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ABSTRACT

As significance testing comes under increasing criticism, some researchers are turning to other indices to evaluate their findings. Included among the alternative options are the interpretation of effect size estimates and the evaluation of sample specificity (invariance testing). Using a hypothetical data set of 64 cases and two predictor variables, this study explains one approach to estimating whether the results in a study are sample specific--the jackknife method. The jackknife technique is useful with many statistical procedures and is especially appropriate when the sample size is small. The results of a descriptive discriminant analysis followed by the application of the jackknife are discussed to illustrate the procedure. When used with discriminant analysis, the jackknife technique evaluates the stability of the discriminant function coefficients. Ten tables and a 23-item list of references are included. (Author/SLD)

Evaluating the Sample Specificity of Discriminant Analysis Results Using the Jackknife Statistic

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Paper presented at the annual meeting of the Southwest Educational Research Association, San Antonio, TX, January 24, 1991.

ABSTRACT

As significance testing comes under increasing fire, some researchers are turning to other indices to evaluate their findings. Included among the alternative options are the interpretation of effect size estimates and the evaluation of sample specificity (sometimes called invariance testing). Using a hypothetical data set, this study explains one approach to estimating whether the results in a study are sample specific; that approach is the jackknife method. The results of a descriptive discriminant analysis followed by the application of the jackknife are discussed so that the procedure will be concrete and understandable.

Researchers often place exaggerated importance on the statistical significance of their results. As a consequence, they neglect to consider other important issues integrally associated with their results, such as practical significance (or effect size) and result replicability (or sample specificity). Carver (1978) and Fish (1986a) argue that statistical significance is largely an artifact of sample size. Thompson (1989) and Welge-Crow, LeCluyse, and Thompson (1990) provide examples that support this argument.

The research community sometimes loses sight of the fact that statistical significance does not imply practical significance; that is, statistical significance does not mean a noteworthy effect has been discovered. As an example used by Welge-Crow et al. (1990) illustrates, a trivial difference of .30 between one IQ mean of 100.15 and another IQ mean of 99.85 will be statistically significant if sample size is large enough, as it might be in a large school district. A result such as this suggests that researchers must also consider the practical significance of their findings. Practical significance can be thought of in terms of effect size, or the percent of variance in the dependent variable accounted for by the independent variables. As Cohen (1977) describes it, effect size is the "degree to which the phenomenon exists" (p. 9). Several methods are available for determining effect size (see Thompson, 1989 for some examples). Effect sizes range between 0% and 100%, but in the social sciences effect sizes of about 30% are generally considered large (Thompson, 1989).

Notwithstanding the importance of practical significance, the most critical aspect of any research study is the evaluation of sample specificity or result replicability. Neither statistical significance nor practical significance provide grounds for assuming that results will generalize to future studies (Carver, 1978; Thompson, 1989). As Stevens (1986) maintains, a result which is sample specific and cannot be replicated, that is, one which lacks "generalizability, ...is of limited scientific value" (p. 58)--no matter how statistically significant nor how practically significant the result may be for the sample employed. Stated another way, study results found to be significant at a stringent alpha level, even if they are also found to have a substantial effect size, do little to advance a body of knowledge if the results cannot be replicated.

Result generalizability, sample specificity, replicability, and invariance testing are often used interchangeably to refer to the assessment of the likelihood of reproducing research results in subsequent studies. The current paper presents an application of one invariance procedure, the jackknife statistic, following a discriminant analysis. To facilitate the reader's understanding of the jackknife results, a brief interpretation of the discriminant analysis results will be provided first.

Discriminant Analysis Results

Discriminant analysis is a multivariate technique that can be used in two ways (Afifi & Clark, 1984; Huberty & Wisenbaker, in press). One application of this technique is classification, sometimes referred to as prediction. Use of this application allows for individuals to be classified into two or more groups. Afifi and Clark (1984) explain that "the [groups] are known to be distinct and each individual belongs to one of them" (p. 247). An example might be classifying voters as Democrats, Republicans, or members of some other political party. By using the discriminant functions developed for making the classification, more accurate predictions can be made about the classification of future voters.

A second application of discriminant analysis is description. When used in this way, the technique "characterize[s] the major differences among...groups" (Stevens, 1972, p. 501). This application is not so much concerned with the accurate classification of cases, as it is with identifying the variables or combinations of variables that separate the groups (Afifi & Clark, 1984) and with examining the extent of group separation (Huberty & Wisenbaker, in press). Again using voters' party affiliation as an example, the descriptive application of discriminant analysis would provide information regarding the particular variables or variable composites that separated the voters by political parties. Such variables might include voter attitudes toward government welfare programs, the military, and the environment. The descriptive application tends to focus on which variables distinguish the groups and on why they do so.

Discriminant analysis employs factor analytic techniques which lead to the extraction of orthogonal synthetic variates (i.e., combinations of the original variables each of which is uncorrelated with the other combinations). In discriminant analysis, these synthetic variates take the form of discriminant functions, and are extracted in such a way that differences among the groups are maximized (Stevens, 1986).

For the present study, a discriminant analysis was calculated from a hypothetical data set with 64 cases and two predictor variables, X and Y. The first 32 cases are from a data set developed by Fish (1988). Four groups of 16 cases each were derived. The data set and the SPSSx commands for the discriminant analysis are contained in Tables 1 and 2, so that the reader is able to replicate and further explore the analysis. Means, standard deviations, and Pearson correlations for the data set are reported in Table 3.

INSERT TABLES 1, 2, AND 3 ABOUT HERE

Several sources of information available from commonly used statistical packages help in interpreting discriminant results. One source of information involves the discriminant functions. Two facts about each function are important; one is the significance of the discriminant function, the second is the percent of variance accounted for by the function. As Table 4 indicates, for this data set, two functions were found to be significant at the $p = .01$ level. The first function accounts for 79% of the variance and the second function for 21%. Because both functions account for major portions of the variance, both are useful in interpreting the discriminant results.

INSERT TABLE 4 ABOUT HERE

A second source of information useful to the researcher is the standardized function coefficients. These function coefficients are analogous to beta weights used in regression analysis. Consulting the standardized function coefficients presented in Table 5, we find that on Function I, the two variables have coefficients that are about equal in magnitude, but with opposite signs (1.55689 and -1.20080 for X and Y, respectively). This feature of Function I suggests that the contributions of X and Y to group separation are roughly equal, given that the coefficients have roughly the same magnitude. The opposite signs carried by the function coefficients indicate that individuals scoring high on one predictor variable and low on the other are members of the same group. With regard to Function II, X and Y are quite different in their respective contributions to group separation. As seen in Table 5, Y, with a standardized function coefficient of .99103, strongly contributes to group separation. On the other hand, X, with a function coefficient of only .01167, plays a very small role.

INSERT TABLE 5 ABOUT HERE

A third source of information useful in interpreting discriminant results is the canonical structure coefficients which are the correlation coefficients between scores on each variable and scores on each function. By consulting the structure coefficients for the present analysis, some interpretative clarity is gained and some is lost. The structure coefficients reported in Table 6 indicate that X has a moderate relationship with Function I ($r = .637$), while Y has almost no relationship with the first function ($r = -.008$). Further, the strength of the relationship between Y and Function II is confirmed; Y has a nearly perfect correlation with scores on Function II ($r = .99997$).

INSERT TABLE 6 ABOUT HERE

Consideration of the structure coefficients, however, clouds the role of X in explaining separation of the four groups on Function II. X has a strong relationship with scores on Function II ($r = .771$), the function on which its weight was negligible. This occurrence points to the need for interpreting both function and structure coefficients. A researcher can be most confident when both coefficients indicate that a variable is important. However, when the coefficients suggest conflicting interpretations, more emphasis is usually placed on structure coefficients for reasons elaborated by Thompson (1990).

A Rationale for Evaluating Sample Specificity

As has just been demonstrated, the interpretation of results from a discriminant analysis may prove difficult. However, regardless of the facility or difficulty encountered in interpretation, a thoughtful researcher will be wary of investing much confidence in the findings unless the generalizability of results can be empirically demonstrated. As Thompson (1989) notes, "when results appear to be replicable, the researcher can interpret the set of results...with more confidence" (p. 4).

Fish (1986b) explains the logic of evaluating sample specificity, noting that these techniques

> attempt to determine how stable the statistical results are likely to be across different samples. In the typical invariance procedure an analysis is performed separately on...subgroups into which the study sample has been divided, and the results are compared. When the results of an analysis are not comparable—i.e, not invariant—serious doubts about the generalizability of the results are in order. (pp. 65-66)

The suggestion that researchers evaluate the sample specificity of their results is based on a problem common to all statistical techniques. Daniel (1989) describes this problem, noting that "there is always the possibility that...results may simply capitalize on artifacts of the sample employed" (p. 1). "Artifacts of the sample" include such features as outliers and the chance selection of an atypical sample which differs substantially from the population. Characteristics such as the ones just mentioned lead to biased results and hence to the reporting of inaccurate conclusions. Compounding the problem, the smaller the sample size is, the greater is the risk of sample specific results.

Evaluating the sample specificity of a study can be accomplished with a number of procedures. Cooil, Winer, and Rados (1987) discuss three invariance methods. The first such approach is frequently called cross-validation and involves splitting the sample into two subsets or invariance groups. The coefficients of concern are determined on one subset and validated on the other (Cooil et al., 1987, p. 272). A drawback of this procedure is the further reduction in sample size that results from splitting the sample. This is especially problematic if the original sample is small (Daniel, 1989).

A second procedure, termed the 'simultaneous' approach by Cooil and his colleagues, "simultaneously estimates parameters and cross-validates the estimates" (Cooil et al., 1987, p. 273). These authors provide an example of this technique, noting that their investigation indicates it "has not been applied...in the social sciences" (Cooil et al., 1987, p. 271).

The third technique explained by Cooil et al. (1987) is the sample reuse method of which bootstrap procedures (see Diaconis & Efron, 1983, for a readable discussion) and the jackknife statistic are examples. Thompson (1989) explains that the bootstrap procedure "involves copying a data set over and over again into a megafile and then repetitively drawing different samples with different combinations of subjects...to determine how sampling influences affect results" (p. 3). The jackknife statistic, on the other hand, involves repetitively eliminating different subsets of cases from the total sample, calculating the statistic of interest on the remaining cases, and then averaging the results across subsets. The jackknife procedure was applied in the present study to evaluate the sample specificity of the discriminant analysis results.

The jackknife is useful with many statistical procedures and is especially appropriate when sample size is small, thus overcoming a problem with cross-validation methods, as noted. When used with discriminant analysis, the jackknife evaluates "the stability of the [discriminant function] coefficients" (Crask & Perreault, 1977, p. 62). This technique was developed by Tukey based on work by Quenouille and Jones (Fenwick, 1979). Tukey chose the name 'jackknife' because like a scout's jackknife, it is "a rough-and-ready instrument capable of being utilized in all contingencies and emergencies" (Miller, 1964, p. 1594). Crask and Perreault (1977) note that in discriminant analysis, the jackknife is useful for "characterizing...the underlying dimensions which discriminate between groups" (p. 63). Stated another way, the jackknife statistic evaluates the synthetic variates that separate the groups. The reader will recall that group separation was the focus of the interpretation of the discriminant analysis results previously discussed.

The jackknife statistic is a versatile invariance technique that is useful with both multivariate and univariate statistical methods. In addition to being applicable when sample size is small (Crask & Perreault, 1977), it is also an unbiased estimator (Quenouille, 1956; Tukey, 1958). On the point of bias, Cooil et al. (1987) explain that "the jackknife is intended to be a bias-reduction technique" (p. 272). Mantel (1967) added that this characteristic of the jackknife is "a particularly desirable property" (p. 570). The capability of producing unbiased estimators is a feature not characteristic of all approaches to assessing replicability. This bias reducing tendency makes the jackknife a particularly useful technique.

Procedure for Calculating the Jackknife Statistic

The jackknife statistic is conceptually and computationally straightforward. The following explanation will be non-mathematical; readers wishing a mathematical treatment of the process are encouraged to read Gray and Schucany (1972). To compute a jackknife, the original sample of size (N) is divided into k subsets of equal size (n) such that ($N = kn$). Each subset may be comprised of as few as one case or may be "as large as the largest multiplicative factor of N" (Daniel, 1989, p. 7). The statistic of interest is computed for the total sample. Then, each subset is deleted in repetition and a new statistic is computed for each of the truncated data sets. The result is k values, and the value derived for the total sample (N). As can be deduced from this step, the smaller the subsets the greater the number of repetitions required. There are several advantages to forming small subsets--the influence of outliers can be more easily detected, any lack of stability in the results can be more easily pinpointed, and the researcher can be more certain of the results. Thus, best practice dictates small subsets.

Having computed the statistic of interest for each truncated data set, the next step involves the calculation of a coefficient which is derived from the values obtained thus far. This new value is called a 'pseudovalue' by Tukey (Miller, 1964) and is the basis on which the jackknifed coefficient is determined. Indeed, the mean of the pseudovalues is the jackknifed coefficient. This mean is computed in the typical manner--the pseudovalues are summed and that sum is divided by the number of subsets (k).

With the jackknifed coefficient determined, the next step is interpretation. Invariance testing is a relatively new field, consequently, criteria for interpreting results are still developing. Because the jackknifed coefficient has been found to have approximately a normal distribution (Crask and Perreault, 1977; Miller, 1964), critical values and confidence intervals can be derived. A standard error of the mean is determined for the pseudovalues and a t-statistic is calculated by dividing the jackknifed coefficient by the standard error. The critical t-value is obtained from the t-distribution, with degrees of freedom equalling the number of subsets minus one ($k - 1$). The stability of the jackknifed coefficient can be evaluated either directly, or through the construction of confidence intervals (Crask & Perreault, 1977; Daniel, 1989). Other information, such as the range of the pseudovalues, can also aid in interpretation.

Application of the Jackknife Statistic

A jackknife statistic was computed to evaluate the sample specificity of the discriminant analysis results presented earlier.

A jackknife cannot be calculated through SPSSx; however, the repetition of runs on each truncated data set can be performed with SPSSx. The pseudovalues and jackknifed coefficient can then be computed with a spreadsheet program. The SPSSx program commands for the repetitions of discriminant analyses used in the present study are found in Table 7. The variable 'Subset' contained in the data set in Table 1 was included to partition the sample into eight subsets for computing the jackknife. As noted earlier, the jackknife statistic is of maximum value when the subsets are small. For heuristic purposes in this example, eight subsets of eight cases each were used so that the explanation would not be unduly complicated and readers would be able to replicate the results with relative ease. Best practice, however, would entail much smaller subsets and, thus, many more discriminant runs.

INSERT TABLE 7 ABOUT HERE

Applying the technique to the hypothetical data in the present study involved the following steps:

(a) a standardized discriminant function coefficient was computed for both variables associated with both functions using the total sample of 64 cases. These function coefficients are presented in Table 8;

(b) the sample was divided into eight subsets of equal size using the variable 'Subset;'

(c) each subset of cases was deleted in repetition and standardized discriminant functions were computed for each of the eight truncated data sets, as reported in Table 8;

(d) pseudovalues for X and Y were computed for both functions for each of the eight truncated data sets using the formula,

$J_i(\Theta') = k(\Theta') - (k-1)\Theta_i'$

$i = 1, 2, 3, \ldots, k$

(Crask & Perreault, 1977, p. 62)

where Θ' is the function coefficient derived from the entire sample, k is the number of subsets, Θ_i' is the function coefficient derived from a truncated data set, and $J_i\Theta'$ is the pseudovalue--these pseudovalues are presented in Table 9;

(e) the jackknifed coefficient for X and Y on both discriminant functions was derived by averaging the pseudovalues using the formula,

$J(\Theta') = \sum J_i(\Theta')/k$

$i = 1, 2, 3, \ldots, k$

(Crask & Perreault, 1977, p. 62)

where $J(\Theta')$ is the jackknifed coefficient. The jackknifed coefficients are presented in Table 9.

INSERT TABLES 8 AND 9 ABOUT HERE

Results

As Table 9 indicates, for Function I, the jackknifed coefficients for X and Y, respectively, are $J(\Theta') = 1.52438$ and $J(\Theta') = -1.16857$. With a standard error for X of $s_{\bar{X}} = .19$ and for

Y of $s_{\bar{X}}$ = .37, the respective *t*-values on Function I are t_{calc} = 8.023 for X, and t_{calc} = −3.158 for Y. The critical value for *t* at $\underline{p}$ = .05 with *df* = 7 is t_{crit} = 2.365. The calculated *t*-values exceed the critical values for both variables on Function I.

Table 9 provides the same type of information for Function II. The jackknifed coefficients for X and Y, respectively, are $J(\Theta')$ = .00944 and $J(\Theta')$ = 1.01644. The standard error was found to be $s_{\bar{X}}$ = .269 for X and $s_{\bar{X}}$ = .213 for Y. The resulting t_{calc} = .035 for X failed to exceed the critical value of t_{crit} = 2.365. However, for Y, the obtained t_{calc} = 4.77 was significant at the $\underline{p}$ = .05.

Table 10 provides confidence intervals for both variables on both functions. In all four instances the original coefficients fall within the bounds of the confidence intervals established.

INSERT TABLE 10 ABOUT HERE

Discussion

Table 9 shows that the calculated *t*-value for three of the four jackknifed coefficients exceeds the critical value; the exception is X on Function II. Fish (1986a) makes a cogent argument that it is somewhat illogical to propose invariance testing as an alternative to strict reliance on significance testing and then to judge the results of an invariance test strictly on statistical significance. Consequently, we can look elsewhere in the data to understand the jackknife results more fully.

For example, the confidence intervals presented in Table 10 include the original function coefficients in every instance, including X on Function II. Further, information in Table 9 alerts us to a problem with X. Note that the jackknifed coefficient for X is remarkably small on Function II. Note further that the pseudovalues for this variable fluctuate more radically (*SD* = .269) than those for the other variables. From these three pieces of information, it can be deduced that while the X is likely to replicate in future research, the replication will involve function coefficients that vary between -1 and +1, and thus are unlikely to be particularly important. This problem with X might have been anticipated based on the results reported in Table 8. As can be seen, the magnitude of the function coefficients derived from the truncated data sets for X on Function II approaches zero. In addition, we find both positive and negative values among these function coefficients.

Interpreting all of the evidence from the jackknife procedure suggests that the sample findings are likely to replicate in subsequent research, but that X will produce erratic results that range near zero and are, therefore, not very noteworthy. Based on the jackknife results, the researcher has reason to believe that the study findings are stable under variations in sampling configuration and, thus, are not sample specific. Stated another way, both X and Y appear to be valid discriminators among groups, with characteristics that generalize to the population. However, the findings also suggest that caution should be exercised in interpreting the discriminant analysis regarding X on Function II.

These jackknife results are interesting. The reader will recall the discriminant analysis results. Those findings indicate that X had the greater association of the two variables with Function I, but that it also had a strong correlation with Function II, as noted in the structure coefficients reported in Table 6. Nonetheless, the relationship between X and Function II the was weaker than that of Y and Function II. In interpreting the results of the discriminant analysis, the almost perfect structure correlation between Y and Function II overshadowed the contributions of X. Thus, the jackknife results in this case suggest that the seemingly contradictory finding regarding the role of X on Function II may be unimportant, since the variable's behavior on this function appears to be erratic.

One final note on the jackknife findings. As explained earlier, a reason for developing small subsets is to provide greater ease in detecting unique cases. The behavior of subset four suggests that there may be outliers in data that are captured by this subset. Subset four manifests unusual results in three of the four sets of pseudovalues: it produced by far the smallest pseudovalue for X on Function I, the only positive value for Y on the same function, and the largest pseudovalue for Y on Function II. This subset may well have one or more unique cases and should be examined for this possibility.

Summary

The jackknife statistic is a tool for evaluating the sample specificity of results. Fish (1986a) counsels that invariance procedures like the jackknife provide information about the reproducibility of research findings, not about the interpretation of those findings. As noted previously, results obtained from evaluating the sample specificity of study findings inform the researcher about the degree of confidence with which the interpretation can be made.

The versatility of the jackknife procedure allows for it to be used with both univariate and multivariate statistical techniques. A reminder is in order, however. Parameters for interpreting invariance coefficients such as jackknifed coefficients have yet to be established (Thompson, 1984). Until practice evolves to the point that parameters can be established, "the interpretation of invariance results [will be] a matter of the researcher's judgment" (Fish, 1986a, p. 16). In the meantime, Thompson (1984) recommends that at least one invariance procedure be included in every research report.

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Table 1
Hypothetical Data Set

Case	Group	X	Y	Subset*
1	1	4	2	5
2	1	5	3	8
3	1	4	4	2
4	1	4	5	3
5	1	3	4	4
6	1	6	5	6
7	1	5	6	7
8	1	7	5	2
9	1	6	6	1
10	1	8	6	8
11	1	7	6	1
12	1	9	7	5
13	1	8	7	4
14	1	8	8	3
15	1	9	8	7
16	1	9	9	6
17	2	1	2	8
18	2	3	3	4
19	2	3	5	3
20	2	3	5	6
21	2	2	5	5
22	2	4	6	4
23	2	4	5	2
24	2	5	6	5
25	2	6	6	6
26	2	6	6	1
27	2	6	7	7
28	2	7	7	8
29	2	7	7	2
30	2	8	9	3
31	2	8	9	7
32	2	9	9	1
33	3	4	1	8
34	3	4	2	6
35	3	3	2	3
36	3	2	4	5
37	3	5	3	2
38	3	7	4	1
39	3	4	5	7
40	3	5	4	5
41	3	7	5	8
42	3	9	5	6
43	3	6	5	4
44	3	5	6	1
45	3	7	6	7
46	3	9	7	3
47	3	8	6	5
48	3	8	5	2

Case	Group	X	Y	Subset
49	4	1	7	4
50	4	1	2	3
51	4	1	1	2
52	4	2	2	8
53	4	2	3	3
54	4	2	3	1
55	4	3	2	7
56	4	3	3	4
57	4	3	4	7
58	4	4	5	6
59	4	4	4	5
60	4	4	5	4
61	4	4	6	2
62	4	5	6	1
63	4	5	7	8
64	4	5	7	6

*Subset was used to separate the cases into groups for the jackknife procedure; it was not used in the discriminant analysis itself.

Table 2
SPSSx Commands for Discriminant Analysis

```
FILE HANDLE DT/NAME='DISCRMNT.DAT'
 DATA LIST FILE=DT/CASE 1-2 GROUP 7 X 12 Y 17 SUBSET 20
LIST VARIABLES CASE TO SUBSET
DISCRIMINANT GROUPS=GROUP(1,4)
 /VARIABLES=X Y
 /STATISTICS
```

Table 3
Group Means, Standardized Deviations, and Correlations

	X		Y		
Group	Mean	SD	Mean	SD	r_{xy}
1	6.38	2.03	5.69	1.89	.833
2	5.13	2.36	6.06	1.98	.923
3	5.81	2.14	4.28	1.67	.657
4	3.06	1.44	4.19	2.01	.643

Table 4
Discriminant Functions

Function	Percent of Variance	Significance
I	78.75	.0000
II	21.25	.0047

Table 5
Standardized Function Coefficients

	Function I	Function II
X	1.55689	.01167
Y	-1.20080	.99103

Table 6
Canonical Structure Coefficients

	Function I	Function II
X	.63652	.77126
Y	-.00750	.99997

Table 7
SPSSx Commands to Generate Data for
Jackknife Calculations

```
TEMPORARY'
DO IF (SUBSET EQ 1)
COMPUTE GROUP=5
END IF
DISCRIMINANT GROUPS=GROUP(1,4)
 /VARIABLES=X Y
 /STATISTICS
TEMPORARY
DO IF (SUBSET EQ 2)''
COMPUTE GROUP=5
END IF
DISCRIMINANT GROUPS=GROUP(1,4)
 /VARIABLES=X Y
 /STATISTICS
```

' The commands to obtain the data for the jackknife immediately follow the program commands found in Table 2.

'' As can be seen, the series of commands that follows TEMPORARY are identical except that the (SUBSET EQ #) changes to delete each of the eight subsets one at a time. Because the commands are the same, all eight will not be listed here.

Table 8
Standardized Discriminant Function Coefficients
for the Jackknife Subsets

Group Deleted	Function I X	Function I Y	Function II X	Function II Y
(none)	1.55689	-1.20080	.01167	.99103
1	1.56334	-1.21857	.02623	.97970
2	1.54426	-1.16118	-.02421	1.01833
3	1.46279	-1.09874	.04494	.96672
4	1.71091	-1.50572	.19439	.83535
5	1.61345	-1.24890	-.02809	1.02189
6	1.52628	-1.05436	-.15854	1.11490
7	1.57848	-1.29292	.11018	.91218
8	1.49277	-1.06311	-.06899	1.05020

Table 9
Pseudov 'es, Jackknifed Coefficients,
and t-values

Group Deleted	Function I X	Function I Y	Function II X	Function II Y
1	1.51174	-1.07641	- .09025	1.07034
2	1.64530	-1.47814	.26283	.79993
3	2.21559	-1.91522	- .22122	1.16120
4	.47875	.93364	-1.26737	2.08079
5	1.16097	- .86410	.28999	.77501
6	1.77116	-2.22588	1.20314	.12394
7	1.40576	- .55596	- .67790	1.54298
8	2.00573	-2.16463	.57629	.57684
Jackknifed Coefficient	1.52475	-1.16834	.00944	1.01644
$s_{\bar{x}}$	.19	.37	.269	.213
t_{calc} (df=7)	8.025*	-3.158*	.035	4.772*
t_{crit} (p=.05)	2.365	2.365	2.365	2.365

*Indicates coefficient stability

Table 10
95% Confidence Intervals for the Jackknifed Coefficients

	Function I		Function II	
	X	Y	X	Y
Original coefficients	1.55689	−1.20080	.01167	.99103
Jackknifed coefficients	1.52475	−1.16834	.00944	1.01644
Lower bound	1.15235	−1.89354	−.51780	.59896
Upper bound	1.89715	− .44314	.53668	1.43392

Note: The formula used for the confidence interval is: $CI = J(\Theta') + zs_J$. For example, the interval for X on Function I is $CI = 1.52475 + 1.96(.19) = 1.15235$ to 1.89715.