadblock to where a student wants to be, the problem(atic) is not given an independent status. The problematic acts as a perturbance, i.e. a call to action." (p. 12, my parentheses)

2) In acting, we "use tools and previously familiar schemes of representation" (1989, p.13). Thus although the actions taken by an individual may appear in the form of socially accepted forms of representation, the actions must take place within the framework of existing schema available to the solver. It is important not to confuse the form of action visible to an observer with its content or intent.

3) Reflection involves "monitoring the results of our actions to see if the problematic has been resolved and equilibrium restored." The successful resolution of a problematic may allow us to construct new schema and allow us to "set apart that action or operation (that resolved the problematic) by various processes of naming it, discussing it, objectifying it or creating it into a tool or representation for further action." In effect, the action of reflection prepares us to take on further, possibly more difficult, problematics.

Central to these constructivist models is the idea of the individual 'problematic'. It is the problem as seen and interpreted by the individual that is the driving force behind the construction of mathematical knowledge.

Despite this emphasis on individual cognitive processes, it is not necessarily the case that constructivism must conflict with concepts of social cognition. Constructivist researchers do recognize the interactional, cultural, social, and political quality of teacher - student and of student - student exchange (Cobb and Steffe, 1983; Confrey, 1990a). Confrey, for example, offers the following:

> Constructivists view mathematics as a human creation, evolving within cultural contexts. They seek out the multiplicity of meanings across disciplines, cultures, historical treatments, and applications. They assume that through the activities of reflection and of communication and negotiation of meaning, human beings construct mathematical concepts which allow them to structure experience and to solve problems. Thus mathematics is assumed to include more than its definitions, theorems, and proofs and its logical relationships-- included in it are its evolution of problems and its methods of proof and standards of evidence."
>
> (1989, p. 6-7).

Given the constructivist emphasis on problematic (or perturbation) and resolution by the individual, it is not surprising that some of the initial work on peer collaboration from a constructivist or Piagetian perspective emphasized the role of cognitive conflict. Perret-Clermont, for example, "concludes that peer interaction enhances the development of logical reasoning through a process of cognitive reorganization induced by cognitive conflict." (from Forman and Cazden, 1985, p. 330). Further, Perret-Clermont claims:

> Of course, cognitive conflict ... does not create the forms of operations, but it brings about the disequilibriums which make cognitive elaboration necessary, and in this way cognitive conflict confers a special role on the social factor as one among other factors leading to mental growth. Social-cognitive conflict may be figuratively likened to the catalyst in a chemical reaction: it is not present at all in the final product, but it is

nevertheless indispensable if the reaction is to take place.

(1980, p. 178)

Forman and Cazden do not dispute that in cases where cognitive conflict is apparent, the Perret-Clermont analysis may be viable, but question whether her analysis is sufficient to cover situations where "mutual guidance and support are evident" (p. 340).

Based on their work in elementary schools, Yackel, Cobb and Wood (1990) have developed a more extensive model of peer collaboration. They describe four general areas which are important: i) the role of cognitive conflict as an opportunity to reconceptualize a problem (pp. 5-6); ii) opportunities to verbalize thinking and explain or justify solutions (p. 7); iii) the role of collaborative dialogue "characterized by a genuine commitment to communicate." (p. 7); and iv) the role of classroom norms, particularly that students work together to solve problems and the they isten to each other's solution attempts (p. 3). Yackel, et al conclude that the establishment by the group of "taken-as-shared" knowledge provides a basis for communication which allows collaborative activity to proceed smoothly and dialogue to become relatively limited. (p. 19). Thus, although Yackel, et al extend the constructivist perspective to allow for "mutual guidance and support", it appears, at least in this early paper, that they have not developed a strong model for incorporating these supportive activities. This suggests that extending the constructivist model to allow for these activities does not necessarily conflict with constructivist principles, rather that, with the previous emphasis of constructivist research being oriented towards the individual learner, the language and concepts to account for these activities have not yet developed in the constructivist tradition.

Social-Historical Approaches to Peer Collaboration

Whereas constructivists have taken the individual problematic as the starting point for investigations into learning, Vygotsky emphasized the role of interactions of the child with adults or more capable others. Of primary importance is the process by which interpsychological processes become transformed into intrapsychological processes. Whereas the Piagetian model has seen stages of development determining the possibility for learning and thus for social interaction, Vygotsky turns this around. The possibility for social interaction leads the level of generalization and levels of generalization can only advance when the possibilities for social interaction are realized through interaction with an adult or more knowledgeable peer. (Cole, 1985, p. 148). It is just this difference between the intrapsychological level of an individual's ability to generalize and the interpsychological processes that can potentially take place through social interaction that Vygotsky called the "zone of proximal development (zpd), defined as:

> ... the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers.
> (from Forman and Cazden, 1985, p.341)

It is important to note that, according to Vygotsky, it is not a simple matter of individual development following social development, but that "the very means (especially in speech) used in social interaction are taken over by the individual child and internalized." (Wertsch, 1981, p. 146). Davydov (1989) summarized Vygotsky's work under four major themes:

1) At any time, human consciousness reflects the cultural conditions.

2) Every psychological process is reflected through the symbols and signs of the culture. These include not only language, but all the other symbolic forms through which communication is possible.

3) Every concept is manifested through social life and social exchange.

4) The human being becomes human primarily through relationships with other human beings.

The process by which the 'symbols and signs of a culture' become transformed from interpsychological forms of communication into intrapsychological processes was given the name "appropriation, initially by Vygotsky's pupil, Leontyev (1981, p. 422). This concept has been further developed recently by Newman, Griffin & Cole (1989). In a learning situation, a task is undertaken by a student. For this task to be in the student's zpd, it will not be something that the student can already accomplish on her[1] own, but the student must be able to "form some notion of what the episode is going to be about." There is, however, "no assumption that all parties involved in a zpd have the *same* notion of what is going to happen." (Newman, et al, 1989, p. 64). The teacher plays the role of socio-historical representative in that he is the one who understands the task in terms of its socio-historical context. In this role, he interprets the student's actions as if they were oriented toward the same goal as his own, thus 'appropriating' the student's actions to his own goals. As a result of this appropriation by the teacher, and his feedback which is given as if

1 Our choice of method for attempting to maintain gender neutrality in our work is to alternate gender roles in successive papers. Thus in this paper we will use male pronouns for the teacher and female pronouns for the student.

the student were working on the same goal as he, the student begins to appropriate the goal of the teacher. Thus the teacher 'appropriates' the actions of the student to his own goals, while the student 'appropriates' the goals of the teacher to her actions. Newman, et al argue that this process allows the student to make cognitive leaps. Concepts are "available in the social system" (1989, p. 67) and, thus, can be appropriated by students without the necessity of a more continuous step-wise construction.

This view of cognition is centered on the role of the teacher (or more knowledgeable other). The teacher must create a problem within the zpd of the student and provide an environment in which the student can work on the problem. The teacher is not as concerned with the student's interpretation as he is with interpreting the student's actions as if they were oriented to solving the problem as he understands it. Finally, it seems that the focus of attention for the student is primarily on the teacher and only secondarily on the problem. Thus it is not as important for the student to pay attention to the development of her own problematic or whether the actions she takes are viable in resolving her own problematic as it is in how the teacher interprets her actions and provides her with constructive feedback. Implicit in this relationship is a mutual understanding of who is expert and who is novice.

It seems apparent that this version of the Vygotskian model cannot be directly applied to situations of peer collaboration. Since there are, by definition, no socially recognized expert/novice role divisions, there is no obvious manner in which the student can judge the viability of her actions through a process of social interaction. Although Vygotsky did not discuss peer collaboration, Forman and Cazden use his analysis of adult-child interactions to devise a possible Vygotskian response to this issue. They state:

> ...the Vygotskian perspective enables us to see that collaborative tasks requiring data generation, planning, and management can provide another set of valuable experiences for children. In these tasks, a common set of assumptions, procedures, and information needs to be constructed. These tasks require children to integrate their conflicting task conceptions into a mutual plan. One way to achieve a shared task perspective is to assume complementary problem-solving roles. Then each child learns to use speech to guide the actions of her or his partner and, in turn, to be guided by the partner's speech. Exposure to this form of social regulation can enable children to master difficult problems together before they are capable of solving them alone. More importantly, experience with social forms of regulation can provide children with just the tools they need to master problems on their own. It enables them to observe and reflect on the problem solving process as a whole and to select those procedures that are the most effective. When they can apply this social understanding to themselves, they can then solve independently, those tasks that they had previously been able to solve only with assistance.
>
> (1985, p. 343)

Forman (1989) develops this idea into a notion of a "bidirectional zone of proximal development" in which students, working cooperatively in peer collaboration, assume: "at different times the role of teacher or student." . Her second distinction is to point out the separate and private stances towards a task that will be held by people entering a communicative situation. Due to relative equality of power, "children in a cooperative peer work group can learn to share the responsibility for establishing a common understanding of task goals and strategies." (p. 59). Similar to Yackel, et al., Forman suggests that "proleptic" or minimally explicit talk may become more prevalent in group interactions as common understandings develop.

The emphasis on the interchangability of roles and the individual interpretation of tasks potentially provides a basis for extending the Vygotskian model to provide more understanding of peer collaboration. However, given the general Vygotskian claim that knowledge exists in the symbols and signs of a culture (cultural tools), it is not clear how peers could make decisions about who should take which role at which times or how an adequate common understanding might be established. Forman acknowledges these issues as key questions, suggesting that research on peer collaboration should be oriented towards finding "if children can create a bidirectional" zpd and towards understanding "how a shared task definition is established." (p. 59)

Both Vygotskyism and constructivism acknowledge the necessity of a "steering mechanism" to guide the creation of knowledge. For Vygotsky, this was provided by a more knowledgeable other, while in constructivism, it is provided by the notion of viability within the experiential world. However, constructivists acknowledge that learning takes place within social contexts in which the role of an expert is often beneficial. If we are to make peer collaboration a viable model in classrooms, however, we must understand what happens among peers when no expert is available. Thus this project is not centered around the idea of investigating only the benefits of peer collaboration but also in understanding the real trade-offs that will take place in situations of student investigation with minimal expert guidance.

Constructing a Synthesis

Although significant progress has been made in the development of explanatory models for peer collaboration from both a Vygotskian and a constructivist perspective, important issues remain to be resolved in both areas. We believe that a model that draws on the strengths of both these perspectives may offer complementary explanatory power.

However, given the fundamentally different starting point of these two perspectives, the following question must be answered: In an educational environment which is centered on problem-solving activities through peer collaboration, where should attention initially be focused to gain an understanding of the learning processes that take place? Given the two research traditions we have examined, the two most obvious answers seem to be the problem and individual problematic vs. the teacher as cultural representative. From a classroom management as well as an educational perspective, it seems that a fundamental goal of collaborative learning is to decenter the role of the teacher in the classroom. Thus, it seems that the successful implementation of collaborative learning depends upon establishing a situation that, from the perspective of the students, is centered on the resolution of a problem.

If we then assume that the activities which occur when peers work collaboratively in problem-solving situations are centered on the individual development of a problematic, followed by interactions among students regarding actions to be taken and the achievement of a group solution, we must examine the ways in which some of the concepts from the constructivist and the Vygotskian traditions may be extended and revised to fit into this model. It is important to point out, however, that the development which we are proposing is our own initial attempt at understanding collaborative learning. This model has grown from our own investigation of constructivist and Vygotskian concepts in combination with our observations of small-group work over the last several years. We expect this model to evolve as we gain more understanding through further analysis of our own data as well as through interactions with others working in this area.

Outline of a Model

Problematic: When looked at in the context of peer collaboration, it is particularly important to keep in mind the social constraints which act upon the individual problematic. Although each student comes to a problem situation with a unique perspective, each perspective has been constructed within the forms of cultural tools. Thus, it would be a mistake to assume that the individual constructive process, what might be considered 'finding the problem within the problem', removes the problematic from the constraints of the social world. Language in its role in intrapsychological thought forms a significant component of the building blocks of the problematic, and communicative acts undertaken. As von Glasersfeld states: "Every individual's abstraction of experiential items is constrained (and thus guided) by social interactions and the need of collaboration and communication with other members of the group..." (1990, p.26). We should also not assume that a problematic remains static throughout the problem-solving process. Through interactions with others, actions taken by others, and cognitive conflict, problematics evolve. What is important is that the individual not 'give up' her problematic by turning authority for what constitutes a solution over to others.

Zone of proximal development: Focusing on the problem in peer collaboration may relieve us of the necessity of viewing peer collaboration as a sequential process of alternating teacher and student roles when two students are interacting within a bidirectional zone of proximal development. Since both problematic and solution are interpreted from the perspective of the student, actions and communicative acts which allow the student to modify and/or resolve her problematic may be seen as taking place within the zone of proximal development. Thus teacher/student roles may occur simultaneously in the same individual. Likewise actions and communicative acts which the student is unable to relate to her problematic are outside her zpd. This could be the case in a situation where a student allows an 'other' to 'take over' her problematic and reverts to rote learning[2] or in a situation where the student simply does not (or is unable to) relate actions and communicative acts of the group to her own problematic. This interpretation of the zone of proximal development makes the student/learner an active participant in its construction. In situations where the student expects actions and communicative acts to be relevant to her problematic, that is to be in the zpd, she is, perhaps more likely to actively attempt to construct meaning for these interactions that allow them to be in her zpd. This active involvement and related expectation in determining the zpd may be an important indicator of effective collaborative learning.

Appropriation: When peer interactions are occurring within a student's zpd, we may expect cognitive change to occur through two mechanisms, appropriation and cognitive conflict. Appropriation occurs when an individual has taken a suggestion, concept, algorithm, or solution process of another and appropriated it to her own problematic. Although an essential part of the process of appropriation will often be taking the suggestion of another and 'trying it on for size', merely trying the suggestion, etc. of another is not sufficient to be termed appropriation. Instead, we see appropriation as an active cognitive activity in which a student is able to interpret an action or communicative act such that she ca conceptually remake it in a way that she is able to use it to modify and/or resolve her

2 Another way to view such a situation would be to say that the problematic of the student becomes finding a solution that satisfies an 'other'.

problematic. However, the student may or may not be aware of this 'remaking' activity. It may appear to the student that she has actually taken over the meaning of another's communicative act. However, individuals have no access to the meanings that others assign to cultural signs and, thus must construct their own meanings (von Glasersfeld, 1990, p. 26). An important aspect of appropriation is that it allows us to understand how the same cultural symbols (or tools) come to be associated by members of a peer group with the representation and/or solution of a problem situation.

Cognitive Conflict: Cognitive conflict arises for an individual when she interprets the communicative acts of another in ways that she sees as in conflict with her own problematic, paths to a solution or proposed solution. In order for cognitive conflict to occur, the communicative act must occur within the zpd of the student, for the student must be able to interpret the act in relation to her own problematic. Cognitive conflict requires that the student's interpretation of the act be valid relative to her own problematic, yet also lead to a sense of conflict. As opposed to expert/novice situations, we expect peer conflict to often be resolved through negotiation, rather than one individual simply finding her own situation to be erroneous In these situations, we may expect a process of negotiation to occur leading to the development of a negotiated consensus, or what Yackel, et al called "taken-as-shared" knowledge. There are, of course, no guarantees that such knowledge will strongly resemble what an expert might have proposed.

Minimally explicit talk: Both Forman and Yackel, et al report the observation that as peers work together over time, the explicitness of talk seems to diminish. Forman (1989) adapts the term, "proleptic instruction", which Stone and Wertsch (1984) originally used to describe instruction in which "the learner must construct ... for himself" (the presuppositions of the instructor (1985, p. 135). Forman describes proleptic talk as dealing with knowledge in a way that is "primarily informal, fluid, and implicit." (p. 58) and later attributes proleptic talk to the ability of two students working collaboratively to "help each other incorporate new problem-attack and reasoning strategies into their repertoire." (p. 67). Barnes and Todd (1977) also stress the importance of an indefinite style of dialogue. Their emphasis, however, is on the possibility for variation in meaning among individuals. They emphasize the importance of both the apparent vagueness of some conversations and the interactive effects of social and cognitive interactions in this process. Thus they say: "The very indefiniteness of the 'meanings' or 'intentions'- their lack of reflective definition, that is- allows participants in discussion to collaborate in developing a thread of meaning which may change many times, and radically, in the course of talk" (p.

102). For them, "Meanings are characteristics of people, not of utterances" (p. 108), and the benefits of indefinite dialogue are primarily the space provided for individuals to construct their meaning within a shared social world. Thus we see two complementary roles that minimally explicit talk may play in peer collaboration: the affirmation of existence within a shared social world, combined with the space to develop and explore autonomous meaning.

Negotiated consensus: Negotiated consensus: Actions or statements produced by the group for which they "agree to agree" (Confrey, 1990a). Negotiated consensus arises through both cognitive conflict and mutual appropriation. The primary difference between negotiated consensus and "taken-as-shared" knowledge as used by Yackel, et al (1990) is in emphasis. Yackel et al tend to use taken-as-shared knowledge as implicit and general agreements within the group that develop and allow for communication among group members. We will use negotiated consensus more for explicit and specific situations, such as proposed solutions to a problem or an aspect of a problem for which we believe we can cite evidence which relates the modelled problematic of individuals to the consensus negotiated

Actions: To understand peer collaboration, we will want to think of actions on two levels. On the individual level, for an activity to be an action it must occur within the zpd of the individual. That is each individual must engage in carrying out the action in connection with the problematic of that individual. This sense of the word 'action' is similar to Confrey's use in her problematic-action-reflection model (see p. 3). However, we will also want to discuss 'group actions' which are observable actions, typically undertaken through the computer, taken by the group. Like minimally explicit conversation, we must remember that each individual constructs her own meaning for the group action and we will often see that the publicly observable process of carrying out actions on the computer allow individuals the 'space' to construct their own meaning for the action.

Cultural tools as individual tools: Newman, et al describe tools as the objects of appropriation which are available in the social/cultural sphere (1989, p. 63). It is through the appropriation of cultural tools that an understanding of problems within their cultural context is achieved. However, to understand how students, lacking a cultural authority, make use of tools during peer collaboration, it seems that we again look to the individual problematic and the ways in which the students construct the tool for themselves through their efforts to resolve problematics. This is an argument that the meaning we construct for a tool is always an individual construction in relation to the problematic situations in which we have found it useful.

Despite the differences in emphasis, there is a strong agreement that, from the perspective of a student, tools are not simply amplifiers of an independently existing subject matter (Pea, 1987), rather that tool and problem (or subject matter) are inherently connected. Thus we reject the idea that representations (including computer interfaces) can be "invisible". We expect the descriptions of the learning process *and* the results of that process to include the forms of representations and tools constructed by the students during that process.

Software and the Use of Computers

In the situations we are investigating, peer interactions are typically centered on the use of a software tool, Function Probe© (Confrey, 1990). Function Probe is a multi-representational software tool consisting of table, calculator and graph. It was designed to be used by students in their problem-solving efforts while learning about functional relationships. There are three obvious roles which such a tool may play in peer collaboration: First, it is a socio-historic tool. Although new in its manifested form, it is built from the representational forms that have been developed as part of the cultural history of western mathematics. Thus the processes by which the tool is appropriated by the students and made part of their understanding of mathematics is an important issue for investigation.

Second, it is a multi-representational tool. The emphasis on multiple representations stemmed both from the desire to allow for the diversity of actions students might take and our conviction that encouraging students to take actions in a variety of representational forms and to coordinate those actions across representations was an essential part of the process of understanding functions (Confrey, 1990b). In peer collaborative settings, the availability of multiple representations may also facilitate appropriation and negotiation. The role of these multiple representation in peer collaboration also needs close attention.

Finally, the computer screen, mouse, and keyboard, as public mediums of action and representation, play an essential role in group interactions. In each of these capacities, as socio/historic tool, multi-representational tool and as communicative tool, developing a better understanding is essential to understanding peer collaboration. Due to space limitations, however, we have chosen not to make this development a major aspect of this paper. We expect to more fully explore these computer roles in future work.

Results

In the rest of this paper we will examine the work of one group of three students who worked together over a ten-week secondary mathematics course which was centered around problem-solving using Function Probe. Despite the difficulties of obtaining detailed records of group interactions in classroom situations, we chose this setting to increase the 'ecological validity' of the study, that is, we wanted to have a record of student interactions in a situation the students viewed as educationally valid. In addition, we wanted to have this record over a long enough period of time to be able to see change and development in the group. All classes were video-taped and, in addition, each student wore a wireless microphone which broadcast to a separate channel of a multi-channel tape recorder. All three students had completed two years of the NY State Regents mathematics curriculum and were taking the third year of that sequence simultaneously with the problem-solving course. Although all were juniors, none expected to take more mathematics after the current year. The curriculum for the course consisted of a series of linear problems, followed by a series of exponential problems. The first of the linear problems will be discussed in this paper.

Although our goal in the larger research project is to develop a description of the whole group as they progress through this curriculum, for this paper we have chosen to concentrate on one student's progress through a problem. At this point our primary interest is in investigating the explanatory power of the theoretical constructs described above for an individual working in a collaborative setting.

The group consists of two females, Tess and Lora, and one male, Don. We will follow Tess most closely in these episodes. Tess was described by her previous mathematics teacher as a serious, but average student. From our classroom observations, we found her to be conscientious and very concerned about her own understanding. She typically had difficulty with standard forms of mathematics, particularly algebraic representations, but was almost always concerned that she be able to make sense of group activities, often seeming to overtly make group actions into her own. In addition, her descriptions were quite visual, and her ways of forming representations often what we might call procedurally iconic, that is she seemed to want to see in a representation the actions she would imagine taking in resolving the problems situation. More than any of the others, Tess used her hands on the computer screen as a means of 'appropriating' representations to her own problematic.

THE STONE PATH PROBLEM

The following example, takes place over the last 35 minutes of one class period and the initial 15 minutes of the next. Since this was the first problem of the course and we did not get a separate audio tape of the group work on the first day , we do not have a complete of the audio record during certain parts of the first day. However, much of the work from that day was observable from the video tape and will be discussed.

<u>The Stone Path</u>

> Suppose that you are asked by your elderly neighbor, Agnes Wholesum, to lay a stone path from the back door of her house to a bird feeder 37 ft. away in her back yard. A local store sells circular concrete "stones" which are 1 foot in diameter. She asks you to make a plan for the walk so that the shortest distance possible is covered and each stone is "about one step apart" (Agnes is 5'6" tall and 68 years old). However, she leaves it up to you to decide what the distance between stones should be. She does have some preferences however:
> 1) The last stone should touch the feeder
> 2) She doesn't care how far the first stone is from the house as long as it is not farther than one step.

<u>Class 1</u>

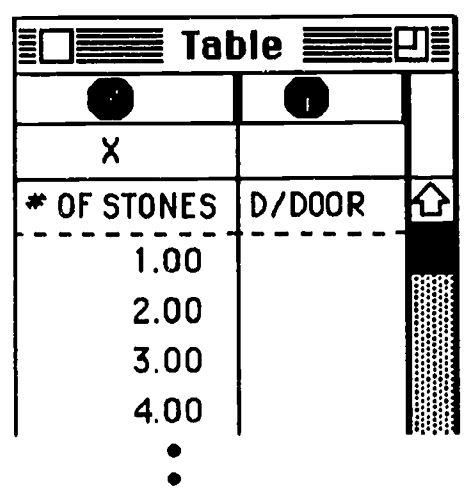


Figure 1

Each group has been asked to be able to present their plan in the form of a table, a graph, and an equation. During the first day, the group decided to place the stones two feet apart (one foot spacing between stones) starting with a one foot space between the door of the house and the first stone. Initially, they open the Table window and label the first two columns as shown in Figure 1. They then use the "Fill" command in the table to have the values 1-19 listed in the first column. This is an operational command in which they indicate the initial value, the final value, and the iterative operation, in this case, 'add 1'. Function Probe would allow the values in the second column, distance from the door, to be added by three methods, typing them one at a time, using the Fill command (adding 2), or by placing an equation in the equation box (next to the variable X in Figure 1). Although Don had originally suggested that they could easily make their table by typing in all the distances, he now proposes that if they enter a formula, the computer will put the values in for them. The possibility of using Fill is never mentioned.

Tess begins: 37 minus.....
David quickly interjects: X-37

Tess turns to David and looks at him incredulously.

Tess: No, 37 - X!

This is accepted by the group and typed into the Table window, producing the table in Figure 2.

Table

X	Y=37-X
# OF STONES	D/DOOR
1.00	36.00
2.00	35.00
3.00	34.00
4.00	33.00
•	•
•	•
•	•
16.00	21.00
17.00	20.00
18.00	19.00
19.00	18.00

Figure 2

We do not know what Tess might have said, had David not interrupted her. In previous discussion, the group has referred to the first stone as the one next to the door and the last (19th) stone being the one next to the feeder. As we have already characterized Tess, she often seems to build representations as action sequences. Thus a plausible suggestion would be that the equation for her represented a series of actions where one started with a 37 foot length, and each stone decreased that length.

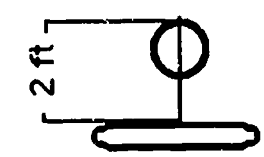
Although the group accepts Tess' equation, they do not, for several minutes, check the validity of the results. Instead, the group initially turns to setting up a third column in the table. After a few minutes, Don states that something is wrong, the first stone is next to the house and cannot be 36 feet from the door. Almost immediately, Tess puts her finger on the computer screen, moving it back and forth between the equation and the table values. This seems to be a situation of cognitive conflict for her and that she is trying to remake for herself the connection between equation and values.

Don first suggests that the equation should be Y=X. After trying some values, Don and Lora say almost simultaneously, and triumphantly, Y=2X. During this process, Tess has not participated.

Don checks out the new conjecture: The first stone is 2 feet, the second stone is four feet. He then starts to type in the new equation. As he is doing so, Tess' hand goes back to the screen, pointing at the three in the first column, and says: No because for the third stone..., oh right, six.

At this point, Don has typed in the equation and the new values appear.

The question we might ask here is whether Tess' initial cognitive conflict has been followed by an appropriation of the equation proposed by Don and Lora. In a local sense, we would say yes, for she first seemed to indicate that the conflict was in her zpd, that is she went to the screen to check out for herself the proposed conflict. Then she again went to the screen in the process of making the new proposed equation meaningful. On the next day, when the group is working on preparing their class presentation, Don suggests that the distance to the first stone should have only been one, because there is only a one foot space between the door and the first stone. Tess quickly makes the drawing below to add to their presentation. This drawing indicates that the 2 feet means the distance to the far side of the stone. There was no apparent discussion in the group of the specific distance being referred to in their table, offering further evidence that Tess appropriated this new equation to her problematic.However if we ask whether she has appropriated an algorithmic concept of an equation to replace the visual/procedural sense that she seems to have expressed earlier, the answer is not so certain. We will see her return to a procedural construction of a function at a later time. However, at this point, as would be expected from students who have gone through a traditional algebra curriculum, functional relationships are strongly equated with equations. Nobody in the group suggested that the table could have been created by Filling the second column (by adding two). The possibility of using a Fill on two separate columns to create a functional relationship is first suggested by David on the next problem. By the time the class reaches the exponential curriculum, this strategy begins to play a major role in their attempts to represent functional relationships.



From this point on, it is difficult to follow the actions of the group in detail. Immediately after entering the equation, as if they have learned from their previous neglect, they decide to scroll to the bottom of the Table window (the bottom values are initially 'below' the screen) to "see if the values came out okay.". After some discussion, they decide that if the last stone is numbered 18.5, rather than 19, that the last distance will be calculated correctly as 37 feet. Although the group has made an interesting adaptation at this point, it is not what might be expected as 'normal' mathematics. Using integers to index a sequence, such as the stones on a path,

Table

X	Y=2X	Z=37-Y
# OF STONES	D/ DOOR	D/FEEDER
1.00	2.00	35.00
2.00	4.00	33.00
3.00	6.00	31.00
4.00	8.00	29.00

Figure 3

precludes the possibility of having a stone #18.5. However, once the equation, Y = 2X, was entered into the table, values in the distance column were calculated automatically. The students did not have the choice of entering a distance of 37 opposite a stone numbered 19.

They then set up the third column for the distance from the feeder (Figure 3). After this, for several minutes they discuss how to get a representation in the graph. They first propose plotting each point by hand using the mouse (FP allows points sets to be entered one at a time through a mouse action on the Graph window). After plotting two points, it is suggested that an equation would be quicker. They type Y=2X into the equation view of the Graph window. The computer will not accept this expression.[3] Finally, they decide to Send the points from the Table window. They make several unsuccessful attempts before being able to carry this out.[4] Eventually they are able to Send the points from the Table window to the Graph window, but in doing so have sent the stone#, X, to the vertical axis and the distance from the door, Y, to the horizontal axis. FP allows students to designate which variable (or column) is to be sent to which axis by dragging the two inverse icons at the top of the Table window (see Figure 3) to the desired columns. By the end of the first day they have sent the points to the graph, and used a Connect Points command to connect the points in the Graph window. The graph is shown in Figure 4[5]. One can see readily that the point representing the last stone (#18.5) is spaced closer to its adjacent point, then are others on the graph. If the group had been able to number that stone as 19 and maintain its distance at 37 feet, the last point would not have 'lined up' with the others.

[3] For pedagogical reasons, certain limitations on the forms of equations were made in the design of the program. We wanted the program to follow conventional forms of representation, yet allow as much flexibility as possible to handle individual preferences. Thus when initially opened, the graph window only allows equations of the form y = (algebraic expression of x), where y and x must be lower case. If students want to use other pairs of variables, a pedagogical decision was made that they be required to designate which variable would go with which axis. They can enter variable pairs in the Rescale box of the Graph window. Also, if points are Sent from the table window. The variables used to label the points in the Table window are automatically allowed in the graph window.

[4] Although the students have previously worked through a tutorial on Function Probe, this is their first use of the program on a problem. Thus they spend some time working through the operations of Function Probe. We suggest, however, that this process of learning the software is not easily separable from the mathematics. Many of the problems encountered by the students had to do with their inconsistency in the use of variables and issues of scale in the graph window. These kinds of questions deserve further investigation, but are beyond the scope of this paper.

[5] The descriptive labels on the graph are for the reader. They do not appear in the graph window of Function Probe

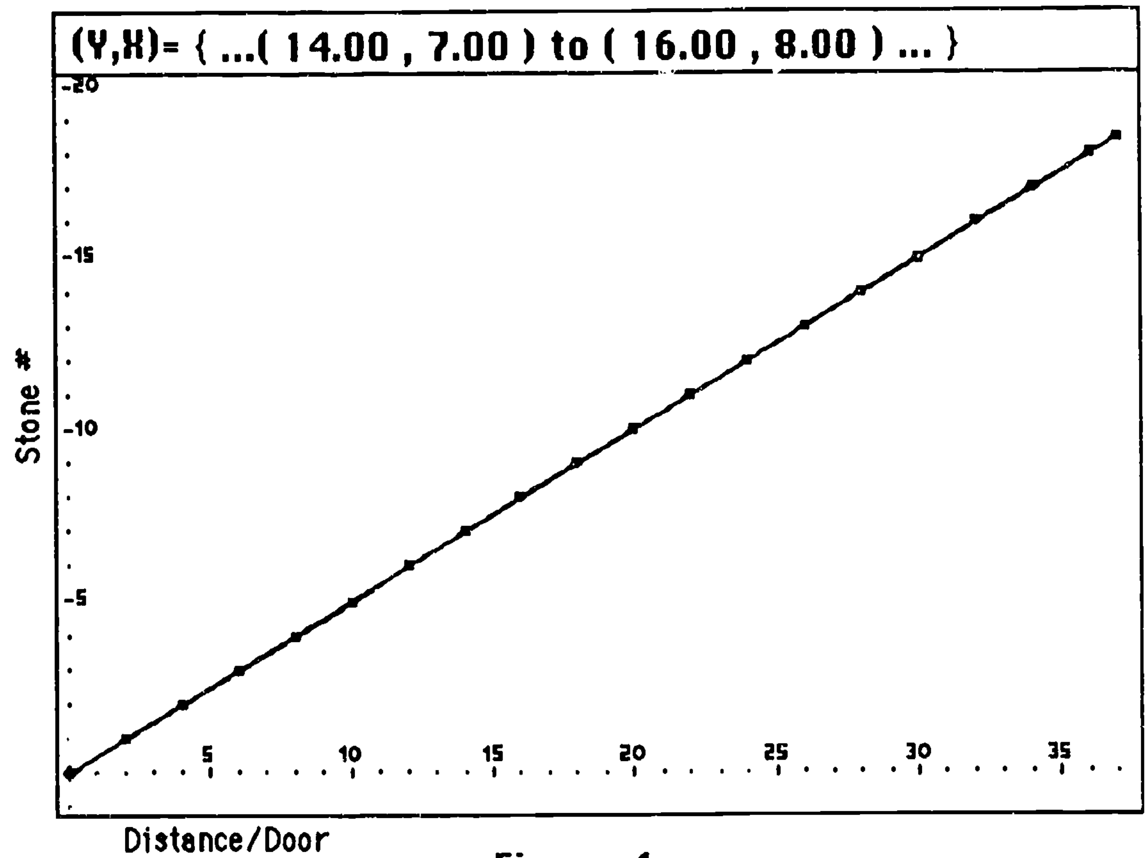


Figure 4

Class 2 (two days later)

At the beginning of day 2, groups are asked to prepare their plans for a public class presentation and be able to describe in each representation (table, graph, equation) how they could tell where any stone should be placed. In the study group, the Graph window is opened, but not the Table window. At this point, there does not seem to be a strong relationship between the table and graph representations. Despite the fact that an equation was used to create the second column in the table, this does not seem immediately important. The discussion starts off:

Lora: We have to write an equation

Don: We have an equation.

Lora: We do?

Don: Like a general equation?

Lora: For when we graphed it, do we have an equation?

Don: I don't know, lets go check.

At this point, Tess steps in with a conjecture:

Tess: Look, for every two, we go up one ... so x= or maybe y = 2x+1

As Tess is describing the graph, she is also running her hand over the graph in what looks like the described pattern (moving it horizontally two units, then vertically one unit). We would again suggest that a reasonable interpretation of her conjecture is descriptive of an action: y = 2x+1 describes an action of going over two and up one, a way of making the graph.

Lora interrupts: No, no, no ...wait a second, x= no no no x = 2y.

Lora and Tess then lean closely together and with their attention focused on the graph the following conversation occurs:

Lora: If x = 2y then if x = 1, I mean if y =1, then x = 2; if y = 2, x = 4 and that's the same. That's what we're doing here.

Because they are leaning together, it is not possible to see what they are doing on the screen. However it looks as if Lora is pointing to points on the graph, almost as if to say to Tess, it's not a pattern that we are describing, it's the values of points. Tess, however, does not readily give up her inclination to look for patterns of change on the graph as a way of describing the equation:

Tess: If y = 2, x = 4?

Lora: No-yeah, y=2...

Tess: 2 is up, 4 is over.

Tess has again appropriated an equation suggested by the others by remaking it in terms of her problematic. However, her problematic also has evolved, for she is now describing the pattern of the graph in terms of the position of points relative to the axis, rather than relative to one another.

Although the equation proposed by Lora is the same one used in the Table window, the Table window has not been mentioned. In addition, although the equation is correct, the Graph window, because of the design of the software, will not accept it in the form x = 2y. AS a default setting, the Graph will accept equations of the form y= f(x). In addition since the group has sent points from the Table, sending X[6] to the y-axis and Y to the x-axis, the Graph will now accept equations of the form $X = f(Y)$. Thus there are two possible equations that could be entered into the Graph window that would produce the desired graph: $y = (1/2)x$ and $X = (1/2)Y$. Of the three students, Tess' is weakest in conventional algebra, and over the next few minutes,, she doesn't participate as Don and Lora together try various combinations of the equation until the agree upon $X = (1/2)Y$. During this period, Don provides a version of the problem from his perspective:

Don: If we type in an equation, it will draw a line for us... So if we type in an equation it will draw this line, but we have a line already. So I think we will have to write down (i.e. off the computer) an equation

(Lora starts to type in an equation)

Don: No, no, no. Don't type it. If you type it up here, it will go nnnnnnnn (running his finger along the line connecting the points) like that.

Tess: That's what we want it to do. (i.e. make the pattern on the table)

(our explanatory comments in parentheses)

At this point there seem to be three distinct problematics. Tess is looking for an equation to match the way she is seeing the patten (or the relationship between points) in the table; Lora is looking for an equation that calculates the numeric values for the points, and Don seems to be saying that the purpose of entering an equation into the Graph window is to create the graph. Since they already have a graph (i.e. connected points) there is no reason to enter an equation in this window. Thus, for him, the current problem of finding an equation is a separate issue from the graph. However, Don acquiesces to Tess' last suggestion (above) and the equation $X = (1/2)Y$ is entered. The result seems to satisfy Don and Lora, but not Tess:

Don: Now it should trace ours ...

Lora: There we go, look. OK, there's our equation.

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[6] Function Probe treats upper case and lower case variables as if they were separate variables. Thus using upper case X makes it a separate variable from lower case x.

Tess: But wait, we want it to stop. It's going to keep going.
(i.e. the graph created by entering the equation keeps going beyond the point representing the last stone.)

Lora: No, it's ok.

Tess: See, it's still going.

Lora: It's ok

Don: Out into infinity

Tess: No, no, no, wait a minute .... we don't want it to go on forever though, we want it to stop at 37!

Don: yeah, um...

For Lora, her problematic was resolved, for the equation calculates values correctly as shown by its passing through the points on the graph. Don would have accepted the graph, although it did not do quite what he had predicted, simply duplicate the line that was already there. For Tess, however, the relationship between the graph and the path is unsatisfactory, for the graph can no longer be made like the path is made. Technically, this is not simply an argument about relating the graph to the formal context of the problem. The only actual values possible for stone number are the numbers, 1, 2, 3, ..., 17, 18, 18.5. Thus, strictly speaking, the graph should consist only of the points, {(2,1),(4,2), ..., (34,17), (36,18), (37, 18.5)}, and not include the connecting line segments. However, if Tess has an inclination to see the graph as an actual representation of the path in a more iconic sense, it is not surprising for her to be satisfied seeing a continuous line, yet want it to stop at 37.

Tess continues: How do we stop it though? Could we put like a less than sign?

After this, for the next ten minutes the group attempts to find a way to make the graph stop a: `7, initially entering a form proposed by Lora: $X = (1/2)Y < 38$[7]. Several unsuccessful attempts are made on variations of this algebraic form. Then:

Tess: What we could do is shorten up our graph so it only goes to 37... Then it would have to stop.

This suggests that part of the issue for Tess was simply visual, i.e. that she wanted the picture on the graph to correspond with her image of the path. To suggest rescaling, however, is also evidence that she is appropriating the software tool into her conceptions of the mathematics. She is visualizing the way she can use this particular tool to create her mathematical representation in a way one typically would not do with pape. and pencil. Don, however, is not happy with this solution:

Don: Well, it just looks like it stops....I'm still not happy with that, there's got to be some way to stop it.

There are two points of interest here: one is the extent to which Don has now appropriated Tess' problem to his own; the second is the contrasting, but seemingly compatible ways in which Tess and Don are viewing the graphing action. Don has now twice expressed a view of the graph as extending to infinity. At another point, he says to Tess: "It will still keep going to infinity..." This does not, however, appear to be a static image of the graph extending indefinitely in both directions. Function Probe draws graph from left to right. Don seems to imagine this action continuing. Thus the graph is not an infinite object, rather an object being continuously constructed from left to right. For Tess, on the other hand, the graph seems to be more iconic and made through the actions she can see on the screen. Equations in graphs and tables are ways to describe how to carry out actions, rather than a relationship of points.

Thus, in response to Don's dissatisfaction with rescaling as a solution to their problem, Tess suggests another action-oriented scheme. If one imagines that graphing is a set of actions which take place within the confines of the Graph window, then it seems one ought

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[7] Function Probe does allow Boolean logic expressions. Thus by entering: '$X =$ if $Y \leq 37$ then $(1/2)Y$', the graph would have stopped at 37. This is not an expression that students would likely find through experimentation.

to be able to control those action by setting up a relational description of the axes. This is what Tess seems to suggest:

Tess: Wait, wait, wait, listen, listen. If we do like x= and then say ... like x= x+10 in parentheses - minus 10. (x=(x+10)-10)......

Then she suggests that for this to work the graph must be rescaled to 47. If one imagines a procedural construction of an equation in which operations inside a parentheses are always carried out first, one could certainly imagine that Tess is suggesting that adding ten to x inside the parentheses will 'make it bigger'. At some point, the computer will set x equal to 37. Then it will add 10, making it 47. Since 47 is the edge of the graph, this will cause it to stop graphing. Finally by subtracting 10, the end of the graph will be plotted correctly, an interpretation which seems consistent consistent with Tess' previous conjectures.

Eventually, being unable to make the graph stop at 37, the group agrees upon Don's original conjecture that since they already have a graph, the equation does not belong in the Graph window at all. Thus they decide not to put an equation in the graph window. At this point, the x-axis is scaled from 0 to 40, the y-axis from 0 to 20. Tess again emphasizes the visual:

Tess: Shouldn't we have put this (the y-axis) up to 40 so we can see our scale *as it is*... Let's just try it, put that up at 40. ...(she rescales so that both axes are scaled from 0 to 40) ...Now it should redo our line too.... Wow, check that out!

Finally, Tess once more uses action and pattern to check the equation as the group attempts to remake the equation off the graph:

Lora: Y=2X ...[8]
x=2y

Tess: X = 1/2 Y

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[8] When students are talking, we, of course, have no way to distinguish between upper and lower case. While using the keyboard, they have always had the 'caps lock' on, thus all variables entered have been upper case. As previously mentioned, the have assigned the variable X to the y-axis and the variable Y to the x-axis. However, in referring to the graph window, they often refer to the x-axis and the y-axis. Thus we have written this last interchange in such a way that every conjecture put forth by Lora and Tess is correct, providing one interprets x vs X and y vs Y correctly. We do this to help clarify for the reader, not to necessarily indicate that this conveys the intentions of the students.

Lora:	yeah, X = 1/2 Y
Tess:	yeah
Lora:	No! x = 2y
Tess:	no, no because for every one y you go up, you're going over 2 x; for every one y, 2 x; for every one y ... X = 1/2 Y
Lora	2y cause
Tess:	2y you're right, you're right, it is 2y.

The nearly parallel structure Tess has in describing the graph at the beginning of the second day (for every two we go up one) and here (for every one y you go up, you're going over 2 x) is striking and yet she reaches quite different conclusions each time ($y = 2x+1$ vs. $x = 1/2\ y$). Tess obviously struggles with conventional algebraic expressions and this suggestion of change in a scheme for describing a graph could be a meaningful case of appropriation which will be important for her. Although we can, in this problem, see this change as 'progress', for it seems more directly related to the needed equation - i.e. she transforms her description into the desired equation, we should note that this strategy, as it is, will only work in cases of direct variation.

Summary

The theoretical constructs developed at the beginning of this paper were based upon the complementarity of two approaches to learning, constructivism and Vygotskyism. Constructivism emphasizes the individual problematic as a starting point for understanding learning while Vygotskyism emphasizes interactions with a more knowledgeable cultural representative. The question we asked was this: In trying to understand peer collaboration, can we appropriate those Vygotskian concepts which seem to capture important aspects of interaction to expand the constructivist model?

We believe that a potential strength of peer collaboration is the space it provides, due to the lack of an authority role, for an individual to maintain her problematic while at the same time developing and resolving it through interactions with others. To exemplify this we focused on one student working with two others through a contextual problem. From this example, we suggest the following:

1) Within the group, Tess had a unique way of understanding and making sense of the problem which was visual, oriented to her way of describing pattern, and often related to the actions she imagined in the contextual situation. She maintained these characteristics of her problematic throughout the problem-solving episode.

2) Tess was vigilant in maintaining her zone of proximal development in relation to the others. We suggested earlier that viewing the zpd as an active construction of the learner rather than as a property of the learner could be an important aspect of understanding peer collaboration. Tess could have easily allowed the actions of the others, particularly their algebraic manipulations, to take place outside her zpd had she not continually involved herself in remaking their actions in relation to her own problematic.

3) Although the actions proposed by Tess, particularly those expressed algebraically, were often based on strong insights into the problem, more often than not they were not productive for her in terms of producing the actions/representations she desired. However, because of her active maintenance of her zpd she was able to reconceptualize through cognitive conflict and to appropriate from others in such a way that she remade the actions of others into actions for herself. This was not a passive activity but an active constructive process.

Although there is much that can be interpreted positively in Tess' experience during this episode, we do not want to neglect the problems that can arise. Two issues that might have been explored more fully under the direction of a teacher/expert are the use of the number 18.5 as a stone number and the use of variables. The students seemed to have an adequate understanding of their use of 18.5, but it did not seem clear that they ever straightened out their mixed use of variables in a strong way. During their public class presentation, they labelled the equation for the table (which they put on the blackboard) as $Y = 2X$. They labelled their equation for the graph as $X = 2Y$. Although they could give quite strong interpretations for the relationship of each equation to its connected representation, they treated the different forms of the equation as unimportant, indicating they had not made a strong connection between the multiple representations of the problem.

As mentioned earlier, we expect to see trade-offs when comparing peer collaboration to interactions with a more knowledgeable other. Thus the inconsistent use of variables, neglect of connections between representations, and only tentative understanding of the distinction between the graph and an iconic representation of the path could have been explored and developed under the probing and guidance of an expert. On the other hand, Tess worked very hard to maintain her ownership of this problem, i.e. to continually remake the actions of the group into actions within her zone of proximal development. Also she appeared to appropriate the actions of others successfully. Thus an essential theoretical question would be: How different would this learning experience have been for her had she worked on the problem with an acknowledged teacher/expert?

Although such a question can never be answered empirically, we hope the development of models such as ours in conjunction with detailed case studies will help us obtain a better understanding of how these processes differ, leading to a better understanding of how successful peer collaborative learning situations may be created in secondary classrooms.

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