DOCUMENT RESUME

ED 333 085 UD 028 108

AUTHOR	Middleton, James A.; And Others
TITLE	Teachers' Conceptions of Mathematics and Mathematics Education. A Report from the Urban Mathematics Collaborataive Documentation Project.
INSTITUTION	Wisconsin Center for Education Research, Madison.
SPONS AGENCY	Ford Foundation, New York, N.Y.
PUB DATE	Nov 90
NOTE	207p.
PUB TYPE	Reports - Research/Technical (143)
EDRS PRICE	MF01/PC09 Plus Postage.
DESCRIPTORS	Attitude Measures; Cluster Grouping; *Mathematics Education; *Mathematics Teachers; Questionnaires; Secondary Education; *Teacher Attitudes; Teacher Morale; *Urban Schools

ABSTRACT

This paper reports the opinions of urban mathematics teachers concerning five areas of school mathematics: mathematics, mathematics teaching, recommended changes in mathematics education, mathematics education, and schooling. Of the 490 secondary mathematics teachers surveyed, 47 percent were frequent participants in the Urban Mathematics Collaborative (UMC), 41 percent were occasional participants, and 8 percent had never participated. An additional group of 40 UMC teachers were asked to respond to corresponding items on the Diary of Professional Relationships survey to provide more personalized and diverse information and to validate findings from group data. Responses indicate that teachers view mathematics primarily as thinking. They want their students to think critically, to understand and use mathematics effectively, and to appreciate the value and beauty of mathematics. Most teachers seemed to hold an eclectic view of mathematics, although one cluster group viewed it as dynamic and changing, while another group viewed it more as a fixed body of skills and rules. These conceptions of mathematics related to the teachers' conceptions of mathematics teaching, recommended changes, mathematics education, and schooling. Frequent participants in the UMC held more favorable views toward recommended changes in mathematics education than the others. The results of this survey are discussed in relation to the UMC's efforts to empower teachers and reduce their feelings of isolation and burnout. Results of the survey are summarized in three tables and one figure. Forty-one references and seven appendixes (with over 100 tables) are included. (CJS)

Teachers' Conceptions of Mathematics
and Mathematics Education

James A. Middleton, Norman L. Webb, Thomas A. Romberg, and Susan D. Pittelman

A Report from
the Urban Mathematics Collaborative Documentation Project

Wisconsin Center for Education Research
School of Education
University of Wisconsin-Madison
November 1990

The authors wish to thank Jack Bard and Margaret Powell for their help in preparing and editing this paper.

The research reported in this paper was supported by the Ford Foundation and by the Wisconsin Center for Education Research, School of Education, University of Wisconsin-Madison. The opinions expressed in this publication are those of the authors and do not necessarily reflect the views of the Ford Foundation or the Wisconsin Center for Education Research.

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Continued)

LIST OF TABLES

LIST OF TABLES (Continued)

Table Page

LIST OF TABLES (Continued)

LIST OF TABLES (Continued)

LIST OF TABLES (Continued)

LIST OF FIGURES

ABSTRACT

Four hundred ninety secondary mathematics teachers at eleven urban sites around the United States were surveyed to discover their conceptions of mathematics, mathematics teaching, recommended change in mathematics education, mathematics education, and schooling. The teachers were situated in sites targeted by the Ford Foundation for its Urban Mathematics Collaborative (UMC) Project. Forty-seven percent of respondents were frequent participants in the UMC project, 4: percent were occasional participants, and 8 percent had never participated in UMC activities. An additional group of 40 UMC teachers were asked to respond in writing to corresponding items on the Diary of Professional Relationships survey. Responses, in general, indicate that teachers view mathematics primarily as thinking. They want their students to think critically, to understand and use mathematics effectively, and to appreciate the value and beauty of mathematics.

Teachers' conceptions were examined in relation to profiles of their responses across six hypothesized conceptions of mathematics: (1) mathematics as a process in which abstract ideas are applied to solve real-world problems; (2) mathematics as a language, used to represent and communicate ideas; (3) mathematics as a collection of concepts and skills; (4) mathematics as thinking in a logical, scientific manner, as a means to develop understanding; (5) mathematics as facts, skills, rules and concepts learned in a particular sequence and applied in work and future study; and (6) mathematics as an interconnected logical system, dynamic and changing, formed by thinking about actions and experiences. Four exclusive clusters of teachers were found to differ in their views toward mathematics. The majority of teachers seemed to hold an eclectic view of mathematics, but two groups in particular differed. One of these groups viewed mathematics as dynamic and changing, while the other viewed mathematics more as a fixed body of skills and rules. These conceptions of the nature of mathematics were found to be related to teachers' conceptions of mathematics teaching, recommended change, mathematics education, and schooling.

In addition, teachers' responses were analyzed by their level of participation in UMC activities. These responses suggest that the UMC project has had a positive impact on teachers' knowledge of, and approach to, teaching mathematics. Frequent participants held more favorable views towards a number of recommended changes in mathematics education than Occasional or Nonparticipants. In addition, Frequent participants reported to a greater extent than both Occasional and Nonparticipants that they enjoy teaching mathematics. These findings were supported by teachers' written responses in the Diary of Professional Relationships.

When examined for differences across the eleven UMC sites, the patterns in teachers' responses point out the importance of considering site-factors in developing teacher empowerment

projects such as the UMC endeavor. Not surprisingly, the role of technology, in particular, seems to be a direct reflection of the quantity of technological materials available for use by teachers--an availability that is highly variable across sites. In addition, teachers from different sites held differing views on the place of competition in motivating students and on issues concerning minimum competency standards and the prognostic testing of students.

The results of this survey are discussed in relation to the effort of the UMC project to empower teachers and ultimately to help reduce their feelings of isolation and burnout.

I. INTRODUCTION

This paper reports the opinions of urban mathematics teachers concerning five areas of school mathematics: Mathematics, mathematics teaching, recommended changes in mathematics education, mathematics education, and schooling in light of the NCTM's new *Curriculum and Evaluation Standards for School Mathematics* (1989) and mathematics education reform. The present document is intended initially as a description of teachers' conceptions of mathematics and the relationship between these conceptions and teachers' opinions in each of the other four areas. It is predicted that differences in teachers' conceptions of the nature of mathematics will reflect differences in opinions on how mathematics is taught, and what changes are deemed necessary for the effective teaching of mathematics. These descriptions will be used in conjunction with a variety of other data to document the evolution of the Urban Mathematics Collaborative project (Webb, Pittelman, Romberg, Pitman, Fadell, & Middleton, 1989) as one facet of the mathematics education movement.

The Urban Mathematics Collaborative Project

The Urban Mathematics Collaborative (UMC) project was initiated in 1984 in order to improve mathematics education in urban schools and to identify new models for meeting the professional needs of high school teachers by (1) exposing them to new trends in the field of mathematics and (2) fostering a sense of support from and collegiality with mathematicians in both business and universities, as well as with other mathematics teachers. Underlying the purpose of the UMC project is the assumption that teachers are the key to educational reform, advancement, and quality. Through support of the Ford Foundation, both of its human and financial resources, it was predicted that local organizations of mathematics teachers in several urban sites across the United States could reduce teacher feelings of isolation, engender a renewed sense of professionalism and enthusiasm, and ultimately encourage innovative teaching practices, such that mathematics education in these sites would be qualitatively improved (Ford Foundation, 1987; Middleton, Webb, Romberg, Pittelman, Pitman, Richgels, & Fadell, 1989).

The Ford Foundation's effort to develop collaborative projects is a direct consequence of the current concern that American education is in a state of crisis. Recent national reports by the government, academic organizations, and private foundations argue that education in the United States has not been successful in providing *all* students with the skills, concepts, and attitudes requisite for personal growth and advancement, and for maintaining the role of the United States as an international leader in business, industry, science, and technology. School mathematics has been the focus of much of this criticism (Romberg, 1984).

The Second International Mathematics Study (SIMS) raised concern about the effectiveness of mathematics education in the United States compared to programs in other countries (Crosswhite, Dossey, Swafford, McKnight, & Cooney, 1985). Results indicated that mathematics students in the United States did not perform well based on international standards. Compared to students in other countries, eighth-grade mathematics students in the United States were found to demonstrate only average proficiency in mathematics. In addition, United States twelfth graders who were enrolled in regular college preparatory precalculus courses scored only at the 25th percentile when compared to the international sample. The study concluded that in the United States, mathematics is taught in a "fragmented" fashion--topics are treated without any attempt to integrate them into a cohesive, unified framework. Further, the SIMS study concluded that many mathematics programs in the United States were "low intensity," not preparing students adequately for further study.

Results from the first and second assessment of the National Assessment of Educational Progress (NAEP) indicated that United States mathematics students had a high degree of proficiency in routine computational skills, but showed severe deficiencies at all age levels studied (9, 13, and 17 years) in solving nonroutine problems. In addition, the majority of students at all age levels showed deficiencies in other basic skill areas including geometry, measurement, and probability and statistics. These studies concluded that students tended to be passive recipients of mathematical knowledge; that they tended to feel as if they had little opportunity to interact with their class and discuss mathematical concepts (Carpenter, Coburn, Reys, and Wilson, 1979; Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981).

Compounding these problems, additional reports have indicated that the majority of secondary school mathematics teachers in the United States do not meet current professional standards (National Research Council, 1989), and that over 15 percent of teachers in the United States teach courses for which they are not certified.

The Holmes Group (1986) emphasized the need for teachers of all disciplines to be thoroughly grounded in their subject matter in order to provide an adequate articulation of knowledge. They recommend that only college graduates with outstanding records in their subject area should be acceptable as teachers. Teachers must understand the nuances of their subject sufficiently to meet the individual needs of all students, from the most basic to the most advanced. Teacher education innovations cannot be implemented only for incoming teachers; individuals who are already certified and teaching need the opportunity to advance their knowledge of mathematics as innovations in the field arise. For those teachers already in the classroom, greater autonomy is more likely to come with increased subject matter knowledge and with greater knowledge of teaching and practical experience in pedagogy (Maeroff, 1988).

One of the initial goals of the Ford Foundation in establishing the UMC project was to enable teachers to enlarge their conception of mathematics and their repertoire of classroom strategies and skills related to mathematics in order to be able to integrate mathematical concepts into a cohesive sequence in their teaching (Romberg, 1984). One intent of the Ford Foundation was to provide nondirective support for each of the collaborative sites in order to respond to the unique needs and potential direction of each individual site.

In 1984, collaboratives were initiated in five cities: Cleveland, Minneapolis/St. Paul, Los Angeles, Philadelphia, and San Francisco. Within 18 months of the conception of the project, six more sites were added--Durham, Pittsburgh, San Diego, St. Louis, Memphis and New Orleans--making a total of eleven Urban Mathematics Collaboratives. Each site is autonomous, yet exists within the support structure of the entire project. Each has the responsibility for gathering support (both financial and human) from local sources so that, eventually, each site will be self-sustaining.

At the start of the project, the Ford Foundation established a Documentation Project at the University of Wisconsin-Madison to chronicle the progress of individual collaboratives and each collaborative's efforts in fulfilling the goals of the project. The Documentation Project was designed to gather information from sites for a period of six years (1985 through 1990), until all of the sites had the potential to evolve to a permanent structure. The Education Development Center, Inc. (EDC), a non-profit research and development organization located in Newton, Massachusetts, was engaged by the Ford Foundation to provide technical assistance to the collaboratives, to disseminate information about the UMC project, and to facilitate expansion of the collaborative concept to other sites. The Documentation Project and the Technical Assistance Project (TAP) located at EDC, link the eleven collaboratives by sharing ideas and individual successes among sites and by providing an informational link for teachers to use in communicating with participating teachers at other sites.

This research report is one in a series of reports designed to provide comprehensive information on the UMC project as a whole as each site continues the effort to meet its own unique needs and to illuminate facets of the project that may be of help to urban mathematics teachers around the country.

Factors Influencing Teachers' Conceptions of Mathematics

History

Teachers are profoundly influenced by the ways in which they were taught mathematics during their own schooling (Maeroff, 1988). Of the target population for the present study, teachers who are frequent participants in the UMC project, 15 percent took their last college course for credit between 1961 and 1970, during the peak of the New Math era; 38 percent took their last college course in the 1970s, at the height of the Back-to-Basics movement; and 46 percent took their last college course after 1980, the period of the current reform movement (Middleton et al., 1989). Such diverse mathematics backgrounds suggest potential differences in teachers' conceptions of mathematics and of mathematics education.

For most of the first half of the twentieth century, behaviorism dominated the education and psychology fields; consequently, the dominant conception of school mathematics was that of a hierarchy of skills that enabled students to perform with a high degree of accuracy on prescribed problems (Resnick & Ford, 1981). This resulted in an approach to teaching that focused on a detailed hierarchy of behavioral objectives, skills at the bottom of the hierarchy requisite for eliciting behaviors higher in the chain. Instruction reinforced lower-order skills until a high degree of accuracy was achieved. Then, behaviors would be combined (such as performing operations within parentheses first, *then* dividing by the common denominator) to solve more complex problems.

During the New Math movement of the 1960s, school mathematics was emphasized as a system of knowledge with a logical structure. Mathematicians and mathematics educators, in response to the post-Sputnik panic, reorganized the content of school mathematics into a framework that stressed the interconnections between mathematical concepts. Although proponents believed that individuals organized their own knowledge, they also believed that the organization inherent in the field of mathematics should be transmitted to students as a framework for attaching new or more specific concepts (NCTM, 1970). In addition, mathematical rigor and precision were seen as central to understanding the logic of mathematics. Note the difference in the objectives between the structuralist position of the New Math researchers and the behaviorists. The latter emphasized learning of discrete mathematical behaviors, while the former emphasized learning the organizational framework of mathematics content.

The disillusionment of some educators and especially the public following the New Math era led to the Back-to-Basics period of the 1970s and early 1980s. Although there was a renewed emphasis on behavioral objectives, successes during the New Math era were also incorporated into the curriculum, such as the use of manipulatives. However, Back-to-Basics appears to have been short-lived primarily due to the reaction to national reports such as the National Assessment of Educational Progress (Carpenter, Coburn, Reys, & Wilson, 1979; Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981), indicating the poor mathematical performance of students in the United States. Associated with this period was increasing pressure for accountability. With the decline in standardized test scores (e.g., SAT), the states reacted by legislating new graduation requirements and mandating the passing of competency tests. By 1984, nearly 80 percent of the states had some form of competency testing program.

The growing dominance of cognitive theories in the behavioral sciences, increased variety of applications of mathematical knowledge, a renewed emphasis on interconnectedness in the mathematical sciences, and the pervasive presence of the computer have served to dramatically change the field of mathematics itself, as well as the approaches taken by the education community to communicate these changes (Hilton, 1987). The concept of mathematics currently gaining in popularity, which forms the basis for the *Standards*, emphasizes the dynamic, changing nature of mathematics--the belief that mathematics is a human construction based on observation and reflection within an activity having some purpose (NCTM, 1989).

In addition, owing to the pervasiveness of a constructivist epistemology in educational psychology, mathematical meaning is seen as being unique to the individual; that is, the individual constructs his or her own conception of mathematics and gives meaning to that construction by attaching new mathematical information to previous knowledge of mathematics and ultimately to his or her way of viewing the world. A large percentage of UMC teachers have taught mathematics for 15 to 20 years (Middleton et al., 1989), and consequently have been exposed to periods of flux in the interpretation of what school mathematics should be. This suggests the potential for a large variation in what collaborative teachers may view as the nature of mathematics and mathematics education.

Conditions of Urban Teaching

Teachers in urban schools face difficulties and challenges unique to their urban environment. Overcrowding, student absences, linguistic factors, and students' lack of prior content knowledge greatly affect the ways in which urban teachers can articulate course content to their students (Middleton et al., 1989; Cole & Griffin, 1987). These contextual determinants of what mathematics is actually taught can affect teachers' views of the nature of school mathematics itself. Differences in the salience of contextual factors in different urban centers, then, should reflect differences in teachers' views about how mathematics should be taught and about the role of the mathematics teacher in both the classroom and the community (Popkewitz & Myrdal, in preparation).

Individual Differences

In addition, the degree of teacher training, professional development, and the autonomy teachers are afforded differ from one urban center to another (Middleton et al., 1989). These situational factors interact with conditions common to urban teachers and the social history of the teacher such that each teacher constructs a personal representation of the nature of mathematics and its relationship to mathematics education within his or her unique environment. Thus, positive, effective change in the ways in which mathematics is taught in our urban schools must focus on the teacher and his or her interaction with students, colleagues, and community.

As previously stated, one of the initial goals of the UMC project was to allow teachers the opportunity to develop their repertoires of mathematical knowledge and skills and to provide them with opportunities to interact with other teachers and with mathematicians in higher education, business, and industry (Romberg, 1984). Implicit in this goal is the assumption that teachers' beliefs about what constitutes the field of mathematics will greatly affect the ways in which they present the field of mathematics to their students. For example, a teacher with a behavioral perspective, i.e., a teacher who sees mathematics as an intricate chain of discrete behaviors, will tend to teach mathematics by drill and practice, while a constructivist will tend to use more discovery learning.

Unfortunately, or fortunately, depending on how you look at it, conceptions of mathematics are not so distinct. Individuals may embody behaviorist principles for certain aspects of their teaching and constructivist principles for others. To assess differences in teacher conceptions of mathematics, one must use a multivariate approach, i.e., one must analyze several factors that may interact in some significant fashion in forming an individual's personal concept. As always, when practices are changed and when emphases shift, teachers are the medium by which these shifts are actualized in the classroom. Historically, when teachers, and their conceptions of a subject and how it is taught and learned, have not been factored into curriculum development, a translation of the new *ideal* curriculum to the *actual* curriculum is made substantially more difficult due to the inertial characteristics of old teaching systems (e.g., Romberg, 1988).

Collaboration and Conceptions of Mathematics

An assumption implicit to the UMC effort is that teacher collaboration will qualitatively affect (i.e., change) individuals' conceptions of mathematics and mathematics education. Depending on the dynamics within each of the eleven UMC sites, participants should affect each others' views through professional dialogue. It was hoped that the opportunities for professional development, consistent with the spirit of mathematics education reform, would give participants flexibility in their thinking about mathematics such that they could model for their students a variety of problem-solving strategies. Thus, it was central to the purpose of this research to assess differences in teachers conceptions by their level of participation in collaborative events and programs.

All eleven UMC sites have sponsored activities emphasizing that mathematics undergoes constant change, and that school mathematics should change accordingly. These activities, combined with professional dialogue with other mathematics teachers, have the potential of impacting how teachers view mathematics as a discipline. Teachers in school districts targeted by the UMC project were surveyed to register their conceptions of mathematics and to detect the relationship, if any, that collaborative participation had in forming these conceptions. The results of the present survey were analyzed to provide a description of teachers' conceptions of mathematics and to understand how collaboration and other contextual factors influence these conceptions. These conceptions were analyzed not only by level of teacher participation, but also by site. In this way, the qualitative differences among UMC sites could be addressed, along with the patterns of participation both within and across collaboratives. The contrast across collaboratives will provide information on the influences of different activities and emphases on the development of the collaboratives that have been significant.

Conceptions of Mathematics

Buck (1965), in reaction to what he felt was an overemphasis on the structure of mathematics promoted by the New Math movement, advanced six universal goals for preparing effective instructional materials that reflect the interrelationship between mathematics and human understanding and action:

1. To provide an understanding of the interaction between mathematics and reality;
2. To convey the fact that mathematics, like most disciplines, is built upon intuitive understandings and agreed conventions, and that these are not eternally fixed;
3. To demonstrate that mathematics is a human activity, that its history is marked by inventions, discoveries, guesses (both good and bad), and that the frontier of its growth is covered by interesting unanswered questions;
4. To contrast "argument by authority" and "argument by evidence and proof"; to explain the difference between "not proved" and "disproved," and between a constructive proof and a nonconstructive proof;
5. To demonstrate that it is important to ask the question "Why?" and that, in mathematics, an answer is not always supplied by merely giving a detailed proof; and
6. To show that complex things are sometimes simple, and simple things are sometimes complex; and that, in mathematics as well as in other fields, it pays to subject a familiar thing to detailed study, and to sometimes study that which seems hopelessly intricate.

The conception of mathematics embodied by these six statements seems to be one of logical questioning and discovery. Although advanced in the early 1960s, these goals permeate the new *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) proposed by the National Council of Teachers of Mathematics as criteria for determining the worth of school mathematics programs and as a framework for developing new curricula. The *Standards*, the credo of the new reform movement in school mathematics, emphasize that students are to know mathematics as problem solving, reasoning, communication, and as a connection to other areas of study. The *Standards* also emphasize that students are to value mathematics and its cultural and historical role and to develop confidence in their own ability to do mathematics. The *Standards* place less emphasis on the behavioral objectives of the Back-to-Basics era and substantially more emphasis on the cognitive processes by which the mathematics student goes about solving problems--both real and theoretical. Yet at the same time, the *Standards* emphasize that students need to be able to *do* mathematical operations in order to solve mathematical problems.

Buck's original six goals were amplified by the NCTM into five ecumenical goals for curriculum and evaluation in school mathematics. According to the *Standards (pp. 5-6)*, students should:

1. Learn to value mathematics;
2. Become confident in their own abilities;
3. Become mathematical problem solvers;
4. Learn to communicate mathematically; and
5. Learn to reason mathematically.

These statements embody a conception of mathematics that is dynamic, creative, and both necessary in meeting and rooted in the real world.

In response to the question, What do teachers think about the current and future status of mathematics education? it is necessary to try to determine teachers' views regarding the nature and relative importance of the different facets of the discipline they teach. The five conceptions of the nature of mathematics discussed below provide an incomplete, but useful, taxonomy by which teachers' views can be assessed. Each conception relates to a specific nonexclusive facet of mathematics, thus we expect that the majority of teachers will agree to some extent with several facets. It is not necessarily the extent to which teachers agree to each facet that is of interest; rather, the pattern of responses across conceptions, both positive and negative, should reveal the ways in which teachers represent and operate on the field of mathematics in their roles of articulating this knowledge. In addition, teachers enter their classrooms with a repertoire of beliefs, attitudes, and skills that they use in determining the ways in which they approach mathematics teaching (McLeod, 1988). It follows that teachers' beliefs about what constitutes the field of mathematics, what constitutes mathematical knowledge, and how mathematical knowledge is acquired should greatly affect the ways in which they interact with students, administration, and other teachers. At the most basic level, mathematics can be conceptualized as either primarily number facts and algorithms, or primarily conceptual problem solving--doing *mathematics* vs. *doing* mathematics (e.g., Resnick & Ford, 1981). It is easy to envision how teachers who hold these differing views of mathematics would teach the same content in vastly different ways.

Mathematics Is a Collection of Skills To Be Used in the Workplace

Historically, mathematics education in the United States has existed for the express purpose of creating a competent workforce. Shopkeepers, clerks, industrial workers, and farmers needed to understand the basics of arithmetic, some Euclidean geometry, and perhaps some basic algebra in order to do their jobs effectively (NCTM, 1970). Only a small elite would advance in their mathematical knowledge at the college level. This approach to teaching mathematics, and consequently the views of a significant proportion of citizens, still centers on Back-to-Basics (Romberg, 1988).

Putnam and Leinhardt (1986) found that many successful mathematics teachers who hold this viewpoint developed "curriculum scripts" based on the skills and concepts they are required to teach. These scripts accounted solely for the content to be covered and differed very little according to the individual differences in their classes. This conception of how mathematics is learned provides predictability and coherence to mathematics teaching by establishing clear, well-defined objectives to be met by both teacher and student (Lampert, 1987).

These basic skills, although necessary, are no longer judged sufficient for adequate functioning in an information society (Romberg, 1988). Therefore, other conceptions regarding the general nature of mathematics interact with the emphasis on skills to provide the student with a workable knowledge of modern mathematics.

Mathematics Is a Language

Studying mathematics has sometimes been likened to the learning of a foreign language--i.e., learning grammar and vocabulary and manipulating them in such a way as to be meaningful to both the speaker and the audience (e.g., Pimm, 1987). Further, if the language metaphor is extended, mathematics may be seen as a medium for logical discourse, and "native speakers" of mathematics will be able to understand and communicate using this language far better than naturalized speakers. Thus, there has been increasing concern on the part of educators who hold this conception about the linguistic properties of mathematics and their influence on teaching.

Learning to communicate mathematically is included as one of the five overriding goals of the *Standards* (NCTM, 1989). The NCTM Commission states that development of students' power to use mathematics involves learning not only the signs, symbols, and terminology of mathematics, but also the oral and written communication of ideas that can be conveyed in the language of mathematics.

Teachers who view mathematics as a language may encourage student discourse and the articulation of mathematical concepts throughout the lesson (e.g., Pimm, 1987). In addition, these teachers encourage active listening, or attempt to ferret out the gist of what another is trying to communicate in a mathematical way.

Mathematics Is Application

There is growing controversy in the mathematics community about the extent to which mathematics *is* the uses to which "pure" mathematical concepts and procedures can be put. Many applied mathematicians in business and industry, as well as in the sciences, feel that the inspiration for all mathematical theories is ultimately grounded in the physical world (e.g., Halmos, 1981). Integral to this notion of the nature of mathematics is the concept of mathematical modeling (Spanier, 1981). The true nature of mathematics, then, is embodied in its capacity to describe, predict, and understand various aspects of the physical world. Although few people deny the fact that "pure" mathematics exists, applied mathematicians view *their* mathematics as a tool for solving the numerous problems associated with today's technological world, rather than as the topic of study itself.

This conception can be distinguished from mathematics as a "collection of skills to be used in the workplace" in that applied mathematicians value the ability to find new, more elegant or efficient ways of solving problems related to their work, while individuals espousing the collection-of-skills conception see mathematics as a tool that needs no modification to do predictable, specific jobs.

Mathematics Is a Way of Thinking

Others view mathematics as an aesthetic, logical way of developing understanding. The pursuit of mathematical knowledge is an end in itself, although the knowledge obtained may be useful in a variety of applications (Halmos, 1981). Mathematics, in and of itself, gives the people who adhere to this view a great deal of intrinsic joy, and this drives them to do more of it (e.g., Dorfler & McLone, 1986). This is perhaps the most classical view of mathematics, since it can be traced back to the Greek notion of mathematics as knowledge. The term *mathematics* is derived from the Greek *mathema*, which means literally "to know."

Mathematics Is a Dynamic System

The changing nature of mathematics is evident in the new fields that have arisen within the mathematical domain in the past hundred years. The computer revolution has provided the number-crunching capability that has transformed the field of statistics, for example, from a collection of relatively simple hypothesis-testing techniques into a powerful field that encompasses mathematical modeling, test theory, and trend analysis as well as other areas. The field of topology was virtually unknown 75 years ago. These innovations in the field of mathematics have engendered a view that mathematics is a dynamic system, drawing from knowledge and innovation in a variety of areas as new problem situations arise. Many of the new recommendations for mathematics education espouse this view (NCTM, 1989). This conception of mathematics is the only one that encompasses aspects of all five views.

The interconnectedness embodied by this conception is apparent in the mathematics community (Hilton, 1987; National Research Council, 1970). The NRC in particular emphasizes that mathematicians from diverse areas of the field are joining forces to develop new areas of study, algebraic geometry for instance. This confluence of topics and theory has opened up vast avenues for inquiry that can be viewed through a variety of mathematical lenses (e.g., NCTM, 1989).

It is expected that teachers in the present study will rate these conceptions in a manner that will reflect their personal conceptions regarding the nature of mathematics. It is assumed that these conceptions influence the way that teachers feel about mathematics instruction, recommended changes in the mathematics curriculum, important facets of mathematics education, and the implications of schooling for children and society. Their conceptions will also provide important information on the nature of their particular collaborative and on collaboration in general.

II. METHOD

Sample and Procedure

For each collaborative, a list of former respondents to UMC Documentation Project questionnaires was generated and distributed to the project coordinator or a designated representative. The coordinator or representative from each site was asked to distribute the *Teachers' Conceptions of Mathematics and Mathematics Education* (TCMME) survey to each teacher on the list. In addition, the representative was asked to distribute questionnaires to teachers who had not completed a UMC survey in the past, such that the ratio of frequent participants, occasional participants, and teachers who had never participated in collaborative activities would be approximately 50:30:10. Of the 990 questionnaires distributed, 490 were returned, for an overall response rate of 49 percent. The total number returned within each participation level was 232 (47%) for frequent participants, 203 (41%) for occasional participants, and 40 (8%) for teachers who never participated. Four percent of respondents did not indicate their level of participation in collaborative activities. See Table 1 for the response rate by collaborative.

After responses had been returned, collaborative representatives were asked to rate the participation level of each respondent as a simple validity check. A nonparametric technique was used to compute an index of similarity between participation level as rated by collaborative representatives and participation level as rated by the respondents themselves. Results indicate moderate agreement ($V = 0.65$, $p < .001$). This moderate value indicates that, in general, collaborative representatives agree with teachers' views of their own level of participation. However, the moderate value indicates that many teachers rate themselves as frequent participants whereas collaborative representatives rate many of those teachers as occasional participants. It is unclear whether the collaborative representatives have stricter standards than the teachers for rating participation level. In addition, in some of the larger collaboratives, representatives may not be able to become familiar with all members on a personal basis, thus may find it difficult to rate them as frequent participants.

The Teachers Conceptions of Mathematics and Mathematics Education Survey

The TCMME Survey is organized into five sections intended to garner information on teachers' conceptions regarding five areas of mathematics education: (1) the nature of mathematics,

(2) mathematics teaching, (3) recommended changes in the mathematics curriculum, (4) mathematics education, and (5) schooling. All items were organized on a 5-point Likert scale (1 = low, 5 = high). Items were randomly ordered within each section of the finished instrument.

Table 1

Number and Percent of Teachers Returning the TCMME Questionnaire for Each Level of Participation by Collaborative

Collaborative	Participation Level		
	Frequent N (%)	Occasional N (%)	Never N (%)
Cleveland	28 (80)	7 (20)	0 (0)
Durham	23 (55)	13 (31)	5 (12)
Los Angeles	34 (53)	18 (28)	12 (19)
Memphis	28 (65)	13 (30)	1 (2)
New Orleans	22 (48)	18 (39)	2 (4)
Philadelphia	16 (38)	26 (54)	1 (2)
Pittsburgh	18 (24)	46 (68)	5 (7)
San Diego	15 (47)	15 (47)	2 (6)
San Francisco	15 (43)	10 (29)	8 (23)
St. Louis	5 (22)	15 (65)	3 (13)
Twin Cities	28 (52)	22 (41)	1 (2)
Total	232 (47)	203 (41)	40 (8)*

*Four percent of respondents did not indicate their level of participation

Conceptions of Mathematics.

Scheding (1981) developed an instrument that assessed the conceptions of teachers and university mathematicians regarding the nature of mathematics. The Scheding instrument cited seven facets of mathematics:

1. Mathematics as an organized body of knowledge, and the generality of mathematics.
2. The nature and attributes of proof, and the roles of induction and deduction in mathematical discovery and proof.
3. The role of insight and intuition in mathematics.
4. Mathematics as an aesthetic, creative art; the beauty of mathematics.
5. The relative importance of massive or complex numerical calculations and abstract or symbolic thought in the work of the mathematician.
6. The relationship between mathematics and the real world; the extent to which applications of mathematics *are* mathematics.
7. The existence of differing views of the nature of mathematics.

Conroy (1987) incorporated views postulated by Howson (1973), condensing and making Scheding's (1981) original seven facets more specific to what it means to *do* mathematics based on a given conception. Conroy's six revised conceptions include:

1. Mathematics is a collection of concepts and skills that can be arranged in a sequence from simple to complex. Mathematical activity consists of the use of these concepts and skills in a search for right answers and elegant solutions to a variety of real and imagined problems.

2. Mathematics consists of a complex and interconnected system of logical structures. These structures are formed initially by observing and reflecting on certain actions and experiences but later emerge as a system of abstractions and generalizations, which are the result of deductive reasoning independent of action.

3. Mathematics is the process of applying abstract ideas and inferences to the solution of real problems in a variety of human endeavors. The mathematician's skill lies in the process of application, rather than in the solutions achieved.

4. Mathematics begins with the intuitive solution of problems, proceeds to the formulation of abstractions and concepts, and finally arrives at a set of consistent ideas and generalizations that can be rigorously demonstrated to be true.

5. Mathematics is a language--a set of signs and symbols with their own concise meanings and grammar that represent abstract concepts and operations. These signs and symbols may be used to communicate mathematical ideas, proofs and solutions, or may be manipulated to reach solutions or to prove theorems.

6. Mathematics comprises a set of fundamental axioms that forms the basis of a structure of theorems. These theorems can be proved logically and used in the solution of real and theoretical problems.

The first section of the TCMME Survey, *Conceptions of Mathematics*, is comprised of a revision of these six concepts (See Appendix A). Statements were edited for clarity and shortened to reduce redundancy and fatigue. Additional statements regarding teachers' views on the nature of mathematics based on the work of Collier (1972) and Thompson (1984) were incorporated into these six items. Teachers were asked to rate each item on the degree to which it reflected their personal concept of mathematics. In addition, teachers were asked to rank order all items to reflect their conception of mathematics. Teachers were also given space to comment on any important aspect of mathematics that they felt was not evident in the six items.

Conceptions of Mathematics Teaching.

Statements in the second section of the TCMME Survey reflect goals for mathematics instruction. Statements are answers, in part, to the global query, What does it mean for a student to know mathematics? Conroy (1987) gleaned information from Romberg (1983) in structuring these goals to correspond roughly to his six conceptions of mathematics. Conroy's original statements, again, were edited and shortened for clarity, and statements from Collier (1972) and Thompson (1984) were included where appropriate. Goals were intended to reflect all six *Conceptions of Mathematics*, but not necessarily any one item exclusively. The goals for teaching mathematics are derived from the six conceptions of mathematics and, thus, are not disjoint. The six goals cover a range of what students are to know about mathematics. These goals for mathematics teaching are:

1. Mastering a sequence of facts, skills, rules, and concepts.
2. Mastering a hierarchy of skills and using them to solve problems.
3. Solving problems and modeling situations.
4. Using mathematics to explain and understand situations.
5. Knowing mathematics as originating in real-world situations.

6. Understanding the meaning of mathematical concepts and communicating these ideas.

Items for the second section of the TCMME Survey reflect each of these six goals. Teachers were asked to rate each item on its importance to their teaching of mathematics. They were also asked to rank order their personal goals for teaching mathematics from the most reflective to the least reflective. Space was provided for teachers to comment on any goals they might have that were not expressed in the context of the six items.

Conceptions of Recommended Change.

In the third section of the survey, teachers were asked to rate how important each of 16 recent recommendations for change were to the mathematics curriculum they use. The items were derived from recent proposals for change in the U. S. mathematics curriculum (NCTM, 1989; National Research Council, 1989). Items covered several broad categories of change including the restructuring of curricula, the role of technology, equity, community involvement, and teacher development. Space was provided to enable teachers to comment on other important aspects they felt needed to be changed in mathematics education.

Conceptions of Mathematics Education.

This section contained questions regarding issues and problems teachers face in their classrooms. The questions addressed teachers' approaches to teaching mathematics, their perceived responsibilities, the role of technology in facilitating the teaching of mathematics, and the role of testing and affective variables (e.g., Sobel, 1981) associated with teaching mathematics. Items regarding mathematics education were adapted from Stake and Easley (1978), Collier (1972), and especially Conroy (1987). Teachers were asked to rate how strongly they agreed with each of the 19 items. Space was provided for teachers to comment on any additional aspect they felt was important to facilitate or improve mathematics education.

Conceptions of Schooling.

The last section of the TCMME Survey was designed to assess teachers' views regarding the purpose, function, and goals of schools in American society. Items were adapted from the Conroy instrument (1987) and focused on the function of schools with regard to children, society, and the academic disciplines. Again, teachers were allowed space to comment on any aspect of schooling they felt was of particular importance.

Diary of Professional Relationships

The Diary of Professional Relationships (Romberg, Pitman, Pittelman, Webb, Fadell, & Middleton, 1988) is an ongoing data collection procedure designed to gather more personalized and direct information from a subset of teachers. The Diary of Professional Relationships is comprised of interview forms, each consisting of a series of four to six questions related to ongoing Documentation Project research. For the present study, two forms were distributed. The first Diary elicited teachers' conceptions of mathematics, goals for mathematics instruction, teachers' own recommendations for change in their schools, key issues directly affecting collaborative schools, and the impact of collaboration and interaction with business and industry on teachers' views pertaining to the nature of mathematics. The second Diary assessed influences on individual teaching practice, collaborative influence on teaching, the purpose of schooling, assignment of students to courses, and equity in mathematics education. The information generated from the Diary is used to validate findings from group data and to emphasize the complexity and diversity of teachers' conceptions.

The project representative (on-site observer) from each site was asked to elicit responses from four or five teachers for each Diary. To ensure the veracity of teachers' responses, results of the Diary of Professional Relations were kept anonymous. Thus, although teacher participation level and whether they completed the TCMME Survey were recorded, a match of teachers' responses to the Diary and to the TCMME Survey was not possible. Responses were obtained from over 40 teachers from the 11 collaboratives. Responses to Diary of Professional Relationships questions are presented in Appendix G.

III. RESULTS

The results of the TCMME Survey are presented in three ways; for the total sample, by cluster membership, and by participation level in the collaborative. In each presentation, items corresponding to the five sections of the TCMME Survey are examined. In addition, any striking patterns in responses encountered by collaborative are reported. Teacher responses to the Diary of Professional Relationships are used as added confirmation of TCMME Survey results.

Teachers' Conceptions for the Total Sample

A complete listing of item means and standard deviations for the total sample is presented in Appendix B. Subjects tended to rate items, for the most part, either neutral or high (Neutral = a rating of 3, High = a rating of 4 or 5). Item means ranged from 2.300 (SD = 1.260) for Item 34, "Mathematics teachers' primary responsibility is keeping order, keeping students busy and productive in the classroom, and covering all the material," to 4.811 (SD = 0.442) for Item 52, "I enjoy teaching mathematics." Item 52 not only was rated highest by participants but also exhibited the smallest standard deviation, indicating extremely high agreement among teachers. In fact, over 99 percent of participating teachers rated this item neutral or higher.

Only three items had a mean rating less than neutral on the entire questionnaire: Item 34 (above), Item 43 (It is difficult to obtain objective evidence of student mathematics achievement. The process of learning mathematics is unique to the individual, and does not lend itself readily to standardized evaluation.), and Item 50 (The greatest influence on my teaching of mathematics was my coursework in college and/or teacher education.). The latter two items were rated on average 2.780 (SD = 1.150) and 2.756 (SD = 1.358), respectively. The fact that teachers rated so few items less than neutral indicates that UMC teachers, in general, view mathematics education as a multifaceted enterprise. Teachers varied according to the emphasis they placed on different items on the survey rather than on whether they agreed with the statements or not.

Teachers' Conceptions of Mathematics.

Results from the first section of the TCMME Survey indicate that teachers believe very strongly that "Mathematics is thinking in a logical, scientific, inquisitive manner, and is used to develop understanding," (Item 4). Teachers rated this item higher than any of the other conceptions of mathematics (M = 4.554, SD = .705), and teachers showed the smallest variation in their responses on this item. Thus, there seems to be a high degree of agreement by UMC teachers in the acceptance of this conception of mathematics.

The conception that seemed to least reflect teachers' views, Item 5, dealt with mathematics as facts, skills, and rules to be learned and applied in future work or study ($M = 3.921$, $SD = 1.068$). This item also showed the greatest variation among the conceptions of mathematics, indicating that, while many teachers agreed with the statement, many others disagreed.

Rank-order data confirm these findings. The largest percentage of teachers ranked Items 4 (Mathematics as logical, scientific thinking) and 6 (Mathematics as dynamic problem-solving) as being "most reflective" of their conception of mathematics while the smallest percentage of teachers ranked Items 3 (Mathematics as a collection of concepts and skills) and 5 (Mathematics as facts, skills and rules to be applied in work and future study, respectively) as "most reflective." In addition, teachers most often ranked Item 5 as "least reflective" of their conception of mathematics with very few ranking Item 4 (Mathematics as logical scientific thinking) as least reflective (see Table B2 in Appendix B for percent ranking items as "most reflective" and "least reflective" of their conceptions of mathematics).

Friedman analysis of variance on ranks (Marascuilo & Serlin, 1988) indicates that Item 4 is ranked significantly higher than Items 3 and 5 for the total sample, X^2 (5 d.f.) = 243.680, $p < .05$ (See Table B2). This would suggest that these three items tend to define the boundaries of teachers' conceptions of mathematics better than the other three items. Whereas the notion of mathematics as a way of thinking seems to be central to teachers' conceptions, the algorithmic, fact-oriented aspect of mathematics appears to be more peripheral, though not antithetical.

Teachers' responses in the Diary of Professional Relationships reflect this emphasis on logical problem solving. Of the 44 responses obtained to the question, "What do you think mathematics is?" (Table G1), 16 directly referenced problem solving. In addition, 12 teachers emphasized the scientific nature of mathematics. However, there were a variety of conceptions expressed by teachers that were multifaceted, emphasizing mathematics as science, problem solving, and communication of ideas. A typical response to this question was generated by a teacher in New Orleans, "Mathematics is the study of numbers and number systems and how they relate, interact, and apply to other areas and to real life situations. This study includes thinking skills, problem solving for everyday living as well as for other areas of science, and basic manipulative skills with the various number systems."

Responses to this question also seem to differ by collaborative. For instance, all teachers in San Francisco responding to the Diary of Professional Relationships emphasized the communicative properties of mathematics, as well as the scientific aspects. Conversely, teachers from San Diego related the logic and skill aspects of mathematics. Although the sample of teachers from these collaboratives is far from random, these results do reiterate the importance of studying the differences between teachers from different collaboratives, and the importance of attending to individual differences when assessing the impact of collaboration on teachers' conceptions.

Based on the results of the survey and the Diary of Professional Relationships, in general more of the responding teachers tend to view mathematics as being mental, thinking, whereas fewer of the responding teachers view mathematics as being bits of information (facts, skills, rules and concepts) to be learned.

Teachers Conceptions of Mathematics Teaching.

It was expected that UMC teachers' conceptions of mathematics teaching would reflect their conceptions of mathematics. Since the majority of teachers tend to view mathematics as a way of thinking, i.e., logical, scientific inquiry, they should tend to have consistent goals for their own teaching. Indeed this seems to be the case. Teachers rated Item 14, which pertained to enabling students to explore situations and to test hypotheses by logical reasoning, as the most important conception of mathematics teaching. Again, as for their most important conception of mathematics, teachers tended to show less variation in their response to Item 14 than in responses to the other items (see Table B1). Also consistent with their conceptions of mathematics, teachers tended to place less importance on Item 13, which dealt with preparing students for work and future study by mastering facts, rules, and paper-and-pencil skills. This conception of mathematics teaching response, like its conceptions of mathematics counterpart, also showed the greatest variation of items from this section of the survey.

Teachers' responses to the rank ordering of their conceptions further strengthens these results. Teachers ranked Item 14 (inquisitive exploration and logical reasoning) more often as "most reflective" of their own conceptions of mathematics teaching, and Item 13 (paper-and-pencil skills) more often as "least reflective." Friedman analysis indicates that Item 14 and Item 11, which deals with enabling students to understand mathematical modeling, were ranked significantly higher than Item 13 for the total sample, X^2 (5 d.f.) = 322.888, $p < .05$ (see Table B3 for mean ranks of Items 9 through 14).

Teachers' statements on the Diary of Professional Relationships regarding their own goals for mathematics instruction are presented in Table G4. The 43 statements provided support the survey results, but also indicate that teachers hope that their mathematics instruction will lead to their students valuing and appreciating mathematics. One-fourth of these responses indicated that teachers hope their students will gain some appreciation of mathematics or self-confidence in doing mathematics. One San Diego teacher wants students to have "a love for math." A teacher in San Francisco wants students to have fun with mathematics. Nearly one fourth of the statements noted that the teachers wanted their students to have "the ability to think." Another fourth of the statements indicated the desire to have students use or apply mathematics. "To have the power to use mathematical notations to express ideas that under normal circumstances would require long sentences," responded one St. Louis teacher to the question of what teachers want their students to get out of mathematics instruction. Less than one fourth of the responses implied the desire for students to have some understanding or mastery of mathematical ideas. One Cleveland teacher wanted students to have an "adequate understanding to do problem solving and be comfortable doing it."

The results of the TCMME Survey and the Diary of Professional Relationships indicate that the responding teachers' expectations for students correlated with their dominant conception of mathematics as thinking. Teachers want their students to think critically, but a number of teachers also want their students to be able to use mathematics successfully and to appreciate its elegance and power.

Teachers' Conceptions of Recommended Change.

The recommended changes presented in the TCMME Survey focused on six areas: Restructuring course content, introduction of core programs/alternative courses, introduction of technology, equity in high school mathematics, teacher development, and community involvement in mathematics education.

Restructuring course content. Items 18, 20, 21, and 22 deal with recent recommendations for restructuring course content in high school mathematics. Results indicate that the responding teachers, in general, are in favor of restructuring some of their own mathematics curricula in order to reflect these recommendations. Teachers rated each of these items as important goals for high school mathematics. In addition, Item 22, which deals with increased attention to mathematical modeling in high school mathematics, appears to be of particular importance. It was rated quite highly by the total sample, and responses showed relatively small variability. This is consistent with the importance teachers placed on the conception of mathematics as scientific inquiry indicated earlier.

Introduction of core programs/alternative courses. The teachers tended to react favorably towards Item 19, which deals with development of alternative mathematics courses for students *not* planning to continue the study of mathematics after high school, although there seemed to be considerable variation in responses.

Items 26 and 30 were rated almost exactly alike by the total sample. Teachers endorse the development of a core mathematics program through Grade 11, as long as students have the opportunity to pursue optional courses and electives, and as long as all students through the sophomore year in high school have the opportunity to prepare for college entry.

Introduction of technology into high school mathematics courses. A large proportion of teachers felt that the introduction of new technologies, including calculators and computers, into the high school curriculum (Item 17) is very important to enhance problem solving and ease the drudgery of computation, as well as to provide a fresh approach to topics.

Equity in high school mathematics. Items 27 and 28 concern the identification of students who are in need of remedial help and the development of remedial materials and programs in mathematics for high school students. Teachers tended to have a somewhat neutral outlook toward the prognostic evaluation of remedial students, although there was considerable variation in responses. On the other hand, respondents felt that increased funding for the development and improvement of remedial programs was of some importance in their own schools.

Teachers strongly endorsed the identification of mathematically talented students, especially students from underrepresented populations, and the encouragement of gifted students to pursue careers in mathematics, science, and mathematics education (Item 31).

Teacher development. Respondents were generally in favor of developing teacher education and improvement programs. Although they felt that inservice and membership in professional organizations were important (Items 23 and 25), they tended to have a somewhat neutral outlook towards career ladders, differential staffing, and appointment of master teachers to develop and supervise new programs (Item 24). Considerable variation in responses indicates that teachers were somewhat divided on the last item. Participating teachers seem to favor professional development for all, but are somewhat less favorable toward having a hierarchy within the ranks of teachers.

Community involvement. Teachers indicated that an effort toward increasing public awareness of the importance of mathematics was important to them (Item 29). However, feelings were divided over the issue of whether parents should have the option of choosing the school their child would attend (Item 32).

When teachers were asked to provide their own recommendations to improve the mathematics curriculum in their schools on the Diary of Professional Relationships (Appendix G, Tables G6 and G7), they provided a variety of solutions to the many problems facing urban teachers. Some teachers focused on the mathematics that is being taught in class and emphasized the introduction of new topics; others focused on making mathematics more interesting by providing a variety of problem-solving situations. Many emphasized the need for increased use of a more varied technology to take the drudgery out of computation, and still others chose to emphasize stricter standards, testing, or alternative courses. Key district issues identified by teachers were numerous. One recurring issue noted by at least a few teachers was the improvement of student achievement.

In summary, results indicate that UMC teachers, in general, believe that changing the current system of mathematics education is important. Further, teachers are favorably disposed to many of the recommendations proposed by the NCTM *Standards*. Conceptions about how these changes are to be incorporated have yet to be studied.

Teachers' Conceptions of Mathematics Education.

Items assessing teachers' views concerning issues and problems affecting mathematics teachers cover five broad areas: teachers' approaches to teaching mathematics, teachers' perceived responsibilities, the role of technology in the classroom, the role of assessment in mathematics education, and the affective characteristics of UMC teachers.

Teachers' approaches to teaching mathematics. UMC teachers, in general, agree with the conception that mathematics should be taught as a combination of basic skills *and* inquiry and problem solving. Teachers rated Items 36 (Special applications, problems, and activities must be tailored to the needs of each student), 38 (Mathematical analysis, interpretation, and inquiry should be taught concurrently with the basic skills), and 40 (Mathematics needs to be discovered through applied problem solving) much higher than Items 34 and 42, which dealt with keeping order, covering all the material, and strict adherence to standard notation. Interestingly, teachers tended to agree with the goal of teaching students to communicate using conventional mathematical signs, symbols, and vocabulary (Item 37), while feeling somewhat neutral towards demanding *strict* adherence to those same conventions (Item 42).

Also interesting is the general trend of participants to agree with the value of allowing students to witness mathematics teachers making mistakes in class (Item 44). This seems to be in accordance with teachers' agreement that teaching mathematics entails a combination of basic skills and inquiry and problem solving, since the process of problem solving is often determined by the approach one takes after discovering mistakes or procedures that did not work.

Teachers' perceived responsibilities. Participants tended to agree that mathematics teachers have responsibility for teaching the requisite skills that their students will need in future courses and in future employment (Items 39 and 46, respectively). However, they seem to be more neutral, in general, toward the notion that mathematics teachers may have to sacrifice the broader aims of the course to bring the entire class up to some minimum competency (Item 41). Teachers also demonstrated a fairly high degree of variability on this item; many teachers agreed strongly with the statement, and many disagreed with the statement. It is hoped that analysis of subgroups of the total sample will reveal the location of these differences.

The role of technology. Teachers agreed strongly that advances in technology enhance mathematics instruction and thus should become an integral component of mathematics courses (Item 35). There seems to be fairly high agreement among participants on this item, as evidenced by the moderate standard deviation (Table B1). This trend also follows from teachers' conceptions of mathematics teaching. Teachers tended to agree most with the goal of enabling students to explore situations and test hypotheses. Calculators and computers provide the speed and efficiency, as well as the learning environment, that can facilitate this kind of instruction (Lesgold & Reif, 1983).

The role of assessment in mathematics education. Teachers in the present sample seem to believe that assessment should play an important role in determining whether or not students should graduate from high school. They tend to agree that all students should be required to pass a minimum competency test in mathematics before receiving their diploma (Item 48). Consistent with this view concerning assessment is the belief that objective testing does indeed measure important aspects of mathematical competency (Item 43); teachers felt fairly neutral toward the idea that learning mathematics is so individual in nature that standardized testing is impractical. In addition, teachers seemed to feel somewhat neutral toward the belief that the results of standardized testing greatly influence the kind of mathematics that is taught in classrooms (Item 47). Many teachers, however, did agree with this statement--thus, it may be that standardized testing affects teachers from different schools and with diverse backgrounds differently.

Affective characteristics of UMC teachers. As stated earlier, teachers feel that teaching mathematics is highly enjoyable. Item 52 (I enjoy teaching mathematics) and Item 45 (Mathematics is an enjoyable discipline) were the two most highly rated items on the entire inventory, and both items exhibited very little variability.

When asked to rate the influence of each of several factors on their own teaching of mathematics, participants tended to rate their colleagues (Item 51) and their own teachers in high school (Item 49) as greater influences than their coursework and teacher education (Item 50), Friedman X^2 (2 d.f.)= 31.614, $p < .05$ (Mean Ranks = 2.10 for Items 49 and 51 and Mean Rank = 1.79 for Item 50, respectively). Particularly interesting are some of the teacher responses to the Diary of Professional Relationships that deal with this issue (Table G8). Teachers in general responded in accordance with TCMME Survey results. The largest proportion of responses to the Diary (40%) indicated that colleagues are the biggest influence on teachers' own teaching of mathematics, followed by high school teachers (34%), and last by college courses (26%). In addition, those teachers who felt that college courses were the most influential emphasized that their college *mathematics* professors greatly influence their own teaching. In reference to the influence of the collaborative, many teachers directly credit their collaborative as influencing their own mathematics instruction. Thus, it appears that teachers tend to look to other teachers, their colleagues, and both their high school and college instructors as role models for their own teaching practice.

Regarding some of the key issues pertaining to the mathematics education in their own districts, respondents to the Diary of Professional Relationships emphasized many of the patterns inferred from the TCMME Survey results. For instance, most teachers indicated that change in some area of mathematics education is necessary. A teacher in San Diego was concerned about equity for non-English speaking students. Another teacher in Pittsburgh stressed the increased use of calculators and the renewed emphasis on problem solving as key issues in their district. Several teachers focused on district policy, standards for student proficiency in mathematics, and their role in motivating students to succeed. From these and TCMME Survey results, it is clear that at each collaborative site there is a unique combination of issues that affects teachers. Indeed, it seems that each teacher faces different issues in his or her own teaching due to differences across districts. Responses to this question are presented in Table G7.

Teachers' Conceptions of Schooling.

Items that assess teachers' conceptions of the purpose, goals, and functions of schooling are organized into those pertaining to children, those pertaining to society, and those pertaining to academic disciplines.

Goals and functions of schooling pertaining to children. Respondents believe that schools provide students with the opportunity for personal fulfillment (Item 60) and the opportunity to enhance their own personal abilities (Item 54). Integral with these conceptions is the idea that schools should be places where children feel comfortable, both with peers and with teachers and administration (Item 67). Teachers rated these items, on average, as having high to very high priority as goals for modern schooling.

Equity seems to be an issue of importance to participating teachers. Teachers agree that schools should allocate resources equally among all students (Item 66), yet they should also offset inequalities by providing special opportunities for the disadvantaged (Item 59). Teachers rated both of these items as important goals of schooling. They seem to be somewhat divided on whether students should be ability grouped or whether students should be grouped according to age (i.e., Item 56, Schools should group students according to similar needs, interests, and abilities, rather than according to age). The mean score for this item was between neutral and agreeing; however, it showed a moderately high standard deviation indicating considerable variation in responses.

Teachers, again, are divided on the importance of training students to follow instructions and to absorb and memorize detail (Item 63). Teachers rated this item, in general, close to neutral. However, they did display moderately high variation in their responses. Further analyses of subgroups of the total sample should reveal the meaning(s) of this variation.

Goals and functions of schooling pertaining to society. Teachers were somewhat neutral toward the role of schooling to preserve the traditions and stability of society (Item 55). Rather, they seem to agree more with Item 58, "Schools must be innovative to ensure that we maintain a dynamic and expanding society." Yet, not all traditions of society were looked on in a neutral fashion Teachers agreed with the goal that children should be taught proper work values and the reward value of hard work (Item 61). Competition and its role in schooling seems to be agreed to somewhat, but it does not have a particularly high value for teachers (Item 62).

A similar pattern emerges for teachers' perceptions of time regulation (Item 64). They seem to agree somewhat that society and schools must regulate much of the time children spend in schooling. However, time regulation as a function of school and teacher responsibility does not seem to have a high priority.

Although teachers tended to agree that schools should be for students who want to work and are not a social institution (Item 68), there was a high degree of variability in responses. This would indicate that many urban teachers perceive a dual role of schooling, one that must meet both the academic and social needs of urban pupils.

Goals and functions of schooling pertaining to academic disciplines. Teachers, in general, tended to place a high priority on the role of schooling both for transmitting the knowledge and skills associated with different academic disciplines (Item 57) and for providing students with the ability to solve problems as a function of training in academic disciplines (Item 65). Both of these items were rated above 4, on average, on the 5-point scale, indicating high agreement.

Teachers' responses to the Diary of Professional Relationships indicate that most teachers feel that the main purpose of high school is to develop students' survival skills, both in future academics and in the adult world. Teachers responses focused on the responsibility of the high school to provide the student a well-rounded education, to provide needed skills, and to make each student a productive member of society (see Table G10).

When asked how students should be assigned to mathematics courses, teachers overwhelmingly focused on ability and interest rather than on age. Almost all respondents emphasized some combination of the two in assigning students to mathematics coursework (see Table G11).

When teachers were asked on the Diary of Professional Relationships to outline their conception of mathematics equity and to indicate whether they thought that schools can achieve equity, teachers seemed to be able to give their definition of equity easily but had a much harder time determining how to achieve it. Most teachers felt that equity consisted of providing equal opportunity to all students. An especially elegant definition was provided by a teacher from Memphis, "Every student, regardless of race or gender, should be given the opportunity to continue their mathematics education on a quality level." Teachers' responses to this question are presented in Table G12.

Discussion

Results indicate that UMC teachers enjoy teaching mathematics and are committed to enlarging their understanding of mathematics and mathematics education. In addition, teachers seem to view mathematics as a way of thinking about the world. They emphasized the importance of mathematical modeling and the introduction of new technologies to help students gain insight into the ways in which mathematics can help explain phenomena through logical, scientific hypothesis testing.

Teachers in the present study also seem to be in agreement that talented mathematics students must be given appropriate programs to help them actualize their mathematical potential. Teachers agreed with the introduction of alternative programs to identify and meet the needs of mathematically talented students, especially students from underrepresented populations.

In regard to the influences on their own teaching, responding teachers emphasized that other teachers, both their own high school mathematics teachers and their teacher colleagues, are more influential than their coursework in college and teacher education. This is an important factor to note in assessing the impact of collaboration on urban teachers since it seems to indicate that, without programs that foster collaboration, teachers would not be able to capitalize on new teaching methods and ways of dealing with students and administration that their colleagues can and do provide. In addition, it says much about the perceived effectiveness of university teacher education programs in actually influencing how teachers approach instruction.

Teachers, in general, tended to perceive school as a place where children should find personal fulfillment and the opportunity to enhance their personal abilities. Congruent with this finding is teachers' commitment to equity in the classroom. Teachers felt that schools should provide additional support for struggling students, as well as additional instruction for the mathematically gifted. They see school primarily as facilitating the dynamics and improvement of society rather than as functioning to maintain the status quo.

These findings appear to contradict many common perceptions that place urban teachers in a somewhat negative light. Teachers in the collaboratives seem to have a high sense of responsibility to their students, to society, and to their academic discipline. They value and enjoy teaching and are committed to the improvement of mathematics education. The question that must be raised is whether participation in collaborative activities has had a role in the development of these attitudes.

Teachers' responses to the question, "How has the collaborative affected your teaching of mathematics?" on the Diary of Professional Relationships Survey, indicate that collaboration does indeed have an impact on teachers' approaches to mathematics instruction (see Table G9). Many teachers emphasized participation in symposia and conferences, as well as networking with other teachers to share problems and successes, as extending their techniques and knowledge of the subject matter. A teacher from St. Louis provides an elegant summary of most responses, "Working with other teachers increases the confidence level at which I approach my work. I do not feel isolated in the system (on a professional level). The collaborative has presented me with opportunities for growth in my profession, with a chance to interact with others in the district and with a chance to voice my ideas. I feel a sense of strength when I know what others are doing around the country (as well as around the schools in my system)."

Results further indicate differences in teachers' conceptions that cannot be detected through an analysis of overall data. The variation exhibited on many of the TCMME Survey items indicates that subgroups exist between and within collaborative sites that could shed light on how collaborative activities affect teachers individually. Thus, the remainder of the Results section will focus on detecting and describing these differences.

Teachers' Conceptions by Their Profile of Responses

Teachers' Conceptions of Mathematics.[1]

Due to the complex nature of teachers' conceptions of mathematics, and owing to the variability found on many of the items on the TCMME Survey, teachers' *profiles* of responses across the six conceptions of mathematics were examined to shed light on how different teachers perceived mathematics and how their perceptions of mathematics affect their views of factors relating to mathematics education. Cluster analysis was used to group teachers according to their pattern of responses across Items 1 through 6. Four groups (clusters) of teachers, each cluster including those who responded similarly, were found. Figure 1 illustrates the profiles of each extracted cluster of teachers over the six items. For a complete listing of means of all TCMME Survey items by cluster membership, please refer to Table 2 at the end of this section (page 42).

Cluster 1 teachers (N = 73) appear to identify with the scientific, logical, factual concept of the nature of mathematics, while disagreeing with the notion of mathematics as a variable discipline that changes as new situations occur. This view is compatible with a belief that mathematics is a fixed body of knowledge, to be learned through disciplined effort.

Cluster 2 teachers (N = 101), on the other hand, agree strongly with the dynamic notion of mathematics and place a great deal of emphasis as well on the scientific and language views. These teachers disagree strongly with the conception that mathematics is primarily facts, skills, and rules to be applied to solving problems.

Teachers in Cluster 3 (N = 109) appear to be nondiscriminatory and more global in their conceptions about the nature of mathematics. These teachers rated all six items higher than any other group. It is unclear at this time whether this profile indicates flexibility in thinking about the different conceptions of mathematics, or whether the nature of the instrument placed a low ceiling on the scores.

---

[1]Ward's method of hierarchical cluster analysis (SPSS-X Release 2.1; SPSS Inc., 1986) extracted four mutually exclusive clusters of teachers who differ on the six proposed conceptions of mathematics. Multivariate methods were chosen simply because teachers' conceptions were expected to be complex, not lending themselves readily to univariate item analysis. Rather, examination of the interaction of teachers' views concerning *several* possible ways of perceiving mathematics was expected to yield a better picture of the complex interactions in teachers' thought processes. For a discussion of the rationale for using cluster analytic techniques to assess teachers' conceptions of mathematics, please refer to Appendix C of this document.

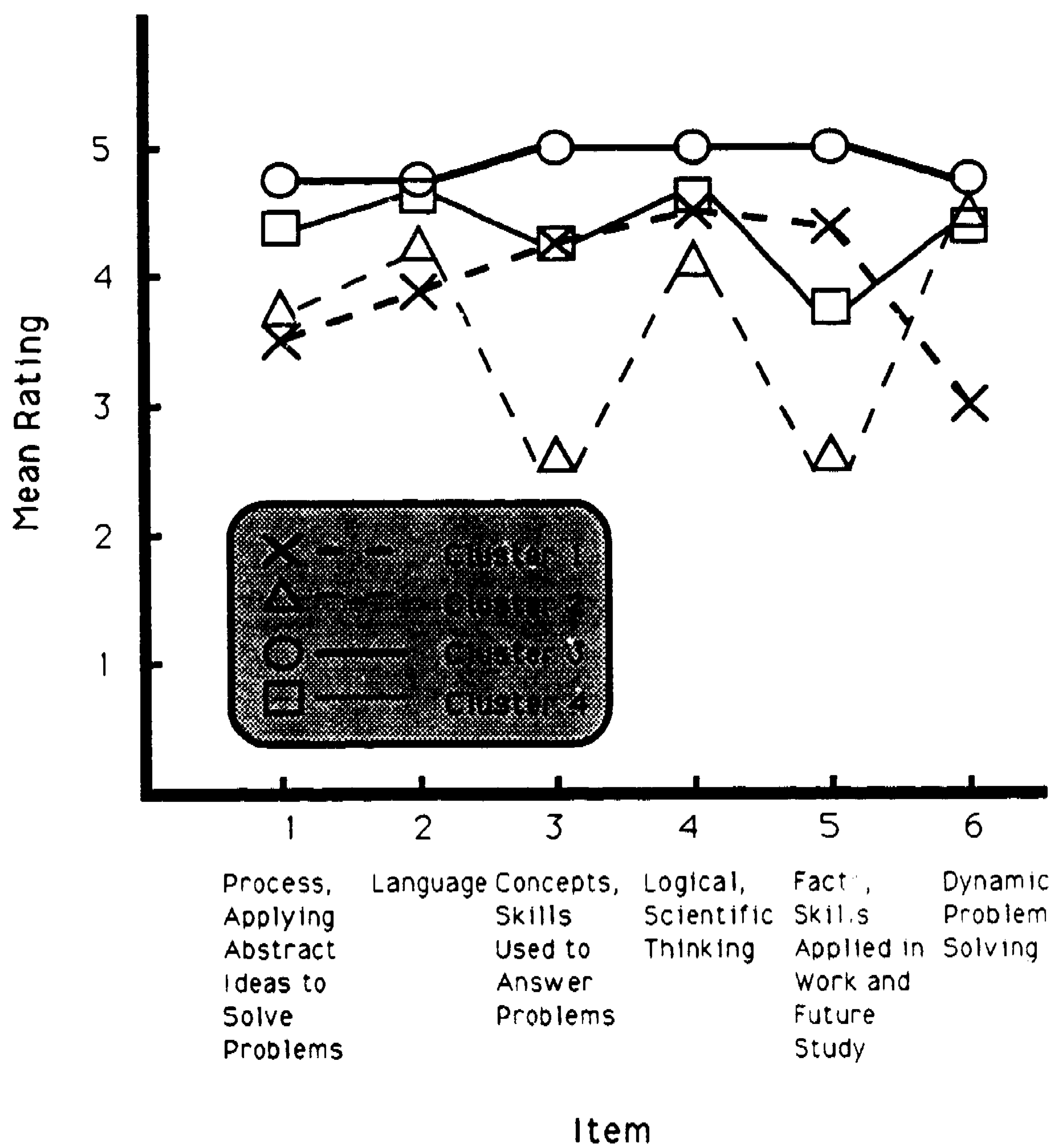

Figure 1. Mean ratings profiles, Items 1 through 6 by cluster membership

The largest group of teachers, Cluster 4 (N = 195), appears to be similar in profile to Cluster 2. However, the entire profile of scores for Cluster 4 is elevated somewhat and displays less extreme differences in ratings across items. This profile seems to indicate a belief that mathematics is primarily a means of thinking in a scientific, inquisitive manner, of communicating those thoughts effectively, and of changing as new information arises. Although mathematics as a collection of facts, skills, and concepts is not rejected, its level of importance is not emphasized in comparison with its dynamic aspects. This conception is consistent with a heuristic, inquiry-based view of mathematics--i.e., that mathematics is primarily a way of thinking, *but* it is also a developed body of knowledge that is useful in the real world.

Data from the rank ordering of items corroborate these findings. It was expected that the percentage of teachers within each cluster who ranked an item as the most reflective of their conception of mathematics would be proportionate to the mean item score for that cluster (see Table D1 for percentage of teachers ranking Items 1 through 6 as "Most Reflective" by cluster). From the mean item score profile (See Table 2), one would predict that teachers in Cluster 1, teachers who view mathematics as a fixed body of knowledge, would rank Item 3 (Mathematics is a collection of concepts and skills used to obtain answers to problems.), Item 4 (Mathematics is thinking in a logical, scientific, inquisitive manner and is used to develop understanding.), and Item 5 (Mathematics is facts, skills, rules and concepts learned in some sequence and applied in work and future study.) as most reflective of their own conceptions of mathematics, while ranking Item 6 (Mathematics is an interconnected logical system, is dynamic, and changes as new problem solving situations arise. It is formed by thinking about actions and experiences.) as least reflective. Friedman analysis of variance by ranks revealed significant differences in the ways in which conceptions of mathematics were ranked by Cluster 1 teachers, X^2 (5 d.f.) = 70.002, $p < .05$. Indeed, the largest proportion of these teachers (36%) did rank Item 4 as most reflective, closely followed by Items 3 and 5 (14% and 17%, respectively), while the smallest proportion ranked Item 6 as most reflective (8%). In addition, Items 3, 4, and 5 were rarely ranked as least reflective by Cluster 1 teachers, while the largest percentage ranked Item 6 least reflective (see Table D2 for rankings of the *least* reflective conceptions). The remaining items fell in the middle, with a modest proportion ranking each as most reflective of their conception of mathematics.

Data from the ranking by the other three clusters of teachers also reflect their mean item profiles. One possible exception would be Cluster 3, teachers who hold a global conception of mathematics. Friedman analysis of variance indicates significant differences in mean rankings of teachers' conceptions of mathematics, X^2 (5 d.f) = 59.394, $p < .05$. Forty-one percent of teachers in Cluster 3 ranked Item 4 as most reflective, while much smaller proportions ranked the other items as most reflective. This would not corroborate profile scores if it were not for the fact that Cluster 3 teachers ranked the other items in nearly equal proportions, as their second most- to least- reflective conceptions.

Teachers in Cluster 2, those who view mathematics as dynamic inquiry, ranked Items 3 and 5 least often as *most* reflective (1% and 2%, respectively) and most often as the *least* reflective of their conceptions of mathematics (38% and 42% respectively), which strongly supports their profile scores. In addition, Item 6, which emphasizes this dynamic view of mathematics, was ranked on average much higher than the other five items, indicating that this view is most central to Cluster 2 teachers' conceptions of mathematics. Friedman analysis confirms significant differences for these items, X^2 (5 d.f.) = 220.400, $p < .05$.

Teachers in Cluster 4 (mathematics as scientific thinking), ranked Items 1, 2, 4, and 6 most often as most reflective of their conceptions of mathematics, and Item 5 most often as least reflective, which would be in concordance with their profile. Friedman analysis revealed significant differences in mean rankings of items, X^2 (5 d.f.) = 115.452, $p < .05$. However, a large percentage (20%) also ranked Item 6 as least reflective of their conceptions. It is unclear what this apparent split indicates. It may be an index of teachers' flexibility in thinking about the dynamic nature of mathematics--i.e., some teachers falling in this cluster may feel thats the dynamic nature of mathematics is more important while others may feel that the known body of mathematics is more important in scientific thought.

It is interesting to note that cluster membership seems to be equally distributed across all collaborative participation levels, X^2 (30 d.f.) = 6.454, $p > .05$. Differences were found, however, in the distribution of cluster membership across some collaboratives, X^2 (30 d.f) = 64.623, $p < .001$. New Orleans, in particular, seems to be different in teachers' conceptions of mathematics compared to the rest of the sample. These will be addressed later, in the section dealing with differences in teacher conceptions by collaborative.

Teachers' Conceptions of Mathematics Teaching.

Teachers' conceptions of mathematics seem to be highly related to their conceptions of mathematics teaching. To determine group differences in conceptions of mathematics teaching, one-way analysis of variance (ANOVA) was performed on Items 9 through 14 with Cluster membership being the grouping variable. Significant differences were found between clusters for Items 9 through 13. Scheffe post-hoc tests determined the specific location of the differences (all p-values < .05, see Tables D3 to D12). In addition, Friedman analysis of variance on rank order of Items 9 through 14 indicate differences by cluster that reflect teachers' conceptions of mathematics profiles (see Table D13).

For instance, teachers who view mathematics as dynamic inquiry (Cluster 2) ranked Item 13 very low, consistent with their disagreement with mathematics as primarily facts, skills, and rules. Conversely, they ranked Item 14 very high consistent with their perception that mathematics encourages inquisitive thinking, X^2 (5 d.f.) = 212.626, $p < .05$. Teachers with the global view of mathematics (Cluster 3) tended to rank each of the six conceptions of mathematics teaching moderately high, although they did rank Item 13 (to prepare students for work and future study via a sequence of facts, paper and pencil skills, rules and concepts) lowest consistently, X^2 (5 d.f.) = 26.303, $p < .05$. Clusters 1 (Mathematics as a fixed body of knowledge) and 4 (Mathematics as scientific thinking) were very similar in the ways in which they ranked the proposed conceptions of mathematics teaching. Both clusters of teachers ranked Item 11 highest and Item 13 lowest, X^2 (5 d.f.) = 24.213, $p < .05$ and X^2 (5 d.f.) = 136.744, $p < .05$, respectively.

Differences on the other conceptions of mathematics teaching also reflect teachers' profiles across the full range of their conceptions of mathematics. For example, one would predict that teachers who agree with Items 3 and 5, which deal with mathematics as a collection of concepts, skills, and rules to be applied, should also agree with Items 9 and 13, which deal with teaching students organized mathematical skills, rules, and concepts. Thus, teachers from Cluster 3 (a global, inclusive view of mathematics), who exhibited the highest scores for Items 3 and 5 should exhibit the highest scores for Items 9 and 13 out of the four clusters. Teachers from Cluster 2 (Mathematics as a dynamic, logical way of thinking) should rate Items 9 and 13 the lowest. And teachers from Cluster 1 (Mathematics as a scientific body of knowledge) and Cluster 4 (Mathematics as scientific inquiry) should fall somewhere in the middle. Tables D3 and D4, and D11 and D12 (Appendix D) show that this is indeed the case. Cluster 3 teachers rated both Items 9 and 13 significantly higher than teachers in all other clusters; Cluster 2 teachers rated these items significantly lower than all the others rated them, and teachers in Clusters 1 and 4 were not found to be significantly different from each other.

Teachers who agree that mathematics is a language and that it is a process of applying abstract ideas to solve problems (Items 1 and 2) should also agree with Item 10, since this conception deals with aspects of both. In fact, teachers with a global view of mathematics (Cluster 3) rated Item 10 higher than both teachers with the conception of mathematics as a scientific body of knowledge (Cluster 1) and teachers with the conception of mathematics as scientific inquiry (Cluster 4). This is only partially consistent with expectations (see Tables D5 and D6). Cluster 1 teachers, since they rated both Item 1 and Item 2 lowest of the four clusters, should also tend to rate Item 10 lowest, and this was indeed the case. However, Cluster 2 teachers, who appear to be similar to those in Cluster 1 with regard to the first two items, showed no significant difference from teachers in either Cluster 1 or 4 for Item 10. It is unclear at this time what this discrepancy means.

For Item 11, which deals with teaching mathematical modeling and real-world problem solving, it would be expected that teachers who agreed with this item should also agree with Item 1, which embodies a similar conception of mathematics. Global teachers, Cluster 3, rated this item significantly higher than teachers who view mathematics as a dynamic way of thinking, Cluster 2. All other group comparisons were nonsignificant (see Tables D7 and D8).

Teachers in Cluster 2, Mathematics as a dynamic, logical way of thinking, rated Item 12 significantly lower than those in Cluster 3, Global view of mathematics, and Cluster 4, Mathematics as scientific inquiry (see Tables D9 and D10). This pattern is consistent with the finding that teachers who believe that mathematics is a dynamic, human creation may not believe that "correct" rules and reasoning exist, because the state of mathematics is constantly in flux. Thus, it makes sense that these teachers would feel that providing "correct" rules and reasoning would be less important than other conceptions.

In addition, it would be expected that teachers in Cluster 3, who exhibit a global view of mathematics, would tend to rate all goals of mathematics teaching higher than the other three groups, who seem to take a narrower view of the nature of mathematics. This proved true, not only for conceptions of mathematics teaching, but also for nearly all of the items that discriminated between clusters of teachers on the TCMME Survey.

Teachers Conceptions of Recommended Change

Restructuring course content. Teachers who view mathematics as dynamic inquiry, Cluster 2, rated Item 21, which recommends the introduction of more topics from discrete mathematics and statistics into the high school curriculum, more important than either teachers who view mathematics as a fixed body of knowledge, Cluster 1, or teachers who view mathematics as scientific thinking, Cluster 4 (see Tables D14 and D15 in Appendix D). In addition, teachers in Cluster 4 rated this item more important than did teachers in Cluster 1. This may be a reflection of the emphasis of teachers in Clusters 2 and 4 on scientific-oriented mathematics, which utilizes techniques from discrete mathematics and statistics more so than other mathematical fields.

Teacher development. Teachers in Cluster 2 also rated Item 24, which recommends career ladders, differential staffing patterns, and appointment of master teachers, higher than teachers in Cluster 1 (see Tables D16 and D17). It is unclear at this time what these differences reflect. Perhaps the emphasis of this item on the development of new programs may appeal more to teachers who feel that mathematics is dynamic than to teachers who feel that it is somewhat fixed, but this is purely speculation.

Introduction of core programs. Items 26 (A core mathemtics program should provide optional tracks and electives, and the opportunity for every student through grade 10 to prepare for college entry) and 30 (A core mathematics program should be established which requires all students to study mathematics through grade 11) were rated consistently higher by teachers with a global view of mathematics than by the other three groups of teachers. This is in accordance with their generally high ratings across the majority of items on the instrument. This may be indicative of Cluster 3 (global, inclusive view of mathematics) teachers' multifaceted conception of mathematics in that new programs would reveal new facets to students, thus giving them a conception closer to the teachers' own. This seems especially evident in the emphasis of Item 26 on providing optional tracks and electives for students. Further, the consistently high ratings across all clusters indicates the importance teachers feel that the study of mathematics has in high school. (See Tables D18 and D19 and Tables D24 and D25 for ANOVA and post-hoc results for Item 26 and Item 30 respectively.)

Introduction of state-level prognostic testing. Teachers across all clusters tended to rate Item 27, which deals with the administration of state-level prognostic tests to determine whether students are ready to continue in further math-related study, fairly neutral. No differences on this item were found except between Global Teachers (Cluster 3) and Dynamic Inquiry Teachers (Cluster 2). Global teachers rated this item closer to "Important," while Dynamic Inquiry teachers rated it closer to "Neutral." It is unclear at this time what these differences signify. They may be artifacts of the Global teachers' tendency to rate most items higher than the other three groups. (See Tables D20 and D21 for ANOVA and post-hoc results.)

Community involvement. All groups of teachers tended to rate "the effort to ... increase the awareness of the importance of mathematics among all members of the community" (Item 29) as important (see Tables D22 and D23 in Appendix D). Global teachers, again, rated this item higher than Dynamic Inquiry teachers (Cluster 2) and Scientific Thinking teachers (Cluster 4). Although these differences are significant statistically, it is unclear whether they have much practical significance, since all teachers seemed to register a high degree of agreement about its importance (Table B1, Appendix B).

Teachers' Conceptions of Mathematics Education.

Teachers' approach to teaching mathematics. Items that differentiated between clusters of teachers centered on the degree of emphasis placed on teaching standard mathematical symbols, rules, and notations (Items 37 and 42), as well as the emphasis placed on teaching mathematics through applied problem solving (Item 40). Dynamic Inquiry teachers, Cluster 2, tended to rate items pertaining to strict adherence to rules and notation lower than Global teachers, Cluster 3, and Fixed Body of Knowledge teachers, Cluster 1 (see Tables D28 and D29 for Item 37, and Tables D36 and D37 for Item 42 ANOVA and post-hoc results). At the same time, Dynamic Inquiry teachers, Cluster 2, tended to emphasize teaching mathematics through applied problem solving more than Fixed Body of Knowledge teachers, Cluster 1, and similarly to Global, Cluster 3, and Scientific Thinking teachers, Cluster 4 (see Tables D32 and D33).

Teachers' perceived responsibilities. Dynamic Inquiry teachers and Scientific Thinking teachers agreed with Items 39 and 46, which deal with teachers' responsibilities to teach the skills necessary for subsequent coursework and subsequent employment, respectively, less than Global teachers. (see Tables D30 and D31, and Tables D38 and D39 for Items 39 and 46, respectively.) However, on Item 41, which pertains to teachers having to sacrifice the broader aims of a course to bring the entire class to a minimum competency level, Dynamic Inquiry teachers agreed less than either Global teachers or Scientific Thinking teachers (Tables D34 and D35).

The role of technology. An interesting deviation from the patterns observed among clusters is evident with Item 35. Teachers who perceive mathematics as primarily dynamic inquiry (Cluster 2) rated the role of calculators and computers the highest of all four groups (see Tables D26 and D27). Dynamic Inquiry teachers agreed with Item 35 significantly more than Fixed Body of Knowledge teachers (Cluster 1).

The role of assessment in mathematics education. All clusters of teachers tended to agree with Item 48, "All students should be required to pass a minimum competency test in mathematics to graduate from high school" (Tables D40 and D41). However, Global teachers tended to rate this item much higher than either Scientific Thinking teachers or Dynamic Inquiry teachers. The results of responses to this item are unclear at this time, especially since teachers exhibited nonsignificant differences on questions pertaining to the validity of standardized testing and the effect standardized testing has on what mathematics is taught in the classroom (Items 43 and 47, respectively).

Teachers' Conceptions of Schooling.

Goals and function of schooling pertaining to children. Dynamic Inquiry teachers rated Item 63 (Schools must train students to learn and apply rules, follow instructions, absorb facts and memorize detail) significantly lower than all other groups (see Tables D50 and D51). In addition, Scientific Thinking teachers rated this item lower than Global teachers. There were nonsignificant differences between Fixed Body of Knowledge teachers and both Global and Scientific Thinking teachers. These findings are in accordance with teachers' profiles on their conceptions of mathematics and mathematics teaching. Dynamic Inquiry teachers, who disagree with the conception of mathematics as a collection of concepts and skills, did disagree with factual training as a primary function of schools.

Scientific Thinking teachers tended to place less emphasis on Item 66, which pertains to the allocation of resources among all students regardless of personal background, than did Global teachers. It is unclear what these differences entail, since all groups tended to place moderately high priority on this conception (see Tables D56 and D57).

Goals and functions of schooling pertaining to society. Global teachers tended to assign moderately high priority to the function of schools to preserve the traditions and stability of society, Item 55, whereas Dynamic Inquiry teachers tended to feel somewhat neutral towards this goal. Significant differences were found between these two clusters of teachers on Item 55 (see Tables D42 and D43),.

All groups of teachers tended to place high priority on Item 51, which deals with developing students' work values, group adaptive skills, and reward values associated with work (see Tables D46 and D47). However, both Global teachers and Scientific Thinking teachers placed significantly higher emphasis on this item than Dynamic Inquiry teachers.

Teachers in the Fixed Body of Knowledge and Global clusters rated the value of competition in schooling (Item 62) higher than either Scientific Thinking or Dynamic Inquiry teachers. While the latter two clusters tended to feel somewhat neutral towards the value of competition, the former groups tended to place moderately high priority on the item (see Tables D48 and D49).

In accordance with other items associated with the function of schooling to maintain stability in our society (Item 64), teachers in the Dynamic Inquiry cluster tended to feel somewhat neutral toward time allocation as a priority for society, teachers, and students (Tables D52 and D53).

Goals and functions of schooling pertaining to academic disciplines. Both Scientific Thinking teachers and Dynamic Inquiry teachers placed moderately high priority on the role of schools as transmitters of knowledge in academic disciplines (Item 57) whereas Global teachers placed significantly higher priority on this function of schooling. It may be that the former two groups of teachers see such disciplines as mathematics and science, where each branch of knowledge benefits the other having "fuzzy" boundaries (Refer to Tables D44 and 45).

Although Global teachers tended to rate Item 65 (Schools exist to develop students' abilities to think, solve problems and make decisions by means of thorough training in academic disciplines) higher than Dynamic Inquiry teachers, all groups placed high priority on this conception. Thus, it is unclear what this difference means as it pertains to the actual practice of teachers (Tables D54 and D55).

Discussion

Results indicate that participating teachers' conceptions of mathematics are highly related to the ways in which they approach teaching mathematics, to the ways in which they view the system of mathematics education and, in particular, to the attitudes with which they view change in that system. Documentation of collaborative impact, therefore, should take these differences into account. Moreover, knowledge of teachers' conceptions of mathematics and how they differ across urban sites may help educators provide a "best fit" of activities for the teachers in their area.

One of the primary goals at the start of the UMC project was to provide urban teachers with a sense of ownership of their teaching (Romberg, 1984). Without attention to differences in the ways in which teachers view mathematics education, reform cannot provide this sense of empowerment.

For instance, Fixed Body of Knowledge teachers, who are not in strong agreement regarding the emphasis on the dynamic nature of mathematics, may not agree entirely with the renewed emphasis on constructivism that pervades the mathematics education literature. Workshops and/or teacher education programs that emphasize the ways in which the Fixed Body of Knowledge approach can be tailored to students' construction of their own knowledge may help these teachers integrate new materials that otherwise might prove frustrating. Likewise, Dynamic Inquiry teachers may put too little emphasis on the facts and skills students need to be effective in mathematical modeling and exploration. Activities that integrate skills and rules with inquisitive exploration may help them provide children with mathematical power.

For the purposes of the Documentation Project, knowledge of the different ways in which teachers from different collaborative sites view mathematics and mathematics education may reveal the effects of different educational policies, differences in available resources, and differences in the characteristics of the involved communities on the process of educating urban children.

Table 2

Summary Table of TCMME Survey Item Means by Cluster

Item	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Conceptions of Mathematics				
1. Mathematics is a process in which abstract ideas are applied to solve real-world problems.	3.5	3.8	4.7	4.3
2. Mathematics is a language, with its own precise meaning and grammar, used to represent and communicate ideas.	3.9	4.2	4.8	4.6
3. Mathematics is a collection of concepts and skills used to obtain answers to problems.	4.2	2.7	5.0	4.2
4. Mathematics is thinking in a logical, scientific, inquisitive manner and is used to develop understanding.	4.5	4.1	5.0	3.9
5. Mathematics is facts, skills, rules and concepts learned in some sequence and applied in work and future study.	4.3	2.6	5.0	3.9
6. Mathematics is an interconnected logical system, is dynamic, and changes as new problem solving situations arise. It is formed by thinking about actions and experiences.	3.0	4.4	4.7	4.3
Conceptions of Mathematics Teaching*				
9. To enable students to master a hierarchy of concepts and skills and to use these in solving problems.	4.3	3.9	4.7	4.3
10. To provide experiences for students to know mathematics as originating in real-world situations and to have the power of using a small set of symbols to represent and solve a wide range of problems.	4.2	4.4	4.5	4.3
11. To enable students to use mathematical procedures to solve problems and mathematical concepts to model both abstract and real-world situations.	4.4	4.3	4.6	4.5
12. To provide students with complete understanding of the meaning(s) of mathematical concepts and enable them to communicate ideas using correct mathematical symbols, rules and reasoning.	4.0	3.8	4.4	4.1
13. To prepare students for work and future study by having them master a sequence of facts, paper-and-pencil skills, rules and concepts.	3.7	2.8	4.3	3.6
14. To enable students to use mathematics to explore situations in an inquisitive manner, and to offer and test hypotheses by logical reasoning, for the purpose of developing a more complete understanding of the situation.	4.3	4.5	4.6	4.4

*Teacher write-in items are not included.

Table 2 (continued)

Summary Table of TCMME Survey Item Means by Cluster

Item	Cluster 1	2	3	4
Conceptions of Recommended Change*				
17. Calculators and computers should be introduced into mathematics courses to enhance understanding and problem solving, and to take the drudgery out of computations. Presentation of topics needs to be revised based on fresh approaches possible with new technologies.	4.4	4.6	4.3	4.4
18. Traditional high school mathematics courses need to be integrated and unified to show interrelationships across topics and applications.	4.0	4.3	4.2	4.2
19. Alternative mathematics courses should be available for students who are planning not to go to college or who are planning not to take a college major with high mathematics content.	4.3	4.0	4.3	4.1
20. More emphasis should be given to simple mental computation, estimation and approximation, and less to practicing lengthy paper-and-pencil calculations.	3.7	4.1	4.0	4.1
21. More topics and techniques from discrete mathematics, statistics and probability should be introduced into the high school curriculum.	3.5	4.2	3.9	3.8
22. Mathematical modeling and problem solving should be incorporated as a central feature in high school mathematics, and should be integrated into other parts of school curricula (such as science and social studies).	4.3	4.5	4.4	4.3
23. Preservice and inservice teacher education programs need to be developed that train teachers in individual and small group teaching, the use of technology, and research.	4.2	4.3	4.3	4.2
24. Schools must adopt differential staffing patterns and career ladders for mathematics teachers by appointing master teachers to develop, coordinate and supervise new programs.	3.2	3.8	3.6	3.4
25. Mathematics teachers should be encouraged to become members of professional mathematical societies and to attend regional and national meetings.	3.9	4.2	4.2	4.1
26. A core mathematics program should provide optional tracks and electives, and the opportunity for every student through Grade 10 to prepare for college entry.	4.1	4.1	4.5	4.1

*Teacher write-in items are not included.

Table 2 (continued)

Summary Table of TCMME Survey Item Means by Cluster

Item	Cluster 1	2	3	4
27. A state-level prognostic test in mathematics should be administered to all students in Grade 9 or 10 to determine if they are ready to pursue further math-related work or study, or if they are in need of remediation or course changes. Results of such tests would not be available for the purpose of college admission or to evaluate teachers.	3.5	3.3	3.9	3.5
28. Increased funding should be made available for the development of improved, appropriate materials, diagnostic techniques and teaching strategies for remedial programs.	4.0	4.0	4.2	4.1
29. Strong efforts must be made to increase the awareness of the importance of mathematics among all members of the community, especially among parents of school age children.	4.4	4.3	4.6	4.4
30. A core mathematics program should be established which requires all students to study mathematics through Grade 11.	4.3	3.9	4.4	4.1
31. Special efforts should be made to identify mathematically talented students, especially minorities and women, and to encourage them to pursue careers in mathematics, science and mathematics education.	4.3	4.4	4.6	4.4
32. Parents should have the option of sending their children to the public school of their choice.	3.2	3.2	3.3	3.3
Conceptions of Mathematics Education*				
34. Mathematics teachers' primary responsibility is keeping order, keeping students busy and productive in the classroom, and covering all the material.	2.3	2.1	2.4	2.4
35. Calculators and computers can facilitate the learning of mathematics. Hands-on experience with changing technology should be incorporated as an integral part of mathematics instruction.	4.3	4.7	4.5	4.5
36. Special applications, real-world problems and extracurricular activities must be tailored to the needs and abilities of each student to help them excel.	4.0	4.1	4.2	4.0
37. Mathematics teachers must teach students to communicate using conventional mathematical signs, symbols and vocabulary.	4.1	3.8	4.4	4.1

*Teacher write-in items are not included.

Table 2 (continued)

Summary Table of TCMME Survey Item Means by Cluster

	Cluster			
Item	1	2	3	4
38. Mathematical analysis, interpretation and inquiry should be taught concurrently with the basic skills. Students must be taught to use mathematics to gain understanding of a variety of phenomena.	4.2	4.4	4.5	4.3
39. Mathematics teachers have the responsibility to teach the requisite skills for subsequent courses.	4.2	4.0	4.5	4.2
40. Mathematical skills and rules should not be taught in isolation. Mathematics needs to be discovered by students through applied problem solving.	4.0	4.3	4.3	4.2
41. Mathematics teachers sometimes have to sacrifice the broader aims of the course in order to spend more time bringing the entire class up to a minimum competency level.	3.4	3.1	3.6	3.5
42. Mathematics teachers should demand strict adherence to methods and notations used in class.	3.2	2.6	3.3	3.0
43. It is difficult to obtain objective evidence of student mathematics achievement. The process of learning mathematics is unique to the individual, and does not lend itself readily to standardised evaluation.	2.7	2.9	2.8	2.7
44. It is good for students to see mathematics teachers make mistakes. It helps them understand that making their own mistakes is part of the science of mathematical thinking.	3.8	4.1	4.0	3.8
45. Mathematics is an enjoyable discipline.	4.6	4.6	4.8	4.6
46. Mathematics teachers have the responsibility to teach the requisite skills for future employment.	3.9	3.9	4.3	4.0
47. The results of standardised tests greatly influence what mathematics is taught.	3.3	3.5	3.7	3.6
48. All students should be required to pass a minimum competency test in mathematics to graduate from high school.	4.3	4.0	4.5	4.1
49. The greatest influence on my teaching of mathematics was my high school mathematics teacher(s).	3.4	3.0	3.1	3.2
50. The greatest influence on my teaching of mathematics was my coursework in college and/or teacher education.	2.8	2.9	2.7	2.7
51. The greatest influence on my teaching of mathematics has been my colleagues who are teachers.	3.3	3.5	3.1	3.2

Table 2 (continued)

Summary Table of TCMME Survey Item Means by Cluster

Item	Cluster 1	Cluster 2	Cluster 3	Cluster 4
52. I enjoy teaching mathematics.	4.8	4.8	4.9	4.8
Conceptions of Schooling*				
54. Schools should provide an opportunity for children to pursue their own talents, interests and creative abilities.	4.3	4.2	4.2	4.2
55. School curricula should function to preserve the traditions of society and the stability of our social institutions. Schools should be accountable to their local community as to how they are achieving these aims.	3.4	3.4	3.8	3.6
56. Schools should group students according to similar needs, interests and abilities, rather than according to age.	3.5	3.6	3.8	3.7
57. A major role of schools is to transmit the knowledge and skills associated with different branches of learning.	4.2	3.9	4.4	4.1
58. Schools must be innovative to ensure that we maintain a dynamic and expanding society.	4.2	4.4	4.3	4.3
59. Schools must offset inequalities by providing special opportunities to disadvantaged students.	3.7	4.1	3.9	4.0
60. Schools should be seen by students as places where they may find personal fulfillment, gain satisfaction from achieving their individual needs, and develop confidence in finding future direction.	4.5	4.6	4.6	4.5
61. To make an easy transition from school to the work place, schools should be places where students develop proper work values and learn to adapt to large groups, and where rewards are seen as both immediate (grades) and future (promise of employment).	4.3	4.1	4.5	4.2
62. Competition is an important component of schooling, both to motivate learning and as preparation for adult life.	4.1	3.2	4.0	3.6
63. Schools must train students to learn and apply rules, follow instructions, absorb facts and memorise detail.	3.7	2.8	4.0	3.4
64. Society must decide which years of a child's life shall be spent in formal learning, teachers must be responsible for determining the appropriate time to allocate materials, and students must learn to use their study time effectively.	3.7	3.4	4.0	3.7

*Teacher write-in items are not included.

Table 2 (continued)

Summary Table of TCMME Survey Item Means by Cluster

Item	Cluster 1	2	3	4
65. Schools exist to develop students' abilities to think, solve problems and make decisions by means of thorough training in academic disciplines.	4.4	4.0	4.4	4.3
66. Schools must allocate resources equally among all students, regardless of social, ethnic or other personal background.	4.1	3.9	4.3	3.7
67. Schools should be places that children feel are safe havens from the streets. Children must feel comfortable with teachers, administration and other students.	4.6	4.6	4.7	4.5
68. Schools should be for students who want to learn and who are willing to work, and not a social agency for attending to all the needs of school-age children.	3.6	3.5	4.0	3.8
N:	73	101	109	195

Teachers' Conceptions by Collaborative Participation Level

A part of the ongoing effort of the Documentation Project is to determine the impact of collaboration on teachers' knowledge, attitudes, and beliefs about their profession. A primary assumption behind the UMC project, then, is that collaboration will positively affect these systems. Thus, teachers who are frequent participants in the collaborative project should exhibit more positive attitudes toward teaching mathematics and should better reflect the spirit of mathematics reform than teachers who have not participated in project activities.

Participants were asked to rate their level of participation in their collaborative, indicating whether they participated frequently, occasionally, or never. A one-way ANOVA was performed on TCMME Survey items across participation level. Scheffe post-hoc analyses determined the location of significant differences. All analyses used the Welsh-Aspin correction for unequal sample sizes (separate variance estimates). Table 3, at the end of this section (page 53), gives the mean response to each item for each of the three levels of participation.

Teachers' Conceptions of Mathematics

Nonsignificant differences were found for all conceptions of mathematics across collaborative participation level except for Item 5. Occasional Participants rated "Mathematics is facts, skills, rules and concepts learned in some sequence and applied in work and future study" significantly closer to their own conception of mathematics than teachers who never participated. Both Frequent and Occasional Participants tended to rate this item close to their own conception while teachers who never participated felt more neutral towards the concept (see Tables E1 and E2 in Appendix E).

Teachers' Conceptions of Mathematics Teaching

Very few differences were found in conceptions of mathematics teaching between collaborative participation levels. Only Item 14 uncovered significant differences between groups. Although all participation levels rated Item 14 (To enable students to use mathematic to explore situations in an inquisitive manner, and to offer and test hypotheses by logical reasoning, for the purpose of developing a more complete understanding of the situation) important as a goal in their teaching, Frequent Participants rated this item higher than Occasional Participants (see Tables E3 and E4).

Teachers' Conceptions of Recommended Change

Since the UMC project is an outgrowth of the desire for change in secondary mathematics education and since the empowerment of teachers involves change in the ways in which they view their profession, it could be hypothesized that a major impact of collaboration should be on participants' conceptions of the importance of recommended change in high school mathematics. Indeed, the majority of group differences between frequent and occasional collaborative participants and those who never participated were uncovered in this section of the TCMME Survey. Results, in general, indicate that individuals who participate in collaborative activities tend to view recommended changes more favorably than nonparticipants.

Introduction of technology into high school mathematics courses. Item 17, which deals with the integration of calculators and computers into high school mathematics courses, was rated significantly more important by Frequent Participants than by Occasional Participants (see Tables E5 and E6)

Restructuring course content. Frequent Participants placed a higher emphasis on the importance of teaching simple mental computation, estimation, and approximation, rather than focusing on lengthy paper-and-pencil calculations (Item 20), than did either Occasional or Nonparticipants (See Tables E7 and E8). In addition, Frequent Participants favored inclusion of more topics from discrete mathematics, probability, and statistics in the high school curriculum (Item 21) than did Occasional Participants (Tables E9 and E10).

Frequent and Occasional collaborative participants rated incorporating mathematical modeling and integration of other academic subjects into the mathematics curriculum (Item 22) much higher than Nonparticipants (See Tables E11 and E12).

Teacher development. Item 23, which deals with development of new inservice programs for mathematic teachers that emphasize individual and small-group teaching, technology, and research, was rated much more important by Frequent Participants than by either Occasional or Nonparticipants. Non-significant differences were found between Occasional and Nonparticipants (see Tables E13 and E14).

Frequent Participants responded more favorably to the adoption of differential staffing patterns and career ladders for mathematics teachers than did Nonparticipants (Item 24). It is unclear exactly what these differences signify. Perhaps collaborative teachers feel that they are exceptional teachers, and therefore may benefit more from such programs (see Tables E15 and E16).

Item 25 is particularly significant to the UMC effort. It deals with teachers' beliefs about the importance of membership in professional organizations, like the collaboratives themselves. A direct positive trend is evident, with Frequent Participants rating this item more important than Occasional Participants, and Occasional Participants rating it much more highly than Nonparticipants. It seems, then, that Urban Mathematics Collaboratives fill an important role in the professional lives of participants (see Tables E17 and E18).

Teachers' Conceptions of Mathematics Education

Only two items in this section revealed significant differences among teachers across participation level in UMC activities. Item 34, which deals with teachers' conceptions that the primary responsibility of mathematics teachers is keeping order and covering all the material, was rated higher by Occasional Participants than by Frequent Participants (Tables E19 and E20). This is some indication that frequent participation in collaborative activities may influence teachers to redirect their instructional priorities toward some of the conceptions involving recommended changes and away from being primarily a disciplinarian.

Item 52, "I enjoy teaching mathematics," was rated higher by Frequent participants than by Nonparticipants (see Tables E21 and E22). Frequent participants felt very strongly that they enjoyed teaching mathematics. This is consistent with results from other Documentation Project surveys (Middleton et al., 1989).

Since the model for collaboration in the UMC project is to provide a variety of experiences for teachers to glean information from business, industry, and other teachers that would influence their own teaching of mathematics, it was hypothesized that collaborative participants would rate the individuals who most influenced their own teaching differently from the way nonparticipants would rate them. Specifically, Frequent participants were expected to rate their colleagues who are teachers higher as an influence than either their coursework in college or their own high school teachers. This, indeed, was the case. A Friedman two-way ANOVA by ranks was performed using Items 49, 50, and 51 as repeated measures (see Table E25). Frequent participants rated their colleagues and their high school teachers significantly higher than their coursework in college and/or teacher education, X^2 (2 d.f.) = 20.989, $p < .0001$. Occasional participants also rated their colleagues and their high school teachers as being more influential than their college courses, X^2(2 d.f.) = 11.308, $p < .01$. Nonparticipants, however, showed no significant difference among ratings of colleagues, high school teachers, and college coursework ($p < .05$).

In addition, if one looks at the magnitude of the mean ranks of Items 49, 50, and 51, an interesting pattern appears. The influence that ranked highest for Frequent participants was their colleagues who are teachers, which is consistent with our hypothesis. However, for Occasional participants, the highest ranking influence was their own high school teachers. Nonparticipants showed very little difference in mean rankings at all. This leads to the conclusion that UMC mathematics teachers are influenced *primarily* by other teachers. If teachers are primarily influenced by their colleagues or their own teachers, then the impact of the collaborative project becomes significant. For instance, if a prospective teacher is influenced primarily by her former high school mathematics teacher, and if that teacher is part of an effort to improve and reform teaching like the collaborative, then the student will be influenced by the collaborative *through her own teacher*. If another prospective mathematics teacher is influenced primarily by her colleagues, then involvement in the collaborative will help her improve and refine her teaching skills.

Both types of teachers will in turn influence their own students and colleagues who are prospective teachers. Collaboration, then, sets in motion a cycle of teacher improvement that continues through future generations of teachers. Without projects that encourage teachers in self-improvement, mathematics teaching may remain resistant to change and may not capitalize on new techniques and activities that can enrich students' mathematical expertise.

Teachers' Conceptions of Schooling

The focus of collaborative teachers on the improvement of their own teaching and schooling in general is evident in teachers' responses to Item 58. Frequent participants placed very high priority on innovation in schooling in order to maintain a dynamic and expanding society. Frequent participants rated Item 58 significantly higher than Nonparticipants (see Tables E23 and E24).

Discussion

Results indicate that participants in UMC activities tend to be more favorably inclined towards recommendations for change in mathematics education than nonparticipants. Frequent participants were inclined toward using mathematics in inquisitive hypothesis-testing situations. Congruent with this conception, Frequent participants also tended to view the use of technology and restructuring course content to emphasize estimation and approximation as desirable change in their own schools. In addition, Frequent participants felt favorably toward increased teacher education and inservice designed to integrate new teaching methods, technology, and research into their professional repertoire. This is consistent with findings from Middleton et al. (1989) that showed UMC teachers as more inclined to continue their education than a national sample of teachers.

In addition, collaborative teachers indicated that they enjoy teaching mathematics more than Nonparticipants. Although it may be that Frequent participants enjoy mathematics teaching more, and thus engage in pertinent activities like those provided by the collaboratives, it also suggests a positive impact of the collaborative project on influencing teachers' affect. Teachers' statements on the Diary of Professional Relationships seem to reflect these patterns. Teachers stated that the collaborative provided them with the opportunity to interact with other teachers, to learn new classroom methods, and to broaden their conceptions of the nature of mathematics education. In addition, teachers reported that the collaborative gave them a fresh outlook on their profession, increased their sense of control and confidence in their abilities, and curbed burnout--in more than just a few cases.

Further evidence supporting the positive influence of the UMC project on teachers' conceptions is that participants tended to rate their colleagues as a greater influence on their own teaching more than nonparticipants. This would indicate that the collaboratives are fulfilling an important objective in providing teachers with the opportunity to interact with each other and share new and varied teaching techniques. Further, it seems that teacher collaboration may initiate a cycle of positive development in mathematics education through the influence of collaborative participants on future teachers of mathematics, their students.

Table 3

Summary Table of TCMME Survey Item Means by Collaborative Participation Level

Item	Participation Level Frequent	Occasional	Never
Conceptions of Mathematics			
1. Mathematics is a process in which abstract ideas are applied to solve real-world problems.	4.2	4.1	4.0
2. Mathematics is a language, with its own precise meaning and grammar, used to represent and communicate ideas.	4.4	4.5	4.5
3. Mathematics is a collection of concepts and skills used to obtain answers to problems.	4.0	4.1	4.0
4. Mathematics is thinking in a logical, scientific, inquisitive manner and is used to develop understanding.	4.6	4.6	4.4
5. Mathematics is facts, skills, rules and concepts learned in some sequence and applied in work and future study.	3.9	4.0	3.5
6. Mathematics is an interconnected logical system, is dynamic, and changes as new problem solving situations arise. It is formed by thinking about actions and experiences.	4.3	4.1	4.2
Conceptions of Mathematics Teaching*			
9. To enable students to master a hierarchy of concepts and skills and to use these in solving problems.	4.3	4.3	4.2
10. To provide experiences for students to know mathematics as originating in real-world situations and to have the power of using a small set of symbols to represent and solve a wide range of problems.	4.4	4.3	4.3
11. To enable students to use mathematical procedures to solve problems and mathematical concepts to model both abstract and real-world situations.	4.5	4.4	4.3
12. To provide students with complete understanding of the meaning(s) of mathematical concepts and enable them to communicate ideas using correct mathematical symbols, rules and reasoning.	4.1	4.1	4.0
13. To prepare students for work and future study by having them master a sequence of facts, paper-and-pencil skills, rules and concepts.	3.5	3.7	3.7
14. To enable students to use mathematics to explore situations in an inquisitive manner, and to offer and test hypotheses by logical reasoning, for the purpose of developing a more complete understanding of the situation.	4.6	4.4	4.4

*Teacher write-in items are not included

Table 3 (continued)

Summary Table of TCMME Survey Item Means by Collaborative Participation Level

Item	Participation Level Frequent	Occasional	Never
Conceptions of Recommended Change*			
17. Calculators and computers should be introduced into mathematics courses to enhance understanding and problem solving, and to take the drudgery out of computations. Presentation of topics needs to be revised based on fresh approaches possible with new technologies.	4.6	4.3	4.2
18. Traditional high school mathematics courses need to be integrated and unified to show interrelationships across topics and applications.	4.3	4.1	4.1
19. Alternative mathematics courses should be available for students who are planning not to go to college or who are planning not to take a college major with high mathematics content.	4.1	4.2	4.1
20. More emphasis should be given to simple mental computation, estimation and approximation, and less to practicing lengthy paper-and-pencil calculations.	4.2	3.9	3.8
21. More topics and techniques from discrete mathematics, statistics and probability should be introduced into the high school curriculum.	4.0	3.7	3.8
22. Mathematical modeling and problem solving should be incorporated as a central feature in high school mathematics, and should be integrated into other parts of school curricula (such as science and social studies).	4.4	4.4	4.1
23. Preservice and inservice teacher education programs need to be developed that train teachers in individual and small group teaching, the use of technology, and research.	4.4	4.1	3.9
24. Schools must adopt differential staffing patterns and career ladders for mathematics teachers by appointing master teachers to develop, coordinate and supervise new programs.	3.6	3.5	3.1
25. Mathematics teachers should be encouraged to become members of professional mathematical societies and to attend regional and national meetings.	4.4	3.9	3.5
26. A core mathematics program should provide optional tracks and electives, and the opportunity for every student through Grade 10 to prepare for college entry.	4.2	4.1	4.0

*Teacher write-in items are not included.

Table 3 (continued)

Summary Table of TCMME Survey Item Means by Collaborative Participation Level

Item	Participation Level Frequent	Occasional	Never
27. A state-level prognostic test in mathematics should be administered to all students in Grade 9 or 10 to determine if they are ready to pursue further math-related work or study, or if they are in need of remediation or course changes. Results of such tests would not be available for the purpose of college admission or to evaluate teachers.	3.6	3.6	3.4
28. Increased funding should be made available for the development of improved, appropriate materials, diagnostic techniques and teaching strategies for remedial programs.	4.1	4.1	3.9
29. Strong efforts must be made to increase the awareness of the importance of mathematics among all members of the community, especially among parents of school age children.	4.5	4.3	4.3
30. A core mathematics program should be established which requires all students to study mathematics through Grade 11.	4.2	4.1	4.1
31. Special efforts should be made to identify mathematically talented students, especially minorities and women, and to encourage them to pursue careers in mathematics, science and mathematics education.	4.5	4.4	4.3
32. Parents should have the option of sending their children to the public school of their choice.	3.3	3.2	3.5
Conceptions of Mathematics Education*			
34. Mathematics teachers' primary responsibility is keeping order, keeping students busy and productive in the classroom, and covering all the material.	2.2	2.5	2.2
35. Calculators and computers can facilitate the learning of mathematics. Hands-on experience with changing technology should be incorporated as an integral part of mathematics instruction.	4.6	4.5	4.4
36. Special applications, real-world problems and extracurricular activities must be tailored to the needs and abilities of each student to help them excel.	4.1	4.0	4.2
37. Mathematics teachers must teach students to communicate using conventional mathematical signs, symbols, and vocabulary.	4.0	4.1	4.2

*Teacher write-in items are not included.

Table 3 (continued)

Summary Table of TCMME Survey Item Means by Collaborative Participation Level

Item	Participation Level Frequent	Occasional	Never
38. Mathematical analysis, interpretation and inquiry should be taught concurrently with the basic skills. Students must be taught to use mathematics to gain understanding of a variety of phenomena.	4.4	4.3	4.2
39. Mathematics teachers have the responsibility to teach the requisite skills for subsequent courses.	4.2	4.3	4.1
40. Mathematical skills and rules should not be taught in isolation. Mathematics needs to be discovered by students through applied problem solving.	4.3	4.1	4.3
41. Mathematics teachers sometimes have to sacrifice the broader aims of the course in order to spend more time bringing the entire class up to a minimum competency level.	3.4	3.5	3.5
42. Mathematics teachers should demand strict adherence to methods and notations used in class.	3.0	3.1	3.1
43. It is difficult to obtain objective evidence of student mathematics achievement. The process of learning mathematics is unique to the individual, and does not lend itself readily to standardized evaluation.	2.9	2.7	2.9
44. It is good for students to see mathematics teachers make mistakes. It helps them understand that making their own mistakes is part of the science of mathematical thinking.	3.9	3.9	4.0
45. Mathematics is an enjoyable discipline.	4.7	4.6	4.5
46. Mathematics teachers have the responsibility to teach the requisite skills for future employment.	4.0	4.0	4.0
47. The results of standardized tests greatly influence what mathematics is taught.	3.6	3.5	3.5
48. All students should be required to pass a minimum competency test in mathematics to graduate from high school.	4.2	4.1	4.4
49. The greatest influence on my teaching of mathematics was my high school mathematics teacher(s).	3.2	3.3	2.8
50. The greatest influence on my teaching of mathematics was my coursework in college and/or teacher education.	2.7	2.9	2.9
51. The greatest influence on my teaching of mathematics has been my colleagues who are teachers.	3.3	3.2	3.1

Table 3 (continued)

Summary Table of TCMME Survey Item Means by Collaborative Participation Level

Item	Participation Level Frequent	Occasional	Never
52. I enjoy teaching mathematics.	4.9	4.8	4.7
Conceptions of Schooling*			
54. Schools should provide an opportunity for children to pursue their own talents, interests and creative abilities.	4.2	4.2	4.2
55. School curricula should function to preserve the traditions of society and the stability of our social institutions. Schools should be accountable to their local community as to how they are achieving these aims.	3.6	3.5	3.6
56. Schools should group students according to similar needs, interests and abilities, rather than according to age.	3.7	3.6	3.7
57. A major role of schools is to transmit the knowledge and skills associatea with different branches of learning.	4.1	4.2	4.2
58. Schools must be innovative to ensure that we maintain a dynamic and expanding society.	4.4	4.2	4.3
59. Schools must offset inequalities by providing special opportunities to disadvantaged students.	4.0	3.8	3.9
60. Schools should be seen by students as places where they may find personal fulfillment, gain satisfaction from achieving their individual needs, and develop confidence in finding future direction.	4.5	4.5	4.5
61. To make an easy transition from school to the work place, schools should be places where students develop proper work values and learn to adapt to large groups, and where rewards are seen as both immediate (grades) and future (promise of employment).	4.3	4.3	4.1
62. Competition is an important component of schooling, both to motivate learning and as preparation for adult life.	3.7	3.7	3.5
63. Schools must train students to learn and apply rules, follow instructions, absorb facts and memorise detail.	3.4	3.5	3.4
64. Society must decide which years of a child's life shall be spent in formal learning, teachers must be responsible for determining the appropriate time to allocate materials, and students must learn to use their study time effectively.	3.7	3.7	3.6

*Teacher write-in items are not included.

Table 3 (continued)

Summary Table of TCMME Survey Item Means by Collaborative Participation Level

Item	Participation Level Frequent	Occasional	Never
65. Schools exist to develop students' abilities to think, solve problems and make decisions by means of thorough training in academic disciplines.	4.3	4.3	4.0
66. Schools must allocate resources equally among all students, regardless of social, ethnic or other personal background.	4.1	3.8	3.8
67. Schools should be places that children feel are safe havens from the streets. Children must feel comfortable with teachers, administration and other students.	4.6	4.5	4.7
68. Schools should be for students who want to learn and who are willing to work, and not a social agency for attending to all the needs of school-age children.	3.7	3.9	3.5
N:	231	202	40

Teachers' Conceptions by Collaborative Site

Data from previous surveys (Romberg et al., 1988; Middleton et al., 1989) indicate that collaboratives vary in demographics, resources available, state and local policy, and the emphasis of collaborative activities. Consequently, some differences among collaborative sites were expected in teachers' responses to the TCMME Survey. How these differences affect teachers' conceptions of mathematics and mathematics education are as yet unclear. However, contrasting collaborative responses should illuminate individual differences in collaboratives and should suggest ways in which projects such as the UMC project can effectively tailor resources to fit the individual needs of different urban areas.

A one-way ANOVA was performed on TCMME Survey responses across collaboratives. Scheffe post-hoc analyses determined the locations of the main effects. It must be noted here that, since the Scheffe procedure partitions the familywise Type I error rate by the number of contrasts, the power of detecting a significant difference for each contrast is significantly reduced. Therefore, differences among collaborative sites other than those reported are likely. Only the most improbable are reported here.

Perhaps the most interesting findings have to do with the relatively few differences detected among collaborative sites. Out of 55 contrasts per item by 64 items (3,520 total possible differences), only 25 contrasts, less than 1 percent, were classified as significant differences. Although this may be a function of the loss of power by dividing up the error rate into 55 contrasts, it also attests to the fact that participation level and the profiles of mathematics conceptions were distributed similarly across most collaboratives. If conception of mathematics and participation level are major influences on teachers' conceptions of mathematics education, then few major differences should be evident across collaboratives except where conception of mathematics and participation level are different across sites. Differences that were detected among collaborative sites were fairly consistant across all contrasts. Teachers in New Orleans, San Francisco, San Diego, and the Twin Cities were found to rate items differently from the way teachers in other collaboratives rated them, for the majority of the significant contrasts.

Teachers' Conceptions of Mathematics

As with the examination of teachers' conceptions by profile of responses and by participation level, Item 5 seems to discriminate among groups of teachers more than the other conceptions of mathematics. Teachers in New Orleans agreed strongly that the conception of mathematics as facts, skills, rules and concepts fit their own conceptions of mathematics, whereas teachers in the San Francisco and Twin Cities collaboratives rated this conception more neutral (see Tables F1 and F2 in Appendix F).

Teachers' responses to the Diary of Professional Relationships seem to reflect these site differences. For example, the five teachers in San Francisco who responded to the Diary all emphasized language and communications as part of the domain of mathematics, whereas the three respondents in New Orleans focused on the relationships and patterns between quantities, consistent with New Orleans teachers' responses to Item 5.

Teachers' Conceptions of Mathematics Teaching

No significant differences were found between any pair of collaborative sites for any of the six conceptions of mathematics teaching.

Teachers' Conceptions of Recommended Change

Although teachers across all collaboratives agreed that technologies such as calculators and computers enhance comprehension of mathematics topics by eliminating drudgery (Item 17), teachers in New Orleans felt this recommendation was only slightly important. Teachers in San Diego and the Twin Cities, on the other hand, felt that the introduction of new technologies was very important (see Tables F3 and F4).

When rating the importance of mathematics teachers becoming members of professional organizations (Item 25), teachers in Pittsburgh tended to feel more neutral than teachers from Cleveland, Philadelphia, and the Twin Cities (see Tables F5 and F6).

Teachers in New Orleans and Pittsburgh tended to feel that the recommendation of a state level prognostic test in mathematics (Item 27) was important. Teachers in the Twin Cities and San Diego, however, felt more neutral toward the item (see Tables F7 and F8).

Teachers' Conceptions of Mathematics Education

Similarly to their conceptions on Item 17, which also deals with the introduction of technology, teachers in New Orleans rated Item 35 lower than teachers in Durham, Philadelphia, San Diego, San Francisco, and the Twin Cities. Teachers at the latter sites agreed fairly strongly that calculators and computers should be an integral part of mathematics instruction, whereas New Orleans teachers, although they agreed with the item, felt less strongly about it (see Tables F9 and F10). These findings may indicate a lack of availability of technology for many New Orleans teachers. Middleton et al (1989) found that the majority of respondents to the Secondary Mathematics Teacher Questionnaire from Durham, Philadelphia, San Diego, San Francisco, and the Twin Cities used calculators in their classes, whereas the majority of respondents from New Orleans indicated that they did not use calculators in their classrooms and that computers were not available for their use in teaching.

Teachers in two of the California collaboratives, San Diego and San Francisco, tended to feel neutral toward Item 41 (Mathematics teachers sometimes have to sacrifice the broader aims of the course in order to spend more time bringing the entire class up to a minimum competency level). New Orleans teachers, however, tended to agree with the statement (see Tables F11 and F12). These conceptions may be due to differences at these sites in the emphasis placed on mandated competency testing. The New Orleans Public School District is under considerable pressure to raise students' test scores.

When asked about the major influences on their own mathematics teaching, teachers in Memphis and New Orleans tended to rate their high school teachers (Item 49) higher than did teachers in San Diego and San Francisco (see Tables F13 and F14).

Teachers' Conceptions of Schooling

Competition (Item 62) as an important component of schooling was rated fairly neutral by the majority of collaboratives. Memphis and New Orleans tended to rate competition more importantly than did either San Francisco and the Twin Cities (Tables F15 and F16).

Teachers in New Orleans agreed with the statement "Schools must train students to learn and apply rules, follow instructions, absorb facts and memorize detail" (Item 63) more than did teachers in the Twin Cities (see Tables F17 and F18). Responses to the Diary of Professional Relationships from teachers in New Orleans seem to mirror responses to the T-CME Survey. One teacher emphasized increasing students' knowledge and mathematical skills, while another stressed that students were not working up to capacity, thus reducing test scores.

Cluster Membership by Collaborative Site

Significant differences were found between the proportion of teachers in each of the four clusters extracted by collaborative site. In particular, New Orleans teachers exhibited a higher proportion of teachers in the Global Cluster and a lower proportion of teachers in the Dynamic Inquiry Cluster than other collaboratives (see Table F19). New Orleans was significantly different from all collaboratives except Memphis, Pittsburgh, and St. Louis in the distribution of teachers across conceptions of mathematics. Additional differences were found between San Francisco and Twin Cities teachers, and teachers in New Orleans, Memphis, Philadelphia, and San Diego. These differences seem to be centered around the higher proportion of teachers in the Twin Cities and San Francisco in the Dynamic Inquiry Cluster and the lower proportion of teachers in the Fixed Body of Knowledge Cluster.

Many of the differences in ratings between collaboratives seem to be reflected in the proportion of teachers within sites within each of the four clusters. For instance, teachers in New Orleans rated Item 5 (Mathematics is facts, skills, rules, and concepts learned in some sequence and applied in work and future study) higher than teachers in San Francisco and the Twin Cities. Since San Francisco and the Twin Cities have higher proportions of teachers in the Dynamic Inquiry Cluster, which tends to contrast with the conception of mathematics as primarily facts, skills, and rules, these teachers should influence the mean score for their collaboratives to be different from the mean score of teachers in New Orleans.

Discussion

Results indicate that participants in the collaboratives, in general, are highly similar in their conceptions of mathematics, mathematics teaching, recommended change, mathematics education, and schooling. The differences that were detected reflect the issues that are influencing mathematics education at each of the different sites. In addition, differences seem to be related to teachers' conceptions of mathematics. It is unclear just how teachers' conceptions are influenced by the various social and policy issues that are in play within each site, or how conceptions influence the articulation of policy in everyday schooling. However, these differences in conceptions do illustrate the individuality of each of the eleven UMC sites.

Particularly interesting are the contrasts between New Orleans and San Diego and between San Francisco and the Twin Cities. The relatively high percentages of teachers in the latter two cities in the Dynamic Inquiry Cluster (41% and 35%, respectively) and low percentages in the Fixed Body of Knowledge Cluster (12% and 2%, respectively) would suggest increased attention to the changing aspects of mathematics education, including the use of technology, attention to individual differences, and decreased attention to norm-referenced testing and strict adherence to rules.

New Orleans, on the other hand, showed high percentages of teachers in the Global and Fixed Body of Knowledge Clusters (40% and 22%, respectively). This would suggest attention on the part of these teachers to more rigid standards of academic progress, teaching the basics of mathematics, and evaluation of student progress to assure articulation of this knowledge.

San Diego, with the highest percentage of teachers in the Fixed Body of Knowledge Cluster, is more difficult to describe. Although cluster membership would suggest teachers are teaching mathematics in a more structured way, with decreased attention to technologies, exactly the opposite is the case. This may be an artifact of the remaining percentage of teachers being distributed across the other three clusters, or it may attest to some other conception of mathematics altogether that was not uncovered by the cluster analysis.

These differences across collaboratives attest to the need for teacher empowerment projects to be sensitive to the individual differences among teachers in different urban centers. The model for the UMC project is to influence schooling at that crucial point where policy and the intended curriculum are translated into instruction: Teachers. Without sufficient attention to the specific characteristics and needs of teachers within the different sites, the UMC project would cease to be a *collaborative* venture. Instead, through its focus on teachers within the context of their own cities and districts, the UMC project can more effectively concentrate effort to meet their specific needs.

IV. GENERAL DISCUSSION

Results indicate that mathematics teachers participating in the Urban Mathematics Collaborative project, as indicated by their responses to the TCMME Survey, appear to embrace the spirit of mathematics education reform in the United States. They seem to hold flexible, multidimensional conceptions of the nature of mathematics and to place particular emphasis on the notion that mathematics is both logical, scientific thinking and a way of communicating ideas. Although the more traditional view of mathematics, that it is a body of facts and skills, is not denied by the responses of teachers, they placed less emphasis on this conception than on those that emphasized mathematics as a way of thinking. Teachers reported on the Diary of Professional Relationships that their involvement in the collaborative had had a significant impact on the ways in which they approach mathematics and mathematics teaching. Their responses indicate that the collaboratives have given them greater flexibility in thinking about mathematics, and more exposure to new ideas and methods and have made them more aware of the needs of their students than before their involvement.

The love of teaching that participating teachers express dispels some of the myths that all urban teachers are complacent and burned out. They are sensitive to the needs and individual differences of their students, yet they also emphasize the need for all students to become more mathematically powerful (e.g., NCTM, 1989). Teachers indicated their belief that the introduction of new teaching methods and technologies enhances mathematics teaching. In addition, teachers appear to be aware of the importance of mathematics education to their students and to society. In addition, the differences in which Frequent participants rated the enjoyment they receive from teaching mathematics over that of Nonparticipants lends strong support for the continuation of teacher empowerment projects such as the UMC effort.

Darling-Hammond and Hudson (1987) report that collaborative, rather than isolated, teaching settings seem to increase teacher learning and commitment to teaching and this results in lower teacher absenteeism and turnover. Conversely, they stipulate that the absence of professional development opportunities contributes to teacher dissatisfaction and attrition. Since UMC Participants get more enjoyment out of teaching mathematics than Nonparticipants, the collaboratives may foster this feeling and are one method of curbing teacher burnout.

One method of increasing teacher professionalism is to provide teachers the opportunity for professional dialogue with their teaching colleagues. This discourse leads teachers to accept new methods of teaching and, perhaps more importantly, to gain new enthusiasm for their profession and respect for themselves as professionals (e.g., Holly, 1983). Participants in collaborative activities in the present study indicated that they are influenced by their peers who are teachers more than by their college and teacher education courses. Thus, UMC teachers seem to be attuned to observation and modeling of effective teaching techniques by their peers. This feeling has been documented in other teachers. Watts (1985), in studying the verbal and written reports and the results of surveys of nearly 300 teachers, reports that teachers not only learn from each other, but that this is their *preferred* mode of learning. Many of her participants felt that no one understands teachers except other teachers.

A teacher not only knows or believes certain things, but applies such knowledge and beliefs in his or her work (Darling-Hammond & Hudson, 1987). Indeed, this statement seems to be reflected in the conceptions of UMC teachers. Differences in teachers' conceptions of mathematics corresponded to differences in the ways in which teachers rated various issues that pertain to their profession. Through an examination of teacher profiles of responses across the six conceptions of mathematics, four clusters of teachers, who were similar in the ways they viewed mathematics, emerged. Although conceptions of mathematics across clusters overlapped, they successfully described differences in the ways in which teachers viewed aspects of mathematics teaching, recommendations for change, mathematics education, and other aspects of schooling. This would suggest that the two types of knowledge perceived to be necessary for effective teaching--knowledge of what to teach and knowledge of how to teach (e.g., Darling-Hammond & Hudson, in press)--interrelate to substantially affect how recommendations for change arise in different situations. And since teachers' views were distributed across the several conceptions of mathematics differently by collaborative site, it would seem likely that these differences would be manifest in how different collaboratives effect change within their infrastructures.

Results further suggest that documentation of the impact of collaboration on teachers' conceptions of their occupation necessitates a multitrait, multimethod approach. Collaboration is a complex interconnected process. It is peculiar to subject area, participation level, and collaborative site, and it is peculiar to the conceptions held by the participating individuals. Thus, assessing the impact of collaboration must address these individual differences. For example, in the present study, differences were found not only between participation level, but between collaboratives and within collaboratives. In addition, striking differences were found by examining the interaction between teachers' conceptions of mathematics and their geographic location. By looking at "collaboratives" in general, these differences would not have been discovered. Further, even if differences across collaboratives were detected, the meaning assigned the differences between, say, New Orleans and Twin Cities teachers would have been difficult to establish. By looking at the individual differences within collaboratives, however, these patterns can be assigned meaning, and attention can be given to the specific characteristics and needs of each collaborative as a unit.

As an integral part of the ongoing effort of documenting the UMC project, the results of this survey will be used in conjunction with both previously acquired demographic and attitudinal data (e.g., Romberg et al., 1988; Middleton et al., 1989) and future data collection to determine the effects of collaboration on teacher professionalism in a longitudinal manner, i.e., on the ways in which collaboration has reduced teacher isolation and fostered a sense of empowerment on the part of urban mathematics teachers. Of particular interest are the ways in which individual collaboratives have addressed the needs of teachers who embody differing conceptions of mathematics and the implications these influences may have on future collaboratives after the termination of the project.

Preliminary results, of which this study on teacher conceptions is but a part, indicate that the Urban Mathematics Collaborative project has served to reduce teachers' feelings of isolation, increase teachers' sense of professionalism, and increase teachers' knowledge of their subject matter, as well as helped to curb teacher burnout and foster enthusiasm towards mathematics education. A statement provided by a teacher in Los Angeles describes this impact quite elegantly, "Working with the Urban Math Collaborative, which takes teachers' positions and views seriously, has helped keep me sane."

REFERENCES

Buck, R. D. (1965). Goals for mathematics instruction. *American Mathematical Monthly, 72,* 949-956.

Carpenter, T. P., Coburn, T. G., Reys, R. E., & Wilson, J. W. (1979). *Results from the First Mathematics Assessment of the National Assessment of Educational Progress.* Reston, VA: National Council of Teachers of Mathematics.

Carpenter, T. P., Corbitt, M. K., Kepner, H. S., Lindquist, M. M., & Reys, R. E. (1981). *Results from the Second Mathematics Assessment of the National Assessment of Educational Progress.* Reston, VA: National Council of Teachers of Mathematics.

Cole, M., & Griffin, P. (Eds.). (1987). *Contextual factors in education: Improving science and mathematics education for minorities and women.* Madison: Wisconsin Center for Education Research.

Collier, C. P. (1972, May). Prospective elementary teachers' intensity and ambivalence of beliefs about mathematics and mathematics instruction. *Journal for Research in Mathematics Education,* 155-163.

Conroy, J. (1987). Monitoring school mathematics: Teachers' perceptions of mathematics. In T. A. Romberg & D. M. Stewart (Eds.), *The monitoring of school mathematics: Background papers. Volume 3: Teaching and future directions* (Program report 87-3 pp. 75-92). Madison: Wisconsin Center for Education Research.

Crosswhite, F. J., Dossey, J. A., Swafford, J. O., McKnight, C. C., & Cooney, T. J. (1985). *Second international mathematics study: Summary report for the United States.* Champaign, IL: Stipes Publishing Co.

Darling-Hammond, L., & Hudson, L. (1987). Indicators of teacher quality for a comprehensive monitoring system of mathematics and science education. Paper prepared for the National Science Foundation project "Monitoring National Progress in Mathematics, Science and Technology Education."

Darling-Hammond, L., & Hudson, L. (in press). Precollege science and mathematics teachers: Supply, demand and quality. *Review of Research in Education.*

Dorfler, W., & McLone. R. R. (1986). Mathematics as a school subject. In B. Christiansen, A. G. Howson, & M. Otte (Eds.), *Perspectives on mathematics education (pp. 49-97). Boston:* D. Reidel Publishing Company.

Halmos, P. R. (1981). Applied mathematics is bad mathematics. In L. A. Steen (Ed.), *Mathematics tomorrow* (pp. 9-20.). New York: Springer-Verlag.

Hilton, P. (1987). Current trends in mathematics and future trends in mathematics education. In T. A. Romberg & D. M. Stewart (Eds.), *The monitoring of school mathematics: Background papers. Volume 1: The monitoring project and mathematics curriculum.* (Program Report 87-1 pp. 149-163). Madison: Wisconsin Center for Education Research.

Holly, M. L. (1983). Teacher reflections on classroom life: Collaboration and professional development. *Australian Administrator, 4*(4), 1-6.

Holmes Group, Inc. (1986). *Tomorrow's teachers: A report of the Holmes Group.* East Lansing, MI: Author.

Howson, A. G. (Ed.). (1973). *Developments in mathematical education.* Cambridge: Cambridge University Press.

Lampert, M. (1987, April). Attending to students' thinking in math classes: Teachers' perspectives. Invited symposium at the annual meeting of the American Educational Research Association, Washington, DC.

Lesgold, A. M., & Reif, F. (1983). Chairmen's Report. In A. M. Lesgold & F. Reif (Eds.), *Computers in education: Realizing the potential. Report of a research conference* (pp. 11-34). Washington, DC: U.S. Department of Education.

Maeroff, G. I. (1988). *The empowerment of teachers: Overcoming the crisis of confidence.* New York: Teachers College Press.

McLeod, D. B. (1988). Research on learning and instruction in mathematics: The role of affect. In E. Fennema, T. P. Carpenter, & S. J. Lamon (Eds.), *Integrating research on teaching and learning mathematics.* Madison: Wisconsin Center for Education Research.

Marascuilo, L. A., & Serlin, R. C. (1988). *Statistical methods for the social and behavioral sciences.* New York: W. H. Freeman and Company.

Middleton, J. A., Webb, N. W., Romberg, T. A., Pittelman, S. D., Pitman, A. J., Richgels, G. M., & Fadell, E. M. (1989). *Characteristics and attitudes of frequent participants in the Urban Mathematics Collaboratives: Results of the secondary mathematics teacher questionnaire.* Madison: Wisconsin Center for Education Research.

National Council of Teachers of Mathematics. (1970). *A history of mathematics education in the United States and Canada.* Washington, DC: Author.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

National Research Council. (1970). *The role of applications in the undergraduate curriculum.* Washington, DC: National Academy Press.

National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education.* Washington DC: National Academy Press.

Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms.* New York: Routledge & Kegan Paul.

Popkewitz, T. S., & Myrdal, S. (Eds.). (in preparation). Case studies of the eleven Ford Foundation Urban Mathematics Collaboratives. Madison: Wisconsin Center for Education Research.

Putnam, R. T., & Leinhardt, G. (1986, April). Curriculum scripts and the adjustments of content in mathematics lessons. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.

Resnick, L. B., & Ford, W. W. (1981). *The psychology of mathematics for instruction.* Hillsdale, NJ: Lawrence Erlbaum Associates.

Romberg, T. A. (1983). A common curriculum for mathematics. In G. D. Fenstermacher & J. I. Goodlad (Eds.), *Individual differences and the common curriculum: 82nd yearbook of the NSSE* (Part I, pp. 121-159). Chicago: University of Chicago Press.

Romberg, T. A. (1984). *Proposal to monitor collaboration projects for the Ford Foundation.* Unpublished proposal for funding, Wisconsin Center for Education Research.

Romberg, T. A. (1988). *Changes in school mathematics: Curricular changes, instructional changes and indicators of change.* New Brunswick, NJ: Center for Policy Research in Education.

Romberg, T. A., Pitman, A., Pittelman, S. D., Webb, N., Fadell, E. M., & Middleton, J. A. (1988). *Mathematics teachers' views about teaching as a profession: An initial assesssment* (Report from the UMC Documentation Project). Madison: Wisconsin Center for Education Research.

Scheding, J. C. (1981). *Teachers' perceptions of the nature of mathematics and the effect of instruction theron.* Unpublished doctoral dissertation, University of Colorado, Boulder.

Sobel, M. A. (1981). Implementing the *Agenda for Action.* In J. Price & J. D. Gawronski (Eds.), *Changing school mathematics: A responsive process* (pp. 188-200). Reston, VA: National Council of Teachers of Mathematics.

Spanier, J. (1981). Solving equations is not solving problems. In L. A. Steen (Ed.), *Mathematics tomorrow* (pp. 21-27). New York: Springer-Verlag.

SPSS Inc. (1986). *SPSSX User's guide* (2nd edition). Chicago, IL: Author.

Stake, R. E., & Easley, J. A., Jr. (1978). *Case studies in science education. Volume I. The case reports. Volume II. Design, overview and general findings.* Washington, DC: U.S. Gov[illegible]nt Printing Office.

Thompson, A. G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics*, *15*, 105-127.

Watts, H. (1985). How teachers learn: Teachers' views on professional development. Keane, NH: Unpublished Manuscript.

Webb, N. L., Pittelman, S. D., Romberg, T. A., Pitman, A. J., Fadell, E. M., & Middleton, J. A. (1989). *The Urban Mathematics Collaborative Project: Report to the Ford Foundation on the 1987-88 school year* (Program Report 89-1). Madison: Wisconsin Center for Education Research.

Appendix A

Teachers' Conceptions of Mathematics and Mathematics Education Questionnaire

Urban Mathematics Collaborative Documentation Project
University of Wisconsin--Madison

Date____________________
(month) (day) (year)

Teacher Survey III

Please fill in today's date in the upper right hand corner of this booklet and your name, school, city and state in the spaces provided below.

Name____
(first) (last)

School____

City, State_______________________________________

Mathematics Courses
You Teach Currently_______________________________

Grade Level_______________________________________

Please circle the letter which best describes your level of participation in your collaborative:

A. Frequent
B. Occasional
C. Never

This survey contains 68 statements designed to gather information about your opinions regarding five areas of mathematics:

I. Your Conceptions of Mathematic
II. Your Conceptions of Mathematics Teaching
III. Your Conceptions of Recommended Changes in Mathematics Curriculum
IV. Your Conceptions of Mathematics Education
V. Your Conceptions of Schooling

Please read each statement carefully, but do not spend too much time on any one item. Remember, there are no right or wrong answers. All responses will be strictly confidential. Only summary information will be shared.

Thank you for your participation in completing this survey.

Conceptions of Mathematics

The statements listed below portray a variety of viewpoints as to the nature of mathematics. Please rate each statement on a 5-point scale according to how strongly you agree that each statement reflects your own concept of mathematics. (The number 5 indicates that you strongly agree with the statement and the number 1 indicates that you strongly disagree with the statement. A rating of 3 indicates that you are undecided whether the statement reflects your concept of mathematics.)

		Agree Strongly		Neutral		Disagree Strongly
1.	Mathematics is a process in which abstract ideas are applied to solve real-world problems.	5	4	3	2	1
2.	Mathematics is a language, with its own precise meaning and grammar, used to represent and communicate ideas.	5	4	3	2	1
3.	Mathematics is a collection of concepts and skills used to obtain answers to problems.	5	4	3	2	1
4.	Mathematics is thinking in a logical, scientific, inquisitive manner and is used to develop understanding.	5	4	3	2	1
5.	Mathematics is facts, skills, rules and concepts learned in some sequence and applied in work and future study.	5	4	3	2	1
6.	Mathematics is an interconnected logical system, is dynamic, and changes as new problem-solving situations arise. It is formed by thinking about actions and experiences.	5	4	3	2	1

7. Please write in the spaces below the numbers of the 6 above statements, in the order that they reflect your belief of what mathematics is:

______ ______ ______ ______ ______ ______

Most Reflective ... Least Reflective

8. Please feel free to comment on any important aspect, not mentioned above, that reflects your concept of mathematics.

Conceptions of Mathematics Teaching

The following statements are sometimes cited as important goals for teaching mathematis in schools. Please rate each goal on the 5-point scale as to its importance to your teaching of mathematics. (The number 5 indicates that the goal is very important to your teaching, and the number 1 indicates that the goal is very unimportant to your teaching. The number 3 indicates that you are undecided as to the goal's importance to your teaching.)

		Very Important		Neutral		Very Unimportant
9.	To enable students to master a hierarchy of concepts and skills and to use these in solving problems.	5	4	3	2	1
10.	To provide experiences for students to know mathematics as originating in real-world situations and to have the power of using a small set of symbols to represent and solve a wide range of problems.	5	4	3	2	1
11.	To enable students to use mathematical procedures to solve problems and mathematical concepts to model both abstract and real-world situations.	5	4	3	2	1
12.	To provide students with complete understanding of the meaning(s) of mathematical concepts and enable them to communicate ideas using correct mathematical symbols, rules and reasoning.	5	4	3	2	1
13.	To prepare students for work and future study by having them master a sequence of facts, paper-and-pencil skills, rules and concepts.	5	4	3	2	1
14.	To enable students to use mathematics to explore situations in an inquisitive manner, and to offer and test hypotheses by logical reasoning, for the purpose of developing a more complete understanding of the situation.	5	4	3	2	1

15. Please write in the spaces below the numbers of the 6 above statements, in the order that they reflect your belief of what the important goals for teaching mathematics are:

_______ _______ _______ _______ _______ _______

Most Reflective ... Least Reflective

16. Please feel free to comment on any important aspect, not mentioned above, that reflects your concept of mathematics teaching.

____

____

____

Conceptions of Recommended Change

Some recent recommendations for high school mathematics are listed below. Please read each recommendation and rate it in terms of its importance to your mathematics curriculum. (The number 5 indicates that the recommendation is very important to your curriculum and the number 1 indicates that the recommendation is very unimportant to your curriculum. A rating of 3 indicates that you are undecided as to the recommendation's importance.)

		Very Important		Neutral		Very Unimportant
17.	Calculators and computers should be introduced into mathematics courses to enhance understanding and problem solving, and to take the drudgery out of computations. Presentation of topics needs to be revised based on fresh approaches possible with new technologies.	5	4	3	2	1
18.	Traditional high school mathematics courses need to be integrated and unified to show interrelationships across topics and applications.	5	4	3	2	1
19.	Alternative mathematics courses should be available for students who are planning not to go to college or who are planning not to take a college major with high mathematics content.	5	4	3	2	1
20.	More emphasis should be given to simple mental computation, estimation and approximation, and less to practicing lengthy paper and pencil calculation.	5	4	3	2	1
21.	More topics and techniques from discrete mathematics, statistics and probability should be introduced into the high school curriculum.	5	4	3	2	1
22.	Mathematical modeling and problem-solving should be incorporated as a central feature in high school mathematics, and should be integrated into other parts of school curricula (such as science and social studies).	5	4	3	2	1
23.	Pre-service and in-service teacher education programs need to be developed that train teachers in individual and small-group teaching, the use of technology, and research.	5	4	3	2	1
24.	Schools must adopt differential staffing patterns and career ladders for mathematics teachers by appointing master teachers to develop, coordinate and supervise new programs.	5	4	3	2	1

		Very Important		Neutral		Very Unimportant
25.	Mathematics teachers should be encouraged to become members of professional mathematical societies and to attend regional and national meetings.	5	4	3	2	1
26.	A core mathematics program should provide optional tracks and electives, and the opportunity for every student through Grade 10 to prepare for college entry.	5	4	3	2	1
27.	A state-level prognostic test in mathematics should be administered to all students in Grade 9 or 10 to determine if they are ready to pursue further math-related work or study, or if they are in need of remediation or course changes. Results of such tests would not be available for the purpose of college admission or to evaluate teachers.	5	4	3	2	1
28.	Increased funding should be made available for the development of improved, appropriate materials, diagnostic techniques and teaching strategies for remedial programs.	5	4	3	2	1
29.	Strong efforts must be made to increase the awareness of the importance of mathematics among all members of the community, especially among parents of school age children.	5	4	3	2	1
30.	A core mathematics program should be established which requires all students to study mathematics through Grade 11.	5	4	3	2	1
31.	Special efforts should be made to identify mathematically talented students, especially minorities and women, and to encourage them to pursue careers in mathematics, science and mathematics education.	5	4	3	2	1
32.	Parents should have the option of sending their children to the public school of their choice.	5	4	3	2	1

33. Please feel free to comment on any important aspect, not mentioned above, that you would recommend as a change in your mathematics curriculum.

____

____

Conceptions of Mathematics Education

Below are listed staatements pertaining to some issues and problems that mathematics teachers face with varying degrees of regularity. Please read each item carefully and rate it on the 5-point scale provided, to the extent that you agree with it. (On this scale, the number 5 indicates that you agree strongly with the item. The number 1 indicates that you disagree strongly. The number 3 indicates that your feelings are neutral towards the item.)

		Agree Strongly		Neutral		Disagree Strongly
34.	Mathematics teachers' primary responsibility is keeping order, keeping students busy and productive in the classroom, and covering all the material.	5	4	3	2	1
35.	Calculators and computers can facilitate the learning of mathematics. Hands-on experience with changing technology should be incorporated as an integral part of mathematics instruction.	5	4	3	2	1
36.	Special applications, real-world problems and extra-curricular activities must be tailored to the needs and abilities of each student to help them excel.	5	4	3	2	1
37.	Mathematics teachers must teach students to communicate using conventional mathematical signs, symbols and vocabulary.	5	4	3	2	1
38.	Mathematical analysis, interpretation and inquiry should be taught concurrently with the basic skills. Students must be taught to use mathematics to gain understanding of a variety of phenomena.	5	4	3	2	1
39.	Mathematics teachers have the responsibility to teach the requisite skills for subsequent courses.	5	4	3	2	1
40.	Mathematical skills and rules should not be taught in isolation. Mathematics needs to be discovered by students through applied problem-solving.	5	4	3	2	1
41.	Mathematics teachers sometimes have to sacrifice the broader aims of the course in order to spend more time bringing the entire class up to a minimum competency level.	5	4	3	2	1
42.	Mathematics teachers should demand strict adherence to methods and notations used in class.	5	4	3	2	1

		Agree Strongly		Neutral		Disagree Strongly
43.	It is difficult to obtain objective evidence of student mathematics achievement. The process of learning mathematics is unique to the individual, and does not lend itself readily to standardized evaluation.	5	4	3	2	1
44.	It is good for students to see mathematics teachers make mistakes. It helps them understand that making their own mistakes is part of the science of mathematical thinking.	5	4	3	2	1
45.	Mathematics is an enjoyable discipline.	5	4	3	2	1
46.	Mathematics teachers have the responsibility to teach the requisite skills for future employment.	5	4	3	2	1
47.	The results of standardized tests greatly influence what mathematics is taught.	5	4	3	2	1
48.	All students should be required to pass a minimum competency test in mathematics to graduate from high school.	5	4	3	2	1
49.	The greatest influence on my teaching of mathematics was my high school mathematics teacher(s).	5	4	3	2	1
50.	The greatest influence on my teaching of mathematics was my coursework in college and/or teacher education.	5	4	3	2	1
51.	The greatest influence on my teaching of mathematics has been my colleagues who are teachers.	5	4	3	2	1
52.	I enjoy teaching mathematics.	5	4	3	2	1

53. Please feel free to comment on any important aspect, not mentioned above, that you would recommend as a change in your mathematics curriculum.

Conceptions of Schooling

The following items are concerned with <u>your</u> conceptions about the purpose, functions and goals of schools in our society. Please read each item carefully and rate it according to the priority <u>you</u> would assign it as it relates to what you see as the overall purpose of schooling. (a 5 indicates that you would assign a very high priority to the item and a 1 indicates that you would assign a very low priority to the item. A rating of 3 indicates that you are undecided where you would assign the item.)

		Very High Priority		Neutral		Very Low Priority
54.	Schools should provide an opportunity for children to pursue their own talents, interests and creative abilities.	5	4	3	2	1
55.	School curricula should function to preserve the traditions of society and the stability of our social institutions. Schools should be accountable to their local community as to how they are achieving these aims.	5	4	3	2	1
56.	Schools should group students according to similar needs, interests and abilities, rather than according to age.	5	4	3	2	1
57.	A major role of schools is to transmit the knowledge and skills associated with different branches of learning.	5	4	3	2	1
58.	Schools must be innovative to ensure that we maintain a dynamic and expanding society.	5	4	3	2	1
59.	Schools must offset inequalities by providing special opportunities to disadvantaged students.	5	4	3	2	1
60.	Schools should be seen by students as places where they may find personal fulfillment, gain satisfaction from achieving their individual needs, and develop confidence in finding future direction.	5	4	3	2	1
61.	To make an easy transition from school to the work place, schools should be places where students develop proper work values and learn to adapt to large groups, and where rewards are seen as both immediate (grades) and future (promise of employment).	5	4	3	2	1
62.	Competition is an important component of schooling, both to motivate learning and as preparation for adult life.	5	4	3	2	1

		Very High Priority		Neutral		Very Low Priority
63.	Schools must train students to learn and apply rules, follow instructions, absorb facts and memorize detail.	5	4	3	2	1
64.	Society must decide which years of a child's life shall be spent in formal learning, teachers must be responsible for determining the appropriate time to allocate materials, and students must learn to use their study time effectively.	5	4	3	2	1
65.	Schools exist to develop students' abilities to think, solve problems and make decisions by means of thorough training in academic disciplines.	5	4	3	2	1
66.	Schools must allocate resources equally among all students, regardless of social, ethnic or other personal background.	5	4	3	2	1
67.	Schools should be places that children feel are safe havens from the streets. Children must feel comfortable with teachers, administration and other students.	5	4	3	2	1
68.	Schools should be for students who want to learn and who are willing to work, and not a social agency for attending to all the needs of school-age children.	5	4	3	2	1

69. Please feel free to comment on any important issue or problem, not mentioned above, that you feel pertains to the purpose, functions and goals of schools in our society.

Appendix B

Statistical Analyses for the Total Sample

Table B1

TCMME Item Means and Standard Deviations for the Total Sample (N = 490)

Item		*M*	*SD*
Conceptions of Mathematics			
1.	Mathematics is a process in which abstract ideas are applied to solve real-world problems.	4.156	0.916
2.	Mathematics is a language, with its own precise meaning and grammar, used to represent and communicate ideas	4.441	0.774
3.	Mathematics is a collection of concepts and skills used to obtain answers to problems.	4.052	0.988
4.	Mathematics is thinking in a logical, scientific, inquisitive manner and is used to develop understanding	4.554	0.705
5.	Mathematics is facts, skills, rules and concepts learned in some sequence and applied in work and future study.	3.921	1.068
6.	Mathematics is an interconnected logical system, is dynamic, and changes as new problem-solving situations arise. It is formed by thinking about actions and experiences.	4.218	0.948
Conceptions of Mathematics Teaching[a]			
9.	To enable students to master a hierarchy of concepts and skills and to use these in solving problems.	4.318	0.745
10.	To provide experiences for students to know mathematics as originating in real-world situations and to have the power of using a small set of symbols to represent and solve a wide range of problems.	4.353	0.702
11.	To enable students to use mathematical procedures to solve problems and mathematical concepts to model both abstract and real-world situations.	4.441	0.689
12.	To provide students with complete understanding of the meaning(s) of mathematical concepts and enable them to communicate ideas using correct mathematical symbols, rules and reasoning.	4.101	0.822

[a]Means could not be computed for Items 7 (rank order) and 8 (text answer).

Table B1 (continued).

TCMME Item Means and Standard Deviations for the Total Sample (N = 490)

Item		*M*	*SD*
13.	To prepare students for work and future study by having them master a sequence of facts, paper-and-pencil skills, rules and concepts.	3.613	1.128
14.	To enable students to use mathematics to explore situations in an inquisitive manner, and to offer and test hypotheses by logical reasoning, for the purpose of developing a more complete understanding of the situation	4.463	0.708
Conceptions of Recommended Change[*]			
17.	Calculators and computers should be introduced into mathematics courses to enhance understanding and problem solving, and to take the drudgery out of computations. Presentation of topics needs to be revised based on fresh approaches possible with new technologies.	4.429	0.896
18.	Traditional high school mathematics courses need to be integrated and unified to show interrelationships across topics and applications.	4.192	0.928
19.	Alternative mathematics courses should be available for students who are planning <u>not</u> to go to college or who are planning <u>not</u> to take a college major with high mathematics content.	4.155	1.010
20.	More emphasis should be given to simple mental computation, estimation and approximation, and less to practicing lengthy paper and pencil calculations.	4.010	0.986
21.	More topics and techniques from discrete mathematics, statistics and probability should be introduced into the high school curriculum.	3.864	0.911
22.	Mathematical modeling and problem-solving should be incorporated as a central feature in high school mathematics, and should be integrated into other parts of school curricula (such as science and social studies).	4.367	0.748

*Means could not be computed for Items 15 (rank order) and 16 (text answer).

Table B1 (continued).

TCMME Item Means and Standard Deviations for the Total Sample (N = 490)

Item		*M*	*SD*
23.	Pre-service and in-service teacher education programs need to be developed that train teachers in individual and small-group teaching, the use of technology, and research.	4.240	0.952
24.	Schools must adopt differential staffing patterns and career ladders for mathematics teachers by appointing master teachers to develop, coordinate and supervise new programs.	3.512	1.177
25.	Mathematics teachers should be encouraged to become members of professional mathematical societies and to attend regional and national meetings.	4.080	0.958
26.	A core mathematics program should provide optional tracks and electives, and the opportunity for every student through Grade 10 to prepare for college entry.	4.175	0.855
27.	A state-level prognostic test in mathematics should be administered to all students in Grade 9 or 10 to determine if they are ready to pursue further math-related work or study, or if they are in need of remediation or course changes. Results of such tests would not be available for the purpose of college admission or to evaluate teachers.	3.578	1.276
28.	Increased funding should be made available for the development of improved, appropriate materials, diagnostic techniques and teaching strategies for remedial programs.	4.080	0.989
29.	Strong efforts must be made to increase the awareness of the importance of mathematics among all members of the community, especially among parents of school age children.	4.395	0.761
30.	A core mathematics program should be established which requires all students to study mathematics through Grade 11.	4.174	0.975
31.	Special efforts should be made to identify mathematically talented students, especially minorities and women, and to encourage them to pursue careers in mathematics, science and mathematics education.	4.427	0.799
32.	Parents should have the option of sending their children to the public school of their choice.	3.276	1.315

Table B1 (continued).

TCMME Item Means and Standard Deviations for the Total Sample (N = 490)

Item		*M*	*SD*
Conceptions of Mathematics Education[a]			
34.	Mathematics teachers' primary responsibility is keeping order, keeping students busy and productive in the classroom, and covering all the material.	2.300	1.260
35.	Calculators and computers can facilitate the learning of mathematics. Hands-on experience with changing technology should be incorporated as an integral part of mathematics instruction.	4.525	0.739
36.	Special applications, real-world problems and extra-curricular activities must be tailored to the needs and abilities of each student to help them excel.	4.081	0.863
37.	Mathematics teachers must teach students to communicate using conventional mathematical signs, symbols and vocabulary.	4.099	0.805
38.	Mathematical analysis, interpretation and inquiry should be taught concurrently with the basic skills. Students must be taught to use mathematics to gain understanding of a variety of phenomena.	4.333	0.762
39.	Mathematics teachers have the responsibility to teach the requisite skills for subsequent courses.	4.242	0.800
40.	Mathematical skills and rules should not be taught in isolation. Mathematics needs to be discovered by students through applied problem-solving.	4.211	0.883
41.	Mathematics teachers sometimes have to sacrifice the broader aims of the course in order to spend more time bringing the entire class up to a minimum competency level.	3.444	1.175
42.	Mathematics teachers should demand strict adherence to methods and notations used in class.	3.027	1.120

[a]Mean could not be computed for Item 33 (text answer).

Table B1 (continued).

TCMME Item Means and Standard Deviations for the Total Sample (N = 490)

Item		*M*	*SD*
43.	It is difficult to obtain objective evidence of student mathematics achievement. The process of learning mathematics is unique to the individual, and does not lend itself readily to standardized evaluation.	2.780	1.150
44.	It is good for students to see mathematics teachers make mistakes. It helps them understand that making their own mistakes is part of the science of mathematical thinking.	3.916	0.977
45.	Mathematics is an enjoyable discipline.	4.645	0.624
46.	Mathematics teachers have the responsibility to teach the requisite skills for future employment.	3.996	0.861
47.	The results of standardized tests greatly influence what mathematics is taught.	3.568	1.119
48.	All students should be required to pass a minimum competency test in mathematics to graduate from high school.	4.211	1.005
49.	The greatest influence on my teaching of mathematics was my high school mathematics teacher(s).	3.176	1.536
50.	The greatest influence on my teaching of mathematics was my coursework in college and/or teacher education.	2.756	1.358
51.	The greatest influence on my teaching of mathematics has been my colleagues who are teachers.	3.232	1.284
52.	I enjoy teaching mathematics.	4.811	0.442
Conceptions of Schooling[a]			
54.	Schools should provide an opportunity for children to pursue their own talents, interests and creative abilities.	4.207	0.838

[a]Mean could not be computed for Item 53 (text answer).

Table B1 (continued).

TCMME Item Means and Standard Deviations for the Total Sample (N = 490)

Item		*M*	*SD*
55.	School curricula should function to preserve the traditions of society and the stability of our social institutions. Schools should be accountable to their local community as to how they are achieving these aims.	3.578	0.983
56.	Schools should group students according to similar needs, interests and abilities, rather than according to age.	3.641	1.048
57.	A major role of schools is to transmit the knowledge and skills associated with different branches of learning.	4.133	0.817
58.	Schools must be innovative to ensure that we maintain a dynamic and expanding society.	4.302	0.785
59.	Schools must offset inequalities by providing special opportunities to disadvantaged students.	3.943	0.954
60.	Schools should be seen by students as places where they may find personal fulfillment, gain satisfaction from achieving their individual needs, and develop confidence in finding future direction.	4.532	0.634
61.	To make an easy transition from school to the work place, schools should be places where students develop proper work values and learn to adapt to large groups, and where rewards are seen as both immediate (grades) and future (promise of employment).	4.265	0.790
62.	Competition is an important component of schooling, both to motivate learning and as preparation for adult life.	3.694	1.061
63.	Schools must train students to learn and apply rules, follow instructions, absorb facts and memorize detail.	3.422	1.170
64.	Society must decide which years of a child's life shall be spent in formal learning, teachers must be responsible for determining the appropriate time to allocate materials, and students must learn to use their study time effectively.	3.664	0.998

Table B1 (continued).

TCMME Item Means and Standard Deviations for the Total Sample (N = 490)

Item		*M*	*SD*
65.	Schools exist to develop students' abilities to think, solve problems and make decisions by means of thorough training in academic disciplines.	4.270	0.774
66.	Schools must allocate resources equally among all students, regardless of social, ethnic or other personal background.	3.932	1.255
67.	Schools should be places that children feel are safe havens from the streets. Children must feel comfortable with teachers, administration and other students.	4.553	0.710
68.	Schools should be for students who want to learn and who are willing to work, and not a social agency for attending to all the needs of school-age children.	3.733	1.260

Table B2.

Percent of Teachers Recording Conceptions of Mathematics as "Most Reflective" and "Least Reflective" of Their Own Conception, and Mean Rank of Conceptions for the Total Sample

Item		Most %	Least %	Mean Rank
1.	Mathematics is a process in which abstract ideas are applied to solve real-world problems.	15	17	3.45
2.	Mathematics is a language, with its own precise meaning and grammar, used to represent and communicate ideas.	16	14	3.68
3.	Mathematics is a collection of concepts and skills used to obtain answers to problems.	6	19	2.86*
4.	Mathematics is thinking in a logical, scientific, inquisitive manner and is used to develop understanding.	27	2	4.55*
5.	Mathematics is facts, skills, rules and concepts learned in some sequence and applied in work and future study.	8	25	2.85*
6.	Mathematics is an interconnected logical system, is dynamic, and changes as new problem-solving situations arise. It is formed by thinking about actions and experiences.	28	23	3.60

* $p < .05$

Table B3.

Percent of Teachers Recording Conceptions of Mathematics Teaching as "Most Reflective" and "Least Reflective" of Their Own Conception, and Mean Rank of Conceptions for the Total Sample

Item		Most %	Least %	Mean Rank
9.	To enable students to master a hierarchy of concepts and skills and to use these in solving problems.	19	11	3.57
10.	To provide experiences for students to know mathematics as originating in real-world situations and to have the power of using a small set of symbols to represent and solve a wide range of problems.	15	8	3.89
11.	To enable students to use mathematical procedures to solve problems and mathematical concepts to model both abstract and real-world situations.	18	4	4.15*
12.	To provide students with complete understanding of the meaning(s) of mathematical concepts and enable them to communicate ideas using correct mathematical symbols, rules and reasoning.	8	15	3.05
13.	To prepare students for work and future study by having them master a sequence of facts, paper-and-pencil skills, rules and concepts.	7	49	2.26*
14.	To enable students to use mathematics to explore situations in an inquisitive manner, and to offer and test hypotheses by logical reasoning, for the purpose of developing a more complete understanding of the situation	33	13	4.08*

$p < .05$

Appendix C

Assessment of Teachers' Conceptions of Mathematics Using Cluster Analytic Techniques

James A. Middleton

The purpose of this paper is to familiarize the reader with the rationale for using cluster analytic techniques in studying teachers' conceptions of mathematics. Rather than focusing on the matrix algebra, or the formula for computing distance metrics, I attempt to present both a general outline of how cluster analysis is performed and, more importantly, *why* cluster analytic techniques are appropriate for uncovering teachers' conceptions. I will give general rationales that apply to most clustering methods, but I will also focus on one of the most popular methods in the behavioral sciences: Ward's method.

Cluster analysis is a relatively new technique in the behavioral sciences that attempts to partition a heterogeneous set of objects (persons or variables) into a smaller number of relatively homogeneous subsets based on the objects' similarity across some measure(s) (Aldenderfer & Blashfield, 1984). Clustering techniques are highly flexible and can be adapted for use in a variety of settings (Aldenderfer & Blashfield, 1984). For example, researchers have utilized cluster analysis to uncover groups of *individuals* based on the similarities in their ratings across several variables, to discover the relationships between *variables* within individuals or groups of individuals to determine the way in which individuals construct their knowledge, and as a test of the validity of a model of human thought.

The procedure for clustering n objects on k variables is as follows (after Kachigan, 1986):

1. Each object is measured with respect to each variable and ordered in an $n \times k$ matrix.

2. Some measure of inter-object similarity is computed. This may be a Euclidean distance, a city-block metric, a correlation coefficient, or other measure of similarity. Deciding on the appropriateness of a metric depends largely on the type of research question to be answered or hypothesis to be tested. Euclidean distances and their derivatives are the most commonly used in the social sciences. They take into account

the relative similarity of each object to its neighbors without imposing linear structure on the cluster solution.

3. Clusters of objects are formed based on their relative similarities. In hierarchical cluster analysis, two approaches to determining cluster membership are used: "Top down" and "bottom up". In the top down approach, the total sample of objects is split into two mutually exclusive groups, and each group is then split into two more groups, and so on until each object stands alone in its own group. The reverse operation, the bottom up approach, can be performed where each object stands alone, and is then added to the object to which it is most similar, and then these two objects are combined with another group of objects that are most similar, and so on until all groups have been combined into a single cluster consisting of the entire sample. Ward's method is a hierarchical algorithm that uses the "bottom up" approach.

4. After clusters have been formed, the researcher must determine the appropriate number of clusters to include for interpretation. This is no easy task. An analysis involving *n* objects has cluster solutions ranging from 1 to *n*. The appropriate number of clusters to interpret depends upon the research question as well as the statistical properties of the clusters. One method of determining "significant" clusters is analogous to the Scree test for factor analysis (Lathrop & Williams, 1987). The curve formed by plotting the fusion coefficient (distance between clusters) at each stage of combining object clusters, by the number of clusters formed (ranging from 1 to *n*), "flattens" out asymptotically as the differences between clusters becomes negligible. By visually inspecting the curve and determining the point where it begins to flatten out, the researcher has a reliable index of the number of significant clusters. For Ward's method, the fusion coefficient minimizes the within-group error sum of squares. Thus, by inspecting the curve, the researcher can determine the point where the curve flattens out (i.e., where the within-group error starts to become negligible) and can use the corresponding number of clusters at that stage as an appropriate number for examination.

5. Lastly, the researcher must examine the properties of the objects that make up each cluster in relation to the criterion variables in order to make "sense" out of the derived

clusters. This is the stage that is most susceptible to experimenter bias, for the researcher must *place* some meaning on the statistically-derived categories.

To illustrate these steps, I will relate the study summarized in this volume.

It was hypothesized that groups of urban mathematics teachers would organize their conceptions of mathematics differently due to differences in historical, environmental, and individual factors. Teachers were asked to rate each of six hypothesized conceptions of mathematics as to the extent that each reflected their own conceptions of mathematics. It was assumed that each subject rated the items based on how he or she constructed their representation of mathematics.

Since each teacher's representation was hypothesized to be different, some kind of *average* needed to be constructed by the researcher in order to evaluate the group. Further, since each conception was unique, at least syntactically, a simple arithmetic average would not have been appropriate (as an analogy, think of the appropriateness of asking, "What is the average of 2 apples and 5 oranges?"). Thus, the Euclidean distance was computed between each subject in six-dimensional space (each dimension corresponding to a conception of mathematics). Teachers who responded most similarly across the six items correspondingly shared the smallest computed distance.

After developing the measure of similarity, Ward's method of hierarchical clustering was chosen as an appropriate algorithm. Although there are many different clustering algorithms, their general function is to *minimize* the distances between objects within a cluster and, at the same time, *maximize* the distances between different clusters (in this instance, the geometric center of each cluster). In essence, algorithms numerically compare and order objects (teachers' patterns of responses in this example) such that they are highly similar to other members of their own cluster, but are highly different from members in other clusters across the criterion variables (Aldenderfer & Blashfield, 1984). Figure C1 illustrates the relationship between clusters and within clusters in two dimensions.

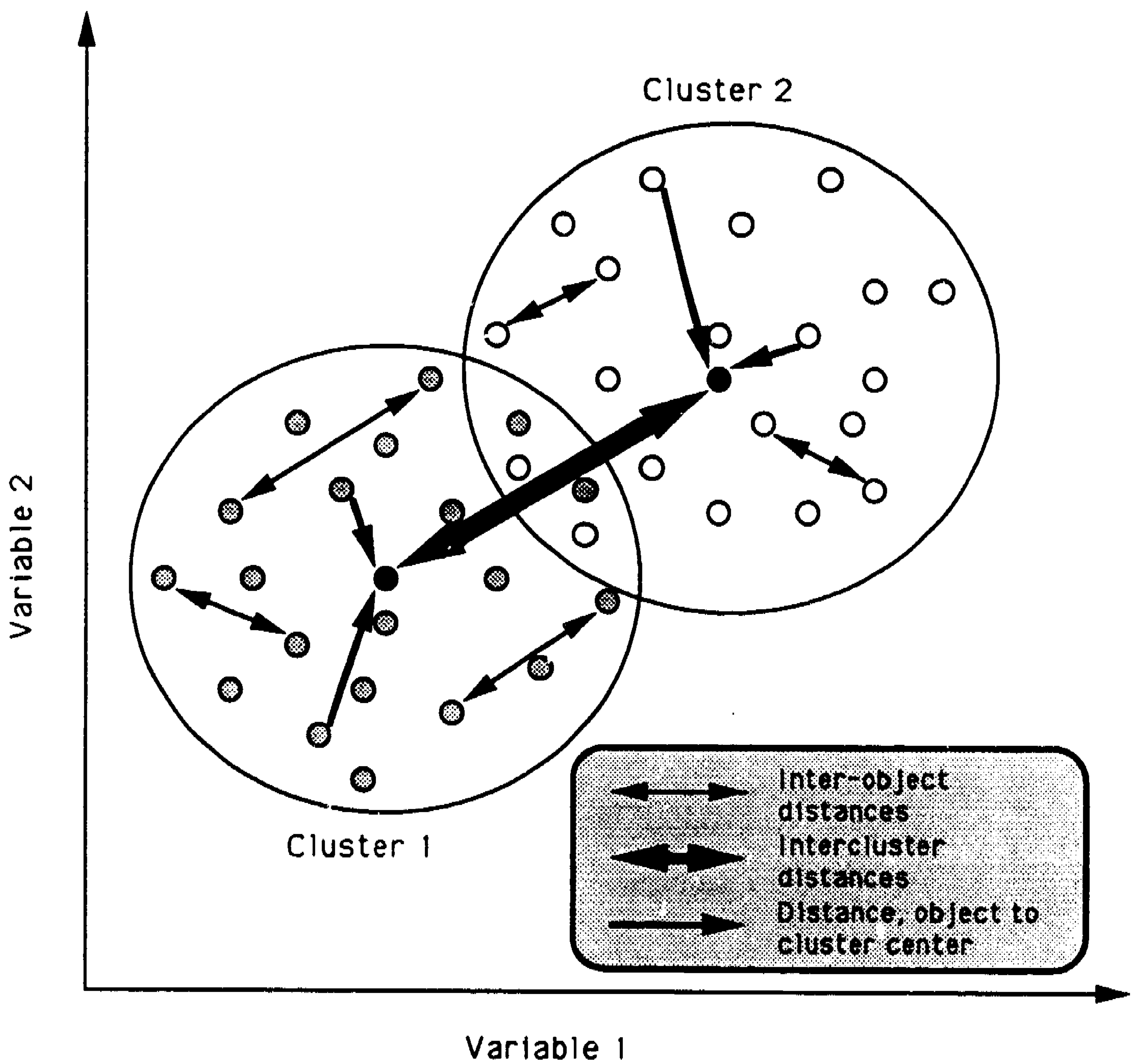

Figure C1. Diagram illustrating a two-cluster solution for two variables. In reality, coordinates represent proximities in multidimensional space.

The flexibility afforded by clustering techniques is stated simply, *You can cluster almost anything*. Whereas factor analysis is concerned with categorizing a large group of *variables* into a number of groups of hypothetical variables (Kim & Mueller, 1978), cluster analysis can categorize *individuals* as well as variables (One method of clustering is to do "reverse" factor analysis in order to find groups of individuals). In addition, since most clustering algorithms are not iterative, cluster analysis can be performed on much smaller sets of data than factor analysis and still be relatively robust. Moreover, cluster analysis does not necessarily impose a linear structure on the data as does factor analysis.

Care must be taken, however, when analyzing the results of clustering methods. Although the rationale for using cluster analysis is to seek for structure in human thinking, the techniques themselves *impose* structure on the data (Aldenderfer & Blashfield, 1984). Further, different similarity metrics and different clustering algorithms may extract highly different solutions. Therefore, *a priori* predictions about the nature of criterion variables should be given wherever possible to prevent experimenter bias.

In addition, because clustering algorithms *create* different groups of objects, i.e., clustering takes a single group and divides it based on values across some measure(s), testing of the hypothesis that cluster members represent different populations with respect to the criterion variables is inappropriate. For instance, imagine taking a population of objects, normally distributed about some mean value, and dividing it into two groups: one with values less than the mean, and one with values greater than or equal to the mean. If an F test were performed on the two groups, they would be found to be significantly different because of the disparity in their respective means. One would tend to conclude that the two groups came from different populations: a Type I error. However, validity of clusters may be established by examining group differences on variables that relate theoretically to the clustering criteria.

The real power of cluster analyses in cognitive science is that they are so adaptable. They are exploratory in nature and are composed mainly of heuristics and rules of thumb that guide the researcher to uncover relationships among entities in a multivariate, and hence, more *human* fashion.

References

Aldenderfer, M. S., & Blashfield, R. K. (1984). Cluster Analysis. In J. L. Sullivan & R. G. Niemi (Eds.), *Series: Quantitative Applications in the Social Sciences*. Beverly Hills, CA: Sage Publications.

Kachigan, S. K. (1986). *Statistical analysis. An interdisciplinary introduction to univariate and multivariate methods*. New York: Radius Press.

Kim, J. O., & Mueller, C. W. (1978). Factor Analysis. Statistical methods and practical issues. In E. M. Uslaner (Ed.), *Series: Quantitative Applications in the Social Sciences*. Beverly Hills: Sage Publications.

Lathrop, R. G., & Williams, J. E. (1987). The reliability of inverse scree tests for cluster analysis. *Educational and Psychological Measurement*, *47*(4), 953-959.

Appendix D

Analysis of TCMME Survey Responses by Extracted Cluster

Table D1

Percentage of Teachers Ranking the Proposed Six Conceptions of Mathematics as "Most Reflective" of Their Own Conception of Mathematics

Item Number	Cluster 1 %	Cluster 2 %	Cluster 3 %	Cluster 4 %	Total %
1	14	10	18	17	15
2	11	21	6	20	16
3	14	1	5	6	6
4	36	14	41	24	27
5	17	2	10	6	8
6	8	51	21	27	28
N	72	96	101	187	456

Table D2

Percentage of Teachers Ranking the Proposed Six Conceptions of Mathematics as "Least Reflective" of Their Own Conception of Mathematics

Item Number	Cluster 1 %	Cluster 2 %	Cluster 3 %	Cluster 4 %	Total %
1	31	10	19	14	17
2	17	5	22	11	13
3	4	38	16	16	19
4	0	1	1	3	2
5	1	41	9	34	25
6	47	2	28	20	22
N	72	97	105	189	463

Table D3

Analysis of Variance: Item 9 by Cluster Membership

Source	Sum of Squares	DF	MS	*F*	*p*
Between	30.4175	3	10.139	20.522	.0000
Within	233.1864	472	.494		
Total	263.6029	475			

Table D4

Scheffe Post-hoc Differences Between Extracted Clusters for Item 9

Cluster	Cluster 2 Mean	Cluster 3 *t*	Cluster 4 *t*	*t*
1	4.29	3.37*	3.98*	0.49
2	3.90		7.28*	4.30*
3	4.66			4.37*
4	4.33			

*$p < .05$

Table D5

Analysis of Variance: Item 10 by Cluster Membership

Source	Sum of Squares	DF	MS	*F*	*p*
Between	6.989	3	2.329	4.811	.003
Within	228.076	471	.484		
Total	235.065	474			

Table D6

Scheffe Post-hoc Differences Between Extracted Clusters for Item 10

Cluster	Mean	Cluster 2 t	Cluster 3 t	Cluster 4 t
1	4.164	1.64	3.09*	1.11
2	4.366		1.87	0.93
3	4.537			3.16*
4	4.290			

*$p < .05$

Table D7

Analysis of Variance: Item 11 by Cluster Membership

Source	Sum of Squares	DF	MS	F	p
Between	6.036	3	2.012	4.402	.005
Within	214.810	470	.457		
Total	220.846	473			

Table D8

Scheffe Post-hoc Differences Between Extracted Clusters for Item 11

Cluster	Mean	Cluster 2 t	Cluster 3 t	Cluster 4 t
1	4.356	0.67	2.40	1.16
2	4.280		3.30*	2.06
3	4.602			1.82
4	4.466			

*$p < .05$

Table D9

Analysis of Variance: Item 12 by Cluster Membership

Source	Sum of Squares	DF	MS	*F*	*p*
Between	19.959	3	6.653	10.644	.000
Within	294.391	471	.625		
Total	314.350	474			

Table D10

Scheffe Post-hoc Differences Between Extracted Clusters for Item 12

Cluster	Mean	Cluster 2 *t*	Cluster 3 *t*	Cluster 4 *t*
1	4.000	1.37	3.51*	0.97
2	3.812		5.30*	2.75*
3	4.417			3.68*
4	4.109			

*$p < .05$

Table D11

Analysis of Variance: Item 13 by Cluster Membership

Source	Sum of Squares	DF	MS	*F*	*p*
Between	113.056	3	37.685	36.283	.0000
Within	488.170	470	1.039		
Total	601.226	473			

Table D12

Scheffe Post-hoc Differences Between Extracted Clusters for Item 13

Cluster	Mean	Cluster 2 t	Cluster 3 t	Cluster 4 t
1	3.658	5.04*	3.78*	0.14
2	2.802		11.09*	6.67*
3	4.269			5.58*
4	3.635			

*$p < .05$

Table D13

Friedman Analysis of Teachers' Conceptions of Mathematics Teaching by Cluster

Item Number	Mean Rank Cluster 1	Cluster 2	Cluster 3	Cluster 4
9	3.74	3.13	3.74	3.63
10	3.71	4.40	3.59	3.84
11	4.09	4.09	3.99	4.28
12	2.96	2.92	3.01	3.13
13	2.83	1.44	2.89	2.16
14	3.68	5.02	3.77	3.96

*$p < .05$ within cluster.

Table D14

Analysis of Variance: Item 21 by Cluster Membership

Source	Sum of Squares	DF	MS	*F*	*p*
Between	18.936	3	6.312	7.948	.0000
Within	371.656	468	.794		
Total	390.591	471			

Table D15

Scheffe Post-hoc Differences Between Extracted Clusters for Item 21

Cluster	Mean	Cluster 2 *t*	Cluster 3 *t*	Cluster 4 *t*
1	3.500	5.03*	2.50	2.73*
2	4.168		2.60	3.22*
3	3.860			0.10
4	3.849			

*$p < .05$

Table D16

Analysis of Variance: Item 24 by Cluster Membership

Source	Sum of Squares	DF	MS	*F*	*p*
Between	17.545	3	5.848	4.281	.005
Within	640.679	469	1.366		
Total	658.224	472			

Table D17

Scheffe Post-hoc Differences Between Extracted Clusters for Item 24

Cluster	Mean	Cluster 2 *t*	Cluster 3 *t*	Cluster 4 *t*
1	3.167	3.43*	2.37	1.63
2	3.772		1.01	2.51
3	3.611			1.18
4	3.438			

*$p < .05$

Table D18

Analysis of Variance: Item 26 by Cluster Membership

Source	Sum of Squares	DF	MS	*F*	*p*
Between	12.279	3	4.093	5.757	.001
Within	334.866	471	.711		
Total	347.145	474			

Table D19

Scheffe Post-hoc Differences Between Extracted Clusters for Item 26

Cluster	Mean	Cluster 2 *t*	Cluster 3 *t*	Cluster 4 *t*
1	4.082	0.10	3.16*	0.19
2	4.069		3.44*	0.32
3	4.472			3.78*
4	4.177			

*$p < .05$

Table D20

Analysis of Variance: Item 27 by Cluster Membership

Source	Sum of Squares	DF	MS	*F*	*p*
Between	17.756	3	5.919	3.705	.012
Within	754.049	472	1.598		
Total	771.805	475			

Table D21

Scheffe Post-hoc Differences Between Extracted Clusters for Item 27

Cluster	Mean	Cluster 2 *t*	Cluster 3 *t*	Cluster 4 *t*
1	3.521	0.87	1.98	0.13
2	3.347		3.23*	1.28
3	3.908			2.43
4	3.544			

*$p < .05$

Table D22

Analysis of Variance: Item 29 by Cluster Membership

Source	Sum of Squares	DF	MS	*F*	*p*
Between	7.177	3	2.392	4.356	.005
Within	259.733	473	.549		
Total	266.910	476			

*$p < .05$

Table D23

Scheffe Post-hoc Differences Between Extracted Clusters for Item 29

Cluster	Mean	Cluster 2 t	Cluster 3 t	Cluster 4 t
1	4.356	0.55	2.38	0.04
2	4.287		3.34*	0.75
3	4.624			3.37*
4	4.361			

*$p < .05$

Table D24

Analysis of Variance: Item 30 by Cluster Membership

Source	Sum of Squares	DF	MS	F	p
Between	16.107	3	5.369	5.759	.001
Within	436.297	468	.932		
Total	452.405	471			

Table D25

Scheffe Post-hoc Differences Between Extracted Clusters for Item 30

Cluster	Mean	Cluster 2 t	Cluster 3 t	Cluster 4 t
1	4.343	2.54	0.62	1.85
2	3.929		3.53*	1.27
3	4.430			3.15*
4	4.098			

*$p < .05$

Table D26

Analysis of Variance: Item 35 by Cluster Membership

Source	Sum of Squares	DF	MS	*F*	*p*
Between	6.407	3	2.136	3.955	.008
Within	253.837	470	0.540		
Total	260.245	473			

Table D27

Scheffe Post-hoc Differences Between Extracted Clusters for Item 35

Cluster	Mean	Cluster 2 *t*	Cluster 3 *t*	Cluster 4 *t*
1	4.315	3.43*	1.38	1.99
2	4.700		2.02	2.23
3	4.491			0.39
4	4.529			

*$p < .05$

Table D28

Analysis of Variance: Item 37 by Cluster Membership

Source	Sum of Squares	DF	MS	*F*	*p*
Between	19.594	3	6.531	10.751	.0000
Within	284.319	468	0.608		
Total	303.913	471			

Table D29

Scheffe Post-hoc Differences Between Extracted Clusters for Item 37

Cluster	Mean	Cluster 2 t	Cluster 3 t	Cluster 4 t
1	4.032	1.95	3.04*	0.04
2	3.820		5.34*	2.38
3	4.430			4.25*
4	4.078			

*$p < .05$

Table D30

Analysis of Variance: Item 39 by Cluster Membership

Source	Sum of Squares	DF	MS	F	p
Between	11.919	3	3.973	6.394	.0003
Within	293.297	472	0.621		
Total	305.216	475			

Table D31

Scheffe Post-hoc Differences Between Extracted Clusters for Item 39

Cluster	Mean	Cluster 2 t	Cluster 3 t	Cluster 4 t
1	4.206	1.30	2.50	0.06
2	4.040		4.29*	1.72
3	4.505			3.27*
4	4.212			

*$p < .05$

Table D32

Analysis of Variance: Item 40 by Cluster Membership

Source	Sum of Squares	DF	MS	*F*	*p*
Between	7.561	3	2.520	3.293	.021
Within	360.536	471	0.765		
Total	368.097	474			

Table D33

Scheffe Post-hoc Differences Between Extracted Clusters for Item 40

Cluster	Mean	Cluster 2 *t*	Cluster 3 *t*	Cluster 4 *t*
1	3.959	2.39	2.68*	1.73
2	4.300		0.43	1.08
3	4.349			1.51
4	4.192			

*$p < .05$

Table D34

Analysis of Variance: Item 41 by Cluster Membership

Source	Sum of Squares	DF	MS	*F*	*p*
Between	17.036	3	5.679	4.201	.006
Within	635.304	470	1.352		
Total	652.340	473			

Table D35

Scheffe Post-hoc Differences Between Extracted Clusters for Item 41

Cluster	Mean	Cluster 2 t	Cluster 3 t	Cluster 4 t
1	3.425	1.82	0.96	0.62
2	3.080		3.13*	3.01*
3	3.596			0.52
4	3.526			

*$p < .05$

Table D36

Analysis of Variance: Item 42 by Cluster Membership

Source	Sum of Squares	DF	MS	F	p
Between	27.310	3	9.103	7.592	.000
Within	563.587	470	1.199		
Total	590.897	473			

Table D37

Scheffe Post-hoc Differences Between Extracted Clusters for Item 42

Cluster	Mean	Cluster 2 t	Cluster 3 t	Cluster 4 t
1	3.181	3.16*	0.77	1.24
2	2.620		4.41*	2.69
3	3.312			2.48
4	3.015			

*$p < .05$

Table D38

Analysis of Variance: Item 46 by Cluster Membership

Source	Sum of Squares	DF	MS	*F*	*p*
Between	9.798	3	3.266	4.443	.004
Within	346.194	471	0.735		
Total	355.992	474			

Table D39

Scheffe Post-hoc Differences Between Extracted Clusters for Item 46

Cluster	Mean	Cluster 2 *t*	Cluster 3 *t*	Cluster 4 *t*
1	3.904	0.23	2.48	0.48
2	3.870		3.42*	0.97
3	4.259			2.98*
4	3.969			

*$p < .05$

Table D40

Analysis of Variance: Item 48 by Cluster Membership

Source	Sum of Squares	DF	MS	*F*	*p*
Between	16.410	3	5.470	5.582	.001
Within	463.477	473	0.980		
Total	479.887	476			

Table D41

Scheffe Post-hoc Differences Between Extracted Clusters for Item 48

Cluster	Mean	Cluster 2 t	Cluster 3 t	Cluster 4 t
1	4.306	1.85	1.36	1.11
2	3.990		3.79*	1.18
3	4.514			3.48*
4	4.144			

*$p < .05$

Table D42

Analysis of Variance: Item 55 by Cluster Membership

Source	Sum of Squares	DF	MS	F	p
Between	10.488	3	3.496	3.641	.013
Within	448.434	467	0.960		
Total	458.921	470			

Table D43

Scheffe Post-hoc Differences Between Extracted Clusters for Item 55

Cluster	Mean	Cluster 2 t	Cluster 3 t	Cluster 4 t
1	3.425	0.09	2.58	1.25
2	3.410		2.79*	1.43
3	3.813			1.92
4	3.578			

*$p < .05$

Table D44

Analysis of Variance: Item 57 by Cluster Membership

Source	Sum of Squares	DF	MS	*F*	*p*
Between	12.563	3	4.188	6.631	.000
Within	294.907	467	0.632		
Total	307.469	470			

Table D45

Scheffe Post-hoc Differences Between Extracted Clusters for Item 57

Cluster	Mean	Cluster 2 *t*	Cluster 3 *t*	Cluster 4 *t*
1	4.178	1.91	2.06	0.80
2	3.930		4.16*	1.50
3	4.410			3.61*
4	4.093			

*$p < .05$

Table D46

Analysis of Variance: Item 61 by Cluster Membership

Source	Sum of Squares	DF	MS	*F*	*p*
Between	11.843	3	3.948	6.710	.000
Within	276.503	470	0.850		
Total	288.346	473			

Table D47

Scheffe Post-hoc Differences Between Extracted Clusters for Item 61

Cluster	Mean	Cluster 2 t	Cluster 3 t	Cluster 4 t
1	4.315	2.14	1.75	0.77
2	4.050		4.00*	1.86
3	4.519			3.05*
4	4.238			

*$p < .05$

Table D48

Analysis of Variance: Item 62 by Cluster Membership

Source	Sum of Squares	DF	MS	F	p
Between	46.079	3	15.360	14.927	.000
Within	484.658	471	1.029		
Total	530.737	474			

Table D49

Scheffe Post-hoc Differences Between Extracted Clusters for Item 62

Cluster	Mean	Cluster 2 t	Cluster 3 t	Cluster 4 t
1	4.069	5.50*	0.22	3.55*
2	3.230		5.69*	2.84*
3	4.037			3.66*
4	3.603			

*$p < .05$

Table D50

Analysis of Variance: Item 63 by Cluster Membership

Source	Sum of Squares	DF	MS	*F*	*p*
Between	80.976	3	26.992	22.814	.000
Within	552.532	467	1.183		
Total	633.507	470			

Table D51

Scheffe Post-hoc Differences Between Extracted Clusters for Item 63

Cluster	Mean	Cluster 2 *t*	Cluster 3 *t*	Cluster 4 *t*
1	3.681	5.08*	1.67	1.75
2	2.750		8.17*	4.63*
3	3.962			4.69*
4	3.399			

*$p < .05$

Table D52

Analysis of Variance: Item 64 by Cluster Membership

Source	Sum of Squares	DF	MS	*F*	*p*
Between	17.060	3	5.687	5.996	.001
Within	442.932	467	0.949		
Total	459.992	470			

Table D53

Scheffe Post-hoc Differences Between Extracted Clusters for Item 64

Cluster	Mean	Cluster 2 t	Cluster 3 t	Cluster 4 t
1	3.616	1.33	2.37	0.41
2	3.396		3.99*	2.09
3	3.963			2.73
4	3.672			

*$p < .05$

Table D54

Analysis of Variance: Item 65 by Cluster Membership

Source	Sum of Squares	DF	MS	F	p
Between	9.039	3	3.013	5.173	.002
Within	273.757	470	0.583		
Total	282.795	473			

Table D55

Scheffe Post-hoc Differences Between Extracted Clusters for Item 65

Cluster	Mean	Cluster 2 t	Cluster 3 t	Cluster 4 t
1	4.370	2.47	0.72	1.01
2	4.050		3.57*	2.02
3	4.449			2.27
4	4.264			

*$p < .05$

Table D56

Analysis of Variance: Item 66 by Cluster Membership

Source	Sum of Squares	DF	MS	*F*	*p*
Between	21.327	3	7.109	4.697	.003
Within	714.360	472	1.514		
Total	735.687	475			

Table D57

Scheffe Post-hoc Differences Between Extracted Clusters for Item 66

Cluster	Mean	Cluster 2 *t*	Cluster 3 *t*	Cluster 4 *t*
1	4.055	0.88	1.35	1.84
2	3.890		2.42	0.89
3	4.287			3.87*
4	3.749			

*$p < .05$

Appendix E
Analysis of TCMME Survey Responses by Collaborative Participation Level

Table E1

Analysis of Variance: Item 5 by Collaborative Participation Level

Source	Sum of Squares	DF	MS	*F*	*p*
Between	7.824	2	3.912	3.431	.03
Within	527.912	463	1.140		
Total	535.736	465			

Table E2

Scheffe Post-hoc Differences by Collaborative Participation Level for Item 5

Participation Level	Mean	Occasional *t*	Never *t*
Frequent	3.917	0.77	1.98
Occasional	3.995		2.33*
Never	3.500		

*$p < .05$
Note: t-value for Frequent vs. Never contrast = .054

Table E3

Analysis of Variance: Item 14 by Collaborative Participation Level

Source	Sum of Squares	DF	MS	*F*	*p*
Between	4.793	2	2.396	4.83	.008
Within	232.231	468	.495		
Total	237.023	470			

Table E4

Scheffe Post-hoc Differences by Collaborative Participation Level for Item 14

Participation Level	Mean	Occasional t	Never t
Frequent	4.56	3.06*	1.25
Occasional	4.35		0.05
Never	4.41		

*$p < .05$

Table E5

Analysis of Variance: Item 17 by Collaborative Participation Level

Source	Sum of Squares	DF	MS	F	p
Between	8.240	2	4.120	5.355	.005
Within	362.362	471	.769		
Total	370.601	473			

Table E6

Scheffe Post-hoc Differences by Collaborative Participation Level for Item 17

Participation Level	Mean	Occasional t	Never t
Frequent	4.569	2.89*	2.08
Occasional	4.325		0.57
Never	4.231		

*$p < .05$

Table E7

Analysis of Variance: Item 20 by Collaborative Participation Level

Source	Sum of Squares	DF	MS	*F*	*p*
Between	12.750	2	6.375	6.717	.001
Within	443.242	467	.949		
Total	455.992	469			

Table E8

Scheffe Post-hoc Differences by Collaborative Participation Level for Item 20

Participation Level	Mean	Occasional *t*	Never *t*
Frequent	4.170	3.24*	2.32*
Occasional	3.865		0.62
Never	3.750		

*$p < .05$

Table E9

Analysis of Variance: Item 21 by Collaborative Participation Level

Source	Sum of Squares	DF	MS	*F*	*p*
Between	8.474	2	4.237	5.114	.006
Within	386.064	466	.829		
Total	394.537	468			

Table E10

Scheffe Post-hoc Differences by Collaborative Participation Level for Item 21

Participation Level	Mean	Occasional t	Never t
Frequent	4.000	3.15*	1.21
Occasional	3.720	0.66	
Never	3.821		

*$p < .05$

Table E11

Analysis of Variance: Item 22 by Collaborative Participation Level

Source	Sum of Squares	DF	MS	F	p
Between	4.341	2	2.170	3.906	.021
Within	259.500	467	.556		
Total	263.840	469			

Table E12

Scheffe Post-hoc Differences by Collaborative Participation Level for Item 22

Participation Level	Mean	Occasional t	Never t
Frequent	4.435	1.08	2.51*
Occasional	4.358		1.95
Never	4.077		

*$p < .05$

Table E13

Analysis of Variance: Item 23 by Collaborative Participation Level

Source	Sum of Squares	DF	MS	*F*	*p*
Between	9.566	2	4.783	5.407	.005
Within	412.209	466	.885		
Total	421.774	468			

Table E14

Scheffe Post-hoc Differences by Collaborative Participation Level for Item 23

Participation Level	Mean	Occasional t	Never t
Frequent	4.377	2.63*	2.26*
Occasional	4.140		1.00
Never	3.947		

*$p < .05$

Table E15

Analysis of Variance: Item 24 by Collaborative Participation Level

Source	Sum of Squares	DF	MS	*F*	*p*
Between	10.917	2	5.454	4.038	.018
Within	630.000	466	1.352		
Total	640.917	468			

Table E16

Scheffe Post-hoc Differences by Collaborative Participation Level for Item 24

Participation Level	Mean	Occasional t	Never t
Frequent	3.639	1.40	2.77*
Occasional	3.483		2.00
Never	3.079		

*$p < .05$

Table E17

Analysis of Variance: Item 25 by Collaborative Participation Level

Source	Sum of Squares	DF	MS	F	p
Between	36.461	2	18.231	22.048	.000
Within	387.801	469	.827		
Total	424.263	471			

Table E18

Scheffe Post-hoc Differences by Collaborative Participation Level for Item 25

Participation Level	Mean	Occasional t	Never t
Frequent	4.351	5.09*	5.02*
Occasional	3.906		2.36*
Never	3.500		

*$p < .05$

Table E19

Analysis of Variance: Item 34 by Collaborative Participation Level

Source	Sum of Squares	DF	MS	*F*	*p*
Between	15.360	2	7.680	4.914	.008
Within	731.383	468	1.563		
Total	746.743	470			

Table E20

Scheffe Post-hoc Differences by Collaborative Participation Level for Item 34

Participation Level	Mean	Occasional t	Never t
Frequent	2.151	3.02*	0.14
Occasional	2.520		1.50
Never	2.180		

*$p < .05$

Table E21

Analysis of Variance: Item 52 by Collaborative Participation Level

Source	Sum of Squares	DF	MS	*F*	*p*
Between	1.625	2	.813	4.132	.017
Within	92.804	472	.197		
Total	94.430	474			

Table E22

Scheffe Post-hoc Differences by Collaborative Participation Level for Item 52

Participation Level	Mean	Occasional t	Never t
Frequent	4.862	1.41	2.39*
Occasional	4.803		1.70
Never	4.650		

Table E23

Analysis of Variance: Item 58 by Collaborative Participation Level

Source	Sum of Squares	DF	MS	F	p
Between	5.738	2	2.869	4.686	.010
Within	287.142	469	.612		
Total	292.879	471			

Table E24

Scheffe Post-hoc Differences by Collaborative Participation Level for Item 58

Participation Level	Mean	Occasional t	Never t
Frequent	4.409	3.04*	0.97
Occasional	4.178		0.68
Never	4.275		

*$p < .05$

Table E25

Mean Ranks of Items 49, 50, and 51 by Collaborative Participation Level

Participation Level	Item Number 49	50	51
Frequent	2.10	1.75	2.15
Occasional	2.15	1.82	2.04
Never	2.00	1.96	2.04

Appendix F
Analysis of TCMME Survey Responses by Collaborative Site

Table F1

Analysis of Variance: Item 5 by Collaborative

Source	Sum of Squares	DF	MS	*F*	*p*
Between	50.381	10	5.038	4.768	.000
Within	496.617	470	1.057		
Total	546.998	480			

Table F2

Scheffe Post-hoc Differences by Collaborative for Item 5[*]

Collaborative	Mean	San Francisco *t*	Twin Cities *t*
New Orleans	4.522	5.090	5.589
San Francisco	3.324		
Twin Cities	3.442		

[*]Note: Only significant ($p < .05$) post hoc *t*-values are reported

Table F3

Analysis of Variance: Item 17 by Collaborative

Source	Sum of Squares	DF	MS	*F*	*p*
Between	29.420	10	2.942	3.880	.000
Within	362.396	478	0.758		
Total	391.816	488			

Table F4

Scheffe Post-hoc Differences by Collaborative for Item 17

Collaborative	Mean	San Diego *t*	Twin Cities *t*
New Orleans	3.783	4.512	3.879
San Diego	4.781		
Twin Cities	4.667		

Table F5

Analysis of Variance: Item 25 by Collaborative

Source	Sum of Squares	DF	MS	*F*	*p*
Between	45.255	10	4.526	5.377	.000
Within	400.621	476	0.842		
Total	445.877	486			

Table F6

Scheffe Post-hoc Differences by Collaborative for Item 25

Collaborative	Mean	Cleveland *t*	Philadelphia *t*	Twin Cities t
Pittsburgh	3.485	5.341	5.353	5.154
Cleveland	4.429			
Philadelphia	4.383			
Twin Cities	4.352			

Table F7

Analysis of Variance: Item 27 by Collaborative

Source	Sum of Squares	DF	MS	*F*	*p*
Between	82.195	10	8.220	5.156	.000
Within	710.846	477	1.490		
Total	793.041	487			

Table F8

Scheffe Post-hoc Differences by Collaborative for Item 27

Collaborative	Mean	San Diego *t*	Twin Cities *t*
New Orleans	4.413	6.275	6.070
Pittsburgh	3.779	3.897	
San Diego	2.625		
Twin Cities	3.170		

Table F9

Analysis of Variance: Item 35 by Collaborative

Source	Sum of Squares	DF	MS	*F*	*p*
Between	22.986	10	2.299	4.508	.000
Within	242.218	475	0.510		
Total	265.204	485			

Table F10

Scheffe Post-hoc Differences by Collaborative for Item 35

Collaborative	Mean	New Orleans t
New Orleans	3.978	
Durham	4.643	3.502
Philadelphia	4.723	3.894
San Diego	4.719	3.571
San Francisco	4.686	3.608
Twin Cities	4.698	3.760

Table F11

Analysis of Variance: Item 41 by Collaborative

Source	Sum of Squares	DF	MS	*F*	*p*
Between	47.149	10	4.715	3.596	.000
Within	622.851	475	1.311		
Total	670.000	485			

Table F12

Scheffe Post-hoc Differences by Collaborative for Item 41

Collaborative	Mean	San Diego t	San Francisco t
New Orleans	4.022	4.322	4.487
San Diego	2.844		
San Francisco	2.882		

Table F13

Analysis of Variance: Item 49 by Collaborative

Source	Sum of Squares	DF	MS	*F*	*p*
Between	101.617	10	10.162	4.629	.000
Within	1047.227	477	2.195		
Total	1148.844	487			

Table F14

Scheffe Post-hoc Differences by Collaborative for Item 49

Collaborative	Mean	San Diego t	San Francisco t
New Orleans	3.761	5.357	4.374
Memphis	3.634	4.950	
San Diego	2.063		
San Francisco	2.314		

Table F15

Analysis of Variance: Item 62 by Collaborative

Source	Sum of Squares	DF	MS	*F*	*p*
Between	63.536	10	6.354	6.250	.000
Within	483.877	476	1.017		
Total	547.413	486			

Table F16

Scheffe Post-hoc Differences by Collaborative for Item 62

Collaborative	Mean	San Francisco t	Twin Cities t
New Orleans	4.283	4.798	5.466
Memphis	4.163	4.615	5.382
San Francisco	3.057		
Twin Cities	3.093		

Table F17

Analysis of Variance: Item 63 by Collaborative

Source	Sum of Squares	DF	MS	*F*	*p*
Between	53.884	10	5.388	4.197	.000
Within	605.955	472	1.284		
Total	659.839	482			

Table F18

Scheffe Post-hoc Differences by Collaborative for Item 63

Collaborative	Mean	Twin Cities t
New Orleans	4.023	4.964
Twin Cities	2.926	

Table F19

Percent of Teachers in Each of the Four Conceptions of Mathematics by Collaborative

Collaborative	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Cleveland	14	23	29	34
Durham	18	23	18	43
Los Angeles	14	26	14	49
Memphis	14	12	23	51
New Orleans	22	2	40	36
Philadelphia	21	26	21	32
Pittsburgh	12	14	31	43
St. Louis	18	14	23	45
San Diego	38	16	19	28
San Francisco	12	41	12	35
Twin Cities	2	35	19	44
Total	15	21	23	41

Appendix G

Responses to the Diary Of Professional Relationships

Table G1

Teacher Responses to the Diary of Professional Relationships

What do you think mathematics is?

Cleveland

Calculations and geometric figures.

A logical process of problem solving.

Study of logic and a symbolic language which all life's problems can be solved with.

Mathematics is the science of solving problems that exist or will exist in the world. It consists of all techniques and algorithms that are used to solve these problems.

A language for problem solving.

Los Angeles

Ability to translate real world events into abstract expressions.

A means to explain how the world around us works.

Mathematics is a process of abstract thinking used in a logical way to communicate ideas and to solve problems.

A systematic way of looking at the world.

Manipulation of numbers, forms and shapes. Creative ideas, manipulation.

Memphis

The study of the axiomatic method, deductive reasoning, logic and the study of rules to understand this axiomatic method.

Exact science; more than adding, subtracting, multiplying and dividing; critical thinking and reasoning.

The science that is the working tool of all other sciences.

Science of patterns, critical reasoning, change. Tool for physical sciences and engineering.

Science of numbers.

Table G1 (continued)

Teacher Responses to the Diary of Professional Relationships

What do you think mathematics is?

New Orleans

Mathematics is the relationship of forms and figures and the relationship between quantities and symbols.

Mathematics is the study of numbers and number systems and how they relate, interact and apply to other areas and to real life situations. This study includes thinking skills, problem solving for everyday living as well as for other areas of science, and basic manipulative skills with the various number systems.

The science in which known relations between magnitude are subject to certain processes that enable other relations to be deduced.

Philadelphia

The study of how numbers are used in the universe.

Geometry is mathematics and mathematics is a way of life!

Mathematics is problem-solving--the application of certain concepts to specific situations in order to discover properties and/or answer questions.

Mathematics is a study of science. It helps us to think logically. It also helps us to understand abstract ideas and apply them to real-world situations.

It is the study of relationships of numbers. It is learning what we can do with numbers. It is problem solving. It is learning to manipulate numbers.

Pittsburgh

Logical thinking, reasoning, and problem solving through the use of the basic skills.

Not arithmetic. A systematic method of dealing with problems using arithmetic and/or logical thinking.

Mathematics is the study of number concepts and its uses and application to work and living.

I see math as the study of relationships between numbers and the application of the relationship to solve problems. (Traditional definition!)

St. Louis

The study of numbers and their relationships and properties.

Table G1 (continued)

Teacher Responses to the Diary of Professional Relationships

What do you think mathematics is?

St. Louis

The science of quantities and magnitudes and the relation between them, and the methods by which knowns can be found from the unknown.

The logical organization of postulate, theorems, and definitions and the use of these ideas to solve problems.

The body of information which allows critical thinking, reasoning, and problem solving, through algebra, geometry, and higher math; utilizing one's own mind and other technological tools.

The science of numbers.

San Diego

Study of patterns, logic, how it relates to numbers, geometry, shapes.

Hard question. It's probably to learn some skills to solve some problems.

Abstract tool to deal with the world around us.

The study of concept of numbers. Manipulation of symbols that represent numbers.

Logical, orderly, symbolic logic.

San Francisco

The language of science. Something to enjoy.

A process, a language--the language of science.

It's a language, a way to communicate about the world around us.

I think it's partly a way to communicate about science, but I think it's also a science in itself, with an intrinsic beauty and value that reflects how our minds make sense out of the world.

It's the study of patterns. Patterns in nature and a way to communicate about them.

Table G1 (continued)

Teacher Responses to the Diary of Professional Relationships

What do you think mathematics is?
Twin Cities
Mathematics is an ever-used skill that permeates our lives. To make that first cup of coffee we measure; we estimate travel time and time for projects; we add, subtract divide, and multiply and mix our way through the day; till we estimate whether we will get enough sleep to be alert tomorrow.
Mathematics is the science of patterns--L. A. Steen (*Science* 29 April, 1988). Mathematics is the study of all possible patterns. W. W. Sawyer.

Table G2

Teacher Responses to the Diary of Professional Relationships

How has the collaborative affected your conception of what mathematics is?
Cleveland
It has not affected my conception.
More practical applications available.
Broadened my knowledge and scope of what mathematics is and it has influenced my approach to problem solving.
The collaborative has presented many problems that are not traditional or found in our math texts, and has developed ways to solve these problems.
The ability to communicate with other teachers, discuss and exchange ideas has given me a clearer outlook on using mathematics to teach problem solving.
Los Angeles
Broadened the scope of math applications.
I have a deeper understanding of how it relates to the world.
It has made me extend my conception beyond my narrower views.
Hasn't changed
It has broadened my knowledge of different topics and phases of math that I have very little time to explore.
Memphis
It hasn't made me more aware of what math is but has broadened my horizons about statistics and other branches of math. MUMC has broadened my perspectives.
Made me realize mathematical thinking and reasoning is more important. More aware of importance of NCTM standards and realize it is critical to incorporate them.
I have seen many additional ways in which math is a tool.
It has made me aware of national trends, to realize that math is changing and that updating is essential.
More insight of what should be emphasized. More involved in using new technology to teach.

Table G2 (continued)

Teacher Responses to the Diary of Professional Relationships

How has the collaborative affected your conception of what mathematics is?

New Orleans

The collaborative has affected my concept of mathematics by extending my definition of mathematics, thus helping me to be more productive.

The collaborative has made me aware of the kinds of mathematics needed to perform certain highly technical jobs through the site visitation programs. Through the symposiums I have amassed quite a few facts concerning the problems in mathematics.

By showing me discoveries and projects.

Philadelphia

It has given me the opportunity to participate in professional activities that have re-enforced my knowledge of mathematics.

It hasn't.

It has broadened my conception of mathematics by exposing me to other teachers' ideas.

It hasn't changed my conception of what mathematics is. It has taught me how to motivate the teachers and how to work in a school system where the administration has no power of imagination of ________________ an educational system.

It has enhanced it.

Pittsburgh

It has shown me problem solving is important to basic everyday living.

The math collaborative has provided me with information and ideas (through speakers, visiting companies, and by acting as a resource center). This has helped me better understand and apply math to concrete application.

Several examples of mathematical applicability in business and industry were most helpful.

Not much, really.

St. Louis

Collaborative speakers have given me a broader insight into math from diverse points of view.

I don't think I have been active enough to see any change. Maybe from a workshop or two.

Table G2 (continued)

Teacher Responses to the Diary of Professional Relationships

How has the collaborative affected your conception of what mathematics is?

St. Louis

I have seen new topics and areas of math that I had never experienced. Discrete math, for example.

Rather than affecting, it enhances--keeping me in touch with current trends--and allows me to interact with colleagues professionally and personally.

It has expanded my idea of how mathematics can be applied.

San Diego

It hasn't affected my conception of what mathematics is. However, it has affected how I teach mathematics.

Changed my perception of math. It used to be paper and pen. Now we go from abstract to concrete. It's applicable and practical.

Reinforced my attitude about what math is . . . more excited about teaching mathematics.

More of an attitude that math is a tool to analyze problems logically and clearly.

It hasn't changed it.

San Francisco

It's given me different peoples' views...the collaborative makes things more human.

It's enabled me to see other people's conception of mathematics. It's given me a chance to share and communicate.

It has really affected me.

I've learned a lot more about how mathematics is used. I never knew much about applications. The dinner lectures and exploratorium taught me a lot about connections between math and science, and math in industry and engineering.

I've learned about what other people think it is. I've had a chance to meet and talk to other teachers and hear different ideas of what math is and how to teach it.

Table G2 (continued)

Teacher Responses to the Diary of Professional Relationships

How has the collaborative affected your conception of what mathematics is?
Twin Cities
It has broadened my perception of mathematics and has provided a forum for discussion of topics with like interested persons. It has also provided a wealth of exciting people.
I have developed and refined my conception of mathematics through reading and study, rather than through collaborative activities. I have been able to learn more about some applications of mathematics through collaborative activities, however.

Table G3

Teacher Responses to the Diary of Professional Relationships

What interactions with representatives from business and industry have you had through the collaborative? How have they affected your conceptions of mathematics?

Cleveland

Through the dinners and symposia. No effect on conception.

Informal discussion at symposia. More practical applications.

It has enriched me professionally. Met with National City Bank for problem solving. Nordson Corp. for an interview, BR America, etc. Symposia have helped.

Most of the interactions have been through the symposiums that I have attended and they have affected my concept of mathematics that it is a never ending change of ideas and methods of solving problems.

A great amount of interactions. I know what the student should learn in school in order to compete with other students for jobs.

Los Angeles

Very supportive of math teachers.

I've attended lectures and workshops of people from industry. It taught me that a great deal needs to be revised in the modern day math curriculum.

Not much interaction. Has not affected any conceptions.

None.

Memphis

I had an internship at St. Jude's last summer and worked in their biostat department. I have a new image of statistics and computer sciences. I see that statistics is really important and I appreciate it more.

Speaker's Bureau and Mathcounts (run by engineers) Engineers show how math is vital for their career and life.

None.

IBM dinner--realized that I could explain some concepts and encourage thinking by using computers in class.

Table G3 (continued)

Teacher Responses to the Diary of Professional Relationships

What interactions with representatives from business and industry have you had through the collaborative? How have they affected your conceptions of mathematics?

Memphis

IBM dinner(1988)--reinforced usefulness of computers in classroom.

New Orleans

I have had an interaction with representatives of a natural gas company in the form of a seminar that related job possibilities in this industry. They informed me to what extent I could expand my instruction for my classes.

I visited Southeastern Regional Research Center last year. I observed some of the chemical tests that were conducted. The representatives lectured at each demonstration and I received some samples of problems that are solved in order to perform the tests. This made me aware of the concepts I needed to be sure that my advanced students understand.

New Orleans Public Service Inc.; Electric and Gas; by being able to read my meters.

Philadelphia

I have not had many interactions with such representatives.

None.

None.

I received a grant to go to Atlanta for the national AMS Conference and learned a lot from attending several lectures. Professor Kemeny's lecture on Teaching Calculus gave me an insight to teach Calculus in a new way. I also came to know Janet Ramsey from IBM. She helped me to get an IBM computer on loan to run a special program on "Artificial Intelligence."

None.

Pittsburgh

None

The math collaborative has provided me with information and ideas (through speakers, visiting companies, and by acting as a resource center). This has helped me better understand and apply math to concrete application.

Table G3 (continued)

Teacher Responses to the Diary of Professional Relationships

What interactions with representatives from business and industry have you had through the collaborative? How have they affected your conceptions of mathematics?

Pittsburgh

Discussion with Blue Cross actuary. Financial planner with local brokerage firm. Westinghouse Nuclear Training Center for utility personnel.

St. Louis

None. I was suppose to visit once, but I was unable to make the engagement.

I have visited IBM. I learned more uses of mathematics that I could relate to the student.

I visited McDonnell Douglas two summers ago. I really enjoyed that visit.

Most specifically technology. IBM kit. Modern technology revolutionizes many pencil calculations. This technology frees one to develop concepts and minimize some of what students have felt "Boring."

None this year.

San Diego

Off hand, I can't think of any. Didn't attend the bank tour.

One visit with the bank. We haven't had much. With the classic calculator, they have affected my conception of mathematics.

I have not had many interactions.

None.

I personally didn't.

San Francisco

I met people from Chevron at those dinners. They saw us as professionals and really treated us well. We could talk to them and see how much math was used.

I've had little interaction. I guess I interacted with some speakers at least in the beginning when Chevron had the dinner lectures. We got a chance to see where people were coming from and what they were looking for.

Table G3 (continued)

Teacher Responses to the Diary of Professional Relationships

What interactions with representatives from business and industry have you had through the collaborative? How have they affected your conceptions of mathematics?
San Francisco
Like when that engineer from the Golden Gate Bridge spoke. That was a nice example of how mathematics is used.
I guess I met a few people from industry at dinner lectures but we didn't really get too much into mathematics. It was more like social pleasantries. It was nice how they seemed to respect what we do though.
Haven't really had any.
Twin Cities
An even wider perspective of use; what we are preparing students to do; the exciting new avenues they will experience.
Dinner meetings and some contact through classes. I have learned how mathematics is applied in noneducational contexts through _______.

Table G4

Teacher Responses to the Diary of Professional Relationships

What do you want your students to get out of mathematics instruction?

Cleveland

Be able to solve basic problems. Should have the math to solve problems.

Adequate understanding to do problem solving and be comfortable doing it.

To be proficient in problem solving and to learn to use and handle math and to grow intellectually.

To think critically about a problem and to develop techniques that they can use to solve these problems.

The ability to think.

Los Angeles

Ability to think.

A fuller appreciation of the importance of math to their future. Also fun out of doing math and a deeper understanding and overview.

I want them to be able to think critically, to be able to formulate questions and follow through on solving a wide range of problems.

I want them to gain self-confidence. I want them to feel that they have the necessary skills to solve new problems.

I would like any students to become efficient enough to be able to get along mathematically in our scciety. To know all of the basic skills and use them as a background to think through and solve problems, thus arriving at logical conclusions.

Memphis

1) To master objectives in the curriculum. 2) Appreciate math, its usefulness and it's necessity, to be successful in our world.

a) Have no fear of math b) be open and willing to try math c) realize math is important for everyone d) do their best.

1) Realization of the importance of math in everyday existence. 2) Appreciation of the beauty involved in mathematical systems. 3) Foundation to be able to achieve satisfactorily at higher levels of math or actual future employment.

Table G4 (continued)

Teacher Responses to the Diary of Professional Relationships

What do you want your students to get out of mathematics instruction?

Memphis

1) Think out math problems instead of rote memory of methods. 2) Appreciate math's beauty and usefulness. 3) Realization that they can work hard and achieve.

1) Dispel myths that women can't do math. 2) Build computational and problem-solving skills. 3) Increase self-esteem.

New Orleans

I want my students to be able to use math in everyday life and to be able to use math in various careers that would allow them to be independent and productive.

I want my students to be able to do the following:

a. Perform basic skills rapidly and mentally, without the use of a calculator.
b. Perform operations on real numbers, solve equations of real numbers.
c. Solve problems with or without numbers that relate to science and social sciences.
d. Solve problems related to everyday living.
e. Develop critical and logical thinking skills.

Develop latent ability in some students and interest many who heretofore have shown little or no enthusiasm for mathematics.

Philadelphia

I'd like them to see the beauty of mathematics--its patterns and its structure. I'd like them to <u>enjoy</u> the material presented to them in class. I'd like them to be able to take the skills that they have learned and apply them to solving problems that they will face in the future.

Everything they can. Thinking processes and questioning techniques plus content. I want them to know enough to be able to judge what is worth knowing and what is not.

I want my students to be able to manipulate mathematical concepts, in an appropriate manner, in order to reach conclusions. I also want them to be able to judge the reasonableness of their conclusions.

a) A good knowledge of History of Math.
b) How to apply math in real life situations.
c) To learn the techniques of Problem Solving.
d) Theorem proving and application.

Confidence in themselves.

Table G4 (continued)

Teacher Responses to the Diary of Professional Relationships

What do you want your students to get out of mathematics instruction?
Pittsburgh
A knowledge of the basic skills and how to apply them to problems that occur in their lives.
Hopefully my experience will be related to my students and any increased appreciation of math will be motivational for my students.
To be able to function in society doing those mathematical concepts necessary for "survival," i.e., checking, banking, measuring, money, estimating, computation and become mathematically literate.
1) Acquire basic math skills. 2) Acquire application skills. 3) Acquire application for "elegance" of math.
St. Louis
1) Ability to do basic math. 2) Ability to evaluate and think logically through any problem. 3) Ability to do above average on the college placement test.
To have the power to use mathematical notations to express ideas that under normal circumstances would require long sentences. To think about relationships among numbers.
To learn enough math to succeed in life and to be able to pursue the career of their choice. To enjoy math and see a real use for it in life.
Have the ability to problem solve, "reason," using whatever tools and techniques that are logically appropriate.
The ability to understand the numbers they hear, read and use. To appreciate how mathematics can help them solve problems. To be able to use mathematics at work and elsewhere.
San Diego
How to access information, technical information. They become masters of the tools themselves. They can teach themselves whatever they need to know.
Some formulas and how to apply them. How to apply them in the future.
A love for math. A feeling of competency and feeling of equality. . . that they can do any math that they are given. . . that math is not a foreign, esoteric, subject. With patience they can learn any math put before them.

Table G4 (continued)

Teacher Responses to the Diary of Professional Relationships

What do you want your students to get out of mathematics instruction?

San Diego

The ability to think clearly. To reason. Being able to decide which approach to take when they're trying to solve a problem.

The ability to carry on in the work world or in college.

San Francisco

I want them to enjoy it--to be less uptight about it and go with the flow.

Critical thinking skills, pleasure, excitement.

Investigation skills. Extension skills so they can go beyond what they've learned and apply it to new situations.

I want them to understand its importance in a technological society, but I'd be happier if I could get them to understand how neat math is just as a way to think about things.

I want them to have fun with it. Really, if you see mathematics as patterns, it can be like a game.

Twin Cities

That mathematics is not static, but an ever-changing, flowing system adapting to the new and innovative.

Table G5

Teacher Responses to the Diary of Professional Relationships

How has your participation in the collaborative affected your goals for teaching mathematics?

Cleveland

Reinforced my feeling that graphing is very important and should be learned by all students. Probability and statistics should be taught.

Not at all.

It enhanced my ability to teach math. I learned different methods to present concepts, work with manipulatives, and it broadened my horizons.

By being a part of the collaborative, I have tried many new ideas and techniques to improve my instruction. Also by relating mathematics to every day world problems, mathematics becomes more interesting and not so abstract to students.

It has increased my participation in other activities concerning mathematics, and it has given me a greater outlook on teaching math.

Los Angeles

Personal goals have expanded and now I want to make changes in district's policy w/r/t math.

Concepts are far more important than the calculations. Taking risks, trying new and exciting things.

My goals now include having students formulate problems, work on open-ended problems, and use appropriate estimation techniques and technological tools in their work.

Hasn't changed.

Yes, in many ways. I am aware of the needs of our students, and the many available resources. It has given me many ideas that might be used in the classwork.

Memphis

I feel probability and statistics should be a goal included in our teaching of math. I see that what I teach is a basic tool for many careers. I realize better that math is useful and necessary.

MUMC has opened new goals. It has made me aware that I should expect more from my students. I am willing to sacrifice time and share knowledge with other teachers.

Table G5 (continued)

Teacher Responses to the Diary of Professional Relationships

How has your participation in the collaborative affected your goals for teaching mathematics?
Memphis
I feel I am more application-oriented and now use computers in class to achieve goals.
Made me aware that I must continually update my methods.--Involve students in thinking rather than placidly accepting notes from the teacher.--Add new technology to classwork.
My involvement keeps my own skills attuned. MUMC helps me sell myself- It shows the students that I'm interested and am updating myself.
New Orleans
My goals have been extended beyond some of my earlier expectations of my students and myself. In other words, the collaborative has afforded me an opportunity to reach far beyond my goals and expectations.
My participation in the collaborative has broadened my goals for teaching mathematics. Instead of aiming for preparing a student to be eligible to enter college, I am now concerned with giving the student a background in mathematics such that he can enter college, be successful and make a choice of career options.
Participation in the collaborative affected my goals for teaching math by arranging on-site visits at different places to see what science can do along with mathematical procedure.
Philadelphia
Participation in the collaborative has helped me focus in on the goals that I have outlined for teaching mathematics.
Participation in the collaborative has opened doors and provided me with all kinds of great materials, and has made me a _______.
Pittsburgh
No.
The collaborative has made me realize that my goals for my students are not in line with goals of the school where I am teaching. At this point, I am very frustrated and unhappy.

Table G5 (continued)

Teacher Responses to the Diary of Professional Relationships

How has your participation in the collaborative affected your goals for teaching mathematics?

Pittsburgh

I've enjoyed all the collaborative activities not only because they enable interaction with my peers, but also they give me a good feeling knowing that I am in a position of service to influence young minds to pursue mathematics and hopefully have the successes that the participants have had. I view the collaborative as a very positive influence upon those in attendance. There has not been one disappointing session that I've attended.

Not much.

St. Louis

The collaborative gave me the incentive to join and become an active member of math professional organizations (NCTM, MCCEL, etc.).

Yes. I now focus more on positive student behavior and [put] less emphasis on negative behavior.

I am able to keep up with modern trends and techniques of teaching math. The workshops I have attended have increased my knowledge of mathematics.

It keeps me focused on national goals--rather than my own goals. So I may provide more efficient, effective teaching methods to counter the element of "burnout" due to many years' experience.

The collaborative has enhanced my teaching by allowing me to see new approaches and new technologies and to network with other mathematicians to obtain additional ideas.

San Diego

It has changed them quite drastically. I think, the learning styles of the children. We need to change our teaching styles.

They changed a lot personally. I use more manipulatives. I try to use cooperative learning. Everything I learned from the collaborative, I use. We're reviewing software and would like to use it in the classroom.

Enthusiasm. A sense of a lack of isolation. I never talked to other math teachers. I used to think that people were erroneous in their teaching. Now others feel as I do.

More of an awareness that the goal is to get students to help one another as well as to receive from me, the teacher. I use cooperative learning. Although I don't use it as much as I'd like to, it's always in the back of my mind.

Table G5 (continued)

Teacher Responses to the Diary of Professional Relationships

How has your participation in the collaborative affected your goals for teaching mathematics?

San Diego

I'm looking at a career change. I am thinking about the community college.

San Francisco

I became more interested in math. It made want to teach higher, more academic levels.

It's validated goals I think I already had.

Not very much. I think I always had the same goals.

It's prepared me a lot more to help students learn about how math is used and how it's connected to other areas. I've gotten a lot of ideas for how to better present or demonstrate concepts.

It's made me want to do more. I've learned about how you can do more hands-on stuff and strengthen students' higher-level thinking skills.

Twin Cities

It has promoted open-mindedness to the changes that are coming faster and faster.

I am much more aware of the need to emphasize problem-solving and applications of mathematics whenever possible. I am trying for more of a balance between theoretical and applied aspects of mathematics than I did previously.

Table G6

Teacher Responses to the Diary of Professional Relationships

What recommendation would you make to improve the mathematics curriculum in your school?

Cleveland

Statistics and probability should be added. Students should not leave school without data analysis. Teachers should make individual curricular decisions. Graphing calculators should be introduced in beginning algebra classes. Analysis of graphs and tell what the results are.

More involvement of the computer into all mathematical topics from remediation through calculus.

Every student should take two math subjects--one for concepts and the other for manipulatives and applications.

We as math teachers must make math interesting for students. Students are turned off if they do not see the relationship of what we teach in math and the real world. We need to use hands-on techniques and ideas to make students get a feel for mathematics.

Increase the courses in mathematics that are needed for graduation. Provide math lab for all students.

Los Angeles

Inservice time for faculty to understand new curriculum strands/courses (i.e., Math A, Math B, etc.).

Incorporate more technology and weed out the bullshit. You know the meaningless math lessons, the busy work.

More choices beyond the Algebra-Geometry-PreCalculus-Calculus track. We have an experimental Finite Math class which started this year, but we are not allowed to call it that or our students will not receive advanced math credits for college application!!!

Teachers need more time during school hours to develop solutions for departmental problems. Bimonthly goals need to be set and evaluated. The department needs to access its accomplishments and failures.

Each student should be pre-tested and the whole curriculum should be reviewed. Departmental test should be given at the end of each semester.

Memphis

Probability/statistics should be included in our curriculum: a) formal courses b) add into other existing courses.

Table G6 (continued)

Teacher Responses to the Diary of Professional Relationships

What recommendation would you make to improve the mathematics curriculum in your school?

Memphis

Teachers should have a positive attitude that they can be successful. Teachers should take students where they are and then move with them, regardless of time pressures set by MCS.

Incorporate more updated materials demonstrating math applications. More available computer software correlated to the curriculum.

Incorporate computers/calculators in class.--More communications with teachers of the same course.--More realistic applications.

Offer a probability/statistics semester twice a year. Open more options for non-math majors. Applications should be emphasized.

New Orleans

I would like to see students scheduled by abilities beyond the honor classes and special programs.

To improve the mathematics curriculum in my school. I would like to see that the objectives we have are implemented more effectively. I would like to see thinking skills and problem-solving skills incorporated in the objectives of all courses taught. Add a course for the students who plan to attend technical schools that is at a higher level than the general mathematics and business mathematics courses.

Activity work books.

Pittsburgh

Keep some form of course for those who still have trouble in computation.

If there could be a more direct relevance of mathematics to the students' future. Maybe a section of each course that would demonstrate this direct applicability would help to motivate students.

We need to make math more relevant to the students--more practical applications, more high-level job-related applications.

St. Louis

More classroom computers (one per classroom). Seminars on better use of (algebra, new math). Writing across the curriculum. College placement test help.

Table G6 (continued)

Teacher Responses to the Diary of Professional Relationships

What recommendation would you make to improve the mathematics curriculum in your school?

St. Louis

More student tutors (Classical academy to help under-classmates.) Students learn well from other students.

It should be geared to the everyday student's need. I think we are trying through the standards and everything to make mathematicians out of everyone.

Give me greater support, disciplinarily so I can focus on math. Support service needed for many low achievers, so learning can take place. Provide more and easier access to technological equipment.

Utilize the computer software available in the teacher's classroom.

San Diego

We talked about this during our minimum day. Sometimes we expect students to reach too far. They're locked into it. Between six and 12 weeks, parents, teachers, and students via their parents, can make the change. We also thought that we would teach the intermediate algebra before geometry.

I would like to see the students use more calculators. We're beginning to require students to. use them more.

Need to get teachers who are sincerely interested in teaching mathematics. We still have teachers who are uninvolved.

Make it more flexible. We see all of these neat, wonderful things, but when you have to be on Chapter 15 by a certain date, you can't be as flexible and implement those ideas.

Our department is in a real battle over that issue . . . 50-50 split. The mathematics major vs. the nonmathematics. Math people want kids to stay in Algebra. until they learn the concepts before moving on. Nonmath people want everyone to be exposed to concepts. I would make the standards uniform. It's a real battle.

San Francisco

More of math lab--things to play with, computers. I think we should try more of an integrated approach.

We're doing it. We have STAMP, IMP, we're trying to bring math to all students. Mission [high school] has virtually every program that's going on in the district.

Table G6 (continued)

Teacher Responses to the Diary of Professional Relationships

What recommendation would you make to improve the mathematics curriculum in your school?
San Francisco
We're trying to focus on problem-solving. We're trying a lot of different things with problem-based curriculums to keep students in math classes.
We've got to make the curriculum less dry. We really need to do something about Math A and Algebra. Kids just don't get it and they can't see what it's good for.
I wish more teachers would be willing to try new things besides just doing what's in the books. There's a lot of stuff we could do that's more interesting and relevant.
Twin Cities
To focus on the understanding and not on the rote material. To promote the creative and innovative ideas that will be needed for the future.
We must find ways to use technology in math classes in such a way that the integrity of mathematics is preserved, but calculators and computers aid in the understanding of mathematics. We must also find ways to increase the exposure of the students to concepts from statistics and probability.

Table G7

Teacher Responses to the Diary of Professional Relationships

What are the key issues regarding mathematics education in your district and how will they affect your school?

Cleveland

1) Look ahead instead of playing catch up. We should not adopt antiquated curricula. 2) Evaluate the curriculum and make changes necessary to conform with our new standards.

1) A definite topical structure must be established in each subject area. 2) Dissolution of credit for remedial mathematics. 3) Requirement of algebra for graduation for all students and algebra should be for two periods. 4) Math lab necessary for learning problem solving by peer tutoring.

1) Have to motivate the students to succeed; 2) Have to give students a positive attitude; and 3) Have to give students confidence.

A key problem that we have is the decentralization of the schools in Cleveland. Each school will do what it wants, books, teacher usage, and we still at the same time need to pull together and help one another. All of us have strong and weak points in our teaching. We need to share our strong points and learn from them to eliminate our weak points. Students change from school to school very often during the school year. We need to cover the same topics so that when changes are made by students from school to school, he will fit into a class at his new school and not be lost.

The new standards. Implementing these standards.

Los Angeles

Lack of student motivation regarding education.

1) District test. It will set a standard. It is badly needed. 2) Purchase of hardware will affect my classroom in a positive manner.

Los Angeles Unified is stifled in every respect by the dead weight of nonteaching administrators who pay no attention to teachers.

School-based management is the most important issue whether it be English, Science, Math or Social Studies.

Working with the Urban Math Collaborative which takes teachers' positions and views seriously has helped keep me sane.

I'm not sure what the district's key issues are, but our school needs to get students to realize how important math education is. And in turn have them become more successful.

Table G7 (continued)

Teacher Responses to the Diary of Professional Relationships

What are the key issues regarding mathematics education in your district and how will they affect your school?

Los Angeles

Try to get more students into Algebra. It puts pressure on our students and school. We had to comply with district trends.

Memphis

1) Non-certified math teachers teaching math (not at my school) 2) Courses required for certification need updating. 3) Curriculum needs to be updated and include technology; learn new methods, use computers and calculators.

Curriculum time frame--deals only with minimums required. Teachers must do more and set higher goals.

1) Role of the computer--more funding is needed for equipment and teacher training. 2) Implementation of the NCTM Standards--updating of curriculum will be necessary.

Curriculum--needs to be updated, teachers need to complete the minimum and do more. Computers--new technology needs to be available to math teachers and classes; software should be tested and evaluated and made available.

Proficiency test--affects curriculum, what is taught/stressed. (My school has a high passing rate). Test scores--Affect our sequencing of courses. Affects what is stressed.

New Orleans

1. To increase students' mathematical skills in many areas.
2. To provide the proper atmosphere for learning these skills.
3. To provide personnel capable of carrying out school district goals.

The above goals will increase students' knowledge, students' skills and thus, a school district with higher standards and productivity.

The key issues regarding mathematics education in my district are lack of money to implement changes and shortage of mathematics teachers. This affects the availability of purchasing books, attracting new teachers and generally purchasing supplementary materials.

Some students are not working up to capacity and test scores will be low.

Table G7 (continued)

Teacher Responses to the Diary of Professional Relationships

What are the key issues regarding mathematics education in your district and how will they affect your school?

Pittsburgh

No longer rote calculating after elementary school, but a more concentrated effort on problem solving. There will be no more basic skills courses. There will be calculator use and emphasis on solving problems.

I feel my school is a "day care center" for teenagers. Our students miss my classes for a huge variety of reasons most of them sanctioned by our administration. Our classes must have such a high priority that they are never interrupted. Maybe if we take this education this seriously, our students and their parents will take it seriously.

I fear that mathematics courses have been viewed by adults as being too difficult for students and as a result they have not become alarmed by decreasing scores on tests. There appears to be an allowing of students to take easy courses rather than the challenging courses. Some administrators feel that success is most important at any cost. An example would be allowing a TAS [Trigonometry, Algebra, Statistics] student to drop TAS and take Applied Math because "He wasn't doing good in TAS"--POOR EXCUSE.

We need to make math more relevant to the students--more practical applications, more high-level job-related applications.

St. Louis

Writing across the curriculum must be done by all students in every class. Test scores.

The new concepts, use of computers, calculators, manipulatives, games, etc., to teach concepts. We are going to have to spend more time in the lab and look for many new teaching strategies.

Better test scores in math. Less complaints from the community. Every student can learn high school courses, not just general math. More students will take algebra and geometry and higher courses.

New texts and curriculum which we were given with no time to peruse. New math techniques call for different types of classroom control. Control has been in form of all quiet, working at seat rather than interactions. This requires new support from administration/counselors and this is not in place yet. Also, [it] requires greater student self-discipline and responsibility.

Table G7 (continued)

Teacher Responses to the Diary of Professional Relationships

What are the key issues regarding mathematics education in your district and how will they affect your school?

St. Louis

All new textbooks in all subjects. Too much to digest at once. Our supervisors do not provide assistance in the delivery system. There needs to be more coordination, more dialogue. In talking with other supervisors at NCTM meetings, they seem to have more contact with teachers.

San Diego

Don't know.

ESL classes. We need to help children whose primary language is not English. We thought that we had only 15 students. Instead we have 40 students, two classes. We need some help. No teacher applied for position. Maybe parents could help out.

I'm not sure if the Sweetwater has any push for mathematics at all. Perhaps the superintendent is more in tune than our principal. She's only interested in roosting.

San Diego

Getting those test scores up! We're still tied to those tests! Need to build in more flexibility.

District-wide they're really messed up. They are resisting implementing the state framework. The curriculum person in the district would not receive anything from the teachers who had attended the San Diego Math Project.

San Francisco

The district doesn't know from year to year what the issues are. They find test scores more important. Math becomes more like a competition instead of something for people to enjoy. I think a key issue for us is whether we'll adopt a more integrated approach and deal with the issue of tracking students. That will have a big effect on how we do things in our school.

The drop-out rate is a big problem the district is trying to address. We ry to address it with our special programs.

The drop-out rate. More centralization is going to be an issue that affects us--choosing common books so when students transfer they're doing the same things. We have a 50% turnover every year.

Table G7 (continued)

Teacher Responses to the Diary of Professional Relationships

What are the key issues regarding mathematics education in your district and how will they affect your school?

San Francisco

Minority achievement in math is a big issue in the district. More and more black and Hispanic students drop out of the math program earlier and their test scores are going down. It happens at our school too. The district needs to do something to prepare them better in the lower levels and we need to find curriculum and ways of teaching that will interest more of our students in math, and help them and their parents see how important it is to learn math.

I guess there's the whole issue of nation-wide math reform and how we need to make math accessible to more students. I don't know how that will affect our school yet. We sort of wait to see what happens other places.

Twin Cities

That we can prepare the student in the use of the tools of the future.

Our school is a self-contained private school. The issues of the school are what we would have to call the issues of the district. Two major issues: More time for mathematics on all levels from K through 12. There is not enough time devoted to mathematics. The goals of the *Standards* cannot be reached in the time we presently have available. Students in the lower grades can and should have a different kind of mathematics preparation if the ideas of teachers in upper grades can be translated for the lower grade teachers. It is possible to help lower grade teachers to become interested in more than computation. They would probably welcome a more diverse and powerful conception of mathematics than they now possess. The teachers in the upper grades can help to supply this new awareness of mathematics.

Table G8

Teacher Responses to the Diary of Professional Relationships

What has influenced your teaching of mathematics (i.e., coursework in college, a high school mathematics teacher, colleagues, teaching experience, the collaborative)?

Cleveland

High school mathematics teacher and the collaborative.

All of the above.

College professor, high school math teacher and teaching experience and the collaborative.

a) My high school algebra teacher.
b) A methods course at Ohio State University.
c) The math collaborative
 1) The resource center
 2) Symposiums.
d) Colleagues.

Colleagues, mainly the department chairman I had at the first school I worked.

Durham

The challenge of a new field [I was trained as an English teacher] plus a chance to be part of the reform movement currently sweeping math.

A high school math teacher gave me an interest in mathematics. I had her for two years and she showed me what fun I could have with math. I also enjoy the course content of high school math courses.

Coursework in college, high school math teacher, colleagues, teaching experience, opportunities offered by DMC.

All of these things at one time have influenced my decision to teach and the method I use in teaching. However, colleagues have recently had the most effect on my views of education.

Mathematics teachers were well liked, especially in undergraduate work. Capability to do math well and succeed. The overall field of math being needed for other fields. Liking children.

All of these have had some influence in my teaching of mathematics. In more recent years, my colleagues have had a lot of influence.

My supervising teacher when I did my student teaching. The collaborative networks and programs. My students and their personalities and needs.

Table G8 (continued)

Teacher Responses to the Diary of Professional Relationships

What has influenced your teaching of mathematics (i.e., coursework in college, a high school mathematics teacher, colleagues, teaching experience, the collaborative)?

Durham

My teaching was most influenced by high school math teachers. Recently, teaching experience and working with the collaborative has influenced my teaching.

Los Angeles

Teacher (high school)
teaching experience
+PLUS+

Colle : professors, colleagues, 22 years experience, opportunity to teach students at different levels.

Workshops, teaching experience, methods used when I was taught math, and students' fear of math.

My college coursework, several of my mathematics teachers, and experience in industry influenced my teaching of mathematics.

Attending conferences and learning from past teaching experiences has influenced my teaching of math.

Memphis

A high school mathematics teacher.

By a high school math teacher who was great. By my teaching experience. By MUMC which gives me the opportunity to attend workshops and meetings where I learn innovative methods.

My graduate work at MSU. Probably the biggest is the lack of teaching I had in high school. I determined a long time ago that my students would get a better math background than I did. I have learned a great deal through experience. The collaborative has been a boost! I don't know that it changed my thinking drastically, but it made me feel more professional, and gave me the incentive to try new things and to think, "anything is possible."

My high school math teacher was a model for me. I always wanted to be the kind of teacher he was, the best math teacher in the world.

1) Teaching experience has influenced my teaching most. 2) College courses. 3) Previous teachers.

Table G8 (continued)

Teacher Responses to the Diary of Professional Relationships

What has influenced your teaching of mathematics (i.e., coursework in college, a high school mathematics teacher, colleagues, teaching experience, the collaborative)?

Philadelphia

My experiences mostly. However, it's been an eclectic process, and many of the experiences and workshops provided by the collaborative and professional organizations (at the local, regional and state level) have had profound influence.

I've always enjoyed mathematics as a student during my early educational years. However, it wasn't until I started my teaching career, that my interest increased greatly.

1) College math teacher, 2) colleagues, 3) collaborative

Experience and collaborative.

Interaction with colleagues combined with my teaching experience has most influenced my teaching of mathematics. The collaborative has provided increased opportunity to interact with colleagues.

Pittsburgh

All of the above.

Probably high school teachers at Schenely High School Teacher Center.

Colleagues, coursework, and teaching experience.

My knowledge and interest in math.

Colleagues.

High school math teachers, coursework in college, teaching experience.

High school teacher and college courses. For 22 years I was a science teacher and the school closed, at which time I was transferred as a math teacher. Now I teach some science courses.

A high school teacher.

My first goal was to work with children as a teacher and my proficiency in mathematics led me into that field. My older brother offered me challenging problems, experiences and puzzles that helped develop my interest in mathematics. After 25 years of teaching and being isolated with my students, the collaborative exposed me to other mathematics teachers who challenged with their problems and puzzles, thus renewing my interest.

Table G8 (continued)

Teacher Responses to the Diary of Professional Relationships

What has influenced your teaching of mathematics (i.e., coursework in college, a high school mathematics teacher, colleagues, teaching experience, the collaborative)?

St. Louis

I have taken many math courses with very good instructors. A and B are two that really influenced my teaching. The collaborative has helped too by providing workshops and helping with math conventions.

The greatest influence has been fellow teachers. I have worked with and talked with some excellent teachers (not only math). My coursework in college has had a lot of influence.

College math teachers, teaching experience, family, colleagues, probably in that order.

Discussion with fellow teachers and workshops and lectures attended at various conventions have influenced me the most. These have produced the most dramatic changes in my style of teaching. The collaborative has produced an avenue that has accelerated some of these changes and allowed me to feel comfortable about changes and allowed me to draw on the expertise of others.

A combination of observations (other teachers), my own experiences, collaborative activities, professional course work.

San Diego

I don't really have an answer. Too many things have influenced me.

Not teaching math now. Running The Excel program. One-to-one interaction with students. Tutoring in math.

Family. Parents always wanted me to do [it]. My sister and I went to school together. Now we teach together. Plan things together. Do curriculum writing together. Work at the same school. I became interested in computers. Now she's interested.

Combination of enjoying kids...influenced by teachers in high school.

Combination of things. I'm a science major but I like math and I take a lot of courses.

Table G8 (continued)

Teacher Responses to the Diary of Professional Relationships

What has influenced your teaching of mathematics (i.e., coursework in college, a high school mathematics teacher, colleagues, teaching experience, the collaborative)?

Twin Cities

My high school math teacher was excellent and very highly dedicated. Much of what I do today is modeled after him.

Much of my teaching style is based on my student teaching experiences. Currently I am attending the University of Minnesota in Math-Ed. Many new ideas have stemmed from that.

All of the above! Plus on-the-job supervision, in-service, reading publications, feedback from students, etc.

Mostly a college mathematics professor.

Primarily my teaching experience now determines how I teach.

Table G9

Teacher Responses to the Diary of Professional Relationships

How has the collaborative affected your teaching of mathematics?

Cleveland

Opened my mind to the innovations occurring in the math teaching profession.

It has improved my teaching.

By the symposiums that I have attended--I have used ideas from them to improve my instructions and techniques of presenting mathematical lessons:

a) By utilizing the resource center
b) By sharing your ideas with colleagues, college professors, and members of industry
c) By keeping up with "new standards" of mathematics.
d) By allowing you to participate in the decision making process.

Provided opportunities to attend conferences and symposia and to get together with math teachers from other schools.

Durham

Provision of materials; workshops, special interest groups, support.

It has given me a way of talking to other teachers about teaching. I also have had an opportunity to see how math is used in "real life" work situations.

I have only been involved with DMC this school year. During that short time, DMC has provided several professional opportunities for growth in teaching mathematics. In particular, the networking meetings have given me the opportunity to interact with other math teachers. This, in turn, has helped me to look at new ways to teach concepts.

The collaborative helps keep me aware of the new ideas and methods in mathematics. It provides me with material and training that otherwise would not be available to me and allows me to update what and how I teach.

New ideas presented on how to teach my subject. New interests and excitement for me as a teacher in discussions with other teachers and any events held.

It has encouraged me to use the shared ideas of other teachers. I am trying to really encourage the use of calculators. I am working to learn to use the graphing calculator I got at a math council workshop.

It has served to "modernize" my techniques and subject matter.

Table G9 (continued)

Teacher Responses to the Diary of Professional Relationships

How has the collaborative affected your teaching of mathematics?

Durham

Workshops provided by the collaborative and the opportunity to attend a national conference provided by the collaborative have given me new ideas and problems to use in my classroom.

Los Angeles

More courses to experiment and be innovative.

Not been involved.

It has reinforced the positive aspects of my teaching and focused my attention on, say, negative aspects that should be reevaluated.

Not really, I still have many students without the basic skills needed for algebra.

As yet, the collaborative has not affected my teaching because we have not been able to implement any of our plans.

Memphis

I was a burned-out math teacher. The collaborative has renewed me through exciting activities, opportunities to broaden my knowledge of math and enhancing my sense of professionalism.

My teaching methods have become more creative. I am placing a greater emphasis on problem-solving methods and applications.

I use technology more extensively. I am trying to find ways to implement the *Standards*. I am using cooperative learning in some instances. I plan to use writing in my classes next year. I also want to try some other learning styles.

The collaborative has made me more aware of certain needs of some groups of students. More applications are needed. The teaching must be applicable to students in every way. There must be meaningful activities.

Because of my exposure to collaborative activities, I have used a computer this year as well as innovative manipulative in my geometry classes.

Table G9 (continued)

Teacher Responses to the Diary of Professional Relationships

How has the collaborative affected your teaching of mathematics?

Philadelphia

Offerings by the collaborative have been important for me, and I've taken full advantage when possible. PEG's have made it feasible for me to attend meetings I definitely would've missed otherwise. I've also established contact and professional dialogue through the collaborative with some outstanding teachers and professional educators.

The collaborative has influenced me to attempt to be creative in the classroom. It has also allowed me to interact with teachers from other schools.

A new enthusiasm, greater feeling of control.

Reinforced attitude of constant professional development.

It's helped avert burn-out, kept me in touch with the latest in software, encouraged me in a leadership role. I'm more excited about teaching than I've been in years.

Pittsburgh

It has expanded my awareness of the problems that students encounter in different areas.

It has let us get together as math teachers and let us compare our teaching of mathematics in the different high schools.

It gives teachers a chance to meet and exchange ideas and techniques. We hear of new programs and listen to speakers that can influence our teaching.

It has broadened my knowledge in the mathematics area by the seminars, workshops, etc. sponsored by the collaborative.

It reinforced the curriculum that was proper for this society.

Some of the concepts and ideas presented by the speakers seem practical and beneficial to the students but I still need more time to evaluate their effectiveness in my classes.

I have been able to get examples of how things operate.

It has affected teachers in that they have a vehicle for discussion.

The teachers I've met have given me varied ideas and points of view that I can use in my classroom, and also have given me a sense of the importance of my profession. The collaborative has facilitated the meeting of these teachers.

Table G9 (continued)

Teacher Responses to the Diary of Professional Relationships

How has the collaborative affected your teaching of mathematics?
St. Louis
The collaborative has given me a positive attitude about my knowledge of math and towards my students. By going to workshops (sponsored or promoted by the collaborative), I have learned new and different ways of presenting my lessons.
It has helped me become aware and stay attuned to current math trends.
The collaborative-sponsored activities help to keep me abreast of the latest research and activities in my field.
Working with other teachers increases the confidence level at which I approach my work. I do not feel isolated in the system (on a professional level). The collaborative has presented me with opportunities for growth in my profession, with a chance to interact with others in the district and with a chance to voice my ideas. I feel a sense of strength when I know what others are doing around the country (as well as around the schools in my system).
By allowing me the opportunity to communicate with other mathematics teachers outside of my department. I have become more involved with my math supervisors and am able to attend more professional workshops that contribute to my growth and development intellectually.
San Diego
Broadened my horizon. I've changed my approach and techniques a great deal since I became involved with the collaborative.
Renewed outlook around me.
Keeps us informed about what's new.
It has afforded me opportunities to develop myself professionally by attending national conferences.
It's helped me to use different strategies and different approaches.

Table G9 (continued)

Teacher Responses to the Diary of Professional Relationships

How has the collaborative affected your teaching of mathematics?

Twin Cities

The opportunities of interaction with other collaborative teachers has kept me alive and motivated as a teacher. Before the existence of the collaborative, we did not have a local function where we could share professionally on a social basis.

Very much. Many of the people I have met through the collaborative have become good friends and have given me ideas on teaching.

Encouraged me to take workshops. Set up networking with peers. Brought me into contact with leaders in math and business and higher education.

It helped me in gaining more insight into the topic of statistics and probability (Woodrow Wilson workshop).

Its sponsorship of the Woodrow Wilson Summer Institutes has had an effect both on what I teach (course content) and how I teach it.

Table G10

Teacher Responses to the Diary of Professional Relationships

What do you think is the main purpose of high school?

Cleveland

Critical thinking and problem solving.

To prepare the students for advanced studies.

To prepare students to fit into society for the betterment of society.

The main purpose of high school is to prepare the students to become self-sufficient in meeting the challenges of life.

To prepare students for the future.

Durham

Preparation for whatever comes of [the] life [course] the student chooses to take.

The main purpose of high school is to help students 1) develop into citizens who are active in and productive for our societ·· and 2) to help students reach their highest potential.

To give students the opportu· / to use (apply) the thinking skills that they have already learned.

To prepare students for the next stage in their life, whether this is work of further education [or not], and to instill in all students the desire to continue the "learning process" throughout the remainder of their lives.

To teach students the skills needed to be a better person, that would be socially, emotionally and mainly intellectually to survive and be a positive contribution to our society.

To teach students to become productive, functioning, thinking, members of society.

To prepare students to be productive citizens who can contribute to society and lead a happy or at least contented life.

The main purpose of high school is to provide an appropriate education for each student, whether it be preparation for higher study, job skills, or survival skills.

Los Angeles

Prepare adolescents for survival in an adult world.

Educate the student to the point he/she is able to be prepared for the next step.

Table G10 (continued)

Teacher Responses to the Diary of Professional Relationships

What do you think is the main purpose of high school?

Los Angeles

To prepare students to be able to take advantage of a variety of options.

The main purpose(s) of high school are 1) to prepare young people to be good citizens, able to live useful productive lives, and/or 2) to prepare students for post-high school education.

The main purpose of high school is to prepare those students wanting to go on to college for success and to prepare those students not opting for college for career skills.

Memphis

To give students a variety of learning experiences for basic knowledge.

High school should be a relevant part of growing up. Its focus should be the preparation of the student for survival in the real world. The high school student should have a storehouse of academic knowledge and skills which he/she should be able to apply in daily life and society.

To prepare students to work in the real world or to pursue their education further. To give students a basic education so they can discuss topics with friends on an intelligent level. To enhance the student's ability to think.

To prepare students to function in a complex society. High schools must adequately prepare those students who desire to enter college so they can be successful. It is the school's responsibility to prepare students to enter the work force or to succeed in college.

High school should provide a bridge between adolescence and adulthood by providing a social and academic atmosphere that allows students to discover their own strengths, weaknesses, and interests.

Philadelphia

A. Babysitting and socializing our youth.
B. Provide a viable option for the opportunity for a quality education for all Americans, regardless of sex, race, or socio-economic status.
C. Provide experiences for youth to discover and practice critical thinking while preparing themselves to be part of an informed and responsible citizenry--capable of exchanging mutual respect and of supporting themselves financially through legal and gainful employment.
D. All of the above.

Table G10 (continued)

Teacher Responses to the Diary of Professional Relationships

What do you think is the main purpose of high school?

Philadelphia

High schools should equip students with solid fundamental skills so that upon graduation they will make positive contributions to society.

To prepare for the next step of life.

Develop people who can be lifelong learners. Develop people who can adjust to change.

The main purpose of high school is to continue the educational process, i.e., expose students to a variety of disciplines, tap into their natural curiosity, build problem-solving skills and ultimately empower them with a sense of control over their destiny.

Pittsburgh

It's the foundation of their future in college or business.

To teach them the fundamentals of mathematics and prepare them to develop the thinking process.

To give students the opportunity to establish their skills and the social values needed to become valuable citizens.

To educate students so that they can function in the outside world.

To give students an education that allows them to fit into our society.

Basically to give students a broad view and understanding of all parts of life.

High school is an experience unto itself. It is to prepare the students for life work and/or more education.

To do something with children between the ages of 14-18.

High school should expose students to various curriculums and ideas that would prepare them for their life experiences and proper preparation would make these experiences more enjoyable.

St. Louis

To learn the basic background in different courses and to grow socially and mentally for the world of tomorrow. To be able to expand one's knowledge in many positive ways for the future.

Table G10 (continued)

Teacher Responses to the Diary of Professional Relationships

What do you think is the main purpose of high school?

St. Louis

Prepare students for whatever their future goals might be, whether it is academic, work or raising a family.

To continue the academic as well as social preparation for "life." Hopefully students will be convinced that education is an ongoing process that never ceases.

To prepare students with the more advanced thinking skills necessary in college and the world of work, and to acquaint them with more advanced topics (facts) that they may need to be productive in our society.

To prepare students for higher education or supply them with technical skills needed to obtain employment.

San Diego

Math is a tool that goes across all disciplines and the youngsters need it.

To equip individuals emotionally, academically, and socially to live a productive life.

Probably two-fold. Kids who aren't going on [to college]. . . Gives them a foundation to go out and get a job. Kids who are going to higher education...it gives them the knowledge they need to succeed later.

To teach kids how to find information that they'll need later on.

All students should take algebra at least and those who are going on [to college] should take more mathematics in high school. I think that all kids can learn algebra. I taught general math students algebra and they liked it.

Twin Cities

To teach students how to learn, to prepare them for the real world, and to prep them for further education.

I believe it is supposed to prepare one for college or an alternative. I don't feel that the current structure is doing the job. Changes need to be made so the schools, parents, and students are accountable. High School should give students a well-rounded education regardless of their aspirations.

Table G10 (continued)

Teacher Responses to the Diary of Professional Relationships

What do you think is the main purpose of high school?
Twin Cities
Help students develop their ability to reason and think, develop their attitudes and morals, project into the future and plan, develop specific skills, build a foundation of knowledge.
To enable students to become thinking citizens with some of the basic facts and skills in place.

Table G11

Teacher Responses to the Diary of Professional Relationships

How should students be assigned to high school mathematics courses (i.e., interest, ability, age)?

Cleveland

1) Ability level; 2) interest.

Interest and ability.

Student should be assigned to math classes based upon ability and ability only.

a) By ability priority, b) By interest

Not by interest, if it were we would all be out of work. Combination of ability and age.

Durham

He seemed to feel that desire and interest were more important than the other criteria. Seemed most concerned about the student who might be a "late bloomer" and so be denied the opportunity to study certain topics because of past test results. All in all he seemed to feel that a student should be allowed to choose his own courses and then helped to succeed in his choice. [On-site observer comments]

Interest and ability should be strong factors. All students should have the opportunity to go as far as they can, but there should be several options. These should include some courses with "real life" consumer focus.

The main criteria should be student interest in a course with some consideration also being given to demonstrated ability. Prerequisites are somewhat important but should not be the deciding factor.

Interest and ability.

Mainly ability, all math courses require prerequisite skills. I do not need to reteach Algebra 1 or an Algebra 2 course. I disagree with the fact we have "let down" our C average as a requirement to take the next course. Students <u>will</u> <u>not</u> <u>strive</u> as hard.

I think they should be assigned according to interests and ability.

Primarily interest and willingness to work and so learn. Ability, of course, matters a great deal but not as much as does interest and ability to work.

On the basis of their interest and ability.

Table G11 (continued)

Teacher Responses to the Diary of Professional Relationships

How should students be assigned to high school mathematics courses (i.e.. interest, ability, age)?

Los Angeles

Ability/interest.

Interest and ability. A student who is not prepared for a math course is programmed to fail

Ability. Interest.

Students should be assigned to high school mathematics based on ability and interest.

Interest, ability, math background, testing for placement.

Memphis

Ability, interest (desire); classes could contain various ages.

By ability and interest. Those students who have the ability and interest to go to college should be directed into college preparatory math courses. Other students should not have to have to take the algebras, geometry, or other higher level math courses.

Every student should be assigned to a math class. Interest is an important consideration, but ability should be the first consideration.

Students should be assigned according to ability and interest.

Students should be assigned by ability. In my opinion, it is ridiculous to require Algebra II for graduation or for entrance into state colleges.

Philadelphia

Students should have access according to their interests and abilities (regardless of age); at some point, it may be necessary to eliminate some students (who show by their lack of "appropriate" participation) that would impede the group progress. We must be careful not to restrict access to courses which are instrumental for other goals, such as Algebra I.

I believe ability and interest should be a priority when selecting math subjects for high school. I also believe a change in the structure and the methods of teaching math at all grade levels [is needed].

Ability and "need to know."

Interest first, ability second.

Table G11 (continued)

Teacher Responses to the Diary of Professional Relationships

How should students be assigned to high school mathematics courses (i.e., interest, ability, age)?

Philadelphia

Ability should be taken into account, especially at the higher levels, so as not to bore students and at the lower levels so as not to frustrate students.

Pittsburgh

Ability and potential and interest. Interest would be third because some students might not realize that a particular course might be a part of their future.

By mathematical ability and interest.

All students who graduate from high school should be able to do mathematics well enough to lead a normal life. However, students should try to attain higher goals. High schools should assign students to courses that will challenge them.

I think they should be pretested for the level they wish to enter.

Definitely interest.

If students are college bound, they should follow the high school curriculum for math students. The undecided students should be assigned to the basic math courses.

Ability and interest grouping.

All factors, in an individual manner that considers all factors.

I would always like to offer challenging courses to students. Students that might not show the necessary skills on standardized tests might, with proper teaching and motivation, find these challenging courses gratifying and thereby add to their own self-esteem. Students that have an avid interest in the subject should be permitted in.

St. Louis

Students should be tested before assigned to a particular math course, that way if they are lacking in basic skills, we will find out before it is too late. If students cannot master the basics they should be in remedial classes, not algebra and geometry, etc.

In this order: 1) ability; 2) interest. They first must have the ability and then the interest to succeed.

Table G11 (continued)

Teacher Responses to the Diary of Professional Relationships

How should students be assigned to high school mathematics courses (i.e., interest, ability, age)?

St. Louis

I feel that students should be assigned to high school math courses by interest and/or ability. Pupils who have high academic ability must take the required coursework. They may also be able to take below-level math courses of interest as electives.

I feel students should be assigned on the basis of their ability. I do not feel interest should play a heavy factor at this early stage. When we have defined what knowledge is necessary to be mathematically literate (not just arithmetically literate) and the thinking skills necessary to be able to solve problems by drawing logical conclusions, we should have students take mathematics until they [these skills] are mastered.

Students should be assigned according to ability.

San Diego

Interest and ability.

If you do it strictly by interest, you will restrict the kids a lot. You have to push them. Ability has to play a part in the process also.

Got to be a combination of interest and ability. Without the interest it's not going to work, and without the ability the kids will get frustrated.

By abilities and then reassigned, so that it doesn't become a track system. They shouldn't be locked into grade level.

Ability and interest are the two things that should be considered.

Twin Cities

If we continue to have the large (over 25 and more likely over 30) class sizes that we have, then we are forced to continue to assign students on the basis of ability. However, if we could reduce class size (under 20), then interest could also be considered.

Definitely ability. Gear the students to the courses. Tracking is necessary so students at the upper echelon can achieve to their highest potential, while lower ability students can get the extra help they need.

Table G11 (continued)

Teacher Responses to the Diary of Professional Relationships

How should students be assigned to high school mathematics courses (i.e., interest, ability, age)?

Twin Cities

Ability--a combination of measured achievement and assessment of aptitude.
Goals--depending on what the student perceives as future needs.

By having the necessary skills to function well in a particular math course.

A. Assuming the student has the prerequisites, placement should be elective on the part of the student.
B. The state/district has the right to mandate either (or both): 1) A minimum number of math credits; 2) A minimum math competency.

Table G12

Teacher Responses to the Diary of Professional Relationships

What do you think equity in mathematics education refers to? Can schools achieve equity? If so, how?

Cleveland

All schools, irregardless of their ability to raise funds, can educate their students equally in math. Yes, greater participation of teachers of mathematics in selection of curriculum and preparation to teach that curriculum.

Providing all mathematics for all students.

Equity in mathematics means that every student has the opportunity to progress as far as he or she can in mathematics based on her or his abilities.

Equal opportunity achievement for sex and racial balance! I don't know.

I would assume it is given that schools can achieve equity, by being fair to all students. Equity is giving the same opportunity to all students.

Durham

Equal opportunity for all. Again, this person was concerned about the student who had not been identified as a "Math student" prior to entry into high school. He felt that under our present set-up true equity will be hard to achieve. He feels that genuine commitment to the standards is our best path to achieving equity IF it begins in kindergarten. [On-site observer comments].

That every student has the opportunity to go as far as he or she can. This push should begin in the lower grades so students are prepared for algebra and geometry in high school. However, there should not be a requirement that ALL students take these courses.

Equity is a term that refers to the opportunity for every student to have access to the same quality math courses without regard to socio-economic status, sex, or race. Some students need the added incentive of motivational intervention by teachers, counselors, and administration. These students who have not had required courses previously should be given the opportunity to fill in the "missing links" with creative scheduling and instruction. Schools can only achieve equity if our concerns for mathematics education are moved to the elementary grades where we need to provide leadership and assistance for the planning and implementation of a curriculum that will not eliminate certain students from math courses later.

Equity in mathematics refers to providing the opportunity to every student to achieve to his/her ability. If it is to be successful, major changes must be made in grammar schools to insist upon math-certified personnel at all levels. Also, the college training of educators needs to help teachers deal with the "real world" of high school where the majority of the students are not mathematicians.

Table G12 (continued)

Teacher Responses to the Diary of Professional Relationships

What do you think equity in mathematics education refers to? Can schools achieve equity? If so, how?

Durham

A) That all students can take courses, but they need to meet requirements first. All schools need to offer the same courses. B) As in anything, as much as they can, considering teacher availability and funds!!!

I think it means all students have a chance to take as much mathematics as they are able. I think schools can achieve equity, but students are going to have to put more effort into their studies than they are currently doing.

Equity means that all have an equal opportunity to enroll in classes and succeed. It does not mean that everyone can be a mathematician nor that everyone can play on the various athletic teams. We can achieve equity if students receive the proper teaching and encouragement from kindergarten on. High school is nearly always too late to start seeking equity.

Equity refers to quotas (female and minorities). Each student should have the same opportunity to advance themselves in math and should be encouraged to do so.

Los Angeles

All students have equal access to the math curriculum adopted by their state. With appropriate counseling and sufficient course offerings, schools provide equal access.

Students of all races, mathematics, both genders should have some opportunity to achieve/to take all levels of math. How?

Giving all students access to the math class appropriate to their ability.

Equity in mathematics education probably refers to students in any school having mathematics classes (and achieving at the same level) with any given school in the district, nation. I doubt that mathematics education can be achieved so long as students pass elementary mathematics without knowing the material and then are placed in algebra and higher math courses without having the necessary background.

Equality for all students in all math classes.

Memphis

All students should have teachers of equal math backgrounds Yes, the school can make an attempt. Teachers should teach where they are best prepared -- not where there is a hole in the schedule.

Table G12 (continued)

Teacher Responses to the Diary of Professional Relationships

What do you think equity in mathematics education refers to? Can schools achieve equity? If so, how?

Memphis

Equity in mathematics education means allowing each student the opportunity to learn mathematics and to be the best that he or she can be. Schools can achieve this equity if they assign students to courses by interest and ability rather than requiring all students, regardless of their ability, to take courses like algebra and trig.

Every student, regardless of race or gender, should be given the opportunity to continue their mathematics education on a quality level. Right now I feel students are turned off to mathematics by the time they get to high school. A greater emphasis on math in the elementary school would help this. Train teachers to "teach the why" of math as well as the "facts of math." Use concrete manipulatives, etc., to improve student understanding. Things of this nature would increase students' interest in math. Then, when they get to the high school level, teachers would have to incorporate this type of motivation into their courses in order to keep student interest. Use real problem situations to increase interest and investigation.

Equity means to ensure equality of opportunity for postsecondary education in math for all students. This can be accomplished by teaching for the transitions from high school to college. Schools must develop the Basic Academic Competencies in Math: 1) reading, 2) writing, 3) speaking, 4) reasoning, 5) studying, 6) mathematics, and 7) observing. There must exist the attitude that all students have worth and dignity and can learn.

a) Every student may be able to take the math courses they would need to pursue their career interests. b) Yes. Students should still need to meet prerequisites for certain courses. Schools could offer more applied courses such as statistics, linear algebra, accounting, and business math.

Philadelphia

I assume this refers to equal availability of all levels of mathematics courses to any student capable of success at those levels. I think this is possible under the direction of "enlightened" leadership which emphasizes a humanistic approach to dealing with <u>students as people</u>, each with a variety of abilities, needs, and goals, and each needing attention, nurturing, and respect.

I'm not sure, so I'd rather not comment.

Getting each student to develop to the best of his/her ability.

Equity means allowing all students the resources and opportunities to develop their full potential.

Table G12 (continued)

Teacher Responses to the Diary of Professional Relationships

What do you think equity in mathematics education refers to? Can schools achieve equity? If so, how?

Philadelphia

I think equity in mathematics education means that all students even those with weak computational skill, should be exposed to advanced mathematical topics and sophisticated problem-solving techniques.

I don't know if it can be completely achieved but it is worth trying!

General Math courses must be revamped and renamed. It should not be a rehash of junior high arithmetic.

Pittsburgh

All students have the right to a properly structured curriculum which would include basic math, algebra, geometry, calculus, and trig, with emphasis on word problems applying to real life situations.

Definitely! We are trying to achieve that equity by having more communication among all department members.

Think all math classes are taught at the same intensity level. No.

We can't have equality at the high school level without the help of the elementary and middle schools. If we are going to improve the Math performance of students in the U.S., we must first have equal opportunities at the early ages. I'm not sure this opportunity doesn't already exist for the student who is motivated. The key is to motivate students early in their education and then to maintain that interest in the middle and high schools.

The question is not clear. The term equity is vague in this statement.

That all students receive the same quality of education. Absolutely. Allow the teacher the freedom to deal with each student as an individual.

That could refer to ability of male or female, or black, or white. I don't know if they can achieve, but our goals are to try to [help them]. Subject classes consist of 50/50, female/male, and black/white ratios.

Define your terms.

Bridging the various gaps such as gender, social, and economical. Only by bringing down the ones that are higher.

Table G12 (continued)

Teacher Responses to the Diary of Professional Relationships

What do you think equity in mathematics education refers to? Can Schools achieve equity? If so, how?

Pittsburgh

I truly feel we have equity in mathematics. It is not prejudiced. A good math student is appreciated by the math teacher regardless of sex, age, or race.

St. Louis

Rules for mathematics education standards. By requiring all students to attend testing before placing students in courses. Then offering the same math courses, using the same textbook and teaching the same concepts.

Equity in mathematics education means that all children regardless of sex, race, or economics have the opportunity to not only learn math concepts, but be able to apply these concepts within their daily lives.

Equity in mathematics education means if pupils have completed a particular math curriculum, the results for individuals, regardless of ethnic background or gender, should be the same. Yes, equity in education can be achieved. It can be achieved by ending the racist and sexist climate that exists in this country. Until there is equal opportunity for all in the lower socio-economic structure and females, there will not be equity. Why strive if you lack hope of being hired. A positive change in climate, with equal distribution of funds for education, will help.

An equal opportunity to acquire the basic mathematical knowledge needed and the thinking skills needed to produce in our society. Also, the opportunities must be challenging so that a student's ability is stretched as far as possible. This does not mean that all students must take the same courses at the same time.

Some ways to have equity:

1) A more accurate means of discerning an individual's deficiency in mathematics and the concepts, facts or thinking skills needed.

2) Making sure all students are aware of the knowledge needed and the types of thinking skills needed. If students have a kind of check list of what is needed and where these are taught or practiced, they may be more conducive to remaining in school.

Opportunities to participate in and be exposed to current trends and methodology in mathematics education without regards to financial constraints; all students should have the opportunity to utilize appropriate software as supplementary tools to instruction. All students should have teachers that have the benefit of district-sponsored seminars and inservice programs.

Table G12 (continued)

Teacher Responses to the Diary of Professional Relationships

What do you think equity in mathematics education refers to? Can schools achieve equity? If so, how?

San Diego

Absolutely no meaning to me. Vague term. Equal opportunity across gender and ethnic groups.

It means allowing every kid to find his/her own niche. You can't make carbon copies of them and force them. You have to provide them with the basics.

That's a tough one. The equity issue has to go back to the home situation. Backing from the home makes the difference. It's a thing where we need to educate parents. Parental education. Mexican American families tell the kids they're adult at age 16 and they drop out.

Every student should be afforded the opportunities to go as far as he/she wants to. . . . They can try. I'm not sure if they're ever going to succeed.

All students should be exposed to at least algebra and not just stay with general math.

Twin Cities

To encourage girls and minorities to the values of mathematics in today's society. We can no longer depend on the white males to supply our needs. How? Help them see how they fit into the picture.

Equity refers to equality among races and genders in the classroom. There must be equal representation in the classroom that is the same as the minority population in the community. Students must also be recognized equally. Curriculum must reflect recognition of students of color.

Equity means each student feels math is accessible and achievable. Each student feels any occupation of interest is available and appropriate--and so is the math preparation. Schools must strive to achieve equity by applying much of the research and proven methods on this topic!

Equity in terms of race and gender. I believe it is possible that schools can achieve equity; however, schools cannot do it alone.

Each student's access to mathematics is limited only by his/her ability. Not alone. To achieve equity a number of our present students must receive additional help. This is costly. Neither our society nor my school district is willing to bear this cost. In fact, this year, my district has eliminated previously available assistance to needy students of math!
