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ABSTRACT

This study examines the relationship between children's procedural and conceptual understanding of mathematics and their accuracy in reporting and interpreting geography text material containing mathematical information. It was hypothesized that (1) children's misconceptions or lack of experience with particular mathematical content areas would be associated with inaccurate interpretations of geography content; and (2) that mathematical competence would not necessarily be applied to reasoning about mathematically related geographical concepts. Sixty-four children, 16 in each of grades 3-6, were interviewed about related topics in mathematics and geography to test these hypotheses. Preliminary data analysis focusing on the correlational relationship between knowledge of mathematics and the attainment and application of geographical concepts tended to be consistent with expectations. First, there seemed to be a positive relationship between overall mathematics and geography performance. However, the data indicate that mathematically inaccurate children had lower accuracy scores on some but not all geographical information as compared to children who were accurate in their mathematical concepts and procedures. This suggested that the positive correlation between mathematics and geographical knowledge scores was not necessarily a function of mathematically competent children applying their knowledge to geographical contexts. There may have been a common non-mathematical component related to performance in both areas that affected some contexts but not others. Appended are examples of mathematical misconceptions and associated geographical knowledge for grades 3-5. (KR)

The Role of Mathematical Knowledge in Children's Understanding of Geographical Concepts

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This study examines the relationship between children's procedural and conceptual understanding of mathematics and their accuracy in reporting and interpreting geography text material containing mathematical information. It was hypothesized that a)children's misconceptions or lack of experience with particular mathematical content areas would be associated with inaccurate interpretations of geography content and that b)mathematical competence would not necessarily be applied to reasoning about mathematically related geographical concepts. Sixty-four children, 16 in each of grades 3 - 6 were interviewed about related topics in mathematics and geography to test these hypotheses.

Preliminary data analysis focusing on the correlational relationship between knowledge of mathematics and the attainment and application of geographical concepts tends to be consistent with expectations. First, there seems to be a positive relationship between overall mathematics and geography performance. However, the data indicate that mathematically inaccurate children have lower accuracy scores on some but not all geographical information as compared to children who are accurate in their mathematical concepts and procedures. This suggests that the positive correlation between mathematics and geographical knowledge scores is not necessarily a function of mathematically competent children applying their knowledge to geographical contexts. Rather, there may be a common non-mathematical component related to performance in both areas that affects some contexts but not others. A three-part-knowledge model is being developed to explain these findings.

Background

In recent years there has been increasing concern about Americans' geographical illiteracy (Daniels, 1988; National Geographic Society, 1988; Solorzano, 1985). No doubt we have all heard reports of students and adults who were unable to identify the United States on a map of the world or who could not name the states that bordered on the Pacific Ocean. At the most obvious level, we might assume that the main reason for this geographical illiteracy is that Americans simply do not spend very much time studying geography in elementary or secondary schools. Currently there are moves underway to correct this deficiency and in fact within the next year or two many states will be mandating the inclusion of some kind of "global" curricular course as a requirement for high school graduation.

There is, however, another way to view the problem of geographical illiteracy. That way is to attribute the difficulty, not to the amount of time spent on geographical content, but to the appropriateness of the geographical content students are expected to learn. This view grows out of a developmental perspective of education in which emphasis is placed on the fact that students' construct different personal meanings from the same objective content depending upon the knowledge and understanding they bring to a task (Piaget, 1929). Appropriateness in a geographical context then means that the learner has developed the necessary cognitive framework into which the geographical material will fit. If there is a mismatch between the information provided and the student's ability to interpret that information in the manner intended by the curriculum developers, that information may be misunderstood and remembered inaccurately. A sensible interpretation of the material, therefore, requires that the student be familiar with and have an accurate and flexible understanding of the non-geographical concepts, i.e, concepts from other academic disciplines, that are embedded in the geography content. One factor contributing to the problem of geograpical illiteracy, then, may be that educators have failed to view the acquisition of geographical concepts in the context of students' existing knowledge of other academic fields (Adler, 1989; Blaut & Stea, 1971; Downs, Liben, & Daggs, 1988).

Purpose of the Study

This study is concerned with the relationship of a particular academic content area, that of mathematics, to students' learning and understanding of geography. An informal survey of geography curricular materials supports the contention that knowledge of mathematical concepts and procedures seems to be a critical variable in developing an appreciation of many geographical ideas. It indicates that many geographical concepts, indeed, presuppose knowledge of particular mathematics concepts. For example, a discussion about the use of map scales and distances between cities on a map presupposes a knowledge of ratio and proportional relationships, knowledge of different units of measurement, and an ability to transpose units from one scale to units in another scale. If these non-geographical mathematics concepts have not yet been developed or have been developed but contain some basic misconceptions, students' recall of the geographical material may be inaccurate or, at best, remembered by rote. Moreover, students' interpretations and ability to apply the material to new contexts will be limited by their lack of a suitable framework for organizing and making sense of the geographical information. In the long run, then, lasting knowledge of geographical concepts will fail to emerge.

Hypotheses

This study examined the relationship between children's procedural and conceptual understanding of mathematics and their accuracy in reporting and interpreting geography text material containing mathematical information. It was hypothesized that:

a)children's misconceptions or lack of experience with mathematical content areas would be associated with inaccurate interpretations of geography content

b)particular types of mathematical competence would be more strongly associated with particular types of knowledge in geography than others

c)mathematical knowledge would not necessarily be applied to reasoning about mathematically related geographical concepts (i.e., students with mathematical skills might not apply them in a geographical context (Bryant, 1985)).

Methods and Procedures

Subjects were students in a middle class public suburban school district in northern New Jersey. In all, 16 students in each of grades 3 - 6 were selected from the participating schools. Within grade and sex, these subjects were haphazardly chosen from among the possible 201 students whose parents signed consent forms to allow their children to participate in the study. The data have not yet been completely analyzed and the results reported here are based on responses of 42 of the 64 subjects. These subjects were 8 boys and 8 girls from each of grades 3 and 4 and 5 boys and 5 girls from grade 5.

Initially several sets of textbooks and supplementary materials in geography were reviewed for the selection of tasks to be used for the interview protocols. After review, the investigator selected excerpts of mathematically loaded samples from the geography textbooks used in the schools of the participating district (Loftin & Ainsley, 1988). Math items were adapted from the district's math text series (Eicholz, O'Daffer, & Fleenor, 1989) and particular items were selected to match the mathematical content of the geography items. Based on these selections, interview protocols were developed for each of the grades.

While the specific task content varied from grade to grade, the general format of the interviews was the same for all subjects. In the first part of the procedure students were asked to read two short excerpts from a grade-level geography text. One contained content dealing with knowledge of maps and the other was related to information about the population, climate, or industry of a given geographical area. For each excerpt students were asked a)factual information questions based on the content, b)interpretive questions that went beyond the given information in the text and that required the application of some mathematical knowledge, c)definition questions about some mathematically loaded terms used in the text, and d)concept extension questions in which the same concepts described in the text needed to be applied to an analogous situation.

In the second part of the procedure students were asked a)to work out computational examples for each mathematical concept embedded in the geography text, b)to demonstrate their understanding of the computational procedures and numerational system, c)to solve word problems involving the application of the same computational procedures, d)to construct the solution to a measurement problem related to a concept raised in the geography text, and e)to interpret a graph problem comparable to the material in the text, but in a non-geographical context.

Each student was individually interviewed outside the classroom for approximately one hour and within each grade all students were asked the same core questions. Clinical interviewing procedures were also utilized, however, to clarify students' responses and identify misconceptions in their reasoning. All interviews were videotaped. (Another aspect of the project is that some of the vidootapes will be edited for use in teacher preparation programs.)

Results: Scoring Procedures

Within both the geography and mathematics tasks, subjects were evaluated on accuracy of answers and/or procedures used as well as on misconceptions expressed.

An accuracy score of 0, 1, or 2 was obtained for each item and total scores were obtained for all items in each domain and for subsections of items within each domain. The geography subsections consisted of factual questions, questions requiring interpretation or application of text material, and definitions of mathematically loaded geographical terms from the text material. The mathematics subsections consisted of computation examples, questions measuring understanding of concepts or procedures used, questions requiring applications of understanding and computational knowledge to a problem situation, and questions involving the ability to read graphs. In general, the criteria for accuracy scores were that (0) indicated a completely wrong response, (1) indicated a partially correct response or a completely correct response obtained after some prompting, and (2) indicated a fully accurate response offered spontaneously. Maximum scores for categories by grade can be found in Table 1.

Misconceptions were identifed within specific content areas of the

Table 1

Maximum and Mean Scores Attained on Mathematics and Geography Tasks

Task	Third		Fourth		Fifth	
	Max	Mean	Max	Mean	Max	Mean
Overall math	52	28.9	68	40.4	60	26.2
Computation	14	9.5	16	12.6	10	3.3
Understanding	18	8.3	20	10.8	18	8.6
Applications	8	4.2	14	6.9	14	5.1
Graphs	12	6.9	18	10.1	18	9.2
Overall geography	42	24.5	62	40.4	60	33.9
Geography facts	12	9.8	24	20.1	20	15.4
Interpretation	18	8.8	24	13.5	26	11.3
Definitions	12	5.9	14	6.9	14	7.2

mathematics interview protocols. Based on an evaluation of subjects' errors in the mathematics protocol, they were identified and grouped as having or not having particular mathematical misconceptions. Because interview protocols were designed to assess parallel mathematical concepts in geography and mathematics contexts, it was not difficult to identify particular mathematical misconceptions and then match them to items utilizing the same kind of knowledge in a geographical context. Matching geographical content areas and the content of mathematical misconceptions for each grade are listed in Table 2.

Each subject's score on the particular geography questions related to the mathematics involved was then obtained and partial group geography means were calculated for subjects previously categorized as having or not having mathematical misconceptions. Example of the kinds of misconceptions and matching geographical items can be found in the Appendix.

Results: Analysis and Discussion

Sign tests were performed at each grade to determine whether students scored systematically higher or lower on either the geography or mathematics tasks. The results indicated that there was no consistent directional difference between the scores, although more students had higher geography than math accuracy scores (z third grade = .25; z fourth grade = 1.60; z fifth grade = 1.58).

Further analyses of the data focused on the correlational relationship between accuracy in the knowledge and use of mathematical contents and accuracy in the attainment and application of geographical concepts within each grade level. Correlations, utilizing the Pearson product moment correlation procedure, were

Table 2

Content of Mathematical Misconceptions and Related Geographical Content by Grade

Content of Mathematical Misconceptions	Third grade	Fourth grade	Fifth grade
	Related Geographical Content		
Numeration (place value, relative magnitude, Latitude/longitude negative integers)	Population information	Population information	Population information
Subtraction (computation, place value)	Map scale Population information	Map scale Populaton information	
Multiplication (representation, computation)	Map scale	Map scale	
Division (representation, computation)		Map scale	Population information
Fractions (concrete form, symbol concept, addition)		Map scale Population information	Population information
Measurement (area, perimeter, proportion)	Map scale	Map scale Population information	
Graph Reading (line graph, circle graph, Latitude/longitude bar graph, grids, numeration)	Population information	Population information	Population information

obtained for overall accuracy scores between domains, for every possible pairing of subsections between domains, and for pairings between domains of overall accuracy scores and subsection scores.

The data analysed so far and reported on here were consistent with two of the three original expectations. First, it was hypothesized that children's misconceptions or lack of experience with mathematical content areas would be associated with inaccurate interpretations of geography content. This hypothesis was confirmed. As reported in Table 3, the data analysis indicated that there was a significant positive correlation between overall performance on the mathematics and geography items at all grades (r grade 3 = .78, $p < .01$; r grade 4 = .78, $p < .01$; r grade 5 = .88, $p < .01$). It also indicated that the relationship between accuracy in overall geographical and mathematical performance tended to increase from third to fifth grade both in overall geography accuracy and within each of the geography subsection areas. In particular, the magnitude of the relationship between mathematics accuracy and being able to define mathematically loaded geographical terms seemed to increase with grade (r grade 3 = .57, $p < .05$; r grade 4 = .55, $p < .05$; r grade 5 = .95, $p < .05$).

These findings suggest that as children go up in the elementary grades, the role of mathematical reasoning and skill become increasingly important in students' knowledge of geography. Whether the knowledge of mathematics concepts and procedures is necessary for learning some kinds of geographical contents or whether some common thinking or studying skill is needed for learning in both areas in not clear from these data.

Second it was hypothesized that particular types of mathematical

Table 3

Correlations Between Overall Mathematics Accuracy and Geography Tasks Accuracy

Task	Grade: Third	Fourth	Fifth
Overall geography	.78**	.78**	.88**
Geographical facts	.65**	.71**	.79**
Interpretation of facts	.78**	.76**	.82**
Definition of terms	.57*	.55*	.95**

* $p < .05$. ** $p < .01$.

competence would be more strongly associated with particular types of knowledge in geography than others. This hypothesis was also confirmed. Tables 4, 5, 6, and 7 indicate the correlations obtained between all possible geography and mathematics subsection pairs. These findings indicate that although correlations within grades tended to be significant between most pairs, they were stronger between some types of questions than for others and varied somewhat from grade to grade.

Computational skill, for example, (See Table 4) appears to be relatively stable and significantly correlated to accuracy in recall and recognition of geographical facts at all grades, dropping only slightly at grade 4 (r grade 3 = .72, $p < .01$; r grade 4 = $p < .01$; r grade 5 = .73, $p < .01$). However, computational skill seemed to have increasing importance in students' ability to interpret geographical information and understand definitions of geographical terms as grade increased from third to fifth (r grade 3 = .66, $p < .01$; r grade 4 = .61, $p < .05$; r grade 5 = .85 $p < .01$). This suggests that as children go up in the elementary grades and increasinly abstract concepts become involved in computation, an appreciation of these concepts facilitates interpreting mathematically related geographical material.

Table 5 indicates a marked increase in the magnitude of correlations from third to fifth grade between accuracy in understanding concepts and procedures of mathematics and all measures of geographical knowledge. The greatest increase, however, occurred between third to fourth grade (overall geography r grade 3 = .44, $p < .05$; r grade 4 = .72, $p < .01$; r grade 5 = .81, $p < .01$). Interestingly, for third graders, little or no relationship was found

Table 4

Correlations Between Accuracy on the Mathematics Computation Subsection and the Geography Tasks

	Grade		
Task	Third	Fourth	Fifth
Overall geography	.63**	.70**	.73**
Geographical facts	.72**	.65**	.73**
Interpretation of facts	.66**	.61*	.95**
Definition of terms	.30	.61*	.77**

* $p < .05$. ** $p < .01$.

Table 3

Correlations Between Accuracy of Understanding of Mathematical Concepts or Procedures and Accuracy on Geography Tasks

Task	Grade: Third	Fourth	Fifth
Overall geography	.44*	.72**	.81**
Geographical facts	.30	.60*	.73**
Interpretation of facts	.49*	.73**	.68*
Definition of terms	.25	.51*	.91**

* $p < .05$. ** $p < .01$.

between understanding concepts and procedures of mathematics and geographical knowledge (r overall = .44, $p < .05$; r facts = .30; r interpretation = .49, $p < .05$; r definitions = .25). This finding is consistent with the overall finding that mathematical competence becomes increasingly important in understanding geographical concepts as children get older. It also suggests that conceptual understanding of mathematics plays a generally less important role in the early grades than it does in later grades.

Table 6 suggests that the ability to apply mathematical knowledge to problem situations while generally related to accuracy on geography tasks (overall r grade 3 = .83 $p < .01$; r grade 4 = .84, $p < .01$; r grade 5 = .78, $p < .01$), may vary in relative importance in its usefulness for particular geographic activities in different grades. For example, it is least related to accuracy in interpreting facts at third grade ($r = .44$, $p < .05$) and most related to it at fourth grade ($r = .87$, $p < .01$). In general, though, these findings indicate that students who are able to problem solve in mathematics are likely to use similar strategies for problem solving in other subject areas.

Table 7 suggests the interesting notion that the ability to read graphs may be largely unrelated to performance on geography tasks until fifth grade at which time it is related to factual knowledge, interpretation of facts, and definition of terms. For third and fourth graders graph reading seems completely unrelated to mastery of geographical facts and the interpretation of these facts (r third grade = .40 & .25; r fourth grade = .30 & .28). These findings, of course, may be a function of the partiular contents chosen from the text material at each grade level. Specifically, for fourth graders and to a lesser extent for third graders, graph reading may not have

Table 6

Correlations Between Accuracy on Applying Mathematics to Problem Situations and Accuracy on Geography Tasks

Task	Grade Third	Fourth	Fifth
Overall geography	.83**	.84**	.78**
Geographical facts	.56*	.80**	.73**
Interpretation of facts	.44*	.87**	.69*
Definition of terms	.66*	.46*	.86**

* p < .05. ** p < .01.

Table 7

Correlations Between Accuracy on Reading Graphs and on Geography Tasks

	Grade		
Task	Third	Fourth	Fifth
Overall geography	.72**	.28	.90**
Geographical facts	.40	.30	.88**
Interpretation of facts	.25	.28	.79**
Definition of terms	.73**	.17	.90**

* $p < .05$. ** $p < .01$.

been necessary for an understanding of the geography text material used. In these grades students may have relied more on reading the text than on interpreting the graph to get information about the geographical concepts in question. Conversely, the fifth graders' questions may have dealt more directly with graphs rather than text. Nevertheless, it may be the case that interpreting graphs becomes a more complex matter as children go up in the elementary grades, and so differences in s ability become increasingly important in learning about other academic concepts involving graphs.

The third hypothesis was that mathematical knowledge would not necessarily be applied to reasoning about mathematically related geographical concepts. This hypothesis was only somewhat confirmed by the data correlating accuracy on math and geography tasks. In examining the few low and nonsignificant correlations in Tables 4 - 7, we can see that competence in mathematics areas is not always related to success in geographical tasks. We see this particularly in third graders whose understanding of mathematics concepts and procedures, (such as numeration and place value, knowledge of graph reading) was not always accompanied by success in using these skills in a geographical context. In general, however, the students who were successful in math were also successful in geography.

This hypothesis was also examined in another way, by looking at students mathematical misconceptions. If the hypothesis was to be confirmed, students with specific math misconceptions should not necessarily have lower scores on mathematically parallel geography tasks than students without misconceptions. Rather the students without misconceptions might fail to apply their knowledge appropriately in a different context and so not do any better in geography than the students who have math misconceptions.

Although these analyses have not yet been completed, the data suggest that children with strong and accurate mathematical conceptions, contrary to expectations, are able to apply their understanding to the geography context. Similarly, students' misconceptions also appear to color their reasoning about geography text material. An illustration of the analysis done on the relationship between particular mathematical misconceptions and performance on parallel geography tasks from each grade level is shown in Table 8. T-tests were utilized to compare the difference between the mean mathematically parallel geography scores of the two groups.

In third grade mathematical misconceptions were identified in representing the operation of multiplication in terms of concrete sets and in terms of addition. Students who exhibited these misconceptions were put in one group ($n = 5$) and those who did not were put in another group ($n = 11$). For each of these groups a mean geography score using items about map scales was computed (misconception group mean = 6.6; non-misconception group mean = 9.18) and found to be significantly different (t, df 14 = 2.97).

In fourth grade mathematical misconceptions were identified as confusions about what multiplication and area measurement actually represent beyond the execution of rote procedures. Again students were grouped according to who had and did not have misconceptions and for each of these groups a mean geography score using items about population density and land area was computed. The mean geography score of the group having mathematical misconceptions in area and multiplication concepts ($n = 8$) was 5.25 while that of the group without misconceptions ($n = 8$) was 8.13 (t df 14 = 2.32, $p < .05$).

Table 8

Comparison of Mean Parallel Geography Scores of Students With and Without Specific Mathematical Misconceptions

	Mean Score		
Grade	Students without misconceptions	Students with misconceptions	t-test
Third grade (multiplication)	9.18	6.60	2.97**
Fourth grade (area/multiplication)	8.13	5.25	2.32*
Fifth grade (fractions)	6.80	3.40	2.54*

* $p < .05$. ** $p < .01$.

Similarly in fifth grade, students were grouped as having mathematical misconceptions about the representation of and operations with fractional numbers. In geography, group mean scores were based on items related to population statistics involving the fractions in circle graphs of immigrants coming from particular places in relation to the total number of immigrants. The students with basic misconceptions in math ($n = 5$) had a parallel geography score of 3.4 while the students who did not have these misconceptions about fractions ($n = 5$) had a significantly higher geography mean of 6.8 (t df $8 = 2.54$, $p < .05$).

Other parallel content questions will be examined in all grades and the final data will include all the concept comparisons indicated in Table 2. The findings thus far suggest that mathematical misconceptions can interfere with the understanding and learning of mathematically related geographical material and that children who have accurate conceptions of the mathematics material were able to apply their knowledge to the geographical context. The third hypothesis of this study, therefore, was not confirmed. In particular, knowledge of the meaning of multiplication seems necessary for an accurate understanding of map scale relationships; knowledge about area measurement seems necessary for understanding population density; and understanding of fraction numbers seems necessary for a sensible interpretation of circle graph representations of population statistics.

Summary and Conclusions

Overall the findings of this study suggest that success in mathematically related geography is strongly associated with children's knowledge of mathematics concepts and procedures and that this relationship tends to increase as children go up in the elementary grades. This, however, does not necessarily imply that mathematically related geographical content is acquired solely as a function of applying mathematical knowledge to geographical contexts. Rather, in some contexts there may be a common non-mathematical component related to performance in both areas.

For example, students who are facile at tuning into relevant information and selecting important facts from incidental ones, may be at an advantage for learning both mathematical and geographical content. Moreover, having a reliable memory would be an asset for acquiring both procedural techniques and number facts in mathematics as well as for retaining the verbal content of geography text material even if its meaning is not clear. Similarly, students who appoach geography text as a reading comprehension task, may be able to accurately repeat mathematically connected geography information as long as it does not require any actual mathematical activity.

The data, however, did suggest that children who are competent in grade-level mathematics will tend to successfully apply their knowledge to another domain, that of geography, while children who are not competent in grade level mathematics may be at a disadvantage for learning geographical content. The results of this study, indicating that children's level of knowledge in one area can affect the acquisition of knowledge in another area, therefore, could be significant for educational practices.

First they add to our general understanding that children's existing knowledge base can interact with school curricular content and remind us that there are a variety of ways in which students can interpret "objective" content. Second, by extending our knowledge about how concepts from one domain can apply to another, this study demontrates that effective instruction must take into account the fact that academic subject areas often overlap with one another. Teaching in one content area, therefore, implies that instruction should precede or be accompanied by instruction in a conceptually related area.

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APPENDIX

Examples of Mathematical Misconceptions and Associated Geographical Knowledge for Grades Three, Four, and Five

A) In third grade children were asked to answer several different types of questions involving the concept or procedures for multiplication. For example, they were asked to draw a picture to represent 4 x 7, to draw a rectangle that was two times higher and two times wider than an original rectangle that was 2 units high and 4 units wide, and to express some single digit multiplication examples as addition. Misconceptions included:

a) representing 4 x 7 as:

```
     o o o o o o o
x          o o o o
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b) indicating that a figure that was two times higher and wider than a 2 x 4 rectangle was 4 x 6 because 2 + 2 = 4 and 4 + 2 = 6

c) saying that 3 x 15 was the same as 3 + 15

The parallel geography questions involved text material about using a map scale. Students were asked to read a few paragraphs about the meaning of "drawing to scale" and shown a map of a playground that was drawn on a scale of one inch to 10 feet. They had to answer some question about distances in the real playground based on this scale.

B) At fourth grade, students were grouped according to the presence or absence of mathematical misconceptions involving area measurement and multiplication-division concepts. These concepts were parallel to geographic text material dealing with land area and population density. In the mathematics interview, children were asked to find the area of a rectangle drawn on centimeter squared paper and to draw a shape that had an area of 15 square units. They were also asked to carry out some written muliplication, express it as addition, compute a division example and prove the answer was correct. Some common misconceptions included:

a) confusing the concept and procedure for finding area with that for finding perimeter as in constructing a shape with an area of 15 by drawing a square with each side measuring 15

b) expressing 6 x 20 as equivalent to 6 + 20

c) solving a division example by using the process of multiplication

The parallel geography content included answering questions about a passage that reported on the populations, land areas, and relative population densities of New Jersey and Alaska.

C) On the fifth grade math interview children were asked questions about the value of fraction symbols ("Which number is larger, 3/5 or 3/8?"), about how to operate sensibly with fraction symbols ("How much is 2/3 of a 6-pack of soda?"), and about how to interpret graphic representations of fractions ("If you added up all the fractions in this circle, how much would you get?"). Among the most common misconceptions was a tendency for some students to treat each part of the written fraction number as a whole mber. For example, when asked about the relative value of 3/5 and 3/8, many children responded that 3/8 was larger because the 3's were the same, but the 8 was larger than the 5. Similarly, when asked to figure out how many cans were in 2/3 of a 6-pack of soda, some students tried to use some kind of whole number operation on the 2 and 3 in 2/3, such as 2 + 3 is 5, so the answer is 5. In a similar way, some students added the numerators and then added the denominators of the fractional parts of a circle in order to determine the total of all the fractions in that circle.

Analogously, on the geography interview, the children were asked to use the population information obtained from text, a bar graph and related fractions in circle graphs to determine some facts about immigration patterns from Europe to the United States over several decades ("In which decade did the most immigrants come?" "What fraction of all European immigrants in the 1860's were from Northern and Western Europe?" "Were there more immigrants from Northern and Western Europe coming to the United States in the 1860's or between 1900 nd 1909?"). To complete the task accurately it was necessary to understand that the circles as a whole represented all the European immigrants and that the fractional parts of the circle represented a fractional part of that whole. Most critical to understanding was the idea that the fractional part did not represent any particular number of immigrants, but could be assigned a value if used in combination with the information in the bar graphs.