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ABSTRACT

The extent to which standardized regression coefficients (beta values) can be used to determine the importance of a variable in an equation was explored. The beta value and the part correlation coefficient--also called the semi-partial correlation coefficient and reported in squared form as the incremental "r squared"--were compared for variables in 2,341 two-predictor equations and 8,670 three-predictor equations to examine the information they provided for evaluating variable importance. A subset of 1,316 two-predictor equations lacking suppression and a subset of 1,127 three-predictor equations lacking suppression were also examined. Results show that beta values can be used for interpreting the importance of predictors within an equation, but the interpretation is complex. Caution is required for three or more predictors. It is contended that when evaluating the importance of a variable, it is not wise to use the beta value alone. Thirty-two tables present the results of the analyses. (SLD)

Interpretation of Standardized Regression Coefficients in Multiple Regression

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Standardized regression coefficients (β's) are one of the most frequently reported summary statistics used with multiple regression. β's are usually interpreted in one of two ways. The most direct interpretation of β is the amount of change that occurs in the predicted value of the dependent variable as a result of a change in an independent variable, assuming the other independent variables remain constant, with the changes expressed in standardized form. This interpretation is accepted as a valid use of β.

A second common use of β is to determine the importance of each of the variables in a regression equation. This interpretation is subject to frequent criticism. The major purpose in this study is to explore the extent to which β values can be used to determine the importance of a variable in an equation.

There are a number of factors that will not be considered in this study that could be dealt with in evaluating β's. Pedhazur (1982) suggests consideration of whether experimental or nonexperimental research was used, the degree of specification and measurement errors and the presence of multicollinearity. These factors will not be addressed here, only interpretations after they have been considered.

It is well known that β's are more influenced by the variability of the variables in the model than are the raw score coefficients (b's). For this reason b is preferred over β by many as an indicator of the "effect" of a variable. To eliminate the influence of variability, this study only used standardized data. The value of β as an indicator of "effect" is not addressed.

Definition of importance

The importance of a variable as a predictor can be viewed in two ways: absolute importance and relative importance.

Absolute importance is comparing β values across equations. If a specified variable had β's of .5 and .7 in two equations, if absolute importance was a valid comparison the variable could be considered to be a better predictor in the second equation.

Relative importance is comparing β values within an equation. If two variables had β values of .5 and .7 in the same equation, if relative importance was a valid comparison the second variable could be considered to be a better predictor in the equation. This study will investigate whether "absolute" or "relative" interpretations of importance are valid when using β values.

As Pedhazur (1982) explains, "the relative importance of the independent variables . . is an extremely complex topic" (p. 63). In this study the number of variables in the equation, the intercorrelations between the predictors, and the correlation of the predictors with the dependent variable will be considered in trying to determine correct uses of β.

Whichever criterion is used to measure importance, importance is relative to the number of predictors in the equation. A variable might be the most important single predictor of a dependent variable when used alone but an unimportant predictor when used in combination with other predictors due to the amount of shared predicted variance.

Regression Statistics to Use To Evaluate Importance

There are six numbers that are routinely reported with regression equations that can be used as indicators of importance in an equation. Table 1 shows a portion of a SPSS Multiple Regression printout for a three predictor equation which gives these six numbers.

Table 1

SPSS Multiple Regression Output

Dependent Variable Y

Multiple R .96709
R Square .93527

------------------------------ Variables in the Equation ------------------------------

Variable	B	Beta	Part Cor	Partial	T	Sig T
X3	-.00268	-.12976	-.04806	-.18560	-.463	.6599
X2	.24233	.15335	.10218	.37266	.984	.3633
X1	.68225	.72551	.25564	.70878	2.461	.0490
(Constant)	4.69636				.604	.5680

B is the raw score regression coefficient which should not be used to evaluate importance since it is so strongly influenced by the standard deviation of the predictor (Pedhazur, 1982, p. 64).

The numbers under the headings "Beta", "Part Cor" and "Partial" are the standardized regression coefficients, part correlations, and partial correlations.

The part correlation coefficient is also called the semi-partial correlation coefficient. It is usually reported in regression analysis in squared form as an incremental r^2, which is the increase in the multiple R^2 due to the variable in question if entered last into the equation. This is sometimes called the increase or change in R^2, contribution to R^2, or unique contribution to R^2. It is equivalent to the amount by which the R^2 would decrease if the variable was removed from the equation. In this study it will usually be referred to as the incremental r^2.

The partial correlation is usually reported in regression analysis in the unsquared form. When squared this is the percentage of the remaining variance of the dependent variable not predicted by the other variables that is predicted by the specified independent variable.

T (t) and Sig T (p value) provide the same information as the incremental r^2 for evaluating importance. The incremental r^2 values for the predictors are proportional to the t values since each incremental r^2 can be converted to an F value (t^2) using the following formula.

$$F = \frac{R^2_{Full} - R^2_{Restricted}}{(1 - R^2_{Full}) / (N - k_{Full} - 1)}$$

Since the denominator in the formula is constant for all predictors in an equation, the incremental r^2 (the numerator) is proportional to the F (t^2) value and the probability associated with it.

The three statistics dealt with in this study are: the standardized regression coefficient (β), the partial correlation coefficient and the incremental r^2.

The notation for the statistics used will be as follows:

zero-order correlation coefficient between Y and predictor 1:	r_{Y1}
intercorrelation between predictor 1 and predictor 2:	r_{12}
standardized regression coefficient for predictor 1:	β_1
multiple correlation coefficient with three predictors:	$R_{Y.123}$
partial correlation coefficient for predictor 1:	r_{Par1}
incremental r^2 for predictor 1:	r^2_{Inc1}

In order to make the comparisons between zero-order correlation coefficients, standardized regression coefficients and incremental r^2 easier, the squared values of each will usually be used.

Partial correlation coefficients are not good statistics to use for determining importance. Their value is more helpful in evaluating the significance of the variable (the degree to which the relationship can be considered to be due to chance) If $r^2_{Y1} = .999$, $r^2_{Y2} = .001$, and $r_{12} = .00$, then $R^2_{Y.12} = 1.00$ and $r^2_{Par1} = r^2_{Par2} = 1.00$. The 1.00 partials would indicate that both variables are extremely good (perfect) predictors, which would be true to the extent that each predicts perfectly the variance that the other does not predict. But the two variables are definitely not equally important in this equation. The variable that explains 99.9% of the variance is more important than the one that explains .1% of the variance, especially since they are not correlated with each other. In this case $\beta^2{}_1 = .999$ and $\beta^2{}_2 = .001$ (both the same as the zero-order correlations) which would be the true importance of the variables.

β and r^2_{Inc} are the two best statistics to use as indicators of importance. β is probably the best single statistic but the interpretation of either statistic is so complex that they should probably not be used alone and if used appropriate caution is necessary. Concerning this situation, Pedhazur (1982) states that "your sense of frustration at the lack of definitive answers to questions about the relative importance of variables is not difficult to imagine. . . it will become evident that there is more than one answer to such questions, and that the ambiguity of some situations is not entirely resolvable" (p. 65).

Procedures

The major technique used in this study is to compare the β and r^2_{Inc} for variables in two and three predictor equations to examine the information they convey for evaluating variable importance. Statistics were computed for a large number of combinations of correlations. All possible different two-predictor equations were computed varying r_{12} from .00 to 1.00 in multiples of .04 and varying r_{Y1} and r_{Y2} from .00 to 1.00 in multiples of .10. A total of 2,341 two-predictor equations were run. A subset of 1,316 of these equations in which there was no suppression were also examined. Suppression was defined as occurring for any equation that had β's of the opposite sign from or greater absolute value than the corresponding zero-order correlation coefficients.

All possible different three-predictor equations were computed varying r_{23} from -.90 to +.90 in increments of .10 and using values of -.90, -.50, -.20, .00, +.20, +.50, and +.90 for r_{12}, r_{13}, r_{Y1}, r_{Y2}, and r_{Y3}. A total of 8,670 three-predictor equations were run. A subset of 1,127 of these equations in which there was no suppression was also examined.

All analyses were done using standardized data. Pedhazur (1982) states that r varies as a function of the variability of X while the raw score coefficient (b) remains constant. Since differences in variability with the predictors affect correlation coefficients and consequently all statistics associated with it, standardized data was used for all comparisons.

Importance will only be considered with a constant number of predictors. There will be separate sections for one, two, and three predictors.

Importance of β in One-Predictor Equations

"Absolute" importance

When evaluating many variables as potential single predictors, the variable with the highest correlation coefficient with the dependent variable is considered to be the best predictor. Since in a one-predictor equation, β is equal to the zero-order correlation coefficient, β can be interpreted directly as indicating the importance of the variable as a single predictor. Comparing β's between equations as indicators of importance is as valid as comparing zero-order correlation coefficients between variables.

In a one predictor equation the zero-order correlation coefficient, β, partial coefficient, and semi-partial coefficient are all equal and thus equally good as measures of importance.

"Relative" importance

Since relative importance compares variables within the same equation there can be no relative importance in a one predictor equation.

Importance of β in Two-Predictor Equations

"Absolute" importance

r^2_{Inc} values can range from .00 to 1.00. Since β's can take values below -1.00 and above +1.00 as a result of suppression, β^2 values range from .00 to >1.00. Since there is no constant upper limit for β values, you cannot make "absolute" interpretations of β's values. You cannot say, for example that .8 is a high β, 1.5 very high, and 2.5 extremely high.

For example in the two situations below, predictor two is much better in equation one than in equation two. Predictor two explains all of the variance of Y in equation one while the two predictors together only predict 54.1% of the variance of Y in equation two. The fact that β_2 is much larger in equation two than equation one is exactly opposite to the true "absolute" importance of predictor two in the two equations.

r_{Y1}	r_{Y2}	r_{12}	$R_{Y.12}$	β_2
.00	1.00	.00	1.000	1.000
.10	.30	.96	.541	2.602

In evaluating how β and r^2_{Inc} are related, correlations between these two statistics (plus the squared partial correlation for comparison) were computed for the total sample of 2,341 equations and the non-suppression sample of 1,316 equations. Table 2 shows the correlations between the three statistics used for determining importance to be evaluated: r^2_{Inc}, r^2_{Par}, and β^2. Statistics for both the first and second predictors are presented.

The correlations between β^2 and r^2_{Inc} for the 2,341 equations were .6389 for predictor one and .5017 for predictor two, indicating large differences between the two statistics. In examining the specific cases the largest differences occurred when suppression was present since β values can range much larger than 1.00 while r^2_{Inc} cannot exceed 1.00. Removing the equations in which suppression existed increased the correlations to .9452 for predictor one and .9529 for predictor two showing a close but not perfect relationship.

Table 2

Correlations Between Importance Statistics -- Two Predictors

All Equations

	r^2_{Inc1}	r^2_{Par1}	β^2_1
r^2_{Inc1}	1.0000		
r^2_{Par1}	.8497	1.0000	
β^2_1	.6389	.4403	1.0000

	r^2_{Inc2}	r^2_{Par2}	β^2_2
r^2_{Inc2}	1.0000		
r^2_{Par2}	.9303	1.0000	
β^2_2	.5017	.4813	1.0000

Equations Without Suppression

	r^2_{Inc1}	r^2_{Par1}	β^2_1
r^2_{Inc1}	1.0000		
r^2_{Par1}	.8704	1.0000	
β^2_1	.9452	.8627	1.0000

	r^2_{Inc2}	r^2_{Par2}	β^2_2
r^2_{Inc2}	1.0000		
r^2_{Par2}	.9143	1.0000	
β^2_2	.9529	.9256	1.0000

For the 1,316 cases without suppression, in every case β^2 was equal to or larger than the corresponding incremental r^2, with the maximum difference being .198. The differences were larger when there were higher intercorrelations between the two predictors and higher correlations between the independent variables and the dependent variable. The 15 largest differences for predictor one are reported in Table 3.

Table 3

Largest Differences Between β^2_1 and r^2_{Inc1} Without Suppression

$\beta^2_1 - r^2_{Inc1}$	β^2_1	r^2_{Inc1}	r_{12}	r_{Y1}	r_{Y2}
.198	.207	.008	.980	.900	.900
.194	.211	.017	.960	.900	.900
.190	.215	.025	.940	.900	.900
.186	.220	.034	.920	.900	.900
.182	.224	.043	.900	.900	.900
.177	.229	.052	.880	.900	.900
.173	.234	.061	.860	.900	.900
.169	.239	.070	.840	.900	.900
.164	.245	.080	.820	.900	.900
.160	.250	.090	.800	.900	.900
.157	.163	.006	.980	.800	.800
.156	.256	.100	.780	.900	.900
.154	.167	.013	.960	.800	.800
.151	.261	.110	.760	.900	.900
.150	.170	.020	.940	.800	.800

The largest differences in Table 3 were due to low r^2_{Inc1} values caused primarily by the fact that predictor two explained most of the variance (high r_{Y2}). In cases of high r_{12} which were also found in the examples in Table 3, β is a better indicator of importance than r^2_{Inc} since any variance of Y that is predicted by both independent variables is not included in either of the two incremental r^2s. If $r_{Y1} = r_{Y2} = .90$ and $r_{12} = .98$, $\beta_1 = \beta_2 = .455$ while $r^2_{Inc1} = r^2_{Inc2} = .01$. .01 indicates that both variables are poor predictors, while .455 indicates more properly that they are good predictors.

If both r_{12} and r_{Y1} were below .70, the maximum difference was .059. The largest of these differences are reported in Table 4.

Table 4

Largest Differences Between β^2_1 and r^2_{Inc1} Without Suppression
When r_{12} and $r_{Y1} < .70$

$\beta^2_1 - r^2_{Inc1}$	β^2_1	r^2_{Inc1}	r_{12}	r_{Y1}	r_{Y2}
.059	.128	.069	.680	.600	.600
.057	.131	.074	.660	.600	.600
.055	.134	.079	.640	.600	.600
.053	.137	.084	.620	.600	.600
.051	.141	.090	.600	.600	.600
.049	.144	.096	.580	.600	.600
.046	.148	.102	.560	.600	.600
.044	.152	.108	.540	.600	.600
.042	.156	.114	.520	.600	.600
.041	.089	.048	.680	.500	.500
.040	.160	.120	.500	.600	.600
.040	.091	.051	.660	.500	.500
.038	.093	.055	.640	.500	.500
.038	.164	.126	.480	.600	.600
.037	.095	.0[illegible]9	.620	.500	.500

With no intercorrelation between the predictors, β^2 and r^2_{Inc} are equal, no matter what the values of r_{Y1} or r_{Y2}. As r_{12}, r_{Y1} and r_{Y2} increase, the size of the difference between β^2_1 and r^2_{Inc1} increases.

As shown in Table 2, β^2 correlates better with r^2_{Inc} than with r^2_{Par}. With suppression cases removed, both correlations are quite high. When there is no suppression with β's remaining below 1.00, β's and partials are usually quite close except when one variable predicts most of the variance and then the r^2_{Par} for the second variable may become very large if it predicts most of the small remaining variance and the β for the second variable will be quite small. For example when $r_{Y1} = .40$, $r_{Y2} = .90$ and $r_{12} = .00$, predictor two explains 81%

of the variance of Y and predictor one explains 16% of the 19% remaining variance giving $r_{Par1} = .918$ ($r^2_{Par1} = 16/19 = .84$), while $\beta_1 = .4$ which is equal to r_{Y1} since $r_{12} = .00$. Here again, β is a good indicator of importance while the partial correlation is not.

Comparisons of situations where $\beta^2{}_1$ and r^2_{Par1} are most different in equations with no suppression are presented in Tables 5 and 6. The largest differences between $\beta^2{}_1$ and r^2_{Par1} are found when r^2_{Par} is larger than β. These are reported in Table 5. The conditions causing most of these large differences are a large $R^2_{Y.12}$, a large r_{Y2}, and a not-so-large r_{Y1}. Because the two variables together predict almost all of the variance (large $R^2_{Y.12}$), either predictor will explain most of the variance in addition to the other variable, therefore giving high r^2_{Par}'s. The large β_2 and smaller β_1 on the other hand more closely reflect the actual contribution of the variables to the $R^2_{Y.12}$.

Table 5

Largest Differences Between $\beta^2{}_1$ and r^2_{Par1} Without Suppression and $r^2_{Par1} > \beta^2{}_1$

$r^2_{Par1} - \beta^2{}_1$	r^2_{Par1}	r^2_{Par2}	$\beta^2{}_1$	$\beta^2{}_2$	r_{12}	r_{Y1}	r_{Y2}
.785	.970	.992	.186	.749	.080	.500	.900
.785	.995	.998	.211	.567	.320	.700	.900
.776	.967	.990	.191	.660	.200	.600	.900
.755	.995	.997	.240	.455	.460	.800	.900
.725	.924	.972	.198	.560	.340	.700	.900
.722	.894	.973	.172	.737	.100	.500	.900
.715	.894	.968	.178	.651	.220	.600	.900
.697	.926	.961	.229	.450	.480	.800	.900
.682	.842	.964	.160	.810	.000	.400	.900
.668	.855	.946	.187	.554	.360	.700	.900
.662	.821	.955	.158	.726	.120	.500	.900
.657	.824	.948	.166	.644	.240	.600	.900
.642	.860	.926	.218	.444	.500	.800	.900
.635	.936	.936	.301	.301	.640	.900	.900
.622	.768	.948	.146	.796	.020	.400	.900

The largest differences between $\beta^2{}_1$ and r^2_{Par1} when β is larger than r^2_{Par} are reported in Table 6. When there is no suppression partial correlations are usually larger than β's (as illustrated by the extreme values in Table 5) except when there is a high correlation between the two predictors as in Table 6. The high intercorrelation may produce extremely small partials while the β's can be quite a bit larger. Here again the β's give a better reflection of the actual importance of the variables.

Table 6

Largest Differences Between $\beta^2{}_1$ and r^2_{Par1} Without Suppression and $\beta^2{}_1 > r^2_{Par1}$

$\beta^2{}_1 - r^2_{Par1}$	r^2_{Par1}	r^2_{Par2}	$\beta^2{}_1$	$\beta^2{}_2$	r_{12}	r_{Y1}	r_{Y2}
.164	.043	.043	.207	.207	.980	.900	.900
.145	.018	.018	.163	.163	.980	.800	.800
.130	.036	.036	.167	.167	.960	.800	.800
.124	.087	.087	.211	.211	.960	.900	.900
.115	.010	.010	.125	.125	.980	.700	.700
.115	.055	.055	.170	[illegible]	.940	.800	.800
.108	.020	.020	.128	[illegible]	.960	.700	.700
.100	.030	.030	.130	.130	.940	.700	.700
.100	.074	.074	.174	.174	.920	.800	.800
.093	.040	.040	.133	.133	.920	.700	.700
.086	.006	.006	[illegible]	.092	.980	.600	.600
.085	.051	.051	.136	.136	.900	.700	.700
.084	.094	.094	.177	.177	.900	.800	.800
.083	.132	.132	.215	.215	.940	.900	.900
.082	.011	.011	.094	.094	.960	.600	.600

"Relative" importance

With two predictors the two β^2's and r^2_{Inc}'sare proportional to each other. The ratio of the two β^2's is equal to the ratio of the two r^2_{Inc}'s. You get the same information concerning the relative importance of the predictors in a two predictor equation by examining the β's or the contribution to the R^2.

$$\beta^2{}_1 / \beta^2{}_2 = r^2_{Inc1} / r^2_{Inc2}$$

It can be seen in Table 7 that the correlation between the beta and incremental ratios is 1.00 for all equations with or without suppression. The ratio of the two partial correlations is not perfectly correlated with either of the other ratios.

Tables 8 and 9 show how varying r_{Y1} and r_{12} while keeping the other correlations constant affects the β and r^2_{Inc} values differently but the ratios remain equal. Table 10 is a random sample of 50 equations from the 2,341 equations used. It can be seen that the ratios are equal for all of the equations. This relationship is not effected by the presence of suppression. This can be seen, for example, in Table 9, where all of the equations with $r_{12} > .32$ show suppression (β_1 is the opposite sign from r_{Y1}) and the relationship holds.

Table 7

Correlations Between Statistical Ratios

All Equations

	$\beta^2{}_1 / \beta^2{}_2$	r^2_{Inc1}/r^2_{Inc2}	r^2_{Par1}/r^2_{Par2}
$\beta^2{}_1/\beta^2{}_2$	1.0000		
r^2_{Inc1}/r^2_{Inc2}	1.0000	1.0000	
r^2_{Par1}/r^2_{Par2}	.9383	.9383	1.0000

Equations Without Suppression

	$\beta^2{}_1 / \beta^2{}_2$	r^2_{Inc1}/r^2_{Inc2}	r^2_{Par1}/r^2_{Par2}
$\beta^2{}_1/\beta^2{}_2$	1.0000		
r^2_{Inc1}/r^2_{Inc2}	1.0000	1.0000	
r^2_{Par1}/r^2_{Par2}	.9604	.9604	1.0000

As shown in the following example, it is possible to have two different equations with equivalent R^2 values and two predictors of equal relative importance (similar $|\beta|$ values) within the same equation but radically different β values across the two equations. This indicates that interpreting β values in a relative way with two predictors is not affected by the lack of ability to deal with absolute importance.

r_{12}	r_{Y1}	r_{Y2}	β_1	β_2	$R^2_{Y.12}$
.95	.20	40	-1.046	+2.154	.492
.55	.60	.65	.348	.459	.507

Table 8

Effect of Changing r_{Y1} With Constant r_{12} and r_{Y2}

r_{12}	r_{Y1}	r_{Y2}	β_1	β_2	$\beta^2{}_1$	$\beta^2{}_2$	r^2_{Inc1}	r^2_{Inc2}	$\beta^2{}_1/\beta^2{}_2$	r^2_{Inc1}/r^2_{Inc2}
.400	.000	.600	-.286	.714	.082	.510	.069	.429	.160	.160
.400	.100	.600	-.167	.667	.028	.444	.023	.373	.063	.062
.400	.200	.600	-.048	.619	.002	.383	.002	.322	.006	.006
.400	.300	.600	.071	.571	.005	.327	.004	.274	.016	.016
.400	.400	.600	.190	.524	.036	.274	.030	.230	.132	.132
.400	.500	.600	.310	.476	.096	.227	.080	.190	.422	.422
.400	.600	.600	.429	.429	.184	.184	.154	.154	1.000	1.000

Table 9

Effect of Changing r_{12} With Constant r_{Y1} and r_{Y2}

r_{12}	r_{Y1}	r_{Y2}	β_1	β_2	β'_1	β'_2	r'_{Inc1}	r'_{Inc2}	$\beta'_1\beta/'_2$	r'_{Inc1}/r'_{Inc2}
.000	.100	.300	.100	.300	.010	.090	.010	.090	.111	.111
.020	.100	.300	.094	.298	.009	.089	.009	.089	.100	.100
.040	.100	.300	.088	.296	.008	.088	.008	.088	.088	.088
.060	.100	.300	.082	.295	.007	.087	.007	.087	.078	.078
.080	.100	.300	.076	.294	.006	.086	.006	.086	.068	.068
.100	.100	.300	.071	.293	.005	.086	.005	.085	.058	.058
.120	.100	.300	.065	.292	.004	.085	.004	.084	.049	.049
.140	.100	.300	.059	.292	.003	.085	.003	.083	.041	.041
.160	.100	.300	.053	.291	.003	.085	.003	.083	.034	.034
.180	.100	.300	.048	.291	.002	.085	.002	.082	.027	.027
.200	.100	.300	.042	.292	.002	.085	.002	.082	.020	.020
.220	.100	.300	.036	.292	.001	.085	.001	.081	.015	.015
.240	.100	.300	.030	.293	.001	.086	.001	.081	.010	.010
.260	.100	.300	.024	.294	.001	.086	.001	.081	.006	.006
.280	.100	.300	.017	.295	.000	.087	.000	.080	.003	.003
.300	.100	.300	.011	.297	.000	.088	.000	.080	.001	.001
.320	.100	.300	.004	.299	.000	.089	.000	.080	.000	.000
.340	.100	.300	-.002	.301	.000	.090	.000	.080	.000	.000
.360	.100	.300	-.009	.303	.000	.092	.000	.080	.001	.001
.380	.100	.300	-.016	.306	.000	.094	.000	.080	.003	.003
.400	.100	.300	-.024	.310	.001	.096	.000	.080	.006	.006
.420	.100	.300	-.032	.313	.001	.098	.001	.081	.010	.010
.440	.100	.300	-.040	.317	.002	.101	.001	.081	.016	.016
.460	.100	.300	-.048	.322	.002	.104	.002	.082	.022	.022
.480	.100	.300	-.057	.327	.003	.107	.003	.083	.030	.030
.500	.100	.300	-.067	.333	.004	.111	.003	.083	.040	.040
.520	.100	.300	-.077	.340	.006	.116	.004	.084	.051	.051
.540	.100	.300	-.088	.347	.008	.121	.005	.085	.064	.064
.560	.100	.300	-.099	.355	.010	.126	.007	.087	.078	.078
.580	.100	.300	-.112	.365	.012	.133	.008	.088	.094	.094
.600	.100	.300	-.125	.375	.016	.141	.010	.090	.111	.111
.620	.100	.300	-.140	.387	.020	.149	.012	.092	.131	.131
.640	.100	.300	-.156	.400	.024	.160	.014	.094	.152	.152
.660	.100	.300	-.174	.415	.030	.172	.017	.097	.175	.175
.680	.100	.300	-.193	.432	.037	.186	.020	.100	.201	.201
.700	.100	.300	-.216	.451	.047	.203	.024	.104	.229	.229
.720	.100	.300	-.241	.473	.058	.224	.028	.108	.259	.259
.740	.100	.300	-.270	.500	.073	.250	.033	.113	.291	.291
.760	.100	.300	-.303	.530	.092	.281	.039	.119	.327	.327
.780	.100	.300	-.342	.567	.117	.321	.046	.126	.364	.364
.800	.100	.300	-.389	.611	.151	.373	.054	.134	.405	.405
.820	.100	.300	-.446	.665	.199	.443	.065	.145	.449	.449
.840	.100	.300	-.516	.734	.267	.538	.078	.158	.495	.495
.860	.100	.300	-.607	.822	.368	.675	.096	.176	.545	.545
.880	.100	.300	-.727	.940	.528	.883	.119	.199	.598	.598
.900	.100	.300	-.895	1.105	.801	1.222	.152	.232	.655	.655
.920	.100	.300	-1.15	1.354	1.313	1.834	.202	.282	.716	.716
.940	.100	.300	-1.56	1.770	2.445	3.132	.285	.365	.781	.781
.960	.100	.300	-2.40	2.602	5.750	6.771	.451	.531	.849	.849

Table 10

Equal β and r^2_{Inc1} Ratios From a Random Sample of 50 of 2,341 Equations

r_{12}	r_{Y1}	r_{Y2}	β_1	β_2	β'_1	β'_2	r'_{Inc1}	r'_{Inc2}	β'_1/β'_2	r'_{Inc1}/r'_{Inc2}
.300	.000	.200	-.066	.220	.004	.048	.004	.044	.090	.090
.220	.000	.300	-.069	.315	.005	.099	.005	.095	.048	.048
.400	.000	.400	-.190	.476	.036	.227	.030	.190	.160	.160
.120	.000	.600	-.073	.609	.005	.371	.005	.365	.014	.014
.280	.000	.600	-.182	.651	.033	.424	.031	.391	.078	.078
.120	.000	.700	-.085	.710	.007	.504	.007	.497	.014	.014
.200	.000	.800	-.167	.833	.028	.694	.027	.667	.040	.040
.180	.000	.900	-.167	.930	.028	.865	.027	.837	.032	.032
.240	.100	.100	.081	.081	.007	.007	.006	.006	1.000	1.000
.040	.100	.200	.092	.196	.008	.039	.008	.038	.220	.220
.200	.100	.200	.063	.188	.004	.035	.004	.034	.111	.111
.960	.100	.200	-1.171	.327	1.377	1.760	.108	.138	.783	.783
.020	.100	.300	.094	.298	.009	.089	.009	.089	.100	.100
.260	.100	.400	-.004	.401	.000	.161	.000	.150	.000	.000
.320	.100	.400	-.031	.410	.001	.168	.001	.151	.006	.006
.480	.100	.500	-.182	.587	.033	.345	.025	.255	.096	.096
.760	.100	.500	-.663	1.004	.439	1.008	.186	.426	.436	.436
.260	.100	.600	-.060	.616	.004	.379	.003	.353	.010	.010
.760	.100	.600	-.843	1.241	.710	1.539	.300	.650	.462	.462
.060	.100	.700	.058	.697	.003	.485	.003	.483	.007	.007
.000	.100	.100	.100	.100	.010	.010	.010	.010	1.000	1.000
.740	.200	.300	-.049	.336	.002	.113	.001	.051	.021	.021
.800	.200	.300	-.111	.389	.012	.151	.004	.054	.082	.082
.320	.200	.500	.045	.486	.002	.236	.002	.212	.008	.008
.580	.200	.500	-.136	.579	.018	.335	.012	.222	.055	.055
.640	.200	.500	-.203	.630	.041	.397	.024	.234	.104	.104
.120	.200	.600	.130	.584	.017	.342	.017	.337	.049	.049
.640	.200	.800	-.528	1.138	.279	1.296	.165	.765	.216	.216
.260	.300	.300	.238	.238	.057	.057	.053	.053	1.000	1.000
.280	.300	.500	.174	.451	.030	.204	.028	.188	.148	.148
.460	.300	.500	.089	.459	.008	.211	.006	.166	.037	.037
.200	.300	.800	.146	.771	.021	.594	.020	.570	.036	.036
.140	.400	.400	.351	.351	.123	.123	.121	.121	1.000	1.000
.380	.400	.400	.290	.290	.084	.084	.072	.072	1.000	1.000
.180	.400	.500	.320	.442	.103	.196	.099	.189	.525	.525
.300	.400	.600	.242	.527	.058	.278	.053	.253	.210	.210
.120	.400	.700	.321	.662	.103	.438	.101	.431	.235	.235
.200	.400	.700	.2[illegible]	.646	.073	.417	.070	.400	.176	.176
.280	.400	.800	.191	.747	.036	.557	.034	.514	.065	.065
.340	.400	.900	.106	.864	.011	.746	.010	.660	.015	.015
.420	.500	.500	.352	.352	.124	.124	.102	.102	1.000	1.000
.900	.500	.500	.263	.263	.069	.069	.013	.013	1.000	1.000
.360	.500	.700	.285	.597	.081	.357	.071	.311	.227	.227
.440	.500	.800	.184	.719	.034	.517	.027	.417	.065	.065
.100	.600	.700	.535	.646	.287	.418	.284	.414	.686	.686
.960	.700	.700	.357	.357	.128	.128	.010	.010	1.000	1.000
.580	.700	.800	.356	.594	.126	.353	.084	.234	.359	.359
.340	.700	.900	.446	.749	.198	.560	.176	.496	.354	.354
.400	.700	.900	.405	.738	.164	.545	.138	.458	.301	.301
.860	.700	.900	-.284	1.144	.081	1.310	.021	.341	.062	.062

The equations with the largest difference between the ratio of the two partial correlations and the ratio of the two β's are reported in Table 11. In these equations the ratio of the two partials are close to one. This is because both partials are close to one due to the high $R^2_{Y.12}$. The β's are quite different in size because in every case predictor two is a much better predictor of Y (higher r_{Y1}).

Table 11

Largest Differences Between Partial Ratio and β Ratio Without Suppression

r^2_{Par1}/r^2_{Par2} - β^2_1/β^2_2	r^2_{Par1}/r^2_{Par2}	β^2_1/β^2_2	r^2_{Par1}	r^2_{Par2}	β^2_1	β^2_2	r_{12}	r_{13}	r_{23}
.730	.978	.248	.970	.992	.186	.749	.080	.500	.900
.687	.977	.290	.967	.990	.191	.660	.200	.600	.900
.686	.918	.233	.894	.973	.172	.737	.100	.500	.900
.676	.873	.198	.842	.964	.160	.810	.000	.400	.900
.649	.923	.274	.894	.968	.178	.651	.220	.600	.900
.642	.860	.218	.821	.955	.158	.726	.120	.500	.900
.627	.811	.183	.768	.948	.146	.796	.020	.400	.900
.626	.997	.371	.995	.998	.211	.567	.320	.700	.900
.611	.869	.258	.824	.948	.166	.644	.240	.600	.900
.598	.801	.203	.751	.937	.146	.717	.140	.500	.900
.597	.951	.354	.924	.972	.198	.560	.340	.700	.900
.580	.750	.170	.698	.932	.133	.784	.040	.400	.900
.573	.815	.242	.756	.928	.154	.637	.260	.600	.900
.567	.904	.337	.855	.946	.187	.554	.360	.700	.900
.556	.744	.188	.685	.920	.133	.708	.160	.500	.900

Given constant zero-order correlations, as the intercorrelation between the predictors increases, the β and r^2_{Inc} values both decrease. Table 12 illustrates how β, r^2_{Inc}, and $R^2_{Y.12}$ are affected by the size of the intercorrelation. The change in r^2_{Inc1} (.160 to .002) is much greater than for β_1 (.400 to .202).

With no intercorrelation between the predictors, β is equal to the zero-order correlation coefficient (top equation in Table 12). The sum of the β^2 is equal to the $R^2_{Y.12}$. This could be interpreted as saying that when the sum of the β^2 is equal to $R^2_{Y.12}$, the β's indicate that each variable is responsible for predicting half of the variance.

As the intercorrelation increases, β's gradually gets smaller until they reach their smallest value when there is a perfect correlation between the predictors (even though a two-predictor equation would have to have $r_{12} <$ 1.00). In this situation each of the predictors also contributes equally to the $R^2_{Y.12}$. Since each of the β^2 at this point is equal to ¼ of the $R^2_{Y.12}$, in effect this could be interpreted as saying that each predictor accounts for ¼ of the $R^2_{Y.12}$ in its combined form with the other variable and ¼ of $R^2_{Y.12}$ by itself.

Table 12

Effect of Changes of r_{12} on β, r^2_{Inc} and $R^2_{Y.12}$

r_{12}	r_{Y1}	r_{Y2}	β_1	β_2	r^2_{Inc1}	r^2_{Inc2}	$R^2_{Y.12}$
.000	.400	.400	.400	.400	.160	.160	.320
.020	.400	.400	.392	.392	.154	.154	.314
.040	.400	.400	.385	.385	.148	.148	.308
.060	.400	.400	.377	.377	.142	.142	.302
.080	.400	.400	.370	.370	.136	.136	.296
.100	.400	.400	.364	.364	.131	.131	.291
.120	.400	.400	.357	.357	.126	.126	.286
.140	.400	.400	.351	.351	.121	.121	.281
.160	.400	.400	.345	.345	.116	.116	.276
.180	.400	.400	.339	.339	.111	.111	.271
.200	.400	.400	.333	.333	.107	.107	.267
.220	.400	.400	.328	.328	.102	.102	.262
.240	.400	.400	.323	.323	.098	.098	.258
.260	.400	.400	.317	.317	.094	.094	.254
.280	.400	.400	.313	.313	.090	.090	.250
.300	.400	.400	.308	.308	.086	.086	.246
.320	.400	.400	.303	.303	.082	.082	.242
.340	.400	.400	.299	.299	.079	.079	.239
.360	.400	.400	.294	.294	.075	.075	.235
.380	.400	.400	.290	.290	.072	.072	.232
.400	.400	.400	.286	.286	.069	.069	.229
.420	.400	.400	.282	.282	.065	.065	.225
.440	.400	.400	.278	.278	.062	.062	.222
.460	.400	.400	.274	.274	.059	.059	.219
.480	.400	.400	.270	.270	.056	.056	.216
.500	.400	.400	.267	.267	.053	.053	.213
.520	.400	.400	.263	.263	.051	.051	.211
.540	.400	.400	.260	.260	.048	.048	.208
.560	.400	.400	.256	.256	.045	.045	.205
.580	.400	.400	.253	.253	.043	.043	.203
.600	.400	.400	.250	.250	.040	.040	.200
.620	.400	.400	.247	.247	.038	.038	.198
.640	.400	.400	.244	.244	.035	.035	.195
.660	.400	.400	.241	.241	.033	.033	.193
.680	.400	.400	.238	.238	.030	.030	.190
.700	.400	.400	.235	.235	.028	.028	.188
.720	.400	.400	.233	.233	.026	.026	.186
.740	.400	.400	.230	.230	.024	.024	.184
.760	.400	.400	.227	.227	.022	.022	.182
.780	.400	.400	.225	.225	.020	.020	.180
.800	.400	.400	.222	.222	.018	.018	.178
.820	.400	.400	.220	.220	.016	.016	.176
.840	.400	.400	.217	.217	.014	.014	.174
.860	.400	.400	.215	.215	.012	.012	.172
.880	.400	.400	.213	.213	.010	.010	.170
.900	.400	.400	.211	.211	.008	.008	.168
.920	.400	.400	.208	.208	.007	.007	.167
.940	.400	.400	.206	.206	.005	.005	.165
.960	.400	.400	.204	.204	.003	.003	.163
.980	.400	.400	.202	.202	.002	.002	.162

The right hand parts of the formula $R^2_{Y.12} = \beta_1 r_1 + \beta_2 r_2$ can be used to indicate the relative importance of each variable in an equation. The value of $\beta_1 r_1$ indicates the value of predictor one in the equation and the value of $\beta_2 r_2$ indicates the value of predictor two. The following two examples use the data of Table 12 ($r_{Y1} = r_{Y2} = .40$) to illustrate this point.

When r_{12} = .00 (top equation), each predictor contributes 16% alone (r^2_{Y1} = r^2_{Y2} = .16) or in combination as shown below. Since the β's and r's are equal, either is an equally good indicator of importance.

Total variance	=	contribution of predictor 1	+	contribution of predictor 2
$R^2_{Y.12}$	=	$\beta_1 r_1$	+	$\beta_2 r_2$
.32	=	.4 x .4 (.16)	+	.4 x .4 (.16)

When r_{12} = 1.00 (would be the bottom equation if possible), each predictor also contributes 16% alone (r^2_{Y1} = r^2_{Y2} = .16) but could be considered to share equally (.08) in the 16% predicted together. Here the β's and r's are not equal, and neither the β's nor the r's could be interpreted as indicating the value of the predictor. Since the two predictors are perfectly correlated, either variable could take all of the credit. The values of β (half as much as the zero order correlations) indicate that each variable is to take half of the credit in the combined form and the other half as a predictor by itself.

Total variance	=	contribution of predictor 1	+	contribution of predictor 2
$R^2_{Y.12}$	=	$\beta_1 r_1$	+	$\beta_2 r_2$
.16	=	.2 x .4 (.08)	+	.2 x .4 (.08)

Summary of the Value of β in a Two Predictor Equation

In a two predictor equation β is a good measure of relative importance of each variable. This will be true whether or not there is suppression. As measures of absolute importance, when suppression exists they should not be used. When suppression does not exist they are probably better indicators than r^2_{Inc}, but should not be used without caution.

Importance of β in Three Predictor Equations

Many of the conclusions reached with two predictors do not hold with three predictors. The relationships between the statistics are much more complex and difficult to determine. Changing one correlation at a time does not allow simple predicting of results because of the effects of the other two predictors.

"Absolute" importance

In evaluating how β and r^2_{Inc} are related, correlations between these two statistics (plus the squared partial correlation for comparison) were computed for the total sample of 8,670 equations of which 1,127 did not have suppression. Table 13 shows the correlations between the three statistics used for determining importance to be evaluated: r^2_{Inc}, r^2_{Par}, and β^2. Statistics for all three predictors are presented.

The correlations between β^2 and r^2_{Inc} for all the equations were .3279, .3456, and .3690 for predictors one, two, and three, indicating large differences between the two statistics. In examining the specific cases the largest differences occurred when suppression was present since β values can range much larger than 1.00 while r^2_{Inc} cannot exceed 1.00. Removing the equations in which suppression existed increased the correlations to .9790, .8867, and .9713.

The same relationship holds here as with two predictors -- there is a very high, but not perfect relationship between β and r^2_{Inc} when there is no suppression.

Table 13

Correlations Between Importance Statistics for Three Predictors

All Equations

	r^2_{Inc1}	r^2_{Par1}	β^2_1
r^2_{Inc1}	1.0000		
r^2_{Par1}	.8872	1.0000	
β^2_1	.3279	.2834	1.0000

	r^2_{Inc2}	r^2_{Par2}	β^2_2
r^2_{Inc2}	1.0000		
r^2_{Par2}	.8060	1.0000	
β^2_2	.3456	.2411	1.0000

	r^2_{Inc3}	r^2_{Par3}	β^2_3
r^2_{Inc3}	1.0000		
r^2_{Par3}	.8916	1.0000	
β^2_3	.3690	.3202	1.0000

Equations Without Suppression

	r^2_{Inc1}	r^2_{Par1}	β^2_1
r^2_{Inc1}	1.0000		
r^2_{Par1}	.8795	1.0000	
β^2_1	.9790	.8721	1.0000

	r^2_{Inc2}	r^2_{Par2}	β^2_2
r^2_{Inc2}	1.0000		
r^2_{Par2}	.6551	1.0000	
β^2_2	.8867	.6845	1.0000

	r^2_{Inc3}	r^2_{Par3}	β^2_3
r^2_{Inc3}	1.0000		
r^2_{Par3}	.8860	1.0000	
β^2_3	.9713	.9146	1.0000

For the cases without suppression, in every case β^2 was equal to or larger than the corresponding r^2_{Inc}, with the maximum difference being .413 for predictor one. The largest differences occurred in many different situations which show no single pattern. The largest differences for each predictor are reported in Tables 14-16. Whereas with two predictors the largest difference occurred with the highest tested values of r_{12}, r_{Y1}, and r_{Y2}, the largest difference with three predictors included correlations of $r_{13} = -.20$ and $r_{Y2} = +.20$.

The relationship between β and r_{Par} was the same for three predictors as it was for two predictors. The largest difference between the β^2 and r^2_{Par} occurred when r^2_{Par} approached 1.00 and β was small. β is a better indicator of importance when no suppression exists. The largest differences between the two statistics are presented in Tables 17-19 ($\beta^2 > r^2_{Par}$) and Tables 20-22 ($\beta^2 < r^2_{Par}$).

Table 14

Largest Differences for Predictor One Without Suppression

$\beta^2_1 - r^2_{Inc1}$	β^2_1	r^2_{Inc1}	r_{12}	r_{13}	r_{23}	r_{Y1}	r_{Y2}	r_{Y3}
.413	.490	.077	-.50	-.20	-.70	-.90	.20	.50
.336	.538	.202	.50	.50	-.20	-.90	-.50	-.50
.336	.538	.202	-.50	-.50	-.20	-.90	.50	.50
.334	.601	.267	.50	.50	-.10	-.90	-.50	-.50
.334	.601	.267	-.50	-.50	-.10	-.90	.50	.50
.320	.640	.320	.50	.50	.00	-.90	-.50	-.50
.320	.640	.320	-.50	-.50	.00	-.90	.50	.50
.303	.667	.364	.50	.50	.10	-.90	-.50	-.50
.303	.667	.364	-.50	-.50	.10	-.90	.50	.50
.302	.423	.121	.50	.50	-.30	.90	-.50	-.50
.302	.423	.121	-.50	-.50	-.30	-.90	.50	.50
.286	.687	.400	.50	.50	.20	-.90	-.50	-.50
.286	.687	.400	-.50	-.50	.20	-.90	.50	.50
.278	.694	.417	-.20	-.20	-.80	-.90	.20	.20
.270	.701	.432	.50	.50	.30	-.90	-.50	-.50

Table 15

Largest Differences for Predictor Two Without Suppression

$\beta^2_1 - r^2_{Incl}$	β^2_1	r^2_{Incl}	r_{12}	r_{13}	r_{23}	r_{Y1}	r_{Y2}	r_{Y3}
.304	.360	.056	.00	.20	.90	.20	.90	.90
.276	.397	.122	.20	.50	.80	.50	.90	.90
.215	.224	.009	-.20	.20	.90	.00	.90	.90
.213	.250	.037	.00	.50	.80	.20	.50	.50
.203	.250	.047	.50	.50	-.40	-.90	-.50	-.50
.203	.250	.047	-.50	-.50	-.40	-.90	.50	.50
.203	.250	.047	.90	.90	.80	-.90	-.90	-.90
.203	.250	.047	-.90	-.90	.80	-.90	.90	.90
.203	.250	.047	.90	.90	.80	.90	.90	.90
.202	.303	.101	.00	.20	.80	.20	.90	.90
.194	.250	.056	-.20	.20	.80	.00	.90	.90
.182	.224	.043	.00	.00	.90	-.20	.90	.90
.182	.224	.043	.00	.00	.90	.20	.90	.90
.182	.224	.043	.00	.00	.90	.00	.90	.90
.181	.223	.042	-.20	-.20	.90	-.20	.90	.90

Table 16

Largest Differences for Predictor Three Without Suppression

$\beta^2_1 - r^2_{Incl}$	β^2_1	r^2_{Incl}	r_{12}	r_{13}	r_{23}	r_{Y1}	r_{Y2}	r_{Y3}
.336	.538	.202	-.20	.50	.50	.50	.50	.90
.326	.627	.301	-.50	.20	.50	.20	.50	.90
.326	.627	.301	-.50	-.50	-.20	-.50	-.20	.90
.320	.640	.320	.00	.50	.50	.50	.50	.90
.304	.810	.506	-.20	.00	.60	.20	.50	.90
.304	.360	.056	-.20	.00	.90	-.20	.90	.90
.286	.687	.400	.20	.50	.50	.50	.50	.90
.278	.694	.417	.00	.20	.60	.50	.50	.90
.276	.397	.122	-.50	-.20	.80	-.50	.90	.90
.263	.563	.300	-.20	-.20	-.60	-.50	-.50	.90
.263	.563	.300	-.20	.20	.60	.50	.50	.90
.247	.718	.471	-.20	.20	.50	.20	.50	.90
.243	.475	.233	-.20	.50	.40	.50	.50	.90
.241	.722	.482	-.50	-.50	.50	-.50	.50	.90
.241	.722	.482	.50	.50	.50	.50	.50	.90

Table 17

Largest Differences Between β^2_1 and r^2_{Par1} Without Suppression and $\beta^2_1 > r^2_{Par1}$

$\beta^2_1 - r^2_{Par1}$	β^2_1	r^2_{Par1}	r_{12}	r_{13}	r_{23}	r_{Y1}	r_{Y2}	r_{Y3}
.167	.250	.083	.00	.90	.10	-.50	-.50	-.50
.167	.250	.083	-.90	.00	.10	-.50	.50	.50
.167	.250	.083	.00	.90	.10	.50	.50	.50
.122	.174	.052	.20	.90	.30	-.50	-.50	-.50
.122	.174	.052	-.90	-.20	.30	-.50	.50	.50
.122	.174	.052	.20	.90	.30	.50	.50	.50
.096	.174	.078	-.50	-.20	-.60	-.50	.20	.20
.090	.210	.120	.20	.50	-.50	-.50	-.20	-.20
.090	.210	.120	-.50	-.20	-.50	-.50	.20	.20
.081	.111	.030	.50	.90	.60	-.50	-.50	-.50
.081	.111	.030	-.90	-.50	.60	-.50	.50	.50
.081	.111	.030	.50	.90	.60	.50	.50	.50
.080	.227	.147	.20	.50	-.40	-.50	-.20	-.20
.080	.227	.147	-.50	-.20	-.40	-.50	.20	.20
.049	.174	.125	.20	.50	-.50	-.50	-.50	.00

Table 18

Largest Differences Between β^2_2 and r^2_{Par2} Without Suppression and $\beta^2_2 > r^2_{Par2}$

$\beta^2_2 - r^2_{Par2}$	β^2_2	r^2_{Par2}	r_{12}	r_{13}	r_{23}	r_{Y1}	r_{Y2}	r_{Y3}
.201	.250	.049	.00	.50	.80	.20	.50	.50
.168	.224	.057	-.20	.20	.90	.00	.90	.90
.153	.194	.041	.00	.20	.90	.20	.50	.50
.142	.207	.065	-.90	-.20	.10	-.50	.50	.50
.107	.250	.143	.00	.20	.80	-.50	-.50	-.50
.107	.250	.143	.00	.20	.80	.50	.50	.50
.092	.128	.036	-.90	-.50	.40	-.50	.50	.50
.092	.128	.036	-.50	.50	.40	.00	.50	.50
.084	.098	.014	.20	.50	.90	.20	.50	.50
.072	.134	.062	.00	.50	.70	.20	.50	.50
.071	.148	.077	-.50	.50	.30	.00	.50	.50
.067	.122	.055	.00	.20	.80	.20	.50	.50
.066	.069	.004	-.20	.20	.90	.00	.50	.50
.063	.360	.297	.00	.20	.90	.20	.90	.90
.057	.077	.020	.90	.90	.80	-.50	-.50	-.50

Table 19

Largest Differences Between β^2_3 and r^2_{Par3} Without Suppression and $\beta^2_3 > r^2_{Par3}$

$\beta^2_3 - r^2_{Par3}$	β^2_3	r^2_{Par3}	r_{12}	r_{13}	r_{23}	r_{Y1}	r_{Y2}	r_{Y3}
.201	.250	.049	-.50	.00	.80	-.20	.50	.50
.200	.250	.050	-.90	.00	.40	.00	.20	.50
.168	.224	.057	-.20	.20	.90	.00	.90	.90
.153	.194	.041	-.20	.00	.90	-.20	.50	.50
.142	.207	.065	.20	.90	.10	-.50	-.50	-.50
.142	.207	.065	.20	.90	.10	.50	.50	.50
.107	.250	.143	-.20	.00	.80	-.50	.50	.50
.095	.250	.155	-.50	.00	-.60	-.20	-.20	.50
.095	.250	.155	-.50	.00	.60	.20	.20	.50
.092	.128	.036	-.50	.50	.40	.00	.50	.50
.092	.128	.036	.50	.90	.40	-.50	-.50	-.50
.092	.128	.036	.50	.90	.40	.50	.50	.50
.090	.210	.120	-.50	-.50	-.20	-.20	-.20	.50
.090	.210	.120	-.50	.50	.20	.20	.20	.50
.090	.210	.120	-.50	-.20	-.50	-.20	-.20	.50

Table 20

Largest Differences Between β^2_1 and r^2_{Par1} Without Suppression and $\beta^2_1 > r^2_{Par1}$

$\beta^2_1 - r^2_{Par1}$	β^2_1	r^2_{Par1}	r_{12}	r_{13}	r_{23}	r_{Y1}	r_{Y2}	r_{Y3}
-.972	.028	1.000	-.50	-.20	.10	-.50	.50	.90
-.843	.099	.942	-.20	-.20	.80	-.50	.90	.90
-.843	.099	.942	.20	.20	.80	.50	.90	.90
-.833	.082	.914	-.20	-.20	.20	-.50	.50	.90
-.833	.082	.914	.20	.20	.20	.50	.50	.90
-.810	.040	.850	.00	.00	.70	-.20	.90	.90
-.810	.040	.850	.00	.00	.70	.20	.90	.90
-.768	.123	.891	.00	.20	.30	.50	.50	.90
-.752	.179	.931	-.20	-.20	-.40	-.50	-.50	.90
-.752	.179	.931	-.20	.20	.40	.50	.50	.90
-.693	.015	.708	-.20	.00	.10	-.20	.50	.90
-.656	.015	.671	-.20	.20	.10	.20	.50	.90
-.590	.143	.732	-.20	-.20	-.10	-.50	-.20	.90
-.578	.105	.683	-.20	-.20	.90	-.50	.90	.90
-.578	.105	.683	.20	.20	.90	.50	.90	.90

Table 21

Largest Differences Between $\beta^2{}_2$ and r^2_{Par2} Without Suppression and $r^2_{Par2} > \beta^2{}_2$

$\beta^2{}_2 - r^2{}_{Par2}$	$\beta^2{}_2$	$r^2{}_{Par2}$	r_{12}	r_{13}	r_{23}	r_{Y1}	r_{Y2}	r_{Y3}
-.889	.111	1.000	-.50	-.20	.10	-.50	.50	.90
-.833	.082	.914	-.20	-.20	.20	-.50	.50	.90
-.833	.082	.914	.20	.20	.20	.50	.50	.90
-.833	.082	.914	.20	.20	.20	-.90	-.50	-.50
-.833	.082	.914	-.20	-.20	.20	-.90	.50	.50
-.809	.152	.961	-.20	.00	.10	-.20	.50	.90
-.770	.195	.965	-.20	.20	.10	.20	.50	.90
-.767	.095	.862	-.20	-.20	-.40	-.50	-.50	.90
-.767	.095	.862	-.20	.20	.40	.50	.50	.90
-.752	.175	.928	.00	.50	.10	.50	.50	.90
-.751	.076	.828	.00	.20	.30	.50	.50	.90
-.726	.172	.898	.00	.20	.10	.20	.50	.90
-.722	.172	.894	.00	.00	.10	.00	.50	.90
-.714	.216	.930	-.20	-.20	.80	-.50	.90	.90
-.714	.216	.930	.20	.20	.80	.50	.90	.90

Table 22

Largest Differences Between $\beta^2{}_3$ and r^2_{Par3} Without Suppression and $r^2_{Par3} > \beta^2{}_3$

$\beta^2{}_3 - r^2{}_{Par3}$	$\beta^2{}_3$	$r^2{}_{Par3}$	r_{12}	r_{13}	r_{23}	r_{Y1}	r_{Y2}	r_{Y3}
-.833	.082	.914	.20	.20	.20	-.90	-.50	-.50
-.833	.082	.914	-.20	-.20	.20	-.90	.50	.50
-.714	.216	.930	-.20	-.20	.80	-.50	.90	.90
-.714	.216	.930	.20	.20	.80	.50	.90	.90
-.673	.280	.953	.00	.00	.70	-.20	.90	.90
-.673	.280	.953	.00	.00	.70	.20	.90	.90
-.666	.069	.734	.20	.20	.30	-.90	-.50	-.50
-.666	.069	.734	-.20	-.20	.30	-.90	.50	.50
-.647	.216	.862	-.50	-.20	-.30	-.90	.50	.50
-.593	.099	.691	.20	.50	-.30	-.90	-.50	-.50
-.573	.164	.737	-.50	-.20	-.20	-.90	.50	.50
-.560	.160	.720	.20	.50	.70	.50	.90	.90
-.535	.340	.875	-.50	-.20	.70	-.50	.90	.90
-.523	.059	.582	.20	.20	.40	-.90	-.50	-.50
-.523	.059	.582	-.20	-.20	.40	-.90	.50	.50

"Relative Importance"

With two predictors the two β^2 's and r^2_{Inc}'s are proportional to each other and the ratio of the two β^2 's is equal to the ratio of the two r^2_{Inc}'s. This is not true with three predictors. It can be seen in Table 23 that the correlations between the beta and incremental ratios are not 1.00 for either sample. As with two predictors, the ratio of the two partial correlations is also not perfectly correlated with either of the other ratios.

Since there were some extremely low β^2 and r^2_{Inc} values that were highly influential with the correlations in Table 23 (forming huge ratios), equations with β^2 and r^2_{Inc} value less than .001 were removed and the resulting correlations between the β^2 and r^2_{Inc} ratios for the three predictors were .854, .891, and .780 for all equations, and .939, .934, and .981 for the equations without suppression.

Differences between the β^2 and r^2_{Inc} ratios for the second and third predictors are shown in Tables 24 and 25. Most of the positive and negative large differences occurred when one of the r^2_{Inc}'s was very small. The β ratios were much less extreme.

Table 23

Correlations Between Statistical Ratios

All Equations

	β'_1/β'_2	r^2_{Inc1}/r^2_{Inc2}	r^2_{Par1}/r^2_{Par2}
β'_1/β'_2	1.0000		
r^2_{Inc1}/r^2_{Inc2}	.9493	1.0000	
r^2_{Par1}/r^2_{Par2}	.6813	.6554	1.0000

	β'_1/β'_3	r^2_{Inc1}/r^2_{Inc3}	r^2_{Par1}/r^2_{Par3}
β'_1/β'_3	1.0000		
r^2_{Inc1}/r^2_{Inc3}	.9490	1.0000	
r^2_{Par1}/r^2_{Par3}	.6859	.6637	1.0000

	β'_2/β'_3	r^2_{Inc2}/r^2_{Inc3}	r^2_{Par2}/r^2_{Par3}
β'_2/β'_3	1.0000		
r^2_{Inc2}/r^2_{Inc3}	.7718	1.0000	
r^2_{Par2}/r^2_{Par3}	.9003	.9322	1.0000

Equations Without Suppression

	β'_1/β'_2	r^2_{Inc1}/r^2_{Inc2}	r^2_{Par1}/r^2_{Par2}
β'_1/β'_2	1.0000		
r^2_{Inc1}/r^2_{Inc2}	.8255	1.0000	
r^2_{Par1}/r^2_{Par2}	.8873	.8358	1.0000

	β'_1/β'_3	r^2_{Inc1}/r^2_{Inc3}	r^2_{Par1}/r^2_{Par3}
β'_1/β'_3	1.0000		
r^2_{Inc1}/r^2_{Inc3}	.8257	1.0000	
r^2_{Par1}/r^2_{Par3}	.8874	.8361	1.0000

	β'_2/β'_3	r^2_{Inc2}/r^2_{Inc3}	r^2_{Par2}/r^2_{Par3}
β'_2/β'_3	1.0000		
r^2_{Inc2}/r^2_{Inc3}	.9963	1.0000	
r^2_{Par2}/r^2_{Par3}	.9748	.9816	1.0000

To evaluate the degree to which β can be considered to be a better indicator of importance than r^2_{Inc},the seventh equation listed in Table 25 will be examined. Each of the three predictors correlated .50 with the dependent variable indicating they were equally good predictors by themselves. Variable two is not correlated with either predictor two or predictor three so it contributes 25% of the variance of Y alone or in combination with predictors two or three ($r_{Y2} = \beta_2$). Variables one and three are highly correlated ($r_{13} = .90$) indicating they largely predict the same variance. The $r^2_{Inc1} = r^2_{Inc3} = .013$ indicating they predict little unique variance of Y. Looking at only the three r^2_{Inc}'s (.013, .250, and .013) would suggest that variable two is a much better predictor than either predictor one or three which is obviously false due to their high intercorrelation. The three β's (.263, .500, and .263) are much closer to indicating the true relative importance of the three predictors. Using $\beta_i r_{Yi}$ as an indicator of importance as shown in the equation below suggests that predictor two ($\beta_2 r_{Y2} = .25$) is about equally as important as predictors one and three which are equal to each other ($\beta_1 r_{Y1} = \beta_3 r_{Y3} = .135$).

$$R^2_{Y.123} = \beta_1 r_{Y1} + \beta_2 r_{Y2} + \beta_3 r_{Y3}$$

$$R^2_{Y.123} = .263 \times .50 + .50 \times .50 + .263 \times .50$$

$$.513 = .1315 + .25 + .1315$$

Tables 26-29 show how changing one or more of the intercorrelations (r_{ij}) or the correlations with the dependent variable (r_{Yi}) while keeping the other correlations constant affects the β's, r^2_{Inc}'s, and their ratios. The coefficient that is changed take all possible values between -.98 and +.98 with increments of .02. Table 26 changes r_{12} while the other correlations are different from each other but remain constant. Table 27 changes r_{12} with the other correlations all having the same constant value. Table 28 changes r_{Y3} with the other correlations all different and constant. Table 29 changes all of the intercorrelations (r_{ij}) equally with the correlations with the dependent variable different and constant.

There are two important things to notice in the tables. First the r^2_{Inc} ratios are usually close to the β' ratios but are seldom equal and sometimes are markedly different. Second, β and r^2_{Inc} change at different rates such that for some equations, the predictor with the higher β may have the lower r^2_{Inc}. Since significance of a predictor is proportional to r^2_{Inc}, it would be possible to have a significant predictor in an equation with a lower β than the β of a non-significant predictor.

Table 24

Largest Positive Differences Between $\beta^2{}_2 / \beta^2{}_3$ and r^2_{Inc2} / r^2_{Inc3} Without Suppression

$\beta^2{}_2/\beta^2{}_3$ - r^2_{Inc2}/r^2_{Inc3}	$\beta^2{}_2/\beta^2{}_3$	r^2_{Inc2}/r^2_{Inc3}	β_2	β_3	r^2_{Inc2}	r^2_{Inc3}	r_{12}	r_{13}	r_{23}	r_{Y1}	r_{Y2}	r_{Y3}
.802	1.000	.198	.455	.455	.038	.191	-.90	-.20	.10	-.50	.50	.50
.802	1.000	.198	.182	.182	.006	.031	-.90	-.20	.10	-.20	.20	.20
.747	1.000	.253	.357	.357	.024	.094	-.90	-.50	.40	-.50	.50	.50
.747	1.000	.253	.143	.143	.004	.015	-.90	-.50	.40	-.20	.20	.20
.240	.961	.721	.379	.386	.095	.131	-.50	.00	.30	-.20	.50	.50
.230	.309	.078	.071	.129	.001	.012	-.90	-.50	.50	-.20	.20	.20
.230	.309	.078	.179	.321	.006	.076	-.90	-.50	.50	-.50	.50	.50
.228	.284	.056	.088	.165	.001	.026	-.90	-.20	.20	-.20	.20	.20
.228	.284	.056	.220	.412	.009	.163	-.90	-.20	.20	-.50	.50	.50
.224	.277	.053	.105	.200	.002	.040	-.90	.00	.00	-.20	.20	.20
.224	.277	.053	.263	.500	.013	.250	-.90	.00	.00	-.50	.50	.50
.216	.865	.649	.339	.364	.068	.104	-.50	.00	.40	-.20	.50	.50
.199	.910	.711	.270	.283	.017	.024	-.50	-.20	.80	-.20	.50	.50
.184	.735	.551	.300	.350	.045	.082	-.50	.00	.50	-.20	.50	.50
.162	.742	.580	.394	.458	.115	.198	-.50	-.20	.00	-.50	.50	.50
.162	.742	.580	.158	.183	.018	.032	-.50	-.20	.00	-.20	.20	.20
.158	.721	.563	.406	.478	.117	.207	-.50	.20	.10	-.20	.50	.50

Table 25

Largest Negative Differences Between $\beta^2{}_2 / \beta^2{}_3$ and r^2_{Inc2} / r^2_{Inc3} Without Suppression

$\beta^2{}_2/\beta^2{}_3$ - r^2_{Inc2}/r^2_{Inc3}	$\beta^2{}_2/\beta^2{}_3$	r^2_{Inc2}/r^2_{Inc3}	β_2	β_3	r^2_{Inc2}	r^2_{Inc3}	r_{12}	r_{13}	r_{23}	r_{Y1}	r_{Y2}	r_{Y3}
-163.	625.1	788.0	-.106	-.004	.007	.000	.20	.50	-.40	-.50	-.20	-.20
-60.1	215.7	275.8	-.331	-.023	.104	.000	.20	.50	.20	-.90	-.50	-.50
-23.0	82.13	105.1	-.426	-.047	.165	.002	.20	.50	-.10	-.50	-.50	-.20
-15.4	3.610	19.00	-.200	-.105	.040	.002	.00	.90	.00	-.20	-.20	-.20
-15.4	3.610	19.00	.200	.105	.040	.002	.00	.90	.00	.20	.20	.20
-15.4	3.610	19.00	-.500	-.263	.250	.013	.00	.90	.00	-.50	-.50	-.50
-15.4	3.610	19.00	.500	.263	.250	.013	.00	.90	.00	.50	.50	.50
-14.3	3.516	17.77	-.165	-.088	.026	.001	.20	.90	.20	-.20	-.20	-.20
-14.3	3.516	17.77	.165	.088	.026	.001	.20	.90	.20	.20	.20	.20
-14.2	3.516	17.76	-.412	-.220	.163	.009	.20	.90	.20	-.50	-.50	-.50
-14.2	3.516	17.76	.412	.220	.163	.009	.20	.90	.20	.50	.50	.50
-10.1	30.25	40.34	-.466	-.085	.171	.004	.00	.50	.40	-.50	-.50	-.50
-10.1	30.25	40.34	.466	.085	.171	.004	.00	.50	.40	.50	.50	.50
-10.1	30.25	40.33	-.186	-.034	.027	.001	.00	.50	.40	-.20	-.20	-.20
-10.1	30.25	40.33	.186	.034	.027	.001	.00	.50	.40	.20	.20	.20
-9.55	3.240	12.79	-.129	-.071	.012	.001	.50	.90	.50	-.20	-.20	-.20
-9.55	3.240	12.79	.129	.071	.012	.001	.50	.90	.50	.20	.20	.20
-9.55	3.240	12.79	-.321	-.179	.076	.006	.50	.90	.50	-.50	-.50	-.50
-9.55	3.240	12.79	.321	.179	.076	.006	.50	.90	.50	.50	.50	.50
-7.00	25.00	32.00	-.333	-.067	.107	.003	.20	.50	.10	-.90	-.50	-.50
-5.38	19.14	24.52	-.149	-.034	.014	.001	.20	.50	.60	-.20	-.20	-.20
-5.38	19.14	24.52	.149	.034	.014	.001	.20	.50	.60	.20	.20	.20
-5.36	19.14	24.50	-.372	-.085	.087	.004	.20	.50	.60	-.50	-.50	-.50
-5.36	19.14	24.50	.372	.085	.087	.004	.20	.50	.60	.50	.50	.50

High intercorrelations (r_{12}) cause inflated β's destroying relative importance interpretations. For the last equation in Table 26 with r_{12} = .96 it appears from the β's as if predictors one and two are much more important than predictor three which is probably a faulty conclusion.

For the 68 equations listed in Table 26, 11 showed inconsistency between interpretations of relative importance based on β and r^2_{Inc} values. The following chart describes these inconsistencies.

r_{12}	β	r^2_{Inc}
-.38 to +.04	$\beta_2 > \beta_3$	$r^2_{Inc2} > r^2_{Inc3}$
.06	$\beta_2 = \beta_3$	$r^2_{Inc2} > r^2_{Inc3}$
+.08 to +.10	$\beta_2 < \beta_3$	$r^2_{Inc2} > r^2_{Inc3}$
+.12 to +.80	$\beta_2 < \beta_3$	$r^2_{Inc2} < r^2_{Inc3}$
+.82 to +.96	$\beta_2 > \beta_3$	$r^2_{Inc2} < r^2_{Inc3}$

All but three of these 11 occur where there is no suppression. Even though suppression occurred in all equations where r_{12} was above .62 (β_1 was the opposite sign of r_{Y1}), inconsistency only occurred with r_{12} values above .80. In many similar equations examined but not listed here, the same pattern existed -- a few small inconsistent values when there was no suppression and many when suppression existed.

Changing r_{12} when the other correlations were equal (Table 27) or changing r_{Y3} (Table 28) did not produce any inconsistent results. The β^2 and r^2_{Inc} ratios were not equal nor perfectly correlated, but always close. β_2 and r^2_{Inc2} were higher than β_3 and r^2_{Inc3} for certain r_{12} values and lower for others.

In Table 29 it can be seen that if all intercorrelations are equal, the β and r^2_{Inc} ratios remained equal as the intercorrelations changed.

There were only 17 equations of the 8,670 tests that showed inconsistent results for predictor one (listed in Table 30). Four equations had predictor one better according to $\beta^2{}_1 - \beta^2{}_2$ and predictor two better according to $r^2_{Inc1} - r^2_{Inc2}$ and 13 equations were in the opposite direction. An example of each type is presented in Table 31 and Table 32 changing r_{12} to see how the inconsistency is affected.

Table 31 uses the third from the bottom equation in Table 30 which has all positive correlations. For all equations, $r^2_{Inc1} > r^2_{Inc2}$. For r_{12} between .20 and .56, $\beta_1 < \beta_2$ which is inconsistent with the r^2_{Inc} interpretation. In this situation, the inconsistency is not caused by high intercorrelation. In fact, the larger inconsistency is with lower intercorrelation.

In Table 32 the top equation in Table 30 is used which has all predictors positively correlated with each other and negatively correlated with the dependent variable. For all of these equations $r^2_{Inc2} > r^2_{Inc1}$ and $\beta^2{}_1 > \beta^2{}_2$ for r_{12} from -.36 to +.74.

Table 26

Effect of Changing r_{12} With the Other Correlations Different and Remaining Constant

r_{12}	r_{13}	r_{23}	r_{Y1}	r_{Y2}	r_{Y3}	β_1	β_2	β_3	r'_{Inc1}	r'_{Inc2}	r'_{Inc3}	r'_{Inc2}/r'_{Inc3}	β'_2/β'_3
-.38	.40	.30	.50	.60	.70	.768	.851	.137	.334	.443	.011	39.00	38.29
-.36	.40	.30	.50	.60	.70	.725	.811	.166	.309	.418	.017	24.62	23.76
-.34	.40	.30	.50	.60	.70	.687	.776	.193	.286	.396	.023	17.07	16.22
-.32	.40	.30	.50	.60	.70	.651	.744	.216	.266	.376	.030	12.61	11.81
-.30	.40	.30	.50	.60	.70	.619	.714	.238	.248	.357	.037	9.750	9.000
-.28	.40	.30	.50	.60	.70	.589	.688	.258	.231	.340	.044	7.795	7.105
-.26	.40	.30	.50	.60	.70	.562	.663	.276	.215	.325	.051	6.398	5.764
-.24	.40	.30	.50	.60	.70	.537	.641	.293	.201	.310	.058	5.362	4.779
-.22	.40	.30	.50	.60	.70	.513	.620	.309	.168	.297	.065	4.571	4.035
-.20	.40	.30	.50	.60	.70	.491	.601	[illegible]	.175	.285	.072	3.953	3.459
-.18	.40	.30	.50	.60	.70	.470	.584	.337	.164	.273	.079	3.460	3.003
-.16	.40	.30	.50	.60	.70	.451	.567	.349	.153	.263	.086	3.059	2.637
-.14	.40	.30	.50	.60	.70	.433	.552	.361	.143	.253	.093	2.728	2.338
-.12	.40	.30	.50	.60	.70	.416	.538	.372	.134	.244	.099	2.452	2.090
-.10	.40	.30	.50	.60	.70	.399	.525	.383	.126	.235	.106	2.219	1.883
-.08	.40	.30	.50	.60	.70	.384	.513	.392	.117	.227	.112	2.021	1.708
-.06	.40	.30	.50	.60	.70	.369	.502	.402	.110	.219	.119	1.849	1.559
-.04	.40	.30	.50	.60	.70	.355	.491	.411	.103	.212	.125	1.701	1.431
-.02	.40	.30	.50	.60	.70	.342	.481	.419	.096	.205	.131	1.571	1.320
.00	.40	.30	.50	.60	.70	.329	.472	.427	.089	.199	.137	1.457	1.224
.02	.40	.30	.50	.60	.70	.317	.463	.434	.083	.193	.142	1.356	1.139
.04	.40	.30	.50	.60	.70	.305	.455	.441	.078	.187	.148	1.266	1.065
.06	.40	.30	.50	.60	.70	.294	.448	.448	.072	.182	.153	1.186	1.000
.08	.40	.30	.50	.60	.70	.283	.441	.455	.067	.177	.159	1.113	.941
.10	.40	.30	.50	.60	.70	.272	.435	.461	.062	.172	.164	1.048	.890
.12	.40	.30	.50	.60	.70	.262	.429	.467	.058	.167	.169	.990	.843
.14	.40	.30	.50	.60	.70	.252	.423	.472	.053	.163	.174	.936	.802
.16	.40	.30	.50	.60	.70	.242	.418	.478	.049	.159	.179	.887	.765
.18	.40	.30	.50	.60	.70	.232	.413	.483	.045	.155	.183	.843	.732
.20	.40	.30	.50	.60	.70	.223	.409	.488	.041	.151	.188	.802	.702
.22	.40	.30	.50	.60	.70	.214	.405	.493	.038	.147	.193	.765	.675
.24	.40	.30	.50	.60	.70	.205	.402	.498	.034	.144	.197	.730	.651
.26	.40	.30	.50	.60	.70	.195	.398	.502	.031	.141	.202	.699	.630
.28	.40	.30	.50	.60	.70	.187	.396	.507	.028	.138	.206	.669	.610
.30	.40	.30	.50	.60	.70	.178	.393	.511	.025	.135	.210	.642	.593
.32	.40	.30	.50	.60	.70	.169	.391	.515	.023	.132	.214	.617	.578
.34	.40	.30	.50	.60	.70	.160	.390	.519	.020	.130	.218	.594	.564
.36	.40	.30	.50	.60	.70	.151	.389	.523	.018	.127	.222	.573	.553
.38	.40	.30	.50	.60	.70	.142	.388	.527	.015	.125	.226	.553	.542
.40	.40	.30	.50	.60	.70	.133	.388	.531	.013	.123	.230	.534	.534
.42	.40	.30	.50	.60	.70	.123	.388	.534	.011	.121	.234	.517	.527
.44	.40	.30	.50	.60	.70	.114	.389	.538	.009	.119	.238	.501	.522
.46	.40	.30	.50	.60	.70	.104	.390	.541	.008	.117	.241	.486	.518
.48	.40	.30	.50	.60	.70	.094	.391	.545	.006	.116	.245	.472	.516
.50	.40	.30	.50	.60	.70	.084	.394	.548	.005	.114	.249	.460	.515
.52	.40	.30	.50	.60	.70	.073	.396	.552	.004	.113	.252	.448	.516
.54	.40	.30	.50	.60	.70	.062	.400	.555	.002	.112	.256	.438	.519
.56	.40	.30	.50	.60	.70	.050	.404	.559	.002	.111	.260	.428	.524
.58	.40	.30	.50	.60	.70	.038	.410	.562	.001	.110	.263	.419	.531
.60	.40	.30	.50	.60	.70	.024	.416	.566	.000	.110	.267	.412	.540
.62	.40	.30	.50	.60	.70	.010	.423	.569	.000	.110	.271	.405	.553
.64	.40	.30	.50	.60	.70	-.005	.432	.573	.000	.110	.274	.399	.568
.66	.40	.30	.50	.60	.70	-.022	.442	.576	.000	.110	.278	.395	.587
.68	.40	.30	.50	.60	.70	-.040	.453	.580	.001	.110	.282	.391	.611
.70	.40	.30	.50	.60	.70	-.061	.467	.584	.002	.111	.286	.389	.640
.72	.40	.30	.50	.60	.70	-.084	.484	.588	.003	.113	.291	.388	.676
.74	.40	.30	.50	.60	.70	-.109	.503	.593	.005	.115	.295	.388	.720
.76	.40	.30	.50	.60	.70	-.139	.526	.598	.008	.117	.300	.390	.776
.78	.40	.30	.50	.60	.70	-.174	.555	.603	.011	.120	.305	.394	.846
.80	.40	.30	.50	.60	.70	-.215	.589	.609	.015	.125	.311	.401	.936
.82	.40	.30	.50	.60	.70	-.265	.633	.616	.021	.131	.318	.411	1.054
.84	.40	.30	.50	.60	.70	-.328	.688	.625	.029	.139	.326	.425	1.212
.86	.40	.30	.50	.60	.70	-.408	.760	.635	.040	.149	.336	.444	1.433
.88	.40	.30	.50	.60	.70	-.515	.859	.648	.054	.164	.348	.471	1.754
.90	.40	.30	.50	.60	.70	-.667	1.000	.667	.076	.186	.365	.509	2.250
.92	.40	.30	.50	.60	.70	-.899	1.219	.694	.110	.220	.390	.564	3.084
.94	.40	.30	.50	.60	.70	-1.30	1.600	.740	.171	.280	.433	.648	4.675
.96	.40	.30	.50	.60	.70	-2.17	2.429	.838	.303	.413	.527	.784	8.397

Table 27

Effect of Changing r_{12} With the Other Correlations Equal and Remaining Constant

r_{12}	r_{13}	r_{23}	r_{Y1}	r_{Y2}	r_{Y3}	β_1	β_2	β_3	r'_{Inc1}	r'_{Inc2}	r'_{Inc3}	r'_{Inc2}/r'_{Inc3}	β'_2/β'_3	$R'_{Y.123}$
-.54	.40	.40	.40	.40	.40	1.714	1.714	-.971	.754	.754	.287	2.626	3.114	.983
-.52	.40	.40	.40	.40	.40	1.500	1.500	-.800	.651	.651	.213	3.054	3.516	.880
-.50	.40	.40	.40	.40	.40	1.333	1.333	-.667	.571	.571	.160	3.571	4.000	.800
-.48	.40	.40	.40	.40	.40	1.200	1.200	-.560	.507	.507	.121	4.207	4.592	.736
-.46	.40	.40	.40	.40	.40	1.091	1.091	-.473	.455	.455	.091	4.998	5.325	.684
-.44	.40	.40	.40	.40	.40	1.000	1.000	-.400	.411	.411	.069	6.000	6.250	.640
-.42	.40	.40	.40	.40	.40	.923	.923	-.338	.375	.375	.051	7.293	7.438	.603
-.40	.40	.40	.40	.40	.40	.857	.857	-.286	.343	.343	.038	9.000	9.000	.571
-.38	.40	.40	.40	.40	.40	.800	.800	-.240	.315	.315	.028	11.32	11.11	.544
-.36	.40	.40	.40	.40	.40	.750	.750	-.200	.291	.291	.020	14.57	14.06	.520
-.34	.40	.40	.40	.40	.40	.706	.706	-.165	.270	.270	.014	19.34	18.37	.499
-.32	.40	.40	.40	.40	.40	.667	.667	-.133	.251	.251	.009	26.71	25.00	.480
-.30	.40	.40	.40	.40	.40	.632	.632	-.105	.235	.235	.006	39.00	36.00	.463
-.28	.40	.40	.40	.40	.40	.600	.600	-.080	.219	.219	004	61.71	56.25	.448
-.26	.40	.40	.40	.40	.40	.571	.571	-.057	.206	.206	.002	111.0	100.0	.434
-.24	.40	.40	.40	.40	.40	.545	.545	-.036	.193	.193	.001	252.3	225.0	.422
-.22	.40	.40	.40	.40	.40	.522	.522	-.017	.182	.182	.000	1022.	900.0	.410
-.20	.40	.40	.40	.40	.40	.500	.500	.000	.171	.171	.000	.	.	.400
-.18	.40	.40	.40	.40	.40	.480	.480	.016	.162	.162	.000	1037.	900.0	.390
-.16	.40	.40	.40	.40	.40	.462	.462	.031	.153	.153	.001	261.0	225.0	.382
-.14	.40	.40	.40	.40	.40	.444	.444	.044	.145	.145	.001	116.7	100.0	.373
-.12	.40	.40	.40	.40	.40	.429	.429	.057	.137	.137	.002	66.00	56.25	.366
-.10	.40	.40	.40	.40	.40	.414	.414	.069	.130	.130	.003	42.43	36.00	.359
-.08	.40	.40	.40	.40	.40	.400	.400	.080	.123	.123	.004	29.57	25.00	.352
-.06	.40	.40	.40	.40	.40	.387	.387	.090	.117	.117	.005	21.79	18.37	.346
-.04	.40	.40	.40	.40	.40	.375	.375	.100	.111	.111	.007	16.71	14.06	.340
-.02	.40	.40	.40	.40	.40	.364	.364	.109	.106	.106	.008	13.22	11.11	.335
.00	.40	.40	.40	.40	.40	.353	.353	.118	.101	.101	.009	10.71	9.000	.329
.02	.40	.40	.40	.40	.40	.343	.343	.126	.096	.096	.011	8.851	7.438	.325
.04	.40	.40	.40	.40	.40	.333	.333	.133	.091	.091	.012	7.428	6.250	.320
.06	.40	.40	.40	.40	.40	.324	.324	.141	.087	.087	.014	6.317	5.325	.316
.08	.40	.40	.40	.40	.40	.316	.316	.147	.083	.083	.015	5.431	4.592	.312
.10	.40	.40	.40	.40	.40	.308	.308	.154	.079	.079	.017	4.714	4.000	.308
.12	.40	.40	.40	.40	.40	.300	.300	.160	.075	.075	.018	4.125	3.516	.304
.14	.40	.40	.40	.40	.40	.293	.293	.166	.072	.072	.020	3.635	3.114	.300
.16	.40	.40	.40	.40	.40	.286	.286	.171	.069	.069	.021	3.222	2.778	.297
.18	.40	.40	.40	.40	.40	.279	.279	.177	.065	.065	.023	2.872	2.493	.294
.20	.40	.40	.40	.40	.40	.273	.273	.182	.062	.062	.024	2.571	2.250	.291
.22	.40	.40	.40	.40	.40	.267	.267	.187	.059	.059	.026	2.312	2.041	.288
.24	.40	.40	.40	.40	.40	.261	.261	.191	.057	.057	.027	2.086	1.860	.285
.26	.40	.40	.40	.40	.40	.255	.255	.196	.054	.054	.029	1.888	1.701	.283
.28	.40	.40	.40	.40	.40	.250	.250	.200	.051	.051	.030	1.714	1.562	.280
.30	.40	.40	.40	.40	.40	.245	.245	.204	.049	.049	.031	1.560	1.440	.278
.32	.40	.40	.40	.40	.40	.240	.240	.208	.047	.047	.033	1.423	1.331	.275
.34	.40	.40	.40	.40	.40	.235	.235	.212	.044	.044	.034	1.300	1.235	.273
.36	.40	.40	.40	.40	.40	.231	.231	.215	.042	.042	.035	1.190	1.148	.271
.38	.40	.40	.40	.40	.40	.226	.226	.219	.040	.040	.037	1.090	1.070	.269
.40	.40	.40	.40	.40	.40	.222	.222	.222	.038	.038	.038	1.000	1.000	.267
.42	.40	.40	.40	.40	.40	.218	.218	.225	.036	.036	.039	.918	.937	.265
.44	.40	.40	.40	.40	.40	.214	.214	.229	.034	.034	.041	.844	.879	.263
.46	.40	.40	.40	.40	.40	.211	.211	.232	.032	.032	.042	.776	.826	.261
.48	.40	.40	.40	.40	.40	.207	.207	.234	.031	.031	.043	.713	.779	.259
.50	.40	.40	.40	.40	.40	.203	.203	.237	.029	.029	.044	.656	.735	.258
.52	.40	.40	.40	.40	.40	.200	.200	.240	.027	.027	.045	.603	.694	.256
.54	.40	.40	.40	.40	.40	.197	.197	.243	.026	.026	.047	.554	.657	.254
.56	.40	.40	.40	.40	.40	.194	.194	.245	.024	.024	.048	.509	.623	.253
.58	.40	.40	.40	.40	.40	.190	.190	.248	.023	.023	.049	.467	.592	.251
.60	.40	.40	.40	.40	.40	.188	.188	.250	.021	.021	.050	.429	.563	.250
.62	.40	.40	.40	.40	.40	.185	.185	.252	.020	.020	.051	.392	.535	.249
.64	.40	.40	.40	.40	.40	.182	.182	.255	.019	.019	.052	.359	.510	.247
.66	.40	.40	.40	.40	.40	.179	.179	.257	.017	.017	.053	.327	.487	.246
.68	.40	.40	.40	.40	.40	.176	.176	.259	.016	.016	.054	.298	.465	.245
.70	.40	.40	.40	.40	.40	.174	.174	.261	.015	.015	.055	.270	.444	.243
.72	.40	.40	.40	.40	.40	.171	.171	.263	.014	.014	.056	.244	.425	.242
.74	.40	.40	.40	.40	.40	.169	.169	.265	.013	.013	.057	.219	.407	.241
.76	.40	.40	.40	.40	.40	.167	.167	.267	.011	.011	.058	.196	.391	.240
.78	.40	.40	.40	.40	.40	.164	.164	.268	.010	.010	.059	.175	.375	.239
.80	.40	.40	.40	.40	.40	.162	.162	.270	.009	.009	.060	.154	.360	.238
.82	.40	.40	.40	.40	.40	.160	.160	.272	.008	.008	.061	.135	.346	.237
.84	.40	.40	.40	.40	.40	.158	.158	.274	.007	.007	.062	.117	.333	.236
.86	.40	.40	.40	.40	.40	.156	.156	.275	.006	.006	.063	.099	.320	.235
.88	.40	.40	.40	.40	.40	.154	.154	.277	.005	.005	.064	.083	.309	.234
.90	.40	.40	.40	.40	.40	.152	.152	.278	.004	.004	.064	.067	.298	.233
.92	.40	.40	.40	.40	.40	.150	.150	.280	.003	.003	.065	.052	.287	.232
.94	.40	.40	.40	.40	.40	.148	.148	.281	.003	.003	.066	.038	.277	.231
.96	.40	.40	.40	.40	.40	.146	.146	.283	.002	.002	.067	.025	.268	.230
.98	.40	.40	.40	.40	.40	.145	.145	.284	.001	.001	.068	.012	.259	.229

Table 28

Effect of Changing r_{Y3} With the Other Correlations Different and Remaining Constant

r_{12}	r_{13}	r_{23}	r_{Y1}	r_{Y2}	r_{Y3}	β_1	β_2	β_3	r^2_{Inc1}	r^2_{Inc2}	r^2_{Inc3}	r^2_{Inc2}/r^2_{Inc3}	β^2_2/β^2_3
.10	.20	.30	.40	.50	-.52	.496	.699	-.829	.236	.444	.605	.734	.711
.10	.20	.30	.40	.50	-.50	.492	.693	-.806	.232	.436	.572	.761	.738
.10	.20	.30	.40	.50	-.48	.488	.686	-.783	.228	.428	.541	.791	.767
.10	.20	.30	.40	.50	-.46	.484	.680	[illegible]	.225	.420	.510	.823	.798
.10	.20	.30	.40	.50	-.44	.480	.673	-.738	.221	.412	.480	.858	.832
.10	.20	.30	.40	.50	-.42	.476	.667	-.715	.217	.404	.451	.896	.869
.10	.20	.30	.40	.50	-.40	.472	.661	-.693	.214	.396	.423	.938	.909
.10	.20	.30	.40	.50	-.38	.469	.654	-.670	.210	.389	.395	.983	.953
.10	.20	.30	.40	.50	-.36	.465	.648	-.647	.207	.381	.369	1.033	1.001
.10	.20	.30	.40	.50	-.34	.461	.641	-.625	.203	.374	.344	1.087	1.054
.10	.20	.30	.40	.50	-.32	.457	.635	-.602	.200	.366	.319	1.148	1.113
.10	.20	.30	.40	.50	-.30	.453	.628	-.579	.197	.359	.295	1.214	1.178
.10	.20	.30	.40	.50	-.28	.449	.622	-.556	.193	.351	.273	1.289	1.250
.10	.20	.30	.40	.50	-.26	.445	.616	-.534	.190	.344	.251	1.372	1.330
.10	.20	.30	.40	.50	-.24	.441	.609	-.511	.187	.337	.230	1.466	1.421
.10	.20	.30	.40	.50	-.22	.437	.603	-.488	.183	.330	.210	1.571	1.524
.10	.20	.30	.40	.50	-.20	.433	.596	-.466	.180	.323	.191	1.692	1.640
.10	.20	.30	.40	.50	-.18	.430	.590	-.443	.177	.316	.173	1.830	1.774
.10	.20	.30	.40	.50	-.16	.426	.583	-.420	.174	.309	.156	1.989	1.928
.10	.20	.30	.40	.50	-.14	.422	.577	-.397	.170	.302	.139	2.174	2.108
.10	.20	.30	.40	.50	-.12	.418	.571	-.375	.167	.296	.124	2.391	2.318
.10	.20	.30	.40	.50	-.10	.414	.564	-.352	.164	.289	.109	2.649	2.568
.10	.20	.30	.40	.50	-.08	.410	.558	-.329	.161	.283	.096	2.958	2.868
.10	.20	.30	.40	.50	-.06	.406	.551	-.307	.158	.276	.083	3.334	3.233
.10	.20	.30	.40	.50	-.04	.402	.545	-.284	.155	.270	.071	3.799	3.683
.10	.20	.30	.40	.50	-.02	.398	.539	-.261	.152	.263	.060	4.382	4.250
.10	.20	.30	.40	.50	.00	.394	.532	-.239	.149	.257	.050	5.132	4.976
.10	.20	.30	.40	.50	.02	.391	.526	-.216	.146	.251	.041	6.118	5.933
.10	.20	.30	.40	.50	.04	.387	.519	-.193	.143	.245	.033	7.456	7.230
.10	.20	.30	.40	.50	.06	.383	.513	-.170	.140	.239	.026	9.340	9.057
.10	.20	.30	.40	.50	.08	.379	.506	-.148	.138	.233	.019	12.12	11.76
.10	.20	.30	.40	.50	.10	.375	.500	-.125	.135	.227	.014	16.50	16.00
.10	.20	.30	.40	.50	.12	.371	.494	-.102	.132	.221	.009	24.01	23.28
.10	.20	.30	.40	.50	.14	.367	.487	-.080	.129	.216	.006	38.64	37.47
.10	.20	.30	.40	.50	.16	.363	.481	-.057	.126	.210	.003	73.66	71.43
.10	.20	.30	.40	.50	.18	.359	.474	-.034	.124	.204	.001	198.6	192.6
.10	.20	.30	.40	.50	.20	.356	.468	-.011	.121	.199	.000	1714.	1665.
.10	.20	.30	.40	.50	.22	.352	.461	.011	.118	.193	.000	1743.	1686.
.10	.20	.30	.40	.50	.24	.348	.455	.034	.116	.188	.001	185.3	179.7
.10	.20	.30	.40	.50	.26	.344	.449	.057	.113	.183	.003	64.67	62.71
.10	.20	.30	.40	.50	.28	.340	.442	.079	.111	.178	.006	32.02	31.05
.10	.20	.30	.40	.50	.30	.336	.436	.102	.108	.172	.009	18.80	18.23
.10	.20	.30	.40	.50	.32	.332	.429	.125	.106	.167	.014	12.21	11.84
.10	.20	.30	.40	.50	.34	.328	.423	.147	.103	.162	.019	8.481	8.224
.10	.20	.30	.40	.50	.36	.324	.417	.170	.101	.158	.026	6.177	5.990
.10	.20	.30	.40	.50	.38	.320	.410	.193	.098	.153	.033	4.661	4.520
.10	.20	.30	.40	.50	.40	.317	.404	.216	.096	.148	.041	3.615	3.506
.10	.20	.30	.40	.50	.42	.313	.397	.238	.094	.143	.050	2.866	2.779
.10	.20	.30	.40	.50	.44	.309	.391	.261	.091	.139	.060	2.312	2.242
.10	.20	.30	.40	.50	.46	.305	.384	.284	.089	.134	.071	1.893	1.836
.10	.20	.30	.40	.50	.48	.301	.378	.306	.087	.130	.083	1.569	1.522
.10	.20	.30	.40	.50	.50	.297	.372	.329	.085	.125	.095	1.314	1.274
.10	.20	.30	.40	.50	.52	.293	.365	.352	.082	.121	.109	1.111	1.077
.10	.20	.30	.40	.50	.54	.289	.359	.375	.080	.117	.124	.946	.917
.10	.20	.30	.40	.50	.56	.285	.352	.397	.078	.113	.139	.811	.786
.10	.20	.30	.40	.50	.58	.281	.346	.420	.076	.109	.155	.699	.678
.10	.20	.30	.40	.50	.60	.278	.339	.443	.074	.105	.173	.606	.588
.10	.20	.30	.40	.50	.62	.274	.333	.465	.072	.101	.191	.528	.512
.10	.20	.30	.40	.50	.64	.270	.327	.488	.070	.097	.210	.462	.448
.10	.20	.30	.40	.50	.66	.266	.320	.511	.068	.093	.230	.405	.393
.10	.20	.30	.40	.50	.68	.262	.314	.533	.066	.089	.251	.357	.346
.10	.20	.30	.40	.50	.70	.258	.307	.556	.064	.086	.272	.315	.305
.10	.20	.30	.40	.50	.72	.254	.301	.579	.062	.082	.295	.279	.270
.10	.20	.30	.40	.50	.74	.250	.294	.602	.060	.079	.319	.247	.240
.10	.20	.30	.40	.50	.76	.246	.288	.624	.058	.075	.343	.220	.213
.10	.20	.30	.40	.50	.78	.242	.282	.647	.056	.072	.369	.195	.189
.10	.20	.30	.40	.50	.80	.239	.275	.670	.055	.069	.395	.174	.169
.10	.20	.30	.40	.50	.82	.235	.269	.692	.053	.066	.422	.155	.151
.10	.20	.30	.40	.50	.84	.231	.262	.715	[illegible]	.063	.450	.139	.135
.10	.20	.30	.40	.50	.86	.227	.256	.738	[illegible]	.060	.480	.124	.120
.10	.20	.30	.40	.50	.88	.223	.250	.761	.048	.057	.509	.111	.108
.10	.20	.30	.40	.50	.90	.219	.243	.783	.046	.054	.540	.099	.096
.10	.20	.30	.40	.50	.92	.215	.237	.806	.044	.051	.572	.089	.086
.10	.20	.30	.40	.50	.94	.211	.230	.829	.043	.048	.605	.080	.077

Table 29

Effect of Changing the Intercorrelations Equally With the Other Correlations Different and Remaining Constant

r_{12}	r_{13}	r_{23}	r_{Y1}	r_{Y2}	r_{Y3}	β_1	β_2	β_3	r'_{Inc1}	r'_{Inc2}	r'_{Inc3}	r'_{Inc2}/r'_{Inc3}	β'_2/β'_3
-.24	-.24	-.24	.30	.40	.50	.689	.769	.850	.402	.502	.613	.819	.819
-.22	-.22	-.22	.30	.40	.50	.632	.714	.796	.350	.447	.555	.805	.805
-.20	-.20	-.20	.30	.40	.50	.583	.667	.750	.306	.400	.506	.790	.790
-.18	-.18	-.18	.30	.40	.50	.540	.625	.710	.269	.360	.464	.775	.775
-.16	-.16	-.16	.30	.40	.50	.502	.588	.674	.237	.325	.427	.761	.761
-.14	-.14	-.14	.30	.40	.50	.468	.556	.643	.209	.295	.395	.746	.746
-.12	-.12	-.12	.30	.40	.50	.437	.526	.616	.185	.268	.367	.731	.731
-.10	-.10	-.10	.30	.40	.50	.409	.500	.591	.164	.244	.341	.716	.716
-.08	-.08	-.08	.30	.40	.50	.384	.476	.569	.145	.224	.319	.701	.701
-.06	-.06	-.06	.30	.40	.50	.360	.455	.549	.129	.205	.299	.686	.686
-.04	-.04	-.04	.30	.40	.50	.339	.435	.531	.114	.188	.281	.671	.671
-.02	-.02	-.02	.30	.40	.50	.319	.417	.515	.101	.173	.265	.655	.655
.00	.00	.00	.30	.40	.50	.300	.400	.500	.090	.160	.250	.640	.640
.02	.02	.02	.30	.40	.50	.283	.385	.487	.080	[illegible]	.237	.625	.625
.04	.04	.04	.30	.40	.50	.266	.370	.475	.071	.137	.224	.609	.609
.06	.06	.06	.30	.40	.50	.251	.357	.464	.062	.127	.213	.594	.594
.08	.08	.08	.30	.40	.50	.236	.345	.454	.055	.117	.203	.578	.578
.10	.10	.10	.30	.40	.50	.222	.333	.444	.048	.109	.194	.563	.562
.12	.12	.12	.30	.40	.50	.209	.323	.436	.043	.101	.185	.547	.547
.14	.14	.14	.30	.40	.50	.196	.313	.429	.037	.094	.178	.531	.531
.16	.16	.16	.30	.40	.50	.184	.303	.422	.032	.088	.170	.515	.515
.18	.18	.18	.30	.40	.50	.172	.294	.416	.028	.082	.164	.500	.500
.20	.20	.20	.30	.40	.50	.161	.286	.411	.024	.076	.157	.484	.484
.22	.22	.22	.30	.40	.50	.150	.278	.406	.021	.071	.152	.468	.468
.24	.24	.24	.30	.40	.50	.139	.270	.402	.017	.066	.146	.452	.452
.26	.26	.26	.30	.40	.50	.128	.263	.398	.015	.062	.142	.437	.437
.28	.28	.28	.30	.40	.50	.118	.256	.395	.012	.058	.137	.421	.421
.30	.30	.30	.30	.40	.50	.107	.250	.393	.010	.054	.133	.405	[illegible]
.32	.32	.32	.30	.40	.50	.097	.244	.391	.008	.050	.129	.389	.389
.34	.34	.34	.30	.40	.50	.087	.238	.390	.006	.047	.126	.373	.373
.36	.36	.36	.30	.40	.50	.076	.233	.389	.005	.044	.122	.358	.358
.38	.38	.38	.30	.40	.50	.066	.227	.389	.003	.041	.119	.342	.342
.40	.40	.40	.30	.40	.50	.056	.222	.389	.002	.038	.117	.327	.327
.42	.42	.42	.30	.40	.50	.045	.217	.390	.002	.036	.114	.311	.311
.44	.44	.44	.30	.40	.50	.034	.213	.391	.001	.033	.112	.296	.296
.46	.46	.46	.30	.40	.50	.023	.208	.394	.000	.031	.110	.280	.280
.48	.48	.48	.30	.40	.50	.012	.204	.396	.000	.029	.108	.265	.265
.50	.50	.50	.30	.40	.50	.000	.200	.400	.000	.027	.107	.250	.250
.52	.52	.52	.30	.40	.50	-.012	.196	.404	.000	.025	.105	.235	.235
.54	.54	.54	.30	.40	.50	-.025	.192	.410	.000	.023	.104	.220	.220
.56	.56	.56	.30	.40	.50	-.039	.189	.416	.001	.021	.103	.206	.206
.58	.58	.58	.30	.40	.50	-.053	.185	.423	.002	.020	.103	.191	.191
.60	.60	.60	.30	.40	.50	-.068	.182	.432	.003	.018	.103	.177	.177
.62	.62	.62	.30	.40	.50	-.085	.179	.442	.004	.017	.103	.163	.163
.64	.64	.64	.30	.40	.50	-.102	.175	.453	.005	.015	.103	.150	.150
.66	.66	.66	.30	.40	.50	-.122	.172	.467	.007	.014	.103	.137	.137
.68	.68	.68	.30	.40	.50	-.143	.169	.482	.009	.013	.104	.124	.124
.70	.70	.70	.30	.40	.50	-.167	.167	.500	.012	.012	.106	.111	.111
.72	.72	.72	.30	.40	.50	-.193	.164	.521	.015	.011	.108	.099	.099
.74	.74	.74	.30	.40	.50	-.223	.161	.546	.018	.010	.110	.087	.087
.76	.76	.76	.30	.40	.50	-.258	.159	.575	.023	.009	.114	.076	.076
.78	.78	.78	.30	.40	.50	-.[illegible]98	.156	.611	.028	.008	.118	.065	.065
.80	.80	.80	.30	.40	.50	-.346	.154	.654	.035	.007	.124	.055	.055
.82	.82	.82	.30	.40	.50	-.404	.152	.707	.043	.006	.131	.046	.046
.84	.84	.84	.30	.40	.50	-.476	.149	.774	.053	.005	.140	.037	.037
.86	.86	.86	.30	.40	.50	-.567	.147	.861	.066	.004	.152	.029	.029
.88	.88	.88	.30	.40	.50	-.688	.145	.978	.083	.004	.169	.022	.022
.90	.90	.90	.30	.40	.50	-.857	.143	1.143	.108	.003	.192	.016	.016
.92	.92	.92	.30	.40	.50	-1.11	.141	1.391	.146	.002	.229	.010	.010
.94	.94	.94	.30	.40	.50	-1.53	.139	1.806	.208	.002	.290	.006	.006
.96	.96	.96	.30	.40	.50	-2.36	.137	2.637	.333	.001	.414	.003	.003

Table 30

Beta and Incremental Ratios of Different Signs

β'_1	β'_2	$\beta'_1 - \beta'_2$	$(\beta'_1 - \beta'_2) - (r^2_{Inc1} - r^2_{Inc2})$	$r^2_{Inc1} - r^2_{Inc2}$	r^2_{Inc1}	r^2_{Inc2}	r_{12}	r_{13}	r_{23}	r_{Y1}	r_{Y2}	r_{Y3}
.227	.185	.042	.049	-.007	.168	.175	.20	.50	.20	-.50	-.50	-.20
.203	.174	.029	.048	-.019	.147	.167	.20	.50	.10	-.50	-.50	-.20
.179	.173	.006	.043	-.037	.127	.163	.20	.50	.00	-.50	-.50	-.20
.128	.109	.018	.025	-.007	.074	.081	.50	.50	.20	-.50	-.50	-.20
.175	.176	-.001	-.009	.008	.164	.156	.20	.20	.30	-.50	-.50	-.20
.026	.035	-.009	-.021	.012	.023	.011	-.20	.00	.80	-.20	.50	.50
.180	.193	-.013	-.024	.010	.170	.159	.20	.20	.40	-.50	-.50	-.20
.055	.077	-.022	-.039	.017	.035	.019	-.50	-.20	.80	-.50	.90	.90
.185	.227	-.042	-.049	.007	.175	.168	.20	.20	.50	-.50	-.50	-.20
.040	.077	-.037	-.049	.012	.040	.028	.00	.00	.80	-.20	.50	.50
.040	.077	-.037	-.049	.012	.040	.028	.00	.00	.80	.20	.50	.50
.040	.069	-.029	-.056	.027	.040	.013	.00	.00	.90	-.20	.50	.50
.040	.069	-.029	-.056	.027	.040	.013	.00	.00	.90	.20	.50	.50
.099	.216	-.118	-.134	.017	.094	.078	-.20	-.20	.80	-.50	.90	.90
.099	.216	-.118	-.134	.017	.094	.078	.20	.20	.80	.50	.90	.90
.105	.193	-.088	-.152	.064	.101	.037	-.20	-.20	.90	-.50	.90	.90
.105	.193	-.088	-.152	.064	.101	.037	.20	.20	.90	.50	.90	.90

Table 31

Changing r_{12} in a Situation With Inconsistent Beta and Incremental Ratios

r_{12}	r_{13}	r_{23}	r_{Y1}	r_{Y2}	r_{Y3}	β_1	β_2	β_3	r^2_{Inc1}	r^2_{Inc2}	r^2_{Inc3}	r^2_{Inc1}/r^2_{Inc2}	β'_1/β_2
.20	.20	.80	.50	.90	.90	.314	.465	.465	.094	.078	.078	1.215	.456
.22	.20	.80	.50	.90	.90	.305	.449	.480	.089	.072	.083	1.232	.462
.24	.20	.80	.50	.90	.90	.297	.434	.493	.083	.067	.088	1.250	.469
.26	.20	.80	.50	.90	.90	.290	.420	.506	.078	.062	.092	1.271	.477
.28	.20	.80	.50	.90	.90	.283	.406	.519	.073	.057	.097	1.293	.485
.30	.20	.80	.50	.90	.90	.276	.393	.531	.069	.052	.101	1.318	.494
.32	.20	.80	.50	.90	.90	.270	.380	.542	.065	.048	.105	1.346	.505
.34	.20	.80	.50	.90	.90	.264	.368	.553	.061	.044	.108	1.378	.517
.36	.20	.80	.50	.90	.90	.259	.356	.563	.057	.040	.111	1.413	.530
.38	.20	.80	.50	.90	.90	.254	.345	.573	.053	.037	.114	1.453	.545
.40	.20	.80	.50	.90	.90	.250	.333	.583	.050	.033	.117	1.500	.563
.42	.20	.80	.50	.90	.90	.246	.322	.593	.047	.030	.119	1.554	.583
.44	.20	.80	.50	.90	.90	.243	.311	.602	.044	.027	.120	1.618	.607
.46	.20	.80	.50	.90	.90	.239	.300	.612	.041	.024	.121	1.693	635
.48	.20	.80	.50	.90	.90	.237	.289	.621	.038	.021	.122	1.785	.669
.50	.20	.80	.50	.90	.90	.235	.278	.630	.035	.019	.122	1.898	.712
.52	.20	.80	.50	.90	.90	.233	.267	.640	.033	.016	.121	2.042	.766
.54	.20	.80	.50	.90	.90	.233	.254	.650	.030	.014	.120	2.228	.836
.56	.20	.80	.50	.90	.90	.233	.241	.660	.028	.011	.118	2.480	.930
.58	.20	.80	.50	.90	.90	.234	.227	.672	.026	.009	.115	2.836	1.063
.60	.20	.80	.50	.90	.90	.237	.211	.684	.024	.007	.111	3.375	1.266
.62	.20	.80	.50	.90	.90	.242	.191	.699	.022	.005	.106	4.271	1.602
.64	.20	.80	.50	.90	.90	.250	.167	.717	.020	.003	.100	6.001	2.250
.66	.20	.80	.50	.90	.90	.264	.134	.740	.018	.002	.093	10.34	3.876
.68	.20	.80	.50	.90	.90	.287	.085	.774	.017	.001	.084	30.40	11.39
.70	.20	.80	.50	.90	.90	.333	.000	.833	.017	.000	.074	.	.
.72	.20	.80	.50	.90	.90	.450	-.200	.970	.018	.001	.063	13.50	5.062
.74	.20	.80	.50	.90	.90	1.174	-1.39	1.778	.035	.019	.064	1.898	.712

Table 32

Changing r_{12} in a Situation With Inconsistent Beta and Incremental Ratios

r_{12}	r_{13}	r_{23}	r_{Y1}	r_{Y2}	r_{Y3}	β_1	β_2	β_3	r^2Inc1	r^2Inc2	r^2Inc3	r^2Inc1/r^2Inc2	$\beta^2 1/\beta_2$
-.36	.50	.20	-.50	-.50	-.20	-1.17	-1.04	.594	.727	.734	.206	.990	1.268
-.34	.50	.20	-.50	-.50	-.20	-1.11	-.990	.555	.680	.688	.183	.990	1.267
-.32	.50	.20	-.50	-.50	-.20	-1.06	-.944	.520	.638	.645	.164	.989	1.266
-.30	.50	.20	-.50	-.50	-.20	-1.01	-.902	.488	.600	.607	.146	.988	1.265
-.28	.50	.20	-.50	-.50	-.20	-.971	-.863	.458	[illegible]	.572	.131	.988	1.264
-.26	.50	.20	-.50	-.50	-.20	-.931	-.828	.431	.533	.540	.118	.987	1.263
-.24	.50	.20	-.50	-.50	-.20	-.894	-.796	.406	.503	.510	.106	.986	1.262
-.22	.50	.20	-.50	-.50	-.20	-.860	-.766	.383	[illegible]	.483	.095	.985	1.261
-.20	.50	.20	-.50	-.50	-.20	-.829	-.738	.362	.451	.458	.086	.985	1.260
-.18	.50	.20	-.50	-.50	-.20	-.799	-.712	[illegible]	.427	.434	.078	.984	1.259
-.16	.50	.20	-.50	-.50	-.20	-.772	-.688	.324	.405	.412	.070	.983	1.258
-.14	.50	.20	-.50	-.50	-.20	-.746	[illegible]	.306	.384	.391	.063	.982	1.257
-.12	.50	.20	-.50	-.50	-.20	-.722	-.645	.290	.365	.372	.057	.981	1.256
-.10	.50	.20	-.50	-.50	-.20	-.700	-.625	.275	.347	.354	.052	.980	1.254
-.08	.50	.20	-.50	-.50	-.20	-.679	-.606	.261	.330	.337	.047	.979	1.253
-.06	.50	.20	-.50	-.50	-.20	-.659	-.589	.247	.314	.321	.043	.978	1.252
-.04	.50	.20	-.50	-.50	-.20	-.640	-.573	.235	.299	[illegible]	.039	.977	1.250
-.02	.50	.20	-.50	-.50	-.20	-.622	-.557	.223	.285	.292	.035	.976	1.249
.00	.50	.20	-.50	-.50	-.20	-.606	-.542	.211	.271	.278	.032	.975	1.247
.02	.50	.20	-.50	-.50	-.20	-.590	-.528	.201	.258	.266	.029	.973	1.246
.04	.50	.20	-.50	-.50	-.20	-.575	-.515	.190	.246	.253	.026	.972	1.244
.06	.50	.20	-.50	-.50	-.20	-.560	-.503	.181	.235	.242	.024	.971	1.243
.08	.50	.20	-.50	-.50	-.20	-.546	-.491	.171	.224	.231	.021	.969	1.241
.10	.50	.20	-.50	-.50	-.20	-.533	-.479	.163	.213	.220	.019	.968	1.239
.12	.50	.20	-.50	-.50	-.20	-.521	-.468	.154	.203	.210	.017	.966	1.237
.14	.50	.20	-.50	-.50	-.20	-.509	-.458	.146	.194	.201	.016	.965	1.235
.16	.50	.20	-.50	-.50	-.20	-.497	-.448	.138	.185	.192	.014	.963	1.233
.18	.50	.20	-.50	-.50	-.20	-.487	-.439	.131	.176	.183	.013	.961	1.230
.20	.50	.20	-.50	-.50	-.20	-.476	-.430	.124	.168	.175	.011	.959	1.228
.22	.50	.20	-.50	-.50	-.20	-.466	-.421	.117	.160	.167	.010	.958	1.226
.24	.50	.20	-.50	-.50	-.20	-.456	-.413	.111	.152	.159	.009	.955	1.223
.26	.50	.20	-.50	-.50	-.20	-.447	-.405	.104	.145	.152	.008	.953	1.220
.28	.50	.20	-.50	-.50	-.20	-.438	-.397	.098	.137	.145	.007	.951	1.217
.30	.50	.20	-.50	-.50	-.20	-.429	-.390	.093	.131	.138	.006	.949	1.214
.32	.50	.20	-.50	-.50	-.20	-.421	-.383	.087	.124	.131	.006	.946	1.211
.34	.50	.20	-.50	-.50	-.20	-.413	-.376	.082	.118	.125	.005	.943	1.207
.36	.50	.20	-.50	-.50	-.20	-.405	-.369	.077	.112	.119	.004	.940	1.204
.38	.50	.20	-.50	-.50	-.20	-.398	-.363	.072	.106	.113	.004	.937	1.200
.40	.50	.20	-.50	-.50	-.20	-.390	-.357	.067	[illegible]	.107	.003	.934	1.195
.42	.50	.20	-.50	-.50	-.20	[illegible]	-.351	.062	.095	.102	.003	.930	1.191
.44	.50	.20	-.50	-.50	-.20	-.377	-.346	.057	[illegible]	.096	.002	.926	1.186
.46	.50	.20	-.50	-.50	-.20	-.370	-.340	.053	.084	.091	.002	.922	1.181
.48	.50	.20	-.50	-.50	-.20	-.363	-.335	.049	.079	.086	.002	.918	1.175
.50	.50	.20	-.50	-.50	-.20	-.357	-.330	.045	.074	[illegible]	.001	.913	1.169
.52	.50	.20	-.50	-.50	-.20	-.351	-.326	.041	.070	.077	.001	.908	1.162
.54	.50	.20	-.50	-.50	-.20	-.345	-.321	.037	[illegible]	.072	.001	.902	1.155
.56	.50	.20	-.50	-.50	-.20	-.339	-.317	.033	.061	.068	.001	.896	1.147
.58	.50	.20	-.50	-.50	-.20	-.333	-.313	.029	.057	.064	.001	.889	1.138
.60	.50	.20	-.50	-.50	-.20	-.328	-.309	.026	.053	.060	.000	.881	1.128
.62	.50	.20	-.50	-.50	-.20	-.322	-.305	.022	.049	.056	.000	.873	1.117
.64	.50	.20	-.50	-.50	-.20	-.317	-.301	.018	.045	.052	.000	.863	1.105
.66	.50	.20	-.50	-.50	-.20	-.311	-.298	.015	.041	.048	[illegible]	.853	1.091
.68	.50	.20	-.50	-.50	-.20	-.306	-.295	.012	.037	.044	[illegible]	.840	1.076
.70	.50	.20	-.50	-.50	-.20	-.306	-.292	.008	.034	.041	.000	.827	1.058
.72	.50	.20	-.50	-.50	-.20	-.294	-.289	.005	.030	.037	.000	.810	1.037
.74	.50	.20	-.50	-.50	-.20	-.289	-.287	.002	.027	.034	.000	.792	1.014
.76	.50	.20	-.50	-.50	-.20	-.283	-.285	-.002	.024	.031	[illegible]	.770	.985
.78	.50	.20	-.50	-.50	-.20	-.276	-.283	-.005	.020	.028	.000	.743	.951
.80	.50	.20	-.50	-.50	-.20	-.270	-.283	-.009	.017	.024	.000	.711	.910
.82	.50	.20	-.50	-.50	-.20	-.262	-.283	-.013	.014	.021	.000	.670	.858
.84	.50	.20	-.50	-.50	-.20	-.253	-.284	-.017	.011	.019	.000	.619	.792
.86	.50	.20	-.50	-.50	-.20	-.242	-.288	-.022	.009	.016	.000	.550	.704
.88	.50	.20	-.50	-.50	-.20	-.226	-.296	-.028	.006	.013	.000	.456	.583
.90	.50	.20	-.50	-.50	-.20	-.200	-.313	-.038	.003	.010	[illegible]	.320	.410
.92	.50	.20	-.50	-.50	-.20	-.143	-.357	-.057	.001	.008	.001	.125	.160
.94	.50	.20	-.50	-.50	-.20	.167	-.625	-.158	.000	.008	.003	.056	.071

Conclusions

β values can be used for determining the importance of predictors within an equation but the interpretation is complex. With three or more predictors more caution is needed in this type of interpretation. In evaluating the importance of a variable it is wise to consider the zero-order correlation coefficient, β, r^2_{Inc} and r^2_{Part} and whether suppression exists. It is especially helpful to evaluate the $\beta_i r_{Yi}$ products as they contribute to R^2.

Reference

Pedhazur, E. J. Multiple Regression in Behavioral Research, 2nd Edition. New York, NY: Holt, Rinehart, and Winston.