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ABSTRACT

This project was a one-year study of California's reform effort in mathematics education. The research focused on two issues of great importance to current efforts to reform mathematics instruction: (1) What distinguishes teaching mathematics for "understanding" from rote performance and what does teaching mathematics for understanding in elementary schools entail? and (2) What can policy do to improve such mathematics teaching and learning? The research addressed these questions by examining effects on classroom practice of a state policy to promote teaching mathematics for understanding. In 1988-89 researchers observed and interviewed 23 second- and fifth-grade teachers concerning their knowledge and beliefs about mathematics teaching and policy initiatives, their efforts to change their practice, and resources and constraints that influenced their ability to do so. Researchers also interviewed administrators and policymakers at the school, district, and state levels. This report focuses on changes in teaching practices related to the state policy of teaching mathematics for understanding. Detailed case analyses are presented of nine elementary teachers in six different elementary schools (three high SES and three low SES schools) in three different school districts in California. Three themes emerging from these cases are discussed in a final commentary. The first theme concerns texts and other curriculum materials as agents of change; the second theme explores the "layers of reform"; the third theme focuses on the profound influence of practice on policy. (KR)

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Effects of State-Level Reform Of Elementary School Mathematics Curriculum On Classroom Practice

Final Report

Submitted by
David K. Cohen and Penelope L. Peterson, Co-Directors
and
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Abstract

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Authors' Acknowledgements

This report is the result of work done by a collaborative team of faculty and graduate students at Michigan State University. Team members included: Deborah Loewenberg Ball, David K. Cohen, Ruth M. Heaton, Penelope L. Peterson, Richard S. Prawat, Ralph T. Putnam, Janine Remillard, Nancy Jennings Wiemers, and Suzanne M. Wilson. This work was sponsored in part by the National Center for Research on Teacher Education and the Center for the Learning and Teaching of Elementary Subjects, College of Education, Michigan State University. It was funded by the Office of Education, Research and Improvement, Grant No. R117 P8000 4, United States Department of Education. The opinions expressed in this report do not necessarily represent the position, policy, or endorsement of the Office or the Department. The researchers wish to thank the teachers whose stories are told here, as well as those whose tales remain untold, for their cooperation, openness, and willingness to share their thoughts and teaching.

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Policy and Practice: An Overview

David K. Cohen and Deborah Loewenberg Ball

Policymakers believe that they can steer school practice and change school outcomes. That conviction is manifest in many past and present efforts to improve education, including competency tests, teacher evaluation systems, state teacher certification codes, mandated curricula, and high school graduation requirements. Yet educational researchers report that state and federal policies have affected practice only weakly and inconsistently (Launor, 1984; Cohen, 1989, 1990; Cuban, 1990; Kennedy, Berman, & Demaline, 1986; Rowan & Guthrie, 1989; Sarason, 1971; Stake & Easley, 1978; Welch, 1979).

Policymaking has not, however, been slowed by such reports. The last decade has seen a bumper crop of state policies aimed at instructional improvement. Many states have launched testing programs that seek to drive teaching and learning in new directions. State-mandated reforms of instructional content also have increased. These include the California curriculum frameworks, the revised Michigan reading assessment, efforts to "align" instruction and assessment in Florida and South Carolina, and many new state teacher education requirements.

Despite this rich harvest of new policies, educational researchers continue to argue that the effects of education policies and programs depends chiefly on what teachers make of them (Elmore & McLaughlin, 1988; Firestone, 1989). Are policymakers mistaken to assume that their policies and programs can change teaching and learning? Or do researchers fail to notice ways in which policy *does* affect instruction?

We know relatively little about the answers to these questions. Many recent state policies have sought to push high school instruction toward an academic core by increasing graduation requirements. And the policies seem to have had some of the desired effects: High school course offerings and students' course taking have changed (Clune, White, & Patterson, 1989). But whether more required courses actually affect what is taught or learned remains an open question. Little is known about how teachers perceive instructional policies, how they interpret them, and how different kinds of policies influence teaching and learning. Many policies and programs have been aimed at classrooms, but what we know about those policies stops at the classroom door, for policy research has seldom investigated the effects of policies on the actual work of teaching and learning. We are only beginning to learn about effects of policy on classrooms, teachers, and students (Cohen, 1990; Porter, Floden, Freeman, Schmidt, & Schwille, 1988; Schwille, Porter, Floden, Freeman, Knappen, Kuhs, & Schmidt, 1983).

This report focuses on the relations between instructional policy and classroom practice. Behind the classroom door, we explore how nine elementary teachers have interpreted and responded to a state-level policy designed to radically change mathematics teaching and learning. The policy in question resembles others in the current reform movement: It is an ambitious effort to shift mathematics teaching from mechanical drill and memorization toward mathematical reasoning and understanding. The policy's framers and advocates want students to learn mathematics in more meaningful ways and, toward this end, they are proposing fundamental revisions in content and pedagogy. Although the policy has a familiar ring to mathematics educators, it is a far cry from modal practice. Mathematicians and mathematics educators have pressed for similar changes for nearly a century but have persistently failed to make much progress. Policy analysts say that endeavors of this sort are not unusual in the so-called "second wave" of current reform efforts, in which state agencies seek fundamental change in instruction (Firestone, Fuhrman, & Kirst, 1989).

The analysis presented in this report centers on case studies of nine teachers. To set the stage, we offer first a cursory discussion of the nature of the policy, its vision, and the changes it implies. We then sketch a few themes that have been evident in the early stages of the policy's implementation and foreshadow threads that run throughout the cases themselves.

A New Approach to Elementary School Mathematics

Just what changes in teaching and learning mathematics does the new California state policy envision? And what does the policy entail for practice? A brief look at the current mathematics education reform movement, of which the California initiative is a part, helps to provide a context for the nature of the changes envisioned.

One impetus for the current reform movement (National Council of Teachers of Mathematics, 1989a, b; National Research Council, 1989, 1990) is the widely-held belief that American mathematics education is failing (Dossey, Mullis, Lindquist, & Chambers, 1988). Highly predictable, mathematics teaching in most elementary classrooms emphasizes rules, procedures, memorization, and right answers (Goodlad, 1984; Stodolsky, 1988). Students seldom confront serious mathematical problems and are rarely expected to reason about mathematical ideas. Teachers stand at the board, show students how to do a particular procedure or type of problem, and assign practice exercises. Students then work quietly on these, asking the teacher for help if they get stuck. When students are done, the teacher checks their answers, marks the ones that are wrong, sometimes goes over the steps once again, and students fix their incorrect answers.

In these classes, mathematics is represented as calculation; learning mathematics as rote memorization. Mechanics dominate: Students do not experience mathematics as ways of thinking about quantitative and spatial patterns nor as a creative endeavor. Mathematical domains such as probability and geometry are typically sacrificed to traditional arithmetic topics such as subtraction and long division. Students produce answers, not questions; "problem solving" most often means symbolizing and calculating routine word problems. And students seem to learn as they are taught: While many are able to perform basic arithmetic calculations, few are able to reason about mathematical questions or to solve even moderately complex problems (Dossey, Mullis, Lindquist, & Chambers, 1988). Viewed in world perspective, American students' performance is notably weak (Tevers & West' ary, 1989). In light of these problems, reformers have argued for major changes in goals, in content, and in pedagogy.

New goals for learning. A consistent theme in current efforts to reform mathematics education is the need for dramatic change in what students learn. One prominent idea is that students should be able to reason mathematically. Another is that they should be able to use such reasoning by applying mathematics to everyday situations. They should be able to understand the conceptual basis of mathematical proceduresas well as be able to evaluate mathematical arguments and quantitative data (California State Board of Education, 1985; National Council of Teachers of Mathematics, 1989a; National Research Council, 1989, 1990).

New conceptions of mathematical "content." A second major theme in current reform efforts is that students will not learn such things unless teachers make significant changes in what they teach. Reformers argue that teachers should help students make sense of traditional topics; in addition, they should offer students opportunities to explore novel topics like probability. And, in addition to teaching mathematical "topics," teachers should focus on helping students learn to reason with, communicate about, and use mathematics. Instead of asking them to memorize algorithms for solving time-speed-distance problems, for example, teachers should help students to figure out how to turn a story about traveling from New York to Chicago into a mathematically solvable problem. Instead of helping students translate "how many more miles" into minus signs or to convert hours methodically into minutes, teachers should help students make and interpret their calculations in the meaningful context of the travelogue itself. Rather than just trying to get students to give the right answers, teachers should help students to reason mathematically about what might be plausible solutions. Teachers should encourage students to offer alternative solutions to problems and invite them to collaborate in figuring out what makes sense and why.

New pedagogy. If teachers taught these kinds of things, mathematics classes would be very different. Rather than relying exclusively on symbolic representations, teachers and students would use blocks, fraction bars, beans, and pictures and diagrams of many different sorts to represent their mathematical ideas. Students would talk much more in mathematics classes--with their peers in small groups as well as in whole-class discussions. Instead of focusing on the "right answer," students would discuss how to frame problems fruitfully and debate the merits of alternative ways of solving them. In classrooms of this sort, teachers' and students' roles would be dramatically different. Students would do much more of the teaching, for they would be working together, filling the air with ideas about how to solve problems, about what made sense, and why it made sense. Teachers would rely less on direct instruction and more on framing lessons, coaching, orchestrating discussion, and the like. Teachers would be no less important, but they would be important in different and unusual ways (National Council of Teachers of Mathematics 1989b).

Teaching and learning of this sort are quite remote from current practice. In most elementary math classrooms teachers have a virtual monopoly on instruction. They tell and show students how to do particular procedures. They monitor students' performance on written assignments. They act as though students learn mathematics through repetition and drill. Students' contributions are limited to doing as they are told, memorizing procedures, learning "facts," and giving brief, unexplicated answers to highly-focused teacher questions.

Great changes therefore would be required to realize the new policy in practice. How did policymakers in California propose to produce such changes?

Early Implementation

The policy was announced in 1985, in a new set of state mathematics curriculum guidelines. State officials had consulted widely with mathematicians, mathematics educators, and teachers in the development of the guidelines, referred to as the *Mathematics Curriculum Framework*. Such frameworks have been issued as advisories for local educators since the 1960s, and the 1985 version echoes many of the earlier *Frameworks'* concerns (Wilson, 1989). But the 1985 *Framework* has played a very different role than its predecessors. State officials used it to press publishers to revise both the mathematical content and pedagogical suggestions in their books, arguing that the texts should conform to the *Framework*. Publishers were exhorted to emphasize understanding rather than rules and to include novel topics such as probability and logic. Publishers were told that if they did not make major changes, their books would be struck from the state's textbook adoption list. This was a bold departure from past uses of state curriculum frameworks. And the State Education Department soon showed that it meant business by rejecting every math textbook that publishers submitted. State officials announced that many revisions were required before the books would be accepted.

Officials in Sacramento then used the new *Framework* to launch a major revision of the state's student achievement testing program. Since 1985, state officials have been developing tests that are intended to assess students' understanding of mathematics rather than just measuring their capacity to remember algorithms and produce correct answers. Policymakers in California believe that these revised tests will be a potent instrument of policy: The redesigned tests are expected to "drive" fundamental changes in teaching and learning mathematics.

This sketch reveals that instructional *alignment* plays a key role in California's efforts to improve mathematics teaching and learning. The idea behind alignment is deceptively simple:

Education agencies should recast curriculum guides, textbooks, and assessments, so that all send the same clear messages to teachers and students. The underlying notion is that if instructional guidance is more consistent, teaching and learning will improve (Finn, 1989). But this approach is quite unfamiliar in American education: Most education agencies set only broad goals, or require courses for graduation (Cohen, 1990; Porter, Floden, Freeman, Schmidt, & Schwille, 1988; Schwille, Porter, Floden, Freeman, Knappen, Kuhs, & Schmidt, 1983). In contrast, education agencies seeking alignment might define rather specific instructional goals; they might recommend particular teaching methods; and they might specify the topics to be covered, as well as their content. Alternatively, education agencies might seek only to influence all of these things by way of powerful assessments (Finn, 1989; Resnick & Resnick, in press; Shanker, 1989). Whatever the mechanisms, an underlying assumption is that instruction is too important to be left to schools and teachers: It must be closely and carefully managed by higher level agencies.

Effects in Practice

What is happening in California classrooms in response to the new policy? The story is just beginning to unfold. This account is a first report and the cases that follow offer only partial and preliminary answers. They are partial because the cases focus on only one aspect of the issues sketched here--what teachers have done with the new guidelines and textbooks. The new state tests had not yet been used in the elementary grades, because their revision was incomplete when we visited the classrooms discussed below. And our answers are preliminary because we focus on teachers' responses only a few years into the reform. But the issue is compelling, nonetheless: What sense do practitioners make of such policies?

One thread in the cases is that teachers have responded to the policy in quite varied ways: Some have made what they see as major changes, while others have changed little or not at all. Another thread, not surprisingly, is that the changes we saw depended partly on what we looked for. Had we attended only to the forms of instructional discourse--asking whether these were traditional lecture, recitation, and seatwork classes--we probably would have concluded that little was new, and that policy had not much affected instruction. Alternatively, we might only have asked if the mathematical content of classrooms measured up to the new *Framework*'s demands. Had we done so, we probably would have concluded that the new practice did not measure up, and that the policy had made little difference.

These inquiries are reasonable, but we did not limit ourselves to them. We also scrutinized the finer texture of the teachers' classroom practice: The particular topics that teachers taught, the content they sought to convey, their pedagogy and classroom organization, and the relations among these things. We also investigated how these compared to teachers' views of their past work. We looked closely in part because our analysis of the policy suggested that it proposed particularly demanding changes--a notion that found support in some other studies of teaching and learning (Cohen, 1989; Lampert, 1988). For one thing, the policy signalled the need for a revolution in most teachers' knowledge of mathematics. For another, the new policy invited basic change in teachers' beliefs about mathematics and in their beliefs about how students learn mathematics. Additionally, the policy called for change in how teachers thought about their role and how they conducted their classes.

Each of these is a crucial dimension of teacher knowledge and practice. Taken together they form an intricate web of ideas and understandings, orientations, habits, and assumptions. None seemed likely candidates for facile change. We suspected that most teachers would have much to learn--and *unlearn*--in order to make such changes. If so, it seemed likely that, like students learning to understand mathematics, California teachers might learn in partial and halting ways.

These seemed sufficient reason to look closely at teachers' work, but there was still another. Researchers have relatively little experience in direct study of how innovations affect teaching practice. We had much to learn from the teachers we proposed to observe, about how teachers respond to policy.

Our studies are an early step in such learning. We turned up evidence that California's new policy had influenced practice, but we also turned up evidence that practice had influenced policy. For this new policy could only affect students' mathematics learning through teachers' extant knowledge and beliefs about mathematics and their practice of mathematics teaching. Policies like this one are made in order to change practice, but they can only work through the practice they seek to change. Teachers are at once the targets and the agents of change.

This point plays itself out in a variety of ways. For example, many teachers see mathematics rather traditionally, as a string of topics to be covered serially: Single-digit addition, two-digit addition, single-digit subtraction, and so on. Some of the teachers whom we observed took that view, and it affected their understanding and enactment of the new policy: They tended to interpret the policy as a set of suggestions for topics that they should add to the string. Estimation, manipulatives, and problem solving became new discrete topics, rather than--as the *Framework*'s authors appeared to intend--elements useful in all sorts of mathematical reasoning. These teachers had implemented an element of the *Framework*, in a sense; their lessons had some of the new content. But that new content was organized within the existing structure of traditional school mathematics. To take another example, teachers often search for pedagogical tactics that will engage students and transmit material more effectively: classroom games, film strips, tricks, concrete models, and the like. Some of the teachers whom we observed seemed to apprehend the policy as a new source for such strategies. For example, the *Framework* had recommended the use of concrete materials so that students would have access to varied representations of mathematical ideas. Several teachers were delighted with the idea; they avidly embraced concrete materials, and used them extensively. In a sense they had implemented the policy. But these teachers' use of the new materials was filtered through their established practice: Some of these teachers offered students no opportunities to figure out what sense they made of the new materials; they used the manipulatives to capture and rivet students' attention on memorizing the traditional rules and procedures. Rather than treating the materials as opportunities to help students construct and articulate their understanding of mathematics, these teachers treated the materials as another didactic agent of direct instruction, presenting traditional menus of mathematical content. Again, the new mathematics instruction was filtered through an older and much more traditional mathematical and pedagogical structure.

Hence, our case studies of how policy influences practice also are studies of how practice influences policy. For instructional policies are filtered through teachers' knowledge and beliefs about academic subjects, and through their established practice. California teachers' mathematical and pedagogical pasts shaped the mathematical future that this new policy invited them to help create. Policies that seek to change instructional practice depend upon--and are changed by--the practice and the practitioners they seek to change.

These comments highlight a key dilemma in recent demands for improved teaching--teaching for "understanding," or "higher order thinking." On the one hand, teachers are central actors in the old school subjects: They offer a mechanical version of reading, or writing, or mathematics, and the result offends reformers. Often it bores students. Teachers are, in one sense, the problem that policy seeks to correct. Yet teachers are the most important agents for improving things: Students' encounters with mathematics in school will not change unless teachers change them (Cohen, 1990).

How can teachers teach a mathematics that they never learned, in ways that they never experienced? That is the dilemma that such reforms pose. Our cases explore it. They suggest the enormous complexity of the changes that many instructional policies imply and the great difficulties of producing such changes from a distance.

One last point: These cases are not an evaluation of California's new policy or of the teachers we portray. Such an evaluation would be entirely inappropriate, for the policy has only just begun and we visited these teachers in their first year of using the newly-revised texts. This is not another effort to pull a tender young thing up by the roots, to see if it is growing. It is only an early report from a few outposts on the front lines. We offer no conclusions about the policy's "impact." But we do offer some insights into how instructional policy and teaching practice affect each other. We think that those will be of interest to researchers and policymakers everywhere, at any stage in the evolution of instructional policy.

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The California Study of Elementary Mathematics:

An Overview of the Methodology

Penelope L. Peterson

This research is part of a larger study of the effects of state education reform in elementary mathematics curriculum on school district policies and practice, with special attention to teaching and learning in elementary mathematics classrooms. In this study, researchers attempt not only to document effects but to also to understand how and why certain effects occur. Additionally, the research is designed to explore the processes that lead to certain effects on teachers' classroom practice of teaching and learning mathematics. To do this, researchers collected data at four levels: state, school district, school, classroom (teachers and students).

Method

Site Selection

State. California was selected as a site for the research because the state is at the leading edge of education reform and is making an unprecedented effort to realize a new vision of mathematics teaching and learning. Looking at the text of the *California Mathematics Framework* (California State Department of Education, 1985) provides some insight into this vision of "teaching mathematics for understanding":

> Those persons responsible for the mathematics program must assign primary importance to a student's understanding of fundamental concepts rather that to the student's ability to memorize algorithms or computational procedures. Too many students have come to view mathematics as a series of recipes to be memorized, with the goal of calculating the one right answer to each problem. The overall structure of mathematics and its relationship to the real world are not apparent to them.
>
> The assumption is sometimes made that students who can perform an arithmetic computation understand the operation and know when to apply it. Teachers know and test results indicate that students are fairly competent at performing computations but have difficulty applying their skills to problem solving situations.

Teaching for understanding emphasizes the relationships among mathematical skills and concepts and leads students to approach mathematics with a common sense attitude, understanding not only how but also why skills are applied. Mathematical rules, formulas, and procedures are not powerful tools in isolation, and students who are taught them out of any context are burdened by a growing list of separate items that have narrow application. Students who are taught to understand the structure and logic of mathematics have more flexibility and are able to recall, adapt, or even recreate rules because they see the larger patten. Finally, these students can apply rules, formulas, and procedures to solve problems, a major goal of this framework (pp. 12-13).

Teaching for understanding, as defined by the *California Mathematics Framework* (California State Department of Education, 1985, pp. 12-13), is contrasted with teaching rules and procedures for their own sake in the following ways:

Teaching for understanding:	Teaching rules and procedures:
Emphasizes understanding	Emphasizes recall
Teaches a few generalizations	Teaches many rules
Develops conceptual schemes or interrelated concepts	Develops fixed or specific process or skills
Identifies global relationships	Identifies sequential steps
Is adaptable to new tasks or situations (broad application)	Is used for specific tasks or situations (limited context)
Takes longer to learn but is retained more easily	Is learned more quickly but is quickly forgotten
Is difficult to teach	Is easy to teach
Is difficult to test	Is easy to test

Districts and Schools. Within California, a large and a moderately-sized school district were selected as sites for the research. The large district, Southdale, has a school-age population of 118,000, and the moderately-sized district, Forest Glen, has a school-age population of 24,706. These districts were selected because they were reported to be engaged in significant efforts to teach mathematics for understanding, and district personnel agreed to participate in the research. A third smaller district, Viceroy, was selected that adjoins the larger district. In contrast to the other two districts, this smaller district was selected because it had a standard mathematics program.

Within Viceroy, two schools were selected that were nominated by a district administrator as "average" schools. Within Southdale and Forest Glen, two elementary schools were selected that were nominated by district administrators as schools in which attempts were being made to teach school subjects for thinking and understanding. Because the schools were selected as part of a planned research design (Elementary Subjects Center, 1987), the schools were selected within each district to represent contrasts in socio-economic status (SES) composition of the student body. The rationale for this design was that differences are likely to exist in the enacted curriculum between classrooms comprised primarily of students from low SES families and classrooms comprised primarily of students from high SES families (e.g., Irwin, Alford, Berge, Floden, Freeman, Porter, Schmidt, Schwille, & Vredevoogd, 1986). Figure 1 shows the SES level and school district for each of the schools in the study.

Figure 1. Socioeconomic status of school population and type of district for each of the schools in the study.

Districts	Socioeconomic Status of School Population: High SES	Low SES
Southdale (large urban)	Johnson School	Columbus School
Forest Glen (moderate-sized urban/suburban)	Davidson School	Valley School
Viceroy (small suburban)	Mt. Vernon School	Highland School

Classrooms and teachers. Within five of the six schools, four elementary teachers and their classrooms and students were studied, two at the second-grade and two at the fifth-grade levels. At Viceroy School, two teachers were studied at the second grade level and one at the fifth-grade level. These grades span the elementary curriculum, and studying two teachers at each grade level permitted us to explore possible differences in implementing change in primary versus upper elementary level work.

Data Collection Procedures

As part of the larger study, data were collected from state-level informants, from school district administrators, and from school principals about state-level and school district policies, procedures, and practices with regard to elementary mathematics curriculum, elementary classroom practice and teaching of mathematics. In the case studies reported here, these data were used to gain an understanding of the policy context in which teachers were working.

State level. On-site and telephone interviews were conducted with key state-level informants in Sacramento, as well as the Bay Area. Informants included individuals who were responsible for writing the 1985 *California Mathematics Curriculum Framework*; individuals within the State education agency who are in charge of curriculum, instruction, and assessment; persons responsible for allocation of resources and training; and people in state associations who have a special interest in curriculum, mathematics, teaching, and teachers (e.g, the California Mathematics Project). The purpose of the interviews was to determine individuals' perceptions of (a) what the policy is; (b) the demands on practice that they think are implied by the policy; and (c) the resources that they imagine it would take to implement the policy.

District Level. District level data were collected through on-site interviews with individuals who were judged to be important in affecting how curriculum content policies, including the allocation of resources. Informants included mathematics and elementary curriculum coordinators, staff development coordinators, relevant staff in the superintendent's office, and superintendents. Interviews were designed to surface personnel's perspectives on the mathematics program, their knowledge of the mathematics *Framework*, their perspectives on what it means to teach mathematics for understanding, as well as asking them questions about testing, textbooks, and teacher education.

School level. The principal in each school was also interviewed as well as any teachers or other staff who were perceived as leaders in mathematics curriculum development or staff development in the school (e.g., "mentor" or "support" teachers). Questions focused on the informants' knowledge and understanding of the policy, perceived demands of the policy, resources needed and provided, perceived utilization of resources, and perceptions of implementation of the policy.

Classroom Level. The classroom level data collection and analyses were directed toward determining whether and how state level mathematics curriculum policies have influenced classroom practice. Information on teachers ($N = 23$) and their classroom practices was collected during two week-long site visits in December, 1988, and again in March, 1989. Researchers conducted observations and interviews in each of the two schools during that week. Substitute teachers were hired so that each teacher could be released for a two-hour interview during each of the site visits. In addition, teachers were paid honoraria for their participation.

In the interviews with teachers, researchers attempted to determine teachers' knowledge of the policy, including what they knew and believed about the policy and about what was implied by the policy. The interviews focused on issues of mathematics curriculum, assessment, teaching and learning in mathematics, including what it means to teach mathematics for understanding. Interviewers attempted to determine the demands placed on practice from a teacher's point of view and the resources that teachers felt were needed to implement the policy, as well as those that the teachers reported had been made available and had been utilized. Information was gathered on what other policies and practices in the school district affected a teacher's ability to implement the instructional reform (e.g., other district policies, working conditions, absence of resources).

In addition, because we hypothesized that teachers' ability to implement the instructional reform in elementary mathematics would be influenced significantly by their knowledge of and beliefs about mathematics and their knowledge of and beliefs about the learning and teaching of mathematics, questions were designed get their understanding about math, learning, teaching, and students. The interview format and approach used for these questions were similar to those used by researchers in the National Center for Research in Teacher Education (NCRTE) (See, for example, NCRTE, 1989). For example, second-grade teachers were asked a set of questions on subtraction with regrouping, and fifth-grade teachers were asked a set of questions on addition and subtraction of fractions. Questions included: "What do you think is important for students to learn about this? Are there things that kids find difficult in learning this? Why? Can you describe to me how you taught this topic this year (or how you have taught it in the past?) Were there things you decided to do differently than in the book? Why? Some teachers make up stories or other gimmicks of some kind to help students get the idea. Do you do anything like this? What is it?" In addition to these questions, the interviewers showed teachers several examples of students' correct and incorrect solutions to problems in either addition and subtraction of fractions or subtraction with regrouping. Then they queried teachers about these solutions, and asked what teachers would do if their students came up with them. Interviewers showed teachers the specific pages of their textbook that dealt with these topics and questioned them about whether and how the teacher used the textbook to teach addition and subtraction of fractions or subtraction with regrouping.

Researchers also observed the teachers teaching mathematics for two days during the first site visit and one day during the second site visit. Researchers conducted a brief pre-observation interviews with the teachers as well as extensive post-observation interviews. During the post-observation interview, the conversation focused on what the teacher was trying to teach, why the teacher was trying to teach it, how the teacher was trying to teach the material, and what the teacher thought the students got out of the lesson. These pre- and post-observation interview procedures and the observation procedures were adapted from those developed by NCRTE (1989).

Classroom discourse was audiotaped using a wireless microphone worn by the teacher. If significant student discourse occurred that could not be picked up by the teacher's wireless microphone, additional tape recorders were used. Portions of the tapes of the classroom discourse were transcribed by the observer for later analyses. Observers also took field notes. Later the same day, observers wrote narrative summaries of their observations of the mathematics lesson and class. The summary included information on the students in the class, the mathematics subject that was being taught, the instructional representations that were used to teach the subject, the teacher's and students' actions, behaviors, and talk during the lesson. In addition, the observer identified and described the major "chunks" of the lesson, the role that the textbook played in the instruction, the use of manipulatives and the use of cooperative groups. The observer also wrote responses to a series of analytic questions on an observation guide. Questions focused on what mathematical content was being taught; what seemed to be the goal; what instructional representations and mathematical tools were used by the teacher and students; what opportunities existed to "unpack" ideas or use mathematical tools; what was the nature of teacher and student discourse; and whether instruction was "mixed" (e.g., of means and ends; of means and means; of ends and ends) and if so, how.

Analyses

From transcripts of interviews with teachers, written field notes, and audiotapes of teachers' and students' classroom discourse, researchers constructed case studies of the teachers whom they had observed and interviewed. The subset of cases presented here illustrate important issues in the implementation of policy in practice.

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Who is Minding the Mathematics?

The Case of Sandra Better

Ruth M. Heaton

Somehow you've got to entertain them and they've got to learn. (interview, 12/88)

Sandra Better wants her 34 fifth graders, mostly white from middle and upper middle-class backgrounds, to learn mathematics in meaningful ways. And she wants all of her students, especially the girls, to find mathematics engaging. "A lot of women," she observes, "end up teaching math and a lot of women never felt good about math. A lot of these kids have been taught by people that don't feel good about math."[1] Sandra thinks she has a positive attitude towards mathematics and she hopes it will spread to her students.

Sandra is young, energetic, and extraordinarily enthusiastic about everything she does. Lively, she moves and talks quickly even when she is not in a hurry. Sandra is a risk taker, eager to try new things. Actually, she is more than willing. She eagerly attends workshops and inservices. She looks for opportunities to be a learner and thinks about ways to turn her own learning experiences into learning activities for her fifth graders. She says she learns best when "I laugh and learn at the same time." She tries to create similar experiences for her own students.

<u>Sandra and the *Framework*</u>

Sandra's familiarity with the *California Mathematics Framework* (1985) comes from a six-week mathematics seminar she attended at a nearby university a couple of summers ago. Sandra interprets the *Framework* as a list of the topics she is supposed to teach: It "tells what kids need to know at each grade level." From Sandra's perspective, the *Framework* defines what she needs to teach, but gives her discretion in setting purposes and choosing methods for teaching mathematics. She says the *Framework* "has opened up worlds" for her within her practice. It does not feel restrictive nor does it create any pressure for her mathematics teaching. Sandra commented:

> I think I was probably doing a lot of the stuff anyway, to tell you the truth . . . it makes me more comfortable now doing what I am doing. (interview, 12/88)

Sandra uses the *Framework* as a guide for what she should do in her classroom. She believes that she is already teaching for understanding. In an interview, Sandra responded to a paragraph from the *Framework* describing teaching for understanding by saying, "When I read it I seemed to be nodding the whole time. I think this is how I teach." Engaging students in real life problem solving situations that are fun is a primary goal for Sandra. Her goals of involvement, enjoyment, and fun within the context of real world mathematical problems emphasize the affective rather than the cognitive nature of mathematics. These goals are consistent with the *Framework*'s goal of having math programs that promote "the understanding and enjoyment of mathematics and provide a high degree of motivation" (California State Department of Education, 1985, p. 25). In addition, Sandra's practice illustrates some of the characteristics of a "high quality" program outlined in the state's *Model Curriculum Guide* (California State Department of Education, 1987). Teachers are to enjoy "engaging in mathematical activities and naturally project an expectation of enjoyment for students" (p. 4). As a teacher, Sandra exhibits an attitude ". . . of exploration and invention, conveying the idea that all students can learn, enjoy, and use mathematics" (p. 6).

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[1] All words in quotations are taken directly from transcripts of interviews or observations.

Sandra is eager to learn and willing to embrace new things that come her way. She likes to teach mathematics, and is viewed as an exemplary teacher within her school district. These attributes would seemingly make her an ideal candidate to teach mathematics for understanding. So it would appear that Sandra's eagerness, enthusiasm, and "fun" aligns her practice with the *Framework*. But something is missing from Sandra's math teaching. Though she is highly committed to making mathematics fun and meaningful, she does not always focus on the integrity of the mathematics. Consequently, the mathematical content is often problematic. In what sense, then, is Sandra teaching mathematics for understanding?

<u>Textbooks and Inservice as Instruments of Change</u>

Within the local school system in which Sandra works, testing, textbooks, and inservice have all been designed to align mathematics instruction with the *Framework*. Textbooks and inservice are especially prominent in Sandra's practice. Sandra's use of each is strongly influenced by her views of teaching and learning. These resources are designed to enhance the teaching of mathematics for understanding although they may become constraints when filtered through Sandra's knowledge and beliefs about teaching and learning mathematics.

Sandra's large, suburban school district in California chose one of the most innovative textbooks of the texts adopted by California, *Real Math* (1987). When I asked Sandra if she felt her district was committed to adopting the *Framework*, she responded affirmatively. She said they had adopted the Open Court textbook as a means of helping their teachers implement the *Framework*. But Sandra does not think Open Court--or any other textbook--is the answer to implementation of the *Framework*. As I talked with her, I got the sense that she and other teachers were concerned less with the textbook than all of the things they are being asked to reconsider:

> There's so much that goes on that teachers are asked to do. It is so hard to be great in everything and get excited about everything. You kind of get to the point where you are thinking, one more change. Who knows how long this is going to be in. (interview, 12/88)

Even if mathematics was the only thing Sandra had to think about, textbooks would not be at the top of her list of valuable resources. In keeping with her ideas of wanting learning to be fun and engaging, she cannot bring herself to teach directly from the book. Doing it, she says, "just bores me to tears."

Sandra described in more detail how she uses the new textbook:

> I used to follow it a lot. I'm trusting myself more to not follow it. I like to follow it because I like to know what things are important, what things are needed. But, I don't like teaching the way that they like to teach because somebody else wrote the book and they are not me . . . their method of teaching is totally different than mine. It doesn't work whenever I try to teach. You know it says do this first, then do this. It doesn't work. Kids don't get it and I don't get it. It's because I can't follow somebody else that well. So, pretty much, I improvise. (interview, 12/88)

Sandra follows the textbook loosely, modifying or omitting entire lessons. In using the textbook as a resource, she is doing what Ball and Feiman-Nemser (1988) found teacher education programs to be explicitly communicating to their students: "textbooks should be used only as a resource . . . following a textbook was an undesirable way to teach" (p. 414). In her use of the textbook, Sandra is making "professional" decisions about content and instruction.

But Sandra's professional decisions are filtered through her personal beliefs about learning, teaching, and mathematics. She wants her students, like herself, to learn as much as they possibly can and have fun in the process. When she deems it appropriate, Sandra combines lessons or skips pages from the textbook. "I can't stand spending two lessons on something that they're going to get in three minutes and I want them to get as much as they can" (interview, 12/88). She talked about wanting to teach her students geometry. "I made a point that I would really get to geometry this year and I had to skip a lot to get there . . . I must have skipped 150 pages in this book." Since becoming aware of the *Framework*, Sandra says she has felt "a lot more comfortable not necessarily going by the book and bringing in other stuff." Sandra's ideas about how children are helped to learn help her decide what sort of learning experiences to create for her students. However, Ball and Feiman-Nemser (1988) point out that "developing one's own plans requires a flexible understanding of the content to be learned as well as ideas about how children might be helped to learn it" (p. 419). Sandra seems to have ideas about how children learn but what is her understanding of the content to be learned?

Inservices, another instrument in place within the system designed to align mathematics instruction with the *Framework*, are a valuable resource for the "other stuff" Sandra brings into her classroom. She is always on the lookout for new activities. Her enthusiasm for teaching seems to be sustained by the activities she encounters and tries out in her classroom. She appears to thrive on these experiences. Her teaching interests shift with her latest inservice. When I asked her about her favorite subject to teach, she responded, "I don't know. It depends on what I have been inserviced in recently that got me on a high . . . whatever I feel I am most up with and I am doing something innovative in." When I asked her about specific inservices she had attended, she said she thought they were all "really good and very similar." They each included "a lot of group activities, a lot of fun activities, and games to get at the concepts." These inservices provide Sandra with "other stuff" to do so that she does not feel dependent on the textbook. But what are they giving her and what is she taking as she filters them through her view of learning and her knowledge of mathematics?

Perhaps it would help to consider what Sandra's teaching looks like. What follows are two snapshots from Sandra's practice. One a textbook lesson on functions, the other Sandra's use of an activity that she learned about in an inservice.

Inverse or Reverse? Sandra's Adaptation of the Text

I saw Sandra teach a lesson on inverse functions from *Real Math* (pp. 154-156). Inverse functions undo what another function does. For example, given $6 \longrightarrow (x4) \longrightarrow 24$ and $24 \longrightarrow (\div 4) \longrightarrow 6$, division is the inverse of multiplication. Traditionally, functions have not been included in an elementary school's math curriculum. The authors of *Real Math* included functions in their textbook and describe them as being "one of the most pervasive and powerful ideas in mathematics (which, for example, helps show the inverse relationships between multiplication and division and between addition and subtraction)" (*Real Math*, p. 114). For Sandra, as it would be for most other elementary teachers using this textbook for the first time, functions are a new topic to teach.

The objective for this lesson, as stated in the textbook, is "given a function rule (such as $x \longrightarrow (+3) \longrightarrow y$) or the equivalent explained in words, the students should be able to find y given x and find x if given y" (p.154). Sandra wrote sample problems from the textbook on an overhead projector. For example, given x=6, and the function $x \longrightarrow (+4) \longrightarrow y$, find y, or given y=18, and the function $x \longrightarrow (x6) \longrightarrow y$, find x.

For students who have difficulty with this, it is suggested in the textbook that the teacher use physical models, like a function machine or a wrapped gift. A function machine is a box that does something. You can put numbers into it and get out other numbers, according to some rule. For example, if the rule for the function machine is +8, the machine adds 8 to every number you put in. To find the inverse, you take numbers that come out of the box and put them back into the box.

The rule for the inverse function becomes -8. The machine now subtracts 8 from every number you put into the box. Thus, the inverse function will undo whatever the function does. A wrapped gift is a way to think about doing and undoing a function. To wrap the gift is to do the function. To unwrap the gift is to undo the function or do the inverse function.

Sandra used the analogy of shifting the gears of a car to indicate the directional movement of forward and reverse within the function. When teaching functions and their inverses, she proceduralized the task by devising a rule: Given x and a function, go "forward" to find y. Given y and a function go in "reverse" and do the opposite function to find x. Sandra came up with the idea of the car analogy and "forward" and "reverse" herself. She appears to have chosen the car analogy because of her concerns about motivating students and herself rather than concerns about the appropriateness of a car as a mathematical representation, for she explained in one interview:

> I mean the car, you know that's a lot more fun for me . . . and to me, if I taught the other way that would have been teaching rules and that wouldn't have been fun. (interview, 12/88)

The car analogy represents the directional movement within the function but does not represent the inverse relationship of the operations within the function unless "being in reverse" implies reversing the operation as well as the direction. Sandra's reasons for representing the concept of inverse in this manner are not derived from her mathematical understanding of inverse or functions. Rather, they seem guided by her interest in making mathematics fun.

> The driving forwards and the driving backwards, you can't teach unless you do that. They're not going to get it and they aren't going to care . . . I try really hard to bring it in, so that they'll have fun with it and they'll have cars in the future . . . you know, the car and all that just keeps a little bit more humor in math. (interview, 12/88)

Sandra's limited understanding of inverse functions becomes apparent in her description of the purpose of this lesson.

> I thought the lesson was to show them when to be in reverse and when to be in forward. One thing I wanted them to learn from this particular lesson was they need to know the reciprocals . . . you check a division problem with multiplication. . . . It's still a very incredible concept for them to learn that they're reversible. . . . Really, the most important thing was the reciprocals. Learning that what you do can be undone after we've already done it. That was really the thrust or the objective of the lesson. (interview, 12/88)

Sandra's use of the word "reciprocal" is problematic unless she meant a reciprocal relationship. In mathematics, *reciprocal* commonly refers to the quotient of a specific quantity divided into 1. For example, the reciprocal of 6 is 1/6; the reciprocal of 2/3 is 3/2. Multiplication and division are not reciprocals, as Sandra implies. Rather, they are inverse operations, as are addition and subtraction. Again--as was the case with her use of inverse and reverse--we see Sandra use language that has particular and precise mathematical meaning in more colloquial ways.

Even though the focus of the lesson was inverse functions and *inverse* is highlighted in bold print throughout the text of the lesson in the textbook, the word was noticeably absent from Sandra's vocabulary during the lesson. In an interview after class I asked, "As I looked at this lesson in the book, I saw the word 'inverse.' Do you use that?" She responded:

> Yes, I said, "What's the inverse?" I hope I did. Maybe I said "opposite." I don't know, I would try to use the language the book would use. I'm sure that would be a real good thing to do. If I'm not real comfortable with it, if it doesn't slip into my mouth when I'm teaching, it doesn't. But I try to . . . I think it's real important. . . . But if I don't, then I don't. But I do try. I probably did call it "opposite." (interview, 12/88)

The textbook uses the word "opposite" in the context of defining inverse: "The inverse of a function does the opposite of whatever the function does." Sandra's omission of the word "inverse," while perhaps a professional prerogative, may also be indicative of Sandra's lack of recognition of the importance of a "distinct mathematics vocabulary" (California State Department of Education, 1985, p. 17) in developing a conceptual understanding of mathematics. She said at one point:

> Part of it is, I can't forget where I came from too. . . . If I completely try to change who I am for this book, the book and I would just not get along. So I take it from the book and I try to use it as much as I can. If it's going to change even my verbiage, I can't use it. (interview, 12/88)

Sandra makes decisions about how to use the textbook. In the process, ironically, the mathematical content is often left behind or changed in important ways.

Trouble in the Park: Sandra's Adaptation of an Inservice Activity

During another one of my visits, Sandra had put aside the textbook. Her students were designing a park, based on a lesson from *S.P.A.C.E.S.* (Solving Problems of Access to Careers in Engineering and Science), an activity she had acquired at a recent inservice.

Sandra had been one of about 75 participants from across the country at EQUALS, a teacher education program at the University of California, Berkeley (UCB). The program, whose primary focus is equity of women and minorities within mathematics, originated at the Lawrence of Hall of Science, UCB, in 1977. EQUALS began as a program to help classroom teachers to encourage young women's interests in mathematics. It quickly became evident to the developers that, compared with women, minority students were at an even greater risk of dropping out of mathematics as soon as they could. Today EQUALS focuses on both women and minorities. The program tries to heighten teachers' awareness of the current state of women and minorities in mathematics. The program encourages teachers' awareness of the aspirations of their own students and offers activities teachers can do with their students to promote equity within the context of mathematics A staff member of EQUALS commented, "The EQUALS program is really an equity program. The mathematics is what we probably are most noted for but in our minds the mathematics is secondary. The mathematics is a window into opportunities for female and minority students" (interview, 8/89).

I was curious about how the people from EQUALS thought about the park project, so I did a telephone interview (8/89) with one of the staff people. I was especially interested in how they thought about the mathematics. I asked the question: What does a teacher need to know about mathematics to do this activity with a group of students? The EQUALS staffer replied, "I don't think a teacher would have to know anything in particular" (8/89). The staff member went on to say that most people who would try this activity would do it for the group work that it promotes. The collaborative nature of the project--not the mathematics--was stressed to the inservice participants. The staffer talked about the way EQUALS thinks about the activities they introduce during their inservices:

> In creating an activity, we always have to start from the position that there are going to be some people who don't know mathematics. Some teachers don't, especially in the elementary level. We also start from the position that teachers need other ways of presenting than they traditionally use. So these are the two things that we probably focus most on in putting an activity together. The activity itself should carry the mathematics and its understanding that it needs. (interview, 8/89)

"The activity should carry the mathematics." AN interesting claim, but one I was led to question as I watched Sandra teach the park lesson.

In the park project, students work together to design a park within a $5000 budget. They must make decisions about what sort of equipment and materials are needed to construct their parks, calculate their expenses, and make models of their parks out of paper.

The desks in Sandra's classroom were clustered into groups of four and five. Materials for constructing the park were piled in the center of the clusters. Materials included: a large piece of paper that covered the tops of the desks, construction paper of varying sizes and colors, scissors, glue, a ruler, a calculator, and a copy of the directions for the activity from the *S.P.A.C.E.S.* book. The directions included a price list for various items that students could use in their parks.

The initial steps for designing the park were to write down the materials and equipment (i.e., bricks, sand, picnic tables, swings, fencing) that each group wanted in their park and then prioritize the items on their list. Once this was done, students were to compute the total cost of their materials, keeping within a budget of $5000. Items were priced per unit and the units varied. For example, things like bricks, trash barrels, and picnic tables were priced individually. Playground equipment, like swings, were priced "per item." Rope was priced "per 10 feet," while wire fencing and asphalt pavement were priced "per 10 running feet." Sand was priced by the "cubic foot." Calculating prices of the various items is a complex task, requiring knowledge of common units of measurement and the ability to convert from one common unit to another. In an interview with Sandra, after the students had prioritized their lists of materials and before they had begun to compute the total cost of their parks, I asked if she anticipated anything becoming troublesome for the students in future lessons. She thought some of the calculations would be difficult, especially since "some things are per feet and some are each and some are pairs and you know I think that will be hard, that will be a struggle." (interview, 4/89). Even though Sandra anticipated problems with calculating the cost of individual items, she apparently did nothing to review her own understanding of the mathematics prior to the lesson.

As Sandra predicted, problems in calculating prices did, indeed, surface the following day. Students raised the problem of how to calculate the cost of fencing for the park whose size was to be 200 feet by 300 feet. The fencing was priced at $30 per ten running feet. Sandra told her students to multiply 200 x 300 to figure out the total amount of fencing they needed. She gave students the answer, "60,000 feet." By multiplying the length times the width, Sandra has calculated the area of the land inside the fence, not the amount of fencing. The perimeter, found by adding the length of the sides of the park (200 + 200 + 300 + 300) is the measurement needed to calculate the amount of fencing. I did not see Sandra or her students try to calculate the cost of the fencing once they thought they had found the amount of fencing. To find the cost, the total amount of fencing needs to be divided by 10 and then multiplied by $30, since the fencing is priced $30 per 10 running feet. If they had used the 60,000 feet, as Sandra suggested, dividing by 10 would equal 6,000 feet. Multiplying 6,000 feet by $30 would equal $180,000. The cost of the fencing would far exceed the $5000 budget of the park. Sandra's method of solving this problem suggests a lack of understanding of area and perimeter and an incomplete understanding of the mathematics involved.

Sandra was also unprepared when the problem arose of calculating the cost of sand, priced per cubic foot. Sandra reflected:

> Something came up today when we were doing the park and they wanted to know what cubic foot was. You know, the thing is that I couldn't really answer that question. Then I thought and I thought, then I remembered how to measure a cube. You know the area of a cube is height times width times length or whatever. So then, we looked around and looked up in the

dictionary and we put it together what a cubic foot. . . . then we went from there. (interview, 4/89)

Sandra seemed content with the way this problem had been solved; the teacher and students working together to come up with the answer. Once "cubic foot" had been defined, Sandra had the idea that groups should measure a large sandbox on the school's playground. The area was sectioned off with railroad ties and served as the foundation for several sets of swings. Sandra gave each of the members of a group a yardstick. Two students worked on measuring the length while the others measured the width. Yardsticks were laid end to end and counted. When they finished, the pair measuring the length reported 46 yards, and those measuring the width reported 10 yards. Several of the students hovered around a yardstick, as Sandra helped them to measure the height. They realized they could not use a yard as the unit of measurement because it was too large. Someone suggested they use inches. They counted up the number of inches and concluded that the height was one foot. Now that they had measurements for height, length, and width, Sandra instructed the students to multiply the numbers 46, 10, and 1 together.

But multiplying these three numbers, two representing yards and one representing feet, is problematic. First, the units of measurement are different. The length and width were measured in yards while the height was measured in feet. 46 yards x 10 yards x 1 foot equals 460, but it is neither cubic yards nor cubic feet. To make sense of these measurements, they must be converted to common units of measurement. For example, 46 yards could be converted to 46 x 3 (3 feet in a yard) or 138 feet and 10 yards could be converted to 10 x 3--or 30 feet. This allows the numbers to be multiplied and labeled cubic feet.

In this lesson, Sandra's efforts to teach for understanding may lead to misunderstanding. While it is true that during this portion of the lesson her students were actively engaged in mathematics in the context of a real life situation, her lack of mathematical knowledge colored what might have been an opportunity to use mathematics in a realistic context into an occasion for mathematical confusion. Not only is she confused herself, but she has passed misinformation on to the children, seemingly unaware of her own misconceptions.

Sandra's understanding of scale drawing also seems somewhat confused. This became evident when Sandra, prompted by my questioning, attempted to draw a connection between this activity and the Open Court mathematics textbook, specifically the topic of geometry and the concept of scale drawing. "We are in geometry in the book. So I am on geometry with the kids and this is where the park came on it. It was scale drawing. I thought this was the perfect time to bring it in" (interview, 4/89).

A scale drawing means that something is made in accordance with a particular proportion or scale. The proposed park is to be 300 feet by 200 feet. The directions for the activity suggest that the children be given a piece of paper cut to three feet by two feet so that the drawing can be done to scale. In Sandra's classroom, each group of students had a large piece of paper that covered the tops of a cluster of desks. When I asked her if the size of the paper each group was given had any relationship to the size of the park, Sandra said she "just grabbed a piece that would fit on their desk." At the same time she assumed that "afterwards we'll talk about what they did with the scale." Mathematically, this does not make sense.

Since Sandra and her students gave no attention to proportions while designing the park, the end product of this activity would not be a scale drawing. In talking with Sandra, it was clear that she had outcomes in mind for this activity which did not include an understanding of scale drawing. Her goals included:

> . . . talking about what goes into landscape and architecture, and money and working together and forming a park. . . that's math, that's reality, it's real life. It's working together, it's what happens in the real world.
>
> The end result will be that they will pick exactly what they want, it will be about five thousand dollars. . . . they will actually draw the material on another paper and physically put it where they want, moving it around . . . they will name their park . . . they'll show their park . . . they will describe the different aspects of the park . . . and why they put things in different places. So they are going to have to rationalize it and I think that will be a real good way to tell, I mean even if they are slobs, you'll be able to see some thought in it. (interview, 4/89)

In this discussion of her outcomes for the park activity, Sandra never mentions the mathematical concept of scale drawing as a goal, nor does she include anything about perimeter or cubic feet. Other goals, such as student involvement, enjoyment, making mathematics real, requiring students to think and problem solve cooperatively are more important to her.

The activity does not carry the mathematics, as the Project staff thought. Sandra did not fully recognize and understand the mathematics involved. (As an observer in Sandra's class, it was only after some reflection that I was able to figure out the intricacies of the calculations myself.) But as far as Sandra was concerned, the activity was a success. "The park thing with math, that is going really well, so I am loving math." Her evaluation is understandable, given the goals she has set for her teaching.

Sandra's efforts to teach for understanding may have resulted in misunderstandings. It is true that her students worked collaboratively and had fun while trying to solve a real life problem. These are all good things. But something is missing: Sandra did not know the mathematics required to teach this activity. If Sandra had been better prepared, this could have been a much more valuable mathematical experience for her students.

Missing Elements

On one level, Sandra's goals for mathematics seem to promote the kind of teaching the *Framework* suggests. Involvement, enjoyment, and fun in the context of real world situations are examples of elements in Sandra's practice oriented to the *Framework*. Knowledge of mathematics is an element of the missing from Sandra's practice.

Sandra, like other teachers, makes many decisions regarding her own practice. Decision-making is inherent in the teaching profession but there is something troublesome in this case. Sandra, repeatedly, makes decisions that influence her teaching of mathematics without sufficient knowledge or concern for the meaning of the mathematics at hand.

Nothing has prepared Sandra to teach mathematics for understanding and nothing keeps mathematics in the forefront of her decision-making. The importance of students actively engaged, having fun, feeling successful. enjoying themselves, thinking and problem solving in the context of real world situations should not be minimized. These are all favorable objectives and stated explicitly within the *Framework*. The mathematical confusion within Sandra's practice illustrates what may happen when a teacher tries to teach for understanding with a limited understanding of mathematics while nothing within the educational system is drawing her attention to the mathematics.

In Sandra's case then, the instruments in place to help align math instruction with the *Framework* do not seem to be working. The district's adoption of an innovative textbook has had little effect on Sandra's teaching. In the textbook lesson on functions, she appears to make content and instructional decisions based on what feels right to her and what she thinks will be fun for her students, with little regard for the mathematics. In the park activity, she was given no guidance in figuring out the mathematics. Her attention was never drawn to just how complex it was. She was given this activity at the inservice and she took it back to her classroom. In many ways, it was just what she was looking for--something fun and engaging for her students. Unfortunately, she overlooked the mathematics. But so, apparently, did the Project staff.

Sandra's case raises questions about what teachers need to understand and consider when teaching mathematics for understanding. The *Framework* hints that teaching for understanding requires more from teachers. It states that teachers need to be more thoughtful about the activities they do and their interactions with students. If teachers are not thoughtful, activities like the park, which has potential for being a powerful mathematical experience, remains merely a fun and interesting activity, and may add confusion to the mathematics. The *Model Curriculum Guide* states then:

> We must be clear about the particular idea or concept we wish students to consider when we present activities or use concrete models. It is not the activities or the models by themselves that are important. What is important is the students' thinking about and reflection on those particular ideas dealt with in the activities or represented by the models. (California State Department of Education, 1987, p. 14)

In both the lesson on functions and the park activity, Sandra was not clear about the mathematics at the heart of the lessons. How can a teacher help the students to think and reflect on mathematical ideas when the teacher is uncertain or unaware of the mathematical ideas?

If one looks at the elements of the *Framework* without considering the implications for what teachers need to know, and if one looks at Sandra's practice without considering her knowledge of mathematics, one would probably say that her practice is aligned with the *Framework*. A closer look tells a different story. On her own, Sandra has figured out important goals for the teaching and learning of mathematics. But she, along with others in the system, do not seem to recognize everything that is required of her to teach for understanding. The *Model Curriculum Guide* hints at what may be required,

> This type of mathematics instruction involves students actively and intellectually requires much from the teacher. Without thoughtful decisions about the activities and without thoughtful interactions with students, potentially powerful mathematical experiences can become little more than interesting activities for students. (California State Department of Education, 1987, p. 13)

The problem is not that Sandra is not a thoughtful teacher. She is. In fact, Sandra seems like an ideal teacher for embracing something new. She is enthusiastic, she is a learner, she loves teaching and she cares about her students. The problem seems to be that there is nothing in the current educational system helping teachers, like Sandra, to be thoughtful about the mathematics. In Sandra's case, no one appears to be minding the mathematics.

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A Conflict of Interests:

The Case of Mark Black

Suzanne M. Wilson

> I started in Arizona. I taught in Arizona in a real redneck type district and I can bring a group of kids in the first day and I can sit them down from the first minute and I can work them solid until the last minute of the last day. And I'll tell you, I've done it but it's no fun. But I can if I had to. I can sit them down, I can shut them up, and I can work the :. (interview, 12/88)

Mark has t teaching fifth grade for ten years. Initially certified in Arizona, he moved to California several years ago. Mark is an energetic and enthusiastic teacher, constantly moving around the room, speaking clearly, encouraging students to ask questions, patting kids on their backs. He is always asking questions, reminding students that it's okay to be confused. After all, what is his job if it is not to help clear up their confusions?

Decisively in control of his class, Mark tolerates no disruptions and maintains a quiet and orderly classroom. Students seem to have a good time: They smile often, are eager to answer his questions, and are willing to ask many of their own. Parents like having their children in his class for he has a reputation in the school for "straightening out" troubled kids. Moreover, his classes have scored high on the California Assessment Program (CAP) tests in the past. In fact, last year his class received the highest scores in the school, scoring even higher than the gifted class. In addition to their achievement, Mark is also concerned with students' self confidence since, as he put it, "you need confidence just to be successful in life." He believes kids acquire confidence through mastery of schoolwork and Mark works hard to help students master mathematics, the subject matter that was focal in our discussions.

The school district in which Mark works adopted *Real Math* (1987) as the textbook best suited to meet the goals of California's mathematics *Framework* (1985). Mark himself had never seen the *Framework* when me met in December; the closest he had come to it was a meeting with a representative from the textbook company in September. According to Mark, the representative, "tried to sell us on the textbook." Mark is certain that the textbook represents the *Framework* authors' intentions, although he is not sure about the specific goals of the *Framework*, having only heard about it in casual conversations with other teachers and school administrators. He's almost certain that he agrees with its spirit which he interprets as making math "real" and "useful" to children. As he explained:

> It's something that I've said for years. People don't sit around--somebody mentioned it the other day, except for accountants--people don't sit around all day and do math problems. This is what they're going to do in real life. They're going to be out somewhere at a pizza party and they're going to have left over pizza and they're going to have to figure out how to divide it up or something along that line. You know what I mean? So, this is the sort of thing that can help them. To me, that's why it's important. To help them understand . . . math in a real setting type thing. (interview, 12/88)

Mark's beliefs about and reactions to the *Framework*, his enactment of the curriculum, and his beliefs about teaching mathematics are the basis of this case. I observed Mark on three separate occasions, twice in December of 1988--four months after he had started using the new textbook adopted by his school district--and then once more in April of 1989, eight months into the academic year. On each occasion, Mark's commitment to helping students learn mathematics was clear. In the first section of the case I describe in detail two instances of his teaching that reflect this concern. I then move on to an analysis of what Mark thinks about teaching mathematics for understanding and how he implemented the new curriculum. I close the case with a discussion of several conflicts inherent in Mark's practice that influence his teaching and his implementation of the policy.

Inside Mark's Classroom

December 1988: Teaching Long Division

Mark teaches in a large suburban school district in northern California in which students attend school all year. His classroom is a rectangular room in a building that looks like a quonset hut. Twenty eight children were present when I visited, half of them were white, the other half a mixture of black, Hispanic, Asian, Filipino, and Indian. Most students in his class come from middle and lower-middle class backgrounds. Their seats were arranged in pairs, all facing the front of the room and the blackboard. Mark stood and spoke from a podium at the front of the room when the whole class was working together. At other times, he wandered throughout the room, checking students' papers and working with students who were working at the blackboard.

Mark's lesson came directly from the teacher's manual of the textbook, *Real Math, Level 5* (pp. 106-107). Class began with a mental math exercise in which Mark read problems from the textbook, e.g., 40 divided by 4, and students signaled thumbs up or thumbs down dependent on whether or not there was a remainder. When students got stuck on the mental arithmetic, Mark had them work the problems out on scratch paper. During these times, he would walk around the room. As students completed problems, Mark would check them for correctness. If a student completed a problem correctly, sometimes Mark would assign him/her the role of student teacher--which meant that the student was free to walk around the room and confer with students who were having difficulties. Each time students worked at their desks, Mark and two or three of these "student teachers" would work with the students as they went through the problem in question. As it turned out, Mark used this system of "student teachers" frequently in his teaching of all subjects. Mark ended the mental math activity with "the problem of the day" which was 2 divided by 3. Most students seemed familiar with the way to solve this problem and no time was spent discussing the answer. Mark simply went through the solution steps at the board, frequently asking students to tell him what to do next.

Mark then moved to a "prophecy activity" in the text. Students read problems from the textbook aloud, and Mark talked them through the solution. The problems required that students predict the solutions to problems, e.g., "If you start at 0 and add 2 each time, will you hit 20?" At this time, he stood at the front of the room with a clipboard on which he had a checklist. As students read the problems, Mark checked off that they had participated in a "speaking" activity. He explained to me later that he also checks them for reading and writing. According to Mark, he is responding to the call for integrating language skills throughout the elementary school curriculum by using this accounting system.

After students had gone through five problems of this sort, Mark moved on to the next page in the text and had another student read the word problem that read as follows:

> One day Laura did a lot of work at the library. She rode her bicycle home for lunch. She rode back to the library after lunch, and in the afternoon she rode home again.
>
> "That's about 4 trips I made today between my house and the library," said Laura.
>
> The odometer on her bicycle showed that she had ridden a total of 6.0 kilometers. "I'm going to figure out how far it is from my house to the library," she said. Laura did the problem this way:

$$4\overline{)6}\quad 1\ R2$$

"So it's 1 kilometer and some more from my house to the library. And it's less than 2 kilometers," said Laura. "But I'd like to get a more exact answer." Here's how Laura can get a more exact answer.

Asking selected students to tell him how to solve the problem at each stage, Mark led the students through this problem, all the while focusing on the procedure for how to do division with decimals (lining up the columns, putting in the decimal, adding zeroes, bringing the zeroes down, subtracting, continuing this process until they reached zero). Clearly, students had learned a set of steps to go through in order to solve these problems, e.g., set up the long division problem, put the decimal in the right place, etc., and the class discussion of the problem went immediately to these steps. Mark spent no time discussing the textual aspects of the problem as it was presented--the scenario, the characters, the problem. Rather, quickly he reduced the word problem to a mechanical division problem. Discourse in the class was characterized by Mark asking pointed questions with right answers and students providing brief, one or two word responses. For example, in one part of the lesson the class was working on the problem: $3 \div 8$. Mark had written on the board:

$$8\overline{)3.0}$$

Mark: Eight into 30, how many times Ashcon?
Ashcon: Three times.
Mark: Three times. Three times eight, Ashcon?
Ashcon: 24.
Mark: [Writing the work on the board.]

$$\begin{array}{r} .3 \\ 8\overline{)3.0} \\ \underline{24} \\ 60 \end{array}$$

Mark: Do you see what I did here? I subtracted, I added a new zero here. I mean, I subtracted, I got a six. I added a new zero, I brought it straight down. Eight into 60, Ashcon? Who knows [students start raising their hands]? Good. Heather, come on, I want you with us today. Ashcon.

$$\begin{array}{r} .37 \\ 8\overline{)3.0} \\ \underline{24} \\ 60 \\ \underline{56} \\ 4 \end{array}$$

Ashcon: Six.
Mark: Six, okay, I believe it would be seven times eight is [writes the problem on the board]. Now, if I subtract, I'm going to get a four, add my next 0, 8 into 40 goes 5 times, there it is. Okay Ashcon? Questions? Okay, what? Do you understand

that one, Ashcon? Did everybody understand it? Give me a yes signal if you understood it. Come on, everybody give me some sort of signal.

```
      .375
8  )3.0
    24
     60        40
     56        40
     40         0
```

In this instance, and in many others that I observed, Mark did most of the talking during mathematics class. When Ashcon proposes "six," Mark barely pauses, sweeping on to "7," the correct answer. During the final portion of the class, Mark had students work on the problems from the textbook. Students who felt comfortable with their skill worked at their desks, occasionally raising their hands with questions. Mark answered individual questions, prompted the next step in the procedure, pointed out mistakes. Most of the time, however, he spent at the chalkboard with students who were still uncomfortable or unsure of how to do the problems. About eight students came up, and worked problems on the board while he looked on. As students became more secure with the procedure, he sent them back to their seats, or they decided to on their own.

The conversations that Mark had with students during this portion of class reveal what his goal for the lesson was: mastery of the procedure for dividing with decimals. When students had difficulty, he would direct them to "put the decimal here," "line the columns up properly," or "bring the zero down." When students were asked to "explain" their answers, it was sufficient and acceptable for students to explicate the steps taken toward the answer. For instance, Nefissa's explanation is typical of what Mark wanted his students to be able to do:

Nefissa: [She has written the following on the board:]

```
     1.2
5  )6.0
    5
    1.0
    10
    00
```

Five goes into 6 once and one times five is five. And then 6 subtract five is one, so you put your decimal point in there, put your zero there. Bring down the zero. Five times, two times five is ten, so you put your ten there. (observation, 12/88)

All of the instruction that Mark provided while the students worked on problems was procedural. His talk was peppered with comments like, "put your decimal in the right place," "bring your zeroes down," "what one little thing did you forget to do?" His comments to Melissa serve as an illustration:

> Melissa, how are you doing? Are you sure? One thing you forgot to do is keep your numbers in a straight column. Right now you've got your 2 right above where the decimal needs to be but your decimal will go between here and here. See, cause after you stop working with the five, then you're in the area where the decimals are, so then the numbers go behind the decimal. (observation, 12/88)

Mark also provided a lot of positive feedback when working with students at the board. He seemed to recognize that several students may have been there because they needed reassurance or attention more than help with the steps. Constantly and cheerfully exclaiming, "Perfect!" "Well done!" "Exactly!" Mark gives his students lots of praise and encouragement.

Mark's focus on the standard procedure is illustrated in his reaction to the book's use of an alternative strategy for notation in long division problems. When Sara noted that the book used a different procedure than the one Mark had taught them, he said to the class:

> I don't understand their procedure either. Let's do it my way for now. Boys and girls, if you look at the examples up there, they do it a completely different way in this book. I'm sorry but I don't know that way, so if you don't know that way, do it as we've been doing it. Alright? I'm sorry, I don't know that way so we can only do it the way I know. (observation, 12/88)

When I asked him about this comment in an interview after the observation, Mark explained:

> I never saw that procedure before. This got really confusing. What they're doing is they're putting their little notation here after you subtract but they don't show the procedure to get it and that's going to lose a lot of these kids. First of all, this is something about this textbook that I'm not happy with--is the fact that this is the first time in my life that I've ever seen this. No introduction, no explanation, not even in the teacher's book. . . . I'm skipping this. . . . I'm forgetting that and I'm going to do what I understand because I can only teach what I understand. (interview, 12/88)

Mark is absolutely right, he can only teach what he understands. In my observations of his teaching in December, it appeared that Mark understood mathematics to be a set of procedures that students needed to master in order to solve exercises involving division, subtraction, multiplication, and addition: tools that could help students solve real world problems like measuring the size of a room or dividing up a pizza. While he was open to the possibility that there were alternative procedures that students could learn to solve those problems, he was aware of the limitations of his own knowledge of alternative procedures and concentrated on teaching students the methods he knew. He neither chose to help students generate their own algorithms, nor did he explain how or why these procedures worked.

One exception to Mark's heavy emphasis on the procedural aspects of solving exercises occurred during my second observation of Mark in December. After my first observation, Mark and I spent several hours talking about the *Framework* and its emphasis on conceptual understanding. Thinking about what we had talked about the night before, Mark decided to spend a little time in class the next day showing the students what was happening when they divided using decimals. He explained to me, "I thought I'd do a little bit of what we were talking about last night." During the lesson, which consisted of solving more exercises involving decimals and division, Mark interrupted the routine and said (observation, 12/89):

Mark: Okay, I'm going to show you something now. Just pay attention for fun. Watch this. [Draws on the board.] We know that we can take 4 into ten and we can get 2 and 5 tenths, right?

$$4\,\overline{)10.0}^{\;2.5}$$
$$\underline{8}$$
$$2.0$$

Everybody understands because I already worked that. And now you're seeing that you can take four into ten and get 2 remainder two. Boys and girls, watch this! Here I have a two [pointing to the numerator of the fraction 2/4] and here I have a five [pointing to the decimal .5 in 2.5] but I can make this two turn into that five. Take this two here, everybody see where I got that two? Nothing up my sleeves. Take this four here, okay? [Writing on the board:]

$$\frac{10}{4} = 2\frac{2}{4}$$

$$\text{and } \frac{2}{4} = .5$$

This is now a fraction, which means, division[1]. You guys remember that? Whenever you have a fraction, it's actually a division problem, you're dividing the bottom number into the top number. Everybody with me on that so far? I've taken this and put it on top of that. Boys and girls, how many times does four go into 20?

Ss: 5.

Mark: Five, yes. Don't I have .5? How many get it? [About half the class raise their hands.] Good, mathematics *works*! There's no secrets, there's no tricks. [Quickly erases everything from the board.]

As an observer, it appeared that students did not understand what Mark was trying to do during these five minutes. Mark explained to me that he didn't have time to always "explain" the underlying rationale for why some mathematical procedure worked, but that he had wanted to show students--if only briefly--that division with decimals and fractions were related. When students asked him questions about his explanation, he repeated what had gone before, going through the same numerical manipulations. Several students continued to ask questions, unclear about the relationship between the numerator 2 in 2/4ths and the decimal .5 in the 2.5, and he eventually said in frustration, "I only showed you that little thing *for fun*. We can't spend all this time talking about it."

Mark's teaching is familiar, his class looks like countless other mathematics classes: children learn how to manipulate numbers, solve problems, practice in class, do homework sets. Talk is teacher-centered, student participation consists of curt responses to simple, informational questions. He is a prototype of the "effective" teacher. Using an old script, Mark is acting out a part that has been well articulated and clearly defined by process-product researchers of teaching. He asks his students dozens of questions, he smiles often, he provides practice and preliminary explanations in which he models the strategy he is about to teach. He energetically walks around the room, patting students on their backs, providing extra help for those who express the most

[1] What Mark is trying to help students see is the equivalence between 2/4 and .5. He assumes that they do not understand that a remainder of 2/4ths is another way of expressing the decimal .5 and that 10 and 2/4 = 10.5.

confusion. But *why* does Mark teach math this way. Is it because he sees mathematics as procedural? Alternatively, does he view the teaching and learning of mathematics in a way that shapes his teaching this way? Or is it because he has developed strategies for teaching familiar content that are habits hard-to-break, methods tried-and-true? A view of Mark teaching another topic, one that is new to the curriculum and to him, may help us begin to explore some of the reasons for Mark's pedagogical style and choices.

April 1989: Teaching Functions

When I returned to observe Mark in April, he was teaching inverse functions, a topic that was new to the fifth grade curriculum in this school and one which Mark had never taught. Since we had last met, he had altered the seating arrangements of his students. Instead of pairs of desks facing the front of the room, the desks were arranged in three large circles of 11 children each. The lesson was a review of the work that they had been doing for the past two days and Mark started the class by writing different functions on the board, asking students to generate the inverse of the function and then solve it. For example, the first ten minutes of class went something like this (observation, 4/89):

Mark began class by writing the following on the board, occasionally glancing in the teacher's guide:

INVERSE FUNCTIONS

The inverse function does
the opposite of whatever
the function does.

Mark: Who can read that to me?

Girl: [Reading from the board] The inverse function does the opposite of whatever the function does.

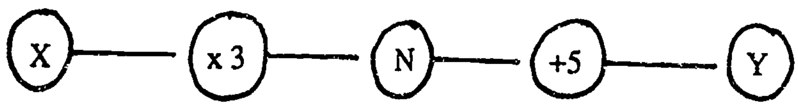
Mark: Alright. Everybody remember this? What was the biggest problem you guys had on last week's quiz? Remember on that paper? I mean the homework paper, not the quiz?

Boy: The arrows.

Mark: The arrows. Remember? Okay, so, let me give you a simple one here and let's begin. [Writes the following on the board:][2]

---

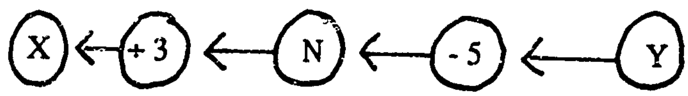
[2] This representation is of the function $3x + 5 = y$. The textbook authors introduce the students to the notion of function by having them create a "function machine." The function machine allows you to put some number into it and get others out. The rules that govern what happens inside the machine are represented by addition, subtraction, multiplication or division symbols. After students have mastered the visual representation of a function in the form of this machine, the textbook authors use the representation of function sentences that consist of circles and arrows. The arrows indicate that a number is being placed in a machine; the circles represent the machine and its special operation. This function, then, has two machines associated with it. First, students are to replace the variable x with a number, say, 3. Three is then placed in the first machine in which it is multiplied by 3. This new number, 9, is represented as N in the number sentence. That number is then placed in the second machine in which 5 is added to it. The product of these operations is the answer, in this case, 14. Students in Mark's class have already learned to substitute numbers in function sentences like these. What they are reviewing in this lesson is the construction, and validation through testing, of inverses of functions.



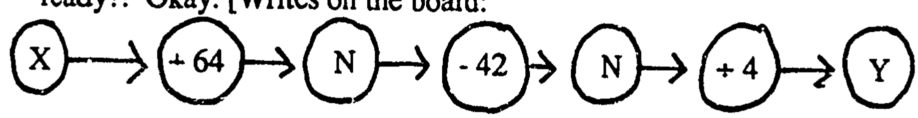
Mark: Watch your arrows, boys and girls! Okay, go ahead and do that. First of all, copy the function then give me the inverse. [Writes the board:]

1. Copy the function
2. Give the inverse

[Mark then walked around the room while students worked the problem. When most were done, he went to the front of the room.] How many of you copied the function? I want you to have that practice. So you copy the function. Then you reverse the direction, don't you? It goes the opposite way. So if you are starting at y, you must subtract five to get your basic number in the middle and then the opposite is divide three. [Writes on the board:]



Alright, any questions?[3] Any questions? I've got a dead group back there not paying any attention. Are there any questions? [continued silence] Okay. I know this is Monday and I know the weather's been warm, but that's okay. Are we ready?! Okay. [Writes on the board:



The class then went through several more examples which followed the same pattern. Mark generated a function and wrote it on the board. He then gave students a couple of minutes to solve it (which consisted of copying the function and finding the inverse). After that, he wrote the correct answer on the board.

The remainder of the class continued in this pattern. Mark would present a problem, students would solve it. Mark would walk around the room checking students' work. He continued the

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[3] The arrows in this inverse function, while reversed, serve the same purpose: They point the student in the direction of the function machine in which to place the number. In constructing the inverse of functions, students must reverse the arrows and decide what function machines would reverse the work done in the original ones. In the case of the function under discussion here, the original machine added 5 to the number. The inverse of that machine would then subtract 5. To use the same example, if Y = 14, the first step of the inverse function involves placing the 14 into the first function machine in which 5 is subtracted, leaving the student with 9 for N. The next arrow directs the student to place 9 (or N) into the second function machine which has been designed to undo what it's counterpart did in the original function, that is, divide by 3. Students divide 9 by 3, obtaining the original used in the first function.

"student teacher" system I had observed in the fall, asking students who had finished their work correctly to help others who were having difficulties.

Like the lessons I observed in the fall, this one came from the textbook. Students were supposed to know that a function involved manipulating a number with a series of operations--addition, subtraction, multiplication, or division--to get a new number. For example, if you start with 5 and add 3, multiply by 2 and subtract 7, you end up with a new number, 11. Students were then supposed to learn that the "inverse" of this function involved coming up with a series of steps that would undo what had been done, that is, add 7 to 11, divide by 2, and subtract 3, to get the original 5. Mark's goal was to have students be able to generate the inverse function of any function that he put on the board. In the lesson, he emphasized the mechanics of the process--reversing the arrows, exchanging multiplication and division signs, and exchanging addition and subtraction signs. There was no discussion of why students might exchange multiplication and division signs, addition and subtraction signs. There was no discussion of function machines, what a "function" was or what an "inverse" was, and the focus of this exercise involved helping students learn to get the right answers, emphasizing the "hows" of generating the inverse without discussing the nature of functions, what was going on with all these numbers, why you would want to know the inverse of a particular function, or what the relationship between a function and its inverse is.

When I asked him later how he felt about the lesson, Mark said he thought this was important content because it was good preparation for pre-algebra since "functions are algebra." More importantly, he thought that these problems gave students practice in the basics--addition, subtraction, multiplication, and division since they had to use all of those operations to solve the problems. The lesson had the potential for communicating aspects of inverse functions that are important for students to know if they are to "understand" the nature of inverses, e.g., *why* the arrows are there in the function, *why* you turn them around in the inverse. Representing functions with arrows helps communicate to students the dynamic aspects of functions--how operations are done to variables, how numbers change. And while the inverses of simple linear functions are equivalent to reversing the operations in a linear fashion, not all inverses are so simply constructed. By defining inverse as "doing the opposite" or "reversing the arrows," Mark oversimplifies the mathematical ideas at the heart of this lesson. "Inverse" for Mark's students was a series of steps, not a mathematical idea. The first steps were, literally, put on the board:

1. Copy the function down
2. Write the inverse function

The second set of steps, just as important, was never written:

1. Go to Y, the end of the function
2. Turn the arrows around
3. Exchange the sign in the circle for its opposite, e.g. substitute a - for a +, a ÷ an x.
4. Turn the next arrow around
5. Exchange the sign in the circle for its opposite
6. Repeat until you reach the X

Math, as represented in this lesson, consisted of a set of steps that must be done in order. If students do all the steps, they will get the right answer. The lesson was bifocal: It provided many occasions for children to practice addition, multiplication, subtraction, and division and it gave students a new procedure, a procedure that would produce something called an inverse function. There was no evidence, though, that students had learned to think about or make sense of functions and their inverses.

Conflicts and Constraints

Teaching for Understanding: Levels of Knowing

From one perspective, Mark appears to be a good teacher. He asks his students many questions in warm and enthusiastic ways, checking their solutions, helping them through the steps of algorithms. He covers the content of the curriculum, making sure that he exposes students to all of the topics in the textbook. Students get a lot of drill and practice with addition, multiplication, subtraction, and division--operations that are considered, by some, the "basics" of elementary school mathematics. But if we switch lenses and look at Mark through the spectacles of the *Framework*'s rhetoric, he looks different. Marks believes that mathematics is the mastery of algorithms. He teaches his students to acquire habits, like lining up numbers and following a set of steps, that allow them to manipulate and conquer algorithms. In one interview, I asked Mark to describe the teaching of procedures and rules. His description captures the essence of what I saw him do in his own classes:

> [Procedures consist of steps...] step by step, by step, by step, you get a result. Rules, I guess are just similar that certain things *have* to occur in order to have a correct answer. So that's it. Step by step, and the correct steps would be rules and procedures. Correct steps to get the correct answers. You blow one of them, you make a mistake and naturally something's going to be wrong. That's why I send them back. I say, "No, you've made an error, go back and see if you can find it." (interview, 12/88)

Mark is committed to his students' mastery of "steps" through lots of practice. Yet consider the *Framework* authors' position on computation and algorithms in mathematics:

> Those persons responsible for the mathematics program must assign primary importance to a student's understanding of fundamental concepts rather than to the student's ability to memorize algorithms or computational procedures. Too many students have come to view mathematics as a series of recipes to memorized, with the goal of calculating the one right answer to each problem. The overall structure of mathematics and its relationship to the real world are not apparent to them. (California State Department of Education, 1985, p. 12)

Although Mark emphasizes the acquisition of algorithmic knowledge, he recognizes that there are other levels of understanding in mathematics. In our interviews, for example, he differentiated between the type of understanding that he aims for which involves "setting" the algorithms in his students' minds so that they can successfully complete problems involving addition, subtraction, multiplication, and division, and the kind of understanding he doesn't have time to teach, one that involves knowing "why" the algorithms work, what a fraction is, or what multiplication means. This distinction surfaced first in a post-observation interview. When I asked him how he thought the lesson went, Mark explained:

> I thought that, overall the lesson went fine because they were doing division and they were comprehending. Now, did they understand deeper the meaning of fractions? I don't think so. Or decimals, I don't think so. But at least they understand the steps. [Interviewer: And what makes you think they did not understand the "deeper meaning"?] First of all, I haven't taught much about the deeper understanding because this book skims over it. I would wait til later in the year to be on this particular concept. And then, by then I would have developed decimals. They haven't developed it very well in this book. That's what I mean by they don't understand the deeper. Probably if you asked them what's a decimal, most of them aren't going to be able to tell you that it's a part of a whole, or, even if they do mimic those words, what does that mean? They can't tell you. (interview, 12/88)

Mark's reaction to the text reflects a fundamental difference between the textbook authors' approach to teaching mathematics and his own. The textbook authors, for example, have structured the curriculum of *Real Math* to develop layers of understanding--beginning with intuitive concepts and slowly moving toward more explicit, sometimes algorithmic knowledge. The curriculum is also structured to interweave related ideas, e.g., fractions and decimals are taught side-by-side instead of as separate and discrete topics within mathematics. They explain their approach to fractions in the teacher's manual:

> Children have intuitive notions about fractions, because fractions are part of people's everyday language. "We're about halfway there"; "only about one-third of these are good." In Real Math we use these intuitive understandings. In fifth grade, for example, the students estimate fractional lengths, areas, and so on to review fractional notation in a way that corresponds to their intuitive notions of fractions. We also do a lot with fractions of numbers, because that is a common use of fractions that the students have encountered often outside mathematics class. Later on in the year, the students add and subtract fractions, including those with unlike denominators, using their intuitive notions to help them add, say, one-quarter and one-half. Then when we develop standard algorithms for adding and subtracting fractions, the students find that these procedures fit well with their understandings of the world and our language. (Willoughby, Bereiter, Hilton, & Rubinstein, 1987, p. xvi)

Rather than think of learning mathematics as the layering of understandings or the gradual development of understandings from intuitive to explicit, Mark has a building-block notion of mathematical understanding. Mathematical concepts rest upon a foundation of mathematical rules and procedures (intuition does not play a role in this conception). Students must first master procedures. They do so by learning about a series of topics, e.g., single digit subtraction, double digit subtraction, and gaining algorithmic mastery over each "type" of problem. After that basis has been laid, teachers can explore more conceptual aspects of mathematics. In the best of all possible worlds, students would learn procedures and concepts because algorithmic knowledge alone, in Mark's opinion, is rather useless:

> If I put numbers on the paper and you can add them up and get a new number--so what? What can you do with it besides write them on the paper and do that. It's like a child who can read out loud but can't understand what they're reading. It's like, I've got a dog who can do certain tricks but she doesn't know what she's doing. So what? (interview, 12/88)

Mark believes that the ability to represent mathematics--with pictures, diagrams, models--is an example of a more advanced and sophisticated level of understanding in mathematics. So he believes that all children should first learn the mechanics, and then some students, if they have the ability and the disposition, may begin to develop the ability to represent those problems. He explained in one interview:

> I wouldn't count on any kids in my class [coming up with a pictorial representation]. You've got to understand one thing, there's no gifted kids in my class because they have been siphoned off. What they do in our district is they siphon off the highest talent, the gifted and put them in their own class . . . So I wouldn't count on anybody in my class coming up immediately with pictures unless we've had a lot of practice. (interview, 4/89)

For Mark, then, only the brightest students develop the cognitive ability to represent mathematical notions in pictorial forms. This believe too seems in opposition with those that undergird the *Framework*. In that document, the authors argue that <u>all</u> students can and should develop "mathematical power and that *no* student should be limited to the computational aspects of the number strand" (California State Department of Education, 1985, p. 4). They also argue that the teacher's eye must always be on the development of conceptual understandings while Mark seems to believe that this is something a teacher should do if there is enough time.

Conflict #1: The press of time, community, and tests. So why does Mark teach for rules and procedures if he recognizes that there are different levels of understanding and he is clearly concerned about students "getting it"? Open about his choice, Mark named three causes: time, tests, and parental pressure. With limited time, for example, Mark believed that he could only work on the basic foundation--the rules and procedures--as reflected in his reactions to the *Framework* authors' claim that teaching for understanding is more important than teaching rules and procedures:

> When do I have the time to teach? Because I barely got through what they would call here rules and formulas and procedures [in today's class]. I didn't have time to get into how to use it. Tomorrow I have another lesson to present. I agree with it, you've got to learn how to apply it, no doubt about it. No doubt about it. That's what math is. But when? (interview, 12/88)

Mark also mentioned parents as a source of pressure for covering the content:

> These kids are going to be dragging their books home and one day a parent is going to look at it and say, "You've been in school 9 months and you're only on page 100? You've got 200 more pages, what's happened?" And they're going to be romping and stomping in here, saying to the principal, "This teacher is not going fast enough." And I'll tell you what. They can make your life very, very sticky and I've had it happen. Where I taught very well and didn't go very fast and parents were screaming and squawking, "They're not going fast enough." You speed it up and then you know what you hear? You hear from the parents who have kids who are going too slow. One parent says you're going too slow, the next parent says you're going too fast. (interview, 12/88)

Finally, Mark remarked that the press to get high test scores on tests like the CAP also limited his ability to teach for deeper understandings. His problems with such tests were twofold. First, he did not think the tests were designed to test the kind of material that was being presented in the *Framework* or the textbook, e.g., conceptual understandings. Second, he believed that the test was one of the factors pushing him to cover content since students who did not know the "basic functions"--addition, subtraction, multiplication, and division--would perform poorly on the tests:

> Teaching for understanding is what we are supposed to be doing. Now, I only have so many minutes of the day. I'm supposed to teach for understanding. Look at the last one. It's difficult to test, folks. That is the bottom line. It's funny they put it at the last one, because the bottom line here is that all they really want to know is how are these kids doing on the tests? They want me to teach in a way that they can't test. Except that I'm held accountable to the test. It's a catch-22. [Rules and procedures are] easy to test. (interview, 12/88)

What is most paradoxical and troubling about Mark's talk is that there seems to be a real distinction in his mind between teaching and teaching for understanding, and even though he wishes that he had more time to teach the material so that his students would learn it, he is willing to simply "teach." According to his own self-reports, as well as the observations I made of his teaching, most of Mark's teaching consists of showing students how to manage the procedures of mathematics. When he asks students whether they "understand" something, he is checking whether they have been paying attention or following his directions, not for the degree to which they have conceptually mastered the material. This is reflected in much of Mark's talk both in and out of the classroom. Recall his comment to students, "Everybody understands because I already worked that." He has taught something if he has told them about it and provided time to practice the steps. This belief has been clarified and reinforced by Mark's experiences in schools in which the press is to cover material and document performance, not to ensure understanding. Consi... his remarks on the *Framework*'s claim that teaching for understanding takes longer that teaching rules and procedures: "[Reading from the *Framework*] 'Teaching for understanding . . . takes longer to learn.' Hey, if I were spending ... time to really get these kids to learn it, I might be several pages back" (interview, 12/88).

Mark made comments like this several times, in which he would explicitly state that his teaching did not involve making sure that students understood the material. He even stated that, given his limited resources and large class size, he didn't try to reach all of his students:

> What with the testing, I know that the top ones are going to pass. What do I need to worry about them for? I've got 33 kids, what do I care? That's terrible to say. I care. But they're going to ace it no matter what. I get kids in there who get straight As no matter what I do. They're going to get straight A's even if I didn't teach them. And I've got kids who are in there flunking, ok? I've got to bring them up, they need the help. And the middle ground are the ones who can do it, aren't really able to, and are going to make the most progress, and that's going to show. So I shoot for the middle ground. They're the ones to show me in a lesson who really got it. Well, there should be a core. There are those who are going to get it no matter what, there are those that will never get it, and there are those that you can move along. And as you move this thing along, you bring them along as best as you can. (interview, 12/88)

Mark's concerns for parental pressure, students' performances on standardized tests, class size, and content coverage combined with his beliefs about how children learn mathematics and their abilities to master some aspects of the subject, have put Mark in a position where he has chosen to teach only knowledge of procedures and skills because that is safer, more efficient, more manageable. Mark portrays himself as a teacher caught in a desperate tug of war: The state wants him to teach conceptual understanding but tests procedural knowledge; teaching for deeper understanding requires compromises be made between breadth and depth--compromises that are often questioned by parents and the community. Mark's concerns about the press of time, parents, and tests are very real and he is right in acknowledging the power they have over the choices that get made by teachers in schools. But Mark's talk also suggests that other factors are influencing his pedagogical decisions, a point that becomes clearer as we examine the ways in which Mark used the textbook.

"Following" the Book

> I pretty much follow it step by step. That's the way I was always brought up in teaching. To me, texts are supposed to be sequenced....but math is generally, the way I understood it, sequenced so it kind of goes in stages. So, I kind of follow it step by step. However, I did skip a little here and there. When you get too long in one thing I move on, I move on to the next thing. (interview, 12/88)

As already noted, Mark had no exposure to the *Framework*, save our conversations about it in interviews. The mathematics *Framework*, for Mark, is but one of a series of curricular chimera introduced by the state to increase student achievement in California schools. Teachers, according to Mark, have had no input into these decisions but are nonetheless supposed to implement the *Framework* by using the textbooks adopted by their districts.[4] Moreover, when the teachers were

[4] Mark is mistaken about the participation of teachers in the development of the state's policy. Teachers are an integral part of the state's policymaking in all arenas, and they hold positions on all essential committees: curriculum, textbook, and testing. Moreover, within Mark's school district, teachers participated in the review of textbooks and the subsequent adoption of *Real Math* by the district.

introduced to the textbook at the beginning of the school year, they were told to follow it page by page. And according to Mark, that is just what he is doing.

But in conversations with Mark, it became clear that his claim to have "skipped a little here and there" is an understatement. For example, the text, which relies heavily on the use of manipulatives and games, is accompanied by a set of materials, materials that Mark has not "seen the need for yet":

> They gave us a big box. But I haven't really had time to look at it. They have little game pieces and they have paper, fake money. I haven't even seen a need for that yet, to tell you the truth. I use the little [response] cubes. We use those quite a bit when the game is lined up with that. They have a whole series of games but it's hard to fit them in, that's the big thing. There's a whole box of materials, as I say I haven't really looked at it. There's a practice book or a work book where you ditto off the pages. I use them to back things up.[5] (interview, 12/88)

To avoid using the materials, Mark either had to skip lessons that have required them or translate lessons into one that he could teach without the materials. This is especially problematic given Mark's claims that the textbook does not teach the conceptual aspects of the topics covered. What Mark fails to realize is that the conceptual territory is often covered through the use of manipulatives and story problems. Following the book, for Mark has meant following the pages in order, but dropping lessons that don't fit with his sense of what students should be learning, adapting ones that require manipulatives so that they can be taught without those materials (perhaps turning them into something entirely different than the authors' intended lessons), and adding "backup" work that has included practice sheets that he has sent home for homework assignments--some of which have come from the workbook in the "box", others that he has from previous years of teaching. In addition, because he only spends about 30 minutes a day on mathematics (and often less), Mark has had to "streamline" lessons to save time. Through his adaptation of these materials, Mark may unwittingly be fulfilling his own prophecy: the students may not be developing deeper understandings of the mathematical content presented in the textbook.

Mark's transformation of the curriculum is not a surprise. We know from research that teacher-proof materials are an illusion, and Mark exemplifies how a teacher's beliefs, knowledge, and concerns influence how curricular materials are used. But Mark's critical use of the textbook is not fueled by some malevolent wish to boobytrap the new mathematics *Framework*. He sincerely believes that he is following the text. And from his perspective, he is. Yet, in many ways, he is not. What is it about Mark, about his knowledge of teaching or of mathematics, about his instructional goals, about his dispositions that contributes to his translation of the curriculum?

Conflict #2: Competing conceptions of learning and teaching mathematics. For one, there is a clear dissonance between Mark's beliefs about how one learns the "basics" and how the textbook presents these basics. Marks believes that students should learn "the basic functions" in sequence: First you introduce addition--all types of addition, single digit and double digit--all the while providing a great deal of practice. Then you move on to subtraction, covering it completely and, again, providing plenty of practice. Once addition and subtraction have been mastered, you move on to multiplication and division--covering each separately and thoroughly. Related activities, e.g., learning about decimals and working through applications of these basic functions, can be

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5 An important element in Mark's theory of what it takes to learn mathematics is the role of practice, or what Mark refers to as "backup." Throughout our interviews, Mark constantly made reference to the textbook's lack of practice problems. Without practice, the knowledge of procedures does not "set" in students' mind and, moreover, the teacher lacks feedback on how much students understand.

added on if you have time. However, the introduction of such activities should be held off until all students have mastered the procedures. And the procedures are best mastered if they are done separately, so as not to contaminate one another since Mark believes that it is easier for students to master a procedure if they concentrate on one at a time. Adding more "facts" to be memorized only confuses students in Mark's eyes, and he sees one of his responsibilities as teacher easing the learning of his students--reducing any potential sources of confusion or conflict.

The book that Mark is using is based on another set of assumptions. Rather than separate operations that are conceptually related, the book interweaves the teaching of addition and subtraction, multiplication and division; the text also starts with an emphasis on the intuitive before it gradually moves to a more explicated version of mathematical concepts. Representations, and the ability to generate and manipulate alternative representations of the subject matter, are central to the curriculum--not an add-on if there is time left after students master the procedures. Mark, while he applauds the "philosophy of the book," in his words, to "teach the meaning of these concepts," is troubled by what he considers a "pinball approach" to teaching. He likens the interweaving and spiralling of the curriculum to the painting of a house:

> It's kind of like a coat of paint. You paint it one time and you let it dry. You paint it again and you let it dry. It might soak in and it might kind of chip off and things like that. So, I think they just figure a little smattering here and a little smattering next year. See, they're assuming these kids have had this since first grade or so and they haven't. [The students] are not used to this pace. They're used to a pace where you do division until you basically have it and then you move on, you know. And they don't have it. And I really learned how to pace things where I could jump over addition and subtraction and just keep smattering that. I'd spend a long time. I usually spent the first two thirds of the year on multiplication, addition, subtraction and--well, addition and subtraction and then get beyond that, multiplication and division can take almost until the last quarter of the year and then you're into fractions and, you know, the other things like that. But, by then, most of your class is able to multiply and divide. (interview, 4/89)

Mark was very concerned about the fact that, in April, his students still didn't know how to multiply and divide large numbers:

> Right now I have kids in my class that don't know how to multiply or divide yet and the text isn't addressing it. Here's a page on multiplying, but it didn't just teach multiplication, it jammed decimals on top of it, too. They're getting confused by the decimals when they don't even know how to multiply well. (interview, 4/89)

In addition to the interweaving of topics, there are other features of the textbook of which Mark disapproves. For example, the textbook uses representations--symbolic, pictorial, and otherwise--throughout to help develop understanding. Mark, on the other hand, believes that the use of representations is an ability or skill that is developed after students master the procedures. He does not believe, for instance, that students should learn to represent mathematical ideas before they learn to manipulate the numbers involved. This is why he has dropped all aspects of lessons that deal with concrete objects or manipulatives. Seems unaware that the "box" does not contain supplementary materials to be used in spare time but instead contains essential tools for much of the teaching that the policy advocates and that in prescribed in *Real Math*. In some very real and fundamental ways, then, Mark's view of mathematics teaching and learning conflicts with the one on which the textbook is based.

So here we see a teacher in conflict with the text: Teacher and text have fundamentally different assumptions about how mathematics is best learned and taught. Mark handles the frustration this clash produces in several ways: He skips parts of the text, he provides extra practice for students, and he peppers his teaching of the textbook with lessons that are taken from the "Scoring High" pamphlet provided by the district. Mark does not seem to recognize that his sporadic and inconsistent use of the text and its accompanying materials might be contributing to the difficulties his students are having with the material. This melange of activities and ideas is, in Mark's eyes, "following the book," for he does cover most of the lessons--dropping aspects that seem unimportant and adding practice and content that will ensure students' success on traditional measures of performance.

Conflict #3: Knowledge of alternative pedagogical strategies. Recall Mark's comment: "I can only teach what I understand." Another factor that appears influential in Mark's selective use of the textbook is his own lack of knowledge about how to teach mathematics in the ways suggested by the textbook. For example, although he applauded the use of manipulatives in mathematics teaching, Mark voiced concern over his own experience and knowledge of how to use such materials:

> So I would work a lot with word problems and manipulatives but I'm not well-trained in manipulatives and to be perfectly honest I don't how, I don't have any idea right off the top of my head how to make any manipulative for what we were doing in today's lesson in division of fractions. (interview, 12/88)

On another occasion he remarked:

> My teaching hasn't been that much different [this year] except following a different text. As I was saying, my biggest hurdle to doing all these new methods--I call them new but some of them are old that I wasn't a part of when they were in before--they're regenerating them because they are finding them to be valuable--is my knowledge of what I've done all these years and I don't know how to make the transition. And I don't completely know all these methods in the math series. (interview, 12/88)

Mark and his students happily go through the motions, enacting the lessons laid out in the textbook. But their interpretation of those lessons is colored by what they know. For instance, Mark doesn't know how to use manipulatives nor does he know why one would use them in particular settings. Since he believes that representing mathematics is a higher order skill, one that follows the proficient use of algorithms and procedures, Mark chooses to skip over lessons that involve manipulatives or drop the manipulatives from the lessons in his race with the curriculum coverage clock.

Because they are simply going through the motions, Mark and his class bear little resemblance to the vision proposed by the *Framework*. Discourse in this class is highly constrained. While Mark invites students' questions, only certain types of questions are allowed: questions about how to do the procedures. When students ask other types of questions, e.g., about why something works, Mark responds by saying, "Remember what I taught you?" or "We don't have time for that," throwing the responsibility for answering the question back in the laps of the students. The only inquiry that is encouraged is that which concerns the "steps" of a procedure. Students do not explore serious mathematical problems, they do not generate multiple solutions to problems, they do not discuss and debate alternative interpretations and answers. The mastery of rules and procedures is the focus of Mark's curriculum; no attention is paid to the underlying conceptual ideas. A good explanation is one that traces the steps of a procedure, not one that traces the student's reasoning through a series of mathematical decisions.

This is not surprising. Mark has had no inservice training in the *Framework* or in the use of the textbook (with the exception of the beginning-of-the-year overview provided by the textbook company). Without the assistance of people who are willing to help teachers learn new ways of approaching mathematics, Mark is left to his own devices. Alone, he does the best he can: skips things he sees as irrelevant, alters assignments and activities to fit his understanding of mathematics and teaching mathematics, interprets the textbook based on his own beliefs and orientations.

While Mark is sensitive to his own limitations, he is also cynical about the "expertise" of some of the individuals who are proposing these curricular changes. Mark resents "outsiders" who "have never been in classrooms" telling him how to teach:

> I guess one thing that really is beginning to drive me up the wall in this business is the fact that every year somebody comes in and says, "Here it is folks, this is the best way to teach. This is it! This is the one that's going to cure everything." They're like the old snake oil salesman. And yet *none* of them--well, I take that back because the last guy that came did--but most of them never say, "What do you do? What works? What doesn't work? What do you need?" *None* of them! (interview, 12/88)

Given his wariness of outsiders, it is not surprising that Mark reacted to the textbook representative in the way that he did:

> They take you through it and they show you a few sample lessons and they try to sell you that what they've sold the district is the best thing in the world. I don't know, to tell you the truth usually I don't listen to them. Because I don't need someone telling me how to work a textbook first or all. And second of all, she wasn't making a lot of sense to me. Most of the people I talked to came out of there saying they would have done better to take the book home and read it. Another problem was, I can't remember now if it was my vacation time or right in there, but I hadn't even had time yet to work with the textbook. So I didn't even have time yet to know good or bad points that I'd like to ask about. I had nothing to go on, so really I got very little out of it. So what I did was that night I went home and read the format, how it works, things like that. (interview, 12/88)

Mark's lack of knowledge about alternative teaching strategies does not put him in conflict with the *Framework* as much as it constrains his ability to implement it in spirit. Mark wants to do the right thing but it remains unclear what the right thing is. Mark read his textbook but the text has been unsuccessful in communicating to him the importance of thinking about mathematics, as well as its teaching and learning, in very different ways. Having a new textbook and a box of materials does not guarantee the appropriate use of them. Mark does not simply lack knowledge of *how* to teach with manipulatives or with cooperative groups, he also lacks knowledge of *why* a teacher might choose to use a particular strategy. Without such knowledge, he is left to interpret the materials in his own way, rejecting those that conflict with his own sense of what should be taught and experienced in a fifth grade math class. While the authors of *Real Math* make an attempt at providing rationales for the choices they made, the authors do not recognize how powerful the lenses of traditional practice can be, nor how much they can influence what teachers read on and between the lines of their teacher's manual.

Conflict #4: Competing calls for reform. Mark has reason to voice concerns; he works in a context in which there are multiple messages and reforms. Although the mathematics *Framework* has been implemented this year, Mark is aware of the impending implementation of similar frameworks in language arts, social studies, and science. Mark is poignantly aware of his own shortcomings as a teacher in these reformed classrooms:

> And it's bad to say, but I'm finding my biggest hurdle right now to integrating language and now one of the big sweeps is literature and I mentioned cooperative learning and now this new. I mean we've got four new sweeps coming at us right now. We've got, as I mentioned, integrating language, we're supposed to do that. We're supposed to do literature and we're supposed to do cooperative learning and now this new math series. That's right now. That's

> on top of us right now. And to tell you the truth, I wasn't trained very well in any of those. (interview, 4/89)

Moreover, Mark knows that the CAP tests have not yet been altered to match the differences in content emphasis:

> I guess another thing is all the stuff they pile on us to do. It's a lot of stress mainly because you know what they look at, they look at test results. And the tests are not written for any of this stuff, the tests are written for the old way. The tests are written for "Open the book boys and girls. Do this activity, learn your nouns, verbs." Now they're saying, "No, don't teach nouns and verbs in isolation, teach writing for competency." But the test isn't for that and if they go down in the test they're going to come to me and say, "Mr. Black.." and I'm going to say, "Wait a minute! You said to teach this but the test is about that!" There is a lot of stress right now. If you were here long enough at this school you would find a lot of deep seeded stress. (interview, 12/88)

Finally, Mark reminds us of the larger context in which all of these teachers work. Concerned about the learning of all school subjects, California has produced a series of frameworks that call for change in all content areas. While these changes occur in cycles, with emphasis and resources being placed on one subject matter each year--teachers like Mark know that it takes more than one year with a new textbook to alter one's teaching.

Conclusion

The drums of reform echo loudly and teachers like Mark feel the press to change their math teaching and their language arts teaching and their social studies teaching and their science teaching. And as he noted, the situation is complicated by the fact that testing remains the same: Teachers are to teach new content but student performance will still be measured with old measures until the new CAP tests are instated. Equally important is the fact that community evaluation is based on traditional conceptions of what and how things are taught in school. Parents expect teachers to teach their children as they themselves were taught. As he makes choices about what to do--what content areas to focus on, what teaching strategies to learn, how to prepare students for CAP tests and cover the new curriculum--Mark's world is a maelstrom of conflicting demands.

Mark needs help. Some of the reasons he needs help are those he himself noted: help in learning about new methods, help in finding time to teach for understanding, resources for evaluating such understanding. But Mark also needs help for reasons he cannot see: While he speaks about different levels of mathematical understanding, Mark's own beliefs about what it takes to learn and know mathematics are in conflict with those that underlie the *Framework*. He needs to learn to think about mathematics as a field of inquiry, not as a body of procedures. He needs to learn to think about the goals of learning mathematics as greater than the mastery of skills computational. And he probably needs to learn new things about the subject matter since his own knowledge of mathematics may be limited to the procedural aspects of the traditional curriculum.

Mark cannot make fundamental changes in his teaching without several kinds of support. First, he needs time and assistance in examining and evaluating his own assumptions about how children learn mathematics, comparing his own assumptions to those that undergird the *Framework*. Assumptions about what it means to know mathematics and how best the subject is taught have changed a great deal since Mark was taught to teach. He is a model teacher in the "effective teaching" paradigm and there is much evidence that he has worked hard to learn to do that teaching well. But the changes encouraged by the *Framework* depend on another image of teaching, one that focuses on the student as well as on the teacher, on conceptual understanding as well as technical mastery. If Mark is to understand the nature of those changes, he needs a chance to examine the central differences between effective teaching and the teaching envisioned by the

Framework authors. He cannot be led to believe that implementing the *Framework* involves adopting a few new activities and instructional strategies for at its heart the *Framework* assumes fundamentally different things about the nature of learning and knowing. Mark's conceptions of learning and teaching mathematics conflict with those inherent in the *Framework*. If we fail to acknowledge that such conceptions act as lenses through which teachers perceive and interpret curriculum, we see teaching like Mark's: an innovative curriculum edited to be familiar.

Second, Mark needs to think about the kinds of pedagogy best suited to facilitate the development of such understanding. This reform does not call for changing teaching across the board, no matter what. Rather, this reform is based on the belief that teaching methods should match educational goals and that teaching requires complex decision-making about the use of a range of alternative pedagogical strategies. Mark needs to learn about that range of methods, including their respective strengths and weaknesses. His lack of knowledge about alternative methods constrains his ability to implement this reform.

Third, Mark needs practice and experience implementing strategies he has never used--gaining familiarity with new materials, adapting old strategies to meet new goals, crafting a version that draws on his strengths and minimizes his weaknesses. Learning to use new methods takes time. As they become more familiar with methods, teachers acquire insight and understanding about each strategy--when and how it is most effective, how students react, what students need to know and be able to do in order to participate in the experience, what the nature of the teacher's and students' respective roles are in the activity. Such understanding is developed over time and best facilitated when teachers are given opportunities to practice, to make mistakes, to reflect on their experiences and those of their colleagues. Again, Mark's lack of skill in the use of new methods restricts how much he can change his own practice.

Finally, Mark needs to work in a context that is sensitive to the complexity of teaching and the factors that influence classroom work. The tests are changing (California State Department of Education, 1989). But the public, including parents, must be re-educated in their own conceptions of what and how students should be learning mathematics. If we want teachers to change their practices, we must provide safe, supportive environments that encourage those changes. Tests must be aligned with the goals of the curriculum, parents must be helped to see the benefits of the new curriculum, and administrators and teachers alike must understand that changes in practice are not easy, are often rocky, and always take time.

These types of support--room to examine beliefs and prior knowledge, new information, practice, and a safe and secure environment--are the kinds of support that we consistently urge teachers to provide students. The *Framework* authors acknowledge the complexity of learning to teach in this way when they state:

> Teachers need the same opportunities to develop their understanding and their ability to apply their knowledge to new situations as students do, and such development does not occur in a one-time two-hour workshop on a single topic. Rather, well-planned, extended programs are needed in which teachers have the opportunity to see new techniques demonstrated in classrooms, try out new methods with their own students, and reflect on the changes in the curriculum. Further, teachers must receive coaching and support over a period of time to build their confidence and to see for themselves how content and methodology are related in their teaching. (California State Department of Education, 1985, p. 6)

Mark has yet to experience such support. What will happen to Mark? Will he continue to adapt the textbook to meet his more traditional vision of mathematics teaching? Will Mark encounter teachers or support staff who can begin to help him develop new skills like using manipulatives or coordinating cooperative learning? Or are Mark's beliefs about the nature of mathematics and how children learn mathematics so ingrained in his pedagogical reasoning that he will be forever unable to implement this textbook--and perhaps the *Framework* authors' vision of mathematics teaching and learning--in ways that are more consistent with those documents?

This introduction to Mark and the changes he is making in his mathematics teaching ends, then, on an ironic note: The policy we are investigating calls for teaching mathematics for understanding, a kind of teaching that respects both the mind, dispositions, and interests of the learner as well as the difficulties inherent in learning anything in meaningful ways. Yet in its first year of implementation, Mark was not treated with a similar sense of respect for his needs as a learner. Instead of working with the *Framework*, a textbook became the messenger of the policy. Instead of being placed in settings where the policy could be explored, questions asked, alternative interpretations made, Mark heard through the grapevine that his teaching was supposed to change. Teachers, like their students, are learners who need to be taught in innovative, flexible ways. How the state of California and the school districts in which teachers like Mark work respond to the needs of the learners who comprise their teaching force will be a critical piece of the story we might someday be able to tell about the connections between this curriculum reform policy and its impact on classroom practice.

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Filtering the Framework:

The Case of Jim Green

Janine Remillard

> The *Mathematics Framework* presents a vision of elementary and high school mathematics programs dependent not on expanded research but on expanded commitment. (California State Department of Education, 1985, p. vii)

Jim Green is an experienced teacher who is profoundly committed to his students. Attentive to unique concerns of low income, minority children, he works hard to help his 34 fifth graders be "successful" in mathematics. Nevertheless, the vision of mathematics instruction to which he is committed does not necessarily mirror that outlined in the *Framework*. While his mathematics teaching has not gone untouched by the efforts behind the *Framework*, Jim's interpretation of its vision has been shaped by his textbook, as well as his views of the learning and teaching of mathematics. The knowledge and beliefs that Jim brings into the classroom act as filters for how he sees and understands the *Framework*; consequently, they help determine how his commitment to his students' learning of mathematics plays out in his teaching. In this case, I explore the ways which these beliefs about mathematics and learning shape his practice.

Jim Green teaches fifth grade at Columbus, a multi-lingual school in a low income neighborhood in Southdale. The students in his mathematics class, 90% of whom are black American, Asian, or Hispanic, comprise the upper ability group of the two "regular" (English-speaking) fifth grade classes. His rapport with students is good, for he is a warm, entertaining and supportive teacher. Nevertheless, Jim approaches his mathematics teaching seriously and firmly, and maintains a vigorous pace. He wants his students to be successful in mathematics, to enjoy it, and to realize its relevance in their lives. Over twenty-five years of teaching, he has developed a repertoire of strategies to meet these goals.

The routines in Jim's class are well established. As soon as the students enter the room after recess they begin "daily drill"--five or six exercises that Jim has written on the board. After several minutes, and well before all the students have completed each exercise, he stops the class and goes through the drill, occasionally collecting the students' work. He then moves the class into the next portion of the lesson; one group (divided according to student ability) works individually on the textbook assignment given the previous day while Jim instructs the other group. Upon completing his presentation, he gives that group an assignment and moves rapidly to the other group, often hurrying students along with him, as if to remind them of the ever-present clock ticking away throughout the 45 minute lesson.

The atmosphere of the class is busy, at times rushed; engrossed in their assignments, students occasionally consult a peer or the teacher's aide. Everyone moves around the room freely, sharpening pencils, collecting supplies. The classroom, with this serious and busy spirit, resembles a crowded business office in which many employees work somewhat diligently, but not silently, giving little regard to a meeting being held at one end of the room.

Jim's Knowledge and Beliefs about Teaching and Learning Mathematics: Filters for Viewing the *Framework*

Jim's beliefs about mathematics teaching and learning act as filters for how he sees the *Framework*. Thus, elements of the *Framework* which are consistent with his views--or that can be compatibly mixed with his practice--flow through, while more dramatic or radical elements get caught in the sieve of his beliefs. I offer several examples here.

Jim's Knowledge of Mathematics: Procedures and Concepts are Inherently Connected

> Mathematical rules, formulas, and procedures are not powerful tools in isolation, and students who are taught them out of any context are burdened by a growing list of separate items that have narrow application. (California State Department of Education, 1985, p. 12)

Jim stands behind the *Framework*'s proposition that children should understand the underlying concepts of mathematical rules and procedures and he sees himself teaching concepts to his students. This stance does not belittle the importance of mastering procedures. Jim most certainly feels his students should become proficient with mathematical procedures, but he also expects that they will understand why these procedures work. As he explains:

> One of our goals, naturally, is to hope they understand why they're doing what they're doing, not just how to do it, but why they're doing it . . . If you know why you're doing it then it's of more value to you. (interview, 12/88)

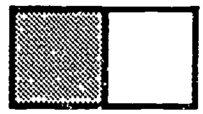
As suggested by the *Framework*, Jim resists separating or isolating rules and procedures from their related concepts. He does this because he sees procedures and their related concepts as being inherently connected and interdependent. In response to a chart in the *Framework* which lists contrasting characteristics of "Teaching for Understanding" and "Teaching Rules and Procedures" Jim said, "You have to have both. You have to have [teaching rules and procedures] to go hand in hand with [teaching for understanding]." (interview, 12/88)

Given his commitment to procedural mastery and conceptual development, one might expect to see Jim give equal airtime to both in his teaching. Watching Jim teach, though, suggests that he places a heavier emphasis on rules and procedures with only scattered attention to conceptual development. He provides his students with the rules necessary to correctly perform the procedures and demonstrates them on the board repeatedly, but does not always explain why. For example, when referring to converting fractions to their equivalents, he stressed the age-old phrase, "Whatever you do to the bottom you do to the top" (observation, 3/89). His explanations in class focus on correct manipulation of algorithms, not on making sense of how and why the procedure works. When reviewing the subtraction of decimals Jim reminded his students, "The main thing to remember is to keep it in the exact column" (observation, 3/89). By following this rule, Jim was sure students would not mix ones and tens and hundreds in their answers. Students were not encouraged to think about why aligning columns was necessary in this procedure.

Observing this emphasis on procedural and mechanical aspects of number manipulation, one wonders how Jim makes sense of his commitment to conceptual understanding. One hypothesis is that Jim's own understanding of mathematical concepts is shallow and disconnected. Thus, his ability to focus on the conceptual may be fragmented and inconsistent. But in our conversations it seemed to me that Jim has a grasp of most of the concepts he is teaching: Comfortable with the mathematics, and facile with the procedures, he was able to speak articulately about various ways of thinking about topics like division and fractions. His knowledge of the mathematics he was teaching seems flexible and detailed.

Alternatively, a second hypothesis might be that his conceptual understanding may be implicit and tacitly tied to his more explicit procedural understandings. This might lead him to believe that by presenting procedures to his students, conceptual understanding naturally follows.

A closer look at Jim's teaching may help uncover the reasons why he places so much emphasis on the procedures. During a lesson in which he introduced a group of students to fractions for the first time, he began by using a visual representation which, like the *Framework* suggests, gives meaning to symbolic notations. He drew a large rectangle, representing a candy bar, on the board and explained that if he did not want to eat a whole candy bar, he would break it in half and just eat one of the halves. He divided it into halves and shaded one of them. He said, "It's a whole candy bar, is it not? And Mr. Green breaks it into two parts (writing 2 as the denominator). And he eats one of the parts (writing a 1 above the 2). So he eats one of the two."



$$\frac{1}{2}$$

He went over this several times emphasizing that the 2 tells how many pieces the candy bar is broken into and the 1 tells how many he ate. He went on.

Jim: If I have this half and I cut that in half again how many pieces do I have? He divided one half into half again.)

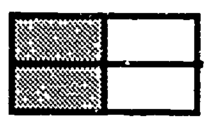
Students: Two.

Jim: I've got two (writing 2 as a numerator). And if they were broken into the same size, how many pieces would there be?

Students: Four.

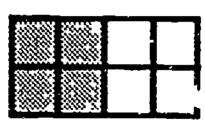
Jim: I have four all together. The two fourths is equivalent to one half.

Jim wrote:



$$\frac{1}{2} = \frac{2}{4}$$

He then divided the candy bar into eighths,



$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$$

He then did the same with sixteenths. His class seemed satisfied with this; no one raised any questions. Jim then turned to his students and announced, "Ladies and Gentlemen, you now know what equivalent fractions are . . ."

Abandoning this representation, Jim erased the board and wrote:

$$\frac{1}{2} + \frac{1}{4}$$

Moving to a rule-based procedure, Jim used common multiples to find a common denominator and rewrite $\frac{1}{2}$ as $\frac{2}{4}$. Looking at his class Jim said, "Now I can add $\frac{1}{4}$ and $\frac{2}{4}$. It is . . ."

Several students said, "3 eighths."

Jim seemed surprised by this error. "No, you don't--I'm going to explain fractions again." He reviewed the previous work, reminding them:

> The bottom number just tells you how many pieces something is broken into. The top number says how many pieces you have. In this fraction (pointing to $\frac{1}{4}$) I have one part of something broken into four. In this one (pointing to $\frac{2}{4}$) I have two parts of something broken into four. So when I add my one part and my two parts together I get three parts of something broken into four. (observation, 3/89)

The connection between the previously used candy bar example of equivalent fractions and the symbolic notation used to represent fractional parts seemed obvious to Jim.[1] It was also obvious to him that in order to add two fractional parts, each needed to be divided into the same size parts or have the same "name." Moreover, when adding two fractions with the same name, the numbers of pieces are added together, but the "name" remains the same.

Jim assumed his students "knew" these things; after all, he had taken them through the candy bar explanation of equivalent fractions. The fact that they did not transfer the "knowledge" they displayed during the discussion of the candy bar to the subsequent discussion of $\frac{1}{4} + \frac{2}{4}$, took him aback. To an observer, it seems a sensible error for students to make. But if Jim's knowledge of fractions is sound and conceptual, he may not recognize the possibility of less well developed understandings in his students. Just as mathematicians dismiss naive questions from students about *why* something is true by claiming--"it's intuitive." The fact that it is intuitive for Jim may make it hard for him to see what makes it *not* intuitive for his students. He was thus surprised when they added two denominators together. The fact that Jim did not spend time on the mechanical steps of "only adding the numerators"--a commonly heard rule-of-thumb in mathematics classes--implies that he really was expecting his students to *understand* fractions, rather than just manipulate them according to given rules. If his students really did "know what equivalent fractions are" to the degree that he had thought, it would not make sense to them to add the denominators.

It seems that the procedures and their underlying conceptual ideas are so interconnected for Jim that he assumes that facility with the procedures indicates conceptual understanding. So, although Jim would agree with the *Framework* that rules and procedures should not be taught in isolation, his view of procedures and concepts as being wrapped together seems to have led him to focusing

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[1] The denominator indicates the number of parts the whole candy bar was divided into and the numerator refers to the number of these parts being eaten.

on procedures, occasionally and sporadically alluding to the conceptual issues. All along, he assumes that his students are developing their understanding of the concept as they master the procedure. To Jim, he is not teaching the procedure in isolation because the procedural draws with it the conceptual.

Consider Jim's response when I asked Jim how he would feel if a student could subtract $\frac{2}{3}$ from $\frac{3}{4}$ correctly, but could not model it with fraction materials. He explained that such a situation was not possible. "By the time they are able to do this we've been through the pie and all that sort of thing. I can't imagine them not being able to" (interview 3/89). He sees facility with the procedure as indicative of understanding its meaning, although he did admit that adding the denominators was a common error among fifth graders. When they make this error repeatedly, he emphasizes that the "bottom number" is "just the name of it" and repeats the process of finding the common denominator. He does not return to the pictorial representation of the candy bar.

Jim's intertwined view of procedures and concepts became even more clear to me in other interviews and observations. In one interview he made mention of drawing pictures on the chalkboard to show students why procedures worked (interview, 12/88) and he talked a great deal about using fraction models when teaching about fractions (interview, 3/89). The drawing of the candy bar to model fractions was, however, the only use of drawings or models that I observed during my visits. I did observe him interweaving conceptual meaning into his lessons in somewhat sporadic ways. During one lesson, for example, he was checking answers with a group of students who had been converting liters to milliliters. Jim's embarking on a conceptually-based explanation was initiated by student-voiced confusion. He had begun the interaction by telling several stories to illustrate when to use the different units of measure:

Jim: If I want to convert this (pointing to .863 liters on the chalkboard) to milliliters, what would I do?

Kevin: Multiply.

Latoya: Move the decimal.

Jim: Which direction do I move it?

Latoya: To the right.

Jim: (Moving the decimal to the right of the 3) Therefore, point 863 liters equals 863 milliliters.

.863 liters = 863. milliliters

Ciapha: Do you always move it to the right?

Jim: If you're changing from the large unit to the small unit, from the liters to the milliliters, you move it to the right because what you are actually doing is you are multiplying . . .

At this point, Jim paused. He seemed unsure of his explanation. Stopping his verbal explanation, he faced the board and multiplied 1,000 x .863.

Jim: See what you're doing? Any questions? To rename smaller units with larger units, you divide.[2]

After a pause he continued:

Jim: So if I have four liters and I want to change it to milliliters, then I would multiply times 1,000 and I would end up with 4,000 milliliters. So if I took one of those milk cartons and divided it into milliliters, I would end up with 4,000 of those little drops. (observation, 3/89)

This spontaneous explanation arose when Jim was confronted with an instance in which his students' conceptual understanding had once again not been carried by their mastery of the procedure. As in the case of adding fractions, it appears that Jim sees inherent connections between rules and procedures and the concepts that underlie them. Thus, it is not possible to teach rules an procedures in isolation. According to Jim, if students master the procedures, they know the concepts. Jim does not distinguish between teaching rules and procedures in isolation and teaching for understanding. Instead, the distinction he makes is "teaching rules and procedures *well*" (which brings with it conceptual understanding) versus teaching rules and procedures *not well*.

Nevertheless, Jim's beliefs about the connectedness of the procedural and conceptual aspects of mathematics are not the only factors that cause Jim to teach in this way, for he has other beliefs about teaching and learning mathematics that lead him to make the pedagogical decisions he does. His knowledge and beliefs interact; no single belief completely determines choices. It is to a related belief--about the usefulness of mathematics--that I now turn.

Jim's Beliefs About Mathematics: Understanding means Application

> The goal of this framework is to structure mathematics education so that students experience the enjoyment and fascination of mathematics as they gain mathematical power. (California State Department of Education, 1985, p. 2)

> To isolate the acquisition of mathematical knowledge from its uses and its relationships is to limit the depth of understanding achieved. (California State Department of Education, 1985, p. 12)

While Jim sees the connection between conceptual understanding and mathematical skills as implicit, he is explicit about not isolating mathematical skills from their potential uses. He views mathematics as fundamentally useful and powerful in his everyday life. This view shapes his practice and how he interprets the call of the *Framework* for "Teaching for Understanding." To Jim, teaching for understanding means making mathematics applicable and relevant to daily life. He stressed:

> It's very important that they learn the mathematical rules and formulas, but the the bottom line is where, when, how to use it. . . . I think just doing all of that in isolation and not dragging in part of your lifestvle or things that you are doing in everyday life would be ridiculous. . . . you have to always apply it to something. (interview, 12/88)

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2 As an observer, it was not clear whether Jim's students understood that moving the decimal resulted from multiplying or dividing by a power of 10 or why multiplying or dividing was the correct way to transiate liters into milliliters, or vice versa.

In Jim's eyes, it is this application to real life that gives mathematics its power and value. Jim wants his students to realize the powe and usefulness of mathematics in their lives; he is worried that they see math as only applicable to school. While distributing practice pages of story problems in one lesson, Jim asked:

Jim: Why do we work on problem so ving all the time?

Vincent: To get a lot of practice.

Han: For the test.

Jim: Why else?

Sherika: So we can do good in 6th grade.

Carl: To be able to go to high school.

Jim: Do we ever use math to solve problems in *real life*?

Robert: Taxes.

Shana: When you go to th y store.

Jim agreed with these two examples and then gave several of his own: increasing a recipe when cooking for a large family during the holiday season and building shelves in the garage. He emphasized and explained particulars about the mathematics in each case: doubling two and a half cups of flour or measuring and calculating how much wood to buy (observation, 12/88).

Jim commented on this exchange in one of our conversations:

Nobody wants to learn anything if they're never going to use it. Just like the answers I got out of them . . . So you can go to the sixth grade? So you can graduate from high school? All the answers had nothing to do with life. . . It was all school. So you can get a good grade. I want them to start--they won't be really enthusiastic about math until they realize, "Hey, I'm going to use this. I might use it tomorrow". . . They can see that it's going to be used. Fractions are going to be used because you just go in your mother's cookbook or ask your mother if she ever puts one half a teaspoon in it if she's going to double the recipe. That's why we use it. (interview, 12/88)

True to his commitment that students will realize the significance and power of mathematics in their lives, Jim's teaching is laced with stories about real-life situations: work with money, adjusting a recipe, determining the amount of wood or paint to purchase for a home repair project, building a dog house. Jim uses these stories to link particular topics or ideas to students' prior knowledge and everyday experience, as well as entertaining illustrations of mathematics' usefulness,. For example, when comparing the relative quantities of a liter and a milliliter, he explained that he deals with liters a lot, as he drinks a lot of soda pop (diet soda only, he assured his class). His wife, on the other hand, who is a nurse, deals more frequently with milliliters when she gives patients shots or medication measured in eye droppers. These anecdotes, which pervade Jim's teaching, are indicative of how he thinks about and represents the power of mathematics.

Jim's concern that his students become disposed to applying mathematical skills to situations in their lives is heightened by his view that the demands to do so are increasing as our society progresses. He sees current trends in our information oriented society as adding complexity to everyday situations, demanding higher levels of thinking and responding. As he put it, "There is so much more demanded and people have to think them out so much more critically that it seems imperative that children are going to have to be trained to think that way or learn to think that way" (interview, 12/88).

The emphasis that Jim places on developing students' mathematical power through realizing its applicability to real life situations reflects the *Framework*'s calls for developing mathematical power and problem solving abilities. Yet Jim's view of the applications of mathematics seems to be limited to instances in which actual mathematical procedures and calculations taught in school can be directly inserted. His hope that his students will move beyond school-related uses of mathematics represents of piece of the *Framework* that fits with Jim's view of mathematics. The larger chunks--that mathematical power "involves the ability to discern mathematical relationships [and] reason logically" or that it "helps students develop thinking skills, order thoughts, develop logical arguments, and make valid inferences" (California State Board of Education, 1985, p. 1)--have not found their way through the filter of his beliefs. Indeed, Jim believes mathematics to be powerful, yet he does not hold this broader definition of mathematical power espoused in the *Framework*. Consequently, through his anecdotes and examples of real-life applications, Jim portrays a utilitarian picture of mathematics to his students.

This utilitarian view of mathematics highlights its immediate and technical uses and is tied to yet another set of beliefs that influence Jim's teaching--beliefs about learning and motivation. These beliefs serve as another filter for how he interprets what it means to teach mathematics for understanding.

Jim's Beliefs About Learning Mathematics

Jim has three significant sets of beliefs about learning mathematics that appear to influence his teaching. These are: the belief that students learn through watching, listening, and practicing; that learning takes time; and that learning mathematics should be made fun. Each of these significantly shapes his practice and his interpretation of the *Framework*. Additionally, pieces of his beliefs seem to fit with pieces of the *Framework*, while others appear to be in direct opposition. I will discuss these particular beliefs below.

Learning through watching, listening, and practicing.

> Many of the basic mathematical concepts are learned in the primary grades. It is a mistake, however, to assume that the initial learning will be retained without reinforcement. Concepts and skills from all the strands of mathematics must be continually reinforced and extended. (California State Board of Education, 1985, p. 13)

Jim is a firm believer that students will not retain concepts and skills unless they are reinforced. Reinforcement, for Jim, occurs through repetition and drill. He credits the daily drill he uses with playing a significant role in reinforcing the concepts he teaches and for successfully getting his students on or above grade level, as determined by the California Test of Basic Skills:

> [T]hat kind of repetitiveness of the things they had two months ago and six months ago . . . Bring it up to them again, again and again, throughout the year . . . and it has paid off . . . I have a whole regular fifth grade class, from the lowest to the top at grade level or well above. I really think that [the daily] drill has a lot to do with it. (interview, 12/88)

In one sense, then, Jim's emphasis on repetition can be construed as being in line with the *Framework*, as he is reinforcing previously taught concepts and skills. In fact, his assumption that knowing the procedure implies understanding the concept would lead him to believe that by providing his students with repeated practice, he is reinforcing their understanding of mathematical concepts. On the other hand, Jim's reinforcement consists of isolated practice of computational skills without connecting them to a conceptual base or placing them within a meaningful context. This seems in opposition to the *Framework*'s commitment to "focusing initially on the basic concept and then repeating experiences with the concept in a variety of new settings" (California State Board of Education, 1985, p. 13). Jim's interpretation of the *Framework* seems to be both line with and at odds with the authors' intentions.

Closely related to Jim's conception of the role of practice in reinforcing mathematical concepts is his apparent belief that watching and listening are powerful modes through which students learn mathematics. A large amount of Jim' s instructional time consists of him demonstrating and explaining procedures at the board. A one-man show, he does most of the talking, example giving, and computing, and at the same time, providing a "play-by-play" account for his spectators who seem to be intent on the action at the board. The account of the action has a familiar ring. They've heard it before, because with each similar algorithm, he uses the same script (Putnam & Leinhardt, 1986). is used repeatedly with each similar algorithm. Occasionally he provided blanks in his script for a student to fill in. But just as often he fills in his own blanks.

The script he followed, for example, when going over 45)6987 was the same one he used in each long division problem I observed Jim teach. He began by reminding his students, "We've been over this many times. This is just the drill to keep you in training for the CTBS which is coming up in about a month." He looked at the problems on the board and began writing in the numbers as he rapidly announced each play.

Jim: Forty-five goes into 69 one time. Forty-five times one is 45. Nine minus five is four. Six minus four is two. Bring down the eight. Now, 45 goes into 248--we have to estimate: We can think about this 45 as being 50 and 248 as being 250. Fifty goes into 250 how many times?

```
      1
45 )6987
    45
    248
```

Tara: Five.

Jim: Five. Five times five is 25. Put down the five and regroup the two. Five times four is 20, plus two is 22. Eight minus five is three. Four minus two is two. Two minus two is zero.

Jim completed the computation in this manner, arriving at 155 with a remainder of 12.

```
      155 r 12
45 )6987
    45
    248
    225
     237
     225
      12
```

He asked a student to read the answer to the class and then continued to the next problem with out further comment or explanation (observation, 3/89).

Jim refers to this aspect of his teaching as "modeling." On several occasions when his class seemed unsure about a particular procedure, he asked, "Would you like me to model it one more time?" Jim assumes that his students will learn mathematical procedures by watching him perform them. Consequently, he insists that his students give him "eye contact" throughout the lesson. As he explained in one interview:

> I like eye contact, especially if I'm modeling at the board or something. At least I got something going there, whether the brain's all here or not. At least if I have eye contact, they're visually seeing something that I want them to see. I pretty much like to concentrate on eye contact. (interview, 12/88)

Although he acknowledges that watching alone does not lead to understanding, Jim's comments and lecture-style presentation suggest that he believes there is much to be learned through watching and listening. Highly experienced with students who are more interested in playing games or sleeping than learning mathematics, Jim insists that they look at him throughout the lesson. He hopes perhaps that by watching him manipulate the procedures, his students might be pushed one step closer to understanding.

The *Framework* authors also note the critical role of engagement in learning. However, they argue for a more *active* stance for students, a stance that goes beyond "eye contact": "Mathematics concepts and skills must be learned as part of a dynamic process, with active engagement on the part of the students" (California State Board of Education, 1985, p. 12). Once again, we see how, in spirit, Jim believes in the principles promoted by the *Framework*. But we also see that, in practice, the correspondence between the *Framework* authors' intentions and Jim's teaching is questionable.

Learning takes time.

> [Teaching for understanding] Takes longer to learn but is retained more easily. (California State Board of Education, 1985, p. 13).

Jim would not dispute that learning mathematics takes a long time. In fact, a significant piece of Jim's beliefs is that an "incubation period" is a requisite part of learning. This is the time a student needs to let an idea "soak in" before being able to understand it. Jim spoke about this period often:

> You find new information and you don't really process it right away. Then about a day or so later, all of a sudden, it dawns on you, or a week later. Certain types of information will take a longer incubation period. (interview 12/88)

In many ways, this particular belief seems to fit well within the confines of the *Framework* which rejects drill and practice as a means of teaching mathematics and emphasizes that students need to take time to explore mathematical concepts in order to develop an understanding of them. But the belief also appears problematic, for it implies a magical connection of sorts between hearing information or using a procedure and understanding it. While being unclear about what occurs during the incubation period, Jim explained that he allows for it in class by continually returning to ideas introduced previously. He expects that students who originally had difficulty will understand them the second or third time around. True to his belief that mathematical procedures carry with them conceptual clothes, Jim returns to these ideas by reviewing procedures.

Making mathematics fun.

> The inherent beauty and fascination of mathematics commend it as a subject that can be appreciated and enjoyed by all learners. (California State Department of Education, 1985, p. 1)

It is extremely important to Jim that his students enjoy mathematics; he wants them to "have fun with it" (interview, 12/88). In fact, he has found that when he puts too much pressure on the students and forgets about encouraging them to have fun, math class becomes less productive. Jim recalled:

> It's very difficult to do for a person like me, especially because I wanted them to get it right. I wanted them to learn it, then I started putting pressure on them and then I'm just defeating my own purpose that way. I have to back off and have a little fun to ease everything off and get us enjoying the time of the day sort of stuff. (interview, 12/88)

The stories with which Jim frequently embellishes his lessons, not only provide examples of applications of mathematics, but also serve the second purpose of making math class fun and enjoyable.

Jim's emphasis on having fun grows out of particular beliefs about learning that are specific to his students learning the subject of mathematics. He has found that math is difficult for the low income students he teaches. Their experiences are limited and as a result they lose interest quickly. Concerned about student engagement, and having "eye contact," Jim has found that he loses his students if he fails to make math fun:

> [I]t can be deadly to children that aren't good at it and don't like it. You have to have some fun. At least you get a chance of grabbing them. Ham it up a little bit. Have some fun with it. Try to make it feel good. (interview, 12/88)

Jim recalls having classes of students for whom the mathematics itself was fun and engaging. In cases where the students do not show a natural interest, it is important, he feels, that he bring the fun to it.

As a teacher of these students, then, Jim carefully considers ways to capture his students' attention and to hold their interest. The stories, as well as gimmicks he uses are intended to serve as eye-catchers. But many also become mnemonic devices simply to help the students remember the procedure. In one class I watched, he used an intriguing gimmick called "naked numbers" as a way to teach his students how to find the lowest common multiple of two denominators being added: He began by explaining that trying to add fractions with different denominators is like trying to add apples and oranges or a Chevrolet and a Cadillac. "If you want to add a Chevrolet and a Cadillac you have to call them both cars; and with fractions this includes finding the lowest common multiple." He then announced that they would be learning to do this by working with something he could never teach in front of the principal or a parent. In a very low whisper he explained that they would be working with, "naked numbers." Several students snickered, but all were intent on Jim as he wrote on the board:

$$\frac{1}{2} + \frac{1}{4}$$

Jim: We can use the naked number method to find the lowest common multiples so we can rename this (pointing to the fractions on the board) in equivalent fractions and then we can add it. Remember, our goal is to find equivalent fractions and to rename one of these two. In order to do that we have to find the lowest common multiple, and we are talking about our (pointing to the denominator)--Do you remember this from 4th grade? This top number is our numerator and the bottom is our denominator. When you add fractions you have to have common denominators--a number that they both go into. In order to add these fractions we have to find a common denominator. In order to do that we have to find the lowest common multiple, and in order to do that we're going to strip one of these numbers down to the bare necessities.

He wrote:

$$\frac{1}{2}\{2, \quad \}$$

$$\frac{1}{4}\{4, \quad \}$$

and explained that "any number lower than another number is totally undressed, and we can not allow it to go that way. So we take multiples of that number until we get it as big or bigger than the other number." He pointed out that the 2 was naked because it was lower than the 4. The next multiple of two is four, so he quickly wrote a 4 to the right of the 2. Once the numbers are equal they are both "dressed." He then explained that this meant that they wanted to make both fractions into fourths. He took the class through the steps of multiplying both the numerator and denominator of $\frac{1}{2}$ by 2, explaining that "What ever you do to the bottom of the fraction, you have to do to the top." He stressed that another name for one half is two fourths (observation, 3/89).

Jim later told me that he had learned about the naked number method in a workshop several years ago and that "For years I have been able to get the kids' attention with it" (interview, 3/89). Jim has come to feel that such representations provide helpful instructional tools in teaching mathematics and uses them for specific reasons. Examining the reasoning he uses in selecting these tools can be revealing of Jim's beliefs about mathematics and learning. In her work, Ball (1988) provides a normative framework of warrants for judging the products and process of pedagogical reasoning that underlie representing mathematics to students" (p. 308), arguing that the justifications used depend on the teacher's view of the goals teaching mathematics. The warrants she proposes derive from the domains of subject matter, learners, learning, and context. Similar to the prospective teachers Ball interviewed, Jim's pedagogical warrants for selecting the naked number representation were not based on the mathematical meaning underlying common denominators and adding fractions or the accessibility of the mathematics in the representation. Instead they reflect his beliefs about learning and mathematics--that the essence of the content is procedural, that learning mathematics should be made fun, and that the students' interests should be piqued. Consequently, Jim's naked numbers, are stripped of their conceptual clothes. Thus, while the representation might be compelling because kids find it engaging, it lacks conceptual mathematical power and represents the essence of the content as procedural. Like the previous candy bar example, which illustrates that $\frac{2}{4}$ is another way of referring to $\frac{1}{2}$, a representation that visually illustrates fractional parts and necessitates renaming them in equivalent forms in order to be added together would have a closer fit to the conceptual essence of the mathematics, making the meaning more accessible to the students.

Like the *Framework* authors, Jim would like his students to enjoy mathematics, yet he does not see its "inherent beauty and fascination" as being powerful enough on its own to bring enjoyment to his students. So he dresses it up for them. Ironically, in so doing, he removes it from its conceptual base, shedding from it the beauty and fascination that are "inherent." This is another example where Jim's intentions and concerns are not fundamentally different from the *Framework*, but his beliefs about learning and his learners lead him down a path that runs counter to teaching for understanding.

Jim's Perspective of the *Framework*: A Once-Removed Interpretation

Jim's interpretation of the *Framework* and teaching for understanding are shaped by his beliefs about the teaching and learning of mathematics. Acting as filters, his beliefs accept recognizable propositions from the *Framework*'s rhetoric, reshape others into more familiar forms and strain out completely foreign or incompatible pieces. Thus, he sees the *Framework* as demanding only minor changes in his practice which are reasonable and realistic goals. He does not perceive ambiguities or complexities involved in changing mathematics teaching. He sees himself as already making or working toward the necessary changes. He summarized his position on the *Framework* with finality, "I am pleased with the *Framework*. I think it's going to work" (interview, 12/88).

Jim interprets the *Framework* as being based on two central ideas: "critical thinking" and "application." He represents these ideas in his practice as pieces which can be smoothly integrated into his teaching as it exists, without causing radical changes in the nature of his teaching or beliefs. Both of these pieces of the *Framework* fit well with Jim's view that understanding mathematics means being able to apply it in real life situations. Jim described the changes he saw in his textbook as having to do with:

> [G]eneral thinking. It makes them think more than the other program we used last year . . . They have to get some basics down, but there is more to it than that. It is the business of thinking "Well, how can I use it?" or "Which part should I use of what I've learned?" . . . So they have to think more, rather than just go step by step and build a little bit on previous skills. (interview, 12/88)

Although Jim sees these changes as appropriate, making changes in his practice is something he does not do readily. "If you can change me, you can change anyone," he said while taking a retrospective look at his first year with a new textbook (interview, 3/89). Thus, the minor changes he has made in his practice as a result of the *Framework* are significant to him.

In talking about the effect the *Framework* has had on his teaching, Jim referred frequently to estimation, perhaps because it is something that he struggled with at the beginning of the year. Initially, he did not see the point of the pages devoted to estimating reasonable answers to specific problems before the procedure for arriving at the exact answer had been introduced. "I said, 'What's this estimating? . . . I want them to know how to add, subtract, multiply and divide" (interview,12/88). He enthusiastically admitted, however, that by working with the text, he realized the value of estimation. He has seen his students use it in their daily work and has recognized the extent to which he makes estimations throughout the day. In this way, changes in his text have caused Jim to make changes in his beliefs about the relevance of some of the mathematical content.

While willing to make minor changes in his practice such as this one, Jim does not see the *Framework* as suggesting radical changes. Take, for example, his interpretation of the idea of "mathematical power." The *Framework* proposes that developing students' mathematical power is the "central concern of mathematics education and . . . the context in which skills are developed" (California State Board of Education, 1985, p. 1). The pieces called for in the *Framework*--estimation, problem solving, reasoning--seem to be means to this end. Jim has added these pieces, but treats them as ends in themselves. Furthermore, the pieces Jim has added to his practice at this point are only several of those included in the *Framework* as part of mathematical power. He does not construe the *Framework* as calling for mathematical discourse, argument, conjecturing and exploration, although they are also pieces of the *Framework*. But these activities do not fit into his view of mathematics. So, Jim's beliefs have led him to a limited and modest interpretation of the *Framework*, resulting in limited and modest changes in his teaching.

In addition to being shaped by his beliefs, Jim's interpretation of the *Framework* is further complicated by the fact that he has learned about the *Framework* through the new textbook adopted by his district. The conclusions he makes about the *Framework* are solely dependent on the representation of the *Framework* in the textbook. The textbook, once removed from the *Framework*, is a critical mediator between Jim and the *Framework*. Jim is aware of the fact that the materials he sees are once removed. When I asked him if he was familiar with the *Framework* he said, "Indirectly I am in this way: The city schools provide all materials and they're right with the *Framework*, so if I know city schools material, I know the *Framework*" (interview, 12/88). It is not surprising, then, that Jim's interpretation of the *Framework* seems different than its authors'.

Blocks in the Filter:
The Context in which Jim Teaches

In these early stages of the *Framework*'s implementation, Jim's teaching has been modestly touched by the *Framework*. His beliefs have acted as filters, selectively accepting pieces that seem familiar and then interpreting these pieces in ways that fit with what he already knows about teaching and learning mathematics. There are also confounding elements to this filtering process. Like all teachers in the state of California, Jim is pulled in different directions by multiple, varying and often contradictory demands. The state of California, with its new policy initiatives, is just one context in which Jim teaches. His district and school also make demands and set standards and policies. Furthermore, the uniqueness of his particular school also puts forth its own agenda. These multiple concerns, often at cross purposes from one another, act as blocks in Jim's interpretive filter.

The Southdale District places many demands on its teachers. To address the problem of low standardized test scores in schools with large minority populations, the district has implemented a carefully paced, mastery of basic skills program that Jim must follow. He uses both the Holt text and the supplementary basic skills materials, adapted from the Holt text. The supplementary materials follow a mastery learning model: teach the skill, practice it, apply it, test for mastery, followed by re-teaching and retesting of the students who did not achieve the required level of mastery on the first test. Such an approach seems to be in opposition to the *Framework*'s focus on understanding and emphasizing the development of skills within meaningful contexts, rather than settling for mere computational mastery. Because these materials were adapted by Holt as supplementary to the text, Jim does not see this mastery approach as conflicting with the *Framework*'s philosophy.

Columbus, as a multi-lingual school in an underprivileged neighborhood also places its own demands on its teachers. Having an 80% student turn over, the school is unimaginably overcrowded with students speaking 18 different languages and from a multitude of different cultures. Like many teachers, Jim relies heavily on the CTBS as a yardstick of his students' progress. He trusts it as an accurate measure of their abilities and his own teaching proficiency. The fact that Jim has been able to get all of his students on or above grade level according to this standardized test is a significant accomplishment given that his population of students traditionally has low scores on such tests. Thus, Jim is not likely to accept changes that he construes as compromising his students' success on standardized tests. This potential block in the filter, together with a host of other related contextual concerns, represent what Jim faces as a teacher. And all of these factors shape how Jim thinks about teaching mathematics for understanding.

Conclusion

An experienced teacher, Jim has established beliefs about learning and teaching mathematics which make sense to him and fit with widely-accepted views about mathematics. Committed to his students, he relies on the knowledge he as accrued and methods he has developed over the years to engage his students and help them find success in school. By many standards, Jim is an excellent and effective teacher: His students are fond of him, they work hard and do well on standardized tests. Yet, the *Framework* suggests another set of standards. The degree to which Jim's teaching meets these standards is a function of how they are interpreted. As we have seen, Jim's teaching can be viewed as simultaneously in line with and at odds with the *Framework* depending on the set of beliefs shaping the interpretation.

How would the *Framework*'s authors assess Jim's teaching? Although he has made changes in his teaching that have been inspired by the *Framework*, his changes are limited. There are many places where his teaching falls short, where Jim's teaching actually runs counter to the *Framework*'s vision. For example, he sees no value in the partner and small group activities, and skips right over these suggestions in his textbook. *Framework* proponents might also question Jim's lecture style for teaching math or his lack of thorough attention to conceptual development. These shortfalls, however, are unsurprising considering Jim's knowledge of the *Framework*, the complex contextual issues he faces, and the lessons he has learned through years of experience. How, then, might state policy makers and advocates of changes in mathematics teaching help Jim make more substantial changes in his teaching?

One possible argument is that Jim is thoroughly entrenched in a view of mathematics teaching and learning that is unmovable and that he is not likely to change. While this may be part of the story, he is not merely a case of a teacher who is set in his ways; for we have seen that Jim is willing to make certain changes in his practice. It might also be argued that Jim must know more about the *Framework*, its underlying philosophy and goals. He needs to attend teacher inservices that will present him with a perspective on the *Framework* that is less second hand and support in using it. Indeed, Jim's thinking may certainly be informed by more direct exposure to the policy document. Yet, even clearer, more encompassing messages of the *Framework*'s vision, together with greater support, are likely to have a limited and unpredictable effect on Jim's interpretation of the policy. How he understands what is being asked of him is necessarily shaped by his knowledge and beliefs about mathematics, teaching and learning. Thus, the likelihood that Jim's practice will change is dependent on the likelihood that his beliefs will change.

The *Framework* is a reform document that challenges common assumptions about mathematics and professes that learning involves a process much deeper than learners being told what to know. Yet the reformers are forced to take a "telling" approach to communicating their vision to teachers, whose experiences with mathematics teaching and learning lay far outside that being proposed. Yet, what Jim hears is passed through a complex filter that is constructed by his various beliefs. So the difficulty is a catch-22: In order to be convinced of the *Framework*'s relevance of teaching and learning, Jim will need to change how he thinks about teaching, learning and what it means to know mathematics. He must have experiences which help confront his beliefs and consider alternatives. But at the same time, the sense Jim makes of these experiences and the meaning he brings to them are determined, less by the intent of the reformers, and more by his current beliefs.

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Are Changes in Views about Mathematics Teaching Sufficient?

The Case of Karen Hill

Richard S. Prawat

Overview

Karen Hill, the subject of this case study, represents a modest success for the *California Mathematics Framework* at this early point in the implementation process. Judging from our extensive interviews, Karen's ideas about mathematics' teaching have undergone significant change as a result of using the new, more conceptually-oriented math curriculum adopted by her district. Although this change appears to be serendipitous, attributable more to Karen's attentiveness to what her students are learning than to any systematic intervention effort on the part of the district or state, it nevertheless represents an important accomplishment for this teacher. Given support and encouragement at the school and district level, it might eventually lead to important changes in her classroom practice. This, at least, is one reading of the data presented in this case study.

If one focuses on what Karen actually *does* in the classroom, however, the situation is more confusing. Important changes in Karen's views about teaching seem not to have influenced her classroom practice. In December, and again in the spring, Karen's teaching of mathematics was very traditional. Although she did show a greater willingness to experiment with some of the innovations called for in the *Mathematics Framework* during the second round of observations, her performance fell far short of the sweeping changes called for in the *Framework* document. At best, Karen's teaching at Time Two represented a hybrid of the old and new. This, then, is the dilemma dealt with in this case study: There is important change in Karen's views about mathematics teaching over the course of the year, but this does not appear to be reflected in her classroom practice.

Context

By any traditional measure, Johnson school, which is where Karen teaches, is one of the most successful schools in the Southdale district. Students, who are primarily from upper middle class families, perform extremely well on the California Test of Basic Skills (CTBS), typically scoring in the seventy-fifth percentile in reading and in the eighty-fifth percentile in math. This probably reflects the socioeconomic status (SES) and ethnic makeup of the school as much as it does the quality of teaching. The principal estimates that the student population, which averages close to 1000, is 80 percent majority and 20 minority in ethnicity.

Johnson opened in September, 1976. It started with approximately 300 students and 9 teachers, and has grown to 1000 students and 35 teachers. The current principal, who enjoys an excellent reputation among district administrators, has occupied her position for five years. When she first came to the school, she indicated, math scores on the CTBS were considerably lower than they are at present (students scored below the seventieth percentile). One of the principal's first decisions was to change the school math program. At that time, the district was going through an adoption: "They were using a program called Southdale Mathematics; and it was either that or they recommended two other ones," the principal recalled. "I let the teachers look at the three programs that they were offering. They decided to go with the Southdale Mathematics Program." Math became a high priority for the teachers at the school. The principal added, "We did a lot of in-services in math; teachers went to workshops . . . The whole school zeroed in on math." This was not without its problems: "Our scores have gone up so high in math that our reading scores now don't look good--because we worked so hard with math."

Test scores are very important in the district as a whole, and in Johnson school in particular. Much of its enviable reputation rests on test score performance. One only has to scan a wall in the principal's office to realize what is at stake in this regard. There is a plaque is from the state of California, naming Johnson as one of the California Distinguished Schools. There are also framed letters from the governor and two state assemblymen congratulating the school for its accomplishments.

The principal and staff continue to carefully monitor test results; this past year, reading and spelling at the third and fifth grade levels have been singled out because, as the principal explains, "test scores were not what we would like them to be." Although the principal appears to strongly endorse the new *Mathematics Framework*, believing that it addresses issues like problem solving and critical thinking that typically get short changed, she is cautious about the Holt (Fennell, Rays, Rays, & Webb, 1988) math series, taking a wait-and-see attitude: "I think it will help with creativity in math. I do not like to say whether it's going to be the series or not until I see how test scores come out and how the students are developing. I think it's just too early to tell."

The principal felt that teachers were happy with the new series. She also thought, however, that teachers were concerned that not enough stress was being placed on computation: "There's a lot of problem solving and critical thinking skills, but they're finding that they are having to do some work with the computation so that doesn't go by the wayside--speed tests, and memorization of facts." This emphasis on computation, in part, reflects the importance the principal places on test score performance. As the principal explains, "I have a concern in that we had something that was working really, really well, and that we might get something that is all right, but that our scores might not show the growth that we have been experiencing so far." As will become evident, Karen Hill has been strongly influenced by this pressure to maintain high scores on the math portion of the CTBS.

The Textbook as *Framework*: Views at Time One

Karen is a teacher with a good deal of prior instructional experience. Although at the time of the study, she was teaching a combination fourth/fifth grade for the first time, she had taught third grade for three years at Johnson school, and prior to that had spent 18 years teaching second grade at another school within the district. Not surprisingly, given this experience, she conveys the impression of someone who has a great deal of confidence in her ability. She does not mince words; on the contrary, she is rather outspoken and assertive. As she was quick to point out, she has been around--she has seen things come and go in education. This does not make her an easy target for reform. For one, she is a bit cynical about the whole process of educational innovation. This came through in her views about Southdale's recent curriculum adoption in mathematics: "Teachers don't count" in this process, according to Karen. In fact, she said, she had heard from other teachers that the adopted series was rated well below a number of the other series that were examined. Her school district errs on the side of going with "the new and fancy." She characterized their thinking in this way: They say, "Let's look around and see what's new. Oh, we like the idea of high level thinking skills. You know, that sounds real fancy. Let's go find out about that and then go with it without really exploring the whole thing."

Karen's views about the new Holt curriculum are important because, for her, as for many of the teachers in Southdale, this series *is* the *Framework*. Unlike the series, any information she has about the *Framework* is secondhand, based on comments made by her husband; he is a high school mathematics teacher who, Karen said, has kept abreast of what is happening at the state level. Karen characterized the *Framework* as advocating "some radical changes" in mathematics teaching. "They are going more for the high level thinking skills," she said, "to keep up with

Japan. The idea is to have more thinking rather than just doing." She indicated that she saw no problem with this in principle; she applauded the effort to bring some coherence to the K-12 curriculum. "I think they're trying to give some direction, which I think we need," she stated emphatically. She saw the *Framework* as an effort to provide some continuity in the curriculum. She added, "I think for an awful lot of years, it's just kind of been, 'Well, here's your book. Now, just go teach it.' And there's no continuity; and I think we need that. I think we need it across the board, whether it's K through high school or whether it's California or whatever state it is." The problem is that the state has gone overboard: "I just think they got a little carried away with some of their objectives. They're starting a little too early with a little too much." What motivates this concern, according to Karen, is that the schools are going to end up with children who "can't do any math."

In this context, Karen talked about a series of conversations she and her husband have had with her best friend's spouse, who is a mathematics professor at a nearby university. This individual questions the emphasis placed on computation in the early grades. She always sides with her husband in these discussions, she said. It is his firm belief, as a high school math teacher, that *his* problems can be attributed to student's failure to master the basics at the elementary and middle school level. Students need a basic foundation "to step off from." Karen shares her husband's perspective, she said.

Her main concern with the *Framework*, and with the new curriculum series adopted by Southdale, is that it is not very practical. To expect students to understand procedures in mathematics when they have not committed them to memory is ridiculous, in her view. This kind of approach, she said, is reminiscent of the new math twenty years ago. As a beginning teacher, she resisted teaching the new math. Instead, she focused on math facts. This approach was successful. By the end of the year, students were ready to understand the reasons for some of what they had learned: ' We picked up on some of these reasons for why this was this and this was that. And they loved it."

According to Karen, the pendulum in mathematics has recently swung too far in the opposite direction. "Too much material is being presented too quickly," in her view. The average student, in particular, has trouble handling the complicated material. She elaborated on this idea,

> I think that is the problem. I have no problem with what they're doing and I have no argument with the idea of introducing concepts. I'm all for that. I'm for a spiral thing. You kind of start and every year you introduce a little more and little more. But I think in here (pointing at her textbook)--I get the feeling they're supposed to have mastered this stuff. No, I don't agree with that.

Karen was concerned about the sheer amount of material that she was being asked to cover in the new series, and also about the nature of the material. She felt there was too much emphasis placed on what she termed "lofty ideas," not enough on mathematical rules and procedures. These problems have created a dilemma for her in her teaching. She felt that, because there was too much material, some of it had to be eliminated, but she was unsure about what. Her major concern was, that if she eliminated something, it might create problems for the students later on: "I don't want to eliminate anything with the idea that it's going to come back and haunt these kids." She thus worries about the consequences of not covering the prescribed material. She asks herself, she said, "Are these kids, when they go to the next grade, going to be lacking?"

This view--that there is more material to cover than one can do justice to--has increased Karen's frustration with the nature of the material contained in the new curriculum series. In this regard, she reflects the views of some of her colleagues. Thus, Karen had talked with teachers at another school who had piloted the new series: "The first comment out of everyone's mouth that I talked to--and I know of two teachers who piloted it--was that there is too much material in too little time . . . They said, 'You'll be lucky if you get half way through the book here.'" More recently, Karen had visited a teacher in a neighboring school. Although the purpose of the visit was to observe a reading lesson, Karen said she ended up talking about mathematics with this individual. Her colleague indicated that she had to push her students to get through the material: "She didn't feel that they had as much concrete math skills as they should have by the end. She said there was a lot of all this high thinking stuff and she tried to do it."

Karen strongly endorsed her colleague's skeptical view: "It sounded like fun when I first read it. At the end of every chapter, they've got this group project thing, and I really wanted to do it." However, she added, "There just isn't time, there just isn't, unless you're going to devote your whole day to math; and at this school, you don't do that." At this point in the conversation, Karen introduced another factor that has influenced her views about the new curriculum: This is the strong pressure to maintain high test scores, especially in math and reading, felt by teachers at the Johnson school: "We're supposed to be tip top at everything, and we're trying to make everything tip-top; and I really, honestly, don't think you can do that."

Karen elaborated on the nature of this pressure in a later interview. Three or four faculty meetings are devoted to test score performance every year, according to Karen. At the first one or two meetings, results from the previous year's CTBS are discussed with someone from the district office: "They had this lady come in twice from the district, and I guess she is the one who compiles all of this stuff. It was a good hour and a half and two of our meetings. She basically put on the board diagrams and bar graphs showing where we rated in the city on everything. We would look at what we did last year and say that our fifth graders needed to come in this area or are doing fine in this area." Asked specifically about the school's performance on the standardized test in mathematics, Karen said, "Actually, we are doing very well in math." She added a cautionary note, however, "But we are all scared, because you know what we think is going to happen." Teachers anticipated a significant drop in students' performance in mathematics, she indicated, because the new curriculum did not place enough stress on computational skills.

Karen had lots of reasons for feeling anxious about the new curriculum at the time of the first interview. Further, it is obvious from this discussion that Karen's views about the *California Mathematics Framework*--and its instantiation as the Holt mathematics series--were more negative than positive. This perspective was influenced by a number of specific beliefs, as well as a number of practical considerations. Foremost among the latter was Karen's concern that the new curriculum tried to cover too much territory. Its emphasis on problem solving was alright in principle. It provided students with a "reason" for learning the necessary facts and procedures in mathematics. Unfortunately, she felt, it went too far in that direction. Students were not being given sufficient opportunity to master the basics. The basics are important not only because that is what is emphasized on tests; the basics provide the necessary tools for problem solving. Karen's views about the new curriculum are thus strongly influenced by her views about the nature of mathematics.

Karen's Initial Views about Mathematics: What It Is, and How It Is Taught

In the first interview, in early December, Karen was quick to point out that mathematics is *not* her strong suit. She felt that she could be more creative in other areas of the curriculum. Her strength, she indicated, is in the language arts domain. As for her math expertise, Karen admits, "I was not the greatest math student in the world." This, coupled with the fact that she was teaching a new fourth/fifth grade math curriculum, made her a little more hesitant in her approach than she might be otherwise. She pointed out, for example, that she had never taught fractions before: "So right there," she said, "we are starting with something where I am going to have to be stretching back and remembering."

Karen's views about math appear to be more the norm then the exception among elementary school teachers. "I enjoy math," Karen said, "but I don't get the thrill out of it that my husband does." At the time of the initial interview, Karen expressed very traditional views of mathematics: Math learning proceeds hierarchically, with children first needing to master the "basics," certain math facts and procedures learned in rote fashion, before getting into problem solving and other application sorts of activities. This came through in her response to one of the interview items, where she was asked to respond to the following quotation taken from the *Framework*:

> Teaching for understanding emphasizes the relationships among mathematical skills and concepts and leads students to approach mathematics with a commonsense attitude, understanding not only how but also why skills are applied. Mathematical rules, formulas, and procedures are not powerful tools in isolation, and students who are taught them out of any context are burdened by a growing list of separate items that have narrow application. Students who are taught to understand the structure and logic of mathematics have more flexibility and are able to recall, adapt, or even recreate rules because they see the larger pattern. Finally, these students can apply rules, formulas, and procedures to solve problems, a major goal of this framework...Teaching for understanding does not mean that students should not learn mathematical rules and procedures. It does mean that students learn and practice these rules and procedures in contexts that make the range of usefulness apparent. (California State Department of Education, 1985, p. 12-13)

Karen responded favorably to this statement. She chose to emphasize, however, the second from the last sentence. She said,

> I like this statement. Teaching for understanding does not mean that students should not learn mathematical rules and procedures. But what I'm afraid happens is the *not* gets left out, and in our great quest for teaching this understanding--whether it's teachers themselves or the books that lead us to think that you can teach things without any skills.

Granted, she added, rules and procedures are not powerful tools in isolation. They must be learned in context: "It's the idea that you just learn to spell a spelling word, and you forget it next week." While Karen agreed with this notion, she also stressed the idea that mastering rules and procedures is a prerequisite to application: "I think we have to be careful that we don't lose sight of the fact that they do need to learn it first . . . It really bothers me. Yes. Yes. We're going to solve all these problems without any tools, just by thinking about them. Well, that's totally absurd, if you think about that statement."

Karen felt that use of the word "context" was problematic in the *Framework* statement. "I think that you have to be careful in defining what the word context means," she added. "I use context all the time." She explained that it is important for students to have reasons for learning rules and procedures: "Most of my lessons, I start out with a statement of why we're doing what we're doing." This, according to Karen, is one way to provide context. Another way is to present real-world situations that illustrate how a particular rule or procedure can be used. In this sense, Karen felt that her teaching was consistent with what the *Framework* espouses.

Karen's hierarchical view of mathematics appears to drive her views about how to teach mathematics. At the time of the first interview, Karen was a strong advocate of the "demonstrate-test-apply" model of instruction. According to this approach, students are first shown the mathematical rule or procedure. At this point, an attempt is made to elicit their feedback (i.e., comments about what they do not understand). Students then practice the skill. Finally, when students demonstrate that they can carry out the procedure, they are taught to apply it. Karen summarized her approach in this way: "I think the teacher should present them (i.e., rules and procedures) in total as to what it is you are doing. This is the way you do this, and you show them. You show them a million times and among those times, you get their feedback. You have them do it sometimes. You do some with them. You have them practice it sometimes and then you come back to it again and you come back to it again." Once a skill has been mastered in this way, Karen indicated, it is important to "spiral" back on it frequently so that students do not forget it: "Learning is very stressful, and when you're constantly learning something new, you're not going to remember anything else. You need to go back and remember it again."

As indicated above, Karen's major complaint about the new mathematics curriculum at the time of the first interview centered on its (for her) obvious departure from the demonstrate-test-apply pattern. There was, she thought, too much emphasis on problem solving; students are simply being thrown into it without being given enough time master the basics. She expressed concern that, by downplaying the importance of math facts, "we'll be a bunch of mindless idiots." According to Karen, the one feeds into the other: "When you start getting into some of this higher level math and stuff, the students who can bring up good math skills like adding, subtracting, multiplying, and dividing, and feel comfortable with that, they do better."

Karen indicated that she was in favor of teaching problem solving, although "it should be done slowly and surely." She was especially concerned with the number of complex, two-step problems that students were being asked to solve: "I think you need to do a lot more one-step problems." Karen also felt that too much time was being devoted to estimation. She had trouble seeing the merit in this procedure; nor was she alone, she said. Her students, and even some of their parents, raised questions about this "anything goes" approach: "It's kind of like, 'Well, it doesn't really matter. We'll just come up with this figure and it doesn't matter if it's too high or too low.'"

Karen elaborated on her concerns about estimation: Talking about her first impression of the textbook, she said, "There was just an extraordinary amount of estimation . . . and I was almost left to feel that, Oh, well. Whether you get the right answer or the wrong answer doesn't really matter. We'll just try to look at this and guess.'" She indicated that, while she taught students to estimate, she also wanted them to actually compute answers to the problems so they could compare the two approaches. This would give them a healthy respect for what might go wrong in the process. She elaborated: "I think it's good for them to see a difference because I've had it happen to me. I've had 20 bucks in my hand; I've walked into a store and I've looked around and picked a few things out, made my estimation, got up there and it was $25."

According to Karen, her students were also having problems viewing estimation as a legitimate part of the math curriculum. "They fought me through two chapters on those estimation things," she said. "They didn't want to do it . . . I think they were nervous--they thought they were going to have a wrong answer." She added,

> They just literally would not round the numbers to make the estimation. It was the funniest thing . . . So I went home and kind of read through all the helpful hints and all that, and I was kind doing what it said to do. So I went back the next day [and] I said, "What's the problem here?" I just asked them. "What's the big deal here with this?" And they really couldn't say. They just kept saying they didn't like those problems and I finally said, "Well. Is it because there's really no exact answer?" And, yeah, they kind of bought that but they never really came out with it. And then, I had parents calling me, too...In fact, the one father who volunteered to come in, I think that's why he started wanting to come in. He wanted to see what was going on in here.

In the December interviews, Karen also expressed some reservations about the use of manipulatives. In fact, she confided, she had not yet used *any* of the manipulatives. "Those nice boxes or stuff," she said, "are still sitting there." She worried about how much time it would take to pass the material out and to collect it when the lesson was done: "I had to really, seriously say to myself, 'Now, how much did we really get out of this lesson because every minute of the time I spend with these kids in here is valuable to me. There just isn't any playtime left.'" She felt that the use of manipulatives was not at the heart of the program; rather, as she put it, "It is the icing on the cake."

One would have a hard time mapping Karen's views about problem solving, estimation, and the use of manipulatives onto the argument laid out in the *Mathematics Framework* document; Karen, of course, had not studied that material. Had she done so, it is likely that she would have interpreted much of what it had to say about these important aspects of mathematics in terms of her own traditional views of teaching and learning. According to Karen, problem solving is an "add-on" in mathematics--it comes at the end of the learning sequence and serves as a test to see if students can apply the mathematical tools they have acquired. Viewed in this way, it makes sense for Karen to insist on mastery of the "basics" first. Estimation and the use of manipulatives are clearly viewed as tangential to the main thrust in mathematics. They may represent interesting activities or things to do in this subject, but not at the expense of more important content. Although not explicit in this regard, Karen appears to seriously question an important assumption underlying the teaching of estimation--the notion that it helps develop a sense of number in students. Instead she equates estimation with "rounding," a common misconception mentioned in the *Framework*. Manipulatives are an even more suspect aspect of the new curriculum, according to Karen. Karen sees little use for manipulatives, or any other type of concrete material for that matter. She terms the use of manipulatives "playtime," apparently placing it in the same category as other "hands on" activity which serves a primarily motivational or interest arousing function.

In the vision of mathematics put forth in the *Framework* document, these three aspects of the curriculum--problem solving, estimation, and the use of concrete material are viewed as all part of a piece. Problems--or more precisely, problem situations--serve several purposes in mathematics, only one of which relates to the application function stressed by Karen. Problem situations also provide a meaningful context for the learning of mathematical skills and concepts. They play an important affective or motivational role, arousing student interest and helping to build self-confidence; according to the *Framework*, problem solving teaches students to overcome obstacles and thus allows them to derive "the reward of arriving at solutions through their own efforts" (California State Department of Education, 1985, p. 13). Last, but not least, problem solving is a way to foster higher order thinking skills:

> In working with more complex situations, students will formulate and model problems, screen relevant from irrelevant information, organize information, make conjectures, and test their validity, analyze patterns and relationships, use inductive or deductive processes, identify or evaluate alternative mathematical approaches, find and test solutions, and interpret results. (California State Department of Education, 1985, p. 3).

According to the *Framework*, estimation and the use of concrete material go hand in glove with the problem solving approach. Estimation plays an important role in computation; for this reason, the *Framework* authors insist, it should not be taught as a separate lesson but as a step in all arithmetic procedures. In addition, estimation plays an integral role in problem solving: "Independent of any attempt to estimate or follow the calculation involved, the student should be taught always to ask, 'Is my answer reasonable? Could it possibly be a solution? Within what range of numbers must my answer lie?'" (California State Department of Education, 1985, p. 4). The use of concrete material, which includes manipulatives and other forms of representation, helps students understand new concepts and deal with difficult problems. In this latter role, drawing pictures and making models are viewed as legitimate steps in the problem solving process.

The analysis offered above differs considerably from the perspective one discerns in listening to Karen talk about the new mathematics curriculum. This is not to say, however, that Karen's views about mathematics--and what it means to teach and learn it--are any less consistent than those offered in the *Framework*. As I will demonstrate in the next section, Karen's didactic approach to instruction in mathematics fits nicely with her hierarchical, learn-the-basics-first view of the discipline. Both, in turn, seem consistent with what might be termed a "multiple exposure," incremental view of learning.

Karen's Practice at Time One

As indicated above, Karen's approach to teaching definitely falls in the traditional, teacher-directed category. In her classroom, there is no doubt about who is in charge. While there is a good deal of verbal give-and-take between her and her students, she is firmly in control at all times. In mathematics, in particular, Karen's approach is very didactic. In this section, I will illustrate what this means in the context of two, fairly typical lessons. In both these lessons, which occurred on consecutive days, Karen was helping students prepare for an important unit test. The classroom environment, although pleasant, was somewhat cramped. Karen taught in an annex and space was at a premium. Perhaps for this reason, students were arrayed in long rows facing toward the front of the room. As the class began, Karen assumed her customary position at the front of the room. To her right was a small desk cluttered with teaching paraphernalia, behind her was a well-used blackboard, on her left, an enormous overhead projector had just been wheeled into action. Several problems had been written on the screen of this projector; they were to be used to prompt students to recall and rehearse the steps involved in the multiplication of whole numbers.

Children had no difficulty in walking Karen through the multiplication procedure. "First you multiply three times eight, and then you put the two in front of the three," a youngster explained in a sing-song fashion. "Then you multiply nine times eight and add the two to seventy-two." The teacher quickly moved through two more problems of a similar ilk before pausing by one that was described as "a little trickier"--100 x 500. The teacher asked if anyone remembered the "shortcut version" for solving this kind of problem. One child, Jennifer, shot her hand into the air and quickly got to the heart of the matter, saying that one "just dropped the zeroes." Before she could develop this thought, another student chimed in, arguing that all they had to do was add a zero and multiply by five. The teacher corrected the second respondent, saying, "Now, in just estimating, we're going to end up with 50. Does that sound right?" This was the clue they were looking for. "Just count the zeroes," a third student interjected at this point, "and add them at the end." The teacher nodded in agreement--then made an effort to clear up an earlier confusion: "Now we come back to what Jennifer said. Is that the reason you said to drop the zeroes?" The child shook her head to signify yes; apparently she hadn't had time to finish her statement.

This heavy emphasis on procedural rules was characteristic of Karen's teaching at mid-year. Except for a passing reference to estimation, the lessons observed at Time One probably differed little from hundreds of other lessons taught by the same teacher. The students were comfortable with this question-answer format. The questions seemed straightforward enough and most of the answers, at least those that were accepted, appeared well- rehearsed. There were few surprises associated with this approach. Like the captain of a cruise ship, Karen seemed determined to minimize the negative effects of what could become a stormy passage. If this meant that she had to occasionally go out of her way--lingering longer on certain aspects of the mathematics curriculum until everyone had mastered the content, so be it. Karen seemed to expect such difficulties, and appeared willing to persist until they were overcome. She clearly was on the students' side when it came to mathematics.

In this context, Karen's concern about estimation and problem solving seemed entirely justified. She barely had time to cover the "basics." Expecting her to go beyond this material was unfair to her and the students. She was willing to do what the textbook required, but not without some resistance. This was obvious during the second observation, on the following day, as students continued to review problems for the upcoming unit test. One of the story problems on the practice sheet read as follows: "On Tuesday 207 people paid $2.50 each to take a tour through the old grist mill built in 1837. How much money was collected on Tuesday?" Karen knew that her students would be asked to estimate answers to problems such as this on the unit test. "I want you to do it both ways," she explained, "to estimate and to compute the actual number."

Karen asked a child to explain why it was important to provide both answers when they were asked to estimate. "Because if you do, and estimate right, and then you do the problem, you can see if the answer is reasonable." Karen accepted this somewhat circular argument, tipping her hand by launching into a story illustrative of the dangers associated with unreliable estimation. "Suppose you're in K-Mart," she said, "and you want to buy some things. What if you estimate and figure you are safe?" A youngster, having heard a version of this story before, completed the thought, "...And you don't have enough money." Karen nodded in agreement, adding that this situation could be very embarrassing. As if to drive home this last point, she told about a recent incident involving her and her husband. They had gone to a restaurant to eat; when it came time to settle accounts, her husband realized that he had forgotten his wallet. Karen explained that she had been "held hostage" by the manager until her husband was able to retrieve the money. Although not directly related to the problems of estimation, this story did reinforce the notion that bad things can happen when one relies too much on unreliable data. This vignette, of course, is entirely consistent with what Karen had to say about estimation at the time of the first interview, in early winter. These views, however, were not cast in concrete. Karen's thinking about the teaching of mathematics underwent considerable change from mid-year to spring.

Karen's Views about Mathematics Teaching at Time Two

As indicated, Karen's views about the new curriculum had mellowed a bit at the time of the second interview. She now saw merit in some of the things that she had been quite critical of only three months before. Regarding estimation, for instance, she admitted that, although she "was kind of down on estimation at first," she now saw advantages in teaching it. For one, students found it a useful tool; she saw evidence of students using it to decide if their answers made sense. This is illustrated in the following exchange,

Karen: They (speaking of the textbook writers) are expecting them to make some judgments and to decide. Like this one (reads from the book), "Sometimes when you make plans you have to provide estimated amounts." I was kind of down on estimation at first. There is something valuable to it, because we estimate all the time. Rather than going five years and finally figuring out how to estimate, why don't I just teach them how to estimate? So that is what I did.

Interviewer: Are they using it?

Karen: Yes, they do.

Interviewer: For what sorts of things?

Karen: When they look at a problem and they know they don't have the right answer. So they come up, and I say, "Let's look at this." There is just silence. They say, "Oh, that can't be."

Interviewer: Because they are doing an estimation?

Karen: Yes. They are quick looking at it and seeing that it is just totally out of the ballpark. They have three digits and they are dividing by two digits. How could they have a six digit answer. Although that is not the kind of estimation that is maybe right here (points to the book), but that is all part of looking at things and making some kind of a reasonable judgment about what you are doing. I see some kids who I don't think were doing very well last year, and they are doing quite well this year. I think a lot of it is they are kind of using it, very slowly. I keep harping on it and I keep making them look at it. They don't do it totally on their own. But if I say, "Let's look at this," they just immediately start thinking of those things.

Interviewer: That wouldn't have happened with the previous program?

Karen: No. It wouldn't have happened at all. It wouldn't even have come up.

This represented a noticeable change in Karen's earlier views about the value of teaching estimation. Where before she questioned its value, feeling that "guessing" (as she termed it) fostered the wrong sort of attitude in students, she now saw it in a different light. At Time Two, estimation was viewed as a useful mechanism for promoting sense-making in students.

Karen had revised her views about the curriculum in other important ways as well. One of the most significant changes relates to her views about the use of manipulatives. At Time One, Karen appeared to put manipulatives in the "extra activity" category: a nice things to do if one has the time. The use of manipulatives was not seen as an essential part of the mathematics curriculum. By Time Two, however, her views had changed significantly. In the second interview, Karen talked about how she now differed from her husband, a high school math teacher, on the value of concrete experience in learning mathematics. "He is not a manipulative person," she said. "I am more manipulative than he is." She then added,

He is saying, "Just teach them five and five is ten, and then go back and look at it and discuss why this and that and the other." We don't totally agree on that. I think there needs to be more manipulatives and more concrete work.

In this exchange, the interviewer then asked, "So you are less inclined to follow his advice to leave out manipulatives?" Karen replied,

Right. I kind of changed on that. Particularly for this book, because of some of the thinking skills that they are expecting them to have. I think before you can get to the abstract, you have to have more concrete. If all I wanted them to do was learn their math facts and I gave them a test every week, fine.

Later in the interview, this issue was revisited. The context was this: Karen had been shown a story problem, along with several hypothetical solutions that students might generate. One of the solutions involved constructing a pictorial representation of the problem. Karen thought that such an approach was "fantastic," although she quickly added that she would encourage the student to also write out the answer to the problem. When asked why, she responded, "I have taught my kids that they need to write answers with labels, and I think that would probably be my reasoning in that. You need to label what you are doing. The picture is fine if it helps you see it, but then I think you need to label what you have done." In this comment, Karen acknowledges the role of concrete representations in fostering student understanding. It is apparent, however, that Karen does not *equate* one with the other. Karen, like many of the teachers we interviewed, views representation primarily as a pedagogical tool--a way to promote understanding in students. Unlike some mathematicians, she does not consider it synonymous with understanding.

Karen was given another opportunity to talk about representation toward the end of the second interview. Her comments at this point in the interview further support the contention that she has altered her earlier views. The question again was hypothetical: "Suppose a child in your class could get the right answer to this: 3/4 - 2/3 = ____. But the child could not show it with fraction materials or draw a picture for it. How would you feel about that?" Karen immediately distinguished between solving the problem "manually" and "having the concept." There is more to mathematical understanding then being able to correctly recall and apply procedures. Although this is highly inferential, it appears that Karen is willing to entertain the possibility that concrete representations constitute useful, although alternative, modes of understanding. Thus, Karen argued, there is nothing wrong, "at the very beginning," with students being able only to do the computation. She added,

> Karen: You see, that is what my husband is arguing. He thinks that is okay.
>
> Interviewer: That it is fine if they can do it at the very beginning?
>
> Karen: I do not think there is anything wrong with just knowing how to do it. But eventually, somewhere down the line, assuming they do not have some learning problem or something, they should be able to draw it so that you know they are really understanding what they are doing.
>
> Interviewer: How far down the line? Let's say this is what you are teaching here and the kids can do the calculations.
>
> Karen: No, I would not expect them to do it yet. Maybe if you work on it. Maybe five or six lessons or so.
>
> Interviewer: You are not talking about two years down the line?
>
> Karen: Oh, no, no. I do not mean anything like that.
>
> Interviewer: Why not?
>
> Karen: Jeepers. Although that is the way we learned. Maybe they may never get back to it again--if nothing else, they may literally never get back to it again. I think that as it gets higher in school, there is less of that, and I do not think they will ever be given the opportunity at that point. If they do not do it now, I do not think they will ever do it.
>
> Interviewer: What is the problem if they can do the calculation?
>
> Karen: I guess there is really nothing wrong with just being able to do the calculation. But if they are ever going to have to go and do anything more in math, I would hope that they would have a little better understanding of what they are doing. That is all--kind of see in their mind what they are trying to do.

The notion that students can know or understand mathematics in different ways is a novel one for Karen. It was not evident at all in her comments at Time One. Although she recognized that the new math curriculum calls for more discourse in the classroom, Karen had reservations about this approach. Thus, in one of the initial post-observation interviews, Karen was asked if she frequently called on students to work through problems on the board. "I only do it when it's review," she said. When presenting something new, she indicated, she prefers to demonstrate how to do it herself. "I don't like to embarrass them." Karen went on to say,

> It's focusing an awful lot on the negative of this child's inaccurate answer, so he's embarrassed . . . So then you get kind of this negative thing, and after a while they don't want to raise their hands because they're afraid they're going to say something wrong.

A second problem with having students discuss their solution strategies is that it frequently misleads other students: "If somebody's giving a wrong answer or they're leading somebody through the problem wrong, I think all you've done is confuse everybody."

Even in the first set of interviews, however, there is some evidence that Karen had doubts about her position. This was the interview context: Karen had been asked to respond to several statements taken from the *Framework* document that contrasted "teaching for understanding" and "teaching rules and procedures." The former, the document indicated, is difficult to test, while the latter is easy to test. Karen agreed with this comparison, but said that it was possible to test understanding. When asked to elaborate, she cited as an example her attempts to assess comprehension in reading. The following exchange ensued,

> Interviewer: What sorts of questions might you ask in reading?
>
> Karen: Very general kinds of questions, and then I will ask them to give me three details to support that statement. And if they can come up with the kinds of details that I'm looking for--and it's a very open ended thing--then that tells me that they're understanding.
>
> Interviewer: There's no right or wrong necessarily.
>
> Karen: Well, there could be a wrong in the sense that the child is totally off base...
>
> Interviewer: Could you do that in mathematics--ask those sorts of questions?
>
> Karen. I think so.
>
> Interviewer: Really? How?
>
> Karen: Well, the story problems do that. . . . You present a problem and you ask a person to decide, first of all, how to solve the problem. The person has to do the steps to solve the problem. I think that's testing for understanding . . . Some kids are quite innovative. We had a problem at first, when they were just learning to do the two-step problems. Some of them were doing them backwards, I guess you could say. But they were coming up the right answer. I was just dumbfounded at first because I thought, "How could they do that." It just didn't seem right because in my mind in had to be in this order. So I went home with a couple of them and I fiddled around with them and I finally figured out how they did it--and it was alright.

Karen went on to explain the strategy used by the students. She also described her way of handling this when students were correcting one another's work: She gave one point if the answer was right, one point if the student used the correct procedure. Some students felt that this system was unfair, arguing that their approach was just as reasonable as the one considered correct. Karen was then asked, "Did the rest of the students find this sort of discussion intriguing?" She replied,

> Yes, actually. You'd be surprised. They're all quite interested in stuff like that because, they're going, "I did it the way you said." You know, they're kind of looking at theirs and kind of comparing the other.

That, however, was as close as Karen came during the first interview to recognizing the legitimacy of student discourse during mathematics. By the time of the second interview, Karen had altered her views on this issue. This is a potentially important change because of the prominence assigned to the discourse process in the *Mathematics Framework* document.

Several sections in the *Framework* highlight the role of student discourse in promoting understanding. For example, the following rationale is provided in the section on cooperative learning groups:

> Students have more chances to speak in a small group than in a class discussion; and in that setting some students are more comfortable speculating, questioning, and explaining concepts in order to clarify their thinking. (California State Department of Education, 1985, p. 17)

In a similar fashion, the importance of verbal give and take is stressed in the section on problem solving: When students explain their thinking about different approaches and results, they become comfortable with the notion that some problems have multiple solutions while others may be unsolvable, a much more realistic view than the "one-best-solution" approach typically reinforced in the mathematics classroom.

Karen's growing appreciation for the role of student discourse during mathematics does not appear to be inspired by arguments such as these. Rather, the impetus for change seems lie in her daily interactions with students as, together, they work their way through the new Holt mathematics curriculum. In the spring, Karen still had reservations about this series: "They present too many different ways to do things too quickly, I think." She then made an important observation, one that is indicative of an important shift in her thinking,

> The idea is--and I agree with that--that there are many different ways to find answers to things or to look at things. I mean, maybe I would come up with the same answer--but there are different ways to look at it. But, I mean, when you just overwhelm kids and give them too many all at once, they don't get it and are just confused.

Note here how the argument has shifted. Karen no longer appears to be questioning whether or not students *should* be exposed to alternative problem solving strategies; the issue for her is more one of when and how many.

At the time of the second interview, Karen more fully accepted the notion that students can, and should, be encouraged to develop their own solutions to problems. In responding to the interview questions described earlier, for example, where different solutions to the same story problem are attributed to students, Karen commented: "I would say that this is the gist of this new math program--that kids are allowed to do things this way." When asked how she might respond if students to this sort of situation in her classroom, Karen commented, "I would say this is fantastic, and why don't you stand up and tell the class how you did this--how you came to this conclusion?" "Why would you do that?" she was asked. "I think it would be good for the kids to see it and hear it and use it." A few minutes later, the interviewer asked,

> Interview: So if you were going over this problem in class, and all of these solutions were brought up by different kids, how would you respond?
>
> Karen: I never say things are wrong. I am trying to accept most things. Usually when you do it that way you start with the ones that are easiest to understand and you just work on down. Or you will even have someone else help them. Say Kathy, she probably cannot explain how she did it and maybe someone else can help her.

As this last example demonstrates, Karen's views about mathematics and the teaching of mathematics appear to have changed in significant ways from Time One to Time Two. Furthermore, there is some evidence that these changes in viewpoint are beginning to have some impact on Karen's teaching. This issue is addressed in the next section of the case study.

<u>Karen's Practice at Time Two: Do Actions Speak Louder Than Words?</u>

The lesson that Karen taught in the spring looked different compared to the two lessons observed earlier in the year. For one, it was one of the few times when, by her own admission, she actually made use of manipulatives in her mathematics instruction. This, combined with the fact that she organized students into cooperative groups--complete with assigned roles like "recorder," "checker," and "praiser"--might constitute evidence of a breakthrough of sorts in Karen's teaching of mathematics. Upon closer examination, however, this lesson, like the two described earlier, appears to contain more elements of traditional math than of the much more open-ended, discourse-driven mathematics envisioned in the *California Mathematics Framework* document.

It is not too extreme a statement to say that, in the lesson described below, Karen seems to be going through the motions of reform, implementing some of the practices called for in the *Framework*--but only in a limited way. There seems to be a lack of clarity on her part about the real intent of the recommended innovations. If, for example, the purpose of small groups is provide students with the opportunity to share knowledge and exchange views about content, then one can judge the success of this instructional activity on those grounds. The same argument can be applied to the use of manipulatives. According to the *Framework*, the intent of this activity is to "provide a way for students to connect their understandings about real objects and their own experiences to mathematic concepts" (California State Department of Education, p. 15). Teachers have an important role to play in this regard. "To be effective," the authors of the *Framework* conclude, "teachers must do more than just provide concrete materials for their demonstrations" (p. 16). They must help students make the connections between concrete and abstract instantiations of the concept. A related assumption is that the connecting process takes time; students need to interact with the concrete material directly and for a prolonged period of time. As will become evident in the description of Karen's teaching, these conditions for the use of manipulatives were not satisfied.

One should keep these observations in mind in reading the field notes from the lesson at Time Two. The goal of the lesson was to introduce fractions.

> The teacher began the lesson at 10:45 by telling the class that she wanted them to work in "cooperative groups." "Before we get into our groups," she said, "I want a quick review of what we're supposed to do." She placed a transparency on the overhead projector with the following roles listed and described: "Recorder--records answers after group reaches agreement; reporter--reports progress to the class; praiser--praises group, looks for sharing, listening, cooperation, compliments; checker--makes sure everyone understands the task--there must be group agreement." She went over each of these descriptions, emphasizing certain things such as the word "progress" in the description of the reporter's responsibility, or the fact that "before the recorder can write down anything, everyone has to agree." In describing the recorder's task, the teacher added, "The activity we're going to be doing today is rather open-ended, so it won't be like we've done in the past where you're actually writing down an answer."
>
> Having passed out sets of pattern blocks to certain individuals, the teacher then asked these individuals to stand and "hold up their cards." They were to form the nucleus of the groups. The other individuals apparently knew which groups they belonged to by the color of the little circles on the 3 by 5 cards the teacher had passed out prior to the observer entering the room. Each card also contained a number which identified the particular role (i.e., recorder, praiser) that each youngster within the group was to play. Telling students not to "move any more chairs than possible," the teacher directed them to seek out their groups. This procedure appeared to be well practiced; there was, for example, a minimum of disruption as students organized themselves. The teacher told the students to "be sure they can see and touch the blocks (pattern blocks)" when they seat themselves. There were seven small groups arrayed around the room. Four of the groups consisted of four students, three were composed of three students. With one exception, the groups were all heterogeneous with regard to gender.

The teacher said, "Now I would like the checker to take--gently--the blocks out of the bag." "The first thing we're going to do is just look at these blocks for a minute," the teacher said. "Feel them, touch them, fool around with them for a minute. What I want you to be looking for are some of the ways that you can describe these blocks to me." The teacher passed out paper for the reporters to use in recording the deliberations of the group. Shortly after this, the teacher rang a small bell to get students' attention. "Let's hear the reporter from the red group. Stand and tell us one thing about the blocks," she requested. This without any obvious consultation on the part of group members. The reporter from the red group observed that the blocks were "glazed." A reporter from another group was asked to respond to this question. She commented that the blocks were "colorful." This apparently was closer to what the teacher had in mind. She said, "Now, let's get to some real facts here," asking the child to elaborate what she had in mind. She asked the class as a whole how many colors there were. After exploring this for a minute, she asked if there were any other physical characteristics. A child suggested that there "were no curvy lines." The teacher suggested that he turn that description around and he said that all the lines were straight. The teacher called for other attributes and a child indicated that the blocks were different shapes. When asked to elaborate, another child volunteered that there was a triangle. The teacher asked the child what the shape and color of the triangle was. One child indicated that there was a diamond. Another pointed out that all the pieces fit together. The teacher asked her to elaborate. She said that the two diamond shapes would fit together. The teacher exclaimed, "Ah. Now we are going to get to something."

Using the overhead projector, the teacher placed an octagon on the screen and asked the groups to find two shapes that would fit together and mirror the shape on the overhead. She instructed the children in the groups to lay the blocks aside and work in the middle of the table. She called on Amy, who indicated that she had found the same shape. The teacher clarified that she wanted two shapes. Another child started to describe the blocks that would fit together but he was struggling with what to call them. The teacher suggested he just use the color to describe the type of block. "When I put these two red ones together," the teacher asked, "what am I going to call the two red ones?" "A half," a child said. "We have how many halves?" "Two," responded the students. "And two halves make a . . ." "Whole." The teacher then asked the groups to find another set of blocks that would "mirror" the next figure she had on the overhead (a hexagon). She called on a reporter who said the triangles would fit together to form this figure. She pressed the class to indicate how many "greens" it would take and they said six. She demonstrated on the overhead that this was correct. After constructing this figure, she removed one of the tiny triangles and asked the class to indicate how many were left when one was removed. They correctly responded "five sixths."

At this point (11:15), the teacher directed the "checkers" to put the blocks back in the bag. As she was collecting this material, the noise level rose slightly. The teacher rang a bell to get the students' attention. She asked the praisers to stand and report on the work of the group. The first child she called on said, "We did everything right. Nothing went wrong. There was no fighting. Similar testimonials were given by the other praisers. The teacher told the students to move back to their own seats. She then moved to the next segment of the forty-five minute lesson.

So much for the "experimental" part of the math lesson. As this long excerpt suggests, Karen's teaching at Time Two had not really changed much from what she did at Time One. Students *were* grouped, but there was little time for them to exchange views before responding to her rapid-fire questions. Even if time for deliberation had been provided, the task--determining which of several sets of blocks would form a predetermined pattern--is not well-suited to group decision making. In fact, Karen's rationale for using small groups appeared to be more managerial than instructional. As she indicated in the post-observation interview, the cooperative group routine is an efficient way to get material into and out of students' hands.

In the lesson cited above, students did have a "hands on" experience, but it is unclear how it has contributed to their understanding of fractions. The use of small groups and of manipulative material is an added wrinkle in Karen's mathematics teaching, but there is little change at a more fundamental level. Her teaching of mathematics at Time Two continues to be didactic and centers on procedural as opposed to conceptual "stuff." This is not surprising. Perhaps one shouldn't expect to see much change in classroom practice at this early point in the implementation process. It may be sufficient that teachers like Karen are reexamining some of their beliefs about the teaching of mathematics.

Conclusion

As indicated in the overview, Karen's case represents a modest success for the *California Framework* as implemented in Southdale. She has changed some of her negative views about the curriculum; she also seems more willing to experiment with some of the instructional innovations called for by the *Framework*. Where before, for example, she had disdained the use of manipulatives, seeing it as too time consuming and of benefit to only a few students, her views had changed a bit by Time Two. She now saw real benefit in this approach; some students may need to experience things concretely before dealing with them at a more abstract level. Karen indicated at Time Two that she also saw merit in having students discuss different problem solving strategies with the class--a marked departure from before when she it indicated that it would probably just confuse the students. Karen thus appears to be accommodating to the new mathematics curriculum. In a sense, she is learning along with her students.

As the above comments suggest, Karen's accommodations appear to be primarily instructional in nature. Her experience with the new curriculum has encouraged her to think more expansively about her own mathematics teaching; her perspective on what constitutes useful experience when learning mathematics has broadened. But her beliefs have only been translated into a few bits of action. More change may require a change in perspective far more sweeping than what has occurred thus far. It seems likely that dramatic change in Karen's teaching could require equally dramatic change in Karen's views about the nature of mathematics and about the nature of the learning process. Previous research points to the centrality of both of these beliefs as they relate to teaching practice (Pope & Gilbert, 1983; Roth, 1987).

While there is reason to believe that Karen's views about mathematics learning have undergone some change, it is probably not enough to constitute a "conceptual shift" of the sort advocated by many teacher educators. At the time of the second interview, Karen did acknowledge that there may be more to understanding mathematics than simply doing calculations. In fact, she explicitly distinguished between being able to do a procedure "manually" and "really having the concept." Apparently, Karen accepted the notion that there is more than one way to understand mathematics. This does not mean, however, that she had become a "constructivist" in her orientation to learning. There is no evidence in the interview or observational record of this sort of change in perspective. This may be significant. According to one interpretation of the *Framework*, constructivist thinking--acceptance of the premise that individuals create their own reality--is an important aspect of the vision of mathematics presented in that document. Much of its rhetoric does appear to reflect a constructivist view of learning. Be that as it may, Karen's accommodations to the new curriculum are not of this theoretical sort. There is no evidence that she has changed her views about the nature of the learning process. This inferential leap may be too great for any teacher to make on his or her own.

In a similar vein, Karen's views about the nature of mathematics as a discipline have not undergone substantial change. To the extent that this is also an important issue addressed in the *Framework*, the implementation effort has not influenced Karen's thinking. Mathematics, for her, continues to be a set of tools and techniques; its value lies in its usefulness in helping people meet the daily demands of life. In the *Framework*, there is a reference to "the inherent beauty and fascination of mathematics . . . as a subject that can be appreciated and enjoyed by all learners" (California State Department of Education, 1985, p. 1). This implies some appreciation for mathematics as a discipline--as an arena of human inquiry. This perspective is totally absent in Karen's interview protocols at Time One and Time Two.

Although one can observe significant change in Karen's beliefs about the teaching of mathematics as a result of her attempts to come to terms with the new mathematics curriculum--change which, for her, is dramatic and painful--it may not be enough. If teachers are to fundamentally alter their teaching of mathematics, they may need to reexamine a whole network of beliefs extending far beyond their views about the craft of teaching, narrowly defined; they may need to significantly change their views about the nature of knowledge and what it takes to acquire that knowledge.

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A Revolution in One Classroom:

The Case of Mrs. Oublier

David K. Cohen

Introduction

As Mrs. Oublier sees it, her classroom is a new world. She reported that when she began work four years ago, her mathematics teaching was thoroughly traditional. She followed the text. Her second graders spent most of their time on worksheets. Learning math meant memorizing facts and procedures. Then Mrs. O found a new way to teach math. The summer after her first year of teaching, she took a workshop in which she learned to focus lessons on students' understanding of mathematical ideas. She found ways to relate mathematical concepts to students' knowledge and experience. And she explored methods to engage students in actively understanding mathematics. In her third year of such work, Mrs. O is delighted with her students' performance, and with her own accomplishments.

Mrs. O's story is engaging, and so is she. She is considerate of her students, eager for them to learn, energetic, and attractive. These qualities would stand out anywhere, but they seem particularly vivid in her school. It is a drab collection of one-story, concrete buildings that sprawl over several acres. Though clean and well managed, her school lacks any of the familiar signs of classy education. It has no legacy of experimentation or progressive pedagogy, or even of heavy spending on education. Only a minority of children come from well-to-do families. Most families have middling or only modest incomes, and many are eligible for Chapter I assistance. A sizable minority are on welfare. The school district is situated in a dusty corner of southern California, where city migrants rapidly are turning a rural town into a suburb. New condominiums are sprouting all over the community, but one still sees pick-up trucks with rifle racks mounted in their rear windows. Like several of her colleagues, Mrs. O works in a covey of tacky, portable, prefab classrooms, trucked into the back of the schoolyard to absorb growing enrollments on the cheap.

Mrs. O's story seems even more unlikely when considered against the history of American educational reform. Great plans for educational change are a familiar feature of that history, but so are reports of failed reforms. That is said to have been the fate of the earlier "new math," in the 1950s and 1960s. A similar tale was told of efforts to improve science teaching at the time (Welch, 1979). Indeed, failed efforts to improve teaching and learning are an old story. John Dewey and others announced a revolution in pedagogy just as our century opened, but apparently it fizzled: Classrooms changed only a little, researchers say (Cuban, 1984). The story goes on. Since the Sputnik era, many studies of instructional innovation have embroidered these old themes of great ambitions and modest results (Gross, Giaquinta, & Bernstein, 1971; Rowan & Guthrie, 1989; Cohen, 1989).

Some analysts explain these dismal tales with reference to teachers' resistance to change: They argue that entrenched classroom habits defeat reform (Gross, Giaquinta, & Bernstein, 1971). Others report that many innovations fail because they are poorly adapted to classrooms: Even teachers who avidly desire change can do little with most schemes to improve instruction, because they don't work well in classrooms (Cuban, 1984; Cuban, 1986). Mrs. O's revolution looks particularly appealing against this background. She eagerly embraced change, rather than resisting it. She found new ideas and materials that worked in her classroom, rather than working against innovation. Mrs. O sees her class as a success for the new Mathematics *Framework* (California State Department of Education, 1985). Though her revolution began while the *Framework* still was being written, it was inspired by many of the same ideas. She reports that her math teaching has wound up where the *Framework* intends it to be.

But as I watched and listened in Mrs. O's classroom, things seemed more complicated. Her teaching does reflect the new *Framework* in many ways. For instance, she has adopted innovative instructional materials and activities, all designed to help students make sense of mathematics. But Mrs. O seemed to treat new mathematical topics as though they were a part of traditional school mathematics. She used the new materials, but used them as though mathematics contained only right and wrong answers. She has revised the curriculum to help students understand math, but she conducts the class in ways that discourage exploration of students' understanding.

From the perspective of the new *California Mathematics Framework*, then, Mrs. O's lessons seem quite mixed. They contain some important elements that the *Framework* embraced, but they contain others that it branded as inadequate. In fact, her classes present an extraordinary *melange* of traditional and novel approaches to math instruction.

Something Old and Something New

That *melange* is part of the fascination of Mrs. O's story. Some observers would agree that she has made a revolution, but others would see only traditional instruction. It is easy to imagine long arguments about which is the real Mrs. O, but they would be the wrong arguments. Mrs. O. is both of these teachers. Her classroom deserves attention partly because such mixtures are quite common in instructional innovations--though they have been little noticed. As teachers and students try to find their way from familiar practices to new ones, they cobble new ideas onto familiar practices. The variety of these blends, and teachers' ingenuity in fashioning them are remarkable. But they raise unsettling questions. Can we say that an innovation has made much progress when it is tangled in combination with many traditional practices? Changes that seem large to teachers who are in the midst of struggles to accommodate new ideas often seem modest or invisible to observers who scan practice for evidence that new policies have been implemented. How does one judge innovative progress? Should we consider changes in teachers' work from the perspective of new policies like the *Framework*? Or should they be considered from the teachers' vantage point?

New Materials, Old Mathematics

From one angle, the curriculum and instructional materials in this class were just what the new *Framework* ordered. For instance, Mrs. O regularly asked her second graders to work on "number sentences." In one class that I observed, students had done the problem: 10+4=14. Mrs. O then asked them to generate additional "number sentences" about 14. They volunteered various ways to write addition problems about fourteen--i.e., 10+1+1+1+1=14, 5+5+4=14, etc. Some students proposed several ways to write subtraction problems--i.e., 14-4=10, 14-10=4, etc. Most of the students' proposals were correct. Such work could make mathematical relationships more accessible, by coming at them with ordinary language rather than working only with bare numbers on a page. It also could unpack mathematical relationships, by offering different ways to get the same result. It could illuminate the relations between addition and subtraction, helping children to understand their reversibility. And it could get students to do "mental math," i.e., to solve problems in their heads and thereby learn to see math as something to puzzle about and figure out, rather than just a bunch of facts and procedures to be memorized.

These are all things that the new *Framework* invited. The authors exhort teachers to help students cultivate ". . . an attitude of curiosity and the willingness to probe and explore. . ." (California State Department of Education, 1985, p. 1). The document also calls for classroom work that helps students ". . .to understand why computational algorithms are constructed in particular forms . . ." (California State Department of Education, 1985, p. 4).

But the *Framework*'s mathematical exhortations were general, and offered few specifics about how teachers might respond. The reform manifesto left room for many different responses. Mrs. O used the new materials, but conducted the entire exercise in a thoroughly traditional fashion. The class worked as though the lesson were a drill, reciting in response to the teacher's queries. Students' sentences were accepted if correct, and written down on the board. But they were turned down if incorrect, and not written on the board. Right answers were not explained, and wrong answers were treated as unreal. The *Framework* makes no such distinction. To the contrary, it argues that understanding how to arrive at answers is an essential part of helping students to figure out how mathematics works--perhaps more important than whether the answers are right or wrong. The *Framework* criticizes the usual memorized, algorithmic approach to mathematics, and the usual search for the right answer. It calls for class discussion of problems and problem solving as an important part of figuring out mathematical relationships (California State Department of Education, 1985, pp. 13-14). But no-one in Mrs. O's class was asked to explain their proposed number sentences, correct or incorrect. No student was invited to demonstrate how he or she knew whether a sentence was correct or not. The teacher used a new mathematics curriculum, but used it in a way that conveyed a sense of mathematics as a fixed body of right answers, rather than as a field of inquiry in which people figure out quantitative relations. It is easy to see the *Framework*'s ideas in Mrs. O's classroom, but it also is easy to see many points of opposition between the new policy and Mrs. O's approach (California State Department of Education, 1987, p. 9).

Make no mistake: Mrs. O was teaching math for understanding. The work with number sentences certainly was calculated to help students see how addition worked, and to see that addition and subtraction were reversible. That mathematical idea is well worth understanding, and the students seemed to understand it at some level. They were, after all, producing the appropriate sorts of sentences. But it was difficult to understand how or how well they understood it, for the didactic form of the lesson inhibited explanation or exploration of students' ideas. Additionally, mathematical knowledge was treated in a traditional way: Correct answers were accepted, and wrong ones simply rejected. No answers were unpacked. There was teaching for mathematical understanding here, but it was blended with other elements of instruction that seemed likely to inhibit understanding.

The mixture of new mathematical ideas and materials with old mathematical knowledge and pedagogy permeated Mrs. O's teaching. It also showed up extensively in her work with concrete materials and other physical activities. These materials and activities are a crucial feature of her revolution, for they are intended to represent mathematical concepts in a form that is vivid and accessible to young children. For instance, she opens the math lesson every day with a calendar activity, in which she and the students gather on a rug at one side of the room to count up the days of the school year. She uses this activity for various purposes. During my first visit she was familiarizing students with place value, regrouping, and odd and even numbers. As it happened, my visit began on the fifty-ninth day of the school year, and so the class counted to fifty-nine. They used single claps for most numbers but double claps for ten, twenty, etc. Thus, one physical activity represented the "tens", and distinguished them from another physical activity that was used to represent the "ones." On the next day, the class used claps for even numbers and finger snaps for odd numbers, in counting off the days. The idea here is that fundamental distinctions among types of number can be represented in ways that make immediate and fundamental sense to young children. Representations of this sort, it is thought, will deeply familiarize them with important mathematical ideas, but will do so in a fashion easily accessible to those unfamiliar with abstractions.

Mrs. O also used drinking straws in a related activity, to represent place value and regrouping. Every day a "student helper" is invited to help lead the calendar activity by adding another straw to the total that represent the elapsed days in the school year. The straws accumulate until there are ten, and then are bundled with a rubber band. One notion behind this activity is that students will gain some concrete basis for understanding how numbers are grouped in a base ten system. Another is that they can begin to apprehend, first physically and then intellectually, how number groups can be composed and decomposed.

Mrs. O's class abounds with such activities and materials, and they are very different from the bare numbers on worksheets what would be found in a traditional math class. She was still excited, after several years' experience, about the difference that they made for her students' understanding of arithmetic. Mrs. O adopts a somewhat cool demeanor in class, which is enhanced by her almost stereotypically Californian appearance: She is a slender blond, with attractively Nordic features. But her conviction about the approach was plain as she worked with the students, and her enthusiasm for it bubbled up in our conversations. After three years, she had only disdain for her old way of teaching math.

Her approach seems nicely aligned with the new *Framework*. For instance, that document argues that "Many activities should involve concrete experiences so that students develop a sense of what numbers mean and how they are related before they are asked to add, subtract, multiply, or divide them (California State Department of Education, 1985, p. 8). And it adds, a few pages further on, that "Concrete materials provide a way for students to connect their own understandings about real objects and their own experiences to mathematical concepts. They gain direct experience with the underlying principles of each concept." (California State Department of Education, 1985, p. 15).

Mrs. O certainly shared the *Framework*'s view in this matter. But it is one thing to embrace a doctrine of instruction, and quite another to weave it into one's practice. For even a rather monotonous practice of teaching comprises many different threads. Hence any new instructional thread must somehow be related to many others already there. Like reweaving fabric, this social and intellectual reweaving can be done in different ways. The new thread can simply be dropped onto the fabric, and everything else left as is. Or new threads may be somehow woven into the fabric. If so some alteration in the relations among threads will be required. Some of the existing threads might have to be adjusted in some way, or even pulled out and replaced. If one views Mrs. O's work from the perspective of the *Framework*, new threads were introduced, but old threads were not pulled out. The old and new lay side by side, and so the fabric of instruction was different. But there seemed to be litt' mutual adjustment among new and old threads. Mrs. O used the novel concrete materials and phys' activities, but used them in a traditional pedagogical surround. Consequently the new mate ial seemed to take on different meaning from its circumstances. Materials and activities intended to teach mathematics for understanding were infused with traditional messages about what mathematics was, and what it meant to understand it.

These mixed qualities were vividly apparent in a lesson that focused on addition and subtraction with regrouping. The lesson occurred early in an eight or ten week cycle concerning these topics. Like many of her lessons, it combined a game-like activity with the use of concrete materials. The aim was to capture children's interest in math, and to help them understand it. Mrs. O introduced this lesson by announcing: "Boys and girls, today we are going to play a counting game. Inside this paper [holding up a wadded up sheet of paper] is the secret message. . ." (observation, 12/88). Mrs. O unwadded the paper and held it up: "6" was inscribed. The number was important, because it would establish the number base for the lesson: Six. In previous lessons they had done the same thing with four and five. So part of the story here was exploring how things work in different number bases, and one reason for that, presumably, was to get some perspective on the base-ten system that we conventionally use. Mrs. O told the children that, as in the previous games, they would use a nonsense word in place of the secret number. I was not sure why she did this, at the time. As it turned out, the approach was recommended, but not explained, by the innovative curriculum guide she was using. After a few minutes taken to select the nonsense word, the class settled on "Cat's eye" (observation, 12/88).

With this groundwork laid, Mrs. O. had "place value boards" given to each student. She held her board up [eight by eleven, roughly, one half blue and the other white], and said: "We call this a place value board. What do you notice about it?"

Cristie Smith, who turned out to be a steady infielder on Mrs. O's team, said: "There's a smiling face at the top." Mrs. O agreed, noting that the smiling face needed to be at the top at all times [that would keep the blue half of the board on everyone's left]. Several kids were holding theirs up for inspection from various angles, and she admonished them to leave the boards flat on their tables at all times.

"What else do we notice?" she inquired. Sam said that one half is blue and the other white. Mrs. O agreed, and went on to say that ". . . the blue side will be the "cat's eye" side. During this game we will add one to the white side, and when we get a cat's eye, we will move it over to the blue side." With that, each student was given a small plastic tub. which contained a handful of dried beans and half a dozen small paper cups, perhaps a third the height of those dispensed in dentist's offices. This was the sum total of pre-lesson framing--no other discussion or description preceded the work.

There was a small flurry of activity as students took their tubs and checked out the contents. Beans present nearly endless mischievous possibilities, and several of the kids seemed on the verge of exploring their properties as guided missiles. Mrs. O nipped off these investigations, saying: "Put your tubs at the top of your desks, and put both hands in the air." The students all complied, as though in a small stagecoach robbery. "Please keep them up while I talk." She opened a spiral bound book, not the school district's adopted text but *Math Their Way* (Baratta-Lorton, 1976). This was the innovative curriculum guide that had helped to spark her revolution. She looked at it from time to time, as the lesson progressed, but seemed to have quite a good grip on the activity.

Mrs. O got things off to a brisk start: "Boys and girls [who still were in the holdup], when I clap my hands, add a bean to the white side [from the plastic tub]."

She clapped once, vigorously, adding that they could put their hands down. "Now we are going to read what we have: What do we have?" [she led a choral chant of the answer] "Zero cat's eye and one." She asked students to repeat that, and everyone did. She clapped again, and students obediently added a second bean to the white portion of the card. "What do we have now," she inquired. Again she led a choral chant: "Zero cat's eye and two." So another part of the story in this lesson was place value: "Zero cat's eye" denotes what would be the "tens" place in base-ten numbering, and "two" is the "one"s" place. Counting individual beans, and beans grouped in "cat's eye," would give the kids a first-hand, physical sense of how place value worked in this and other number bases.

In these opening chants, as in all subsequent ones, Mrs. O performed more like a drill sergeant than a choir director. Rather than establishing a beat and then maintaining it with her team, she led each chant and the class followed at a split-second interval. Any kid who didn't grasp the idea needed only to wait for her cue, or for a table-mates' cue. There were no solos: Students were never invited or allowed to count on their own. Thus, while the *leitmotif* in their second chant was "zero cat's eye and two," there was an audible minor theme of "zero cat's eye and one." That several repeated the first chant suggested that they did not get either the routine or its point.

Mrs. O moved right on nonetheless, saying that it ". . . is very important that you read the numbers with your hands." This was a matter to which she returned many times during the lesson; she kept reminding the children to put their little paws first on the beans on the white square, and then on the little cups on the blue square, as they incanted the mathematical chants. It was essential that they manipulate the concrete materials. Whenever she spotted children who were not palpitating beans and cups, she walked over and moved their arms and hands for them.

Mrs. O led the bean adding and chants up to five. Then, when the first five beans were down on everyone's card, she asked: "Now think ahead; when I clap my hands this time, what will you have on the white side?"

Reliable Christie Smith scooped it up and threw smoothly to first: "Cat's eye."

Mrs. O led off again: "When you get a cat's eye, put all the beans in a paper cup, and move them over." She clapped her hands for the cat's eye, and then led the following chant: "Put the beans in the cup and move them over."

"Now let's read what we have." The chant rolled on, "one cat's eye and zero. A puzzling undercurrent of "one cat's eye and one" went unattended. She then led the class though a series of claps and chants, leading up to two cat's eyes. And the claps and chants went on, with a methodical monotony, up to five cat's eyes and five. The whole series took about fifteen minutes, and throughout the exercise she repeatedly reminded students to "read" the materials with their hands, to feel the beans and move their arms. By the time they got to five cat's eyes and five, her claps had grown more perfunctory, and many of the kids had gotten the fidgets. But Mrs. O gave no ground. She seemed to see this chanting and bean-pawing as the high road to mathematical understanding, and tenaciously drove her team on.

"Now, how many do we have? "Five cat's eye's and five beans," came the chant. "Now we will take away one bean" [from the "ones" side of the board]. "How many do we have?" Again the answering chant, again led by her, a fraction of a second early, "five cat's eyes and four."

This was a crucial point in the lesson. The class was moving from what might be regarded as a concrete representation of addition with regrouping, to a similar representation of subtraction with regrouping. Yet she did not comment on or explain this reversal of direction. It would have been an obvious moment for some such comment or discussion, at least if one saw the articulation of ideas as part of understanding mathematics. But Mrs. O did not teach as though she took that view. Hers seemed to be an activity-based approach: It was as though she thought that all the important ideas were implicit, and better that way.

Thus the class counted down to five cat's eyes and zero. Mrs. O then asked, "What do we do now?" Jane responded: "Take a dish from the cat's eye side, and move it to the white side." No explanation was requested or offered, to embroider this response. Mrs. O simply approved the answer, clapped her hands, and everyone followed Jane's lead. With this, Mrs. O. led the class back through each step, with claps, chants, and reminders to "read" the beans with their hands, down to zero cat's eye and zero beans. The entire effort took thirty or thirty five minutes. Everyone was flagging long before it was done, but not a chant was skipped or a movement missed.

Why did Mrs. O teach in this fashion? In an interview following the lesson I asked her what she thought the children learned from the exercise. She said that it helped them to understand what goes on in addition and subtraction with regrouping. Manipulating the materials really helps kids to understand math, she said. Mrs. O seemed quite convinced that these physical experiences caused learning, that mathematical knowledge arose from the activities.

Her immediate inspiration for all this seems to have been *Math Their Way*, a system of primary grade math teaching on which, Mrs. O says, she relies heavily. *Math Their Way* announces its purpose this way: ". . . to develop understanding and insight of the patterns of mathematics through the use of concrete materials" (Baratta-Lorton, 1976, p. xiv). Concrete materials and physical activities are the central features of this primary grade program, because they are believed to provide real experience with mathematics. In this connection the book sharply distinguishes between mathematical symbols and concepts. It criticizes teaching with symbols, arguing that symbols--i.e., numbers--"... are not *the concept* [emphasis in original], they are only a representation of the concept, and as such are abstractions describing something which is not visible to the child. Real materials, on the other hand, can be manipulated to illustrate the concept concretely, and can be experienced visually by the child... The emphasis throughout this book is making concepts, rather than numerical symbols, meaningful" (Baratta-Lorton, 1976, p. xiv).

Math Their Way fairly oozes the belief that physical representations are much more real than symbols. This fascinating idea is a recent mathematical mutation of the belief, at least as old as Rousseau, Pestalozzi, and James Fenimore Cooper, that experience is a better teacher than mere books. For experience is vivid, vital, and immediate, while books are all abstract ideas and dead formulations. Mrs. O did not mention these sages, but she certainly had a grip on the idea. In this she resembles many primary school teachers, for the view that concrete materials and physical activities are the high road to abstract concepts has become common currency in nursery school and primary grade teaching. Many primary grade teachers have long used physical activities and concrete materials elsewhere in instruction.

In fact, one of the chief claims in *Math Their Way* is that concrete materials are developmentally desirable for young children. Numbers are referred to many times as an "adult" way of approaching math. And this idea leads to another, still more important: If math is taught properly, it will be easy. Activities with concrete materials, the book insists, are the natural way for kids to learn math: "... if this foundation is firmly laid, dealing with abstract number will be *effortless*" (Baratta-Lorton, 1976, p. 167, emphasis added).

Stated so baldly, that seems a phenomenal claim: Simply working with the proper activities and materials assures that math will be understood. Materials and activities are not only necessary for understanding mathematics, but also sufficient. But the idea is quite common. Pestalozzi might have cheered it. Many other pedagogical Romantics, Rousseau and Dewey among them, embraced a version of this view. Piaget is commonly thought to have endorsed a similar idea. So when *Math Their Way* argues that the key to teaching math for understanding is to get children to use the right sorts of activities and materials, it is on one of the main tracks of modern educational thought and practice. The book's claim also helps to explain why it gives no attention to the nature of mathematical knowledge, and so little attention to the explanation of mathematical ideas. For the author seems convinced that such things are superfluous: Appropriate materials and activities alone will do the trick.

In fact, the book's appeal owes something to its combination of great promises and easy methods. For it offers teachers a kind of pedagogical special, a two-for-the-price-of-one: Students will "understand" math without any need to open up questions about the nature of mathematical knowledge. The curriculum promises mathematical understanding, but it does not challenge or even discuss the common view of mathematics as a fixed body of material--in which knowledge consists of right answers--that so many teachers have inherited from their own schooling. The manual does occasionally note that teachers might discuss problems and their solutions with students. But this encouragement is quite modestly and intermittently scattered through a curriculum guide that chiefly focuses on the teaching potential of concrete materials and physical activities. The book presents concrete representations and math activities as a kind of explanation sufficient unto themselves. Discussion of mathematical ideas has a parenthetical role, at best.

All of this illuminates Mrs. O's indebtedness to *Math Their Way*, and her persistent praise for it. She used the guide to set up and conduct the lessons that I saw, and referred to it repeatedly in our conversations as the inspiration for her revolution. My subsequent comparisons of her classes with the manual suggested that she did draw deeply on it for ideas about materials, activities, and lesson format. More important, her views of how children come to understand mathematics were, by her own account, powerfully influenced by this book.

Math Their Way thus enabled Mrs. O to whole-heartedly embrace teaching math for understanding, without considering or reconsidering her views of mathematical knowledge. She was very keen that children should understand math, and worked hard at helping them. But she placed nearly the entire weight of this effort on concrete materials and activities. The ways that she used these materials--insisting, for instance, that all the children actually feel them, and perform the same prescribed physical operations with them--suggest that she endowed the materials with enormous, even magical instructional powers. The lack of any other ways of making sense of mathematics in her lessons was no oversight. With *Math Their Way*, she simply saw no need for anything else.

In what sense was Mrs. O teaching for understanding? The question opens up a great puzzle. Her classes exuded traditional conceptions of mathematical knowledge, and were organized as though explanation and discussion were irrelevant to mathematics. But she had changed her math teaching quite dramatically. She now used a new curriculum specifically designed to promote students' understanding of mathematics. And her students' lessons were very different than they had been. They worked with materials that represented mathematical relationships in the concrete ways that the *Framework* and many other authorities endorse. Mrs. O thought the change had been decisive: She now teaches for understanding. She reported that her students now understood arithmetic, while previously they had simply memorized it.

Is there a solution to this puzzle? It is a nice question. But first consider two other features of her teaching.

<u>New Topics, Old Knowledge</u>

Mrs. O taught several topics endorsed by the new *Framework*, that would not have been covered in many traditional math classes. One such topic was estimation. Mrs. O told me that estimation is important because it helps students to make sense of numbers. They have to make educated guesses, and learn to figure out why some guesses are better than others. She reports that she deals with estimation recurrently in her second grade classwork, returning to it many times in the course of the year rather than teaching a single unit. Her reason was that estimation could not be learned by doing it once or twice, and, in any event, is useful in many different problem-solving situations. Her reasoning on this matter seemed quite in accord with the *Framework*. It calls for "Guessing and checking the result" as an important element in mathematical problem solving (California State Department of Education, 1985, p. 14). In fact, the *Framework* devotes a full page to estimation, explaining what it is and why it is important (California State Department of Education, 1985, pp. 4-5).

But the teaching that I observed did not realize these ambitions. In one lesson, for instance, the following problem was presented: Estimate how many large paper clips would be required to span one edge of the teacher's desk (observation, 12/88). Two students were enlisted to actually hold the clips so that students could see. They stood near the teacher's desk, near enough to visually gauge its width in relation to the clips. But all the other students remained at their tables, scattered around the room. None had any clips, and few could see the edge of the teacher's desk that was in question. For it was a side edge, away from most of the class.

So only two members of the class had real contact with the two key data sources in the problem--visible, palpable clips, and a clear view of the desk edge. As a consequence, only these two members of the class had any solid basis for deciding if their estimates were mathematically reasonable. Even Mrs. O was seated too far away to see the edge well, and she had no clips either. The problem itself was sensible, and could have been an opportunity to make and discuss estimates of a real puzzle. But it was set up in a way that emptied it of opportunities for mathematical sense-making.

Mrs. O did not seem aware of this For after she had announced the problem, she went on to engage the whole class in solving it. The two students were told to hold the clips up for everyone to see. Seated at the back, with many of the kids, I could see that they were the large sort of clip, but even then they were barely visible. Mrs. O then pointed to the desk edge, at the other end of the room, easily twenty feet from half the class. Then she asked the students to estimate how many clips it would take to cover the edge, and to write down their answers. She took estimates from most of the class, wrote them on the board, and asked class members if the estimates were "reasonable."

Not surprisingly, the answers lacked mathematical discrimination. Estimates that were close to three times the actual answer, or one-third of it, were accepted by the class and the teacher as "reasonable". Indeed, no answers were rejected as unreasonable, even though quite a few were far off the mark. Nor were some estimates distinguished as more or less reasonable than others. Mrs. O asked the class what "reasonable" meant, and one boy offered an appropriate answer, suggesting that the class had some previous contact with this idea.

There was nothing that I could see or imagine in the classroom that led inexorably to this treatment. Mrs O. seemed to have many clips. If eight or ten had been passed around, the kids would have had at least direct access to one element in the estimation problem--i.e., the length of the clip. Additionally, Mrs. O could have directed the kids' attention to the edge of the desk that they could see, rather than the far edge that they could not see. I knew that the two edges of the rectangular desk were the same length, and perhaps some of these second graders did as well. But her way of presenting the problem left that as a needless, and mathematically irrelevant barrier to their work. Alternatively, Mrs. O could have invited them to estimate the length of their own desk edges, which were all the same, standard-issue models. That, along with passed-around clips, would have given them much more direct contact with the elements of the problem. The students would have had more of the mathematical data required to make sound estimates, and much more of a basis for considering the reasonableness of those estimates.

Why did Mrs. O not set the problem up in one of these ways? I could see no organizational or pedagogical reason. And in a conversation after the class, when I asked for her comments on that part of the lesson, she did not display even a shred of discomfort, let alone suggest that anything had been wrong. Mrs. O seemed to understand the broad purpose of teaching and learning estimation (interview, 12/88). But this bit of teaching suggests that she did not have a firm grip on the mathematics in this estimation example. She taught as though she lacked the mathematical and pedagogical infrastructure--the knowledge of mathematics, and of teaching and learning mathematics--that would have helped her to set the problem up so that the crucial mathematical data were available to students.

An additional bit of evidence on this point concerns the way Mrs. O presented estimation. She offered it as a topic in its own right, rather than as a part of solving problems that came up in the course of studying mathematics. After ending one part of the lesson, she turned to estimation as though it were an entirely separate matter. When the estimation example was finished, she turned the class to still another topic. Estimation had an inning all its own, rather than being woven into other innings' work. It was almost as though she thought that estimation bore no intimate relation to solving the ordinary run of mathematical problems. But this misses the mathematical point: Estimation is useful and used in that ordinary run, not for its own sake. The *Framework* touches on this matter, arguing that "... estimation activities should be presented not as separate lessons, but as a step to be used in all computational activities" (California State Department of Education, 1985, p. 4).

When detached from regular problem solving, estimation may seem strange, and thus isolated may lose some of its force as a way of making sense in mathematics. I wondered what the students might have learned from this session. They all appeared to accept the lesson as reasonable. No students decried the lack of comprehensible data on the problem, which they might have done if they were used to such data, and if this lesson were an aberration. No-one said that they had done it differently some other time, and that this didn't make sense. That could mean that the other lessons on estimation conveyed a similar impression. Or it may mean that students were simply dutiful, doing what they had been told because they had so often been told to do so. Or it may mean only that students took nothing from the lesson. Certainly school is full of mystifying or inexplicable experiences, that children simply accept. Perhaps this struck them only as another such mystification. It is possible, though, that they did learn something, and that it was related to Mrs. O's teaching. If so, perhaps they learned that estimation was worth doing, even if they didn't learn much about how to do it. Or perhaps they acquired an inappropriate idea of what estimation was, and what "reasonable" meant.

Was this teaching math for understanding? From one angle, it plainly was. Mrs. O did teach a novel and important topic, specifically intended to promote students' sense-making in arithmetic. It may well have done that. But the estimation problem was framed so that students had no way to bring mathematical evidence to bear on the problem, and little basis for making reasonable estimates. It therefore also is possible that students found this puzzling, confusing, or simply mysterious. These alternatives are not mutually exclusive. This bit of teaching for understanding could have promoted more understanding of mathematics, along with more misunderstanding.

New Organization, Old Discourse

Mrs. O's class was organized to promote "cooperative learning." The students' desks and tables were gathered in groups of four and five, so that they could easily work together. Each group had a leader, to help with various logistical chores. And the location and distribution of instructional materials often were managed by groups rather than individually. The new *Framework* endorses this way of organizing classroom work. It puts the rationale this way: "To internalize concepts and apply them to new situations, students must interact with materials, express their thoughts, and discuss alternative approaches and explanations. Often, these activities can be accomplished well in groups of four students" (California State Department of Education, 1985, p. 16).

The *Framework* thus envisions cooperative learning groups as the vehicle for a new sort of instructional discourse, in which students would do much more of the teaching. In consequence, each of them would learn from their own efforts to articulate and explain ideas--much more than they could learn from a teacher's explanations to them. And they would teach each other as well, learning from their mates' ideas and explanations, and from others' responses. The *Framework* explains: "Students have more chances to speak in a small group than in a class discussion; and in that setting some students are more comfortable speculating, questioning, and explaining concepts in order to clarify their thinking" (California State Department of Education, 1985, pp. 16-17).

Mrs. O's class was spatially and socially organized for such cooperative learning. But the instructional discourse that she established cut across the grain of this organization: The class was conducted in a highly structured and classically teacher-centered fashion. The chief instructional group was the whole class. The discourse that I observed consisted either of dyadic exchanges between the teacher and one student, or of whole-group activities, many of which involved choral responses to teacher questions. No student ever spoke to another about mathematical ideas as part of the public discourse. Nor was such conversation ever encouraged by the teacher. Indeed, Mrs. O specifically discouraged students from speaking with each other, in her efforts to keep the class orderly and quiet.

The small groups were not ignored. They were used for instructional purposes, but they were used in a distinctive way. In one class that I observed, for instance, Mrs. O announced a "graphing activity" about mid-way through the math period. She wrote across the chalk board, at the front of the room "Letter to Santa?" Underneath she wrote two column headings: "Yes" and "No." Then she told the children that she would call on them by groups, to answer the question.

If she had been following the *Framework*'s injunctions about small groups, Mrs. O might have asked each group to tally its answers to the question. She might then have asked each group to figure out whether it had more "yes" than "no" answers, or the reverse. She might then have asked each group to figure out how many more. And she might have had each group contribute its totals to the chart at the front of the room. This would not have been the most challenging group activity, but it would have meaningfully used the small groups as agents for working on this bit of mathematics.

Mrs. O proceeded differently. She used the groups to call on individual children. Moving from her right to left across the room, she asked individuals from each group, *seriatim*, to come to the front and put their entry under the "Yes" or No" column, exhausting one group before going on to the next. The groups were used in a socially meaningful way, but there was no mathematical discourse within them.

Mrs. O used the small groups in this fashion several times during my visits. The children seemed quite familiar with the procedures, and worked easily in this organization. In addition, she used the groups to distribute and collect instructional materials, which was a regular and important feature of her teaching. Finally, she regularly used the groups to dismiss the class for lunch and recess: She would let the quietest and tidiest group go first, and so on through the class.

Small groups thus were a regular feature of instruction in Mrs. O's class. I asked her about cooperative grouping in one of our conversations: Did she always use the groups in the ways that I had observed? She thought she did. I asked if she ever used them for more cooperative activity, i.e., discussions and that sort of thing. She said that she occasionally did so, but mostly she worked in the ways I had observed.

In what sense was this teaching for understanding? Here again, there was a remarkable combination of old and new math instruction. Mrs. O used a new form of classroom organization that was designed to promote collaborative work and broader discourse about academic work. She treated this organization with some seriousness. She referred to her classwork as "cooperative learning," and used the organization for some regular features of classroom work. When I mentally compared her class with others I had observed, in which students sat in traditional rows, and in which there was only whole-group or individual work, her class seemed really different. Though Mrs. O runs a tight ship, her class was more relaxed than those others I remembered, and organized in a more humane way. My view on this is not simply idiosyncratic. If Larry Cuban had used this class in the research for *How Teachers Taught* (1984), he probably would have judged it to be innovative as well. For that book relies on classrooms' social organization as an important indicator of innovative-ness.

Mrs. O also judged her classroom to be innovative. She noted that it was now organized quite differently than during her first year of teaching, and she emphatically preferred the innovation. The kids were more comfortable, and the class much more flexible, she said. But she filled the new social organization with old discourse processes. The new organization opened up lots of new opportunities for small group work, but she organized the discourse in ways that effectively blocked realization of those opportunities.

Reprise

I have emphasized certain tensions within Mrs. O's classes. But these came into view partly because I crouched there with one eye on the *Framework*. The tensions I have discussed were not illusory, but my angle of vision brought them into focus. Another observer, with other matters in mind, might not have noticed these tensions. Mrs. O certainly didn't notice them, and things went quite smoothly in her lessons. There was nothing rough or ungainly in the way she and her students managed. They were well used to each other, and to the class routines. They moved around easily within their math lessons. The various contrary elements of instruction that troubled my mental waters did not disturb the surface of the class. On the contrary, students and teacher acted as though the threads of these lessons were nicely woven together. Aspects of instruction that seemed at odds analytically appeared to nicely co-exist in practice.

What accounts for this smoothness? Can it be squared with the tensions that I have described within these classes?

Part of the answer lies in the classroom discourse. Mrs. O never invited or permitted broad participation in mathematical discussion or explanation. She held most exchanges within a traditional recitation format. She initiated nearly every interaction, whether with the entire class or one student. The students' assigned role was to respond, not initiate. They complied, often eagerly. Mrs. O is an attractive person, and was eager for her students to learn; in return, most of her students seemed eager to please. And eager or not, compliance is easier than initiation, especially when so much of the instruction is so predictable. Much of the discourse was very familiar to members of the class; often they gave the answers before Mrs. O asked the questions. So even though most of the class usually was participating in the discourse, they participated on a narrow track, in which she maintained control of direction, content, and pace.

The *Framework* explicitly rejects this sort of teaching. It argues that children need to express and discuss their ideas, in order to deeply understand the material on which they are working (California State Department of Education, 1985, pp. 14, 16). But the discourse in Mrs. O's class tended to discourage students from reflecting on mathematical ideas, or from sharing their puzzles with the class. There were few opportunities for students to initiate discussion, explore ideas, or even ask questions. Their attention was focused instead on successfully managing a prescribed, highly structured set of activities. This almost surely restricted the questions and ideas that could occur to students, for thought is created, not merely expressed, in social interactions. But even if the students' minds were nonetheless still privately full of bright ideas and puzzling mathematical problems, the discourse organization effectively barred them from the public arena of the class. Mrs. W employed a curriculum that sought to teach math for understanding, but she kept evidence about what students' understood from entering the classroom discourse. One reason that Mrs. O's class was so smooth was that so many possible sources of roughness were choked off at the source.

Another reason has to do with Mrs. O's knowledge of mathematics. Though she plainly wanted her students to understand this subject, her grasp of mathematics seemed to restrict her notion of mathematical understanding, and of what it took to produce it. She had taken one, or two, mathematics courses in college, and reported that she had liked them; but she had not pursued the subject further. Lacking deep knowledge, Mrs. O seemed unaware of much mathematical content and many ramifications of the material she taught. Many paths to understanding were not taken in her lessons--as for instance, in the Santa's letter example--but she seemed entirely unaware of them. Many misunderstandings or inventive ideas that her students might have had would have made no sense to Mrs. O, because her grip on mathematics was so modest. In these ways and many others, her relatively superficial knowledge of this subject insulated her from even a glimpse of many things she might have done to deepen students' understanding. Elements in her teaching that seemed contradictory to an observer therefore seemed entirely consistent to her, and could be handled with little trouble.

Additionally, however much mathematics she knew, Mrs. O knew it as a fixed body of truths, rather than as a particular way of framing and solving problems. Questioning, argument, and explanation seemed quite foreign to her knowledge of this subject. Her assignment, she seemed to think, was to somehow make the fixed truths accessible to her students. Explaining them herself in words and pictures would have been one alternative, but she employed a curriculum that promised an easier way--i.e., to embody mathematical ideas and operations in concrete materials and physical activities. Mrs. O did not see mathematics as a source of puzzles, as a terrain for argument, or as a subject in which questioning and explanation were essential to learning and knowing--all ideas that are plainly featured in the *Framework* (California State Department of Education, 1985, pp. 13-14). *Math Their Way* did nothing to disturb her view on this matter. Lacking a sense of the importance of explanation, justification, and argument in mathematics, she simply slipped over many opportunities to elicit them, unaware that they existed.

So the many things that Mrs. O did not know about mathematics protected her from many uncertainties about teaching and learning math. The limitations of her knowledge also made it difficult for her to learn from her very serious efforts to teach for understanding. Like many students, what she didn't know kept her from seeing how much more she could understand about mathematics. It also kept her from imagining many different ways in which she might teach mathematics. These limitations on her knowledge meant that Mrs. O could "teach for understanding," with little sense of how much remained to be understood, how much she might incompletely or naively understand, and how much might still remain to be taught. She is a thoughtful and committed teacher. But working as she did near the surface of this subject, many elements of understanding and many pedagogical possibilities remained invisible. Mathematically, she was on thin ice. But she did not seem to know it, and so skated smoothly on with great confidence.

In a sense, then, the tensions that I observed were not there. They were real enough in my view, but they did not enter the public arena of the class. They lay beneath the surface of the class's work; indeed, they were kept there by the nature of that work. Mrs. O's modest grasp of mathematics, and her limited conception of mathematical understanding simply obliterated many potential sources of roughness in the lessons. And those constraints of the mind were given added social force in her close management of classroom discourse. Had Mrs. O known more math, and tried to construct a somewhat more open discourse, her class would not have run so smoothly. Some of the tensions that I noticed would have become audible and visible to the class. More confusion and misunderstanding would have surfaced. Things would have been rougher, potentially more fruitful, and vastly more difficult.

Practice and Progress

Is Mrs. O's mathematical revolution a story of progress, or of confusion? Does it signal an advance for the new *Mathematics Framework*, or a setback?

These are important questions, inevitable in ventures of this sort. But it may be unwise to sharply distinguish progress from confusion, at least when considering such broad and deep changes in instruction. After all, the teachers and students who try to carry out such change are historical beings. They cannot simply shed their old ideas and practices like a shabby coat, and slip on something new. Their inherited ideas and practices are what teachers and students know, even as they begin to know something else. Indeed, taken together those ideas and practices summarize them as practitioners. As they reach out to embrace or invent a new instruction, they reach with their old professional selves, including all the ideas and practices comprised therein.

The past is their path to the future. Some sorts of mixed practice, and many confusions, therefore seem inevitable.

This point often goes unnoticed by those in the throes of change, as well as by those who promote and study it. The changes in Mrs. O's teaching that seemed paradoxical to me seemed immense to her. Remember that when she began teaching four years ago, her math lessons were quite traditional. She ignored the mathematical knowledge and intuitions that children brought to school. She focused most work on computational arithmetic, and required much classroom drill. Mrs. O now sees her early teaching as unfortunately traditional, mechanical, and maladapted to children's learning. Indeed, her early mathematics teaching seemed exactly like the sort of thing that the *Framework* criticized.

Mrs. O described the changes she has made as a revolution, I do not think that she was deluded. She was convinced that her classes had greatly improved. She contended that her students now understood and learned much more math than their predecessors had, a few years ago. She even asserts that this has been reflected in their achievement test scores. I have no direct evidence on these claims. But when I compared this class with others that I have seen, in which instruction consisted only of rote exercises in manipulating numbers, her claims seemed plausible. Many traditional teachers certainly would view her teaching as revolutionary.

But all revolutions preserve large elements of the old order as they invent new ones. One such element, noted above, was a conception of mathematics as a fixed body of knowledge. Another was a view of learning mathematics in which the aim was getting the right answers. I infer this partly from the teaching that I observed, and partly from several of her comments in our conversations. She said, for example, that math had not been a favorite subject in school. She had only learned to do well in math at college, and was still pleased with herself on this score, when reporting it to me years later. I asked her how she had learned to do and like math at such a late date, and she explained: "... I found that if I just didn't ask so many why's about things that it all started fitting into place ..." (interview, 12/88). This suggests a rather traditional approach to learning mathematics. More important, it suggests that Mrs. O learned to do well at math by avoiding exactly the sort of questions that the *Framework* associates with understanding mathematics. She said in another connection that her view of math has not changed since college. I concluded that whatever she has learned from workshops, new materials, and new policies, it did not include a new view of mathematics.

Another persistent element in her practice was "clinical teaching," that is, the California version of Madeline Hunter's Instructional Theory Into Practice (ITIP). Hunter and her followers advocate clearly structured lessons: Teachers are urged to be explicit about lesson objectives themselves, and to announce them clearly to students. They also are urged to pace and control lessons so that the intended content is covered, and to check that students are doing the work and getting the point, along the way. Though these ideas could be used in virtually any pedagogy, they have been almost entirely associated with a rigid, sonata-form of instruction, that is marked by close teacher control, brisk pacing, and highly structured recitations. ITIP appears to have played an important part in Mrs. O's own education as a teacher, for on her account she learned about it while an undergraduate, and used it when she began teaching. But she also has been encouraged to persist: Both her Principal and Assistant Principal were devotees of Hunter's method, and have vigorously promoted it among teachers in the school. This is not unusual, for ITIP has swept California schools in the past decade. Many principals now use it as a framework for evaluating teachers, and as a means of school improvement. Mrs. O's Principal and the Assistant Principal praised her warmly, saying that she was a fine teacher with whom they saw eye to eye in matters of instruction.

I asked all three whether clinical teaching worked well with the *Framework*. None saw any inconsistency. Indeed, all emphatically said that the two innovations were "complementary."

Though that might be true in principle, it was not true in practice. As ITIP was realized in Mrs. O's class among many others, it cut across the grain of the *Framework*. For like many other teachers, her enactment of clinical teaching rigidly limited discourse, closely controlled social interaction, focused the classroom on herself, and helped to hold instruction to relatively simple objectives.

As Mrs. O revolutionized her math teaching, then, she worked with quite conventional materials: A teacher-centered conception of instructional discourse; a rigid approach to classroom management; and a traditional conception of mathematical knowledge. Yet she found a way to make what seemed a profound change in her math teaching. One reason is that the vehicle for change did not directly collide with her inherited ideas and practices. *Math Their Way* focused on materials and activities, not on mathematical knowledge and explanation of ideas. It allowed Mrs. O to change her math teaching in what seemed a radical fashion while building on those old practices. This teacher's past was present, even as she struggled to renounce and surpass it.

Mrs. O's past also affected her view of her accomplishments, as it does for all of us. I asked, in the Spring of 1989, where her math teaching stood. She thought that her revolution was over. Her teaching had changed definitively. She had arrived at the other shore. In response to further queries, Mrs. O evinced no sense that there were areas in her math teaching that needed improvement. Nor did she seem to want guidance about how well she was doing, or how far she had come.

There is an arresting contrast here. From an observer's perspective, especially one who had the new *Framework* in mind, Mrs. O looks as though she may be near the beginning of growth toward a new practice of math teaching. But she sees the matter quite differently: She has made the transition, and mastered a new practice.

Which angle is most appropriate--Mrs. O's or the observer's? This is a terrific puzzle. One wants to honor this teacher, who has made a serious and sincere effort to change, and who has changed. But one also wants to honor a policy that supports greater intelligence and humanity in mathematics instruction.

It is worth noticing that Mrs. O had only one perspective available. No-one had asked how she saw her math teaching, in light of the *Framework*. She had been offered no opportunities to raise this query, let alone assistance in answering it. No-one offered her another perspective on her teaching. If no other educators or officials in California had seen fit to put the question to her, and to help her to figure out answers, should we expect her to have asked and answered this difficult question all alone?

That seems unrealistic. If mathematics teaching in California is as deficient as the *Framework* and other critiques suggest, then most teachers would not be knowledgeable enough to raise many fruitful questions about their work in math by them selves, let alone answer them. We can see some evidence for this in Mrs. O's lessons. Their very smoothness quite effectively protected her from experiences that might have provoked uncertainty, conflict, and therefore deep questions. But even if such questions were somehow raised for Mrs. O and other teachers, the deficiencies in their practice, noted in many recent reports, would virtually guarantee that most of these teachers would not know enough to respond appropriately, on their own. How could teachers be expected to assess, unassisted, their own progress in inventing a new sort of instruction, if their mathematics teaching is in the dismal state pictured in the policy statements demanding that new instruction?

Additionally, if teachers build on past practices as they change, then their view of how much they have accomplished will depend on where they start. Teachers who begin with very traditional practices would be likely to see modest changes as immense. What reformers might see as trivial, such teachers would estimate as a grand revolution--especially as they were just beginning to change. From a perspective still rooted mostly in a traditional practice, such initial changes would seem--and be--immense. That seemed to be Mrs. O's situation. She made what some observers might see as tiny and perhaps even misguided changes in her teaching. But like other teachers who were taking a few first small steps away from conventional practice, for her they were giant steps. She would have to take many more steps, and make many more fundamental changes before she might see those early changes as modest.

So, if California teachers have only their subjective yardsticks with which to assess their progress, then it seems unreasonable to judge their work as though they had access to much more and better information. For it is teachers who must change in order to realize new instructional policies. Hence their judgement about what they have done, and what they still may have to do, ought to be given special weight. We might expect more from some teachers than others. Those who had a good deal of help in cultivating such judgment--i.e., who were part of some active conversation about their work, in which a variety of questions about their practice were asked and answered, from a variety of perspectives--would have more resources for change than those who had been left alone to figure things out for themselves.

The same notion might be applied to policies like the new *Framework*, that seek to change instruction. We might expect only a little from those policies that try to improve instruction without improving teachers' capacity to judge the improvements and adjust their teaching accordingly. For such policies do little to augment teachers' resources for change. In Mrs. O's case, at least, the *Framework* has been this sort of policy. We might expect more from policies that help teachers to cultivate the capacity to judge their work from new perspectives, and that add to teachers' resources for change in other ways as well. The new instructional policy of which the *Framework* is part has not done much of this for Mrs. O.

What would it take to make additional, helpful, and usable guidance available to teachers? And what would it take to help teachers pay constructive attention to it? Neither query has been given much attention so far, either in efforts to change instruction or in efforts to understand such change. But without good answers to these questions, it is difficult to imagine how Mrs. O and most other teachers could make the changes that the *Framework* seems to invite.

Policy and Practice

Mrs. O's math classes suggest a paradox. This California policy seeks fundamental changes in learning and teaching. State policymakers have illuminated deficiencies in instruction and set out an ambitious program for improvement. Policy thus seems a chief agency for changing practice. But teachers are the chief agents for implementing any new instructional policy: Students will not learn a new mathematics unless teachers know it and teach it. The new policy seeks great change in knowledge, learning, and teaching, yet these are intimately held human constructions. They cannot be changed unless the people who teach and learn want to change, take an active part in changing, and have the resources to change. It is, after all, their conceptions of knowledge, and their approaches to learning and teaching that must be re-vamped.

Hence teachers are the most important agents of instructional policy (Lipsky, 1980; Cohen, 1989). But the state's new policy also asserts that teachers are the problem. It is, after all, their knowledge and skills that are deficient. If the new mathematics *Framework* is correct, most California teachers know far too little mathematics, or hold their knowledge improperly, or both. Additionally, most do not know how to teach mathematics so that students can understand it. This suggests that teachers will be severely limited as agents of this policy: How much can practice improve if the chief agents of change are also are the problem to be corrected?

This paradox would be trivial if fundamental changes in learning and teaching were easy to make. Yet even the new *Framework* recognizes that the new mathematics it proposes will be "...

difficult to teach" (California State Department of Education, 1985, p. 13). Researchers who have studied efforts to teach as the *Framework* intends also report that it is difficult, often uncommonly so. For students cannot simply absorb a new "body" of knowledge. In order to "understand" these subjects, learners must acquire a new way of thinking about a body of knowledge, and must construct a new practice of acquiring it (Lampert, 1988). They must cultivate strategies of problem solving that seem relatively unusual and perhaps counter-intuitive (DiSessa, 1983). They must learn to treat academic knowledge as something they construct, test, and explore, rather than as something they accept and accumulate (Cohen, 1989). Additionally, and in order to do all of the above, students must un-learn acquired knowledge of math or physics, whether they are second graders or college sophomores. Their extant knowledge may be naive, but it often works.

A few students can learn such things easily. Some even can pick them up more or less on their own. But many able students have great difficulty in efforts to "understand" mathematics, or other academic subjects. They find the traditional and mechanical instruction that the *Framework* rejects easier and more familiar than the innovative and challenging instruction that it proposes.

If such learning is difficult for students, should it be any less so for teachers? After all, in order to teach math as the new *Framework* intends, most teachers would have to learn an entirely new version of the subject. To do so they also would have to overcome all of the difficulties sketched just above. For, as the *Framework* says of students, teachers could not be expected to simply absorb a new "body" of knowledge. They would have to acquire a new way of thinking about mathematics, and a new approach to learning it. They would have to additionally cultivate strategies of problem solving that seem to be quite unusual. They would have to learn to treat mathematical knowledge as something that is constructed, tested, and explored, rather than as something they broadcast, and that students accept and accumulate. Finally, they would have to un-learn the mathematics they have known. Though mechanical and often naive, that knowledge is well-settled, and has worked in their classes, sometimes for decades.

These are formidable tasks, even more so for teachers than students. For teachers would have a much larger job of un-learning: After all, they know more of the old math, and their knowledge is much more established. Teachers also would have to learn a new practice of mathematics teaching, while learning the new mathematics and un-learning the old. That is a very tall order. Additionally, it is difficult to learn even rather simple things--like making an omelette--without making mistakes. But mistakes are a particular problem for teachers. For one thing they are in charge of their classes, and they hold authority partly in virtue of their superior knowledge. Could they learn a new mathematics and practice of mathematics teaching, with all the trial and error that would entail, while continuing to hold authority with students, parents, and others interested in education? For another, teachers are responsible for their students' learning. How can they exercise that responsibility if they are just learning the mathematics they are supposed to teach, and just learning how to teach it? American education does not have ready answers for these questions. But there was no evidence that the *Framework* authors, or educators in Mrs. O's vicinity had even asked them. It is relatively easy for policymakers to propose dramatic changes in teaching and learning, but teachers must enact those changes. They must maintain their sense of responsibility for students' accomplishments, and the confidence of students, parents, and members of the community. But most schools offer teachers little room for learning, and little help in managing the problems that learning would provoke.

The new *Mathematics Framework* seemed to recognize some problems that students would have in learning a new mathematics. But the state has not acted as though it recognized the problems of teachers' learning. Mrs. O certainly was not "taught" about the new mathematics in a way that took these difficulties into account. Instead, the California State Education Department taught her about the new mathematics in a way that closely resembled the very pedagogy that it criticized in the old mathematics. She was told to do something, like students in many traditional math classrooms. She was told that it was important. Brief explanations were offered, and a synopsis of what she was to learn was provided in a text. In effect, California education officials offered Mrs. O a standard dose of "knowledge telling." The state acted as though it assumed that fundamental instructional reform would occur if teachers were told to do it.

If, as the *Framework* argues, it is implausible to expect students to understand math simply by being told, why is it any less implausible to expect teachers to learn a new math simply by being told? If students need a new instruction to learn to understand mathematics, would not teachers need a new instruction to learn to teach a new mathematics? Viewed in this light, it seems remarkable that Mrs. O made any progress at all.

What more might have been done, to support Mrs. O's efforts to change? What would have helped her to make more progress toward the sort of practice that the *Framework* proposed? It is no answer to the question, but I note that no-one in Mrs. O's vicinity seemed to be asking that question, let alone taking action based on some answers.

* * * * *

This new policy aspires to enormous changes in teaching and learning. It offers a bold and ambitious vision of mathematics instruction, a vision that took imagination to devise and courage to pursue. But this admirable policy does little to augment teachers' capacities to realize the new vision. For example, it offers rather modest incentives for change. I could detect few rewards for Mrs. O to push her teaching in the *Framework*'s direction--certainly no rewards that the state offered. The only apparent rewards were those that she might create for herself, or that her students might offer. Nor could I detect any penalties for non-improvement, offered either by the state or her school or district.

Similar weaknesses can be observed in the supports and guidance for change. The new *Framework* was barely announced in Mrs. O's school. She knew that it existed, but wasn't sure if she had ever read it. She did know that the Principal had a copy. The new *Framework* did bring a new text series, and Mrs. O knew about that. She knew that the text was supposed to be "aligned" with the *Framework*. She had attended a publisher's workshop on the book, and said it had been informative. She had read the book, and the teachers' guide. But she used the new book only a little, preferring *Math Their Way*. The school and district leadership seem to have thought *Math Their Way* was at least as well aligned with the *Framework* as the new text series, and permitted its substitution in the primary grades.

Hence the changes in Mrs. O's practice were partly stimulated by the new policy, but they were weakly guided and supported by it, or by the state agencies that devised it. There was a little more guidance and support from her school and district: She was sent to a few summer workshops, and she secured some additional materials. But when I observed Mrs. O's teaching there seemed to be little chance that she would be engaged in a continuing conversation about mathematics, and teaching and learning mathematics. Her district had identified a few "mentor teachers" on whom she could call for a bit of advice if she chose. But there was no person or agency to help her to learn more mathematics, or to comment on her teaching in light of the *Framework*, or to suggest and demonstrate possible changes in instruction, or to help her try them out. The new mathematics *Framework* greatly expanded Mrs. O's obligations in mathematics teaching without much increasing her resources for improving instruction. Given the vast changes that the state has proposed, this is a crippling problem.

Mrs. O's classroom reveals many ambiguities, and, to my eye, certain deep confusions about teaching mathematics for understanding. But she has been more successful in helping her students to learn a more complex mathematics than California has been in helping her to teach a more complex mathematics. From one angle this situation seems admirable: Mrs. O has had considerable discretion to change her teaching, and she has done so in ways that seem well-adapted to her school. Though I may call attention to the mixed quality of her teaching, her superiors celebrate her work. But from another angle it seems problematic. For if we take the *Framework*'s arguments seriously, then Mrs. O should be helped to struggle through to a more complex knowledge of mathematics, and a more complex practice of teaching mathematics. For if she cannot be helped to struggle through, how can she better help her students to do so? Some researchers and other commentators on education have begun to appreciate how difficult it is for many students to achieve deep understanding of a subject, an appreciation that is at least occasionally evident in the *Framework*. But there is little appreciation, in the *Framework* or anywhere else that I could find in California education, of how difficult it will be for teachers to learn a new practice of mathematics instruction.

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Transformation and Accommodation:

The Case of Joe Scott

Nancy Jennings Wiemers

Introduction

Interviewer: And what does it mean to you to make them feel successful? Two questions I guess. What does it mean to you and what do you think it means to the kids?

Joe: For me basically it means that I'm doing a good job. If they're successful, then I'm doing what I'm supposed to be doing. For them I think it's probably more complex and it depends on each youngster. For them it means getting an E. For them it means doing about half the problems. For them it means making Mr. Scott happy and that I share a good word with them. . . . For maybe some of them that are very sensitive, [it means] `Gosh, I didn't know how to do this stuff and now I know how to do it.'

Joe Scott has been teaching fifth grade in Viceroy for fourteen years. He is considered an excellent teacher by his principal and colleagues. A self-described "traditional" teacher, Joe's instruction is predominantly teacher-centered: classroom discourse consists of students answering questions Joe asks. Lessons are drawn from the textbook with a large amount of class time being devoted to helping students do exercises. Joe calls himself "the captain of his ship" in his classroom and without question, he is the man in control.

Joe is enthusiastic and committed to teaching. Confident of his own teaching ability, Joe also firmly believes in his students' abilities to learn. Joe wants his students to gain "insight" into mathematics, which for him means an understanding of when and how to use the mathematical knowledge they possess. He is confident all his students can gain this insight, if he provides the right instruction and compelling incentives. Enabling students to feel good about themselves and about their mathematical competence are Joe's main goals. And in watching Joe interact with his students, it is obvious that he knows them well and is concerned about their learning.

Energy and anxiety, quickness and competition are constant features of Joe's mathematics class. Students hurry to finish timed drill tests, compete with each other in computational games, and work to get good grades which carry with them the reward of less work on the next day's homework. Joe is familiar with the *California Mathematics Framework* (California State Department of Education, 1985) but disagrees with some of its underlying principles such as *how* mathematics should be taught and what knowing mathematics means. He particularly disagrees with what he views as the *Framework*'s dismissal of the importance of rote memorization and computation skills. Both of these are important elements of Joe's mathematics instruction, even though by using them he realizes he "may be off on a tangent somewhere--away from the *Framework*" (interview, 12/89). Being on a tangent is okay with Joe who sees himself as a maverick anyway--someone who follows his own lights and is not easily convinced by new ideas. His teaching emphasizes rules, memorization, "tools," mechanical mastery, competition and facts. The *Framework* has not altered these beliefs or practices.

But despite Joe's overt resistance to many of the new ideas about mathematics, Joe's teaching has changed because of the reform. His very traditionalism and commitment to finding better ways to teach students have led him to try out some ideas that to Joe are novel. Like many elementary teachers, Joe follows the lessons in his mathematics textbook when he plans his instruction. Since the new textbook includes topics and ideas that Joe previously ignored, his "traditional" reliance on the textbooks as a lesson planner has led him into uncharted territory. Some of the new ways have worked very well and Joe speaks of them with great enthusiasm. From this perspective, he is an established, well-regarded teacher who is conscientiously attempting to make sense of the new directives and integrate them into his practice.

Yet, the nature of the changes which have found their way into Joe's practice is still unclear. Would an observer looking for examples of teaching mathematics "for understanding" find any in Joe's instruction? Are the changes limited to the content taught and small changes in pedagogy--how best to teach content--or do they represent a movement toward fundamental change in Joe's ideas about mathematical knowledge and learning? Are these changes constrained because they primarily spring from Joe's commitment to use the textbook rather than from a fundamental commitment to new perspectives on teaching mathematics?

A View of Joe's World

Joe has a self-contained, mixed-ability classroom with more girls than boys. Most of his students are white, with a few Hispanics and Blacks. One child in Joe's class receives extra help in mathematics in the school's resource center. A few other children are designated "gifted" by the school and receive extra instruction in a variety of areas. At the time of my observations, only one student had a less than full command of English.

The school exhibits many characteristics associated with upper-middle class schools. There is a great deal of parental interest and involvement; aspirations of academic achievement pervade the school (principal interview, 12/88). In one classroom, students changed the song "It's a Grand Old Flag" to say: "It's a grand old school, it's a high-ranking school . . ." Both the school's principal and Joe talked about parental interest in mathematics and about parental pressure for student achievement in mathematics. Achievement meant, it seemed, getting high scores on standardized tests.

Joe's classroom is neat and orderly. Desks are lined in rows facing the front chalkboard. A few desks are pushed together, out of alignment; Joe explained that he allows new students for whom English is difficult to "team up" with others for help. There are charts on bulletin boards to record students' achievement on timed, math-fact tests. Students who receive perfect scores have stars next to their names, others have blank spaces. Student work in social studies and language arts fills other bulletin boards in the room.

Joe's Own Learning of Mathematics

Mathematics is Joe's favorite subject to teach and was his favorite subject as a student. He said he likes to "figure things out" and has always been successful in doing so. Joe told a story about a favorite mathematics teacher that he had for high school geometry:

> I liked math as a student. I particularly liked geometry. When I was in high school there were two of us, Wayne and I, and we would get Es on the test. I had a male teacher whose name was Mr. Walker and who was also a coach in basketball. And he was less concerned about all the practice as long as we got Es on our tests. So that if we got Es on the chapter tests, then we could go through the entire next chapter and we didn't have to do the work . . . Wayne and I would [do that.] We'd be selective and we would probably do a couple at the beginning and a couple at the end. But, we didn't have to do the work until we didn't get an E. . . . And that's a lot of incentive. But, the bottom line is that we could do less work as long as we were successful. (interview, 3/89)

Joe follows the same principle of reducing work in exchange for good grades as an incentive for his students to work hard. He tells his students often that he wants them to be "successful" in mathematics, as Wayne and he were, which means finishing work and finding correct answers quickly. Joe is confident about his own knowledge of mathematics as well as his ability to teach it. The key to teaching, in Joe's eyes, is providing students with the right tools to understand mathematics and offering them incentives to invest the time and hard work it takes to acquire these tools.

Ideas about the *Framework*

Joe first learned about the *California Mathematics Framework* through his work on a district committee. Because of his reputation as a good teacher and his interest in mathematics, Joe's opinions of the *Framework* and of the textbooks they were reviewing were sought frequently by teachers. Joe's principal commented:

> He knows exactly what is required and we use him to tell the people on the book committee what's wrong with the book--how this matches the CTBS or something else. He analyzes all of that stuff and he's a real professional. (principal interview, 12/88)

Joe sees the goal of the *Framework* as helping students to learn how to apply mathematics; that is, to take the mathematical knowledge they learn in school and use it in their everyday lives. For instance, one of the difficulties that Joe mentioned in teaching fractions to fifth graders in ways as the *Framework* would advocate, is that fifth graders don't often use fractional parts to figure out things outside of school. This makes it difficult for him to make the lessons "relevant" with believable examples. Although he finds it difficult, he doesn't disagree with this perspective of the *Framework*: "I think their [the *Framework*'s developers'] intent . . . is that they want us to reach toward children having an understanding of applications. . . . That's the bottom line, is that they want us to be more accountable in that way" (interview, 12/88). Teaching for understanding, then, in Joe's eyes means understanding applications of the traditional mathematics learned in school.

Even though he disagrees with some of the claims of the *Framework*'s authors, Joe does intend to modify his instruction so that students reach his perception of the *Framework*'s "bottom line". However, he intends for them to reach it using his own methods. Joe talks about "getting his students there" using the "tools that I have access to and the insights that I may have in dealing with these kids for 14 or 15 years" (interview, 12/88). He feels accountable to attend to the *Framework*, but his concept of what he needs to attend to is limited to what he views as the end product of the *Framework*, "teaching for relevance and application." If he can help his students achieve an understanding of application, then he is doing what the state requests, but doing it in ways he finds most effective.

Where Joe disagrees most strongly with the *Framework* is in the recommendations the authors make about instructional strategies that are linked to teaching for understanding (e.g., cooperative learning, the use of manipulatives). And even though the *Framework* authors baldly state that "delivery of instruction is inseparable from curricular content," Joe continues to separate the two (p. 12). He knows the connection the *Framework* draws between content and delivery, but he disagrees with it. After reviewing the *Framework*'s list which contrasts characteristics of teaching for understanding and teaching rules and procedures (see Table 1), Joe commented that the "writers painted a kind of black and white picture" (interview, 12/88) instead of recognizing the need to blend features of the two lists to meet different students' needs. Different students respond to different methods and Joe feels more comfortable with his methods, techniques he has developed through years of experience. His continued use of rote memorization of rules and competition rather than cooperative learning are examples of intentional resistance to what Joe knows the *Framework* advocates: "I'm very comfortable with my approach, whether or not it is consistent with the state's" (interview, 12/88). Joe is willing to accommodate what he sees as the state's goal of application, but at this point he is not willing to abandon his own beliefs about *how* to get there. This is not unreasonable given his past success with such practices.

Table 1

Teaching for understanding	Teaching rules and procedures
Emphasizes understanding	Emphasizes recall
Teaches a few generalizations	Teaches many rules
Develops conceptual schemas or or interrelated concepts	Develops fixed or specific processes or skills
Identifies global relationships	Identifies sequential steps
Is adaptable to new tasks or situations (broad application)	Is used for specific tasks or situations (limited context)
Takes longer to learn but is retained more easily	Is learned more quickly but is quickly forgotten
Is difficult to teach	Is easy to teach
Is difficult to test	Is easy to test

(California State Department of Education, 1985, p. 13)

Accommodating his practice to meet what he sees as the *Framework*'s goal for mathematics instruction while resisting the *Framework*'s messages about delivery may not, however, be Joe's final response to the reform. Joe thinks the *Framework* developers are naive in thinking that changes in practice of the magnitude they are calling for could happen quickly. According to Joe, changes in elementary mathematics instruction need to be woven into extant practice slowly, not only because of teachers, but students as well:

> I think it's a little naive that we could get a good percentage . . . of kids to be able to apply at this point, in the first year of adjustment to the text--to be able to apply those things and get across all of the things that they may be deficient in right now. When you deal with fifth graders and they haven't had access to this new approach. . . . They don't have the background that kids do in first or second grade. They're [1st or 2nd graders] going to be introduced into this, which is great for them, but what do you do with fourth and fifth graders? (interview, 12/88)

Joe sees changes--such things as increased use of manipulatives or diagrams-as occurring slowly over time as students go through the early elementary grades learning mathematics differently. He sees himself having to change his practice to be more aligned with the *Framework*'s vision in order to meet his future students' needs. In the first year then, Joe's response to the *Framework* has been to guide his instruction more toward application skills than he has in the past, but he is very aware that this response to the *Framework* may not be sufficient in the future.

This sense that deeper and more extensive change in the future may be necessary springs out of Joe's great sense of accountability; accountability not only to his own beliefs about how students should learn mathematics, but also to what the school district and the state expect of him as a teacher. When asked why he thought the state developed the *Framework*, Joe replied that it was a way for the state to set standards that it expects teachers to reach:

> I think to know what the state is saying, this is what we expect you to teach, and that message gets to teachers and hopefully staff is professional enough that they feel compelled--well, if this is what the children are supposed to know, then I have a responsibility to get them there. (interview, 12/88)

Joe feels responsible to pay attention to the *Framework*, to "what the children are supposed to know." But since he separates the issue of goals from method, Joe has chosen to focus on the *Framework*'s messages about *what* to teach, avoiding at present its recommendations about how to teach. He is willing to fiddle with the topics he teaches and adjust his instruction toward application as long as he doesn't have to give up his notions about *how* best to teach or how best children learn. So far he sees his use of the district's newly adopted textbook as providing him with a way to be responsible to state expectations. If he teaches the content in the text--since it is aligned to the *Framework*--then he is teaching as the state wishes. In Joe's 14 years of teaching mathematics, he has always used textbooks as his major source of instructional guidance, so using the new text is not a new pattern for Joe. He is able to comply with state expectation in his view by doing something he has always done.

Joe's Mathematics Instruction

Mathematics is first on the docket in Joe's classroom each day. When the students enter the classroom, they find computational exercises already written on the board. Instead of milling about and chatting with friends, most of the students sit down, get out their pencils and paper, and begin working on the exercises. Joe quickly reminds the few who resist this pattern to get to work and then efficiently takes care of attendance and other record-keeping tasks while the students finish. After a few minutes, Joe tells them to stop and exchange papers to correct each other's work. Every day, the work is corrected, graded, and handed in. While correcting each other's papers, the classroom noise level is uncharacteristically loud with students asking their graders how they are doing on their papers. Exasperated gasps or sighs of relief punctuate the lesson. How well they do on the exercises is clearly important to the students.

During two of my observations, Joe gave students one-minute tests on the "rules of divisibility" for the numbers 2, 3, 4, 5, 9, and 10, including such things as "If a number has a 5 or 0 in the ones column it is divisible by 5" or "If the digits of a number add up to a number divisible by 3, it is divisible by 3." Students were to have memorized these rules and, on these tests, were asked to decide which 3-digit numbers from a list of 10 were divisible by the numbers whose rules of divisibility they knew. During these tests, students' anxiety was even higher than during other opening exercises. Not only would they get a grade on this test but, if they got all of them correct, a star would be placed next to their name on a chart displayed prominently in the front of the room.

After the opening exercises, Joe moves on to the lesson. All three lessons I observed were taken from the text. For one lesson on least common denominators, Joe wrote on the board: "Least Common Denominator = Least Common Multiple. To find: Count by the larger denominator until you have a multiple of the smaller denominator." He then had students read together the words on the board and he repeated them often during the lesson. Joe would read an example from the book--the least common denominator of 1/4 and 1/6--recite the procedure written on the board, and then coach students step-by-step through the procedure:

Joe:	The multiples of four are what, Darcy?
Darcy:	4,8,12,16. . .
Joe:	Matthew, the multiples of six are what?
Matthew:	6,12,18. . . .
Joe:	Good. . . The smallest number, the smallest denominator, shared by both fourths and sixths, are what? What is the smallest number that is shared by both?
Class:	12.
Joe:	The smallest one that is shared by both is 12. So then twelve becomes the least common denominator, or least common multiple. Any questions? Okay. (observation, 12/88)

A firm believer in the Madeline Hunter method of direct instruction, Joe follows Hunter's guidelines closely. For the above lesson, the advance organizer written on the board was an example of finding the least common denominators. Next was direct instruction on the procedures for finding least common denominators: Joe made sure that each procedural step was clearly articulated. This was followed by guided practice of text exercises which required students to use the procedure. The class ended with Joe assigning similar problems as homework, due the next day. All the lessons I observed followed this pattern. After one lesson, Joe expressed concern that he hadn't followed the final step in Hunter's program, closure:

> Until it fell down there was a little poster that was up there, that had to do with teaching delivery. [You] try to focus the children's attention and ask them basically direct questions to get as many of them involved as you can. . . . Basically, you guide the practice that they work on with you and then you go into independent practice and then try to bring closure. I didn't bring closure today, as a matter of fact, at all. I'll probably have to. I'll have to come back to them and say--although we did give some rationale why we do it, I don't think we brought factoring to a close. . . . I was disappointed a little bit in the lesson itself, and that was partly me. (interview, 12/88)

Much of mathematics time is given to practice with new content. Joe goes over sample questions in the text with students walking them through each step. His focus is on getting students to memorize the procedure and to learn the different ways in which to apply the procedure. Although all the exercises in the least common denominator lesson required students to use the skill of finding least common denominator for two fractions, each section's exercises were written slightly differently. With the whole class, Joe went over at least two exercises in each section to make sure students would be able to do the work. The last section of problems in the lesson was written in the following form: "2/3 = n/6. Find n." Concerned that the unfamiliar format would confuse his students, Joe wrote on the board 2/3 x _/_ = n/6. He then said: "Isn't this the problem? What do we have to multiply 3 by to get 6, Shanna?" The student answered "2." Joe wrote 2s in the blanks next to 2/3 and then wrote n = 4. "And that's the answer" (observation, 12/89).

After practicing together, students are given time during class to work on problems on their own. During this time Joe's concern for students is most clear. In all three lessons I observed, Joe talked to every student about what they were doing, often asking them what they did the night before or how they were getting along with their families. He would gently ask them questions about the exercises they were doing to check for understanding. The students know that Joe is committed to their mastery of mathematics. He often spends recess and lunch time on the playground helping students with work. His room is a buzz with students before and after school who need help. Joe told me a story about a student who was having a hard time passing the rules of divisibility test:

> I talked to her at lunch time, no, maybe it was morning recess, and she showed a concern that she didn't quite grasp what we were doing and I had her come in and asked her what she understood about the rules and asked her how I could help her. (interview, 12/88)

Joe has genuine concern for his students "being successful" and works hard to ensure that success.

Although students talk easily with Joe, student discourse in this class is limited. Students ask or answer questions about particular algorithms and problems. At no time during the lessons did students explain how they came up with answers--what strategies they used or what they were thinking. Students eagerly competed with one another to be the one to provide the short answer. Joe never lacked for volunteers. In one lesson, the class was correcting a worksheet on factors on which they were given a number to factor. First, Joe announced the number to be factored and then a student was chosen to call out the factors. If the student giving the factors made a mistake, Joe yelled "STOP!" and called on another student to continue:

Joe:	Okay, you'll continue to call the numbers until you make a mistake. The key to factoring is that you have them all. These problems will be scored right or wrong, 1 or 0. That if you have 10 out of 11 factors right, that would be wrong. You've been given information, now you are responsible to use it . . .[Problem] #6, Matt?
Matt:	1,2,3,6 . . .
Joe:	Stop! Aimee?
Aimee:	1,2,3,4,6 . . .(observation, 12/88)

Matt was not asked to explain his mistake nor Aimee her correct answer. Even though the risk of failure seemed rather high to me, students seemed eager to call out answers throughout the lesson. Even Matt raised his hand to be given another opportunity. Attentive and engaged, students appeared energetic and interested during the whole lesson.

Joe frequently sets up activities which are competitive and demand fast recall of facts. Timed math-fact tests are frequent and successful scores decorate the room. As a special treat at the end of one mathematics lesson, students played "Round the World" competing against each other to shout out division facts first. The ultimate winner in the class got to compete against Joe. He commented about the value of the game in an interview later:

> I think they like to do it. They like to compete against me and I've been beaten. I've told them that I've been beaten before, but that they are going to have to be real good. Of course, I'm competing against kids, but they realize that the bottom line is that I will get involved as well. It was interesting. It was nice to see that they were kind of knocking each other off so that a lot of them felt like they were competitive. (interview, 12/88)

Competition is not limited to games. Students who get Es--for excellent--on their assignments, only have to do every other question on the next night's homework. After students graded the daily opening exercises, "E" papers were handed in first. Failing papers, however, were not handed in at all. Joe spoke of all of these techniques as incentives to get kids to try harder and to do the "hard work" of mathematics, memorizing rules. With the shouting of answers and the focus on speed and competition, the class atmosphere was like a benevolent boot camp.

<u>Mathematics as Tools</u>

> As I see it the mechanics of math is, and I think the state would agree, is the lowest level [of math]. But, if I can show them insight and say there are other techniques that we can learn in order to give you insight, then I think I'm doing a service to the children and that's why I'm here. (interview, 12/88)

Joe spends much of his mathematics time teaching students things he calls techniques or "tools." The rules of divisibility mentioned previously are tools. They give students "insight" and help them go beyond the "mechanics of mathematics." For instance, students operating at the mechanical level of mathematics would have to divide a number by 3 to discover if 3 was a factor. Students operating at the "insight" level would be able to use the rule of divisibility--"If the digits in a number add up to a multiple of three, that number is divisible by three"--to figure out the answer. Joe commented about the value of tools:

> Let's say 54. I would like for them to be able to look at 54 and say, `I know just by looking that 5 + 4=9 and because the sum of the digits is 9 that I know . . . 3 is going to work.' [Others] will have to sit down and do the division to see if it will work. I don't want them to do that. I want them to find, to know that there are short cuts and that as long as they're willing to put in the knowledge and to learn these rules, that it will . . . give them insight that they ordinarily wouldn't have. (interview, 12/88)

Tools also include mathematical vocabulary. Arranged on one bulletin board is a group of prefixes and suffixes with their Latin or Greek meanings attached, for example, peri = around, meter = measure, chrono = time. Joe said that students were always getting confused between terms such as perimeter and area and then didn't know how to figure out answers to problems. He thought if they spent some time memorizing important roots, then when they came across words they weren't sure of, they could decode them and "literally interpret exactly what it is they want you to do" (interview, 12/88). Tools then allow students to do things faster and more accurately.

For Joe, the only way to learn the tools of mathematics is to memorize them. The *Framework*'s discounting of learning by rote memory is one of his major disagreements with it:

> I'm not willing yet to let go of some of the rote memory that you see in here. . . well, I think, for certain children it's . . . going to reach some of those kids. [It] provides them an avenue-- `Oh, I see that I can use this tool.'--and it is a tool, that's all I intend it to be. . . . [If] they invest some time to do, let's say the rules of divisibility, it will give them insight into problem solving. (interview, 12/88)

Memorizing rules and learning techniques allow students to put more tools in their tool chests. These tools are handy because "if you take the time, invest the time to learn these rules, then it's going to save you time later on."

For Joe, mathematics tools lead to insight into "problem-solving." Insight means the ability to do mathematics work quickly and correctly so as to be successful as a mathematics student. In December, Joe's students were memorizing the rules of divisibility. In March, they were beginning a unit on adding and subtracting unlike fractions. During one lesson in March Joe asked them to remember the rules of divisibility to help they figure out common multiples:

Joe:	Remember the rules of divisibility. Is 5 a factor of 60,Darcy?
Darcy:	Yes
Joe:	How do you know?
Darcy:	Because there is a zero in the ones place.
Joe:	Good! . . . Use the tools you know or after a while they rust (observation, 3/89).

Joe connected this tool to factoring numbers and finding least common multiples. Students who remembered the tool had greater insight into the exercises they were working on than those students who had forgotten the tool. He expected students to be able to do the factoring procedures quickly because he had given them the necessary tools. This is what Joe means by "doing well" in mathematics: having enough mathematics tools and knowing how to use them to find quickly and correctly answers to questions in the book, on homework, and on classroom and standardized tests. Memorization and drill exercises are useful for they keep these tools sharp.

Like the expression "You don't need to know how a carburetor works to drive a car," for Joe, students do not need to know how or why mathematics tools work, they only need to be able to use them. And he emphasizes technical mastery of tools, not conceptual understanding. Mathematics tools are the rules and procedures that comprise the fixed body of mathematical knowledge students must learn. These tools include number and vocabulary facts, algorithms, procedures, and heuristics. Joe's job is to help students acquire as many tools as they can.

Confidence and Competence

Part of the reason mathematics tools are important to Joe is because they help accomplish one of his major objectives: preparing students for sixth grade mathematics Joe talks of his job as "getting [the students] from here to there and I'll do everything I can to get them ready for where those teachers next year are going to pick up the ball and say okay!" (interview, 12/88). He is confident that he can prepare students: "I see math for me as a strength, not a liability. . . . I've felt comfortable with what I've been doing . . . and I feel comfortable that children who come out of here will have a good background in math" (interview 4, p. 12). He intentionally tells students that mathematics is hard work; by raising their anxiety level through competition, he motivates them to strive for mastery. Joe helps students automatize their use of tools. For example, he uses the timed mathematics tests to raise "their level of anxiety probably a little bit in that they need to be able to be so familiar with the rule that they can apply it" (interview, 12/88). His reliance on competition certainly makes students anxious, but in Joe's view it is helpful in getting them to think quickly.

But Joe isn't only interested in making students anxious and having them compete so that they will have agile mathematical minds. He is also committed to developing students' confidence. So while statements concerning the difficulty of mathematics may initially make some students nervous, their eventual success (which he ensures by teaching them well) increases their sense of accomplishment. If they make it in Joe's class, they can make it anywhere:

> I think they feel an accomplishment that they realize it was hard to do, but they're successful and that's what I want to do. I do want to raise their level of anxiety a little bit, but they've realized that once they've done it and they're successful, that it is really meaningful to them. (interview, 3/89)

Joe talked about how he wants his students to feel. He is convinced that if he is doing his job as he should and if students are investing time to learn the rules and procedures of mathematics as he presents them, then all of his students will be successful:

> For me, I want them to come into math . . . feeling good about it and looking forward to the math period--'Hey, we're going to do this today and then I'm going to know how to do it.' . . . I had some recreation theory years ago in college and a third of the enjoyment of participating in recreation is the anticipation of the event itself. You know, a third is the event and another third of it is looking back at the memory of doing it. . . . So the same thing is true in school. . . . It's looking forward to coming back the next day and saying and being successful and their memory of being successful and all of that built in. . . . That's the way I want kids to feel. That's success for them, I think. (interview, 3/89)

All of his students will be able to go into sixth grade and handle the work. This is Joe's goal for mathematics instruction and, as he said--"in my heart I think that they all can do it."

The Textbook as Change Agent

Clearly, Joe has a vision of what mathematics should be and how to teach it. But as a committed teacher, he is "always learning new things and the new text provides me with new insight, new approaches and a new focus" (interview, 3/89). He has been following the new textbook closely this year to find whatever new insight it might provide. Even when the text presents topics in an unfamiliar sequence, Joe has been willing to try things out, see what happens, see what kids learn with new ways. Because his willingness to "try things out" in the text, Joe articulated two major changes that have occurred in his teaching. For one, he believes that he teaches problem-solving exercises more than before. He also believes he has started using more pictorial representations. After talking to Joe and observing him teach, it is clear that these features are now part of his mathematics instruction. It is not clear though, what these changes mean to Joe. Neither seem to challenge his fundamental beliefs about mathematics as a fixed body of procedures which students need to learn, nor do they contradict his emphasis on mathematics as tools. They instead seem to be just more tools which Joe can now give his students to pack in their tool chests.

The new textbook includes at least one lesson in every unit on problem solving strategies--such things as "guessing and checking" or "making a picture." In addition, many lessons conclude with Challenge or Think problems which ask students to apply deductive reasoning to figure out solutions. Joe says that he does almost all of the problem solving exercises with his students "one by one in order" because he sees them as helpful in applying mathematics to "real" situations. Unfortunately, his students find the problems difficult and, although he knows it is probably contrary to the textbook's intentions about problem-solving, Joe often outlines for students the procedural steps necessary to answer the problems. For example, Joe's students find problems like: "John and Mark solved 34 problems together. Mark solved 3 more than John, how many problems did John solve?" difficult. Baffled by such problems, and frustrated with Joe when he attempted to explain it to them by outlining the problem as x+ (x+3)=34,[1] Joe left one such lesson frustrated by his failure at helping students master the problem. He was sick the next day, so a substitute wrote on the board "(34-3)/2= ?" Students happily solved it. Joe said he finds himself simplifying problem solving exercises in this manner often for his students, breaking the problem down into steps which can be solved by computation.

Joe has resorted then to teaching problem-solving as a discrete skill which students need to practice. Like any other mathematics skill, Joe attempts to break it down into procedural steps so that students can "learn" how to find solutions. The problems become no more "problematic" than other exercises in Joe's classroom. Getting the right answer and getting it quickly is still a part of problem-solving lessons.

Another, perhaps more dramatic, change in Joe's practice involves his use of pictorial representations. While Joe recalled using pictures of circles and squares divided into equal parts to show fractions during his first few years as a teacher, he eventually stopped using such representations because they did not seem necessary. In the new text, students are asked to draw pictures of fractions, so Joe started using pictures again. In one lesson, Joe was teaching students how to add unlike fractions. The problem was 1/4 + 1/2:

Joe: Could we draw a picture of that? [Draws two squares on the board--one divided into halves with one half shaded, the other into fourths with one fourth shaded. The

[1] What Joe has done with this representation is use the variable x to represent John. Thus, Mark, who solved three problems more than John is x+3. Since they solved 34 problems altogether, John + Mark = 34, or x + (x + 3) = 34.

two squares are set up like numbers in a number sentence, with a plus sign between them and followed by an equal sign.] Isn't that the problem? . . . How many parts are in the whole here Anthony ?[pointing to the first square]

Anthony: 2.
Joe: How many parts do you have that are shaded?
Anthony: 1.
Joe: Well, you all agree that this is 1/2?
Class: Yes.
Joe: How many parts are in the whole? [Joe points to the second square.]
Class: 4.
Joe: How many do you have that are shaded?
Class: 1.
Joe: Will you all agree that this is 1/4?
Class: Yes.
Joe: There is another way that we can express this. If we can take the same thing, and we can divide it into four parts, did I change the value of the fraction? [Joe divides the halved-square into fourths.] Did I make it any bigger?
Class: No.
Joe: No, it's still the same, isn't it? But, what I did do was to change the whole and I divided it not into halves, but I divided it into what?
Class: Fourths.
Joe: Now, Darcy, how many fourths do I have that are shaded?
Darcy: 2.
Joe: Good. They are equal aren't they? All of a sudden I now have like fractions. (observation, 3/89)

After drawing the pictures on the board and seemingly getting students to understand the idea, Joe said to the class:

> Pictures are wonderful if you know how to use them. It's like a tool, if you can draw pictures of things using symbols. That's why we show you pictures of things on the board, especially of fractional parts--because fractions are abstract : 'Well, what's 5/8? Something that is divided into 8 equal parts and you have five of them, right?' But, if you can draw it, picture it, suddenly it becomes simple. (observation, 3/89)

Joe's pronouncement had all the conviction of a religious conversion. Excited about using pictures in teaching and having students draw pictures themselves to show fractions, Joe believed that more students were catching on. He credits the textbook for being "right on target" with this idea. But to Joe, pictorial representation is not a different way of conceiving of fractional parts, it is just a different convention which may help some students gain "insight." He walked students through the procedural steps of converting the picture of the halved-square to a quartered-square in the same manner that he walked them through the algorithmic representation. In that way, the pictorial representation was indistinguishable from the rules of divisibility or knowing Greek roots. As one more tool, pictures were helpful devices for finding the right answers quickly.

<u>What Kind of Change is This?</u>

Looking at change in Joe's practice, one wonders if this cup of change is half-full or half-empty. From one perspective, Joe's practice is full of change. He is drawing pictures to represent mathematical ideas and is having students engage in "real-life" uses of mathematical knowledge through problem-solving activities. These are not small changes for Joe. Looking at them from his vantage point they are radical additions. Yet another perspective might suggest very little change. Joe still is "the captain of his ship" in his classroom. Students still give answers, even to problem-solving questions, in boot camp fashion neither explaining their strategies nor validating divergent ideas. Joe's beliefs about what it means to know mathematics and how mathematics is best learned seem untouched by the new policy. Changes in his practice originate not from changes in fundamental beliefs but from his desire to provide students with effective mathematics tools.

The central concern of the *Framework* is to develop students' mathematical power "which involves the ability to discern mathematical relationships, reason logically, and use mathematical techniques effectively" (p. 1). It calls for "attitudes of exploration and invention" (p. 6), for students to understand "not only how but why skills are applied" (p. 12), and to develop a "spirit of inquiry and intellectual curiosity toward mathematics"(p. 1). Even granting that these ideas are ambiguous and might manifest themselves differently in different teachers' practices, Joe's understanding that the *Framework* means teaching kids mathematics application skills seems short-sighted. Even though he knows the *Framework* advocates pedagogical methods which differ from his, his reading of it allows him to dismiss these differences as acceptable as long as he arrives at what he conceives to be the *Framework*'s big idea--teaching applications. In other words, although Joe has read the *Framework* and is familiar with its approach, through his lenses he sees that which he wants to see: recommendations for making mathematics more real for students.

His limited interpretation of the *Framework*'s content and spirit is probably exacerbated by the fact that his textbook serves as the principal carrier of the *Framework*'s message. Even though policymakers in California initially rejected the textbooks proffered by publishers in an attempt to effect major change in their content and organization, officials remained dissatisfied with the revised textbooks. Like Joe, many texts have simply added new pages to old books, and this strategy may reinforce teachers like Joe who see the changes advocated by the state as add-ons, not fundamental reorganization.

The case of Joe raises a host of questions about the effects of policy on teaching practice and about the role textbooks play as change agents. As the state's efforts to affect more change in textbooks gain momentum perhaps the texts that teachers like Joe use will reflect even more radical departures from traditional mathematics curriculum and instruction. If Joe were to use a textbook more divergent from his own vision of mathematics, how would he deal with it? Would his practice change more? Instead, would he either interpret the textbook so as to make it less divergent or just skip more pieces of text in his teaching?

Alternatively, since Joe is committed to students' success and their development of confidence, it is likely that he will continue to pull more pieces out of the *Framework* when he thinks they help him make students "successful" in mathematics. Will the accumulation of these little changes lead to more basic change in Joe's mathematics instruction? And if the suggestions of the *Framework* actually affect change in his class, will Joe begin to adopt more pieces of the policy, perhaps changing his beliefs about how to teach and how children best learn mathematics?

Finally, this case raises questions about the types of change policies like the *California Mathematics Framework* is aimed at provoking, as well as the state's timeline for such change. From Joe's perspective, he has changed in some important ways. Yet measured against the complete vision, albeit ambiguous, of mathematics teaching and learning portrayed in the *Framework*, Joe comes up short in many significant ways. From whose perspective are we to look to judge if Joe has responded to the *Framework* adequately? As he himself noted, changing practice takes time. Moving oneself from traditional, familiar, and successful practice to innovative, and radically different teaching with an adequate, but incomplete textbook as the sole guide is difficult work. And the key to making such difficult changes might be a complete, genuine, and passionate commitment to the learning of all one's students. If that be the case, teachers like Joe will most assuredly succeed if given adequate time and the intellectual and organizational supports essential for such change.

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Doing More in the Same Amount of Time:

The Case of Cathy Swift

Penelope L. Peterson

Looking into Cathy's Classroom: Time 1

When I observed Cathy Swift teach for the first time, she was teaching a 34-minute mathematics lesson on "fact families" to her second-grade class. Her class was ethnically diverse class--22 students from predominantly low income families, including 7 black children, 4 Hispanic children, 5 Asian children, 6 white children. As she taught the lesson, Cathy relied heavily on her new textbook--the California edition of *Mathematics Unlimited* by Holt, Rinehart, and Winston (1988). She began with a 10-minute warm-up activity from the text which consisted of "number puzzles" "to review addition facts to 18" (p. 93). She wrote the following boxes on the board:

4	3	(7)	0	8	(8)	2	6	(8)
5	3	(8)	9	0	(9)	2	3	(5)
(9)	(6)	(15)	(9)	(8)	(17)	(4)	(9)	(13)

Children came up one at a time and wrote the sum--the numbers in parentheses--in one of the empty boxes. Cathy called on the first child and after that each child picked the next child to come to the board. Throughout the activity students were quiet, but they appeared to be actively engaged as they were watching the child at the board. Cathy had told the children to try to solve the number fact in their heads while they were watching. At any given time, eight to ten children had their hands in the air, eager to be called upon next. At two points, children put in a wrong answers so Cathy had each child "check" and then correct the answer by counting on his fingers. Rather, Cathy led the class through "checking" and subsequently confirming a right answer using the number line posted over the blackboard.

Cathy then used the overhead projector to do a two-minute review of the term "fact families" by calling on students to tell her the other addition fact for 9 + 7 = 16 (7 + 9 = 16) and the subtraction facts in the "family" (16 - 7 = 9; 16 - 9 = 7). During the warm-up, the review, and while giving the assignment, Cathy had the teacher's edition of the textbook in her hand and referred to it or read from it. She then assigned four pages of seatwork on fact families, two from the Holt textbook (pp. 93 and 94) and two pages from the ABS/Holt Curriculum Supplement prepared by the school district (pp. 8 and 9). The worksheets consisted of computation problems in addition and subtraction with sums and differences up to 18. As the children began working individually on their worksheets, Cathy set a kitchen timer to ring after twenty minutes.

For the remainder of the period (approximately 20 minutes) students worked quietly at their seats on their seatwork. As they worked, some children whispered to neighbors about their work or counted aloud. Many children were noticeably counting on their fingers to compute the solutions. At one point, a child was having trouble so Cathy told the child to get counters and then said that any children who wanted counters should come up and get them. Eight children came and got counters: Unifix cubes stuck together in columns of ten cubes. While students were working on seatwork, Cathy and her aide walked around and monitored continuously giving students help when needed through convergent questioning and directed prompting. In these interactions with individual students, Cathy focused on getting the right answer, writing the correct number fact, and getting the work done in time.

Cathy told three students who were the first to finish (two girls and a boy) that they could walk around and help other children who were not yet finished. She directed other students who finished thereafter to keep busy by drawing a picture or coloring. The children followed Cathy's directions and stayed on task until the end of the period. At 11:24 the timer bell went off, and Cathy asked the children to hand in their math worksheets. The children quickly and quietly handed in their work to her, and Cathy turned immediately to a social studies lesson.

This year Cathy feels a "lot of pressure" with the "faster pacing" and the "harder material." She acknowledges that "you only have so much time," and she admits that if left to her own devices, she would probably "spend way too much time on one thing," and she wouldn't move on. This lesson as well as the others that I observed Cathy teach were briskly, sometimes breathtakingly paced. Consistently teacher-directed with little student talk except when children "worked with a partner," Cathy uses a kitchen timer to signal the end of seatwork within the lesson "because otherwise it just goes on and on . . . and the timer seems to keep some of them (the children) on task a little better."

On the face of it Cathy's math lesson is a smoothly and swiftly-paced model lesson in the tradition of effective teaching for basic skills--warm-up, review, and seatwork with continuous monitoring by the teacher: direct instruction and directed prompting when a student needs help; impressive content coverage with students completing four computation worksheets in 20 minutes; and high student engagement and task orientation throughout the lesson. Yet Cathy says that this year she is implementing in her classroom a "new" mathematics program that is connected with the new state-level *Mathematics Framework*. So what's "new" here? *Is* Cathy implementing the new state-level *Mathematics Framework*? If so, in what way?

A View of Cathy's Practice Through the Lens of Mathematics Reform

> Those responsible for the mathematics program must assign primary importance to student's understanding of fundamental concepts rather than to the student's ability to memorize algorithms or computational procedures. Too many students have come to view mathematics as a series of recipes to be memorized, with the goal of calculating the one right answer to each problem . . . Teaching for understanding emphasizes the relationships among mathematical skills and concepts and leads students to approach mathematics with a commonsense attitude, understanding not only how but why skills are applied. Mathematical rules, formulas, and procedures are not powerful tools in isolation, and students who are taught them out of any context are burdened by a growing list of separate items that have narrow application (California State Department of Education, p. 12).

In her lesson Cathy focuses on getting students to learn their basic addition and subtraction number facts and to see that groups of four facts (e.g., 8 +9 = 17; 9 + 8 = 17; 17 - 9 = 8; 17 - 8 = 9) form a "family." Cathy's goal is for students to calculate the right answer for each computation and for students eventually to have memorized each number fact. The "number puzzles" are not really "puzzling" in a narrow sense. With a single right answer easily derived through computation, students are not required to puzzle through genuinely problematic situations.

Throughout the lesson, Cathy teaches as though she thinks the way students learn number facts is by drilling on the basic facts over and over on written worksheets, on the blackboard, or in response to teacher questions such as, "What's 16 minus 8? If 8 plus 8 is 16, what's 16 minus 8?" When asked what she hoped kids would get out of the lesson, Cathy explained, "Just practice of their basic facts." On the one hand, Cathy seems to be trying to make number facts conceptually coherent by using the "families" idea, yet on the other hand, she presents number facts as barren entities, decontextualized or devoid of a context in which students might apply them.

The authors of the state level *Mathematics Curriculum Guide* (California State Department of Education, 1987, p. 21) suggest that children can and should learn addition and subtraction number facts so that they connect it to situations that occur naturally in their daily lives. For example, a child takes two trucks outside and then goes into the house to get two more so that they she will have four to play with altogether. A boy who has four cookies sees that he has two left if he gives two cookies to his friend. It is important the children learn to connect what they know to the symbolic representations (e.g., 2 + 2 = 4; 4 - 2 = 2). One way to facilitate children making such connections is to have children "act out" such story problems in mathematics using cubes to represent the trucks of the cookies in the problem or to have children make up stories that go with number sentences such as 2 + 2 = 4 or 4 - 2 =2. Interestingly, according to this perspective, the "abstract" entities are the symbolic representation of the number fact--the sole focus of Cathy's lesson--and not the story problem or word problem in mathematics. In contrast, as we shall see later, Cathy's thinks that the abstract entities are the story problems or word problems. From her point of view the "abstractness" of the word problems account for why her children are having so much trouble solving mathematics word problems.

Cathy does attempt to make number facts more meaningful to her students by suggesting that they use their fingers, the Unifix cubes, or the number line. Cathy's use of manipulatives is the major evidence we see in her first lesson of ideas from the *Mathematics Framework*. The *Framework* authors emphasize extensive use of concrete materials to build a strong base of student's understanding of procedures for how numbers are put together and taken apart and the symbolic representations for these procedures (California State Department of Education, 1985, p. 4). Cathy has her students use their fingers or counters because "it is more concrete, and they can see it . . . until, eventually, if they do it enough times, they'll have it memorized." Here Cathy seems to take an associationist view of learning as occurring through practice. According to this view, learning of numbers occurs through practice by strengthening the associations between concrete representations of the mathematical quantity and the abstract numerical symbols that stand for the quantities. In contrast, the *Framework* authors seem to view learning as the process of developing students' understanding. They discuss the use of concrete materials as a way of building students' understanding by providing a way for students to connect their own experiences to mathematical concepts and to the abstract mathematical symbols.

Cathy's use of manipulatives this year has been a direct result of the new mathematics program adopted by her school district. At the beginning of the year Cathy received a big kit of manipulatives along with the textbooks and materials in the new math program. Cathy sees the manipulatives kit as a Godsend because now she has some of the kinds of concrete materials that she has been wanting to use in her mathematics teaching. Heretofore she has had neither the time nor the resources to make or buy such materials.

For Cathy the use of manipulatives constitutes only one of the major elements that she has added to her mathematics teaching. As we shall see, there are several others that she has added. What are these new elements in Cathy's math teaching and how has she come to add them? How does Cathy think about the changes that she is making in her classroom practice and why she is making them?

Cathy's View of the State Policy--the *Mathematics Framework*

Cathy sees her mathematics teaching this year as reflecting a new mathematics program. Cathy believes that she is implementing the new state-level *Mathematics Framework* although she says that she has seen neither the state-level *Mathematics Framework* booklet nor the state-level *Curriculum Guide*. Cathy's view of the state policy, then, is through the new Holt mathematics program and materials, one of the several textbooks approved by state-level policy makers.

But Cathy also sees the *Framework* through an instructional guide, worksheets, and mastery tests developed by the school district to accompany the textbook. These school district materials were developed specifically for use by teachers such as Cathy who are in low SES Schools designated as in the Achievement of Basic Skills (ABS) program. The Achievement of Basic Skills (ABS) instructional model used by the school district includes the components of content coverage and pacing using a pacing guide, mastery testing and re-teaching if students do not pass the mastery test; maximizing students' time on task and engagement in the mathematics activities, and use of direct instruction (active mathematics teaching).

When we talked about the *Framework*, Cathy remembered something she was told at a workshop for basic skills teachers two years before. The workshop leader said that they were trying to emphasize problem solving rather than computational skills because "they figured that everybody in the future would be able afford a calculator so it wasn't so important to know how to do long division as it was to be able to figure problems out." Cathy sees an emphasis on problem solving as one of the major ways in which her new math program differs from the old. The other major additions include the use of manipulatives and the use of "partner and group work." Cathy's practice reflects her attempts to add these "new" elements to what she is already doing.

Cathy construes the situation as one in which she is being asked to do more--teach problem solving in addition to computation and teach using manipulatives and group work in addition to having students complete individual written seatwork--within the same, amount of time and with virtually the same resources that she had before for teaching mathematics. She feels the same pressure in teaching literature, since her school has also begun the "Wednesday Revolution" in which teachers teach literature to their children on Wednesday mornings. As she struggles to change her classroom practice, Cathy is trying to make sense of how her experiences a basic skills teacher and a regular classroom teacher in a school that serves disadvantaged students mesh with these additional demands. Understanding the changes in Cathy's practice requires understanding something about the context in which she works as well as something about the challenges Cathy faces.

Facing New Challenges

Cathy teaches in Columbus School, a low-SES school in one of the largest school districts in California. Nearly all the children in the school are from low income families with over ninety percent of the children receiving free or reduced lunch. The school has an incredibly high annual mobility rate--only one-eighth of the children who begin in September remain in the school for the whole year. Substantial ethnic and linguistic diversity exists within the school with twenty different languages being spoken by children who are enrolled. Signs posted in the building and information for families in the staff lounge on interpretation of students' standardized test scores are in English, Spanish, Lao, Vietnamese, Cambodian, and Hmong.

Faculty and staff in Columbus school are proud of their students' performance. Columbus has been named one of the "California Distinguished Schools" because the school's scores on the California Assessment Program (CAP) test have been consistently in the top 75% of their "band" based on ethnic background and income level of children in the school. Cathy teaches mathematics to second-grade students on the "regular" track (not "sheltered"--taught in their native language; or "transition"--bilingual) so only English is spoken in her classroom. In order to accommodate a teeming population of 900 children in a building that was built for 350, eighteen bungalows have been added adjacent to the main building. Cathy teaches in one of these bungalows.

Upon meeting Cathy, one is impressed by her shy, soft-spoken but gentle and caring manner. She is a teacher who could have chosen to teach elsewhere--she came from out of state--but she teaches at Columbus because she cares about these kids. At the same time one is arrested by Cathy's apparent level of fatigue, striking in one so young. Cathy is only in her fourth year of teaching. She taught for two years as a basic skills teacher in the district. Then two years ago, Cathy began teaching second grade at Columbus School, first as a substitute then as a regular classroom teacher. Now after only four years as a teacher, Cathy seems frazzled. She is a teacher who is working as hard as she can , yet now she feels she is being asked to do more. She's not sure where and how she can find the extra time and energy to do it. Moreover in mathematics, she's not sure that she *knows* how to do it. While she is excited about the changes she has been asked to make in literature, her response to the new mathematics program has been diffident. As Cathy puts it, "What I am really having fun with this year is the Wednesday revolution where we are using trade books . . . We do some reading and then we can do art projects, or they (the children) make books, or they write, or do all kinds of different things." In math, she says, "I just follow the lessons, and I do what they say."

Unsure of Her Knowledge about Mathematics Teaching

Why has Cathy responded in different ways to the district-wide reform initiatives in mathematics and literature? There are several possible reasons: first, differences in Cathy's knowledge and interests in these two subjects; second, her different conceptions of these two subjects; and finally, her perceptions of the differences between the district's instructional policies in mathematics and literature and the instructional guidance and guidelines that she has been given to teach each of them.

Cathy considers herself more knowledgeable in literature than in mathematics. She says that literature is her favorite subject and that she was an English major in college. In contrast, she feels less knowledgeable and confident in mathematics. She says she feels like she needs "a book to go by" in math; and when describing her math teaching she admits, "I'm still learning every day. You know I'm just a pretty new teacher . . ." In addition to feeling less confident and knowledgeable about mathematics teaching, Cathy also seems to feel that mathematics as a subject is less "creative" and open-ended than literature. She views math as " a lot more sequential" than literature and seems to see math as a fixed body of content to be covered. Cathy's different views of these two subjects are related to the way she thinks she is supposed to teach these subjects. With literature she says, "We get to do all this fun stuff. I get to think of things to do myself, and I do not *have* to teach the basal and (have kids) do the worksheets."

Relying on What She Knows and What Others Tell Her

Unsure of her own knowledge of how to teach mathematics, Cathy relies on that with which she is familiar--the ABS model--as well as on the knowledge and guidance provided by those whom she judges to be more expert at mathematics teaching than she. In mathematics Cathy "follows the lessons" because she feels that the authors who wrote the textbook and materials know more about mathematics teaching than she does. Although she says that she feels that she is not free to teach mathematics the way that she wants to as she is with literature, Cathy seems unsure of how and whether she would teach mathematics differently if she felt she were free to do so.

Tightly wrapped around one another, Cathy's lack of confidence and her feelings of lack of autonomy are hard to disentangle. Cathy relies on the explicit instructional guidance she has been given by the district for how she should teach elementary mathematics in the form of the district's Achievement of Basic Skills (ABS) model which includes a pacing guide for content, specially developed ABS materials (mastery tests and worksheets for re-teaching and enrichment), and support and monitoring by the ABS resource teacher in her school. The ABS model was developed in the late 1970's to improve the achievement in mathematics and reading of children in low SES schools. The district is attempting to implement the new *Mathematics Framework* (program) in low SES ABS schools without modifying or changing the basic elements of the ABS model. In her mathematics teaching, Cathy relies on the teacher's guide that comes with the textbook and the ABS guide and materials developed by the school district. Having been a Basic Skills teacher in the district for two years before assuming her present position, Cathy is both familiar with and comfortable with the ABS model.

What math class has become in Cathy's room is the ABS model with the addition of the new components of use of manipulatives, use of partner and group work, and emphasis on problem solving. Her program is consistent with the views expressed by the district's two staff developers who described the major components of the new math program as: cooperative learning, using manipulatives and calculators, estimating and problem solving. As outsider looking in, it's unclear whether this mixture of ideas is a laundry list or a more coherent, integrated set of practices.

It is to this issue we not turn. How are these ideas related to one another and to learning mathematics with understanding? To answer this question, we examine each of these new components from the perspective of Cathy 's classroom practice and how she thinks about it. Although we saw evidence of use of manipulatives in the first day's lesson, we didn't see Cathy make use of partner and group work until the second day. On that day we also saw extensive use of manipulatives by all the children.

Looking in Cathy's Classroom: Time 2

The second time I observed Cathy teach, her lesson was on "missing addends" in addition and subtraction. Relying on the teacher's guide, she began with a 10-minute warm-up taken from the textbook. She wrote on the board:

8 + 3	14	8 + 8	13
6 + 9	16	7 + 6	17
9 + 7	15	7 + 7	16
8 + 6	11	9 + 8	14

She then called on one student (Martha) to match "this" (Cathy pointed to one of the columns of addends) "with the answer." Martha drew a line connecting 6 + 9 to 15. Martha then called on another student to come up. Four children later, Cathy noticed that girls had only called on girls thus far. She admonished a girl to "choose a boy" next. Boys then called on boys for three times until on the last turn, Cathy intervened again and directed the last boy to call on a girl.

Cathy and her aide then passed out 18 counters to each child. Directing the children to choose a partner (next to them), Cathy gave a brief demonstration of what she wanted the kids to do by calling on Ally and Linda to model the partner activity. The partner activity consisted of having one child show a number of counters, and the partner show a greater number of counters. Then the pair of children were to determine how the first child could make his or her set match the second child's.

The children then worked in pairs for 7 minutes. As they worked on the activity, few children used the counters to directly represent the problem to help them solve it. Only a few pairs of children used the counters to count on or count back to solve the problem. Some children played with the counters or made "red teams" and "yellow teams" with the counters. One pair of students used their counters to form numerals (i.e., one student formed "12" with his counters, and the other student formed "10"). When a pair of children did use their counters, one child generally would count out a number of counters (e.g., 12) and then the second child would count out a number (e.g., 10). Then the children would solve the problem by doing it "in their heads." During the activity, Cathy walked around the room and helped pairs of students who seemed to be having difficulty. Cathy encouraged students to use the counters to find a solution. Her questioning and prompting were convergent, however, aimed at getting students to use a subtraction strategy for solving the problem rather at getting students to develop their own various strategies as the teacher's manual suggests:

> Have children work in pairs, taking turns going first. One child shows a number of counters. The other child shows a greater number of counters. Then they are to determine how the first child can make his or her set match the second child's. *Encourage them to try different strategies each time to find what number of counters must be added to the first set so that it matches the second. Have children share their strategies with the rest of the class* (Italics added).

Cathy summarized this part of the lesson on the board by saying that they had been working on "missing addends." She wrote on the board two number sentences [4 + 4 = 8 and 8 - 4 = 8] to show that "when Ally put out 4 chips and Linda put out 8 chips, we had to figure out the difference between 4 and 8, and we know that 4 was the missing addend." She also said that another way is subtracting to find the missing addend. Given that the children did not seem to use the counters as "instructional representations" it is not clear that the children made the connection between Linda and Ally's problem situation and the number sentences given by the teacher to represent problem. Cathy did not "check for understanding" but presented the number sentences as representations of the situation.

The students then worked on pages 95 and 96 from the Holt textbook. Children worked together in partners on p. 95 for 5 minutes until the timer went off. Cathy read the following directions from the worksheet: "Show 10 counters. Cover some of the counters. Ask how many are covered. Ask what your friend did to find the answer." During this partner activity, many children seemed to guess to find the answer. Children seemed to be engaged in guessing and in trial and error on the task where they were to guess the number of hidden counters. Rather than observing or discussing children's strategies with them, Cathy directly instructed or prompted students. The teacher's guide for the Holt textbook suggests that the teacher observe the children as they work in pairs, keep a list strategies the children are observed to use, and discuss them later with the class.

> Possible strategies include: subtracting the visible number from the beginning number, counting on from the visible number to the beginning number, building another set of counters and taking away the number visible and counting what is left. (Holt, 1988).

When the timer went off, Cathy directed the children to work individually on p. 96. The worksheet showed a missing addend equation (6 + ____ = 14) with the first addend completely represented by a picture (e.g., 6 fish). The children were to draw fish to represent the missing addend and write the number in the equation to show how many fish they drew. The teacher told the children that they could use their fingers or counters to "count on" instead of drawing the pictures to represent the missing addends on the worksheet. Children were observed to be using both counters and fingers to represent the missing addends on the worksheet page. Both representations seemed accessible to children. However, I observed a number of incorrect answers particularly when children hadn't drawn pictures or used counters but apparently had "done it in their heads" instead.

Cathy and her aide walked around monitoring continuously during individual seatwork. Throughout the seatwork portion of the lesson, whenever individual children had trouble or needed help, Cathy directly instructed them how to find the solution or to use a procedure. For example, she often directly instructed a child to count the chips or "count on" by counting aloud with them or telling them to count using their fingers or their counters. Children worked quietly, individually on the workbook page until the end of the period.

Partner and Group Work as "Opportunities"

Cathy sees the use of "partner and group work" as one of the major additions that she has made to her teaching this year. Indeed, Cathy's use of partner work does reflect a major emphasis on cooperative learning that appears in the *Framework*. The *Framework* authors contend that:

> To internalize concepts and apply them to new situations, students must interact with materials, express their thoughts, and discuss alternative approaches or explanations . . . Students have more chances to speak in a small group than in a class discussion; and in that setting some students are more comfortable speculating, questioning, and explaining concepts in order to clarify their thinking. (California Department of Education, 1985, pp. 16-17)

The vision conjured up by these authors is one of children verbalizing their thoughts, discussing mathematical strategies with peers, and sharing alternative mathematical explanations and approaches to problem solving. Such a vision is not what we see enacted in Cathy's class. Instead, we see children barely talking with their peers, focused on getting the right answer. We also see convergence toward a single strategy that comes from or is taught by the teacher rather than consideration of alternative strategies that come from the children, are shared among the children, or taught by one child to another.

These discussions did not occur, perhaps because Cathy did only minimal modeling of verbalization of solution strategies for the children. She did give a brief introduction as follows: "One partner is going to show a certain number of counters. For instance, I see that Linda has 8 counters. And her partner, Ally, is going to count out a number that is bigger. Ally is going to count out more than 8 counters. Let's see you do that, Ally, count out more than 8."

Cathy: OK, how many do you have?

Linda: 12.

Cathy: OK, how many more do you have than Linda has?

Ally: 4

Cathy: 4 more, OK, how did you figure that out? There are different ways you can figure it out. How did you think of that in your head?

Ally: I counted Linda's, and I counted mine, and I have 8 and then I took away 4, and I count 4 back.

Cathy: You took away 4 from your pile? OK, so you found the *difference* between yours and Linda's.

Cathy: OK, go ahead and trade off. One person shows a number and the other persons shows more. Then figure out the *difference* with your partner.

In the above example, Cathy accepts the one strategy that Ally used, but does not ask for others, probably because this is the strategy that Cathy is looking for to solve the compare problem--subtracting to find the difference. She also does not ask students what they think of Ally's strategy--whether it is a good one or not or whether it makes sense--but rather, goes ahead and acknowledges the strategy as correct.

In what ways did Cathy respond to students when they did verbalize solution strategies? Consider the following example:

Cathy: [to Eric who is partners with Crystal]: How many are you showing? Put them out here so she can see them. Now you count out a certain number that's more. (Eric counts out chips). OK, now how many more do you have? No, here. How many more are you showing? No, Crystal, right here. These are the ones that are showing. Stop it and pay attention, Eric.

Crystal: One.

Cathy: Is that right, Eric? She said she had one more. Is that right, Eric?

Eric: Yeh.

Cathy: OK, how did you know?

Eric: She had 3, and I had 2.

Cathy: And oh, three minus two is one?

Eric: No. I added.

Cathy: You added one to his. Oh! You took two like his and then you added one? OK, so you had one. One is the *difference*.

In the above example, Cathy assumes that Eric used subtraction to get the answer--the strategy that represents the way that she herself thinks about the problem. Although Cathy then acknowledges Eric's "counting on " strategy as correct, she does not follow up either with questions comparing Eric's strategy with hers or asking for alternative strategies. As was the case with Eric, when a student did verbalize a solution strategy, Cathy tended to respond with the strategy she thought should be used or to get students to converge on one strategy that she was trying to get across. In interacting with pairs of students or individuals, Cathy does the talking and directly instructs the students in a strategy or uses prompting questions (lower-order-questions) to get students to say the "right" answer.

In sum then, Cathy's practice reveals a mixture of the old and the new. Students work in pairs which provides them with an opportunity to discuss strategies and thinking. Although the potential for such conversations exists, such discussions occur rarely. While Cathy encourages students to verbalize solution strategies, she asks lower-order questions or directly instructs them so that the students verbalize the solution strategy that Cathy wants them to use. Although Cathy says she values the use of multiple strategies to solve problems, she presses toward convergence in her discourse with students. On the whole, discourse in Cathy's classroom is minimal. Indeed, Cathy's lessons are striking for their lack of verbal discourse--students work silently. Even in the third lesson when we will see students "challenge "one another's solutions, students do so silently, in writing. They never verbalize the "why's and wherefores of their challenges," nor does Cathy ask them to do so.

Another aspect of the *Framework* that appears in this lesson is the use of manipulatives. I turn now to the question of how Cathy thinks about manipulatives and the role they play in children's mathematics learning.

Cathy's Use of Manipulatives

Cathy was thrilled with the kit of manipulatives that she received in September, although she admits to that she really needed more than just the three-hour introductory inservice workshop to understand their use. She has gotten comfortable with the manipulatives by "using them" and by "watching the kids. Cathy says that she uses the manipulatives all the time when she is starting to teach a new mathematical skill. Thereafter she "tapers off when it seems like they (the children) can do it on paper." She encourages the children to use the manipulatives, their fingers or "whatever they need to."

In her thinking about why manipulatives help children learn mathematics, Cathy takes a developmental view that children move from a concrete stage of understanding mathematics ideas to a more abstract stage. As she puts it, "Using the manipulatives helps them because we don't start out just doing it on paper, in the abstract." In describing why she thinks it's good for and why she encourages children to use their fingers, she suggests that "it's more concrete," and they can "see it." She gives the same rationale when she explains the role of pictures or pictorial representations:

> It is more concrete, they can see what it is . . . It is just easy for them to deal with it; it is not abstract. They don't have to listen to the directions and try to figure out what they're supposed to do.

She describes how this year she used the Unifix cubes from the manipulatives kit to teach the concept of regrouping. She used a place value mat with a "tens side" and a "ones side." For example, in teaching the concept of regrouping, Cathy gave each child a column of ten Unifix cubes stuck together, asking the children break them up into ones. When she taught subtraction with regrouping, she said that she had the children put a number on their place value mat, for example, 23. "So they would put down 2 tens, and 3 ones. And then I'd say, 'Now how can you take away 4?' Then the children would just break apart one ten to make enough ones to take away 4." Cathy thinks that this is the best way to teach the concept of regrouping "because it's concrete." She says that children "need to know that even though they are changing the form of it, it's still the same number." As Cathy explains it, "That's just the number sense again--some of them haven't developed that well. But this is the kind of thing they need to do to get it." However, she worries that there's just not enough time to do this--to spend enough time with them on manipulatives.

Cathy sees the child's ability to show the answer with Unifix cubes as the very essence of mathematical understanding. For example, when I asked her, "Suppose a child in your class could get the right answer to this problem--53 - 28, but could not show it with base ten materials or Unifix cubes or sticks and bundles. How would you feel about that?" Cathy replied that she would think that the child really did not understand. She thought that although the child would have learned to do it on paper, he did not have a real understanding of why he was doing what he was doing. She said that she would then get some base ten materials and try to teach the child with the manipulatives what the child had done on the paper. She felt that if the child could do it on paper, it wouldn't take much for the child to understand using the manipulatives. The *Framework* authors would agree with Cathy that the child did not really understand if he could only compute the answer without showing it with manipulative materials. However, they might be less sanguine about the rapidity with which the child would come to understand. If the child cannot use concrete materials to show the solution to the problem, he probably does not understand place value. He may have learned subtraction and regrouping operations by memorizing them as unrelated facts and procedures without developing a firm understanding of place value.

In addition to believing that the manipulatives help children learn because they are "concrete," Cathy also thinks that the manipulatives help a lot because "it is more interesting for them, it is just more involving for them to have something to work with. Whereas before they might have just gotten frustrated and given up, the manipulatives keep them involved." She said she uses the kit of manipulatives that came with the program because she believes that using the manipulatives "makes mathematics a lot more real f the children because it's still like a game--it is like playing for them, and that's a big way that children learn."

Has having the manipulatives changed Cathy's teaching? On the one hand, Cathy's teaching seems to have changed little because she still views teaching as telling. She has simply incorporated the use of manipulatives into her direct instruction procedures. On the other hand, Cathy believes the use of manipulatives has changed her teaching because the addition of manipulatives has allowed the children to become more actively involved. For example, Cathy commented that having the manipulatives this year enabled her to teach subtraction with regrouping differently than she had the previous year. The previous year, she had just drawn pictures of "ten little boxes in a stack, and then drawn a picture of breaking up the boxes and moving them into the ones place." This year each child actually has "concrete" objects to manipulate for himself or herself.

By far the most important outcome of Cathy's use of manipulatives may be that children's solution strategies for solving some problems have become "visible" to her. Now that she's seen a bit of how kids are thinking, she's puzzled and perplexed.

Thinking About Children's Thinking

With the use of manipulatives this year, children's solution strategies for solving problems have suddenly become visible to Cathy, and as a result, Cathy has begun to think about children's thinking. For example, in talking to Cathy about her lesson on missing addends she commented spontaneously that it was "real funny" that some children will count out nine counters and then will count out eight counters and then they will count them all again separately. They will go "1, 2, 3, 4" until they get to 18 and "they won't count like 9 and then figure out you don't have to count them again and that you can just count the other stack." She described it by saying "it is funny . . . I don't know why they are doing that, it seems funny to me." When I followed up by asking Cathy if she had some kids in her class who were able to "count on", she replied that she thought most of them could. Then she said the ones who can count on, "just do it and some of them just aren't at that stage yet." She also noted with surprise that it was not something that she had sat down and taught the children how to do except sometimes individually when she was working with them.

Rather than taking a view of the child's solution strategy as an attempt to understand or "make sense," Cathy seems to view the child's "counting all" strategy as funny and perplexing and indicating a lack of readiness to go on to the next stage. In my observations of Cathy's mathematics teaching and her use of manipulatives with the children, I never observed instances of Cathy using questioning or probing to find out the way that the child verbalized a strategy or would use the manipulatives to solve the problem. Rather, Cathy would use the manipulatives to directly instruct the child in a strategy such as "counting on" to solve the problem. Further, although Cathy seems to view children's solution strategies as measures of their true understanding, she does not behave as though she is aware of the idea that students construct their own mathematical understanding. In the tradition of effective teaching, she strives to *teach* students to understand. For example, I showed Cathy the following word problem:

Penelope had 21 stickers. After Penelope visited her grandmother, she had 27 stickers. How many stickers did she get from her grandmother?

Then I showed Cathy several strategies that children might use to solve this problem and asked Cathy first, what she would thought of these strategies and second, if she had ever seen children in her class use these strategies. Cathy described all the strategies as "great" and said that she did not "really care" how children got their answer as long as they understood what they were doing. Further, she commented that if kids brought up these solution strategies she would judge that they were "paying attention and thinking about what they were doing." Thus, although Cathy saw these solution strategies as indications of children's understanding, she defined understanding narrowly, and she did not take the broader and deeper constructivist view implied in the *Framework*. For example, when I presented Cathy with the strategy where the child thinks to herself, "twenty-one and four is twenty-five. Twenty-five and two is twenty-seven; two and four is six." Cathy described this strategy as "pretty abstract," commenting that it seemed really sophisticated to her for a second grader to do so many steps and that she never heard any of her children verbalize a strategy like this. In studies of children's mathematics learning, when researchers have interviewed children individually, they have found such "derived fact" strategies to be commonly used by most children at least some time in grades 1 to 3 (c.f., Carpenter & Moser, 1984). However, Cathy's response makes perfect sense when considered within the context of her classroom practice. Given that Cathy seldom asks students to verbalize their solution strategies or questions them on why or how they get their answers, she would not be likely to hear students verbalize such strategies. The solution strategies that children used in Cathy's class were ones that Cathy directly instructed the students to use such as "counting on."

In sum, although the use of manipulatives is like a window that has opened up for Cathy and given her a glimpse into her students' ways of thinking and problem solving, it is a window that has opened only a crack. What is the likelihood of the window opening further to provide Cathy with the possibility of further insights into her children's thinking?

Time and time again, time is mentioned as a factor by Cathy. Given that Cathy feels the constant pressure of time, the need to maintain pacing, and to cover the content, she worries about spending precious time asking students how they solve a problem and taking the time to listen to students' varieties of responses. Thus, Cathy seems likely to take the familiar and more efficient approach of instructing the students in one or two strategies that she thinks are important for them to use in problem solving. Through her own teaching then, she may unwittingly restrict her own access to knowledge of her children's thinking strategies and remain blithely unaware of children's abilities to make sense of problems which she oftentimes regards as "too abstract"or "too difficult." On the other hand, the possibility exists that Cathy will continue to learn by observing, listening to, and watching children's strategies and attempts to make sense of the mathematics especially as they use their manipulatives. The possibility exists that Cathy will realize that thinking and problem solving take time and that both she and her students need to spend more thoughtful time on understanding mathematics.

This possibility seems unlikely in light of Cathy's concerns about time. For example, Cathy had dropped the use of manipulatives to save time when I returned to see her in March. Although she is a lot more comfortable with the manipulatives than she was earlier in the year, Cathy admitted that she had stopped using them routinely. But the children "can have them if they want them." Cathy explained that she was "trying to wean the children away from the cubes because if they can do it on the paper, it is faster . . . If they count out every single one, it takes a lot longer, and they do not get through their page."

Because manipulatives are an add-on to Cathy's teaching rather than an integral part of her mathematics teaching for understanding, they are easy for Cathy to drop. In this same lesson in March we are intrigued by other deletions that occur even as Cathy follows her text.

Looking in Cathy's Classroom: Time 3

A striking difference in classroom atmosphere was evident between this time compared to the December observations. In December students were very quiet and controlled, yet happily task-oriented and busy. On this day the class was still task-oriented, busy, and happy, but students also seemed bursting with energy. Cathy herself commented that students were "charged up," perhaps because they were looking forward to next week's spring vacation.

The subject of this lesson was two-digit subtraction with regrouping. The specific objective of the lesson was to check subtraction by addition. With the exception of the warm-up activity, the lesson was taken from the book (pp. 187-188). However, Cathy did not hold the text in her hands or refer directly to the teacher's edition during the lesson as she had during my previous visits.

Cathy taught the lesson using direct instruction: The major lesson segments included a fifteen - minute warm-up activity, a two-minute explanation, thirteen minutes of controlled practice, and thirty minutes of seatwork. The lesson began when Cathy wrote on the board nine two-digit subtraction problems that required "regrouping". The computation problems were arranged in a 3 X 3 tic-tac-toe matrix.

The activity involved competition between the boys and the girls to win the tic-tac-toe game by solving correctly three problems in a row in the matrix on the board. A student from one team at a time came to the board and selected a problem to work. After the student wrote the answer, the teacher asked the class, "Any challenges?" A member of the opposing team could challenge the student's answer and go to the board and work the problem correctly. The word "challenge" was used to convey that a child could contend the correctness of another child's answer. The child who "challenged" was to erase the child's incorrect answer and "work" on the problem on the board and make it "correct" by doing the correct work (e.g., cross out for borrowing, write a "1" to show the "ten" regrouped to one's place) and write the correct answer. No verbal discourse was involved.

If no one challenged the solution, the teacher put either a "X" or "O" over the problem depending on the team of the student who had solved it. Then a student from the opposing team was called on to solve a problem. The game continued until the "O's" (the girls' team) won. During the game, the noise level was high, kids were actively engaged in watching, calling out suggestions, and "rooting for their team members." Kids raised their hands and made noises to be called on. When the girls won, the girls cheered so loudly that the teacher held up her hand for silence. Then each student held up a hand until all was quiet. The teacher said, "Let's practice a silent cheer." The class members gave a silent cheer. After the game, Cathy pointed out one problem for which the girls' team had received a point. She said that "no boys challenged this one" and that the boys might have won the game if they had challenged this answer on the board because it was not right.

Before launching into controlled practice, Cathy gave a brief explanation of how to check subtraction by adding. She wrote the following problem on the board and worked it as follows:

$$\begin{array}{r} 46 \\ \underline{-17} \\ 29 \end{array}$$

She then "checked" the problem by writing the following addition problem on the board:

$$\begin{array}{r} 29 \\ \underline{+17} \\ 46 \end{array}$$

Cathy then handed out blank paper, had the students write down a two-digit subtraction problem (32 - 19), solve the problem and check it. While students worked the problem, Cathy walked around monitoring student's work and checking to see if they had the correct answer. She then called on a student to work the problem on the board so other students could check their answers against the student's answer on the board. Then she wrote two more two digit problems (55 - 28 =; 73 - 22 =) for the students to work as controlled practice. Again Cathy monitored and checked to see if students were working problems correctly. When a student had trouble, she typically responded by directly showing or telling the student what to do or by asking a lower-level or factual question (e.g., What's three plus nine? What's one plus one? What did you write down? Is that the same number?) The noise level in the room was high as students were working; many students were talking. Some students were talking about math, but most students were not.

During the last part of the lesson, the students worked on two workbook pages from the text. The two pages had 28 two-digit subtraction problems that students were to solve, write the answer, and then check their answer by writing and solving the corresponding addition problem. At the bottom of the second page were three word problems.

Although the noise level was high during seatwork, children appeared to be actively engaged in computing the answers. No manipulatives were in evidence, but many children were counting on their fingers. When Cathy gave individual help, she often encouraged students to count on their fingers by modeling and counting aloud on her fingers herself. As in previous observations, Cathy and her aide tended to be directive when helping individual students, typically leading the child to the correct answer through prompting, imperatives, and lower-level questioning. For example, at one point the aid said to a child who was having trouble, "You need to borrow right here. If you've got four cookies, you can't eat nine cookies." She repeated these same statements when she helped other students.

This third lesson is interesting for the kind of problem solving that did and did not occur. As in previous lessons, Cathy emphasized the solving of computation problems. Moreover, she gave little attention to the several word problems in the text and on the students' workbook page. Why might this be the case? How does Cathy think about problem solving? Does she see problem solving as a major part of the new mathematics program? If so, how? How does Cathy's view compare with that of the *Framework* authors?

Cathy's Thinking About Problem Solving

Cathy does see problem solving as a major part of *Framework*. Indeed, the *Framework* authors declare that problem solving is "a process, with solutions coming often as the result of exploring situations, stating and restating questions, and devising and testing strategies over a period of time" (California State Department of Education, 1985, p. 13). Further, they assert that "Real problems, mathematical or otherwise, do not usually exist in simple, easy-to-identify forms. More often, they are embedded in descriptions of puzzling or complex situations" (California State Department of Education, 1985, p. 14).

When I asked Cathy if she were familiar with the *Framework*, she said, "What I do remember is that they are really trying to emphasize problem solving, rather then computational skills." And she says that the new mathematics program in the textbook involves "much more emphasis on problem solving." When asked what kinds of problem solving, Cathy says, "There are a lot more word problems."

Interestingly, in spite of this presumed emphasis in the textbook on solving word problems, I saw little evidence of children engaging in word problem solving during the mathematics lessons we observed. Instead I noted selective omissions in Cathy's "following the book." For example, in the third lesson three word problems appeared at the bottom of the second page of the worksheet. Two of the three word problems were of the type that have been referred to as more complex word problems by researchers on children's mathematics learning (see, for example, Riley & Greeno, 1988; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). The word problems on the page consisted of a separate result unknown problem; a compare problem; and a join result unknown problem with four quantities to be joined. The separate problem involved a direct or implied action in which an initial quantity is decreased as a subset was removed. The join (also called combine) problem involved a direct or implied action in which a quantity was increased by a particular amount. In the case of both the separate and join word problems on this page, they were the most common type--result unknown--where the unknown was the total of the joined or separated quantities. The compare problem involved a relationship between quantities rather than a joining action--more specifically a comparison of two, disjoint sets.

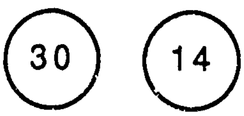
By the end of the lesson, most students in the class had not solved the word problems. When Cathy announced, "Put your work away," she also added, "Don't worry about the word problems now." Students turned in their worksheets with the computation problems completed, but the word problems unattempted.

Like manipulatives, problem solving for Cathy is an add-on to her curriculum rather than an integral part of her teaching of mathematics for understanding. Thus, it is easy for Cathy to drop. Further, she admits that she has found that problem solving is difficult for her children. We saw this difficulty played out in the case of one student, Felicia, who got to the word problems and struggled as she tried to solve them. Cathy attempted to help Felicia solve the compare problem: "There are 30 funny clowns. There are 14 sad clowns. How many more funny clowns are there?"

Cathy: What do you need to do here--adding two groups, taking away, or comparing? What?

Felicia: [no response]

Cathy: [draws on her paper]



Cathy: Think about what you will do.

Felicia: Add.

Cathy: Are you going to add two groups together? Are you going to compare?

Felicia: [Shakes head, "Yes,", but then starts to add on her paper.]

Cathy: [interrupts Felicia's adding] Do you add when you compare?

Felicia: [Shakes head, "No."]

Cathy: So you *take away.*

The instructional representation drawn by Cathy for the word problem did not appear to be helpful to Felicia. The "representation" merely showed the two numbers in the problem with a circle around each one. Perhaps Cathy intended to show that each is a quantity and to highlight that "comparison" or subtraction is needed. However, the representation just as easily might be perceived as highlighting the need to add or put together the two quantities. The representation does not seem to represent the mathematical ideas in the problem . Further, because Cathy drew the representation before Felicia spoke, she was guessing at what might make sense to the child. Felicia then said that she thinks she should add, suggesting that she was thinking of solving the problem by adding on to 14 until she gets 30. Cathy had the opportunity to question Felicia before drawing the picture, to get a sense of how Felicia was thinking about the problem, and then to work with Felicia to construct a representation that would be tied to Felicia's thinking about the problem.

Cathy's interaction with Felicia about the compare problem with the clowns is strikingly reminiscent of her interaction with Eric in December when Eric and Crystal were trying to solve a compare problem (i.e., Crystal had 3 chips. Eric had 2 chips. How many more chips did Crystal have?) In both cases rather than attempting to understand and work from the student's sense of the problem as an addition problem, Cathy tried to teach the students to see a *compare* problem as a *subtraction* problem where you find the *difference*. In both cases, Cathy's attempts to get students to "see" the compare problem as a subtraction problem rather than attempting to find out how and what students are thinking and go from there.

Although Cathy seems unsuccessful in the above interaction, it is not for a lack of trying. This year, Cathy has been thinking a lot about problem solving, not only about what problem solving is, but also about how to teach mathematics problem solving.

Cathy believes that the major aspect that makes the new mathematics program more difficult is the emphasis on problem solving. But what Cathy is perplexed about and unsure of is why the problem solving is difficult for children in her class. She speculates that perhaps it's because "it's more abstract." She's also not sure exactly how to teach mathematics problem solving. Because of her uncertainty, Cathy is doing some questioning and learning from her own teaching. For example, at one point in the interview, she describes a procedural approach that she has used to teach her class how to solve story problems:

> When we first started doing story problems, we went through these five steps: What Does This Story Tell? What Does This Story Ask? What Is [sic] The Data? What Are The Numbers? What Operation Do You Use? And then finally, How Do You Work For The Answer?

She says that on one hand, she thought that this procedure was really good because it broke down the steps that you go through to solve the word problem but, Cathy comments, "You know. It didn't work for some of them. It just didn't, because they didn't understand the whole--the big picture--how to figure it out somehow. Somehow that didn't help them figure it out." Thus, Cathy on her own has discovered what researchers in mathematics education such as Schoenfeld (1987) have recently concluded. Such procedural approaches to problem solving *do not* seem to facilitate mathematical thinking and problem solving when they are conceived of as "training in particular heuristic strategies, teaching a prescriptive "managerial strategy" needed for competent problem-solving performance (Schoenfeld, 1987, p. 213).

When asked why she thought it was that these procedures didn't work, Cathy said she wasn't really sure. On the one hand, these five problem solving steps seemed really logical to her. But on the other hand, she said finally, "You know, I don't think that's how *I learned* to solve story problems." But when she tried to introspect on how she learned how to solve story problems or how she solves them, she's not really sure. She just says, "You know, you just read it, and you can just see what it's supposed to be . . . Somehow, it comes together in your mind."

Cathy also seems to have a developmental theory of learning mathematical problem solving. For example, when asked why children found a particular problem hard, Cathy explained:

> There's a lot of problem solving and abstract reasoning, and that is hard for some of them, either because they aren't used to it, or they are just not developmentally ready to do that kind of reasoning. So I try to walk them through it as much as I can, but I think for some of them, until they are ready to get it, they are just not going to get it.

Cathy's View of Children's Mathematics Learning and Development

Although Cathy does not make her view of learners explicit, her comments suggest that she is thinking of the development of students' mathematical knowledge in terms of Piagetian-like stages in which the student moves from a concrete stage to more abstract stages. She believes that children's mathematical problem solving depends on their developmental level and that students can't solve some problems until they reach certain levels of readiness. In contrast, the *Framework* authors suggest that whenever possible, teachers should "teach the mathematical ideas through posing a problem," and that teachers can and should do this with children of all ages and levels. Thus, not only do Cathy and the *Framework* authors seem to have different views of problem solving, they also seem to hold different views of children's mathematics learning and development. Where did Cathy get her ideas and where did the *Framework* authors get theirs? Cathy's ideas might be traced back to a course she took at the university that she attended for her undergraduate and her Master's degrees. As Cathy recollects:

> When I was in school, I think we did some kind of Piagetian-like experiments. I think one of them was with real kids, having sticks in different sizes, and seeing if they could put them in order--just things like that. I guess that it is problem solving, but is also just seeing what developmental level they are at. I think that is really important. I think it's kind of a mistake to think that you can not over come that if they are not ready for it, I don't think. You can't speed up their development somehow. I don't know if that is really desirable. But it seems like that is what a lot of people want to do.

Cathy's conception of children's development of mathematical thinking reflects her knowledge of Piaget's developmental learned in a course in educational or developmental psychology more than five years before. Since the early 1980's, psychologists interviewing children and studying the development of children's mathematical knowledge have become increasingly impressed by the informal mathematical knowledge that children do have rather than by their lack of knowledge and readiness. In interviewing children, psychologists have become increasingly intrigued with trying to understand how children are thinking and with trying to make sense of how children are making sense. An underlying assumption is that mathematical knowledge--like all knowledge--is not directly absorbed but is constructed by each individual. This constructivist view is consistent with the theory of Jean Piaget but comes in many varieties and does not necessarily imply either a stage theory or the logical determinism of orthodox Piagetian theory (Resnick, 1989, p. 162). However, most teachers do not have access to this recent knowledge; and most teacher education programs probably still teach Piaget the same way that Cathy learned it.

The *Framework*'s View of Children's Mathematics Learning and Development

A constructivist view of children's mathematics learning and development seems to underlie much of the thinking and ideas expressed in the state-level *Mathematics Framework*. These include the ideas that:

We must be interested in what students are really thinking and understanding . . . it is through the probing of the students' thinking that we get the information we need to provide appropriate learning experiences" and "It is not the activities or the models by themselves that are important. What is important is the students' thinking about and reflection on those particular ideas dealt with in the activities or represented by the models." (California State Department of Education, 1987, p. 14).

Juxtaposing the ideas expressed by the *Framework* with the most current ideas expressed by psychologists studying children's mathematics learning and the development of mathematical knowledge, we see some important commonalties. For example, an underlying assumption is that "mathematical knowledge--like all knowledge--is not directly absorbed but is constructed by each individual. Resnick and others (Resnick, 1989; Gelman & Gallistel, 1978; Ginsberg, 1977) have argued for a general reorientation of early mathematics instruction that stresses concepts, explicitly engages children's informally developed mathematical knowledge, and focuses less on computational drill and more on understanding why arithmetic procedures work. For example, when teachers see children as constructing knowledge, filling in gaps, and interpreting in order to understand, then they may view children's "wrong" mathematics answers in a new light. Mathematical "errors" are no longer just mistakes, something to be gotten rid of, but provide insight into how the child is trying to make sense of something. Thus, "it follows the teachers may not always want to teach the rules 'tricks of the trade' that get rid of errors, because they might be getting rid of the clues they need in order follow their students' thinking" (Resnick, 1988-89, p. 15).

The authors clearly have this perspective, but many teachers--having no access to new work in psychology and mathematics education--hold different views of learning. This clash of perspectives might result in frustration for teachers like Cathy who are trying to enact a new curriculum while wearing their old spectacles. To what extent do the *Framework* authors recognize the possibility of teachers experiencing frustrations and what do they see as factors contributing to teachers' frustrations?

The authors of the state-level *Mathematics Model Curriculum Guide* talk about some kinds of frustrations that teachers like Cathy migh experience. They suggest three contributing factors. First, teachers often teach as though symbols have obvious and inherent meaning. Second, teachers need to consider the students' level of cognitive maturity and recognize that what seems obvious to adults may not be obvious to the child. Third, in their search for "the ever clearer explanation" teachers often overlook the importance of the students' "need to construct their own understanding." (California State Department of Education, 1987, p. 10).

To what extent does Cathy seem to know, recognize, or be aware of the these factors that often "contribute to the frustrations of teachers and students involved in the teaching and learning of mathematics"? Interestingly, Cathy seems very aware of the first two factors. Throughout her discussion of the difficulties that her children experience with problem solving, she refers repeatedly to the symbols as being "too abstract." Cathy recognizes that numerical symbols do not have an obvious inherent meaning for her second-grade children. Also, she tends to focus on the her students' cognitive immaturity and lack of "readiness." Thus, Cathy, like many other teachers, takes a conservative view of her children's abilities to know, understand, and make sense of mathematics (c.f., Monk & Stimpson, 1989; Peterson, Carpenter, & Fennema, 1989).

In addition to being influenced by her own implicit views of children's mathematics learning and development, Cathy is also influenced by the way children's mathematics learning is conceived of in the district's ABS model. The ABS model incorporates components of mastery testing and learning and direct instruction to promote students' engagement with and time on mathematics tasks. One of the things Cathy is becoming uncertain about is the "fit" between the ABS model of mathematics teaching and "teaching mathematics for understanding."

Contemplating Uncertainties

Although Cathy expresses uncertainty about her mathematics knowledge and her knowledge of how to teach problem solving, Cathy is knowledgeable and confident of her ability to implement the district's ABS model. The model emphasizes content coverage and pacing, maximizing students' engagement and time on task, using active mathematics teaching, and mastery testing and re-teaching. However, Cathy senses possible conflicts between given elements of the ABS model and "teaching mathematics for understanding." She describes potential conflicts with mastery testing, time and pacing, and direct instruction.

In the course of the interview about her thinking about teaching mathematics for understanding, Cathy began to question whether the kind of paper-and-pencil mastery testing that she is doing as part of the ABS model really assessed students' mathematical understanding. She admitted:

> I don't know how well it measures their understanding of concepts. But you know it (students' mathematical understanding) is extremely difficult to test. I don't know how to test that really so I can't say. I would have to sit down with each kid, and it seems to me and do it . . .Test individuals somehow and ask them. Give them little problems to solve and see if they can do it.

A second conflict she feels involves time and pacing. She senses that students need more time to learn. "It's hard . . . students are not used to thinking ," she said, "and there isn't enough time to let them sit there and figure it out." At times she also feels some incompatibility between the new math program and some elements of the existing ABS model, particularly the emphases on pacing and time: "I suppose ideally, if you are going to emphasize the problem solving, I just think you need more time to let them sit there and kind of "stew it over" more, so I think there is kind of a conflict there. Also as a teacher, she needs more time to figure out *how* to teach, and time to reflect on her own teaching. "There is never enough time. I don't usually have time to sit down and really go through the lesson and figure it out before I teach it." Cathy rightly senses that a key element of problem solving is uncertainty and that problem solving under uncertainty requires time to wrestle with and think about the problem. Students need such thoughtful time to solve mathematics problems and so do teachers who are contemplating the dilemma of how to teach mathematics for understanding under conditions of uncertainty. In her reflections Cathy expresses some of the same sentiments as the authors of the *Mathematics Framework* that she hasn't seen. As the Frameworkers put it:

> The teacher needs to create an atmosphere in which students understand that being temporarily perplexed is a natural state of problem solving. Adequate time for problem solving must be provided because the students, not the teacher, must do the thinking, make decisions, and find successful means to solve problems" (California State Department of Education, 1985, p. 14).

A third tension Cathy feels is between direct instruction and a problem solving approach because *"you can't really teach anybody to think.* Somehow it has to *evolve out of themselves."*

Although Cathy expresses eagerness to learn and try out new strategies in her classroom, she is conflicted as she thinks about new practices. For example, she describes how ABS put on an in-service about reciprocal teaching. She says that one of the things that was said was: "This is much slower. You just go slower, and you really delve into it deeply, and you don't divide it into all these sequential skills."

In a later conversation with me, Mr. Metzenbaum, the head of ABS, confirmed that ABS is adding to its research-based model new strategies such as reciprocal teaching because research has shown this strategy to be effective in promoting thinking (c.f., Palincsar & Brown, 1984). He noted that some teachers have begun using reciprocal teaching in reading, and a couple teachers have also applied reciprocal teaching to mathematics. Metzenbaum did not indicate how reciprocal teaching might fit with or conflict with the existing ABS elements of pacing, mastery testing, time on task, and direct instruction. He described reciprocal teaching as:

> A strategy where all the responsibility for understanding the process moves to the student. The teacher initiates it, models it, and then the student ultimately accepts it, is able to, can intervene to solve the problem or comprehend the paragraph more on their own after they understand the strategies--the metacognitive concepts of thinking about what you're doing, thinking out loud about it, and modeling it.

However, in her conversation with me, Cathy recalls that, at the ABS in-service on reciprocal teaching, the big question from teachers was, "How in the world are we going to do this and be accountable with our ABS pacing charts? And Cathy remembers that the reply to the teachers was,"Well, if you are really doing this, throw the ABS charts out the window." Cathy concludes by turning to me and saying, "Now that is what he said. I don't know if we really believe that, or if we are willing to take a risk."

Unsure of how to teach mathematics problem solving, and unsure of whether the ABS model is compatible with teaching mathematics for understanding, Cathy concludes that she doesn't know whether she is actually doing what she believes in--teaching mathematics for understanding. Cathy's uncertainty became most apparent when I showed her a paragraph from the *Framework* (California State Department of Education, 1985, p. 12) which describes what is meant by teaching for understanding and asked her which of the ideas in the paragraph were consistent with her view of teaching and which ones were inconsistent. After Cathy had read the paragraph, we talked:

Cathy: Yeah, I agree with that. I mean I think it is theoretically sound.

Penelope: What do you mean by theoretically sound?

Cathy: I mean there needs to be a reason for the children to be learning these things, and I think it is easier for them to learn if they know *why they are learning it* and what it is good for . . . I mean I agree with this, but you know when I look at my everyday teaching, I'm not sure if I am applying what I think to what I'm doing.

Penelope: Why do you say that?

Cathy: Well, I just follow the lessons, and I do what they say.

Unsure of Her Knowledge: The Poignant yet Hopeful Case of Cathy Swift

In our study of policy and practice Cathy Swift is a teacher who stands out because she readily admits that she doesn't know if she is teaching mathematics for understanding and she is unsure of how to teach children to solve mathematical problems and to think. Because she so readily admits her uncertainty and lack of knowledge and yet is trying so hard, Cathy at times seems a poignant case. She feels the pressure to do more without adequate resources at this point--either internal (e.g., the knowledge and confidence) or external (e.g., staff development and support) to do so. Her own sense of having inadequate internal resources to meet the new demands comes across clearly in the following statements that Cathy made during one interview:

Cathy: The textbook itself is great. It's hard. The word problems are hard.

Penelope: In what way or how?

Cathy: They are just hard for the kids.

Penelope: Why?

Cathy: I don't know. They are not used to thinking, I guess. And it's really hard to teach although I do my best. And there isn't enough time to let them sit there and figure it out. I guess that's something I could use help with. I mean--how do you teach problem solving? But I don't know if anybody's come up with a great way to do that. But I think too, what they (district folks) are saying (is valid): If they (the children) are used to doing this kind of thing, the kids will get better at it. It *is* our first year.

At present in addition to lacking knowledge and confidence to teach mathematics for understanding, Cathy also lacks time, support, access to new knowledge, and encouragement to learn and develop her own knowledge. Although Cathy says that she has not gotten the help that she needs, she also admits that she has not really asked for help. She tries to "pick things up" just from her own reading. When asked what she feels she needs, Cathy says she need more planning time and more time to teach mathematics so that kids really will have time to think and play around with the manipulatives. Cathy wants more planning time so that she will have time to figure out the mathematics lesson from the book before she teaches it, and she also wants to use her extra planning time to make math games and manipulatives. Cathy's favorite kind of inservice workshops are "make it-take it" workshops in which teachers make materials or activities that can actually be used in teaching.

Though a poignant case, Cathy is far from hopeless. Indeed, in recognizing and knowing that she doesn't know and in questioning, thinking and beginning to examine her own teaching, Cathy is taking a first but important step toward growth and development of her own knowledge and toward change and reform of her classroom practice. Over and over in the course of the interview, Cathy asks, "How do you teach problem solving?" Partly, she seems to view these as rhetorical questions which she then answers herself by saying, "I do not know." But the mere act of querying herself seems to cause her to reflect and to ponder so that I could almost hear her thinking. Thus, Cathy's more complete answer might be "I don't know *now* but I'm thinking, and I'm learning. I need to know more and a year from now I will. I'm a relatively new teacher, you know. And I'm still learning all the time." For Cathy to continue to change, she will need to continue to learn. For Cathy to continue to learn, she will need help, support, access to knowledge, and above all, time.

How will Cathy learn what she needs to know to teach mathematics for understanding? And will she? Who will decide what knowledge of mathematics teaching and learning Cathy needs? How will she get it? What kind of knowledge does Cathy need--knowledge of how children learn to solve mathematics problems, knowledge of mathematics or both? Procedural knowledge of how to teach mathematics problem solving? Research-based knowledge on how to teach mathematics for understanding? How will the knowledge that Cathy develops or is given be related to the policy of teaching mathematics for understanding? These are important questions, and they are central to the issues of whether and how Cathy will continue to reform her mathematics teaching practice over the next few years.

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Learning the Hows of Mathematics for Everyday Life: The Case of Valerie Taft

Ralph T. Putnam

Valerie Taft teaches fifth grade in Forest Glen, a large suburban school district in California. Valerie enjoys teaching mathematics, considering it one of her strongest subjects. She cares deeply that her students learn mathematics that will be useful to them in their lives. Her fundamental goal for students is "to make them prepared for the world of math in everyday life experiences" (interview 1, 12/88). For Valerie, this means helping students master the computational procedures of arithmetic--the "hows"[1] of mathematics--and knowing how to apply these procedures to situations in "real-life." Thus she views mathematics as a set of procedures and concepts whose ultimate importance and relevance are determined by their usefulness in everyday situations. This linking of mathematical skills and procedures with situations is in some ways consistent with California's *Mathematics Framework* (1985), which state and district level educators intend to help shape classroom practice. But Valerie has never seen the *Mathematics Framework*, and her views about mathematics are in other ways at odds with the vision of mathematics teaching it holds forth. Valerie's view of mathematical knowledge is more mechanical than the *Framework*'s. For her, the steps of various procedures are to be learned and then applied in a relatively straightforward fashion to situations in which they are appropriate. Understanding the "whys" of mathematics--why various mathematical procedures work or how they are related to one another--is not a part of her picture of useful and important mathematical knowledge. This rather mechanical view of mathematics, along with beliefs about teaching and learning, shape Valerie's pedagogy: In her classroom there is a clear emphasis on computational procedures being learned and applied to "everyday life" situations. But Valerie's views, along with limitations in her knowledge of the mathematics she is being asked to teach also contribute to some rather serious difficulties in trying to help students learn to apply the mathematical procedures they are learning to everyday situations.

Mathematics as Useful Tools

Valerie's fundamental goal for students is "to make them prepared for the world of math in everyday life experiences" (interview 1, 12/88). For Valerie, this means mastering the computational procedures of arithmetic--the "hows" of arithmetic--and being able to *apply* them appropriately in various situations. Valerie's primary goal here is for students to learn to carry out the steps of the various computational procedures of arithmetic, but also to perceive the usefulness of the procedures in future contexts, especially in everyday life:

> I really feel that the purpose is to make them prepared for the world of math in everyday life experiences. It should be used in that context. They should be able to realize that this is something that you will need later on. (interview 1, 12/88)

Students need to know about specific instances in which the procedures and concepts they are learning can be applied. For example, when asked about what it is important for fifth graders to learn about fractions, Valerie said,

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[1]Words and phrases in quotes are Valerie's terms.

Examples of when they might use it. If you are going to double a recipe or lessen a recipe, cut material, you need to know because you [will] not always have whole numbers. Sometimes that line is going to be farther than one but not quite to two, you need to know then what fraction of that line that you have there. So I could see it being used in that way. (interview 2, 12/88)

Valerie feels it is important that students have opportunities to apply the procedures and concepts they have learned to everyday situations, drawing upon their own experience when possible. When commenting positively on the large number of application problems in *Real Math*, the newly adopted textbook in her district, Valerie noted that the new text has

. . . a lot of word problems and I like that. And because that's basically how kids have to learn how to think. Basically, that's what those problems do for them. So I really do like that a lot. . . . a lot of the problems that are given are very good. They are the kind of problems that even I find challenging sometimes. . . . They don't give them all the information. The kids should know that there are 60 minutes in an hour. Or if someone works 8 hours a day and they are paid so much money for overtime they should be able to compute that. (interview 2, 12/88)

Valerie went on to praise the textbook's problems for being like situations encountered in everyday life in often requiring students to use knowledge acquired at some other time, not just the procedure or concept they happen to be working on that day in math class:

And it gives them opportunities to go across the board to see that just because I had measurements years ago doesn't mean I can't use it anymore until I get to measurements themselves. That sometimes I may have to use them anytime. And it's giving them that kind of practice and I like that. (interview 2, 12/88)

Thus, Valerie clearly values having students be able to apply the mathematics they are learning to everyday situations. She could be said to view mathematics as a set of useful tools--procedures that can be applied in the solving of problems arising in everyday life. Students need first to master the tools (the computational procedures) and then have opportunities to apply these tools in a variety of problem situations. For Valerie, this application involves retrieving the particular computational procedure or fact for the task at hand, for example using multiplication to compute overtime or knowing that there are 60 minutes in an hour.

Valerie is not alone in this view of mathematics as tools to be learned and applied in various situations. The idea is at least a part of the picture of teaching mathematics for understanding painted in the *Framework*:

Teaching for understanding does not mean that students should not learn mathematical rules and procedures. It does mean that students learn and practice these rules and procedures in context that make the range of usefulness apparent. (California State Department of Education, 1985, p. 13)

As students progress through elementary school, they should increasingly be expected to exercise independent judgment in selecting the computational procedure and tool for a given problem. Before the end of the sixth grade, the student should have all procedures and tools continuously available and be responsible for making a choice among them based on the nature of the problem and the numbers involved. (California State Department of Education, 1985, p. 4)

But whereas the *Framework* emphasizes the use of mathematical tools in exploratory and open-ended ways, Valerie's view of mathematical tools is a more mechanical one. She emphasizes learning the steps of various computational procedures. When asked to define *procedures*, Valerie said they were the "steps that you take. You would need to know the steps to get to the end of what you did. If the division problem is three steps, you need to know those procedures" (interview 1, 12/88). Valerie characterizes this focus on steps as an emphasis on the "how" of arithmetic, not the "why." In part, this emphasis is rooted in Valerie's beliefs about the kind of mathematical knowledge that is useful. For example, when asked to comment on a passage from the *Framework* that emphasized not teaching procedures and skills in isolation, Valerie agreed, but with the caveat that procedures themselves are important:

> If they are all just scattered there, then when they get out here in practicality, it will not help them. They have to learn how to bring all of that together to use to their advantage. I do believe that *but there are some times that just the procedure itself can help you through.* (interview 1, 12/88, emphasis added)

Valerie's emphasis on the mechanical steps of procedures reflects, in addition to her beliefs about what mathematics is important, a belief that focusing on the conceptual underpinnings of mathematical procedures--the "why" of mathematics--often confuses students and interferes with their learning of important procedures. When asked to comment on a paragraph from the *Framework* describing the idea of teaching for understanding, Valerie responded,

> I strictly believe that they have to learn math and need to learn *how* it is being done. I think sometimes though they are not always ready to learn exactly the *why*. Sometimes they get bogged down and ask lots of questions. I think sometimes that starts to more confuse them than help them. . . . I believe that they should learn just the procedures sometimes and then later when their mind is more mature, they can understand the why's of why you do what you do. . . . the emphasis should be on learning the skill and then they ask when they are ready. However, when you see that they are getting confused this is when you have to leave it alone. (interview 1, 12/88)

Valerie feels that the *why* is difficult to teach and that students often aren't ready to understand. She gives the example of teaching *why* you move the decimal points in a division problem in which both the divisor and the dividend have a decimal point: "That is just mind boggling; it is just trying to absorb too much and too deeply. They are not ready for it and they cannot focus on it." She goes on to say, "sometimes it is just not necessary for the moment for them to know the why. Lots of times the kids will figure out the why themselves if they are ready for it." For Valerie, it is important that students, rather than getting bogged down in the conceptual mire, learn how to carry out computational algorithms and know when to use them.

For Valerie, problem solving becomes a matter of applying these learned algorithms to "everyday life" situations; problems are sites for the relatively straightforward application and practice of mathematical procedures and concepts that have already been learned. Valerie does not appear to view problem solving as the more open-ended and exploratory process described in the *Framework*: "a process, with solutions coming often as the result of exploring situations, stating and restating questions, and devising and testing strategies over a period of time" (California State Department of Education, 1985, p. 13). Indeed, Valerie seems uncomfortable with open-ended problems that do not have clear solution paths. Although she likes and uses the application word problems in the new textbook, she generally skips the *Thinking Stories*--richly developed situations in which a variety of mathematical problems are posed for students to reflect upon and discuss:

> they have some stories in there and they want to spend a whole day on what they call a *thinking story*. The conversation or questions that go with the story, the kids are getting just like that. I do not feel that I have to keep going over that and keep trying to draw out more and more when there is not more to draw out. . . . I personally do not like them; they are really kind of ridiculous. However, I think that is the whole point; they are just trying to show that some people do not think well and so you are supposed to think for them. (interview 1, 12/88)

The notions of exploring situations for different interpretations or approaches to thinking about a problem are foreign to Valerie's view of mathematics and thus seem out-of-place or "ridiculous" in her classroom. Problem solving, for Valerie, means having well practiced computational skills that can be applied when appropriate to solving everyday problems.

As one would expect, Valerie's beliefs about mathematics and her goals for students are intertwined with what goes on in her classroom. I observed Valerie teaching on three occasions during the 1988-1989 school year--twice in December and once in April. In this paper I describe in some detail here the two lessons I observed in December because they so vividly illustrate how Valerie's knowledge and beliefs unfold in her teaching. The paper concludes with some reflections on those lessons and on Valerie's responses to messages for change in her mathematics teaching.

A Lesson on Averages

Valerie's fifth-grade class consisted of about 30 students--about 60% white, with the remainder being black, Hispanic, and Asian. Students sat at desks arranged in rows facing the blackboard at the front of the room. Valerie took the lesson directly from the lesson plan in the textbook (Willoughby, Bereiter, Hilton, & Rubinstein, 1987, pp. 208-207), but with some modifications and embellishments. She began by writing the word *average* on the board and asking, Does anyone know what this word is?" Students suggested a variety of meanings and uses of averages: regular (not out of the ordinary), the average number of kids in the class, that an average person drinks a certain number of gallons of water a year. Valerie accepted each contribution without much comment until one of the students mentioned the averages appearing in the baseball statistics on her notebook. Valerie picked up the idea of ERAs (earned run averages) and talked about lower averages being better. She had the students compare one team's ERA of 2.03 with a previous year's of 3.25, making the point that the pitching was better (or the other teams were worse). It was evident throughout this episode that Valerie wanted students to realize that averages can be useful and interesting to them; *average* is not simply a textbook concept. After the lesson Valerie commented that she wanted students to realize that averages "are something that they use more often than they think, or hear more often than they think, then they should try to related it to everyday life" (interview 1, 12/88).

Then Valerie had the students turn to page 194 in their textbooks, which described a procedure for finding the average of a list of numbers. Valerie carefully led the students through the two steps, which she wrote on the board:

1. add all numbers given
2. divide the sum by the number of digits[2] added

Consistent with Valerie's emphasis on the *hows* of mathematics, these steps became the primary focus for the rest of the lesson. Valerie led students through the addition and division calculation for these two steps to find the averages for the three lists of numbers on the textbook page, emphasizing that both the addition step and the division step must be taken to find the average.

Then Valerie had the students turn to the next page, which presented a chart of a student's 16 grades in various subjects, along with a series of questions about various averages that could be calculated from the information in the chart. Valerie read the following introduction from the top of the page: "Amalia kept a record of her test scores at school and recorded them on the following chart. On all tests a perfect score is 100" (Willoughby, Bereiter, Hilton, & Rubinstein, 1987, p. 210). Valerie then asked the students what information they saw in the chart, to which various

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[2] Although Valerie wrote "digits" to refer to numbers, this unorthodox use of the term did not seem to create any confusion for the class in carrying out the procedure.

students responded with (a) when Amalia took the test, (b) what subject the tests were in, (c) and what grade she got on each test. Valerie directed attention to the numbered questions on the page and read the first one: "What was the average of all Amalia's test scores?" Prompting students for the procedure they had just learned, Valerie asked them what to do, to which several responded as expected: "Add all the scores." Valerie continued: "Then what?" to which students responded, "divide." The students were now ready to begin the calculations involved in these steps for computing the average. Valerie had them get up and get paper because this is "very difficult to do mentally." (The textbook does not mention the possibility of using calculators for carrying out these rather lengthy computations and, in any case, Valerie would not be likely to allow students to use them because she feels they interfere with students' learning of computational procedures.) The students filed to the back of the room to get paper, returning quickly to their seats to begin working on their calculations. As they worked, Valerie wrote on the board the scores from the chart in two columns of eight numbers and had two students come to the board to add them:

91	79
48	55
65	100
95	90
53	96
49	60
75	95
+97	+96
573	671

The room was quiet as students worked. After a few minutes, when the majority of students had finished their calculations, Valerie announced that there were several ways to add these numbers and gave the sums students should have gotten if they added these separate columns (573 and 671). She also commented that she liked the way some students were adding numbers "two at a time" and described briefly how they added first two numbers--91 and 48--then the summed to the next number--65--and so on. Then on the board, Valerie added 573+671 (the sums of the two columns) to get 1244 and wrote

$$16\overline{)1244}$$

She told students to carry out the division individually. After a bit, Valerie said to the class, "Some of you are saying, 'I haven't had 2-digit division.' It doesn't matter. Try it. It's the same steps." While students were working, Valerie had Benjamin come up to the board to do the long division; he produced the answer: 77r12. Valerie told Benjamin to "go another place" and, after a moment of confusion, he continued to work. Benjamin had some trouble, in part because he had written the problem at the very edge of the blackboard and ran out of space to write. When Benjamin finished, Valerie wrote 77.75 in middle of board and said to the class, "Once you got through dividing, this is the decimal you should have ended up with." Then she proceeded to "talk through" the steps of the long division as she wrote the solution:

T: Okay, what we're doing here, I'd like everyone's attention up front, please. (writes on board:)

$$16\overline{)1244}$$

T: Okay. 16 can't go into 12, so you took 16 into 124, correct?
Ss: Yes
T: And Benjamin did correctly, 7 times. The 7 *should* be over the 4 because you're dividing into 124. (Benjamin had written the digits in the answer further to the left than they should have been) 7 times 6 is what class?
Ss: One, 12
T: 7 times 6?

Ss: Oh, 42
T: 42. Put down the 2, carry the 4. 7 times 1 plus 4 is?

Valerie continued talking through the division procedure in this fashion, finishing with: "So, the decimal you come up with is seventy-seven and seventy-five hundredths." Then Valerie explained how to round off the answer:

T: Now, doing an average for a grade, though, what you would do is, take the number in the hundredths place (circles 5) and round *up*. Since that is 5 or more, we're going to make this place value (points to 7) one digit higher, so it becomes what?
S: 8
T: 8
T: So the average of her test scores is 77?
S: Point 8
T: Point 8, or 77 and 8 tenths is the average of her test scores.

Having successfully computed the average of the test scores, Valerie turned to the next discussion question in the textbook: "How well does the average describe Amalia's test results?" A comment in the margin of the the Teacher's Guide points out about this question:

> The average for all of Amalia's test scores does not describe her performance well; there is much more information on the chart. Because Amalia does better in some subjects than in others (which is not unusual), one average is not too useful, particularly since it won't enable you to tell in future tests whether Amalia is improving in that subject. (Willoughby, Bereiter, Hilton, & Rubinstein, 1987, p. 210)

Valerie converged quickly on this point, rather than using the question in the student text as a site for eliciting students' thoughts on the question of how well the single average describes Amalia's performance. She asked some questions about which subjects Amalia did better or worse in, then through statements and structured queries to students, made the point that Amalia's overall average doesn't accurately reflect her performance in all subjects. If, for example, she was placed in a math class because of her overall average, she would probably be placed in too low a group. Reflecting her emphasis on knowing how to do the procedures of mathematics and knowing how to apply them to real-life situations, Valerie pointed out that "You have to be very careful when you use averages. You still have to know how we get them, then decide how to use them."

Valerie skipped the remaining questions on the student page--questions that asked students to find Amalia's averages for individual subject matters and reflect on how well they describe Amalia's performance. Valerie had students turn their attention to the next page, which presented a chart of long-jump times by two boys, Tomeo and Barry:

Distance of Jump (measured to nearest centimeter)		
	Tomeo	Barry
First jump	140	143
Second jump	141	158
Third jump	139	144
Fourth jump	143	102
Fifth jump	138	149

Valerie had a student read from the text at the top of the page:

> Tomeo and Barry had a contest to see who could jump farther. They decided to make 5 jumps each. The winner would be the one with the longest average of the 5 jumps. Study the results of this contest. Then answer and discuss the questions that follow. (Willoughby, Bereiter, Hilton, & Rubinstein, 1987, p. 211)

Valerie instructed the students on one side of the room to find the average of Tomeo's five jumps and the students on the other side to find the average of Barry's jumps, taking their answers to two decimal places. After students had been adding for a while, Valerie asked for the sum of Tomeo's jumps. Different students gave the answers 690, 701, and 711, which Valerie listed on the board. She then told the group to go back and check their adding. The second group gave her 696 and 676; she also told them to "check yours again; I got two different answers". After the students had worked a bit longer, Valerie announced to the first group, "Okay, for Tomeo, your answer should be 701. Now find the average of *that*." To the second group she said, "696 is the sum for Barry. Find the average of *that*." While students did the division at their seats, Valerie had individuals come up to the board to carry out the computation publicly. When they were done, Valerie asked who won the contest--the boy with the highest score. She pointed out that Barry jumped highest one time, but Tomeo won, because Barry's low jump of 132 really hurt his average.

Valerie summarized the lesson by asking students to restate the two steps for finding the average. She then passed out a ditto on which students were asked to calculate the average and the range for several sets of about five one-digit numbers. After working through the first set together, Valerie told the students to finish the rest for homework.

Reflections on the First Lesson

By some standards, this was a good lesson. Valerie was clear about what students should be learning and the majority of class time was spent in active instruction. Student engagement was high, and management routines were smoothly carried out so that little time was wasted. Valerie worked to help students see the relevance of the mathematical procedures they were learning to everyday life situations. But throughout the lesson, Valerie's primary emphasis was on correctly carrying out the steps of the computational procedure for finding an average. Being able to carry out these steps is, indeed, an important part of what is to be learned about averages. A comment in the margin of the page of the teacher's guide in which the two-step procedure for finding an average is first introduced suggests:

The definition given on the page is an operational one. ("Follow the steps and what you get is the average.") At this time concentrate on these steps rather than on a more elaborate definition. Throughout this unit and later units, the students will work with averages in different contexts (average speed, average height, average score, average consumption). In particular, pages like 196-197 provide a basis for a discussion of using averages sensibly. Thus the students will gradually develop a solid understanding of the meanings of average and the ability to work with averages.

Valerie did, indeed, concentrate on the computational steps, probably more so than the textbook authors intended. Consistent with her desire for students to apply mathematical tools in everyday settings, she also provided some discussion about the usefulness of averages (at the beginning of the lesson) and about "using averages sensibly" based on the problem situations in the textbook (Amalia's grades and the long-jump contest) But this *discussion* involved Valerie *leading* students to the point that averaging all of Amalia's grades together did not reflect her performance in individual subjects and that a single low performance (in the long jump contest) can significantly lower an average. The way Valerie made these points raises an interesting paradox in her teaching. Valerie clearly wanted students to understand how averages relate to everyday situations and she touched on potentially important and powerful ideas. But because Valerie explained them by *announcement*, or by carefully leading students to *fill in the blank* with the expected response, it is not at all clear whether students understood the explanations, or indeed, if they heard them at all. Valerie presumably assumed that since these important points have been *covered*, students could be expected to have understood and learned them. Indeed, communicating through ordinary conversation would be impossible if we did not assume that others understand much of what we say. But unlike most ordinary conversation, discourse in classrooms presumably has the goal, not of communication *per se*, but of student *learning* (cf. Bereiter & Scardamalia, 1989). Because students in Valerie's class had little opportunity to ask further questions or offer their ideas about the points being made, it was not at all clear whether the assumed learning had taken place. Valerie's classroom is not a place to raise and explore questions. It is a classroom in which the teacher announces the content to be learned and the students are expected to practice and learn it.

Valerie's emphasis on learning the steps of computational procedures and being able to apply them reasonably to problems in everyday life seems a reasonable, albeit limited, perspective for teaching mathematics. Students in the lesson just described seemed to be learning the target procedure and would likely be able to compute an average at some time in the future if directed to do so. One could make a fairly persuasive argument that these are valuable things for students to be learning. But most mathematics educators, including the authors of the *Framework*, envision a more flexible and reflective approach to applying mathematical tools to solving problems. Indeed, Valerie's focus on the mechanical steps of computation, along with some limitations in her knowledge of the mathematics she was teaching, led directly to some rather major difficulties in the lesson she taught the next day.

<u>A Second Lesson on Averages: Building on a Classroom Situation</u>

The lesson was a follow-up lesson on averages, this time *not* taken from the *Real Math* textbook. Valerie had commented in an interview after yesterday's lesson that she thought students needed more practice with averages, in part because the numbers used in the textbook examples were too many and too large for students to work with easily. The lesson again illustrates Valerie's view of mathematics as mechanical procedures, and problem solving as the relatively straightforward application of these procedures to appropriate situations. But it also offers a case of some of the difficulties that can arise when a teacher teaches content about which she has only limited knowledge.

Valerie began the lesson, as she did yesterday, by emphasizing the potential utility of averages in everyday life. She began, "Yesterday, we talked about averages. What's an average?" A student offered, "it's the middle of things," to which Valerie responded that yes, most students yesterday had said it was the middle. Then Valerie asked why we would ever need averages, why you would ever use them in everyday life. Students generated some examples, including the average amount birds eat, the average size of a person, the average amount of hamburgers people eat, and grade averages. Valerie picked up on grade averages and commented, "that's probably the most important in your life right now." She went on to say that she had been doing grade averages and sending home deficiency notices, pointing out that she averages grades for each subject, not across subjects (revisiting the idea from yesterday that overall averages may obscure performance in individual areas). Again, all this seemed directed at helping students appreciate the utility of the procedures they were learning in everyday life.

Valerie then spent the bulk of the lesson conducting a series of four small surveys to serve as the site for finding averages. (She said later in an interview that she thought these up on the spot--she had not planned them in advance.) First Valerie polled the students about their favorite flavors of ice cream. She asked how many students liked each flavor and recorded the number of respondents for each on the board:

chocolate	18
vanilla	5
strawberry	10

There was really nothing to average here, so Valerie said, "That just shows what most of us like. The most favorite is?" and students responded, "chocolate." Then Valerie surveyed students about how many times they had eaten ice cream since last Friday (i.e., during the past week). She asked how many students ate ice cream zero times, one time, two times, etc., recording the responses on the board (the left column shows the number of days someone has eaten ice cream; the right column shows the number of students who ate ice cream that many times):

0	-	14
1	-	2
2	-	5
3	-	4
4	-	2
5	-	3
6	-	0
7	-	0

To make a meaningful average from these data would entail either summing each student's response individually (e.g., adding fourteen 0s, two 1s, five 2s, etc.) and dividing the sum by the total number of students who responded, *or* weighting each number of days by multiplying it by the number of students in that category before summing and dividing by the total number of students (i.e., (0 X 14) + (1 X 2) + (2 X 5) + . . .). Either of these approaches would result in 1.57, the average number of days ice cream was eaten by individual students in the class. But the class's procedure for finding averages (presented in the textbook and by Valerie) did not contain such complexities. So when Valerie asked the class what to do to find the average, a student dutifully responded, "add," and Valerie led the class through adding the right-hand column of numbers. After getting 30 as the result, Valerie went to the second step for finding averages by writing

$$8\overline{)30}$$

on the board and talking step by step through the long division. The result was 3.7, which Valerie said can be rounded off to about 4 days.

But Valerie and her students did *not* compute the average number of days a student in the class ate ice cream last week (that number is 1.57). Rather, they computed the average number of respondents in each category--a pretty much meaningless average in this setting. The focus in this classroom was so much on carrying out the steps of the procedure for calculating an average that no one--teacher or students--noticed that the answer they have arrived at doesn't make much sense. Had they reflected more carefully on the numbers in their table, someone might have noticed that only 5 of the 30 students in the class said they ate ice cream on four or more days; thus it doesn't make sense that the *average* would be as high as four days. This is the sort of reflection the *Framework* authors probably had in mind when writing about the importance of estimating results:

> Another important skill allied to that of numerical estimation is the ability to determine whether a particular numerical solution to a problem is reasonable. Independent of any attempt to estimate or follow the calculation involved, the student should be taught always to ask, "Is my answer reasonable? Could it possibly be a solution?" (California State Department of Education, 1985, p. 4)

But the focus in this classroom was not on reflection or estimation; it was on learning the steps of procedures that can be carried out to produce correct answers. And the answer the class produced here was *correct,* in the sense that the calculations had been performed correctly. These calculations were not performed, however, on the appropriate numbers to produce the desired average.

After completing the calculations for the ice cream survey, Valerie moved on to poll the students about how many rooms they had in their homes, again recording the data in two columns (with the number of rooms on the *right* and the number of students responding for each on the *left*):

1	-	2
1	-	5
4	-	6
8	-	7
1	-	8
6	-	9
6	-	10
1	-	11
1	-	12

As before, Valerie led the students through adding the numbers in the right-hand column and dividing to get an average. This time, however, because Valerie put the number of respondents on the left and the response categories on the right, the class averaged the numbers representing how many rooms there were in students' homes, but the average was not weighted for the number of students with each size home.[3] The resulting answer for the "average size home in this class" was 8.5. (In this case, the answer derived from the inappropriate procedure was close to the actual average, which is 8. Thus, reflecting on the size of the numbers involved would not help signal

[3]As in the ice cream survey, the appropriate average here could be obtained here either by summing the students' responses individually (adding one 2, one 5, four 6s, etc.) or by weighting each number of rooms by the number of students giving that response (i.e., (1X2) + (1X5) + (4X6) + . . .).

that an inappropriate procedure had been used.) When a student commented on 8.5 not being "even" (meaning a whole number), Valerie gave a brief explanation that a lot of times averages don't come out even, giving the example of an average family size of 2.4. This explanation provides another example of Valerie announcing but skirting discussion on a potentially important and powerful idea--in this case the idea that an average often does not reflect an actual value possible in the data (you can't have 2.4 people in a family, and you can't have 8.5 rooms in a house).

The example of average size led to a final survey: determining the average family size in the class. There was a good deal of confusion in the class as students raised questions about whom to count as family members. Valerie handled all of these by decreeing various decision rules, such as: start with who's in your house; then add parents, brothers, or sisters who don't live with you. Again, Valerie touched on potentially important ideas: those dealing with how to define categories for taking a survey or compiling data--establishing the operational rules. But students did not grapple with these problems; they were told what the rules would be. Once all the numbers were up on the board, Valerie led students through the addition and division steps to find the average of one of the columns, again an unweighted average of the categories--the number of family members. This time, as Valerie started leading through the addition of the column of numbers, a student questioned which numbers the teacher was adding. Valerie, sounding a bit annoyed at being questioned, responded by saying "This is the number of people (pointing to the left-hand column) who said 3 rooms (pointing to the right-hand column); this (pointing to the right-hand column) is the side I want to add," and went on. In doing the division calculation, which was

$$9\overline{)66}$$

Valerie got to the answer 7.33 and said "this is going to go on," without further comment or notation of the repeating decimal. She then asked students about whether to round up or down and ended up with 7. (Note that there was no explanation given for why this number was rounded to a whole number whereas the previous answers had not been.)

Valerie spent the last part of the lesson going over homework by having students come up to the board to compute the averages from yesterday's ditto. Two episodes during this homework check again highlighted Valerie's emphasis on the mechanical steps of computational procedures. In the first episode, Jimmy had done the following division on the board:

```
   7.4
5)370
  35
   20
   20
    0
```

Valerie, pointing to the decimal point in 7.4, said, "Where did this decimal come from?" Jimmy said, "I don't know," and Valerie countered with "I won't accept 'I don't know.'" Valerie asked again, then explained that if there isn't a decimal point below (in the 370, then there should not be one above (in the answer): "you don't put a decimal point up here unless there's one down here." Jimmy finally muttered something inaudible and corrected his work to get the answer, 74. Valerie asked the class, "Raise your hand if you understand how he got 74," to which about three-fourths of the students raised their hands. Valerie then proceeded to the next student at the board.

The second episode occurred when a student was averaging the numbers 41, 41, 41, 41, and 41. The student had shown her work on the board for summing the numbers and going through long division to find the average. When Valerie got to this problem she said to the class, "Since all of them are 41, what's the average?" Someone in the class says "41". Valerie then said to the girl who had worked the problem out at the board, "The important thing is you went through the steps . . . but when the numbers are all the same, you don't have to do all the work."

The lesson concluded with a basic skills test, of the sort that Valerie gives weekly. Valerie wrote about 10 computational items on the board, including addition, subtraction, long division, and multiplication. Students were to copy the problems and work them. The last part of the quiz was a ditto involving the use of key words as clues for which operations to use in solving word problems.

Reflections on the Second Lesson

This lesson represented an important attempt by Valerie to go beyond the textbook, to provide students with needed additional practice on the procedure they were learning in the context of meaningful situations. By building the lesson around surveys conducted in class, Valerie illustrated the potential usefulness and relevance of averages and capitalized on student interest. These features of the lesson were clearly consistent with Valerie's goals that students learn mathematical procedures and how to apply them in everyday situations. Students indeed had the opportunity to practice the two-step averaging procedure presented in the textbook within the context of realistic settings. But the averaging procedure was *not* applied appropriately. In all three surveys Valerie and her students applied the averaging procedure they knew without taking the additional critical step of making sure that *each* student's response was included in the list of numbers to be summed and divided. This raises two important issues. The first concerns the role of the teacher's knowledge of the mathematical content being taught, which becomes even more critical when departing from the textbook. The second issue revolves around classroom norms and expectations about what is correct or reasonable in mathematics. Is a particular mathematical procedure applied in a situation because the teacher or the textbook says this is the appropriate procedure, or is it important that one reflects on whether the procedure makes *sense* in the situation at hand?

First, regarding subject-matter knowledge, Valerie is not ignorant of averages. She seems quite comfortable with averages as presented in the textbook--situations in which the relevant average is obtained by summing individual numbers or scores of some sort. The difficulties Valerie ran into with the class surveys were due to fact that multiple people had the same numbers, or responses--number of days eating ice cream or number of rooms in a home. These situations would have been more equivalent to those in the textbook--and the sum-divide procedure for finding an average would have been appropriate--had Valerie simply listed each student's response separately rather than tallying the number of students with each response. But Valerie's knowledge of averages was not richly developed enough for her to be aware of these complexities--at least not as they came up during instruction, when there were many other things to attend to as well. Had a similar example been presented in the textbook, with pointers either to the teacher or to students to be careful about which numbers were being averaged, the errors of the day probably would not have occurred.

But Valerie had ventured into a situation that was not treated in the textbook. This raises an important dilemma in teachers' use of textbooks. Many educators want teachers to be empowered to exercise professional judgement, including departing from the textbook when that seems appropriate for students. Indeed, many of the recommendations like those in the *Framework* calling for a more exploratory stance toward solving problems and learning mathematics require a good deal of flexibility and discretion on the part of the teacher. These things cannot be scripted in a textbook, no matter how *teacher-proof* it seems or its authors hope it to be. In Valerie's case, she seemed quite justified in supplementing the textbook with this additional lesson on averages. She felt that her students needed more practice with the procedures and she wanted to provide that practice in a meaningful setting. But to the extent that flexibility and discretion on the part of the teacher are called into play, appropriate knowledge and beliefs about the content being taught will also be required. Because of her limited knowledge about the content she was teaching, Valerie ran into some rather serious difficulty in the lesson, without even realizing that she had treated the content incorrectly.

This failure of Valerie and her students to realize that anything was amiss in the averages they computed leads to the second important set of issues: norms and beliefs about what is correct or appropriate in mathematics. Valerie's view of mathematical procedures that can be rather mechanically applied to solving problems resulted in an instructional environment in which the steps of procedures were presented by Valerie or the text and practiced by students. There were not opportunities to explore possible alternative strategies to solving the problems at hand--only the straightforward application of previously learned procedures. Either you know what procedure to apply in a particular situation or you do not. Reflecting on whether a procedure or answer indeed made sense in a particular situation was not something that happened in this classroom. Thus neither Valerie nor her students thought to question whether they were applying their averaging procedure appropriately in the survey problems. Creating a more exploratory and sense-making orientation to mathematical problem solving is an important goal espoused in the *Framework* and reflected in the following statements about problem solving:

> [Students] must explore and experiment, ask appropriate questions, and bring forth the mathematical knowledge that will enable them to progress toward answering the questions. (California State Department of Education, 1985, p. 3)

> Students must be actively involved in the processes of problem solving. The teacher should encourage students to think through these processes, foster discussion of ideas and approaches, and guide students to consider the reasonableness of their procedures. (California State Department of Education, 1985, p. 15)

But these are not attitudes or expectations that are easily captured in the pages of a textbook or a set of prescriptions for teachers. For Valerie to foster this kind of mathematical learning environment would require some rather fundamental shifts in her beliefs about the nature of mathematical knowledge and how it is learned.

Valerie's Response to Messages for Change

How is Valerie responding to attempts by her district to move mathematics teaching in the directions outlined in the *Framework*? Valerie's teaching seems relatively impervious to these messages for change, in part because her exposure to the messages has been minimal, and in part beacuse, like any teacher, she filters and adapts input for change through her own knowledge and beliefs. Her strongly held beliefs about the importance of students learning computational skills and their relevance for everyday situations provide a substantial buffer against outside press for change in her teaching.

Valerie had never seen a copy of the California Mathematics *Framework* or Curriculum Guide. These were things for district people or others to deal with: "They just send our instructions to us and we just follow them" (interview 1, 12/88). For Valerie, the only real exposure she has had to the ideas in the *Framework* have been through the *Real Math* textbook that her district recently adopted and a brief workshop to introduce teachers to the new text. Valerie does not concern herself with why the district might have chosen this particular textbook. When asked if she knew why *Real Math* was adopted, Valerie responded, "I guess because they liked it" (interview 2, 12/88).

From the perspective of curriculum and staff development personnel in Valerie's district, the recently adopted *Real Math* textbook is, indeed, a major vehicle for communicating to teachers the kinds of shifts in mathematics teaching outlined in the *Framework*. They believe that the *Real Math* textbook is "aligned" with the *Framework*, and that if teachers follow this new textbook quite closely, their mathematics teaching will approach the vision of teaching mathematics for understanding depicted in the *Framework*. Valerie, like other teachers in the district, reported that she had been instructed in inservice workshops to follow this textbook closely, "exactly as it is written, page by page by page" (interview 2, 12/88).

But although Valerie sees some differences in this new textbook (she's concerned with the minimal amount of practice and skill development for slower students; she likes the varied application problems), she does not see it as calling for much change in the way she teaches mathematics. When I asked her if using this new textbook had made her think differently about teaching math to fifth graders, Valerie responded:

> No, not really. I enjoy teaching math and I enjoy it [the new text] because most kids like it, when they understand it of course. . . . I don't teach it a whole lot different than the way I used to. (interview 2, 12/88)

Valerie reports following the textbook:

> I follow the book as much as possible. As much as I see the kids able to work in it, or need to. To follow through with what it has. Pretty much, I follow it. (interview 2, 12/88)

But "following the book" for Valerie means exercising a certain amount of professional judgment. She does not follow it "page by page" as she says she was directed in district inservices:

> No, because I do not do it. I feel as if there are some things that just cannot be covered because it either gets monotonous or it is not all necessary. If I feel I have to come out of that book to find extra work for my students then that is what I do. I do what I feel my students need and if the book meets those needs then fine and if it does not then I do what I feel is best. (interview 1, 12/88)

The major departures from the textbook that Valerie mentioned explicitly were supplementing with more skill work, omitting the *Thinking Stories* (because "kids don't really have to think much for those" [interview 2, 12/88]), and skipping the activities requiring calculators. These departures could result in a significant shift in the balance intended by the textbook authors between learning computational procedures on the one hand, and reflecting on those procedures and other mathematical ideas on the other.

But Valerie also supplements the textbook in ways intended to help students see the usefulness of the mathematics they are learning in everyday situations. We saw one example of this in Valerie's lesson on averages based on class surveys. This lesson represented a rather major departure from the textbook; the entire lesson was essentially a supplement or addition to the content as presented in the book. But this kind of supplementing is not the only way in which Valerie, or any teacher, adapts or interprets a mathematics textbook. Because a textbook can provide only words and pictures on a page--not actual classroom instruction or dialog--virtually everything in it is interpreted or filtered in some way in the course of instruction. Although it may be tempting to think of a textbook as determining much of what happens in the course of a lesson, in reality, it is only one source of influence on how a lesson actually unfolds.

In the textbook lessons that she does follow fairly closely, Valerie highlights the procedural aspects of the content and downplays the opportunities for students to reflect upon and discuss mathematical ideas. We saw this procedural emphasis in the textbook-centered lesson on averages described above. Valerie omitted a number of the questions in the student text that asked students to reflect about the adequacy of averages for various purposes, and the loss of such information as change over time. Instead, she spent the more lesson time on carrying out the computational steps for getting an average.

Both the major departures and the more subtle interpretations of the textbook that Valerie makes are filtered by her view of mathematics as mechanical procedures to applied in everyday life. In making these interpretations, Valerie is sustained by the confidence of her views about what students should be learning. She expressed this confidence when I asked her if she felt pressure from the district or parents for her students to do well on standardized tests in math:

> I know what is on it and I know that as far as fifth graders are concerned they are learning what they need to know and in a lot of cases beyond that. (interview 1, 12/88)

Similarly, when I asked her whether she felt pressure to cover everything in the textbook during the year, she responded:

> No, I don't let that pressure me. . . . I don't feel that they can rightly say that I have to cover all the material in this book when they don't give me all the time I really need to teach math. (interview 2, 12/88)

In short, Valerie perceives no press to change her mathematics teaching. She is satisfied that her students are learning the important skills they need and that the new textbook is providing ample opportunity for them the practice applying these skills to everyday situations. She is pleased with her students' performance standardized tests; they are learning what they should be learning.

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Reflections and Deflections of Policy:

The Case of Carol Turner

Deborah Loew .berg Ball

Mrs. Turner points at the problem on the chalkboard.

$$\begin{array}{r} 30 \\ \underline{-16} \end{array}$$

Thirty-five pairs of eyes are fixed on her, awaiting the cue.
"Tell me what to do here, class!" she demands.
"Borrow!" choruses the class.
"I don't say 6 take away 0, do I? You're so smart!" she exclaims, obviously pleased. "What do I say?"
"Borrow a ten from three tens," comes the quick response.

A dedicated teacher who deeply wants her students to succeed, Carol Turner is lively and enthusiastic in the classroom.[1] She delivers lessons with a brisk flair and virtually insists that the students learn. Center stage, she keeps all eyes on her and all minds on the curricular track.

Carol want her children to master the mathematical content of the second grade curriculum--telling time to the hour and half hour, counting by fives and tens, reading a calendar, adding and subtracting two-digit numbers--and she pushes them to participate in the classroom activity and discourse. Sometimes the math period lasts past recess--over 75 minutes--so bent is she on accomplishing her goals. With all kinds of devices--little stories, mnemonics, play-acting, and concrete materials--she marches her charges through the domain of second grade mathematics. A special education teacher for a number of years, Carol believes that her strengths--patience, the ability to "break down" the content, the use of a wide variety of imaginative teaching devices to suit different "learning modalities"[2]--grow out of this background.

Carol is highly regarded by both parents and administrators. Her classroom, one of about a dozen portables clustered on the school grounds, is crowded like most of the other rooms in this large suburban California district. Her 35 pupils, who attend school year-round, include black, Asian, and Hispanic children from middle and lower-middle class backgrounds. Children sit at desks grouped in fives, reflecting the current vogue for "cooperative learning." The children in each group share a small plastic basket with paper, pencils, glue, and bags of popsicle sticks. The classroom's walls and bulletin boards are cheerfully decorated with holiday and seasonal motifs--Santa Claus with toys, promises, and cotton snow in December, the Easter Bunny and springtime blossoms in March.

Carol believes that she teaches mathematics for understanding. She sees mathematics as a set of sensible tools and her version of teaching for understanding puts her at the helm, in charge of equipping pupils with these tools. "Teaching without understanding is not teaching, " she asserts (interview, 12/88). She says that emphasizing the underlying meanings of the mathematical procedures is essential in teaching and learning mathematics. She contrasts this with "teaching

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[1] Carol Turner is a pseudonym.

[2] Words in quotation marks are the teacher's terms.

rote," something she openly disdains. Understanding"--knowing the reasons and the purposes and being able to "apply math skills to everyday living"--are her fundamental goals.

Perhaps more than any other teacher we visited, Carol Turner was disposed to teach in ways that seemed consistent with the *Framework*. This case explores her approach to teaching mathematics and the deeply-held convictions that underlie it. Interestingly, Carol's approach--grounded in her conceptions of mathematics and her notions about how children learn mathematics--seem to have been virtually untouched by the policy initiative. Her practice, reflecting glimmers of the new ways, somehow also deflected them. We use this case as an opportunity to explore why a teacher like Carol, so apparently disposed toward some aspects of the *Framework*'s vision of mathematics teaching and learning, could remain so apparently unaffected by it.

Carol's Practice: "Teaching Without Understanding is Not Teaching"

Carol demands that her students understand. She makes sure they attend and the mark of a successful teaching moment is that "every eye was on me." Syllogistically, she reasons: When the children are looking at her, they are probably engaged. If they are engaged, then they will understand. They will understand because she carefully structures their activities so that they will develop the *correct* understandings: giving them templates for explaining procedures, guiding their work with manipulatives, leading them with questions.

> *With popsicle sticks bundled in groups of ten, the children are trying to solve 53 - 28. They record their work on small scratch pads. After a couple of minutes, she asks one boy to explain what he did.*
> *S: 13 minus 8.*
> *T: What did you do when you subtracted the 8?*
> *S: Wrote a 5.*
> *T: Yes, but what did you have to do to take the 8 away? You must have had to do something before you could take those 8 sticks away from 3 sticks.*
> *S: Unbundle.*
> *T: Yes. We haven't had to do that much yet because we've just been getting ready to do the subtracting, just setting things up. (She looks at the rest of the children.) How many of you had to take the rubberband off to take the 8 away?*
> *About half the children raise their hands. She directs the others to remove the rubberbands from their bundle of sticks; she waits before continuing.*
> *T: Now, boys and girls, tell me what to do.?*
> *Ss: Cross out the 5 and put a 4.*
> *Carol (crosses out the 5 in 53 and writes a 4 above it): Yes, we have unbundled one of our tens and so we have 4 left.*
> *T: What's 13 take away 8?*
> *Ss: 5.*
> *T: (turns back to the boy whom she had originally called on): Carl, what did you do next?*
> *S: 4 minus 2 .*
> *T: (looks expectantly at the class)*
> *Ss: 2.*
> *T: (writing the 2) Okay!* (observation, 12/88)

In this segment, Carol leads her students through the explanation that underlies the subtraction. She focuses on the link between the bundled popsicle sticks and the conventional algorithm--"What do you have to do to 3 to take 8 away?" This is one of a series of carefully-planned steps through which she marches her students, shepherding them carefully toward the destination of mastering subtraction with regrouping, or "borrowing."

Carol sees herself as different from her colleagues. Many teachers believe, she remarked, that "it's okay if [the children] don't know *why* to borrow--let them learn rote and then later the concept will come." (interview, 12/88). She says she has never agreed with this stance: "If they really haven't pictured that and done it with a base ten [material], you are making [the students'] job harder. I can't buy it when [teachers] say rote is okay. Somewhere along the line they are going to get it? When? When they are 35 years old?" (interview, 12/88). Saying that she hopes she is not "hurting anybody's feelings," Carol says that teachers who aim merely for rote learning are lazy. Explaining that "useless information is skills you give kids and they don't know what you do with them," she describes her goals:

> I want to empower them with a sense of survival skills that they will be an educated adult who can function with math and use that math and not be afraid to use it. When they come to a problem they need to build a door or they need to figure out how much they need carpeting or go purchase something. That they can approach that. They can't approach it if they don't have the concept of area. They can't approach it if they don't have a *concept* of measurement using the length of feet or yards or whatever you're using. (interview, 12/88)

For Carol, mastering mathematics involves understanding the "whys," and she is convinced that manipulatives are essential in helping children develop these kinds of mathematical understanding. She explained that she has "*always* used a lot of manipulatives," and her storehouse of devices includes not only manipulatives, but also stories, metaphors, and gimmicks.

Carol's beliefs about learning mathematics as well as her convictions about teaching mathematics both contribute to her commitment to using such models. First, she believes that students must be "actively engaged." Active engagement entails moving objects--or one's body--around, following a lead, and, above all, *watching* and listening. Experience has convinced her that "children learn through doing. They don't learn through sitting and doing workbook pages." She draws an analogy to underscore her point: "Lincoln didn't really learn law through reading. He really learned it by practicing it and doing it. And actually manipulating things and getting into a courtroom. That's where you learn. So with math it's the same thing." (interview, 12/88). She explained that "research supports the fact" that children learn when they are actively involved, when they're moving things around, when they can see what is going on. Carol's strong beliefs about how and what children need to learn lead her to choose teaching strategies that afford her students opportunities to manipulate objects, watch eye-grabbing demonstrations, and act out stories.

Carol also believes that helping children to develop mathematical understandings depends on "breaking down the content": analyzing and identifying the concepts, steps, and processes, taking them apart, and working on them sequentially. "You really analyze every step you do. When they have grasped that, you go on to the next and you build." So, in teaching "borrowing," Carol focuses initially on place value in two-digit numbers. Next, students regroup two-digit numbers: 57 = 4 tens + 17 ones. Then she has them examine subtraction exercises to decide whether regrouping is necessary or not. After this step, students begin performing the calculations, often in application contexts. In this way, Carol helps her students master one bit of the overall topic at a time.

Carol makes a clear distinction between two qualitatively different ways of "breaking up the content." The point of "breaking down a topic" into its components--a strategy she embraces--is to help children "get to that global understanding." What Carol disapproves of is what she calls "segmenting" content. When mathematics is segmented, topics that are fundamentally connected are taught separately with little reference to one another--for example, when money is separated from place value. Making connections among mathematical ideas is something she sees as central.

Carol's beliefs about teaching mathematics by breaking down the content reinforce her commitment to using a wide variety of manipulatives and other devices. Such representations, she believes, help students focus on the underlying concepts and meanings. Across her development of subtraction with regrouping, for example, Carol uses many concrete and other models. Her view is based on a "a concrete philosophy," developed while she was working with special education children.

Carol uses many different representations to teach a topic because she thinks that helps to "send home the messages." Students are there to receive ideas and skills; she aims to get those into their minds. Carol also believes that children learn best in different ways, in different "modalities." Thus, the more representations, the better, for one device will work best for one child, while another will click for a second.

Carol has a storehouse of models for the content she teaches and she uses these different representations deliberately. When her new text suggests that she use "base-ten materials" for teaching multi-digit subtraction, for instance, Carol has a repertoire of alternatives on which to draw and which she is disposed to use: money, "Mr. and Mrs. Tens and Ones" (a dramatization of place value and regrouping), popsicle sticks and rubberbands, base-ten blocks, a tens and ones pocket chart, and Unifix cubes (plastic interlocking blocks). Not only does she have this extended repertoire, but she differentiates among the models for place value and regrouping. She explains that some are better for some purposes than others. For example, popsicle sticks have a practical advantage over Unifix cubes: they don't fall apart. Popsicle sticks also model grouping by tens especially well, since the children have to put a rubberband around each time they get a group of ten sticks.

Carol is deliberate about using the different models. When she uses popsicle sticks to represent two-digit subtraction, for example, she has the students leave the rubberband on the bundle at first so that they can "see that it is a group of ten and when they put the little 1 on their paper, that still is a group of ten." Later she has them unbundle the sticks so that they can see how you go from 5 to 15 in the ones' column. Money, she explains, makes a good model because it is so familiar; students' everyday experience with and knowledge about money equip them to use it effectively to look closely at the number system. "Mr. and Mrs. Tens and Ones," a dramatic play, is good because "children love play acting and role playing." She makes hats for the children and they pretend to live in either the tens or the ones "house." They make up stories together about children playing in the ones house; when there are ten children in the house, Mrs. Ones must tell them that they can no longer play there--they must go to Mrs. Tens' house. The children must figure out how to do this: "They *physically* move their bodies" (interview, 12/88). Carol emphasizes that "the kinesthetic is probably the strongest modality in children and if you use that, you're going to get a lot further."

In addition to possessing a well-established and articulated repertoire of representations, Carol also invents models on the spot. During one lesson I observed, she was trying to help her students understand "Mrs. Turner's law of math": "Never subtract the top number from the bottom." To illustrate that doing this was "a magic trick," a move that didn't make sense, she spontaneously pulled out a plastic baggie and held it up in front of the class. Dramatically, she challenged the children, "Can you take 4 candies out of this bag?"(observation, 12/88). Later she explained that she had come up with this idea in order to get them to "look at the total situation" and analyze it. She was pleased with how this worked, for "every eye" was on her while she was showing the baggie. "You know, if you keep bringing it back to something concrete," she explained, "they understand. If I had said it or put it on the chalkboard, I wouldn't have had their attention" (interview, 12/88). Here, Carol's belief in the value of looking, of seeing, comes through.

Carol thinks it critical that children make links between the various models and the conventional numbers and symbols. In fact, one key criterion for "understanding" is that students should be able to carry out procedures on paper and also model those procedures:

> I usually use the manipulatives right up front until I feel they have mastered the manipulatives and then I tie the manipulatives into the written. We'll do a manipulative simultaneously with the board. And then from that I want them to show me the manipulative and simultaneously be able to do it on paper. And then take away the manipulative and see if they can do it on paper. Now doing both together is the hard part. They could do it on the paper and they could do it to show but if they really can do both at the same time then I really know they are understanding. And it's not just rote. And for me that's important. (interview, 12/88)

In helping her students make those links, Carol leads the students smoothly between the objects and the symbols using conceptually-focused language, the referential vocabulary of the concrete models, and stories:

T: Okay, let's do 60 - 37, 6 tens and 0 ones minus 3 tens and 7 ones. Rachel, help me out here. What should I do?

$$\begin{array}{r} 60 \\ \underline{-37} \end{array}$$

S: Borrow.
T: You mean I don't go 7 take away 0? . . . You're right! Tell me what to do.
S: Cross out the 6 and write 5 and put a 1 next to the 0.
T: (tries to focus the children on the meaning of these symbols) And that 1 represents what? A group of --?
Ss: Ten!
T :(nods) Okay, so 10 take away 7.
S: 3.
T: (writing this down and moves to the next column. Again she carefully uses the language of place value) 5 tens take away 3 tens?
S: 2.
T : (looking out at the class) What does 2 tens and 3 ones equal?
Ss: 23!
Later, Carol discovers a child working alone who is automatically regrouping on every example. She finds him on 74 - 43.
T (stooping over): Can I take 3 away from 4?
The child, silent, stares at her.
T : (linking it to a real-world context): You have 4 cookies? Can you eat 3 cookies?
S: Yes.
T: How many will be left over?
S: One.
T: So there's no reason to borrow there. Be careful. (observation, 12/88)

Carol aims to *give* her students mathematical understanding. She provides them with representations and guides their use of them. She connects the models to the symbols and oversees the children's linking of these. She directs and shapes the classroom discourse. The children, actively talking, moving objects, and writing, participate in highly structured and controlled ways.

Carol has the children get out scratch paper and number it from 1 to 5. She writes a two-digit subtraction exercise --

$$\begin{array}{r} 65 \\ \underline{-29} \end{array}$$

--on the board and tells them to write Y if they would need to borrow and N if they wouldn't. She reminds them that they worked on this yesterday with sticks. She asks the class for the answer and then asks one boy why. He gives the correct reason ("you can't subtract 9 from 5") and the teacher continues with four more examples. For each one, she asks for the answer and the reason for the answer. Each reason given is short and standardized.

When this is finished, Carol has the students solve the exercises they have been discussing. She goes over each problem with the entire class, focusing both on whether or not borrowing is required and why, and what to do to get the answer:

$$\begin{array}{r} 81 \\ \underline{-56} \end{array}$$

T: Do you need to borrow here?
Ss: (chorus) Yes.
T: Why? Jared?
S: Because you can't take 6 away from 1, And you can't put the 6 on top.
T: Did everyone hear that? Beautifully said! You can't do that magic trick. Now, class, tell me what to do and why.
Ss: (chorus): Cross out the 8, put a 7. Put a 1 next to the 1.
T: (writing this on the board) 11 take away 6 is--?
Ss: 5!!
T: 7 take away 5 is--?
Ss: 2!! (observation, 12/88)

Carol knows where she wants her students to be, both conceptually and procedurally, and she shapes their activity and discourse to make sure they get there. Carol wants her students to master the mathematics of second grade. But to her this means not only mastering, for instance, the algorithm for subtraction. It also means that understanding how and why the algorithm works. In Carol's mind, however, there is *an* explanation, or "why," of borrowing and she wants to make sure her students get it. With a template for the correct reasoning, then, she leads the children to provide the explanations and justifications for their work. The template provides the form and the content of the explanation; students fill in the blanks. Just as there is a right answer, there is also a right explanation.

Carol asks one small group of boys to explain to the rest of the class how they solved 43 - 28. She calls on Phillip, one of the group's members.
S: I crossed out the 4 and put a 3.
T: No, what did you say in the ones place?
S: (puzzled): Unbundle a ten?
T: No. First, what did you say in the ones?
S: (pause) I don't know.
T: Yes, you know. You did the problem. What did you do?
T: (pointing at the problem on the board and prompting) Tell me, can you take 8 away from 3?

S: (lights up) No.
T: So what did you do?
S: I borrowed!
T: (smiles at him) See? You <u>can</u> tell me. Now, where did you borrow from? (in a teasing tone) From Santa Claus?
Ss (giggling)
S: From the tens.
T: You borrowed this from the tens, leaving how many? (She crosses out the 4 on the 43.)
S (pauses).
T (persisting): Did you throw it up in the air or did you do something with that group of tens? Where did you put it?
She writes a 3 above the 4 and a 1 next to the 3. Then she explains, gently: You borrowed from the 4, leaving that a 3 and putting your group of ten here (with the ones).
(observation, 12/88)

This routine of explanation was repeated throughout the lesson, rehearsing the steps and their reasons. Carol pushed and pulled, firmly and insistently, directing the children toward her goal for them: that they be able to perform the procedure of two-digit subtraction, knowing when and how to borrow and why. When Phillip, above, could not fill in the template correctly, Carol completed it for him: "*You borrowed from the 4, leaving that a 3 and putting your group of ten here.*"

In Carol's classroom, children's opportunities to talk are structured and shaped to help them converge on the correct understandings. They are helped to know the right reasons as well as the right answers. Mathematical speculation. conjecture, and invention are not a part of the discourse. One day I observed Carol using a "storybook" from *Real Math* program. She read the children a story about a girl who had a job walking dogs. The problem was: Is 15 minutes enough time to walk ten dogs one at a time if it takes 10 minutes to walk one dog? Carol told the class that "there is no right answer--it's just what you think." Then she led the children step by step to a precisely calculated answer to the unasked question, exactly how long does it take to walk the ten dogs?

T: How many dogs does she have to walk?
S: Ten.
T: Ten. And how many minutes does it take to walk one dog? Douglas?
S: Ten.
T: Ten. So, ten groups of ten. Let's count by tens.
(The class counts together: 10, 20, 30, . . ., 100).
T: So that's a hundred minutes altogether.
Gasps and oohs are heard around the class.
T: How many minutes in an hour?
S: 60.
T: 60. Is 100 larger or smaller than 60?
Ss: Larger.
T: Larger. So does she need more time or less time than an hour?
S: More.
T: More. You are right. She couldn't possibly do that in 15 minutes. <u>Good</u> listening!!
(observation, 12/88)

Carol's introductory comment ("there's no right answer--it's just what you think") may have been intended to encourage student to volunteer ideas and to not fear being wrong. But there was a right answer. And although Carol wanted students to participate in solving the problem, their participation was structured and limited. This was consistent across observations of Carol's teaching, for although students' talk is a regular part of the classroom discourse in Carol's class, their comments rarely consist of more than one or two words and the substance of their talk is highly controlled. They fill in blanks in the templates Carol provides: 5 *what*? Is 60 more or less than 100? 7 take away 4 is____? 50 has how many tens? and so on. These templates model and direct the kind of thinking and the ways of knowing that she is trying to foster. The dog story held opportunities not pursued by Carol: to formulate problems and questions, to experiment with alternative approaches to answering them, to compare estimated with precise answers and to consider the need for either in this context, and so on. Mathematical understanding has for Carol a convergent quality that seems to foreclose a more open-ended search for meaning. With her energies focused on mastery, Carol loves the storybooks because "they tie in very closely with the lessons and [they] really make the kids think and reason," using what they have learned "and not just rote." According to Carol, they offer opportunities to apply mathematical skills to real-life contexts. But note that the contexts are highly constrained. Instead of allowing students the opportunity to create and solve problems in diverse ways, Carol carefully walks her students down the "right" solution path.

Although she does not want to be seen as "bragging," Carol is comfortable with and proud of her approach to teaching mathematics. Unlike many of her colleagues, she believes, she works hard, stresses meaning and understanding, and her students learn. If Carol worries, it is about assessment. It concerns her on two levels: (1) deciding for herself if her students are "mastering" what she is teaching and (2) having her students do well on standardized tests. Behind her classroom door, Carol feels quite able to determine what students are learning--for most of her goals. "Monitoring" their written work, she can see if they are making any errors. She can ask them to "show me with blocks and verbally explain" (interview, 4/89). If children can do at least two of these three "ingredients" (do, model, explain), then Carol feels confident that they understand. She is less sure about how to tell if children are mastering reasoning with the new text's storybook problems:

> There isn't really a guideline to tell you how to build proof that these children are getting some kind of an idea about these problems because the problems are all done orally. Always ask why when they come up with an answer... 'How did you get that?' But I always feel like there is one or two kids maybe I'm missing.

Two concerns surface in Carol's comments. What counts as evidence that students are developing good reasoning skills? And how can one "monitor" students' understanding with a class size of 35?

Still, Carol worries more about whether the annually-administered standardized test will produce evidence that she has taught. She knows that her district places a high premium on good test scores and that her class's results and her school's will be scrutinized by administrators. "In this district it's a big deal. It hits the papers--it's big stuff. Even [administrators'] jobs come and go." She worries that her students will not understand the test's format because they have never had to "bubble in" answers before. She is anxious about making sure that she has covered the necessary content before the test is administered--multiplication, for instance. Consequently, Carol prepares her students to take the standardized test, using a test skills booklet entitled "Scoring High." She uses its contents to review the mathematics she has taught, to give them test-taking "survival skills" (such as eliminating unreasonable answers), to prepare them for the format, and to cue them that the test writers may try to "fool you." Carol does not seem confident that her new approach alone can empower students to succeed on the standardized tests, so she supplements her teaching with specific preparation for such assessments.

Carol and the *Framework*: Take #1

Carol's approach to teaching mathematics reflects a number of specific features central to the *Framework*'s vision of practice. She uses manipulatives and other representations thoughtfully and deliberately. She emphasizes meanings, aiming to help her students know not just how to perform procedures but why. Her goals focus on the meaningful application of mathematical skills in everyday life and she stresses the need to analyze situations carefully and to monitor the reasonableness of answers. Carol is concerned with connections: the relationships among alternative representations as well as among mathematical ideas and skills.

Her role in all this is decidedly directive: orchestrating students' use of a variety of representations, teaching them the explanations for what they are doing, modeling problem solving, and guiding students' practice with all of these. Carol takes pride in her ability to teach for understanding with her large and diverse class. Steeped in techniques of "effective instruction," she is conscious of providing her students with "anticipatory sets," of using both guided and independent practice, and of "checking for understanding." She sees these strengths as consistent with her focus on understanding.

Carol has not actually spent any time studying the *Framework* or thinking about its implications for her practice. Because she is doing her job, she says, her principal would not care that she does not refer to the document. Carol thinks of the *Framework* as a manual for *what to teach* in mathematics, and she has been wholly unconcerned with its contents: She stores it in a box in her room. In her mind, it seems undifferentiated from the school district's curriculum guidelines and, although she feels accountable to those guidelines, she believes that she knows what her students need to learn. "I know what my goals and objectives are, what my job is. This [the *Framework*] is a piece of paper. It's not my Bible. . . . Even if they burned it tomorrow, I could still teach." (interview, 12/88). Carol, assuming that mathematics "hasn't changed over the years," knows that her second graders are not supposed to learn multiplication and division, but that they do need to learn to tell time, to read a calendar, to add and subtract. Additionally, Carol believes that no document can tell her what to teach her particular students because the "basic truth about teaching is that you look at where your children are and you take them where they are and move them on up."

When shown an excerpt from the *Framework* on "teaching for understanding" ,Carol read it over carefully and then exclaimed, "Really this says what I've been saying." She noted the excerpt's emphasis on meaningful application, on fundamental concepts, on making sense of answers. "It really says beautifully what I was aiming for. 'Mathematical rules, procedures, and formulas are not powerful tools in isolation.'. They're *not*. They're meaningless. 'Students who are taught them out of context are buried by a growing list of separate items that have narrow application.' That's why I am saying that that categorization and building of skills [is so important]." Carol was not surprised to find, on reading some of the document, that her approach to teaching fit with the *Framework*; she had, of course, always assumed that it did.

<u>Carol: Is There a Message of Change for Her?</u>

From Carol's point of view, she perceives no real mandate to change what she does to teach mathematics. She is aware that there is a new state framework for mathematics but assumes that it has little, if anything, new to say to her. In fact, when asked to examine it, she says that "it says the things I've been saying." She also knows that her district has just adopted a new textbook series that she is expected to follow closely. Although she believes that this text is "aligned" with the *Framework*, she sees this as an issue for district administrators, not teachers like herself. "[The program] is following the *Framework* and I guess if we don't follow the *Framework*, we lose funding." Still, she does not perceive the textbook adoption as linked to the implementation of the new *Framework*. When asked why her district chose a new series this year, Carol said she did not know: "I really couldn't tell you the whole reason. I think it was time, you know. I think every few years they evaluate the books and decide what's working and not working. Some surveys were done and they probably looked through thousands of programs and decided on this one." (interview, 4/89).

Despite her detachment from the policy and implementation issues, Carol is delighted with *Real Math*. "It's a wonderful program," she says with enthusiasm, for she sees it as fitting with her approach and beliefs much better than texts she has used in the past. What she likes especially about it is that it "builds concepts" at the same time that it develops skills and procedures: "I think when you do that, they're going to retain it more What they're doing has *meaning*." She also likes how well the book breaks down the skills and processes: it reminds her of the kind of "task analysis" and teaching that they do in special education: "In special ed, you don't take anything for granted with the children and you really analyze every step. This program does the step-by-step no matter what and insures that each child is building those skills. In special ed, they also do a "lot of manipulative work, a lot of multi-sensory things." She is impressed with the spiral organization of the content because if a child is having trouble with a concept or a skill, it keeps coming up, in different ways and using different models. "The program doesn't just go money, money, money, time, time, time. It kind of interweaves all those concepts," she observes. And, she adds, "Someone has really thought about the process by which children learn."

Carol is following the textbook closely this year because, she said, "we're supposed to adhere strictly to it. They said for it to really work for you, we want you to use it just as it says." Carol teaches all the lessons, including the games and the storybook discussions which many of her colleagues perceive as "extras." To Carol, these are not dispensable; they are key components of the program. When she alters the text, the changes seem subtle. For example, she chose to have her students keep their popsicle sticks bundled when they first started regrouping for subtraction. Although the book directed her to have them unbundle the sticks, Carol wanted her children focus on the fact that they were borrowing a *ten* and so she modified the book's plan. Other modifications seem more dramatic: Her reshaping of the dog story into a specific question with a specific right answer.

Although she says that many of the other teachers do not like *Real Math* because it has changed their teaching strategy, Carol has not yet found anything she does not like about the program; she believes that it fits her beliefs about learning and teaching mathematics--or at least she sees it that way. Although she does not consider herself a textbook-driven teacher, she likes this book so much that she finds herself sticking to it: "It works beautifully". She points to a few new ideas she has picked up from it. One is teaching children strategies for the basic addition and subtraction facts, to decrease the memory load entailed. Some of these included skip counting, using doubles, and groups of tens. "Ten and six are sixteen, and all of a sudden the light bulbs are going on,where before ten plus six, they'd go, eleven, twelve, thirteen, fourteen, fifteen, sixteen. It's so simple. It's a group of ten and six ones, sixteen. That was fantastic. I've seen a lot of speeding up the action there" (interview, 12/88). The book has also given her a better gauge of where she should be at a given point in the year. In general, though, the book does not seem different or new; it is "pretty much doing the same things" that she has always done--using manipulatives and breaking down the content.

Carol and the *Framework* Revisited: Take #2

Although Carol does not see the textbook adoption as connected to the *Framework*, it is nonetheless her district's vehicle for initiating and implementing change. Yet Carol perceives *RealMath* as congruent with her current approach. Nothing in the new text seems to have, as yet, led her to a sense that she is being asked to rethink and reshape her practice.

From an outsider's perspective, Carol Turner's practice reflects--and thereby also perhaps deflects--the *Framework*. One reading of the *Framework* would suggest that this is a teacher who is doing it all. Following the textbook closely and quite happily, Carol probably includes the seven strands of mathematical content highlighted in the *Framework*. She teaches for understanding, emphasizing underlying concepts and stressing applications. She reinforces and connects concepts and skills, provides opportunities for problem solving, and uses concrete materials. She attends to a variety of learning styles, provides remedial help when needed, has students work in groups, and leads classroom discourse with carefully-planned questions. And she believes that students learn best when they are "actively involved." Examining the ideas embedded in this last phrase reveals how Carol's approach, although it resembles the *Framework*'s vision, also differs in some fundamental ways from it.

What students actively engage *in*. Carol conceives of mathematics as a body of concepts, topics, and reasonable procedures that are useful in everyday life. This knowledge is "out there" to be learned; mathematics is not something that human beings have created and continue to construct. If students engage in a variety of activities that involve them in manipulating objects, acting out and telling representational stories, giving reasons, responding to questions, and, above all, watching closely, they are more likely to master and retain the mathematical concepts and procedures they need. Carol does not think of mathematics as inherently beautiful and fascinating, as a domain of inquiry with intrinsic intellectual value--a view suggested by some of the *Framework*'s text (California Department of Education, 1985, p. 1). She does not think of mathematical activity in terms of formulating problems, making and pursuing conjectures, or evaluating alternative mathematical claims (California Department of Education, 1985, p. 3). Instead, Carol's conception is of mathematics as a set of tools and she engages her students in activities designed to help them learn to use those tools.

What it means to be actively engaged. Carol believes that children learn best when they do--when they can manipulate objects, act out situations, talk. When they do these things, they are more likely to be paying attention, which is key. She contrasts these sorts of activities with doing workbook pages and dittoes, which do not actively involve learners. Carol carefully structures her students' activities so that they will develop the *correct* understandings: giving them templates for explaining, guiding their work with manipulatives, leading them with questions. She also manages time efficiently, moving along briskly and keeping children focused. That the teacher should "serve as a facilitator rather than a directive group leader" (California Department of Education, 1985, p. 13) does not fit her assumptions about the teacher's role in promoting active engagement. She is not inclined to let students be stuck and take time to puzzle and be confused (California Department of Education, 1985, p. 14). Her role is to make sure they "get" the content are supposed to be learning. The *Framework* urges teachers to encourage alternative approaches and divergent ideas rather than pressing for single right answers:

> [Good questions] encourage students to explain, experiment, explore, and suggest strategies. The way in which a teacher responds to students' answers can influence the answers as much as the questions do. If a teacher reacts to an answer in a way that signals conformity (through praise, criticism, or other value judgment, students will perceive that their thinking process is not valued as much as the answers the teacher has in mind. (California Department of Education, 1985, p. 17)

Carol, however, decidedly lets children know if their answers are correct, prodding and praising along the way. She also actively presses for convergence and conformity with standard mathematical procedures, solutions, and explanations, considering that a key aspect of her responsibility.

Carol's approach to teaching mathematics includes innovative practices and materials. She uses manipulatives, emphasizes meaning, and wants students to be able to apply mathematics to real-world situations. Consequently, her classroom appears to reflect key dimensions of the *Framework*'s vision of practice. Still, her conception of mathematics and her beliefs about knowing and learning mathematics are rooted in the traditional epistemology of school mathematics: Mathematics is a body of knowledge, consisting of concepts and procedures. Skill with these mathematical procedures is the central goal. The teacher dispenses the essential knowledge, the children receive it. There is a right way to use and to do mathematics. Because this traditional orientation to knowledge lies under the veneer of Carol's use of manipulatives and focus on the "whys," the nature of her children's encounters with mathematics may differ from the sort implied by a deeper reading of the *Framework*'s vision.

Dilemmas in Communicating About Change

Carol, unwittingly, does not realize that the *California Mathematics Framework* outlines a vision of practice that might suggest a fundamentally different classroom epistemology than the one she enacts. Although she has received a new textbook and has been told to adhere to it closely, she perceives the text as fitting with what she already does well: using concrete materials, breaking down the content, making connections among mathematical topics. Thus, she does not think that her district is expecting her to change the way in which she teaches mathematics to her second graders--and, in fact, they may not be: District personnel may believe that Carol is enacting the vision of the *Framework*. Moreover, the district's press for high standardized test scores only reinforces her sense that nothing is changing. She is still accountable for what seems like the same content, measured in the same ways.

In Carol's district, the new text series has been designated as the primary messenger of change. How well it serves the role of communicating and fostering change is an open question. It clearly can provide guidance to teachers in using manipulatives, in selecting better mathematical tasks, and in creating different kinds of activities. What is less clear is whether or not a text, as the primary vehicle of change, can provide guidance for teachers on basic questions of knowledge and learning. In Carol's case it seems to fail. *Can* Open Court--or any of the "approved" texts--do the job of helping teachers come to understand and begin to implement a different classroom epistemology? Can a textbook provide sufficient vision and guidance for teachers to take on new roles in helping children to construct knowledge? Can a textbook challenge teachers' assumptions about knowledge and begin to help them develop different understandings of mathematics as a domain of inquiry and of knowing?

The promise and pitfalls of textbooks as messengers of change

Those who would try to change what goes on in schools must figure out how to communicate about change in a way that makes sense and respects where teachers are and yet makes them realize that they are being asked to rethink what they do, and provides guidance. for that change. Because many teachers rely on textbooks as a core for their teaching, a textbook is a reasonable candidate for communicating and providing guidance for change. Yet teachers never literally "follow" textbooks: They necessarily interpret and depart from them. Deliberately and unintentionally, teachers adapt and modify the suggestions in the text to suit their own orientations and the needs of their particular students. However, a textbook is still more steadily and consistently available than a resource teacher and more concrete and specific than a set of curriculum statements. From a pragmatic standpoint, teachers are far more likely to read a textbook and consider its contents than they are to seriously engage a policy document. Textbooks can also be significant resources for teachers whose subject matter or pedagogical understandings are thin. Still, in the case of the kinds of changes suggested by the *Framework*, textbooks present problems of two kinds--problems in the message and problems with the messenger.

Problems in the textbook's message. Mathematics textbooks, however radically revised (and the California ones are not), tend to comprise a melange of old and new, of the traditional and the novel. Patched together through market-spurred revisions, mathematics textbooks include pages with unstructured and non-routine problems interwoven with pages with algorithmic presentations of procedures (e.g., multiplication of decimals). The message is thereby easily garbled: What exactly is this new approach to teaching mathematics for understanding? Is it adding manipulatives and calculators? Is it sprinkling "problem solving" into the curricular stew? The classrooms envisioned by the *Framework*'s writers are coherent. Textbooks, given the politics of their revision and change, are not.

Problems with the textbook as messenger. Texts, by their very nature, focus on the substantive--on the topics and procedures of the subject: fractions and decimals, measurement and geometry, functions and multiplication. A mathematics text containing both familiar and not-so-familiar topics can help teachers reconsider *what* to teach. The pedagogical suggestions lining the margins or buried in the recesses of the teacher's guide can influence *what* teachers use to represent those topics. Far less probable, however, is it that a marketable textbook can package a different orientation to knowing or encourage a different role for the teacher. These are deep-seated dispositions, simmered over the years of a teacher's experience and seasoned by cultural assumptions about and images of teaching and learning (Cohen, 1989). As a map of a curricular voyage through mathematics, a textbook suggests the pathways and the sights to be seen, but is less likely to influence the nature of the trip. Changing the role the teacher plays in leading the trip--*how* the teacher helps students learn--is not merely a matter of changing the suggestions in the teacher's guide, for shifting the balance of authority for answers from the teacher and the text to the students cannot be directed from the margins. Changing ways in which children encounter mathematics in school so that they might develop understandings through mathematical conjecture and argument, for example, requires something that likely goes beyond written texts. It requires changed views of what mathematics is and what it means to know and do mathematics, as well as changed assumptions about students and how they learn. These are complicated changes, often underestimated by reformers.

Carol and the Winds of Change

In order to help a successful teacher like Carol Turner consider changing what she does in her classroom, the messages she encounters must engage her where she is--they must both make sense and be compelling. They must give her an alternative vision and give her considerable guidance about what that alternative practice would look like and entail. This does not seem to have happened in Carol's case. Two reasons seem to account for this.

First, what she does reflects enough of some features of the *Framework* that any new ideas she has encountered--using manipulatives, for example--seem familiar and comfortable. The ideas purveyed by the new textbook and the district's messages to teachers do not challenge her accustomed ways of helping students learn mathematics. She hears that she should stress concepts, meanings,and applications. She should use cooperative grouping and manipulatives. All this, Carol thinks to herself, she has always done.

A second and related reason is that Carol perceives no mandate to change or rethink her practice. A question we must a[illegible] the *Framework* is whether or not a teacher like Carol is being asked to change. *Is* she teaching in the spirit of the *Framework*? How would the *Framework*'s authors and advocates assess her approach to teaching mathematics? Like any text, the *Framework* is open to multiple interpretations. In this case, Carol's teaching looks different n light of different readings of the policy statement. Which one is right? Is there *a* "right" interpretation, from anyone's perspective in California?

Carol's case highlights the conceptual difficulty of communicating an alternative vision of teaching to those who would enact it. For teachers, as the implementors of policy, the message must be sufficiently clear. They must be able to understand the direction and substance of the policy. This means that the policy must make sense from the teachers' current frames of reference. From this standpoint, textbooks make good policy messengers because they can represent ideas in a forum that is familiar and concrete. Carol was enjoying and paying careful attention to her new textbook. Still, for change to occur, the message must seem to be outlining a direction and a practice that seems different from the status quo. The policy must be seen to be advocating something that would require some change. Here textbooks may fall short, for creating texts that can represent a significantly different sort of classroom practice is difficult. Carol's interpretation of the policy's thrust was that it was asking for what she already did. Carol's case helps to illustrate a central problem of change: how to communicate and provide guidance for change in a way that is comprehensible and yet challenges current practice.

Carol's case also highlights questions concerning the targets of reforms like the California *Framework*. Carol is a thoughtful teacher whose practice, while traditional in many ways, goes far beyond typical mathematics instruction. Her students do learn that there are reasons for the mathematical moves they learn to make and they can represent their written work with models. But Carol's students have never been given the opportunity to frame their own problems, or explore multiple, and valid, solution paths. They have not learned to take charge of their own learning, or to participate as full members in classroom discourse. The visions of mathematics in the *Framework* are multiple and Carol could be seen at once as complying with the *Framework*, or as subtly contradicting it. This raises questions both about the intentions of the *Framework* and about the nature of this reform. Would a state like California be happy if they could move all teachers to where Carol is? Alternatively, do policymakers want to change all teachers--those in the mainstream and on the fringe? Cases like Carol's suggest that policy instruments must pack a more powerful punch if they want to challenge both pedestrian and accomplished practice.

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Relations Between Policy and Practice:

A Commentary

David K. Cohen and Deborah Loewenberg Ball

Our portraits of nine elementary teachers show that instructional policy makes a difference. Yet they also highlight the fact that the policy has been interpreted--and thus enacted--in a variety of ways. Here we explore some of the factors that seemed to contribute to the striking variations in interpretation and implementation that we saw. We begin with a closer look at the *Framework* (California State Department of Education, 1985) and at the reform movement within which it is rooted. Next, because revised elementary textbooks were the primary messengers of the *Framework*, we consider their role as messengers in changing practice. Then we examine the intellectual and political context of this policy and conclude with some observations about the relations between practice and policy.

The Language and Rhetoric of Reform

In communicating the spirit and direction of reform, policy statements vary in their effectiveness. How does this policy work as a messenger of vision for elementary mathematics instruction? One clue lies in the language of the reform. As is often the case with ambitious social reforms, the central ideas of the current movement to improve mathematics instruction seem particularly open to multiple interpretations. The language of this *Framework* is, in some ways, vague. For example, few people would disagree with goals such as "understanding" and "problem-solving." That students need experiences with "concrete materials" or that they should be able to "apply" mathematical ideas will also not generate debate. Yet what people mean by these can--and does--differ wildly.

From one perspective, such vagueness in the policy's language is a strength: It may broaden the appeal of the reform movement, allowing recruits of rather different persuasions to join what they imagine to be the same parade. As one rather traditional teacher commented, "What do they think we've been doing--teaching for *mis*understanding?" But from another perspective such vagueness is a defect: After collecting varied and even contrary tendencies under a single banner, the parade may become a melee.

One point seems plain The current reform movement in mathematics instruction has collected quite a variety of vogue-ish ideas and practices: from manipulatives to cooperative learning, from calculators to problem solving, from an emphasis on student talk to the addition of probability and estimation. These disparate pieces seem to lend themselves to being picked up in random bits and then enacted in variously interpreted permutations of each bit. The leading ideas of this movement do not yet cohere in an integrated conception of mathematics teaching and learning, rooted in a distinctive epistemology and framing a distinctive practice. This quality of the movement's leading ideas makes it more difficult for followers and bystanders to understand how the new instruction might look if enacted well. Indeed, it increases the likelihood that each teacher will apprehend and enact the ideas in his or her own terms.

Producing Instruction: The Relation of Texts and Teachers

Texts have so far been the key instrument in California's plan to align mathematics instruction. Like policy statements, texts also vary in their effectiveness as agents of change. In some nations, the school system offers very prescriptive guidance for content coverage. Not only are courses required but topics within courses also are prescribed. Sometimes methods of teaching are suggested or even strongly recommended. Curriculum guides that discuss topics and the means of treating them often are provided. In such cases, textbooks and curriculum guides can offer extensive and focused guidance about instructional content. And in such systems it appears that teachers attend to the guidance, and that topic coverage is relatively homogenous (Porter, Floden, Freeman, Schmidt, & Schwille, 1988; Robitaille & Garden, 1989; Schwille, Porter, Floden, Freeman, Knappen, Kuhs, & Schmidt, 1983; Travers & Westbury, 1989). Textbooks might be quite a potent agent of policy in school systems of this sort.

But prescriptions for instructional content tend to be quite vague in the United States (Floden et al., 1988; Porter, Floden, Freeman, Schmidt, & Schwille, 1988; Schwille, et al., 1983; Travers & Westbury, 1989). Prescription is strongest at the most general level, in course requirements: All states require various subject matter courses for elementary and secondary schools. But students and teachers have unusual latitude, even within these requirements. Mandatory high school English courses often can be satisfied in different ways, for instance (Powell, Farrar, & Cohen, 1985). Additionally, there is little prescription in topic coverage within courses in the U.S. Few state and local systems issue detailed curriculum guides, let alone prescribe topics within courses or curricula. Guidelines for how to deal with specific topics are equally rare. Hence there is relatively weak guidance about course content in most U.S. classrooms.

It often is said that U.S. teachers nonetheless teach more or less the same thing, because they all use textbooks, and all textbooks are quite similar. Many states and most localities do officially adopt textbooks, of course, and the texts are similar in some ways. But there are few state or local guidelines for what topics texts must cover and publishers compete. Ordinarily, several different texts are available for each subject at each grade level. Consequently there is only moderate overlap in content coverage among texts within fields (Freeman, et al., 1983). This should not be surprising: Textbooks after all are published by private firms that operate in relatively unregulated markets.

Many observers nonetheless argue that teachers teach more or less the same thing, at least when they use the same textbooks. Though there is little research on this matter, the available evidence contradicts common belief. Even when teachers use the same required texts, the content that they cover varies considerably from one teacher to the next (Porter, et al., 1988; Putnam, Wiemers, & Remillard, 1989; Schwille, et al., 1983). The authors of one study concluded that "The results of this investigation challenge the popular notion that the content of math instruction in a given elementary school is essentially equal to the textbook being used" (Freeman & Porter, 1990, p. 418). American teachers typically have much discretion in their decisions about instructional content--including their use of texts--and they exercise it (Cohen, 1990).

Hence textbooks are rather rubbery agents of policy, as things now stand in U.S. education. They differ substantially across publishers, and teachers make use of them in ways that fit with their assumptions and orientations. We certainly saw many different versions of "following the textbook" in the California classrooms that we observed. All but one of the teachers we visited claimed to be following the newly revised textbooks that their districts had adopted. Each one used and conscientiously attended to the text. But the teachers attended in their own ways, adapting the text to their own approach to teaching as well as their own view of mathematics. Some of these adaptations were relatively subtle. For instance, one teacher followed the text's suggestion that the class discuss alternative solutions to problems. This was a new element in the text, added in response to the *Framework*. The suggestion was intended to get divergent ideas out on the table so that students could explore alternatives, discuss what made sense, and explain their answers. But, in following the text's advice, this teacher conducted a traditional discussion in which the goal was to converge on a single right answer--thus frustrating the text's intention. Some other teachers adapted the texts in much more blatant ways. For instance, one simply omitted a novel topic--probability--that was in the new text, because she felt that students should learn "the basics" first.

But teachers' adaptations came in many varieties: We also saw teachers learning from the new textbooks. For instance, one rather traditional teacher made some important instructional changes precisely because his sense of professionalism led him to faithfully follow the new text. The book contained an innovative approach to fractions which he dutifully used with his students. The students began to reason about fractions in quite sophisticated and unexpected ways. The teacher was amazed but pleased. He confessed that he learned something from the experience: He never had imagined that his fifth graders could think and reason in such advanced ways. It seemed to him that if he continued to have such experiences, his teaching might change more fundamentally. Another teacher introduced new topics and many more concrete materials into her class because she was using a supplementary text that emphasized the use of manipulatives: Every lesson entailed students' use of beans or paper strips or counters or blocks. This teacher also seated her students in groups because the text suggested that "groupwork" was helpful to students. One moral of this story, then, is that texts and other curriculum materials can be important agents of change. But another is that in the very loose structure of U.S. education, these agents work only as teachers are able or inclined to let them work. Teachers and texts interact in diverse ways, and the results are often at odds with the developers' intentions.

The Context of Practice: Layers of Reform

While policies regularly announce a new instructional order, the classroom slate is never clean. Whatever novelties policymakers embrace, teachers must work with residues of the past. The *Framework* is part of an infatuation with "higher-order thinking" and "teaching for understanding" that began to grip Americans in the late 1980s. But between the mid-1970s and the mid-1980s Americans had been flailing schools for flabby teaching and sagging SAT scores. Improvement of "basic skills" was a national enthusiasm. State competency tests were a popular way to improve teaching and students' test scores. Chapter I was pressed into service in the same cause, as were other state and local programs. In addition, state agencies, county school districts, local schools, and herds of educational consultants eagerly advertised several new schemes for more didactic teaching: direct Instruction, "effective schools," and Madeline Hunter's Instructional Theory Into Practice chief among them. These schemes, all intended to improve students' scores on standardized tests, were urged on teachers across the country. California policymakers and practitioners embraced them enthusiastically. Teachers were trained in one or more of these approaches and administrators were trained to evaluate teachers' performance in the prescribed pedagogy. Schools and classrooms abounded with fresh signs of these reforms in the districts we visited. The older reforms were still alive and kicking as the new *Framework* was making its way toward classrooms.

The rapid succession of policy innovations is nothing new in U.S. education, but this seemed a particularly striking case. On the one hand, the new mathematics *Framework* exhorted teachers and students to become serious and independent thinkers and to understand big mathematical ideas. This seemed to imply that students should be encouraged to come up with divergent approaches to solving problems. It also meant that students often would be puzzled, even stuck. But on the other hand, state, federal, and local policymakers had just spent more than a decade pressing teachers to implement a direct and managed approach to instruction that focused on accuracy and convergent thinking. These approaches to instruction trafficked in utterly conventional and familiar conceptions of knowledge. Students and teachers were rewarded for attention to standard mathematical rules and routine procedures, getting the right answer was the order of the day. Successful teachers avoided puzzling students or letting them be stuck. Successful students solved routine problems with speed and efficiency.

We thought that there was a real conflict here: Yesterday's didactic policies and programs seemed sharply at odds with today's press for mathematical understanding. All of our teachers had been urged to adopt the earlier approaches and it looked very much as though they had, in one way or another. How could they adopt the new policies if they held onto the old ones? These earlier approaches had an additional advantage: The pedagogy and views of knowledge for which they stood were quite congruent with traditional values and experiences of teachers, administrators, and parents.

We were surprised to find that none of this seemed to be at issue for the teachers whom we observed. They appeared not to notice any contradictions between the two sets of policies, and seemed entirely untroubled by their juxtaposition. They spoke and acted as though the two sets of policies were entirely compatible. Indeed, when questioned, they freely expanded on how well the new policy fit with the earlier initiatives. Some were positively eloquent on the subject. When they commented on the combinations of policies, it was to remark on the number, not the nature, of the demands to which they were being asked to respond.

How did this work out in practice? As a descriptive matter, the new policy that stressed understanding, explanation, cooperative work and extended discourse with students was apprehended through the lens of older policies that stressed learning skills and facts, didactic teaching, individual work, and highly-focused recitation. Many of the teachers whom we observed did change their practice in response to the new policy. But the frame for those changes was the pedagogy that had been pressed by the older policies. New wine was let in, but only in the old bottles.

The result was a curious blend of direct instruction and teaching for understanding. Initially it seemed odd, but upon reflection it made sense. An observer has the leisure and independence to savor contradictions between today's and yesterday's policies, among other curiosities of classrooms. But teachers are busy and engaged actors, who must make their classrooms work: To do so, they must balance all manner of contrary tendencies (Fenstermacher & Amarel, 1983; Berlak & Berlak, 1981; Lampert, 1985). If teachers could not make past and present cohere, they would be unable to do anything at all in their classes.

It costs state legislators and bureaucrats relatively little to fashion a new instructional policy that calls for novel sorts of classroom work. These officials can easily ignore the pedagogical past, for they do not work in classrooms, and they bear little direct responsibility for what is done in localities--even if it is done partly at their instance. But teachers and students cannot ignore the pedagogical past, because it is their past. If instructional changes are to be made, they must make them. And changing one's teaching is not like changing one's socks. Teachers construct their practices gradually, out of their experience as students, their professional education, and their previous encounters with policies designed to change their practice. Teaching is less a set of garments that can be changed at will than a way of knowing, of seeing, and of being. And unlike many practices, teaching can only work if it is jointly constructed by the people on whom teachers work. So if teachers are to significantly alter their pedagogy, they must come to terms not only with the practices that they have constructed over decades, but also with their students' practices of learning, and the expectations of teachers entailed therein.

We were surprised that the teachers with whom we spoke seemed so little distressed about the many different signals that they have been sent by government. All of them saw themselves as independent professionals, and thought that they had considerable autonomy from state government. But they seemed genuinely respectful of the many different policies and programs that have been aimed at their classrooms. Only two teachers were frankly cynical.

The Influence of Practice on Policy

Our discussion so far has focused on the impact of state policy on teaching practice. But another theme in these cases is that practice has a profound influence on policy. One might summarize our argument by saying that teachers do not simply assimilate new texts and curriculum guides, altering their practice in response to externally envisioned principles. Rather, they apprehend and enact new instructional policies in light of inherited knowledge, belief, and practice. Moreover, teachers' interpretations are diverse. Some of the teachers whom we observed missed the new *Framework's* message; they thought they already did everything that the new policy exhorted them to do. Others changed their teaching, but in so doing they reframed the new policy in terms of various pre-existing ideas and practices. Hence the teachers whom we observed produced some remarkable mixtures of old and new mathematics instruction. For instance, in some of the cases presented above, teachers tended to mechanize elements of the policy that they adopted, turning mathematical themes such as estimation or problem solving into discrete topics or activities. But even their mechanizations varied, according to past practice and current preferences. Others ignored such topics in favor of more "basic" matters. Some teachers gave a little attention to the *Framework's* call for greater use of "manipulatives," using them only occasionally. Others absolutely doted on the new concrete materials, turning them into a central instructional activity. Some teachers seemed to see the materials as a species of mathematical salvation, as though children would learn complex ideas simply by moving little plastic cubes around on pieces of paper.

It is worth recalling, in this connection, that many observers believe state and federal policies tend to homogenize instruction. After all, they argue, all teachers receive the same governmental policy messages (Wise, 1978). But if our account is roughly correct, this new instructional policy probably has increased variability in mathematics teaching in California. One reason is universal: Any teacher, in any system of schooling, interprets and enacts new instructional policies in light of his or her own experience, beliefs, and knowledge. Hence to argue that government policy is the only operating force is to portray teachers as utterly passive, agents without agency. That is empirically unsupported by our investigations. Even the most obedient and traditional teachers whom we observed not only saw and enacted higher level policies in their own way, but were aware and proud of their independent contributions.

This general tendency is greatly amplified by two specific features in the present case. For one thing, we already noted that the policy is a bundle of disparate ideas, many vaguely stated, and thus is open to many different constructions. The new *Mathematics Framework* has been relatively weakly specified so far, and it therefore offers teachers abundant opportunities to see the new policy as they like, and make of it what they will. Furthermore, the policy subsists in a system of education that is distinctive for the weak and inconsistent instructional guidance that it offers teachers (Cohen, 1990; Schwille, et al., 1983). Even if the new policy were utterly coherent and highly specified, the governmental and extra-governmental systems through which it passes on the way to teachers are so fragmented that many different sorts of advice are mixed together and few priorities are set among the many messages launched toward teachers. The organization and governance of U.S. schooling creates myriad opportunities for teachers to pick and choose among sources of advice, and it offers them a distinctively great and diverse menu of advice.

Hence the U.S. schooling system amplifies the impact of practice on policies in several different ways. Perhaps other new instructional policies have increased rather than reduced variability in practice. But that result seems particularly ironic in this case, since the new policy sought not only dramatic changes in instruction, but greater coherence as well. "Alignment," after all, is a watchword in California's efforts to improve teaching and learning. Although, state officials do not want to homogenize instruction, they do hope to press teaching and learning toward greater consistency as they press it toward greater intelligence. Yet whatever its effects on intelligence in teaching and learning, this policy seems likely to make instruction more variable rather than more consistent.

In considering the impact of practice on policy, one might wonder if the various changes were just a perversion of the *Framework's* goals? Or might they be a step in the direction of more sophisticated mathematics teaching? Or could both be true? The queries are reasonable. But in response we would ask: Could traditional math teachers avoid such fragmentary and confused work as they tried to change their teaching? Could they somehow simply abandon their old knowledge and practices in one moment and produce a brand-new approach to instruction in the next? It seems unlikely. Most teachers probably could only slowly unseat old knowledge and instructional practices as they gradually construct new ones.

In this connection it is worth recalling the dilemma that we mentioned a few pages earlier. We accept that mathematics teaching and learning generally are poor, and, like the reformers in California, we believe that radical change is needed. But, if learning is to improve, the teachers who teach mathematics badly today are the ones who must do a different and better job tomorrow. New policies can only reach the practice that they seek to correct by way of the teachers who have fashioned the practices that want correction. Teachers are at once the agents who cause the instructional problems that state and federal policies of this sort seek to correct and the agents for their correction.

So if our early snapshots portray a muddle, it seems unavoidable. Teachers have picked up pieces of the reform, and interpreted and enacted them in light of what they know and can do, as well as what they believe they must do. There are deep problems with what most have done, but also some elements of promise. The policy appears to be in its early stages, but no one knows how much further implementation will go. At least one of our teachers feels that she is finished: She has responded to the policy and adapted her practices in light of new directions and goals. Several others, who were surprised to find themselves learning from the policy, seemed to think that their practice might change even more. Others are unsure.

California has launched a reform of great ambition, and noble purpose. But its demands are imposing. So far teachers have been asked to make great changes, but they have not been offered many of the resources that might support such change. Few teachers have had opportunities to see examples of the sort of teaching that the state thinks it wants. Few have been offered opportunities to learn a new mathematics. Few have been given opportunities to cultivate a new sort of teaching practice, and even fewer have been offered assistance in the endeavor. In a word, teachers have not yet been engaged in the sort of conversation--with themselves, with other teachers, with university mathematicians and many others--that would support their efforts to learn a new mathematics, and a new mathematical pedagogy.

We are left with questions. Will teachers carry these changes further on their own? Will state and other educational agencies help teachers to capitalize on the changes, by deploying resources to support and advance them? Can state or local agencies, or universities, or other institutions help teachers to learn from such early efforts at change, and to push beyond them? If they can, will they? And if such help is not forthcoming, how far can teachers be expected to get on their own?

The answers remain in the future, part of a story yet to be written. Indeed, in the erratic politics of U.S. education, it remains to be seen whether more of a story will be told at all. Perhaps policymakers will devise another script entirely: It all remains to be seen. State officials have made some encouraging changes, and have displayed political courage and intellectual ambition that are rare in American public education. Much work is left to be done, though, if teachers are to have the wherewithal to make something of these hopeful initiatives. Real pedagogical change would thrive on the creation of a rich conversation, in and around classrooms, about mathematics, teaching, and learning. But creating such a conversation would be costly, time-consuming, and difficult for many teachers and policymakers to enter into. Teaching would thrive on such a rich and slow enterprise. But policymaking seems to thrive on schemes for swift change in instruction, swiftly adopted and often just as swiftly forgotten.

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