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AUTHOR	Elliott, Ron; Barcikowski, Robert
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ABSTRACT

Different ways that dependent variables can cause multivariate significance are illustrated. It is argued that univariate tests should not be used as follow-up procedures to identify the variables that can contribute to multivariate significance. Methods that can be used to determine the contributions individual variables make to multivariate significance are discussed. Two-group multivariate analysis of variance (MANOVA) with two dependent variables is described; a significant multivariate test under three conditions of univariate significance is considered: (1) with both dependent variables significant; (2) with one dependent variable significant; and (3) with both dependent variables not significant. The relationships among the variables in these conditions yield 11 different situations. The data illustrate: multivariate significance can be caused under a variety of relationships; it is improper to interpret univariate analysis of variance results to find the causes of multivariate significance. The use of discriminant analysis and stepwise discriminant analysis is demonstrated as a valuable tool in considering the contributions of dependent variables to multivariate significance. Three tables illustrate the 11 situations, and Appendix A details the process used to create each situation. (SLD)

Causes Of Multivariate Significance In A Multivariate Analysis Of Variance

Ron Elliott
and
Robert Barcikowski

Ohio University

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Causes Of Multivariate Significance In A Multivariate Analysis Of Variance

Objectives

The purpose of this paper is threefold: first, to illustrate the different ways that dependent variables can cause multivariate significance; second, to reinforce the position that univariate tests should ***not*** be used as follow-up procedures to identify the variables that contribute to multivariate significance; third to discuss methods that can be used to discern the contributions that individual variables make to multivariate significance.

Perspectives

Many of the most popular multivariate analysis of variance textbooks in the social sciences indicate that the importance of a variable to multivariate significance can be found by selecting the dependent variables that have significant univariate tests. Quotes from a few of these books are given below.

> *The **DVs** [dependent variables] that have significant univariate F's are the important ones, and they can be ranked in importance by strength of association (Tabachnick & Fidell, 1989, p. 399).*
>
> *Wilks' Λ is significant at .05 ($F = 4.361$, $p < .000$), indicating that the three groups differ on the set of 4 variables. The univariate F's are all significant at the .05 level, showing that all of the variables contributed to the multivariate significance (Stevens, 1986, p. 160).*
>
> *When the hypothesis of no difference is rejected, it is often informative to examine the univariate test results to get some idea of where the differences may be (Norusis, 1985, p. 207).*

We have found that students and faculty alike will look at the univariate tests that follow the overall multivariate test in a multivariate analysis of variance (MANOVA) to find what *caused* the multivariate significance. Although we feel that the above authors are well aware of the problems associated with interpreting multivariate significance on the basis of univariate significance, statements like those above encourage researchers to make improper interpretations of multivariate significance.

The focus of this paper is on the circumstances under which a set of dependent variables cause a given multivariate test to be significant. In the process of examining these circumstances it is obvious that the preceding quotations are not necessarily true. Follow-up procedures that can help educational researchers identify the causes of multivariate significance are suggested.

Methods

Part 1: Two Group MANOVA

The basic process followed was to initially consider the two group MANOVA with two dependent variables and a significant multivariate test under three conditions of univariate significance: both dependent variables significant (condition 1), one dependent variable significant (condition 2), both dependent variables not significant (condition 3). Within each condition different relationships among the dependent variables and between the dependent and independent variables were considered. This yielded a total of 11 different situations that are discussed and illustrated.

The formulas used are presented in Appendix A. Although there are other situations that could have been presented, we feel that the 11 we selected provide excellent support for our arguments.

The analyses in this part were performed using SPSSX (SPSS Inc., 1988) using summary (matrix) input and the programs MANOVA and REGRESSION. The follow-up procedure considered was discriminant analysis. (Step-Down analysis was also considered as part of the output, but was not as informative as the discriminant analysis.) The results of the analyses are given in Appendix B which is available from the first author.

Definitions

What do researchers mean when they say that a dependent variable *caused* multivariate significance? We are not sure, but the following definitions are what we mean by the terms *caused* and *contributed to* multivariate significance.

> A dependent variable is said to have *caused* a multivariate test to be significant when the part of the variable left after the effects of the other dependent variables in the analysis have been partialed from it would by itself yield a significant multivariate test.
>
> A dependent variable is said to have *contributed to* the significance of a multivariate test when the part of the variable left after the effects of the other dependent variables in the analysis have been partialed from it is determined, on the basis of a rationale provided by the researcher, to be important in identifying group differences.

Part 2: Factorial MANOVA

In Part 2 the material developed in Part 1 was generalized to two groups and three dependent variables and to multiple groups and multiple dependent variables.

Part 1 Results: Two Group MANOVA With Two Dependent Variables

For all of the conditions for the two group MANOVA with two dependent variables we found that the canonical discriminant analysis results (Barcikowski, 1983, p. 47; Green, 1979), and not the univariate tests alone, clearly illustrated the contributions of the dependent variables to multivariate significance. Our criterion was the relative sizes of the standardized discriminant function coefficients. We found that the relative sizes of the standardized discriminant function coefficients allowed us to determine the relative contributions of the dependent variables to multivariate significance. We also found that the discriminant function variable correlations (i.e., the discriminant structure coefficients) mirrored the univariate test results.

Condition 1: Multivariate Significance And Both Univariate Tests Not Significant

If one is to interpret the causes of multivariate significance by considering the significant univariate tests (as is suggested by the textbook authors at the beginning of this paper), what does one do when the multivariate test is significant but both of the univariate tests are not significant? As it turns out, this case is the easiest to interpret because with only two dependent variables multivariate significance must have been caused by contributions of *both* dependent variables *in some way*. We can say this because if only one variable was causing multivariate significance, that variable would have to be significant at the univariate level.

Table 1 illustrates four situations for condition 1 (multivariate significance and both univariate tests not significant). In the **first situation** both dependent variables were uncorrelated and both dependent variables were related to the independent variable. In this situation both variables independently "gang up" to cause multivariate significance.

In the **second situation** the dependent variables were correlated. Here, the first dependent variable was correlated with the independent variable and the second dependent variable was not correlated with the independent variable. However, the first dependent variable acted as a suppressor variable to the second dependent variable in the relationship between the second dependent variable and the independent variable. In the example for this situation the second variable shows a *p*-value of 1.0, but when the first variable was partialed from the second variable what was left of the second variable "ganged up" with the first variable to cause multivariate significance.

The **third and fourth situations** were variations of the first two situations. In the third and fourth situations both dependent variables were correlated, and both

Table 1
Description of the Correlations Among the Variables and of Example Discriminant Analysis Output for Condition 1: Overall Multivariate Significance ($p < .05$) And Both Univariate t Tests Not Significant ($p > .05$)

Relationship Between The Dependent Variables (1 & 2)	Relationship Between The Independent Variable (X) And The Dependent Variables (1 & 2)	Explanation	Example Discriminant Function Coefficients Variable	Coefficient[1]
		Situation 1		
$\rho_{12} = 0.0$	$\rho_{x1} \neq 0.0$ and $\rho_{x2} \neq 0.0$	Both dependent variables are related to essentially independent parts of the independent variable	1	.708
($r_{12} = 0.00$)[2]	($r_{x1} = .255$; $r_{x2} = .254$)		2	.706
		Situation 2		
$\rho_{12} \neq 0.0$	$\rho_{x1} \neq 0.0$ and $\rho_{x2} \approx 0.0$	One dependent variable, 1, is arbitrarily chosen as a suppressor variable. When 1 is partialed from 2 the relationship between 2 and x is increased.	1	1.412
($r_{12} = .706$)	($r_{x1} = .255$; $r_{x2} = 0.00$)		2	-.997
		Situation 3		
$\rho_{12} \neq 0.0$	$\rho_{x1} \neq 0.0$ and $\rho_{x2} \neq 0.0$	Both dependent variables contribute to multivariate significance.	1	.651
($r_{12} = .100$)	($r_{x1} = .260$; $r_{x2} = .274$)		2	.697
		Situation 4		
$\rho_{12} \neq 0.0$	$\rho_{x1} \neq 0.0$ and $\rho_{x2} \neq 0.0$	Both dependent variables contribute to multivariate significance, but 1 also acts as a suppressor in the relationship between 2 and x.	1	.730
($r_{12} = -.100$)	($r_{x1} = .250$; $r_{x2} = .262$)		2	.760

[1] Standardized canonical discriminant function coefficients.

[2] Example values used to illustrate a given situation and to create the discriminant function coefficients are given in parentheses.

dependent variables were related to the independent variable. In the third situation both variables "ganged up" to cause the multivariate significance. In the fourth situation both dependent variables contributed to multivariate significance, but the first dependent variable also acted as a suppressor variable in the relationship between the second dependent variable and the independent variable. Therefore, in the fourth situation the second dependent variable contributed to the overall multivariate significance in a stronger way than if the first dependent variable was not present.

Discriminant Analyses: Standardized Discriminant Function Coefficients. For all of the examples used to illustrate the preceding situations the discriminant function coefficients were relatively large (e.g., greater than .65) for both dependent variables. These coefficients are partial coefficients in that they are derived so that the contribution of the other dependent variable is partialed (removed) in their formation. In this way the discriminant function coefficients take into consideration the relationships both between the dependent variables and between the dependent variables and the independent variable. Therefore, when these coefficients are both large this indicates that both dependent variables contributed to the overall multivariate significance.

Discriminant Analyses: Discriminant Function Variable Correlations. In the examples used for this condition the discriminant function variable correlations conveyed information similar to that provided by the univariate tests. *In general the smaller the p-value for a univariate test the larger is the corresponding discriminant function variable correlation.* For example, in the data analyses for situations 1 and 3 the univariate tests had relatively low *p*-values (e.g., $.054 \leq p \leq .075$) and the discriminant function variable correlations were relatively large (e.g., $.706 \leq r \leq .762$). In situation 2 the univariate test for the first dependent variable yielded a low *p*-value ($p < .074$), but the univariate test for the second dependent variable yielded a large *p*-value ($p < 1.0$). These univariate results were conveyed by the discriminant function variable correlations in situation 2 where the discriminant function variable correlation was relatively large (.708) for the first dependent variable and was zero for the second dependent variable.

Condition 2: Multivariate Significance And One Univariate Test Significant And One Univariate Test Not Significant

The four situations within this condition are illustrated in Table 2. According to our textbook authors, in this condition we would attribute the significance of the multivariate test to the single variable whose univariate test is significant. Of the four situations we chose to illustrate this condition, in only one of them (situation 5) would the textbook authors be correct according to our definition of *caused*.

From our examples you can see that a basic question is: "When does the contribution of the nonsignificant dependent variable to overall multivariate significance become an *important* contribution?". (Perhaps this question could be answered by comparing the correct classifications of the discriminant function based on only the significant dependent variable with the classifications found

Table 2

Description of the Correlations Among the Variables and of Example Discriminant Analysis Output for Condition 2: Overall Multivariate Significance ($p < .05$), First Univariate t Test Significant ($p < .05$) And Second Univariate t Test Not Significant ($p > .05$)

Relationship Between The Dependent Variables (1 & 2)	Relationship Between The Independent Variable (X) And The Dependent Variables (1 & 2)	Explanation	Example Discriminant Function Coefficients: Variable	Coefficient[1]
	Situation 5			
$\rho_{12} \approx 0.0$ ($r_{12} = 0.00$)[2]	$\rho_{x1} \neq 0.0$ and $\rho_{x2} \approx 0.0$ ($r_{x1} = .350$; $r_{x2} = 0.00$)	The first dependent variable is responsible for multivariate significance.	1 2	1.000 .000
	Situation 6			
$\rho_{12} \approx 0.0$ ($r_{12} = 0.00$)	$\rho_{x1} \neq 0.0$ and $\rho_{x2} \neq 0.0$ ($r_{x1} = .300$; $r_{x2} = .199$)	Both variables contribute to multivariate significance but variable 1 contributes more than variable 2.	1 2	.840 .543
	Situation 7			
$\rho_{12} \neq 0.0$ ($r_{12} = .600$)	$\rho_{x1} \neq 0.0$ and $\rho_{x2} \approx 0.0$ ($r_{x1} = .290$; $r_{x2} = 0.00$)	Both dependent variables contribute to multivariate significance. Variable 1 acts as a suppressor in the relationship between 2 and x.	1 2	1.250 -.750
	Situation 8			
$\rho_{12} \neq 0.0$ ($r_{12} = .250$)	$\rho_{x1} \neq 0.0$ and $\rho_{x2} \neq 0.0$ ($r_{x1} = .300$; $r_{x2} = .268$)	Both dependent variables contribute to multivariate significance.	1 2	.695 .566

[1] Standardized canonical discriminant function coefficients.

[2] Example values used to illustrate a given situation and to create the discriminant function coefficients are given in parentheses.

using both dependent variables.)

In **situation 5** the dependent variables were uncorrelated and only the first dependent variable was related to the independent variable, the second dependent variable was not. This was the only situation within condition 2 where one could clearly decide on the basis of the univariate tests that the first dependent variable was causing the multivariate significance. This same information was conveyed by the discriminant function coefficients and the discriminant function variable correlations. Indeed in this case the discriminant function coefficients and the discriminant function variable correlations were equal. Both the discriminant function coefficient and the discriminant function variable correlation were 1.00 for the first dependent variable and 0.00 for the second dependent variable.

Situation 6 is similar to situation 5 in that the two dependent variables are uncorrelated, however in situation 6 both dependent variables were related to the dependent variable and both may be described as contributing to the multivariate significance. In the example for this case the discriminant function coefficients (.840, .543) and the discriminant function variable correlations (.840, .543) were again equal, and their magnitudes indicated that both dependent variables were contributing to the overall multivariate significance.

In **situation 7** both dependent variables were correlated. Here, the first dependent variable was significantly correlated with the independent variable and the second dependent variable was not correlated with the independent variable. However, the first dependent variable acted as a suppressor variable to the second dependent variable in the relationship between the second dependent variable and the independent variable. In the example for this situation if you only examined the univariate tests you would surely think that only the first dependent variable contributed to the multivariate significance because the *p*-value for the first univariate test was significant ($p < .041$) while the *p*-value for the second univariate test was 1.000. Here, the discriminant function coefficients, with each dependent variable having relatively large coefficients (1.250 and -.750), indicated that both dependent variables contributed to the overall multivariate significance.

Situation 8 is similar in description to that of situation 3 and we could have created a situation similar to that of situation 4. The basic idea of these situations is that both variables contributed to multivariate significance even though only one of them is significant at the univariate level.

Condition 3: Multivariate Significance And Both Univariate Tests Significant

Three situations were considered in Table 3 under the condition where the multivariate test is significant and both univariate tests were significant. In two of the three situations (9 and 10) the univariate tests do reveal the contributions made to overall multivariate significance. However, in the last situation (11) only one dependent variable was contributing to the multivariate significance. Another situation, similar to situation 7 could have been created to consider the situation

Table 3
Description of the Correlations Among the Variables and of Example Discriminant Analysis Output for Condition 3: Overall Multivariate Significance ($p < .05$) And Both Univariate t Tests Significant ($p < .05$)

Relationship Between The Dependent Variables (1 & 2)	Relationship Between The Independent Variable (X) And The Dependent Variables (1 & 2)	Explanation	Example Discriminant Function Coefficients: Variable	Coefficient[1]
		Situation 9		
$\rho_{12} \approx 0.0$	$\rho_{x1} \neq 0.0$ and $\rho_{x2} \neq 0.0$	Both dependent variables are related to essentially independent parts of the independent variable	1 2	.707 .707
$(r_{12} = 0.00)$[2]	$(r_{x1} = .280;\ r_{x2} = .280)$			
		Situation 10		
$\rho_{12} \neq 0.0$	$\rho_{x1} \neq 0.0$ and $\rho_{x2} \neq 0.0$	Both variables contribute to multivariate significance but variable 1 contributes more than variable 2.	1 2	.654 .610
$(r_{12} = .250)$	$(r_{x1} = .290;\ r_{x2} = .279)$			
		Situation 11		
$\rho_{12} \neq 0.0$	$\rho_{x1} \neq 0.0$ and $\rho_{x2} \neq 0.0$	Only the first dependent variable contributes to multivariate significance.	1 2	1.001 -.001
$(r_{12} = .756)$	$(r_{x1} = .360;\ r_{x2} = .276)$			

[1] Standardized canonical discriminant function coefficients.

[2]Example values used to illustrate a given situation and to create the discriminant function coefficients are given in parentheses.

where a variable (say, variable 2) is contributing to multivariate significance but is not significant when the other dependent variable (say, variable 1) is partialed from it.

In **situations 9 and 10** both dependent variables were significantly contributing to overall multivariate significance. The difference between these situations is that in situation 9 the dependent variables are uncorrelated and in situation 10 they are correlated.

In **situation 11** both univariate tests were significant but only the first dependent variable was causing the multivariate test to be significant. This was because when the first dependent variable was partialed from the second dependent variable the second dependent variable was found to be unrelated to the independent variable. This result is found through the discriminant function coefficients where the coefficient for the first dependent variable was 1.001 and the coefficient for the second dependent variable was -.001. As in the previous situations the discriminant function variable correlations of 1.000 and .756 mirrored the significant univariate test results.

Part 2 Generalization: Two Group MANOVA With Three Dependent Variables

The preceding situations generalize in many respects to the two group MANOVA with three dependent variables. We will discuss some of the situations that differ from the preceding situations but that conflict with the textbook authors' assertion to look at the univariate tests to find out which variables caused the multivariate significance.

Consider the conditions where all of the dependent variables are not significant at the univariate level. In this condition all of the variables may be contributing in some way to the multivariate significance or only some two of them may be contributing.

In the condition where one variable is significant and two are not, it may be the case that the significant variable can be explained by the two variables that are not significant. In this case it could be that the two variables that are not significant at the univariate level actually cause the multivariate significance.

In the condition where two variables are significant and one is not, it may be the case that the significant variable and the insignificant variable "gang up" to cause multivariate significance and the other significant variable does not contribute to multivariate significance.

In the condition where all three dependent variables are significant it may be that only one, only two, or all three dependent variables contribute to multivariate significance.

Generalization: Multiple Group MANOVA With Multiple Dependent Variables

As can be seen from the preceding two discussions, in a multiple group MANOVA with multiple dependent variables there would be a large variety of situations where the causes of multivariate significance would not be obvious by looking at the univariate test results. In these situations we suggest that researchers, armed with theory and *a priori* predictions, consider discriminant and stepwise discriminant analysis information to find the causes of multivariate significance.

Educational Importance of the Study

The results clearly indicate the ways that multivariate significance is caused under a variety of relationships. The results also indicate why it is improper to interpret univariate ANOVA results to find the causes of multivariate significance. Finally, the use of discriminant analysis is illustrated as a valuable tool in considering the contributions of dependent variables to multivariate significance.

The social science literature provides many examples of studies (e.g., Brandon, Newton, & Hammond, 1987; Stahl & Clark, 1987) where researchers follow-up an overall MANOVA with univariate ANOVA's on each of the dependent variables to discern which of the dependent variables caused the multivariate test to be significant. This paper illustrates the fallacy of this logic and presents educational researchers with illustrations of follow-up methods that help to discern what caused the multivariate significance.

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Appendix A
Process Used To Create Situations 1 Through 11

The process used to create situations 1 through 11 was based on the relationship between a multivariate analysis for two groups (MANOVA) and the regression approach to this same design. In the MANOVA we assumed a large effect size of .80 (Cohen, 1988), a cell size of 25 ($N = 50$), and multivariate significance with a beginning overall F statistic at approximately 3.5 (except for situation 9).

[An F value of 3.23 or greater is necessary to have multivariate significance ($p <$ .05) with 2 and 40 degrees of freedom. The F required for significance in our examples would actually be slightly less than 3.23 because the degrees of freedom for all of our MANOVA's were 2 and 47. The F is described as "beginning" because during the process the F was recalculated and changed slightly, but was always significant at the multivariate level.]

After selecting an F value that would be significant, we calculated the corresponding multiple R by use of the formula:

$$R = \sqrt{\frac{2F}{(N-3) + 2F}}$$

In all of the situations we selected values for the correlation between the dependent variables in the MANOVA and the correlations between the independent variable, x, and one or both dependent variables (denoted by 1 & 2). Using the multiple R and the correlation of the independent variable with the first dependent variable, r_{x1}, the formula

$$R^2 = r_{x1}^2 + r_{x(2.1)}^2$$

was solved for the part correlation $r_{x(2.1)}$.

The part correlation was then used in the formula

$$r_{x(2.1)} = \frac{r_{x2} - r_{x1}\ {}_Rr_{12}}{\sqrt{1 - {}_Rr_{12}^2}}$$

to find r_{x2} the correlation between the independent variable and the second dependent variable. In most situations the correlation between the dependent variables in the multivariate analysis ${}_Mr_{12}$ was initially set equal to the correlation between the variables in the regression analysis, ${}_Rr_{12}$. This could be done because ${}_Mr_{12}$ is generally close to ${}_Rr_{12}$. In some situations r_{x2} was set to zero and ${}_{R}r_{12}$ was found directly.

Obtaining the proper correlations was the key to the process. Once they were found the formulas

$$C_1 = \sqrt{\frac{4\, r_{x1}^2 (N-2)}{N(1 - r_{x1}^2)}}$$

and

$$C_2 = \sqrt{\frac{4\, r_{x2}^2 (N-2)}{N(1 - r_{x2}^2)}}$$

were used to find the means C_1 and C_2 of the first and second, respectively, dependent variables for group 1. The means in group two were set at zero for both dependent variables. The values of C_1 and C_2 were those necessary to produce multivariate significance ($p < .05$).

The formula

$${}_{R}r_{12} = \frac{(N-2)\ {}_{M}r_{12} + N\left(\frac{C_1}{2}\right)\left(\frac{C_2}{2}\right)}{\sqrt{\left[N\left(1+\left(\frac{C_1}{2}\right)^2\right) - 2\right]\left[N\left(1+\left(\frac{C_2}{2}\right)^2\right) - 2\right]}}$$

was then used to find the regression correlation ${}_{R}r_{12}$ between the two dependent variables. This correlation was then used to compute a new part correlation which in turn was used to find the final multiple R.

The results were checked by using the resultant summary information as input for the SFSSX programs REGRESSION and MANOVA. For each situation the F statistics and univariate test information from these programs agreed.

Three situations used a minor variation of the preceding process. For example, in situation 4 we began by setting the part correlation to be larger than the correlation between the independent variable and the second dependent variable.

Situation 9 demanded the correlation between the two dependent variables be set to 0.0, but the correlations between each of the dependent variables with the independent variable had to be large enough to make both of the dependent variables significant at the univariate level. To meet these conditions a beginning F of 4.0, instead of 3.5, was used.

For situation 11 it was necessary to assume the part correlation was 0.0. This caused r_{x1} to be equal to the multiple R. The value of r_{x2} was chosen large enough to make the second dependent variable significant at the univariate level.