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ABSTRACT

The comprehension processes of adult students solving two-step problems of comparison were studied using eye-movement experiments based on the assumption that eye fixations are synchronous with internal cognitive processes. Twenty university students each solved 24 two-step mathematical word problems of three sentences each with consistent language (CL) or inconsistent language (IL) structures. In CL problems, the unknown variable was the subject of the second sentence; in IL, the unknown was the object of the relational sentence. Raw eye movement data were transformed into consecutive fixations. Overall results were generally in line with the model of A. Lewis and R. Mayer (1987) (LM model) suggesting the increased difficulty of IL problems as indicated by increased time required to process the sentences and the number of errors of reversal of correct mathematical operation. Apparently, just as it is the case for the performance variable error rate, the effect of inconsistent language on aspects of the solution process (such as total solution time, initial reading time, and fixation time on the relational clause) is exhibited only when the compare problems have to be processed and solved under rather heavy cognitive demands. Some theoretical conceptions are developed as alternatives to parts of the LM model. One table and one bar graph supplement the discussion. (SLD)

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Objectives and theoretical background

At the 1990 AERA Conference, we presented two eye-movement experiments (De Corte, Verschaffel & Pauwels, 1990), that tested Lewis and Mayer's (1987) model of comprehension processes when solving compare problems. This theoretical model focusses on two forms of compare problems, namely consistent and inconsistent language problems. In a consistent language problem (e.g., "Joe has 5 marbles. Tom has 3 more marbles than Joe. How many marbles does Tom have?") the unknown variable is the subject of the second sentence, and the relational term (in this case "more than") is consistent with the required arithmetic operation (namely addition). In an inconsistent language problem (e.g., "Joe has 8 marbles. He has 5 marbles less than Tom. How many marbles does Tom have?") the unknown variable is the object of the second sentence, and the relational term ("less than") is in conflict with the arithmetic operation (namely addition). Most empirical studies on word problem solving revealed that inconsistent language (IL) problems are more difficult than consistent language (CL) problems. To explain this finding, Lewis and Mayer (1987) put forward the <u>consistency hypothesis</u>: Problem solvers would have a preference for the order in which the relational information is presented. In particular, they would prefer the order of consistent language problems, in which the unknown variable is the subject of the second sentence. When given an inconsistent language problem, in which the unknown variable is the object of the relational sentence, pupils are assumed to mentally rearrange the relational sentence until it fits the preferred format.

This rearrangement procedure consists of reversing the subject and the object of the relational sentence, as well as the arithmetic operation suggested by its relational term. Because the comprehension and solution process may be more error prone when information must be rearranged, the probability of a reversal error (subtraction instead of addition, or the reverse) will be greater for IL than for CL problems.

In an attempt to test their hypothesis, Lewis and Mayer (1987) asked a group of adult students to represent and to solve two-step compare problems. The results were in line with their prediction: The subjects were more likely to miscomprehend the relational statement, and to commit a reversal error when the unknown variable was the object of the relational sentence than when it was the subject. Moreover, they found that the students made more reversal errors on IL problems when the necessary operation was addition or multiplication rather than subtraction or division. This interaction was explained by the presence of marked terms (e.g., "less") in inconsistent language addition and multiplication problems. Since marked terms are more salient than unmarked terms (e.g., "more"), the subjects would be more likely to resist reversing them.

In order to provide a more straightforward test of the hypotheses concerning the internal processes involved in this model, we carried out two eye-movement experiments in which adult students and third-graders were administered one-step addition and subtraction compare problems. The use of eye-movement registration in problem-solving research is based on the general assumption that internal, cognitive processes are synchronous with eye fixations (Just & Carpenter, 1987). In the context of the present studies, this means that the fixation time spent on a particular sentence of a word problem reflects the time needed to process that clause.

While the data of the first experiment with adult subjects revealed no evidence in favor of the consistency hypothesis, the results of the second study with third-graders provided good support of the Lewis and Mayer (1987) model. The most plausible explanation for this contradiction is that one-step addition and subtraction problems are too easy problems for adults. Probably, Lewis and Mayer's model only reflects the solution processes of subjects for whom the compare problems that are administered have a certain level of difficulty. Therefore, we decided to collect eye-movements of university students while solving more difficult two-step compare problems.

Hypotheses

The hypotheses that we derived from the LM-model concern (a) the number of reversal errors, (b) response times, and (c) fixation times on the different sentences in a problem.

First of all, the LM-model predicts that problem solvers will make more reversal errors on IL than on CL problems. Furthermore, an interaction effect between language consistency and the operation to perform was predicted: More reversal errors would occur on IL problems involving addition or multiplication (= problems containing a marked term) than on IL problems involving subtraction or division (= problems containing an unmarked term); this difference would not exist for CL problems.

With respect to the response times the following hypothesis was stated. If students indeed have to reorganize the relational sentence in an IL problem to make it similar to their preferred format, this rearrangement will necessarily lead to greater response times for IL that for CL problems.

The LM-model involves that the difference in the solution process between CL and IL problems concerns the processing of the second, relational sentence: In the case of IL problems this sentence has to be reorganized, in the case of CL problems not. Assuming that there is a close link between what is fixated and what is internally processed, we predicted that the longer response times for the IL problems would be due to longer fixation times on the second, relational sentence and that the fixation times on the other two sentences would not differ significantly for both kinds of problems.

According to LM-model the rearrangement subprocedure is already initiated immediately after the first reading of the second sentence, i.e., before the subject turns to the third sentence in the problem. This implies that the above-mentioned difference in the response time and the fixation time between CL and IL problems will already have shown up during the initial reading of the first and the second sentence. In the present article, we call this initial reading of the first and the second sentence - which constitutes the core of the LM model - the first phase of the word problem solving process.

Method

Subjects

Twenty university students participated in the experiment.

Task

Each subject was given 24 two-step compare problems. The first part of the problem had a compare structure, the second part was a direct variation (e.g., the price of a particular amount of pounds or litres, given the price of one pound or litre). Half of the problems had a CL compare structure, half of them an IL compare structure. Within both categories there were 3 addition and 3 multiplication problems (we will refer to these problems as increase problems) and 3 subtraction and 3 division problems (we will refer to these problems as decrease problems). So there were eight different types of problems. All problems consisted of three sentences, which were controlled for the number of characters. Moreover, the same cover stories were used in the different problems types. Because the available eye-movement-registration equipment requires that the subject's head remains relatively stable during the whole solution process, we had to work with particular number triples, so that the correct answer could be easily obtained by mental computation. Examples of the problems are given in Table 1.

Insert Table 1 about here

Procedure

All subjects were tested individually. They were asked to give the correct answer for each problem, and they were allowed to do this at their own pace. The word problems were presented on a television screen placed 2 meters form the subjects. While they were reading and solving the problems, their eye-movements were registered with DEBIC 80, a system that is based on the pupil center-corneal reflection method. Every 20 milliseconds the spatial coordinates of the subject's point of regard, and a time code are stored on an on-line PDP-11 computer. Furthermore, a videorecording of the subsequent points of regard on the displayed problems is also available. These points are represented as the intersection of a vertical and a horizontal axis superimposed on the problem. (For a detailed description on the eye-tracking system, see De Graef, Van Rensbergen, and d'Ydewalle, 1985).

The raw eye-movement data were transformed into consecutive fixations. A fixation was operationally defined as a time period of a minimum of 100 milliseconds during which the eye is close to immobile at a particular location (Rayner, 1978). Starting from those data, the absolute fixation times on the three different problem sentences were calculated. These variables, as well as the response times, were subjected to an ANOVA with language consistency (consistent versus inconsistent) as the independent variable. The number of reversal errors was subjected to an ANOVA with language consistency and the direction of change (increases or decreases) as independent variables (2 x 2 repeated measures factorial design).

We point to the fact that response time is not the sum of the fixations on the three different sentences of the problem. Indeed, response time includes also the saccades (the time intervals between two successive fixations), the so-called "missings" (short periods during which there is no eye-movement registration, e.g., because the subject blinks eyes or looks away from the television screen), and all fixations above, under and besides the three areas on the screen which contain a problem sentence.

To test the hypotheses concerning the first phase of the solution process, we tried to decompose each eye-movement protocol in two phases. The <u>first phase</u> consists of the initial reading of the first two problem sentences; it starts from the presentation of the problem on the screen, includes all the fixations on the first two sentences, and ends just before the first fixation on the third sentence. The <u>second phase</u> starts from that first fixation on the third sentence and ends at the moment the answer is given. Eye-movement protocols which did not show a series of (at least two) subsequent fixations on the first <u>and</u> on the second sentence before the reading of the third sentence began, were excluded from this analysis.

Results

Number of reversal errors

The results of the analysis on the number of reversal errors were completely in line with those obtained by Lewis and Mayer (1987). The total number of reversal errors for the different types of problems is summarized in Figure 1.

Insert Figure 1 about here

As predicted, the ANOVA revealed a significant effect of language consistency, $\underline{F}(1,19) = 8.39$, $MS_e = 0.29$, $\underline{p} < 0.01$, in which IL problems produced more reversal errors that CL problems. Furthermore, the analysis revealed a significant main operation effect, $\underline{F}(1,19) = 8.64$ $MS_e = 0.14$, $\underline{p} < 0.01$: increases produced more reversal errors than decreases. Finally, a significant interaction between both independent variables was found, $\underline{F}(1,19) = 15.55$, $MS_e = 0.26$, $\underline{p} < 0.01$: The difficulty of overcoming inconsisent language was enhanced when an increase was also present.

Total solution process

Duration. Based on the LM-model, we predicted that the students would need more time to solve the IL problems than to solve the CL ones. The results of the ANOVA were in line with this prediction. We found a significant main effect of language consistency, $\underline{F}(1,19) = 5.73$, $MS_e = 32.6$, $\underline{p} < 0.05$: IL problems elicited longer response times than CL problems (an average of 12.25 s to 11.01 s).

Fixation time on the different sentences. According to the model, the longer response times for IL problems are especially due to the reorganization of the relational sentence. Therefore, we predicted longer fixation times only on the second, relational sentence of the IL problems.

The ANOVA of the second sentence indeed revealed a significant effect of language consistency, $\underline{F}(1,19) = 5.59$, $MS_e = 3993755.1$, $\underline{p} < 0.05$: The students spent an average of 2476 msec on the relational sentence in an IL problem, and only 2044 msec on that sentence in a CL problem. The fixation times on the first sentence (1,865 msec for CL and 2,074 msec for IL, $\underline{F}(1,19) = 2.34$, $MS_e = 2{,}245{,}041$, $\underline{p} > 0.05$) and on the third sentence (1,286 msec for CL and 1,379 msec for IL, $\underline{F}(1,19) = 2.54$, $MS_e = 413{,}707$, $\underline{p} > 0.05$) did not differ significantly for both kinds of problem.

First phase of the solution process

The LM-model assumes that the rearrangement of the relational sentence in an IL problem takes place before the subject reads the third sentence of the problem. In view of testing this hypothesis, we tried to decompose the eye-movement protocols into two different phases. We decided to include in this analysis only those eye-movement protocols which showed several subsequent fixations on the first <u>and</u> on the second sentence before the eye moved to the third sentence. A significant number of eye-movement protocols did not meet this criterion, namely 80 out of 480. In a number of cases the eye-movement pattern was so chaotic that there were no successive fixations neither on the first nor on the second sentence; in other cases, there were several fixations on the third sentence before the first and/or the second one was being fixated. The 400 protocols that did meet the creterion were equally divided among the CL and the IL problems: 198 for the CL problems and 202 for the IL problems.

<u>Duration of the first phase</u>. In accordance with the prediction, we found a main effect of language consistency, F (1,19) = 4.71, MS_e = 15.6, $p < 0.05$. The initial phase lasted significantly longer for IL than for CL problems (8.37 s compared to 7.61 s).

<u>Fixation time on the different sentences during the first phase</u>. In line with the LM-model, we expected that during the initial stage of the solution process, the effect of language consistency on the absolute fixation times would appear only on the second, and not on the first sentence. Mean absolute fixation time on the first sentence was 1,800 msec for CL and 1,960 msec for IL problems, F (1,19) = 1.48, MS_e = 90,592, $p > 0.05$. For the second sentence the means were 1,842 msec and 2,100 msec respectively, F (1,19) = 3.43, MS_e = 293,291, $p > 0.05$. While the difference between the means was somewhat larger for the IL than for the CL problems, the effect of language consistency on the absolute fixation time on both sentences failed to reach significance (for the second sentence: $p < 0.08$).

Discussion

Taking into account the results of the present experiment, it seems reasonable to conclude that the lack of confirmation of the LM-model observed in our earlier study with adults was mainly due to the extremely low difficulty level of the problems for the subjects. Indeed, in the second experiment with third graders and in the present investigation, in which problems were substantially more difficult to the subjects than in the first study, the results were very well in line with the LM-model and with the performance data collected by Lewis and Mayer (1987) themselves. Apparently, just as it is the case for the <u>performance</u> variable 'error rate", the effect of inconsistent language on such aspects of the solution <u>process</u> as "total solution time", "initial reading time" and "fixation time on the relational clause", only shows up when the compare problems have to be processed and solved under rather heavy cognitive demands.

While the overall results of our eye-movement studies can be interpreted as additional empirical evidence for the LM-model, further theoretical and empirical work is certainly warranted.

From a theoretical point of view, there is a strong need to analyse the relationship between Lewis and Mayer's conception and operationalisation of the CL/IL distinction for compare problems on the one hand, and several more general theoretical notions on the other. In order to initiate a critical discussion of the LM-analysis and to provide a perspective for further empirical work, we will briefly present these alternative theoretical conceptions.

First, there is Van Dijk and Kintsch's (1983) idea of text coherence. According to this notion, interpretation and integration of new textual information about an agent is facilitated when this new information starts with the same agent. This idea leads to predictions that are incongruent with the LM-model. For example, one could predict that subjects will have less difficulty in reading and understanding the first two sentences in a IL problem (where the relational sentence starts with a different agent). The results of the present study and of our earlier experiment with third graders contradict this alternative prediction, but those of our first investigation with adults are in line with it. We remind that in the latter study, university students needed not less but significantly more time to read the first two sentences of CL problems than of IL problems; this outcome is in line with the prediction derived from the theory of Van Dijk and Kintsch (1983).

Related to Van Dijk and Kintsch's (1983) idea of text coherence is Reusser's (1989) notion of the narrative focus of a word problem, which refers to the perspective from which a word problem is stated. Reusser (1989) distinguishes between the narrative focus of the story episode (i.e., the first part of a word problem containing the givens) and of the final question. We restrict ourselves to the first aspect. The narrative focus of a story episode can be either protagonist-related or coactor-related. It is protagonist-related if the same person or thing in every partial action or state is in topic position, i.e., serves as the grammatical subject of all sentences of the story episode (e.g., "Yesterday Rudy got 8 marbles from Daniel; today Rudy got some marbles from Hannah; now Rudy has 11 marbles"). The narrative focus is called coactor-related if different persons or things serve as the grammatical subject of the respective sentences of the story episode (e.g., "Yesterday Daniel gave 8 marbles to Rudy; today Hannah gave some marbles to Rudy; now Rudy has 11 marbles"). According to Reusser (1989), story episodes with a protagonist-related narrative focus are easier than episodes with a coactor-related narrative focus. Applied to our compare problems, this would imply again that the first two sentences of an IL problem (which are protagonist-related) should be easier to understand than those of a CL problem (which are coactor related). The unexpected results of our first experiment with adults are in line with this alternative prediction.

Besides these two theoretical notions involving predictions that are incongruent with the LM-model, there are also two other interpretative concepts which lead to predictions that are in accordance with it, and can therefore be considered as alternative theoretical accounts for the positive results obtained in the present study and in the previous experiment with third graders.

First there is the concept of pronominal reference. A closer look at the word problems used by Lewis and Mayer (1987) and in our studies, reveals that only the IL problems contain pronouns. It is generally acknowledged that resolving the problem of pronominal reference may cause special difficulties, especially for young, inexperienced readers. So, one could argue that the systematic difference in the use of pronouns between IL and CL problems may have contributed to the positive results, especially in the experiment with third graders.

Second, a basic assumption underlying the LM-model is that reversal errors on IL compare problems are due to difficulties in understanding and representing the relational information in the problem statement. However, reversal errors on word problems may also originate differently. Take, for example, the so-called key-word strategy. Subjects applying this strategy do not <u>try</u> to understand and represent the problem statement; they simply look for key words, i.e., words with which a particular arithmetic operation is associated. For example, the words "altogether" and "more" are associated with addition; the words "lost" and "less" with subtraction (Nesher & Teubal, 1975). For word problems containing a key word that is associated with the correct operation, the key-word strategy yields success, but for those having a key word that operates as a distractor, this strategy results in reversal errors. In the context of the present studies, applying a key-word strategy would lead to correct answers on all CL problems, but would produce reversal errors on all IL problems. Therefore, one could argue that the differences in success rate between CL and IL problems observed in the present investigations and in the study with third graders, may have been at least partially due to the application of this strategy by some subjects.

Taking all this into account, it seems necessary to critically reconsider the LM-model. However, this theoretical re-analysis should be accompanied by further empirical research aiming at elucidating the relevance of the alternative concepts mentioned above. In this respect, investigators should pay special attention at systematically controlling and/or varying all possibly relevant aspects of the statement of compare problems, such as the presence of pronouns, the order of presentation of the subject and the object in the relational statement, the place of the question sentence, etc. But investigators should also try to complement the kind of data used so far (i.e., solution accuracy, solution time, and fixation times on different parts of the problem statement) with other kinds of empirical findings. In this respect, we think not only of other types of eye-movement data (such as the amount of so-called "regressions" within and between problem sentences as an indication of comprehension difficulty), but also of verbal data such as thinking-aloud protocols, retrospective reports, or retelling data (see e.g., De Corte & Verschaffel, 1987).

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Table 1

Examples of the two-step Compare problems presented in Experiment 3

Operation	Problems
	<u>Consistent Language</u>
Addition	At Aldi a pound of butter costs 80 Bfr At GB butter is 20 Bfr. more per pound than at Aldi How much do 5 pounds of butter cost at GB?
Subtraction	At GB a litre of milk costs 25 Bfr. At Aldi milk is 25 Bfr. less per litre than at GB. How much do 10 litre of milk cost at Aldi?
Multiplication	Nopri sells 150 eggs a day. GB sells 3 times as many eggs as Nopri. How many eggs does GB sell on 2 days?
Division	Nopri sells 50 pounds of tomatoes a day. GB sells 1/5 as many tomatoes as Nopri. How many pounds of tomatoes does GB sell on 4 days?
	<u>Inconsistent Language</u>
Addition	At Nopri a pound of pears costs 65 Bfr. That is 15 Bfr. less per pound than at Colruyt How much do 10 pounds of pears cost at Colruyt?
Subtraction	At Colruyt a pound of sugar costs 40 Bfr. That is 10 Bfr. more per pound than at Delhaize. How much do 5 pounds of sugar cost at Delhaize?
Multiplication	Delhaize sells 50 newspapers a day. That is 1/5 as many newspapers at GB sells. How many newspapers does GB sell on 4 days?
Division	Aldi sells 120 litre of water a day. That is 2 times as many litres as Nopri sells. How many litres of water does Nopri sell on 5 days?

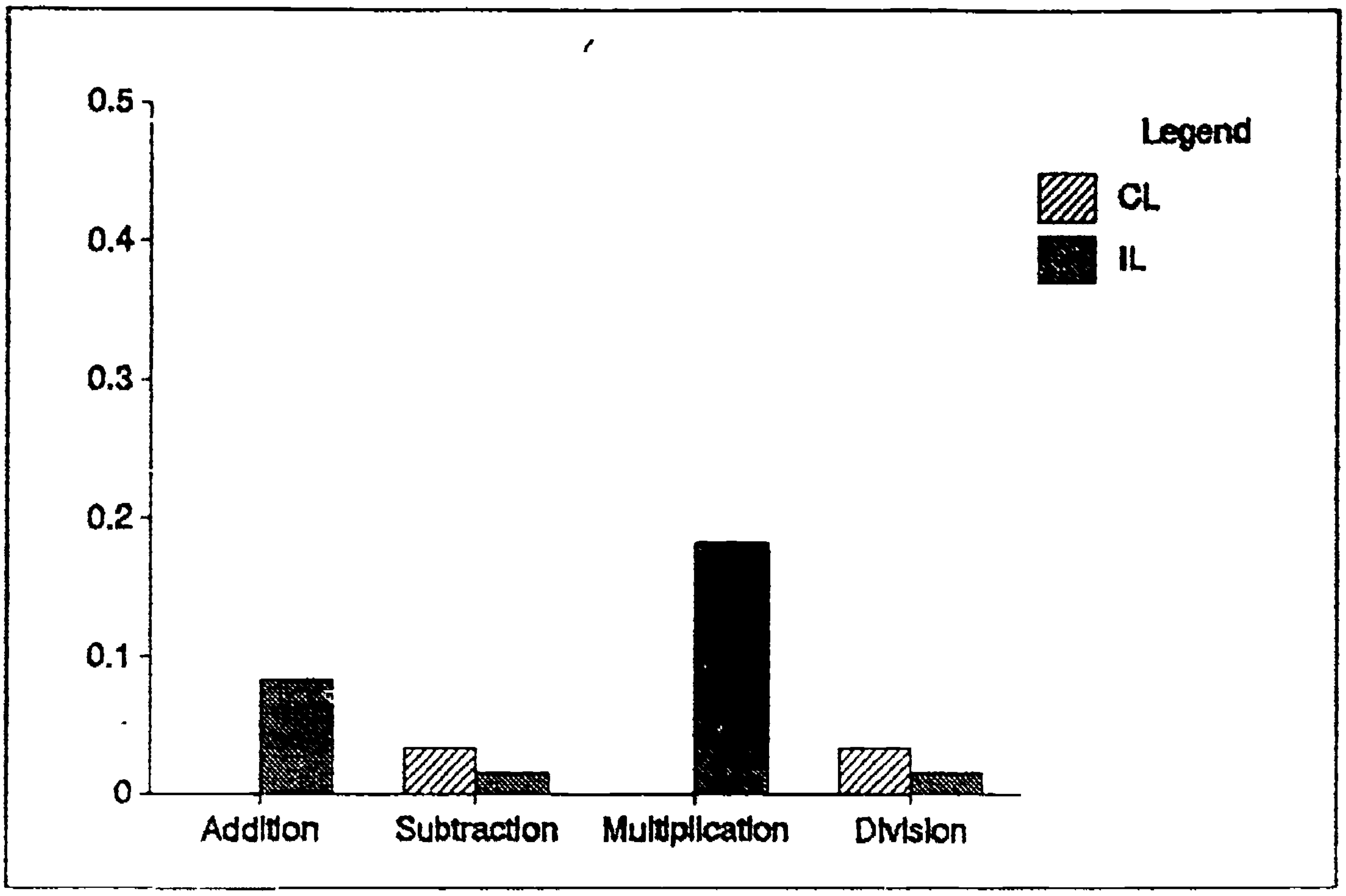

Figure 1. The proportion of reversal errors on CL and IL problems for each of the 4 operations