DOCUMENT RESUME

ED 324 242 SE 051 633

TITLE	Statement on Competencies in Mathematics Expected of Entering Freshmen.
INSTITUTION	California Community Colleges, Sacramento. Academic Senate.; California State Univ., Sacramento. Academic Senate.; California Univ., Sacramento. Academic Senate.
REPORT NO	ISBN-0-8011-0835-5
PUB DATE	Oct 89
NOTE	61p.
AVAILABLE FROM	Bureau of Publications, Sales Unit, California State Department of Education, P.O. Box 271, Sacramento, CA 95802-0271 ($4.00 plus sales tax for California residents).
PUB TYPE	Guides - Non-Classroom Use (055)
EDRS PRICE	MF01/PC03 Plus Postage.
DESCRIPTORS	Academic Standards; *Algebra; Calculus; College Bound Students; *College Freshmen; College Mathematics; Course Content; Course Descriptions; Curriculum Guides; *Geometry; Higher Education; *Mathematics Achievement; Mathematics Curriculum; Mathematics Education; *Minimum Competencies; Program Content; Secondary Education; *Secondary School Mathematics
IDENTIFIERS	*California

ABSTRACT

This book is intended for everyone in California concerned about how well students are prepared for college. It describes the competencies in mathematics necessary for success in college and university work. This statement discusses the following: (1) an emphasis on encouraging students to experience the beauty and fascination of mathematics; (2) stress on building students' confidence in their reasoning power; (3) insistence on developing analytic reasoning ability; (4) advocacy of students' application of the concepts of algebra and geometry to the solution of unfamiliar problems; (5) recognition of the importance of students' developing complete understanding as a way of building their confidence in mathematics; (6) a call for students to have a history of successful experiences in finding solutions when encountering new mathematical situations; (7) stress on the need for students to discuss and write about mathematical ideas; and (8) an elaboration of appropriate uses of calculators. The book is divided into four parts: (1) "Expectations for Entering Freshman"; (2) "Prerequisites to Algebra 1"; (3) "Course Outlines for Algebra 1, Geometry, and Algebra 2"; and (4) "Outlines for Advanced Courses in Mathematics." The appendixes include: sample problems; competencies in speaking, listening, reasoning, and studying; the recommendation of the Board of Governors; and a discussion of the problems with calculus at the high school level. (KR)

STATEMENT ON COMPETENCIES IN MATHEMATICS EXPECTED OF ENTERING FRESHMEN

The Academic Senates of the
California Community Colleges
The California State University and
The University of California

This document was developed by the Academic Senates of the California Community Colleges, the California State University and the University of California. It was distributed by the Intersegmental Coordinating Council and the California State Department of Education.

Copies of this publication are available for $4.00, plus sales tax for California residents, from Bureau of Publications, Sales Unit, California State Department of Education, P.O. Box 271, Sacramento, CA 95802-0271. (916) 445-1260

Information about the development c ° of this statement or about other Competency Statements is avail. ; through the Intersegmental Coordinating Council, 501 L Street, Suite 101, Sacramento, CA 95814. (916) 324-8593

ISBN 0-8011-0835-5

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A PROJECT OF
THE ACADEMIC SENATES OF THE
CALIFORNIA COMMUNITY COLLEGES
THE CALIFORNIA STATE UNIVERSITY AND
THE UNIVERSITY OF CALIFORNIA

DISTRIBUTED BY
THE INTERSEGMENTAL COORDINATING COUNCIL AND
THE CALIFORNIA STATE DEPARTMENT OF EDUCATION

OCTOBER 1989

Printed by the California Community Colleges
Chancellor's Office

INTERSEGMENTAL COMMITTEE OF THE ACADEMIC SENATES
of the
California Community Colleges, The California State University
and the University of California

June 21, 1989

Dear Colleague:

This Statement on Competencies in Mathematics Expected of Entering Freshmen updates the one first published in 1982 and, like the earlier one, has been produced jointly by the Academic Senates of the California Community Colleges, the California State University, and the University of California with the cooperation of high school teachers of mathematics. Its primary intent is to state clearly and concisely the fundamental competencies in mathematics expected of students who embark upon any baccalaureate degree programs.

This statement, while not fundamentally changing the recommended curriculum, goes beyond it in many ways, specifically by its emphasis on encouraging students to experience the beauty and fascination of mathematics and its stress on students' ability to solve problems with confidence. The examples included in Appendix A illustrate the types of problems students should be able to attack without serious difficulty after having acquired the suggested competencies. Appendix A, therefore, should be considered an integral part of this document.

The statement sets forth the position of postsecondary mathematics faculty that students entering college should have studied mathematics throughout their high school years, including both semesters of their senior year.

Both CSU and UC require three years of mathematics - elementary algebra, geometry, and intermediate algebra - for admission. Our faculty statement does not establish admission requirements but rather advises on the optimum preparation in mathematics for entering freshmen.

The California Education Round Table and the State Department of Education have encouraged the work of the faculties in preparing the statement; they have endorsed the statement and have assisted in its publication and dissemination. Many faculty members and teachers have contributed to the development and review of the document.

As you read the document, bear in mind that it is an expression by the faculties of our colleges and universities as to the mathematics education students should have <u>prior to</u> college entrance. Different expressions as to what is ideal or feasible are expected, and modifications to and refinement of the statement are anticipated in the future.

It is important that the statement be widely disseminated and discussed in seminars, conferences, and workshops. We seek your support in arranging for these activities, which we trust will lead to further curriculum development in mathematics throughout the State. Your comments will be welcomed by the faculties and by the Round Table.

Richard W. Gable
Chair
Academic Senate - UC

Ray Ceigle
Chair
Academic Senate - CSU

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President
Academic Senate-CCC

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Introduction

This Statement on Competencies in Mathematics Expected of Entering Freshmen is intended for everyone in California concerned about how well students are prepared for college. It describes the competencies in mathematics necessary for success in college and university work. It has been prepared by a committee of college and high school teachers of mathematics, based primarily upon the college teachers' observation of their students, and endorsed by the Academic Senates of the University of California, the California State University, and the California Community Colleges.

The first Statement on Competencies in Mathematics Expected of Entering Freshmen was written in 1982. The California State Department of Education has since published the *Mathematics Framework for California Public Schools Kindergarten through Grade Twelve* and the *Model Curriculum Standards Grades Nine through Twelve,* both published in 1985. In recognition of these documents (briefly described in Appendix E) and the exciting curriculum in mathematics education that they recommend, the Intersegmental Committee of the Academic Senates asked that the original Statement be reviewed and revised. The writing committee notes that such groups as the Mathematical Sciences Education Board are likely to advocate changes in the college preparatory curriculum in the next few years; we recognize that this Statement is a product of its time and presume that it, too, will be reviewed and revised as such changes gain consensus in the mathematics community.

This Statement updates the previous statement by elaborating its intent and taking account of the1985 State Department documents. It does not fundamentally change the recommended curriculum; it does go beyond the original statement, however, in

- its emphasis on encouraging students to experience the beauty and fascination of mathematics,
- its stress on building students' confidence in their reasoning power,
- its insistence on developing analytic reasoning ability,
- its advocacy of students' application of the concepts of algebra and geometry to the solution of unfamiliar problems,
- its recognition of the importance of students' developing complete understanding as a way of building their confidence in mathematics,
- its call for students to have a history of successful experiences in finding solutions when encountering new mathematical situations,
- its stress on the need for students to discuss and write about mathematical ideas, and
- its elaboration of appropriate uses of calculators.

We hope that students, teachers, and administrators will do everything that they can to ensure that all of the expectations are realized.

The descriptions of qualitative expectations in Section I are intended to be read by students, parents, mathematics teachers, curriculum specialists, principals, and school board members. The more detailed descriptions of specific topics and competencies in Sections II-IV are intended as a curriculum guide for mathematics teachers, department chairs, and curriculum specialists; we urge them to develop and maintain programs and courses that prepare students to meet all expectations in this Statement. These programs will also benefit students whose current plans do not include immediate enrollment in a postsecondary institution. We encourage principals and school boards to accept responsibility for promoting the goals of this Statement by becoming familiar with its goals and disseminating it among the mathematics teachers in their schools.

Section I characterizes the type of high school program that will achieve the expectations. Section II describes the kindergarten through grade eight program that is aligned with the *Mathematics Framework for California Public Schools Kindergarten through Grade Twelve* and the *Mathematics Model Curriculum Guide, Kindergarten through Grade Eight.* Sections III and IV list topics of basic and advanced college preparatory courses in mathematics. These course descriptions are illustrated and brought to life by sample problems in Appendix A; students possessing the ability to solve such problems with confidence certainly have acquired the expected competencies of this Statement. Appendix A should, therefore, be considered an integral part of this document.

Students who have successfully completed the mathematics program we describe have the necessary mathematical foundation for success in college. We therefore must communicate these expectations in order to have them realized. Students who do not achieve these expectations for whatever reason will be considerably disadvantaged in their efforts to deal with quantitatively based college courses. Very often they will be assigned to remedial courses in college for which they will not receive baccalaureate credit and may be denied timely entry into many undergraduate majors. These are some of the reasons why we hope that these expectations will be realized.

Section I. Expectations for Entering Freshmen

The first part of this section enumerates the characteristics of entering freshmen well-prepared to study college mathematics. It describes students who have attained sufficient mathematical maturity to be able to achieve success in a first college mathematics course, be it calculus, statistics, or discrete mathematics. The second part of this section offers suggestions to teachers of ways to present mathematics to help their students develop these characteristics. The third part of this section provides recommendations for the design of secondary school mathematics programs, including enrollment and counseling policies, that will support the efforts of students and teachers.

This section is limited to expectations that relate particularly to mathematics. The committee recognizes that there are general academic competencies, especially reading and writing, required for successful work at the college level and endorses the work of the College Board in articulating basic academic competencies. Selected portions of their statement are included in Appendix B.

In preparing this Statement, the Committee also reviewed the general recommendations of the Board of Governors of the Mathematical Association of America and the Board of Directors of the National Council of Teachers of Mathematics. Our recommendations are consistent with, though not identical to, those of these two organizations, which are reproduced in Appendix C.

Part 1. Characteristics of students who are mathematically well-prepared for college work

Students should have studied mathematics throughout their high school years, including both semesters of their senior year. Their choice of courses should take account of the requirements of different undergraduate majors. From their mathematics courses, students should have gained:

- a sense of number and the ability to discern whether a proposed numerical answer to a problem is reasonable.
- the ability to use their mathematical knowledge to confront unfamiliar problems both in concrete and abstract situations. They should be able to formulate hypotheses about the problem at hand and then test them against available data. They should be aware of the analogy between mathematical structures and phenomena in the real world.
- the ability to discuss the mathematical ideas involved in a problem with other students and to write coherently about mathematical topics and their interrelation.

- both informal and analytic reasoning abilities. They should view mathematics as an interplay between intuition and reason. They should understand mathematical implication and know why various mathematical statements follow from more basic ideas.

- general algebraic proficiency. They should be able to manipulate algebraic expressions and to check these manipulations. They should have a feel for what manipulation is necessary to convert a complex algebraic expression to one that is manageable.

- a well-developed geometric intuition. They should be able to visualize problems geometrically. They should also understand intuitively the qualitative and geometric properties of elementary functions. They should have an understanding of the connections between algebra and geometry.

- an understanding of the way the various parts of mathematics, e.g., algebra and probability, are interrelated.

- a view of mathematics as a creative way of thinking. They should not view mathematics as the mechanical application of an assortment of algorithms to be applied to a collection of problems.

Part 2. Suggestions for mathematics teachers to help students to achieve these expectations

Regardless of the content of the mathematics course being taught, there are certain features of exemplary mathematics teaching that promote the development of the student characteristics described in Part 1. Not only must teachers emphasize the content of the course, but also they must communicate the beauty and fascination of mathematics, the reasoning and questioning in mathematics, and the confidence needed to overcome uncertainty. Finally, they must show that judicious use of calculators will support the learning and doing of mathematics.

Beauty and Fascination

The teaching of mathematics must permit students to experience the beauty and fascination of mathematics. An appreciation of the aesthetics of mathematics should permeate the mathematics curriculum and should motivate the selection of some topics. Students need opportunities to enjoy mathematics.

Building Confidence

Students' successful use of mathematical ideas and techniques to solve problems and develop complete understanding builds confidence in their reasoning power. Mathematical experiences ought to increase students' willingness to experiment with mathematical ideas and to tackle uncertainty in new and unexpected situations. The mathematics classroom must be an environment that supports and rewards inquiry, exploration, persistence, and reasonable risk-taking. A positive instructional climate avoids the development of negative feelings about mathematics and can help to cultivate a quality of analytic thinking that is required for undergraduate study.

Development of Analytic Ability and Logic

The instructional emphasis at all levels should be on a thorough understanding of the subject matter and the development of logical reasoning. Students should be expected to question and to explore why one statement follows from another. Experimentation and discovery should be emphasized, and students should be encouraged to explain ideas in their own language. The development of analytic and logical reasoning is, in fact, more valuable than the mastery of any particular topic or algorithm.

"Problem Solving"

Students entering college should be prepared to solve problems. By this we mean that students encounter new mathematical situations with confidence that they will be able to proceed with a likelihood of success. They should know from experience that learning mathematics includes guessing, analyzing, and revising; and they should understand that attempts may not work out readily, requiring time as well as trial and error before an adequate solution is reached. They should also know that there is often more than one successful approach to a solution and sometimes more than one valid solution. Students should be flexible in their thinking and creative in their approaches, have the ability to construct mathematical models (including approximations or simplifications) of situations, and to approach sensibly the mathematical questions posed by those models. Students must realize that an understanding of the results obtained is an essential component of every solution to a mathematical problem.

Problem solving is <u>not</u> a collection of specific techniques to be learned; it cannot be reduced to a set of procedures. Students entering college should have had experience solving a wide variety of mathematical problems, thus building a history of successful experiences. The goal of teaching problem solving is the development of open, inquiring, and demanding minds. Experience in solving problems gives students the confidence and skills to approach new situations creatively, using the tools available to them by modifying, adapting, and/or combining these tools, and gives students the determination to refuse to accept an answer until they can explain it and understand how it contributes to a solution of the problem.

Calculators

High school mathematics courses should make full and appropriate use of hand calculators. When properly incorporated in the curriculum, the calculator:

- removes student proficiency in lengthy arithmetical computations as a barrier to success both in mathematics and in the application of mathematics to other subject areas.

- provides students additional time to think through problems and to organize their thinking better as they make explicit the organization and procedures needed to solve a particular problem.

- permits students to concentrate their attention on such important aspects of problem solving and theory development as formulating questions and conjectures, devising and evaluating strategies, and verifying and interpreting solutions.

- enables students to deepen their understanding of the function concept, particularly the elementary functions of algebra and trigonometry.

Students should be expected to determine when the use of the calculator will be effective and efficient. The student should develop and maintain an understanding of the meaning of arithmetic operations and the common algorithms used to perform these computations. Successful use of calculators, in fact, requires that students estimate the magnitude of a result and perform elementary mental arithmetic. These skills, as well as the operation of the calculator itself, must be explicitly taught to students, preferably in the context of instruction on a mathematical topic. The development of the concept of variable, for example, is a good example of a vehicle for instruction in these skills.

Part 3. Characteristics of secondary school programs that will enable students and teachers to achieve these expectations

Students will be able to achieve the expectations set out in Part 1 and teachers those in Part 2 only if they are taking part in a mathematics program with policies that provide support for their achievement. Thoughtful and consistent enrollment, counseling, and curriculum policies are essential.

Enrollment policies

- Students should begin the study of algebra only after they have developed the mathematical understandings from the Kindergarten through grade 8 curriculum outlined below in Section II.

- For proper preparation for baccalaureate level work, all students should take a mathematics course every semester in high school. The topics covered in these courses should include those typically contained in one-year courses in elementary algebra (Algebra 1), geometry of two and three dimensions, and advanced algebra (Algebra 2). Content recommendations for these courses are contained in Section III. It should be noted that the traditional course sequence is not the only acceptable sequence for covering the material in these course outlines. Schools are encouraged to seek alternative course sequences that accomplish the same goals.

- Taking a mathematics course in the senior year is particularly important. This course could be Algebra 2 or a more advanced course depending on the student's background. Suggestions for advanced courses are contained in Section IV. These traditional courses are also only suggestions, not the only possible advanced courses in mathematics.

- Entering students who intend to pursue a baccalaureate degree in fields requiring the study of calculus in college should complete the courses in trigonometry, analytic geometry, and mathematical analysis outlined in Section IV in high school. Alternative course titles and organizations are possible for preparing for calculus as long as the topics are covered and the competencies are developed.

- Students should be encouraged to take other courses that make significant use of mathematics.

- Calculus should be taken by high school students only if they have demonstrated mastery of algebra, geometry, trigonometry, and coordinate geometry. The calculus course should be treated as a college level course and should prepare students to take one of the College Board's Advanced Placement Examinations, with the expectation that those who are performing satisfactorily in the high school course will place out of the comparable college calculus course. A joint statement of the Board of Governors of the Mathematical Association of America and the Board of Directors of the National Council of Teachers of Mathematics concerning calculus in the secondary school is included as Appendix D.

Counseling Policies

- Counseling of students and their parents concerning college preparation should occur as early as possible to provide a foundation for successful college and university study and to broaden the students' career choices. Students should be encouraged to take mathematics every year; college-bound students should be especially encouraged to take Algebra 2 or more advanced mathematics throughout their senior year. Early counseling is needed especially to encourage students from groups underrepresented in quantitative majors in California colleges and universities.

- Every effort should be made to assure that students are adequately prepared for the mathematics courses that they take. Special materials and programs should be offered to students who are almost ready for the next college-preparatory mathematics course. Students whose performance is quite poor should not be forced to repeat courses, but should be placed in alternative courses that seek to prepare them for a return to the standard mathematics curriculum. One alternative curriculum for some students is the Math A, B, C sequence described in the *Mathematics Framework for California Public Schools, K-12.*

- Broad-based diagnostic examinations should be given regularly to assess student competencies in mathematics. Every student should take at least one such examaination no later than the end of the student's junior year. The results of such examinations should be one important factor in counseling students concerning their mathematics studies. Students who are not adequately prepared to go on to the next mathematics course should be provided alternative programs that strengthen their weaker skills and prepare them for further study . Although diagnostic tests alone are not adequate tools for evaluating programs, results from diagnostic tests should be considered in mathematics program assessments, especially if there are substantial numbers of students who are not making adequate progress.

Curriculum Policies

Probability and Statistics

Adequate preparation for many careers, not only in the natural and social sciences, business, and education, but also in nursing and other professional fields, now requires at least one college course in probability and statistics. This makes it desirable for students to acquire probability and statistics concepts throughout the entire mathematics curriculum starting as early as possible. The inclusion of probability and statistics in the mathematics curriculum should enable students to collect, organize, analyze, and interpret data, to draw valid conclusions from the analysis of the data, and to recognize and avoid misuses of statistics.

To have these concepts permeate all parts of the mathematics curriculum will also be of substantial value to students not expecting to take a college course in quantitative subjects since probability and statistics have applications that all students are likely to encounter throughout their lives. In addition, simple concepts can be used to solve realistic problems without an extensive background in mathematics. For slightly more advanced students, topics from probability and statistics can also serve as an excellent device to use mathematics previously learned and thereby motivate and reinforce it in an effective manner.

Computer Programming

To the extent that familiarization with the computer is part of the high school mathematics program, computers should be used to enhance the learning of the mathematics curriculum. Programming instruction should not displace essential mathematics topics or courses.

Compression of Courses

The courses detailed in Sections III and IV contain many essential topics. It is not desirable to attempt to cover more material in a year than is contained in any of these course outlines. It is more important to ensure that students understand the material called for in the courses and the other competencies enumerated in this Statement.

The content of an Algebra 2 course taught over 36 weeks should comprise the topics listed in Section III. In particular, none of these topics should be compressed or omitted to allow for a more substantial treatment of Trigonometry than listed in the last item of the content of Algebra 2 in Section III. Trigonometry should comprise a full semester's introduction, either alone or as suggested in the Mathematics Framework as part of a one-year course in Mathematical Analysis.

The content of Algebra 2 is critical for student success in more advanced mathematics. Therefore, adequate time should be allowed for its topics to be mastered. Attempts to compress this year's worth of work into a six- or eight-week summer course should be avoided. Offering both the first and second semester courses every semester is a far better solution to the problem of students failing one semester and having to repeat it.

It should not be necessary for Algebra 2 to include a substantial review of topics from Algebra 1. Topics from Algebra 1 should be kept fresh by their continued use in Geometry.

The Mathematical Analysis course also contains a large number of critical topics. The practice of introducing several weeks of calculus material into this course so that students will have a period of review when they take their first calculus course provides only short term gains and weakens the foundation that a full year of Mathematical Analysis will provide for the full array of college mathematics courses. Further rationale for this statement is included in Appendix D.

Other Desirable Practices

Grades in high school mathematics courses should be based upon achievement rather than upon effort or attendance so that students will receive accurate assessment of their competencies and be in a good position to plan their academic programs.

Each college preparatory mathematics course should include a comprehensive final examination. The implementation of this recommendation can take several forms, depending upon the final examination procedure of the school, and may have to be built into a sequence of regular class periods. In any case, students need to have the experience of consolidating the material of the entire course.

Applications of mathematics should be included in the mathematics curriculum on a regular basis.

Time and effort should be given to assure continuity of the mathematics curriculum. In addition to the topics listed for a particular course, concepts from previous courses should be reviewed and applied so they remain fresh in the student's mind.

No matter how the topics listed in this document are packaged into courses at a particular educational institution, mathematics departments should make sure that all of the topics are covered adequately. To this end, departments should periodically review their course outlines and, through the use of clear prerequisites, assure their students a good chance of success in their own courses as well as in transfer institutions.

Section II. Prerequisites to Algebra 1

The high school college preparatory curriculum defined in this document assumes a kindergarten through eighth grade curriculum that is consistent with the *Mathematics Framework for California Public Schools Kindergarten through Grade Twelve* and the *Mathematics Model Curriculum Guide, Kindergarten through Grade Eight*. The *Framework* addresses the importance of mathematical power, the ability to discern mathematical relationships, reason logically, and use mathematical techniques to solve problems for which the method of solution is not clear at the outset. By the end of the eighth grade, students should be able to pose, argue, prove, or demonstrate mathematical statements. Students should know from the context of the problem and the numbers involved how and when to estimate and to use a calculator or paper and pencil to perform the necessary calculation. They should also have studied all of the "strands" of mathematics. In particular, this means that students should have:

- a sense of number and an understanding of the basic operations with integers, fractions, decimals, and percent and their relationships to one another, and be able to compute with positive exponents and square roots of perfect squares.
- sufficient experience with the measurement of length, area, volume, and angles of regular and irregular figures and solids that they recognize the considerations in selecting appropriate units and tools of measurement according to properties of the quantity being measured and the need for accuracy.
- an appreciation of the approximate nature of measurement & an understanding of scale.
- an ability to analyze real objects and abstract figures in two and three dimensions, including the ability to identify attributes and properties by which they are classified and named.
- an ability to demonstrate relationships among geometric elements, to perform transformations and constructions, and to express observed patterns as formulas.
- a view of functions as an extension of their experience of patterns from a variety of settings, and an understanding that the value of the first quantity determines the corresponding value of the second.
- a recognition that representation of a function by means of a table, graph, ordered pairs, verbal statements, or algebraic rules allows it to be extended indefinitely.
- an ability to frame questions about which they can gather data, design a suitable method for collecting data from a sample or census, compute measures of central tendency and other simple statistics, represent the data in formats that communicate meaning, interpret their results, and identify possible sources of error in their interpretatior
- a substantial background in informal probability so that they can determine relative likelihood of simple events, compute empirical probabilities, and systematically generate combinations and permutations.
- an ability to classify real objects and some abstractions in multiple categories.
- an ability to assess the validity of simple arguments, and determine the equivalence of logical expressions.
- an understanding of variables that permits them to represent mathematical patterns with variables, perform substitutions, simplify algebraic expressions, solve linear equations and simple inequalities.
- a recognition of the relationship between the properties of operations on numbers and the properties of operations on variables.
- an ability to translate verbal problems into mathematical statements, and vice versa.

Section III. Course Outlines for Algebra 1, Geometry, and Algebra 2

The course outlines that follow have been taken directly from the *Mathematics Framework for California Public Schools Kindergarten through Grade Twelve* (see Appendix E for ordering information). As the *Framework* indicates, these traditional courses are only one of the possible ways of organizing the topics. Other approaches that cover the content of the traditional sequence are possible. These topics are essential, but the course organization is flexible.

These course descriptions are illustrated and brought to life by sample problems in Appendix A; students possessing the ability to solve such problems with confidence certainly have acquired the expected competencies of this Statement.

Algebra 1

- Variables; algebraic symbols and expressions; evaluation of expressions and formulas; translation from words to symbols
- Integers, rational numbers, real numbers, and the properties of each; absolute value
- Solutions and applications of linear equations and linear inequalities
- Polynomial expressions and operations; factoring techniques
- Rational expressions and operations; rational equations (including proportions) and applications
- Radicals, operations, and simplifications; algebraic expressions and equations involving radicals
- Solutions and graphs of linear equations and inequalities: concepts of slope, y-intercept, parallelism and perpendicularity
- Linear systems in two variables—solution by graphical and algebraic techniques
- Solutions (by factoring, completing the square, and using a formula) and applications of quadratic equations
- Application of quadratic solutions to maximum, minimum problems
- Operations with expressions involving integral exponents
- Applications of right-triangle trigonometry
- Development of the concept of function and graphs of functions
- Solution of word problems, including geometric as well as algebraic applications
- Introduction to probability; statistical measures of central tendency and dispersion

Geometry

- An adequate set of postulates to support the proof of geometric theorems
- Perpendicularity, parallelism, congruence, and similarity relationships in two and three dimensions
- Mensuration theory, as applied to developing the formulas for measuring lengths, perimeters, areas, and volumes of common geometric figures

- Coordinate geometry, including slope of a line, midpoint of a segment, distance formula, point-slope and intercept forms of the equation of a line, and equation of a circle
- Relationships in circles and polygons—algebraic applications
- Introduction to transformational geometry, including translations, reflections, rotations, and similarity transformations
- Original proofs and proofs of theorems including direct, indirect, coordinate, and transformational techniques
- Introduction to symbolic logic
- Locus, geometric constructions, and concurrence theorems
- Use of algebraic concepts and skills in geometric situations, including solution of problems involving the Pythagorean theorem
- Introduction to the trigonometric functions of angles greater than 90° and special angle relationships

Algebra 2

Note: The topics of Algebra 2, here listed as a one-year course, are also offered in some institutions as two one-semester courses with names such as Intermediate Algebra, Advanced Algebra, College Algebra, etc. The topics constitute, however, a full-year's work, and should not be compressed by the addition of a Trigonometry course.

- Simplification of algebraic expressions, including fractional exponents and radicals
- Solution of linear equations and inequalities, including those involving absolute value and rational expressions
- Operations on polynomials
- Solution of quadratic equations by factoring, completing the square, and using a formula; properties of roots
- Solution of quadratic inequalities
- Solution of poylnomial equations; rational roots; Descartes' Rule of Signs
- Solution of systems of linear equations with two and three variables; homogeneous, dependent, and inconsistent systems; use of determinants and matrices
- Linear, quadratic, and polynomial functions—their graphs and properties
- Introduction to graphing of quadratic equations in two variables—conic sections
- Permutations and combinations; the binomial theorem
- Arithmetic and geometric sequences and series
- Function concept, including notation, composition, inverse, and arithmetic operations on functions
- Graphs of exponential and logarithmic functions; solution of exponential and logarithmic equations
- Solution of word problems, including estimation and approximation
- Complex numbers—notation, graphical representation,and applications
- Solution of systems of quadratic equations in two unknowns
- Mathematical probability; finite sample spaces; conditional probability
- Trigonometric applications—solution of oblique triangles through the laws of sines and cosines

Section IV. Outlines for Advanced Courses in Mathematics

Mathematical Analysis

Note: The topics of Mathematical Analysis, here listed as a one-year course, are also offered in some institutions as two one-semester courses with names such as Trigonometry, Math Analysis, Analytic Geometry, etc. The topics constitute, however, a full-year's work, and should not be compressed by the addition of Calculus topics.

- Development of the trigonometric functions using the concept of circular functions
- Graphical characteristics of the trigonometric functions—including translations, amplitude, change of period, domain, range, sums and differences of functions
- Inverse trigonometric functions—notations and graphs
- Trigonometric identities, including addition and double-angle and half-angle formulas
- Use of degree and radian measures
- Solution of trigonometric equations
- Polar coordinates and vectors; graphical representations
- Solution of problems related to force and navigation
- Trigonometric form of complex numbers and de Moivre's theorem
- Mathematical induction
- Analytic treatment of the conic sections, including translations and rotations
- Rational functions and their graphs
- Parametric equations and their graphs
- Lines and planes in space—three-dimensional coordinate geometry
- Introduction to vectors in the plane and space
- Characteristics of graphs of functions
- Concept of a limit: definition, application, convergence/divergence

Advanced Placement Calculus AB or BC

The content of these courses has been established by the College Board. Copies of these course outlines may be obtained directly from the College Board.

Linear Algebra (Semester Course)

Suggested Topics

- Matrix operations: order, equality, addition, multiplication by a scalar, matrix multiplication
- The algebra of 2 x 2 matrices: determinant function, invertible matrices, matrix representation of complex numbers
- Matrices applied to systems of linear equations

- Column matrices as geometric vectors: algebra of vectors, vectors and their geometric representation, inner product of two vectors, vector spaces and subspaces
- Transformations of the plane: rotations and reflections, linear transformations, matrix representations of transformations, characteristic vectors and values

Probability and Statistics (Semester Course)

Suggested Topics

- Permutations
- Combinations
- Empirical determination of probability
- Inclusive and exclusive events
- Dependent and independent events
- Bayes' theorem
- Measures of central tendency and dispersion
- Binomial and normal distributions
- Sampling techniques
- Testing simple statistical hypotheses
- Regression, correlation, and the significance of a correlation coefficient

Appendix A. Sample Problems

Students should understand and be able to solve most – although not necessarily all – the problems of this Appendix as they complete coursework in preparation for college. Students must be able to recognize when they have obtained a correct solution to a problem. Some of these problems have more than one suitable solution. These are two of the reasons why answers are not provided to the problems of this Appendix. The problems are organized to match the description of Sections III and IV. This set of problems is not intended to illustrate the full depth and range of a good high school mathematics curriculum.

Some problems, by virtue of their richness, cut across the categories and concepts of the curricular descriptions. Examples of such non-routine problems are presented for each course. Frequent experience with problems of this type helps students develop problem solving and thinking abilities.

ALGEBRA I

A. Variables; algebraic symbols and expressions; evaluation of expressions and formulas; translation from words to symbols

1. Given the expression $\frac{1}{2 - x}$,

   (a) choose five values of x and evaluate the expression for each;

   (b) explain why there is or is not a largest value for the expression; and

   (c) describe what happens to the expression when $x = 2$.

2. The braking distance of a car (how far it travels after the brakes are applied) is proportional to the square of its speed. Write a formula expressing this relationship and explain the meaning of each term in the formula.

B. Integers, rational numbers, real numbers, and the properties of each; absolute value

3. $(4)^{-2} =$

4. $\frac{3}{2} - \frac{5}{6} =$

5. $\frac{4}{7} + \frac{8}{9} =$

6. $7 - |3 - 5| =$

7. (a) If the product of two numbers is positive, what can you say about the numbers?

   (b) If the product is zero, what can you say?

C. Solutions and applications of linear equations and linear inequalities

8. Solve for x and give a reason for each step: $\frac{2}{3x+1} + 2 = \frac{2}{3}$

9. Given $3x + 2 \leq 5x - 7$, solve for x and give a reason for each step in your solution.

10. An airplane is flying west at a constant speed. At 1:00 p.m., the plane is 250 miles west of Chicago; at 3:00 p.m., it is 1200 miles west of Chicago.

    (a) Write a linear equation which expresses the distance d of the plane in miles west of Chicago in terms of the time t in hours past noon.

    (b) Where is the plane at noon?

D. Polynomial expressions and operations; factoring techniques

11. Expand $(3x - 1)^2$.

12. Expand $(x^2 - x + 5)(3x + 4)$.

13. Factor completely $27x^3 - 3x$ and $3x^2 + 20x + 12$.

E. Rational expressions and operations; rational equations (including proportions) and applications

14. Simplify: $\frac{2}{x^2 - 9} + \frac{1}{x+3} - \frac{3}{3-x}$.

15. Find the value of x which satisfies the equation $\frac{2x+1}{3x+2} = \frac{4}{5}$.

16. If y varies directly as x, and y is 36 when x is 4, find y when x is 6.

F. Radicals, operations, and simplifications; algebraic expressions and equations involving radicals

17. $\sqrt{25x^4} =$

18. Solve $\sqrt{x-1} + 5 = 11$.

G. Solutions and graphs of linear equations and inequalities: concepts of slope, y-intercept, parallelism, and perpendicularity.

19. A line L passes through points (1,2) and (3,–2).

    (a) Write an equation for L.

    (b) What is the slope, the x-intercept, and the y-intercept for L?

    (c) Write an equation for the line parallel to L with a y-intercept of 1.

    (d) Write an equation for the line with the same y-intercept as L, but perpendicular to L.

    (e) Is the point (1,1) above, below, or on L?

    (f) How can you determine if a point (h,k) is above L?

H. Linear systems in two variables -- solution by graphical and algebraic techniques

20. (a) Solve the system of equations $\begin{cases} 2x + y = 2 \\ x - 2y = 1 \end{cases}$ for x and y.

    (b) Interpret this solution graphically.

I. Solutions (by factoring, completing the square, and using a formula) and applications of quadratic equations

21. Find all values of x such that $2x^2 - x = 0$.

22. Find all values of x such that $3x^2 - 11x - 20 = 0$.

23. For what values of x is $f(x) = 2x^2 - 3x + 4 \geq 0$?

J. Application of quadratic solutions to maximum, minimum problems

24. An item that costs $13 is sold at a price s. The number sold, n, depends on the price according to $n = 100 - k(s - 15)$, with $n = 0$ when $s = 20$. What sales price will maximize the profit?

K. Operations with expressions involving integral exponents

25. Express $(x^4 y^{-3})(x^{-7} y^2)^{-1}$ with no parentheses and no negative exponents.

L. Applications of right-triangle trigonometry

26. A rope from the top of a flagpole is 10 feet longer than the pole. When stretched out the end of the rope touches the ground 25 feet from the pole. How high is the flagpole?

27. A person standing on level ground away from a building observes that the angle of elevation of the top of the building is 30°. After moving 526 feet closer to the building, the angle of elevation of the top of the building is observed to be 45°. How tall is the building?

M. Development of the concept of function and graphs of functions

28. Let $f(x) = \dfrac{3x + 2}{x - 1}$.

    (a) Determine $f(3)$.

    (b) Determine the value of x for which $f(x) = 1$.

29. Let $f(x) = x^2 + 1$ for $1 \le x \le 3$.

    (a) Sketch the graph of this function.

    (b) What is the domain of this function?

    (c) What is the range of this function?

N. Solution of word problems, including geometric as well as algebraic applications

30. A company currently owns a copy machine that takes 5 hours to print 5,000 copies of a newsletter.

    (a) If the company buys a second copier that prints 1,500 copies per hour and uses both machines, how long will it take to print the 5,000 copies of the newsletter?

    (b) Illustrate the solution graphically.

31. (a) Should an $8^1/_2$ by 11 inch sheet of paper be rolled lengthwise or widthwise to create the cylinder of greater volume?

    (b) If the cylinder will be covered top and bottom, which way of rolling the cylinder will give the greater surface area?

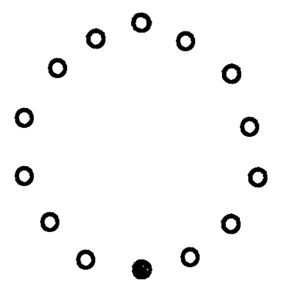
    (c) Generalize to any rectangular sheet of paper.

O. Introduction to probability; statistical measures of central tendency and dispersio

32. Construct a possible histogram of students' ages for a group of 18 students if the mode is 15, the median is 14, and the mean is between 13.5 and 14.

P. Non Routine Problems

33. A ring of "stepping stones" has 14 stones in it, as shown in the diagram.



A boy hops around the ring, stopping to change feet every time he has made 3 hops. He notices that when he has been around the ring three times, he has stopped to change feet on each one of the 14 stones.

(a) The boy now hops around the ring, stopping to change feet every time he has made 4 hops. Explain why in this case he will not stop on each one of the 14 stones no matter how long he continues hopping around the ring.

(b) The boy stops to change feet every time he has made n hops. For which values of n will he eventually stop on each one of the 14 stones to change feet?

(c) Find a general rule for the values of n when the ring contains more (or less) than 14 stones.

34. Prepare a report for the sponsor of a concert to be attended by 150,000 people that will guide the design and selection of a suitable site. Explain your assumptions, the reasoning, and evidence upon which they are based, and the quantitative consequences of errors in the assumptions. Explain the formulas you use to make calculations. With tables, graphs, diagrams, and explanations show the effects of various decisions on

(a) the area and shape of seating area;

(b) the distances of sight lines to the stage for various shapes and slopes of seating areas; and

(c) the waiting time at admission gates for various numbers of gates and at parking lot entries for various numbers of access roads.

GEOMETRY

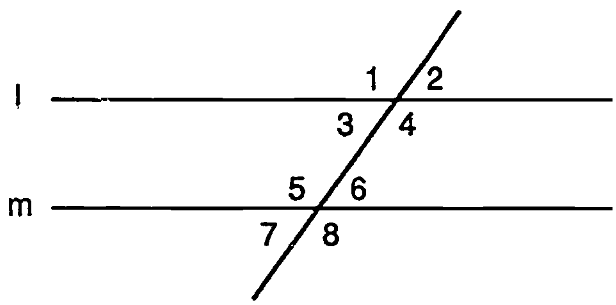
A. An adequate set of postulates to support the proof of geometric theorems

1. (a) Explain how a postulate differs from a theorem.

   (b) Give three examples of the postulates of Euclidean Geometry.

B. Perpendicularity, parallelism, congruence, and similarity relationships in two and three dimensions

2. The lines l and m in the figure below are parallel:



   (a) Which angles are equal to each other?

   (b) How many of the angles would you need to know in order to determine all the angles?

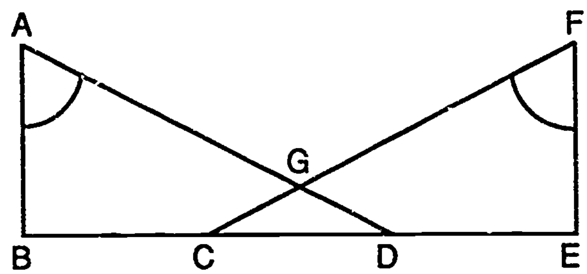
   (c) Explain your reasoning.

3. In the figure below: $\angle A = \angle F$

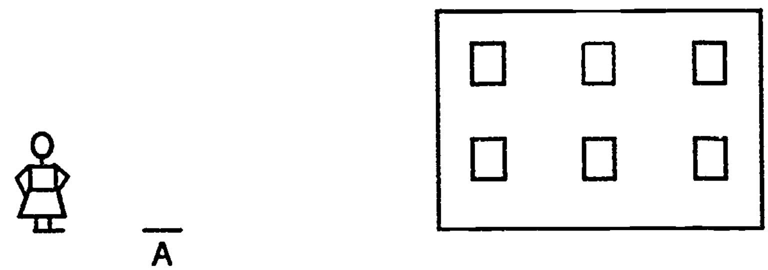
$\angle B$ and $\angle E$ are right angles

$BC = DE$

What can you say about the relationship between ΔABD and ΔFEC? Explain your reasoning.



4. The woman in the picture below wishes to know the height of the building. She can see the top of the building in the middle of her mirror lying on the ground at point A. She knows the distance from her feet to her eyes.



(a) What other distance(s) can she most easily measure that would enable her to compute the height of the building?

(b) Explain your reasoning.

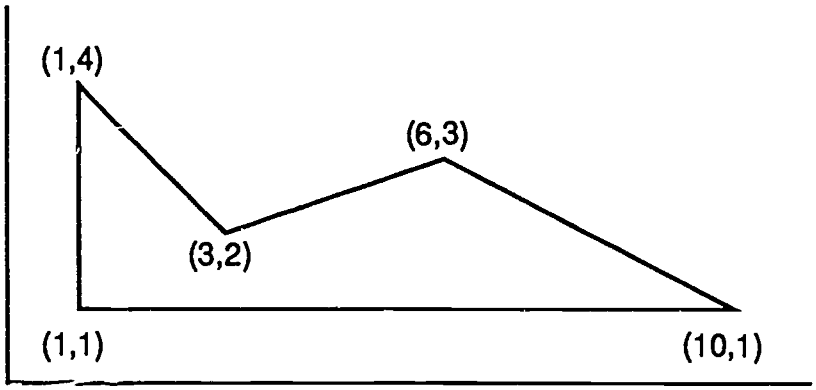
(c) Write a formula for computing the height of the building in terms of these distances.

29

C. Mensuration theory, as applied to developing the formulas for measuring lengths, perimeters, areas, and volumes of common geometric figures.

5. What is the surface area of a cube with a volume of 27 cm^3 ?

6. In the graph below, what is the area enclosed by the figure? The coordinates of the vertices are given in parentheses.



7. The center of a sphere of radius 14 cm. is at a distance of 5 cm. from a plane. What is the area of the plane inside the sphere?

D. Coordinate geometry, including slope of a line, midpoint of a segment, distance formula, point-slope and intercept forms of the equation of a line, and equation of circle

8. (a) Graph $5x + y = 8$.

   (b) Find the distance along this line from the point where $x = 3$ to the point where $y = 1$.

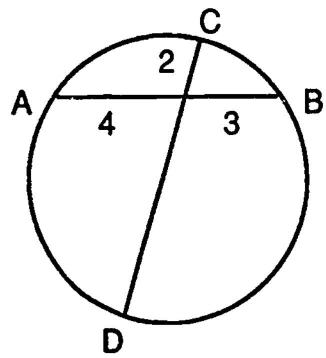
   (c) What are the coordinates of the midpoint of this segment?

9. What are the coordinates of the intersection of the line $x + 2y = 10$ and the circle $x^2 + y^2 = 25$?

E. Relationships in circles and polygons--algebraic applications

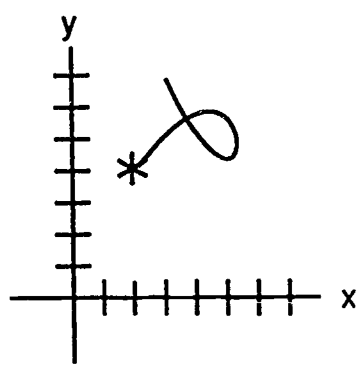
10. Explain why you can tile a plane with regular hexagons but not with regular pentagons.

11. In the figure shown to the right, AB and CD are chords of a circle. Find the length of CD.



F. **Introduction to transformational geometry, including translations, reflections, rotations, and similarity transformations**

12. The star in the picture shown to the right is at (2,4). Sketch the following transformations of the star and its attached curve and give the coordinates of the images of the star.



(a) reflection through the y-axis

(b) 180° rotation around (0,0)

(c) translation of –2 on the x-axis and –8 on the y-axis

13. (a) Sketch a figure other than a circle which has rotational symmetry through a rotation of 60°.

(b) Does your figure have any symmetry of reflection through some line?

(c) Describe the reflection and explain your reasoning.

G. Original proofs and proofs of theorems, including direct, indirect, coordinate, and transformational techniques

14. Prove the following theorem. Two lines which are not colinear and are perpendicular to the same plane do not intersect.

15. Use coordinate geometry to prove that the diagonals of a parallelogram bisect each other.

H. Introduction to symbolic logic

16. (a) If a implies b, can you deduce that b implies a? Explain.

    (b) If a implies b and b is false, what can you say about a? Explain.

I. Locus, geometric constructions, and concurrence theorems

17. Sketch the locus of points on a plane which are:

    (a) equidistant from a point;

    (b) equidistant from a line and a point;

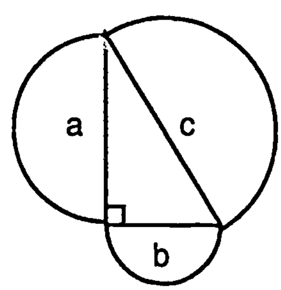
    (c) equidistant from two points;

    (d) the vertices of a triangle rotated around the intersection of the perpendicular bisectors of its sides.

18. Using straight edge and compass only, trisect a given line segment $\overline{AB}$.

J. Use of algebraic concepts and skills in geometric situations, including solution of problems involving the Pythagorean theorem

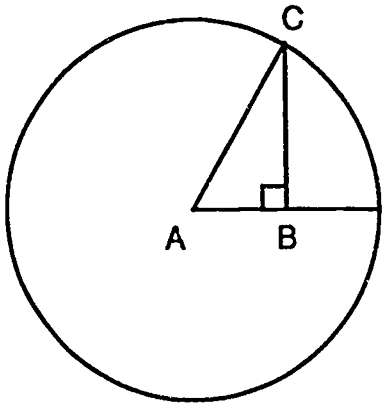
19. In the figure shown to the right, what is the relationship among the areas of the three semicircles? Explain your reasoning algebraically.



K. Introduction to the trigonometric functions of angles greater than 90° and specia angle relationships

20. (a) As AC rotates around A in the figure shown to the right, describe wh * happens to the sine, the cosine, and the tangent of $\angle A$.

    (b) Sketch a graph of the values of sin A as $\angle A$ goes from 0° to 360°.



L. Non-Routine Problems

21. The strength of animal bones is proportional to their cross-sectional area; the weight of animals is proportional to their volume. We have unearthed the femur bones of two animals known to be geometrically similar (for our accuracy of measurement). We find that one bone is four times as long as the other.

    (a) What can we infer about the relative strengths of the bones of the two animals?

    (b) Suppose we knew that the bones of the larger could support twice the animal's weight, how many times its weight could the bones of the smaller animal support?

    (c) What would you speculate about the size and skeletons of creatures on a planet more massive than the earth?

22. A goat is tethered at one corner of a barn which has a square base and measures 20 feet by 20 feet. A compass and a ruler may be helpful in solving this problem.

    (a) If there is grass on all sides of the barn, and the goat is free to graze to the limit of the rope which ties him to the barn, what is the grazing area in square feet if the rope is 20 feet long?

    (b) What is the grazing area if the rope is 40 feet long?

    (c) What is it if the rope is 50 feet long?

23. A goat is tethered at one corner of a barn whose base has dimensions 25 feet by 15 feet. A compass and a ruler may be helpful in solving this problem.

(a) If there is grass on all sides of the barn, and the goat is free to graze to the limit of the rope which ties him to the barn, what is the grazing area in square feet if the rope is 15 feet long?

(b) What is the grazing area if the rope is 25 feet long?

(c) What is it if the rope is 40 feet long?

(d) In the case of a 50 foot rope, obtain an approximation to the grazing area. How accurate is your estimate? Can you justify your answer?

ALGEBRA 2

A. Simplification of algebraic expressions, including fractional exponents and radic

1. Simplify:

(a) $\dfrac{12a^3bc^2}{3ab^2c}$

(b) $6x - \{2x - [2(6 - 3xz)] - 5y\}$

(c) $\dfrac{1}{a-b} - \dfrac{1}{a}$

(d) $\left(\dfrac{9^5x^{3/2}y^{5/2}}{81x^{-1/2}y^{-1}}\right)^{1/2}$

2. Simplify:

(a) $\dfrac{3x-1}{(x-1)(x+1)} - \dfrac{x+3}{(x+1)(x+2)} - \dfrac{1}{x+2}$

(b) $\dfrac{\dfrac{1}{x+1} - \dfrac{1}{x+2}}{\dfrac{1}{x+2} - \dfrac{1}{x+3}}$

(c) $\dfrac{\dfrac{x^2-x-2}{x^2-x-6}}{\dfrac{x^2+8x+7}{x^2-6x+9}}$

3. Express with fractional exponents $\sqrt[3]{\sqrt{a^5}}$

4. $N = \dfrac{a + \dfrac{b}{c}}{\dfrac{x}{y}}$

Which one of a, b, c, x, or y should be doubled to double N?

5. Simplify $\sqrt[n]{3^{n-1} + 3^{n-1} + 3^{n-1}}$.

6. Simplify $\left(\dfrac{3^x + 3^{-x}}{2}\right)^2 - \left(\dfrac{3^x - 3^{-x}}{2}\right)^2$.

7. Simplify $\dfrac{1}{x+\sqrt{x^2 + 1}} \left(1 + \dfrac{x}{\sqrt{x^2 + 1}}\right)$.

8. Which of these have errors? If there are errors, explain what is wrong and change the right hand side to obtain a correct identity.

(a) $-3^2 = 9$

(b) $\dfrac{2A - k}{2} = A - k$

(c) $(a + b)^2 = a^2 + b^2$

(d) $\sqrt{a^2 + b^2} = a + b$

(e) $\sqrt{a^2 + 2ab + b^2} = a + b$

B. Solution of linear equations and inequalities, including those involving absolute value and rational expressions

9. Avid Readers' Club is having membership brochures printed. Fine Print Company gave them estimates of \$63 for 200 copies, the number that they need immediately, or \$132 for 500 copies, a year's supply. Assume a linear function model.

   (a) Which is the independent variable, the dependent variable? What are the data points?

   (b) Write an equation expressing price as a function of number of copies.

   (c) Graph the function.

   (d) What is the y-intercept? In terms of the problem, what does this mean?

   (e) What is the slope of the graph? In terms of the problem, what does this mean?

   (f) A new member of A.R.C. reported that he had 350 copies of a very similar brochure printed for \$100 at Discount Print Company. By plotting this point on your graph, how do you think the prices of printing at the two shops compare?

10. For which sets of real numbers is $|x| \geq x$?

11. Solve:

   (a) $3x - 2 = 5x + 5$

   (b) $\frac{3x - 1}{3x + 2} = 3$

   (c) $\frac{3y + 5}{2y - 1} = \frac{6y + 1}{4y - 1}$

   (d) $3x - 3 < 5x - 4$

   (e) $\frac{2x - 1}{3x + 2} < 1$

   (f) $3 \leq |3x-1| \leq 7$

12. Abacus College admitted students based on the following criteria:

    1) all students with a G.P.A. in college prep courses above 3.2 and combined scores on the five College Board aptitude and achievement tests above 3000 were admitted;

    2) students with $1000 \times (\text{G.P.A.}) + \text{combined scores} \geq 6500$ were admitted; and

    3) students with either a combined score above 3000 and G.P.A. above 2.8 or a G.P.A. above 3.2 and a combined score above 2500 were put in a pool and compared on other factors (e.g. awards, special talents, leadership, etc.).

    Show graphically and algebraically who would be admitted or rejected outright and who would be in the "other factors" pool. Give examples of students from each category. Make up your own admission procedure and express it mathematically.

C. Operations on polynomials

13. Factor $x^4 - 16$ over the complex numbers.

14. What is the coefficient of the x^3 term in the product $(x^3 - 5x^2 + 2x - 1)(x^2 + 7x - 3)$?

D. Solution of quadratic equations by factoring, completing the square, and using a formula; properties of roots

15. Solve by completing the square: $2x^2 + 4x + 1 = 0$

16. Solve: $\sqrt{2x - 5} - \sqrt{x - 2} = 2$

17. Determine the values of m for which the equation $5x^2 - 4x + 2 + m(4x^2 - 2x - 1) = 0$ will have equal roots.

18. A grocer had sold oranges at a dollar a bag, but then raised the price per dozen by 10 cents by reducing the number of oranges in a bag by 4 without changing the price per bag. Find the original number of oranges in a bag, and the original price per dozen.

19. If $x^2 + ax + b = 0$ has $x = 2$ and $x = 3$ as roots, what are a and b?

E. **Solution of quadratic inequalities**

20. Solve: $x^2 - x - 3 > 0$

21. For which interval(s) of the set of real numbers is $x^2 < x$?

22. What condition on the constants a and b implies that

    $x^2 + ax + b > 0$ for all values of x?

23. Solve: $\begin{cases} 9x^2 + 25y^2 \le 225 \\ x < y^2 - 1 \end{cases}$

F. **Solution of polynomial equations: rational roots; Descartes' Rule of Signs**

24. Solve the equation $x^3 - 2x + 1 = 0$ given that $x = 1$ is a root.

25. Find the rational roots (if any) of the equation $2x^3 - x^2 - 9x - 4 = 0$.

26. How many positive, negative, and complex roots does the equation $x^4 + 3x^2 - 7x - 5 = 0$ have? Discuss your reasoning.

27. Plot the graph of $f(x) = x^3 - 5x^2 + 4x + 10$ for x in the domain $-2 \le x \le 5$. Find all zeros, and use synthetic division.

28. Find an equation of the cubic function containing (1,3), (–1,9), (2,9), and (–2,3).

29. List the set of possible rational solutions of $5x^3 - 2x^2 + 7x - 3 = 0$.

G. **Solution of systems of linear equations with two and three variables; homogeneous, dependent, and inconsistent systems; use of determinants and matrices**

30. Solve $\begin{cases} x + 2y - z = 6 \\ 2x - y + 3z = -13 \\ 3x - 2y + 3z = -16 \end{cases}$

31. For what values of a and b in the pair of equations

    $$3x + 2y = 5$$
    $$6x + ay = b$$

    will there be no solutions, precisely one solution, infinitely many solutions? Why?

32 If either 25 pounds of flour and 10 pounds of sugar or 16 pounds of flour and 16 pounds of sugar can be purchased for \$3.20, find the price of each per pound.

33. A dressmaking shop makes dresses and pantsuits. The equipment in the shop allows for making at most 30 dresses and 20 pantsuits in a week. It takes 10 worker-hours to make a dress and 20 worker-hours to make a pantsuit. There are 500 worker-hours available per week in the shop.

(a) Write the inequalities and graph the feasibility polygon.

(b) If the profit on a dress and the profit on a pantsuit are the same, how many of each should be made to maximize the profit?

(c) If the profit on a pantsuit is three times the profit on a dress, how many of each should be made to maximize the profit?

34. Can you find a value of k such that the system

$$\begin{aligned} x + ky &= 1 \\ x - ky &= 2 \end{aligned}$$

(a) has no solution

(b) has an unlimited number of solutions

(c) has exactly one solution

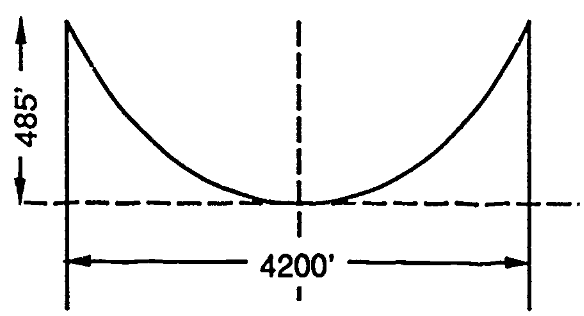
Explain in each case.

H. Linear, quadratic, and polynomial functions -- their graphs and properties

35. Determine the slope of the line whose equation is $2x + 3y - 6 = 0$, and sketch the line.

36. Write an equation of the line which passes through the points $(-1,3)$ and $(1,-2)$ and sketch its graph.

37. Each cable supporting the main span of the Golden Gate Bridge is approximately a parabola. If the supporting towers are 485 feet above the midpoint of the cable and are 4200 feet apart, find an equation approximating the curve described by a supporting cable.



38. Graph $2x + y = 7$ for (x, y) satisfying $|x - y| \leq 10.33$.

39. Write an equation of a parabola which has a vertex at $(3,4)$, axis of symmetry $x = 3$, and passes through the point $(-1,2)$.

40. (a) A rocket is shot vertically with an initial velocity of 40 ft. per second. Its height above the ground after t seconds is given by

$h(t) = 40t - 16t^2$. What is its maximum height?

(b) When will it return to earth?

I. Introduction to the graphing of quadratic equations in two variables -- conic sections

41. Sketch the graphs of the following equations:

(a) $x^2 + y^2 + 4x = 0$

(b) $2x^2 + 3y^2 = 6$

(c) $x^2 - y^2 = 4$

42. If the axis of symmetry of a function is $x = -3$, and the point $(-7,81)$ is on the graph, what is another point on the graph?

43. (a) Will the curves $x^2 + y^2 + 4x - 6y - 29 = 0$ and $x^2 + y^2 - 4x + 6y - 29 = 0$ intersect?

(b) If so, where?

J. Permutations and combinations; the binomial theorem

44. (a) Write the expansion of $(n^2 - 3m)^6$ and simplify the terms.

(b) Find the coefficient of x^3 in the expansion of $(x^3 - \frac{1}{x^2})^{13}$.

45. If $\binom{n}{s} = \frac{n!}{(n-s)!s!}$, then solve for n : $\binom{n+1}{n-1} = 55$.

46. What is the seventh term in the binomial expansion of $(x - 2y)^{10}$?

47. Evaluate $(1.07)^5$ to the nearest thousandth using the bionomial theorem.

48. Solve for n :

$$\binom{n}{2} = \binom{n-1}{1} + \binom{n-1}{2}$$

K. Arithmetic and Geometric Sequences and Series

49. A ball dropped from a height falls 16 feet during the first second, 48 feet the second second, 80 feet the third second, and so on. How far does the ball fall during the sixth second? How far does it fall during the first six seconds?

50. The number of bacteria in a certain culture doubles every 3 hours. If there are N bacteria now, how many will there be in 24 hours?

L. Function concept, including notation, composition, inverse, and arithmetic operations on functions

51. If $f(x) = 3x^2 + 4x + 1$, compute

(a) $f(3)$

(b) $f(x+2)$

(c) $\frac{f(2+h) - f(2)}{h}$

(d) the value of x for which $f(x) = 2$

52. Let $f(x) = x^3 - 5$.

(a) Determine $f^{-1}(x)$.

(b) Graph both functions.

53. Determine the domain and range of $f(x) = \sqrt{9 - x^2}$.

M. Graphs of exponential and logarithmic functions; solution of exponential and logarithmic equations

54. Solve:

(a) $4^{x+2} = 16^x$

(b) $\log_3 x = 1$

(c) $\log_x 32 = \frac{5}{3}$

55. Sketch:

(a) $f(x) = 10^{x+2}$

(b) $f(x) = 5^{-x}$

(c) $f(x) = \frac{1}{3}(3^x)$ for $-3 \le x \le 2$

56. Describe and sketch what happens as x varies from 1 to 0 for

(a) x^3

(b) 3^x

(c) x^x

57. In 1976 the world population was about four billion. In 1983 it reached five billion. Write an exponential equation expressing this growth.

N. Solution of word problems, including estimation and approximation

58. If $\log_2 15.9 = \log_3 x$, which multiple of 10 gives the best approximation to x?

59. A b- ·eria population grows exponentially. At the start of an experiment the bacteria population numbers 1000. Two days later the population numbers 2000. What is the size of the population 5 days after the start of the experiment?

60. The sum of the base b and the height h of a triangle is 24. What is the maximum area of a triangle satisfying these conditions?

61. How many times will the hour hand and the minute hand of a clock overlap in a 12-hour period?

62. Suppose that a line is drawn through the origin at an angle of 30° from the positive x-axis. Will it pass through any points other than the origin whose coordinates are both rational numbers? Explain.

O. Complex numbers -- notation, graphical representation, and applications

63. Simplify $\sqrt{-5}\ \sqrt{-10}$

64. Calculate $(4 + 7i)(5 - 3i)$

65. Solve $x^2 - x + 3 = 0$.

66. Solve $x^3 - 1 = 0$. Represent solutions graphically and in polar form.

67. Write a quadratic equation one of whose roots is $4 - 5i$.

68. Show that $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ is a square root of i.

69. Find the reciprocal of $3 + 2i$.

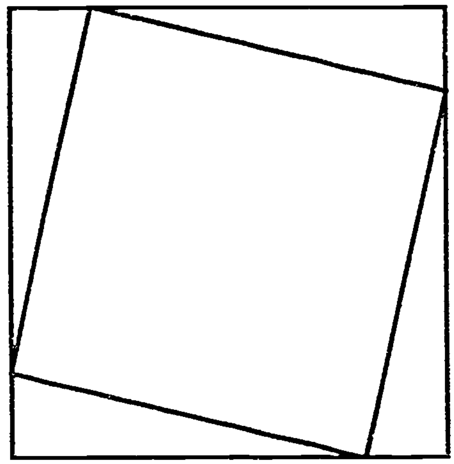
P. Solution of systems of quadratic equations in two unknowns

70. (a) Solve the system $\begin{matrix} x^2 + y^2 = 4 \\ x^2 - y^2 = 1 \end{matrix}$ and

(b) interpret the results graphically.

71. Find the dimensions of a rectangle whose area is 18 ft^2 and whose diagonal is 6 ft.

72. In the figure shown to the right, the area between the squares is 11 sq. in. The sum of the perimeters is 44 inches. Find the length of a side of the larger square.



Q. Mathematical probability; finite sample spaces; conditional probability

73. A man purchased 3 tickets for $10 each in a lottery in which 50 tickets were sold. Two prizes, one of $100 and one of $50, will be awarded in a random drawing. What is his "expected" gain or loss?

74. A batter is now batting .300. (The probability of getting a hit is 3 out of 10.) In the batter's next 4 at-bats, what is the probability of getting at least 3 hits?

75. A clown has 8 balloons, each with a different color. There are 6 children. How many ways can the clown give each child a balloon?

76. There are 10 multiple choice questions on an exam. Each question has 4 alternative answers and you do them by guessing. What is the probability that you will be correct on at least 7 out of 10 questions?

77. Henry wants to maximize the probability of getting to work on time. If he jumps in the car and drives off without checking everything, he has an 85% chance of getting there ahead of time, and a 15% chance of being late due to car trouble. If he takes 10 minutes to check everything, the car is 95% likely to make the trip without a problem and get there just on time, but there is a 30% chance of a ten minute traffic delay because he's into the heavier commuter traffic.

(a) Should Henry take the time to check the car before he leaves?

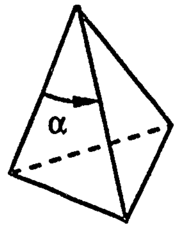
(b) If he were to check the car every day, how many minutes late to work would he be on the average?

78. The probability that a basketball player makes a basket when she takes a shot is 0.6. Find the probability that in 10 consecutive shots she makes exactly 6 baskets. Give your answer in decimal form to the nearest hundredth.

R. Trigonometric applications -- solution of oblique triangles through the use of the law of sines and cosines

79. An airplane is moving with an air speed of 100 mph. The wind, blowing from the north at 20 mph, causes the plane to travel with a ground speed of only 90 mph. Find the direction the plane is heading and the direction it travels.

80. The pyramid shown in the figure drawn to the right has an equilateral triangle as a base and three congruent triangles as sides. The area of its base is 2 square meters. $\angle\alpha = 40°$. What is the volume of the pyramid?



S. Non-Routine Problems

81. Assume that the rate r at which the sun radiates energy is directly proportional to the fourth power of its temperature T.

    (a) Write the general equation expressing r in terms of T.

    (b) Assume that the sun's temperature varies inversely with the number of years y since the universe was created. Write the general equation expressing T in terms of y.

    (c) By combining the equations from parts (a) and (b), write a general equation expressing r in terms of y. Give your answer in simplest form.

    (d) Tell in words how r varies with y.

82. Devise a mathematical model that predicts the proportion of Americans remaining alive at various ages based on the following information: out of 1,000,000 Americans living at age 1, those remaining alive at age 40 number 883,342; at age 60, there are 677,771 and at age 70 there are 454,548.

83. Pick an odd number, square it, then subtract 1. Note that the resulting number is divisible by 8. Prove that this is true no matter what odd number you start with.

MATHEMATICAL ANALYSIS

A. Development of the trigonometric functions through the use of the concept of circular functions

1. If θ is an angle with vertex at the origin, initial side on the positive x-axis , and terminal side through $(-5,6)$, find

   (a) $\sin\theta$

   (b) $\sec\theta$

2. If $\csc\theta = -2$, and θ is in quadrant III, find

   (a) $\tan\theta$

   (b) $\cos\theta$

B. Graphical characteristics of the trigonometric functions -- including translations, amplitude, change of period, domain, range and sums and differences of function

3. Sketch the graph of

   (a) $x = 8\cos t$

   (b) $y = 2\sin\left(\frac{1}{2}x + \pi\right)$

   (c) $y = \frac{1}{2}\tan\left(x + \frac{\pi}{4}\right)$

   (d) $y = \sin 3t$

C. Inverse trigonometric functions -- notations and graphs

4. Find the value of $\cos\left(2 \text{ Arcsin } \frac{3}{5}\right)$.

5. Solve for x: $\text{Arcsin } 3x + \text{Arcsin } x = \frac{\pi}{2}$.

6. Sketch the graph of $y = \text{Arcsin}\frac{1}{2}x$.

D. Trigonometric identities, including addition and double-angle and half-angle formulas

7. If θ is in quadrant III and $\cos\theta = -\frac{5}{9}$, find

   (a) $\sin\frac{1}{2}\theta$

   (b) $\tan\frac{1}{2}\theta$

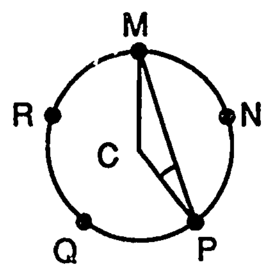
   (c) $\cos 2\theta$

8. Prove the following identity: $\frac{\sin 2x}{1 - \cos 2x} = \cot x$.

9. Express $\cos^4 x$ in a form that does not involve any power >1 of the trigonometric functions.

E. Use of degree and radian measures

10. Five points are equally spaced on the circle with center C as shown in the figure to the right. Determine the degree and radian measure of the angle CPM.



F. Solution of trigonometric equations

11. Find all values of x (using radian measure) for which $0 \le x < 2\pi$ and $\tan^2 x - 3\tan x + 2 = 0$.

12. If $x = \sin\frac{t}{2}$ and $y = \sin 2t$,

    (a) when will $x = y$?

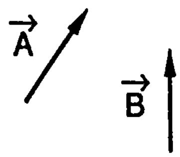
    (b) When will $x = -y$?

    (c) When will $x^2 + y^2 = 1$?

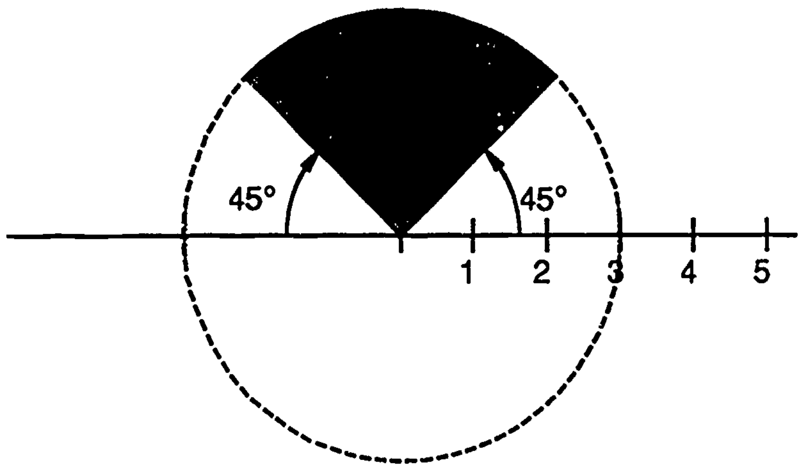
G. Polar coordinates and vectors; graphical representations

13. Write the polar form of the equation $x^2 - 2x + y^2 = 3$.

14. Sketch the vectors $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$.



15. Express in polar coordinates the three curves that bound the shaded region in the figure shown below.



H. Solution of problems related to force and navigation

16. (a) A boat sails 3 miles southeast and then 2 miles northeast. Describe the resulting position of the boat relative to its starting point.

(b) If the boat is to then sail to a point 5 miles east of its starting point how far must it sail and in what direction?

I. Trigonometric form of complex numbers and DeMoivre's theorem

17. Use the polar form of $1 - i\sqrt{3}$ to find $(1 - i\sqrt{3})^6$.

J. Mathematical induction

18. Prove that for any positive integer n,

$$1 + 2^2 + 3^2 + \ldots + n^2 = \frac{n\,(n+1)\,(2n+1)}{6}.$$

19. Prove that for any positive interger n,

$$\frac{n^3}{3} < 1 + 2^2 + 3^2 + \ldots + n^2 .$$

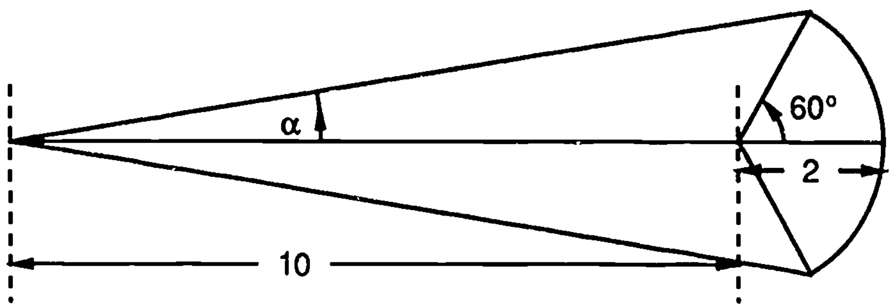
20. Prove by mathematical induction that the sum of the interior angles of an n-sided polygon is $(n-2) \times 180°$ for $n \geq 3$.

K. Analytic treatment of the conic sections, including translations and rotations

21. Find the center and radius of the circle whose equation is $x^2 + y^2 - 10x + 4y + 17 = 0$.

22. Give the coordinates of the end points of the major and minor axes of the ellipse whose equation is: $9x^2 + 108x + 4y^2 - 56y + 484 = 0$.

23. If the curve shown in the figure below is part of a circle of radius 2, find α.



L. Rational functions and their graphs

24. Graph: $y = \dfrac{3}{x(x - 1)}$

M. Parametric equations and their graphs

25. Write an equation in x and y for the curve whose parametric equations are $y = 3 \cos t$, $x = 2 \sin t$.

26. Write a set of parametric equations for the line which passes through the points $(5, -1)$ and $(1,7)$.

N. Lines and planes in space – three-dimensional coordinate geometry

27. Write an equation for the plane parallel to the z-axis that intersects the x-axis at $x = 1$ and intersects the xy-plane in a line making an angle of 45° with the x-axis.

28. Write an equation for the plane that passes through the points $(2,1,0)$, $(3,-2,1)$, and $(0,3,4)$.

29. Write a system of equations for the line that joins $(-1,-2,1)$ and $(1,0,-1)$.

30. Find the coordinates of the point where the line

$$\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{3}$$ intersects the plane $3x + 2y - z = 5$.

31. Write the equations of four planes that intersect to form a tetrahedron.

O. Introduction to vectors in the plane and space

32. Determine the vector $a\vec{i} + b\vec{j} + c\vec{k}$ from the point $(2,-1,3)$ to the point $(5,-3,-2)$.

33. If $\vec{A} = 2\vec{i} + 3\vec{j} - 4\vec{k}$ and $\vec{B} = -3\vec{i} + 4\vec{k} - 5\vec{j}$, determine the angle θ between these two vectors.

34. Write:

    (a) Five vectors that can be used to construct a pentagon.

    (b) Twelve vectors that can be used to construct a prism.

    (c) Six vectors that can be used to construct a tetrahedron.

P. Characteristics of graphs of functions

35. Determine the intercepts, horizontal and vertical asymptotes, and the extent of the graph of $y = \frac{x-1}{x^2-4}$.

36. Given the plane $3x - 7y + 24z = 6$, write an algebraic expression for z that is satisfied by every line through the origin parallel to the given plane.

Q. Concept of a limit: definition, application, convergence/divergence

37. For each of the following, determine whether or not the limit exists. Explain your answer and compute the limits that do exist.

    (a) $\lim\limits_{x\to 0} \frac{|x|}{x}$ and

    (b) $\lim\limits_{x\to 1} \frac{x^2 + x - 2}{x - 1}$

38. As $t \to \infty$, does each of the following expressions converge, diverge, or neither? Why?

(a) $\dfrac{\sin 3t}{\sqrt{t}}$

(b) $\sin 3t \cos 3t$

(c) $t \sin 3t \cos 3t$

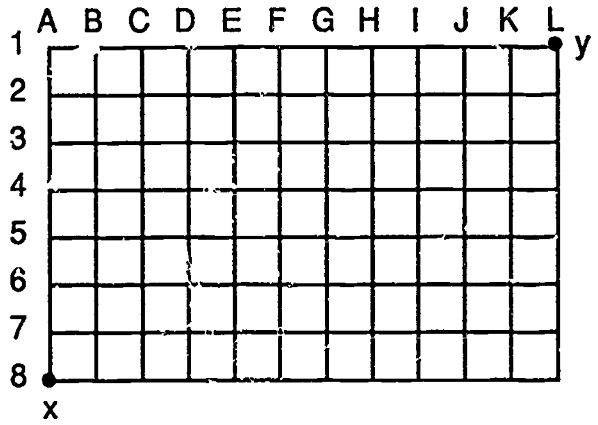
(d) $\dfrac{e^t}{t^4}$

39. If $y = x^2$ for $x > 0$ and $y = x^3$ for $x < 0$, does the limit of y as $x \to 0$ exist? Explain.

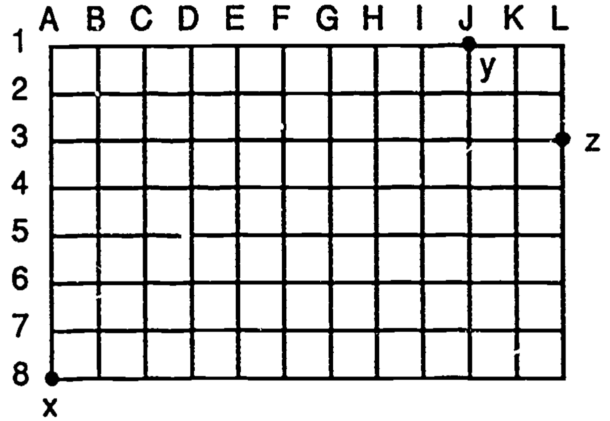
R. Non-Routine Problem

40. A telephone central office is to be located at an intersection in the region containing streets A through L and avenues 1 through 8. All blocks are square. Underground telephone wires from the office to subscribers must run along public streets and cannot cut diagonally through private properties. It is desired that the sum of the lengths of the wires to the subscribers be the least.

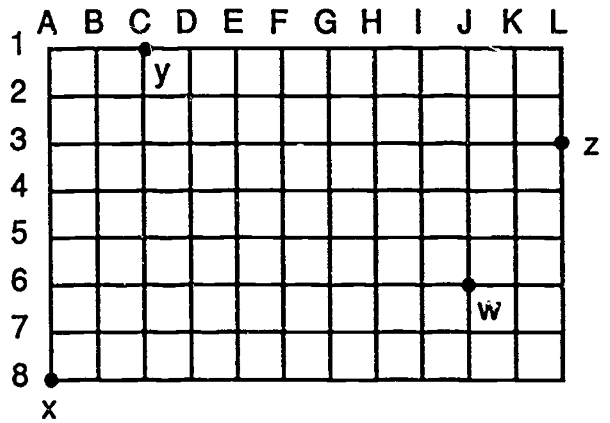
(a) Where could the central office be located for the wires to have a minimal path if there are two subscribers x and y located at the intersections shown below? What is the least total length of the wires in blocks?



(b) Where could the central office be located for the wires to have a minimal path if there are three subscribers x, y, and z located at the intersections shown below? What is the least total length of the wires in blocks?



(c) Where could the central office be located for the wires to have a minimal path if there are four subscribers, x, y, z, and w located at the intersections shown below? What is the least total length of the wires in blocks?



Appendix B

Academic Competencies in Speaking and Listening, Reasoning, and Studying

In preparing the 1982 statement on preparation in mathematics, the Intersegmental Committee reviewed the recommendations on academic preparation developed under the auspices of the College Board's Project EQuality.The Academic Senates endorsed the basic academic competencies contained in the College Board's Preparation for College in the 1980's; we reprint the rationale here:

- Basic Academic Competencies are developed abilities; they are the outcomes of learning and intellectual discourse. They are acquired when there are incentives and stimulation to learning and when there is an encouraging learning environment. There are different levels of competency; they can be defined in measurable terms. They are academic. They are basic.

- Basic Academic Competencies are interrelated to and interdependent with the basic subject-matter areas. Without such competencies, the knowledge of literature, history, science, languages, mathematics, and all other disciplines is unattainable.

- Basic Academic Competencies are not substitutes for drive, motivation, interest, intelligence, experience, or adaptablility. Nor are basic academic competencies social coping skills even though we recognize that coping skills are crucial to success in school, life, and work. Coping skills are simply another important matter which we have decided not to subsume under the area of Basic Academic Competencies.

- Basic Academic Competencies provide a link across the disciplines of knowledge although they are not specific to the disciplines. Teaching that is done in ignorance of, or in disregard for, such competencies and their interrelationships to each of the subject-matter areas is inadequate if not incompetent.

- Basic Academic Competencies provide a way to tell students what is expected of them. The knowledge of what is expected is crucial to effective learning; its absence dooms much of learning to inanity.

The speaking and listening, reasoning, and studying competencies listed below are taken from Preparation for College in the 1980's. The Academic Senates believe that these skills also deserve wide dissemination among California high school students and recommend that the skills be developed and strengthened in all high school courses.

Speaking and Listening Competencies

- The ability to engage critically and constructively in the exchange of ideas, particularly during class discussion and conferences with instructors.

- The ability to answer and ask questions coherently and concisely, and to follow spoken instructions.

- The ability to identify and comprehend the main and subordinate ideas in lectures and discussions, and to report accurately what others have said.

- The ability to conceive and develop ideas about a topic for the purpose of speaking to a group; to choose and organize related ideas; to present them clearly in Standard English; and to evaluate similar presentations by others.

- The ability to vary one's use of spoken language to suit different situations.

Reasoning Competencies

- The ability to identify and formulate problems, as well as the ability to propose and evaluate ways to solve them.

- The ability to recognize and use inductive and deductive reasoning, and to recognize fallacies in reasoning.

- The ability to draw reasonable conclusions from information found in various sources, whether written, spoken, tabular, or graphic, and to defend one's conclusions rationally.

- The ability to comprehend, develop, and use concepts and generalizations.

- The ability to distinguish between fact and opinion.

Study Competencies

- The ability to set study goals and priorities consistent with stated course objectives and one's own progress, to establish surroundings and habits conducive to learning independently or with others, and to follow a schedule that accounts for both short- and long term projects.

- The ability to locate and use resources external to the classroom (for example, libraries, computers, interviews, and direct observation), and to incorporate knowledge from such sources into the learning process.

- The ability to develop and use general and specialized vocabularies, and to use em for reading, writing, speaking, listening, computing, and studying.

- The ability to understand and follow customary instructions for academic work in order to recall, comprehend, analyze, summarize, and report the main ideas from reading, lectures, and other academic experiences; and to synthesize knowledge and apply it to new situations.

- The ability to prepare for various types of examinations and to devise strategies for pacing, attempting or omitting questions, thinking, writing, and editing according to the type of examination; to satisfy other assessments of learning in meeting course objectives such as laboratory performance, class participation, simulation, and students' evaluations.

- The ability to accept constructive criticism and learn from it.

The abilities listed above constitute the key abilities in learning how to learn. Successful study skills are necessary for acquiring the other competencies—reading, writing, speaking and listening, mathematical, and reasoning. Without good study skills, students are apt to be inefficient in their work. Moreover, students should understand the importance of attitude in acquiring the basic study competencies. Students who show a desire to take personal responsibility for their own progress, who understand the value of making full use of teachers as resources, and who conduct themselves in ways that make learning possible for their classmates are more likely to succeed in their own efforts to acquire the basic competencies.

Appendix C

Recommendations of the Board of Governors of the Mathematical Association of America and the Board of Directors of the National Council of Teachers of Mathematics

The Board of Governors of the Mathematical Association of America and the Board of Directors of the National Council of Teachers of Mathematics make the following recommendations:

1. Proficiency in mathematics cannot be acquired without individual practice. We, therefore, endorse the common practice of making regular assignments to be completed outside of class. We recommend that parents encourage their children to set aside sufficient time each day to complete these assignments and that parents actively support the request of teachers that homework be turned in Students should be encouraged to develop the ability to read mathematics.

2. Homework and drill are very important pedagogical tools used to help the students gain understanding as well as proficiency in the skills of arithmetic and algebra; but students should not be burdened with excessive or meaningless drill. We, therefore, recommend that teachers and authors of textbooks step up their search for interesting problems that provide the opportunity to apply these skills. We realize that this is a difficult task, but we believe that providing problems that reinforce manipulative skills as a byproduct should have high priority, especially those that show that mathematics helps solve problems in the real world.

3. We are aware that teachers must struggle to maintain standards of performance in courses at all levels from kindergarten through college and that serious grade inflation has been observed. An apparent growing trend to reward effort or attendance rather than achievement has been making it increasingly difficult for mathematics teachers to maintain standards. We recommend that mathematics departments review evaluation procedures to insure that grades reflect student achievement. Further, we urge administrators to support teachers in this endeavor.

4. In light of 3 above, we also recognize that advancement of students without appropriate achievement has a detrimental effect on the individual student and on the entire class. We, therefore, recommend that school districts make special provisions to assist students when deficiencies are first noted.

5. We recommend that cumulative evaluations be given throughout each course, as well as at its completion, to all students. We believe that the absence of cumulative evaluation promotes short-term learning. We strongly oppose the practice of exempting students from evaluations.

6. We recommend that computers and hand calculators be used in imaginative ways to reinforce learning and to motivate the student as proficiency in mathematics is gained. Calculators should be used to supplement rather than to supplant the study of necessary computational skills.

7. We recommend that colleges and universities administer placement examinations in mathematics prior to final registration to aid students in selecting appropriate college courses.

8. We encourage the continuation or initiation of joint meetings of colleges and secondary school mathematics instructors and counselors in order to improve communication concerning mathematics prerequisites for careers, preparation of students for collegiate mathematics courses, joint curriculum coordination, remedial programs in schools and colleges, and exchange of successful instructional strategies, planning of in-service programs, and other related topics.

9. Schools should frequently review their mathematics curricula to see that they meet the needs of their students in preparing them for college mathematics. School districts that have not conducted a curriculum analysis recently should do so now, primarily to identify topics in the curriculum which could be either omitted or de-emphasized, if necessary, in order to provide sufficient time for the topics included in the above statement. We suggest that, for example, the following could be de-emphasized or omitted if now in the curriculum:

a. logarithmic calculations that can better be handled by calculators or computers,
b. extensive solving of triangles in trigonometry,
c. proofs of superfluous or trivial theorems in geometry.

10. We recommend that algebraic concepts and skills be incorporated whenever possible into geometry and other courses beyond algebra to help students retain these concepts and skills.

Appendix D

Calculus in the Secondary School

To: Secondary School Mathematics Teachers

From: The Mathematical Association of America
The National Council of Teachers of Mathematics

D 'e: September 1986

R· Calculus in the Secondary School

D ar Colleagues:

A single variable calculus course is now well established in the 12th grade at many secondary schools, and the number of students enrolling is increasing substantially each year. In this letter we would like to discuss two problems that have emerged.

The first problem concerns the relationship between the calculus course offered in high school and the succeeding calculus courses in college. The Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM) recommend that the calculus course offered in the 12th grade should be treated as a college-level course. The expectation should be that a substantial majority of the students taking the course will master the material and will not then repeat the subject upon entrance to college. Too many students now view their 12th grade calculus course as an introduction to calculus with the expectation of repeating the material in college. This causes an undesirable attitude on the part of the student both in secondary school and in college. In secondary school all too often a student may feel "I don't have to master this material now, because I can repeat it later;" and in college, "I don't have to study this subject too seriously, because I have already seen most of the ideas." Such students typically have considerable difficulty later on as they proceed further into the subject matter.

MAA and NCTM recommend that all students taking calculus in secondary school who are performing satisfactorily in the course should expect to place out of the comparable college calculus course. Therefore, to verify appropriate placement upon entrance to college, students should either take one of the Advanced Placement (AP) Calculus Examinations of the College Board, or take a locally-administered college placement examination in calculus. Satisfactory performance on an AP examination carries with it college credit at most universities.

A second problem concerns preparation for the calculus course. MAA and NCTM recommend that students who enroll in a calculus course in secondary school should have demonstrated mastery of algebra, geometry, trigonometry, and coordinate geometry. This means that students should have at least four full years of mathematical preparation beginning with the first course in algebra. The advanced topics in algebra, trigonometry, analytic geometry, complex numbers, and elementary functions studied in depth during the fourth year of preparation are critically important for students' later courses in mathematics.

It is important to note that at present many well-prepared students take calculus in the 12th grade, place out of the comparable course in college, and do well in succeeding college courses. Currently the two most common methods for preparing students for a college-level calulus course in the 12th grade are to begin the first algebra course in the 8th grade or to require students to take second year algebra and geometry concurrently. Students beginning with algebra in the 9th grade who take only one mathematics course each year in secondary school should not expect to take calculus in the 12th grade. Instead, they should use the 12th grade to prepare themselves fully for calculus as freshmen in college.

We offer these recommendations in an attempt to strengthen the calculus program in secondary schools. They are not meant to discourage the teaching of college-level calculus in the 12th grade to strongly prepared students.

Appendix E

Annotated References

California State Department of Education, (1985), *Mathematics Framework for California Public Schools, K-12,* discusses the goals of mathematics education; describes teaching strategies that will accomplish these goals; outlines the curriculum in seven strands (Number, Measurement, Geometry, Patterns and Functions, Statistics and Probability, Logic, and Algebra;) and contains the textbook standards that were used in the K-8 textbook adoption. ($3.00 plus sales tax for California residents from Publications Sales, P.O.Box 944272, Sacramento, CA 94244-2720)

California State Department of Education, (1987), *Mathematics Model Curriculum Guide, K-8,* provides a summary of important characteristics of a strong elementary mathematics program, enumerates essential understandings to be developed by such a program, and portrays classroom experiences that will lead to their development. ($2.75 plus sales tax for California residents from Publications Sales, P.O.Box 944272, Sacramento, CA 94244-2720)

California State Department of Education, (1985), *Model Curriculum Standards* does not present the high school curriculum by titles of courses, as do the earlier documents, but rather by program standards, and organizes the core content by concepts and skills, problem solving, and applications, to emphasize the importance of higher order thinking skills. Particularly valuable are the carefully selected examples for each concept and skill and the five applications that are discussed in considerable detail. ($5.50 plus sales tax for California residents from Publications Sales, P.O.Box 944272, Sacramento, CA 94244-2720)

The College Board, 1985, "Academic Preparation in Mathematics" New York. ($6.95 from College Board Publications, Box 886, New York, New York 10101)

Appendix F

Acknowledgements

In August 1986, the Intersegmental Committee of the Academic Senates appointed the following Special Committee to Revise the Statement on Preparation in Mathematics Expected of Entering Freshmen:

Henry L. Alder, Chair; University of California, Davis; Past-Chair of the University's Board of Admissions and Relations with Schools (BOARS), and recent member of the California State Board of Education.

Norbert S. Bischof, Merritt College; President, District Academic Senate, Peralta College District, and Past-President, Academic Senate for the California Community Colleges.

Patricia Deamer, Skyline College, San Bruno.

Walter Denham, Director, Mathematics Education, California State Department of Education.

Arthur P. Dull, Diablo Valley College, Pleasant Hill; Past-President, California Mathematics Council for Community Colleges.

Thomas E. Hale, California Polytechnic State University, San Luis Obispo; former member of the CSU Academic Senate Educational Policies Committee.

Alfred B. Manaster, University of California, San Diego; Administrative Coordinator, UC/CSU Mathematics Diagnostic Testing Project.

Edward Matzdorff, California State University, Chico; representative to the CSU Academic Senate, member of CSU Entry Level Mathematics Advisory Committee.

Virginia A. Nicholas, Department Executive, Mathematics, Lincoln High School, Stockton; Chair, Mathematics Curriculum Committee K-12, Lincoln Unified School District, Stockton.

Albert R. Stralka, University of California, Riverside.

Dorothy Wood, Redwood High School, Larkspur.

All of the Special Committee's meetings were also attended by the following three representatives of major mathematical constituencies in the State:

Philip C. Curtis, Jr., University of California, Los Angeles; Chair, UC/CSU Mathematics Diagnostic Testing Project.

Philip Daro, University of California, Executive Director, California Mathematics Project.

Elizabeth K. Stage, Director of Mathematics Education, Lawrence Hall of Science, University of California, Berkeley; President, California Mathematics Council.

The Special Committee is very grateful to Elizabeth Stage for serving as Editor of this document. She was ably assisted by an Editorial Committee consisting of Arthur P. Dull, Alfred B. Manaster, Edward Matzdorff, and Virginia A. Nicholas.

The sample problems in Appendix A were prepared by a Subcommittee on Problems consisting of Philip Daro, Chair, Philip C. Curtis, Jr., Walter Denham, Thomas E. Hale, and Dorothy Wood. This Subcommittee had a difficult assignment of trying to seek out problems which, both in content and spirit, properly reflect the Special Committee's views of what to expect of entering freshmen. It carried out its tasks with imagination, thoroughness, and a great amount of energy which the Special Committee wishes to acknowledge with great gratitude.

During its deliberations, the Special Committee consulted with a number of other groups and individuals whose contributions the Special Committee appreciates greatly. The chairs of university and college mathematics departments devoted a meeting at the Northern California Section of the MAA on February 28, 1987, to discussion of this Statement which provided particularly helpful input.