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ABSTRACT

One of the most common misconceptions about probability is the belief that successive outcomes of a random process are not independent. This belief has been dubbed the "gambler's fallacy". The belief that non-normative expectations such as the gambler's fallacy are widely held has inspired probability and statistics instruction that attempts to counter such beliefs. This study presents an investigation of student performance pre and post instruction on problems dealing with these kinds of statistical misconceptions. Instruction consisted of 10 laboratory sessions, 1.5 hours each, delivered to 16 high school students attending a summer mathematics program at Mount Holyoke College (Massachusetts). The instruction included computer simulations that were intended to provide students with sufficient data to refute expectations based on the representativeness heuristic, as well as other misconceptions about chance. Student performance suggests that a belief in representativeness may not be as widespread as thought, and that curriculum development aimed at countering this belief should proceed cautiously. In addition, student who apparently do not have a well-developed understanding of independence in random sampling may nevertheless answer such problems correctly based on reasoning that is fundamentally non-probabilistic. Thus, many items currently being used to assess conceptual development may be insensitive to certain misconceptions about probability. Student misconceptions about probability need to be better understood if more appropirate mathematics instruction is to be achieved. (KR)

Belief in Equally-likely vs. Unequally-likely Events

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Draft: June, 1988

Belief in Equally-likely vs. Unequally-likely Events

One of the most common misconceptions about probability is the belief that successive outcomes of a random process are not independent -- that a long runs of heads, for example, increases the likelihood that the next trial will produce a tail. This belief has been dubbed the gambler's fallacy," presumably since people at the roulette table tend not to bet on numbers that have recently come up.

There are several possible explanations for why people might think that a Tails is more likely after a run of Heads. First, they may believe that a series of random events is governed by a self-correcting process. This belief has sometimes been referred to as a belief in "active balancing." Certainly they are examples of phenomena of this type (e.g., thermostats, a swinging pendulum) and it may be that people assimilate random events to such phenomena. Also, random sampling that involves non-replacement behaves in a compensating way such that the longer a student waits for his or her name to be called when the teacher is passing back shuffled-up exams, the more likely it is that the student's name will be called next.

A slightly different explanation for the gambler's fallacy is that people may believe in, but misunderstand, the Law of Large Numbers. People may believe that as the sample size increases the number of heads and tails become closer. The fallacy in this argument is that in the long run the number of heads and tails do not get closer; in fact, the numbers in general get more and more divergent. Rather it is the relative frequency, or percentage of heads and tails, that converges according to the Law of Large Numbers. The contribution to this percentage of a surplus of one of the outcomes is simply swamped out by the large number of trials.

A third possible basis for the gambler's fallacy is based on what Kahneman and Tversky (1972) call the "representativeness heuristic." According to this heuristic, the likelihood that a given sample came from a particular population is judged by comparing the similarity of the sample to the population. Given a choice after a run of four Hs between the two samples HHHHH and HHHHT, the latter is judged as the more likely since it is closer to the ideal population distribution of 50% Hs, 50% Ts.

Use of the representativeness heuristic is often elicited by asking people to chose among possible sequences the most likely to occur. In the case of five flips of a fair coin, all possible ordered sequences are in fact equally likely, the probability of each being $.5_5$. However, given several options people reasoning according to the representativeness heuristic will chose THHTH over THTTT and HTHTH. Kahneman and Tversky (1972) argue that this answer is consistent with the representativeness heuristic in that it reflects both the fact that heads and tails are equally likely and that random series should be "mixed up."

There are other possible rationales for this incorrect answer. It is possible that people do not attend to the specified order of the events in the sequences. If so, they would be correct in selecting THHTH over THTTT in that it is more likely to get three heads than one head if order is ignored. However, that people will select THHTH as more likely than HTHTH or HHHTT suggests that order is not being ignored. It is also possible that the selection is based on a belief in active balancing. People might reason that the sequence THHTH looks like a sequence to which the active balancing mechanism has been applied.

The belief that non-normative expectations such as the gambler's fallacy are widely held has inspired probability and statistics instruction that attempts to counter such beliefs. Curriculum designed by Shaunghnessy (1985) and Beyth-Maron (1983) both include units intended to confront and correct judgments based on informal judgment heuristics. (Nisbett) However, a better understanding the basis of peoples' non-normative beliefs about probability is critical for the design of teaching interventions that can alter those beliefs. Results reported by Pollatsek, Konold, Well, & Lima (1984) suggest, for example, that representativeness and not active balancing is the basis for the majority of incorrect answers to questions like those above. They presented subjects with a situation in which a sample of ten scores was being random chosen from a population with a known mean. Given the first element of the sample, subjects were asked to make their best guess as to the mean of the sample. Even though the first score was an extreme, subjects tended to predict the population mean for the mean of the sample. Furthermore, when asked what the mean of the next nine scores would be, the majority of these subjects did not say that it would be in the direction opposite the first score. This response would be consistent with a belief in active balancing. Rather, consistent with the representativeness heuristic, they chose the population mean as their best guess for the mean for the next nine. The authors point out that several statistics texts have, in the past, tried to counter the gambler's fallacy by trying to convince student that the active balancing theory of coin flipping is untenable by pointing out that "coins don't have memories." However, if people's incorrect answers are not actually based on the belief in active balancing, such explanations will be unhelpful and perhaps confusing.

The present study is an investigation of student performance pre and post instruction on problems like those presented above. The instruction included computer simulations that were intended to provide students with sufficient data to refute expectations based on the representativeness heuristic, as well as other misconceptions about chance. Student performance pre and post instruction, along with performance on the same problems by students in two other educational settings, suggests that a belief in representativeness may not be as widespread as thought, and that we therefore ought to proceed cautiously in developing curriculum aimed at countering the belief. In addition, students who apparently do not have a well-developed understanding of independence in random sampling may nevertheless answer such problems correctly based on reasoning that is fundamentally non-probabilistic. Thus, many items currently being used to assess conceptual development may be insensitive to certain misconceptions about probability.

Method

Instructional Treatment

The instructional treatment consisted of 10, 1-1/2 hour lab sessions on probability conducted each day over a two-week period during the summer of 1987. Sixteen high school girls signed up to take the probability workshop, which was one of several workshops that participants of "Summermath" could elect to take. Summermath is a six-week residential math program for high school girls sponsored by Mount Holyoke College. Summermath recruits nationwide, and participants represent a range of mathematical ability. The workshop on probability consisted of various lab exercises which focused on concepts of independence, randomness, frequency interpretation of probability, and calculating the joint and conjunctive probability in fairly simple situations. Students worked in pairs at an IBM-AT using accompanying software programmed in APL that allows the user to build a variety of sampling models by specifying outcome names and the number of elementary events of each name. Having built such a model, the program can then be used to draw large random samples, perform a variety of analyses, and display the results. A nine-item pretest was administered on the first day of the workshop, and a parallel-version posttest was administered on the last day.

Problems

The items each appeared on a separate page, and students were instructed not to return to a page once it had been turned. This report includes information only two of the items used on the pretest and their parallel versions on the posttest. The pretest items are listed first below.

Four-heads Problem.

A fair coin is flipped 4 times, each time landing with heads up. What is the most likely outcome if the coin is flipped a fifth time?

a. Another heads is more likely than a tails.
b. A tails is more likely than another heads.
c. The outcomes (heads and tails) are equally likely.

H/T Sequence Problem.

1. Which of the following is the most likely result of 5 flips of a fair coin?

   a. H H H T T
   b. T H H T H
   c. T H T T T
   d. H T H T H
   e. All four sequences are equally likely.

2. Which of the above sequence would be least likely to occur?

B/G Sequence Problem.

All families of 5 children in a large city were surveyed. The exact order of births of boys and girls in each family was recorded.

1. Which of the following exact order of births do you think occurred the most frequently?

   a. B B B G G
   b. G B B G B
   c. G B G G G
   d. B G B G B
   e. All four exact orders occurred with approximately equal frequency.

2. Which of the exact orders above do you think occurred the least frequently?

Die Problem.

A fair die is rolled 5 times. The exact order of the outcomes of the rolls was 4, 6, 5, 6, 4. What is the most likely outcome if the die is rolled a fifth time?

a. A number that is 3 or below.
b. A number that is 4 or above.
c. It is equally likely to be a or b.

In addition to choosing an option, students were asked for each problem to "give a brief justification" for their answer. Of the other problems not reported here, two involved computing joint and conjunctive probabilities, two with interpreting meanings of probabilities, such as the meaning of a 70% chance forecast of rain, and one each with a) the effect of sample size on variability of the sample mean, b) producing a hypothetical random string of coin flips, c) choosing between two options the most likely result of drawing randomly from an urn with a known population.

The pretest items were also administered to two other student populations.

Remedial Math

Twenty five undergraduates students enrolled in the Spring 1987 semester of remedial-level mathematics course at the University of Massachusetts, Amherst, volunteered to participate in a study on probabilistic reasoning. They received class credit for their participation. Probability was not a topic covered in these courses. The H/T Sequence and Four-heads Problems were among the 14 items they completed.

Statistical Methods Course

The H/T Sequence and Four-heads Problems were also administered as part of a pre-course survey for a graduate-level statistical methods course in the

College of Education, University of Minnesota, in the Fall of 1987. This course is the first of a three-semester methods sequence required of all advanced-degree candidates in Psychology and Education. Dr. Joan Garfield was the instructor, and administered the survey. [Did I get this right, Joan?]

Results and Discussion

Results for the three groups on the Four-heads Problem is summarized in Table 1. Over 11, 86% of the students responded correctly that both heads and tails were equally likely. Not surprisingly, the performance of the Remedial students was the poorest (70% correct) and the Methods students the best (96% correct). After instruction, the Summermath students all answered the parallel Die Problem correctly. As expected, the most popular alternative answer was the one consistent with the gambler's fallacy, that a tails is more likely after a run of heads. This option was selected by 22% of the Remedial students, 19% of the Summermath students, and 4% of the Methods students. Surprisingly, these result suggest that even without instruction, the majority of students do not commit the gambler's fallacy on this particular problem.

Insert Table 1 about here

Performance on the H/T Sequence Problem is summarized in Table 2. Looking first at the responses to the question of which sequence is the most likely, it appears that a majority of students have a correct intuition that all the sequences are equally likely. Overall, 62% of the students responded correctly, with the Remedial students again performing the poorest (61%) and the Methods students the best (79%). The most popular incorrect alternative overall was the sequence THHTH. This sequence is perhaps the response most consistent with the representativeness heuristic, in that it has roughly equal numbers of heads and tails and they are adequately mixed up. The alternative "f" in the table was added to the possible options by 4% of the Remedial and

6% of the Stat Methods students. They indicated that options a, b, and d were equally likely and that option c was least likely to occur.

Insert Table 2 about here

The most interesting aspect of these data is the percent of correct responses to the question of which sequence is least likely. Overall, only 30% of the students responded that all four sequences were equally unlikely. Thus, roughly half of the students who selected the correct option e for the question regarding the most likely sequence went on to select one of the sequences as least likely rather than respond in a consistent manner that all four sequences were also equally unlikely. Additionally, the post instruction results of the Summermath students shows virtually no improvement in this pattern of inconsistent results on the parallel B/G sequence problem: roughly 40% of the students who responded correctly to the question of the most likely sequence switched to an inconsistent answer in the case of the least likely sequence (see Table 3). These result suggests that even though students may respond correctly to the question of which sequence is most likely, their answers may not be based on normative reasoning

Insert Table 3 about here

The performance of three different groups of students on problems that have been regarded in the literature as prototypical instance for revealing the representativeness heuristic suggests that such reasoning is used by a minority of the respondents. In fact, more than half of the responses would appear to be normative and thus reflect an understanding of independence in random sampling. These results are consistent with performance on an item almost identical to the Four-heads Problem which was used in the most recent National Assessment of Education Progress (as reported by Brown, Carpenter, Kouba, Linguist, Silver, and Swafford, 1988.) Forty-seven percent of the 7th graders and 56% of the 11th graders selected the correct alternative that heads and tails were equally likely after flipping four Tails with a fair coin The percent of responses consistent with the representativeness heuristic was 38% for the 7th graders and 33% for the 11th graders. Given that probability is still infrequently taught at the secondary level, these percentages reflect pre-instruction performance.

However, the results of this study also suggest that perhaps as many as 50% of the student who answer correctly on problems of this type may be doing so based on non-normative reasoning, in that their answers for the questions regarding the most likely- vs. the least-likely sequences are logically inconsistent. Given that the "equally likely" response on the Sequence Problems is of questionable meaning, the equally likely response on the Four-heads should also be held suspect. That is, if asked the parallel follow-up question on the Four-heads Problem, "Which result is least likely to occur?" it is reasonable to expect that a large percentage who chose the correct response would then respond that another heads was least likely.

An Outcome Approach to Probability

One possible explanation for these results is that students who give inconsistent responses are reasoning according to the "outcome approach" (Konold, in press). People who reason according to the outcome approach do not see their goal in uncertainty as specifying probabilities that reflect the distribution of occurrences in a sample, but of <u>predicting</u> results of a single trial in a yes/no fashion. When given a probability such as "there is a 70% chance of rain tomorrow," outcome-oriented individuals adjust the number given to one of three decision points: 100%, which means "Yes," 0%, which means "No," and 50%, which means "I don't know " Thus, many subjects reported that the forecast 70% rain meant, "It will rain tomorrow," and if it did not rain, that the forecast would have been "wrong." Several subjects spontaneously reported that a forecast of 50% chance of rain would mean that the forecaster had no idea of what was going to occur.

For a person who reason according to the outcome approach, the "50/50 chance" associated with coin flipping may not necessarily imply that the person expects roughly 50% heads in a sample, but that he or she has "no way to know" what the outcome of a trial will be. Given the Four-heads and H/T Sequence Problems, outcome-oriented individuals may believe that they are being asked to predict what will happen, and since the probability is 50/50, may respond by choosing the "equally likely" alternative, and by that really mean that they have no basis for making a prediction. When subsequently asked in the H/T Sequence Problem which sequence is least likely, they may no longer frame the question as asking for a prediction of what will occur, and may then chose an alternative based on the representativeness heuristic. A rough wording of their thinking might be, "I can't say for sure what will happen, but I don't think this will."

The written responses that students gave to these problems provides some indication that a few of the subjects may have been reasoning as described above. The responses of four students whose responses to the H/T Sequence Problem were inconsistent are given below. Each excerpt is preceded by a subject code that specifies whether the subject was from the Summermath (S) or Remedial (P) group. The answers the student gave to the multiple-choice question are given in parentheses.

> S.5: (e,a). [For e] Anything can happen with probability. The chances of some of the examples are least likely to occur (a,c), but it can happen. [For a] This chance is least likely to occur because they happen the same side in a row.

S16: (e,c). [For e] They all could occur. [For c] Because it is least likely to occur when you have almost a perfect score.

R2: (e,a). It's a chance game. Receiving 3 heads in a row seems unlikely, but could very well occur. No skill is involved, therefore all could likely occur by chance.

R14: (e,[illegible]). One never knows which way the coin will drop.

Similar reasoning is suggested in the written justifications to correct responses to the Four-heads and Die Problems. These rationales were coded as belonging to one of four categories as indicated down the left column of Table 4. Overall, about 40% of the correct answers were accompanied by justifications that indicated some notion of the independence of trials. [illegible] of the written justifications are given below:

S[illegible]: [illegible] Even if the fair coin was flipped 1,000 times and the outcome of it was all heads, the 1,001 time it still has the probability of [illegible] outcomes, heads or tails. The chance of either heads or [illegible].

[illegible] There is the same chance every time you flip it no matter how many [illegible].

R[illegible]: (c) The chance (50%) remains the same no matter how many times the [illegible].

R[illegible]: (c) The outcome is still 50/50. It could go either way regardless of the last outcome.

Insert Table 4 about here

[illegible] of the correct answers were justified with responses that seem indicative of the outcome approach -- that there was no way to predict what will happen or that one cannot rule out either possibility. A few excerpts are given below as examples of this type of response:

S5. (c) It's hard to tell what side you're going to end up with, so in order to be sure you have to think of either one landing."

S[illegible] (c) If four heads came up it is still possible that a H or T will come up.

R[illegible] (c) It is basically left to chance. There are no definite answers.

R8: (c) There is no way one could ever know whether a coin is going to be heads or tails. You just have to go with it.

Another [illegible]0% of written justifications for the correct answer to the Four-heads Problem were simply that the probability of H and T was 50/50. Examples of justifications coded as instances of this response are provided below:

S1: (c) Because when you flip a coin it's just chance, and it's the same weight on both sides.

R1: (c) There is a 50-50 chance for heads or tails. The chances are equal.

[illegible] are equally likely due to the fact again that it is chance.

The "equally likely" justification is an ambiguous answer in that it may merely be a restatement of the selected option c, it may include the concept that the probability of H is independent of past outcomes, or it may reflect the outcome-oriented interpretation that no prediction can be made. One reason to question the possibility that the subjects who give these justifications hold a notion of independence is that there was a high consistency across Summermath students in their responses to the Four-heads and Dice Problems: if a student mentioned independence explicitly in one problem they were very likely to mention it when appropriate in the other. [illegible] Summermath students whose responses explicitly mentioned independence, all but one of them mentioned independence on the Dice Problem on the posttest. (The other student gave an equally-likely rationale together with mentioning that it was a with-replacement sampling model.) Furthermore, four of the five students who gave "equally likely" justifications on the Four-heads Problem took the posttest, and all four of them gave similar justifications to their answers on the Dice Problem. One reason to suspect that a written justification that the chances are 50/50 reflect an underlying outcome-orientation is that a logical connection was made in some of the justifications between this notion and the assertion that no prediction could be made. These instance were coded under the "either-could-happen" category, and are exemplified below.

> T14: (c) Can't predict something that has equal possibilities.
>
> R25: (c) If both sides are equal in weight then the outcome cannot be judged with conviction.
>
> S14: (c, Die Problem) Each number has an equal chance to be rolled next, so either [option] A or B could be rolled.

It is interesting to note that all three of the students who incorrectly answered that a T was the more likely outcome after four Hs, gave correct answers with justifications including the concept of independence on the posttest Die Problem. This may have been because the labs were developed with the objective in mind of countering the gambler's fallacy and not with countering the outcome orientation. That the labs may have been more effective for those committing the gamblers fallacy is further indicated by the pre-post performance on the H/T Sequence Problem. As shown in Table 5, of the four students who gave correct responses on the posttest, three had given an answer consistent with the gambler's fallacy on the pretest. Furthermore, only one of the six students who gave inconsistent responses on the pretest responded differently on the posttest, switching to a gambler's fallacy response.

Insert Table 5 about here

Conclusion

Problems regarded as prototypical situations for eliciting the gambler's fallacy have been shown in this study to elicit such responses less frequently than it might have been expected. It is not clear to what extent these results might differ from previous findings. For while Kahneman and Tversky frequently cite these problems as exemplars of the gambler's fallacy and representativeness, they do not report performance data on these particular problems. Nor have I yet been able to find published experimental results on similar problems, other than the NAEP results cited earlier.

These results have some fairly direct implications for curriculum development and testing in probability. The belief that the majority of novices faced with these type of problems will commit the gambler's fallacy has helped to shift the focus in probability instruction away from computational skills towards conceptual development (cf. Garfield & Ahlgren, [illegible]). This shift has been accompanied by curriculum aimed at the development of concepts such as independence and randomness and the design of items to test for conceptual understanding. Given this focus, problems like those used in this study are likely to become standard fare on course and national exams of mathematical achievement. The results of this study suggest that a sizeable percentage of correct responses to such problems are spurious and reflect an approach to uncertainty that is perhaps more pernicious than the representativeness heuristic. Problems need to be developed that can discriminate individuals who reason according to what has been described here as the outcome approach from those with a normative concept of independence. One possibility is the inclusion of the option, "The most probable outcome cannot be determined," to the Four-heads Problem and the H/T Sequence Problem. Outcome-oriented individuals ought to prefer this option to the "equally-likely" option.

It needs to be stressed that to characterize people as outcome-oriented, representative, or normative with regards to their orientation toward probability is an oversimplification. It is clear that a particular person can adopted a normative approach to a problem in one context and in another, related problem, reason non-normatively (see, for example, K&T). Nisbett et al. (1983) have suggested that problem characteristics such as an ambiguous sample space and non-repeatability of trials can influence people to take an heuristic approach. Similarly, Konold (in press) has suggested that similar variables, especially repeatability of trials and symmetric elementary outcome probabilities, may be a factor in inducing the outcome approach. Indeed, it appears that the wording change from "most likely" to "least likely" in the H/T Sequence Problem may be sufficient to effect a shift for some people from an outcome orientation to a representativeness framework.

The development of such curriculum for the secondary and even elementary levels has become part of the agenda of current efforts to reform mathematics education in the United States. As mentioned above, one of the directions of new curriculum being developed is to help students overcome various of the well-documented misconceptions about probability and statistics. If however, there are other misconceptions, such as the outcome approach, that a fair percentage of students hold, such curriculum will probably be of limited success. Indeed, the problems used in the educational treatment reported in this study were being used to assess the effectiveness of curriculum designed primarily to disabuse students of the representativeness heuristic. This was to be accomplished by first having students make predictions about situations like the Four-heads Problem after which they collected data via computer simulation. Ideally, observations based on these simulations would motivate reformulation of students' expectations and beliefs. According to the pretest, however, only 3 of 16 students began the labs with expectations about the Four-heads Problem consistent with the representativeness heuristic, 5 of 16 in the case of the H/T Sequence Problem. For them, the labs appeared to be effective. For those who did not employ the representativeness heuristic in thinking initially about these problems, the labs appeared to have little effect, at least in inducing a notion of independence. One explanation for this is that expectations based on the outcome approach were never called into question by the data they collected. Indeed, outcome-oriented students may pose more of a challenge to educators that those who harbor a belief in representativeness, both because there may be more of them and also because it is hard to think of empirical results that would challenge their beliefs. Virtually every result of a simulation with coins or die would appear to support the expectation, based on the outcome approach, that "anything could happen." In fact, the variation in results of replications might serve to strengthen rather than undermine such a belief.

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Table 1. Four Heads Problem

Response	Group Remedial (N=23)	 Summermath (Pre) (N=16)	 Stat Methods (N=49)
a. Heads	2 (9%)*		
b. Tails	5 (22%)	3 (19%)	2 (4%)
c. Equal	16 (70%)	13 (81%)	47 (96%)

*Number of students responding (with percent in parentheses)

Table 2. H/T Sequence Problem

	Group					
	Remedial		Summermath (Pre)		Stat Methods	
	(N=23)	(N=23)	(N=16)	(N=15)	(N=47)	(N=41)
Sequence	Most	Least	Most	Least	Most	Least
a. HHHTT	4 (17%)	2 (9%)	0	1 (7%)	0	3 (7%)
b. THHTH	3 (13%)	1 (4%)	4 (2%)	0	1 (2%)	1 (2%)
c. THTTT	1 (4%)	2 (9%)	0	5 (33%)	1 (2%)	11 (27%)
d. HTHTH	0	10 (43%)	1 (6%)	6 (40%)	5 (11%)	7 (17%)
e. Equal	12 (61%)	8 (35%)	11 (69%)	3 (20%)	37 (79%)	19 (46%)
f. a,b,d	1 (4%)				3 (6%)	

Table 3. Summermath (Post test: B/G Sequence Problem (N=14)

Response	Percent of Responses	
	Most likely	Least likely
a. BBBBG	7	0
b. GBBGB	21	0
c. GBGGG	0	29
d. BGBGB	0	43
e. Equal	71	29

Table 4. Summary of Justifications of Correct Responses: Four Heads and and Die Problems

Justification	Four Heads Summermath (Pre)	Four Heads Remedial	Die Summermath (Post)
Independent trials	6 (46%)	2 (13%)	8 (57%)
Elementary outcomes are equally likely	5 (39%)	7 (44%)	5 (36%)
Either could happen	2 (15%)	4 (25%)	1 (7%)
Other or blank		3 (19%)	

Table 5. Pre/post Performance on H/T Sequence Problem

Answer: Posttest	Answer: Pretest Fallacy	Inconsistent	Correct	Total
Fallacy	1	1	1	3
Inconsistent	0	5	0	5
Correct	3	0	1	4
Total	4	0	2	

Appendix 16

END

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