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ABSTRACT

The application of the Minnesota Adaptive Instructional System (MAIS) decision procedure by R. D. Tennyson et al. (1975, 1977) is examined. The MAIS is a computer-based adaptive instructional system. The problems of determining the optimal number of interrogatory examples in the MAIS can be formalized as a problem of Bayesian decision making. Two features of the MAIS decision procedure can be improved by using other results from this decision-theory approach. The first feature deals with the determination of the loss ratio "R." A lottery method for assessing this ratio empirically is discussed. The second feature concerns the choice of the loss function involved. It is argued that in many situations, the assumed threshold loss function in the MAIS is an unrealistic representation of the loss actually incurred. A linear utility function is proposed to meet the objections to threshold loss. Whether or not these two innovations are really improvements of the present decision component in the MAIS in terms of student performance on posttests, learning time, and amount of instruction must be decided on the basis of experiments. Research projects for these areas have already been planned. One table and one figure illustrate the decision theory approach. A 38-item list of references is included. (SLD)

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Research Report
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in the
Minnesota Adaptive Instructional System

Hans J. Vos

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Abstract

The purpose of this paper is to review the Minnesota Adaptive Instructional System (MAIS). It will first be indicated how the problem of determining the optimal number of interrogatory examples in MAIS can be formalized as a problem of Bayesian decision making. Subsequently, it will be shown how two features of MAIS can be improved by using other results from this decision-theoretic approach. The first feature deals with the determination of the loss ratio R . A lottery method for assessing empirically this ratio is discussed. The second feature concerns the choice of the loss function involved. It is argued that in many situations the assumed threshold loss function in MAIS is an unrealistic representation of the loss actually incurred. In view of this, a linear utility function is proposed. Finally, some new lines of research are suggested arising from the application of decision theory to the MAIS model.

The Use of Decision Theory in the
Minnesota Adaptive Instructional System

Introduction

The term "adaptive instruction" has been in widespread use for over a decade (Farley, 1983; Hansen, Ross & Rakow, 1977; Holland, 1977; Landa, 1976; Park, 1982; Tennyson & Breuer, 1984). Although different authors have defined the term in a different way, most agree that it denotes the use of strategies to adapt instructional treatments to the changing nature of student abilities and characteristics during the learning process (Tennyson & Park, 1984). In the context of computer-based instruction (CBI), adaptive instructional programs are often qualified as individualized study systems (ISS). Examples are the Pittsburgh Individually Prescribed Instruction (IPI) project (Glaser, 1968) and Computer-Assisted Instruction (CAI) (Atkinson, 1968; Suppes, 1966).

In a special JCBI issue on Educational Research and Computer-Based Instruction, Tennyson, Christensen and Park (1984) describe a computer-based adaptive instructional system, the Minnesota Adaptive Instructional System (MAIS). The authors call the system an intelligent CBI-system, because it exhibits some of machine intelligence, as demonstrated by its ability to improve decision making over the history of the system as a function of accumulated information about previous students. In the literature many successful research projects have been reported on MAIS

(e.g., Park & Tennyson, 1980; Tennyson, Tennyson & Rothen, 1980).

Initial work on MAIS began as an attempt to design an adaptive instructional strategy for concept-learning (Tennyson, 1975). According to Merrill and Tennyson (1977) concept-learning can be conceived as a two-stage process of formation of conceptual knowledge and development of procedural knowledge (see Tennyson and Cocchiarella, 1986, for a complete review of the theory of concept-learning).

In MAIS, eight basic instructional design variables directly related to specific learning processes are distinguished. In order to adapt instruction to individual learner differences (aptitudes, prior knowledge) and learning needs (amount and sequence of instruction), these variables are controlled by an intelligent tutor system (Tennyson and Christensen, 1986). Three of these variables are directly managed by a computer-based decision strategy. The functional operation of this strategy was related to guidelines described by Novick and Lewis (1974).

The purpose of this paper is to review the application of the MAIS decision procedure by Tennyson and his associates. First, it will be indicated how this procedure can be situated within the general framework of (empirical) Bayesian decision theory (e.g., DeGroot, 1970; Ferguson, 1967; Keeney & Raiffa, 1976; Lindgren, 1976), and, what implicit assumptions have to be made in doing so.

Using a Bayesian approach, the decision component in MAIS can be improved. As an example, it will be shown how

two features of MAIS can be improved by using other results from decision theory. The first feature is the determination of the loss ratio R . The second feature is the use of the linear utility function instead of the assumed threshold loss function in MAIS.

The paper concludes with some new lines of research arising from the application of decision theory to the MAIS model. We shall confine ourselves in this paper only to one of the three instructional design variables directly managed by the decision component in MAIS, namely determining the optimal number of interrogatory examples (question form).

A Framework of Bayesian Decision Theory

The derivation of an optimal strategy with respect to the number of interrogatory examples in a concept-learning lesson requires an instructional problem be stated in a form amenable to a decision-theoretic analysis. Analyses based on decision theory vary somewhat from field to field, but the following formal elements can be found in most of them:

1. A nonempty set, Θ , of possible states of nature.
2. A nonempty set, Ω , of actions available to the decision-maker.
3. A loss function, $l(a, \theta)$, i.e., a real-valued function defined on $\Omega \times \Theta$.

4. A probability function or psychometric model, $f(x|\theta)$, relating observed values x of a stochastic variable X to a given value $\Theta = \theta$ for the state of nature.

These basic elements have been related to decision problems in educational testing by many authors, particularly in the context of computer-based adaptive instructional systems (e.g., Atkinson, 1976; Swaminathan, Hambleton & Algina, 1975; van der Linden, 1981). As the use of the decision component in MAIS refers to sequential mastery testing, we shall discuss here only the application of the basic elements to this problem.

The first element concerns the student's true level of functioning $\pi \in [0,1]$. In the present problem, there are two possible states of nature: a student is a true master (θ_1) if his/her true level of functioning exceeds the criterion level $\pi_0 \in [0,1]$, and he/she is a true nonmaster (θ_0) otherwise. The criterion level π_0 —the minimum degree of mastery required—is set in advance. Unfortunately, due to measurement and sampling errors, the true level of functioning is unknown. All that is known is the student's observed test score X from a small sample of n interrogatory examples ($x = 0, 1, \dots, n$).

The second element pertains to the following two available actions: advance a student (a_1) to the next concept if his/her test score x exceeds a certain cut-off score c on the observed test score scale X , and retain (a_0) him/her otherwise. Students with test score x below the cut-off score

c are provided with additional expository examples (statement form). A new interrogatory example is then generated. This procedure is applied sequentially until either mastery is attained or the pool of test items is exhausted. Now, the sequential mastery decision problem can be stated as choosing a value of c that, given the value of π_0 , is optimal in some sense.

The third element describes the loss $l(a_i, \theta_j)$ incurred when action a_i ($i=0,1$) is taken for the student who is in state θ_j ($j=0,1$). The loss must be measured on at least an interval scale. In Tennyson's approach the loss function is supposed to be a threshold function. The implicit choice of this function implies that the "seriousness" of all possible consequences of the two available actions can be summarized by four constants, one for each of the four possible decision outcomes (see Table 1).

Insert Table 1 about here

In Table 1 it is assumed that no losses occur for correct decisions and, therefore, the losses associated with correct advance and retain decisions (l_{11} and l_{00} , respectively) are set equal to zero.

In the decision component of MAIS, a loss ratio R must be specified. R refers to the relative losses associated with advancing a learner whose true level of functioning is below

π_0 and retaining one whose true level exceeds π_0 . From Table 1 it can be seen that the loss ratio R equals l_{10}/l_{01} for all values of π .

The last element relates the observed test score X to the true level of functioning π . In MAIS this is done by using the beta-binomial model.

$$f(x|\pi) = \binom{n}{x} \pi^x (1-\pi)^{n-x}.$$

Within a Bayesian decision-theoretic framework the sequential mastery decision problem is solved by minimizing the "Bayes risk", which is minimal if for each value x of X an action with smallest posterior expected loss is chosen. The posterior expected loss is the expected loss taken with respect to the posterior distribution of π .

It can be seen from the loss table that the decision rule which minimizes posterior expected loss is to advance a student whose test score x is such that

$$l_{01} \text{Prob}(\pi \geq \pi_0 | x, n) \geq l_{10} \text{Prob}(\pi < \pi_0 | x, n),$$

and to retain him/her otherwise. Since $l_{01} > 0$, this is equivalent to advancing a student if

$$\text{Prob}(\pi \geq \pi_0 | x, n) \geq R/(1+R),$$

and retaining him/her otherwise. $\text{Prob}(\pi \geq \pi_0 | x, n)$ denotes the probability of the student's true level of functioning

exceeding π_0 , given his/her test score x on a test of length n . In fact, this probability is one minus the cumulative posterior distribution of π . In MAIS this quantity is called the "beta value" or "operating level" (Tennyson, Christensen & Park, 1984).

It should be noted that, as can be seen from the decision rule, the decision-maker need not specify l_{10} and l_{01} in any absolute value. He need only specify their ratio l_{10}/l_{01} .

In order to initiate the decision component in MAIS, three kinds of parameters must be specified in advance (Rothen & Tennyson, 1984). Beside the parameters π_0 and \bar{K} , a probability distribution representing the prior knowledge about π must be available. In MAIS a beta distribution, $B(\alpha, \beta)$, is used as a prior distribution and a pretest score together with information about other students is used to specify its parameter values. It follows that the posterior distribution of π is easily obtained. From an application of Bayes's theorem, the posterior distribution will again be a member of the beta family (the conjugacy property). In fact, if the prior distribution is $B(\alpha, \beta)$ and the student's test score is x from a test of length n , then the posterior distribution is $B(x+\alpha, n-x+\beta)$. The beta distribution is extensively tabulated (e.g., Pearson, 1930). Normal approximations are also available (Johnson & Kotz, 1970, sect. 2.4.6).

The MAIS decision procedure for adapting the number of interrogatory examples can now be summarized as follows: If a

student's beta value exceeds the quantity $R/(1+R)$, (s)he is passed to the next instructional unit (i.e., next concept) or final (summative) posttest. However, if his/her beta value is below this quantity, his/her posterior distribution is used as a prior distribution in a next cycle. A new interrogatory example is then generated. The procedure is applied iteratively until either the beta value exceeds the quantity $R/(1+R)$ or all interrogatory examples in the pool for that particular concept have been presented.

In the next two sections two features of the MAIS decision procedure will be critically reviewed. Subsequently, it will be shown how they can be improved by using other results from decision theory.

Determination of the Loss Ratio

We start with a point already raised by Tennyson and Park (1979) themselves. It concerns the manner in which the loss ratio is determined. In fact, MAIS does not include any techniques to determine the loss ratio. It only assumes that, prior to administering the pretest, the loss ratio is adjusted on the basis of a task-related aptitude score. A high score on the aptitude score is used to adjust the ratio so that the losses associated with the error of a false retain are decreased relative to the error of a false advancement (Rothen & Tennyson, 1978). However, as Tennyson and Park (1979) have pointed out, this procedure seems to be

without any theoretical base. In particular, a loss function should be independent upon the test data.

As mentioned before, a loss function specifies the total costs of all possible decision outcomes. These costs may concern all relevant psychological, social, and economic consequences which the decision brings along. An example of economic consequences is extra computer time associated with presenting additional instructional materials.

For assessing loss functions, or the more generally applicable utility functions, most texts on decision theory propose lottery methods (see, for example, Luce & Raiffa, 1957, Chap. 2; for a recent modification, see Novick & Lindley, 1979). But in principle any psychological scaling method can be used. Although helpful techniques are available, this does not mean that, for example, in programs of individualized instruction, assessment of utilities is always a simple matter. In this paper, we shall consider one method that works in decision problems with a finite number of outcomes such as in the sequential mastery decision problem.

Generally speaking, utility theory uses the notions of desirability of outcomes to scale the consequences of each pair of action and state of nature. Surely, the most desirable outcomes in MAIS are the true positives and true negatives. In either case, we correctly classify the student. For this reason, it is assumed that both outcomes are equally preferred. Furthermore, if a true nonmaster is advanced, (s)he will not only lose the time required to complete the

next concept, but may also become frustrated and discouraged. Therefore, let us assume that misclassifying a true master is much more desirable than misclassifying a true nonmaster.

The set of the possible outcomes associated with making decision a_i when θ_j is the true state of nature, will be represented by O_{ij} . Let O_{\max} be the most preferred outcome and O_{\min} be the least preferred outcome in this set. So, in our notation, we have $O_{00} = O_{11} = O_{\max}$ and $O_{10} = O_{\min}$. Let u_{ij} describe the utility of O_{ij} . These utilities are measured numerically on an interval scale by the following device.

Let u_{\max} and u_{\min} , the utilities of the most preferred and least preferred outcome of the possible outcomes, be assigned values 1 and 0, respectively. Further, suppose the decision-maker is indifferent between O_{01} for certain and a conceptual lottery which has probability p of realizing O_{\max} and $(1-p)$ of realizing O_{\min} . It is assumed (Luce & Raiffa, 1957, Chap. 2) that this indifference will occur if and only if the person has the same utility for O_{01} for sure and for the specified lottery. The expected utility of the lottery is then $p \cdot u_{\max} + (1-p)u_{\min}$ which equals p , since $u_{\max} = 1$, $u_{\min} = 0$. Hence the utility of O_{01} is operationally defined as p , that is $u_{01} = p$. This procedure can then be followed for each O_{ij} in turn until utilities have been assigned to each of the possible outcomes.

As an aside, we note that for each possible outcome O_{ij} one can always define a suitable loss by taking the difference between the utility of the most preferred outcome and O_{ij} (Lindley, 1972).

Admittedly, specifying utility functions is not an easy task, but practical experience with the above method shows it can be done (e.g., Vrijhof, Mellenbergh & van den Brink, 1983). If, for example, correctly classifying a student gives our decision-maker 10 "utils" more than misclassifying a true nonmaster and only 5 "utils" more than misclassifying a true master, then u_{01} would be 0.5. This means that misclassifying a true master is half-way between misclassifying a true nonmaster and correctly classifying a student on an interval utility scale. It follows that

$$R = l_{10}/l_{01} = (u_{\max} - u_{10})/(u_{\max} - u_{01}) = 2.$$

In the MAIS decision procedure, it is assumed that the form of the utility structure involved is a threshold function. Therefore, only u_{01} has to be assessed empirically. Using the described techniques, however, a decision-maker's utility structure can be completely assessed without making any assumptions about the form of the utility function. Only minimal axioms from utility theory have to be assumed. However, as van der Linden (1981) has pointed out, these techniques do not automatically lead to elegant utility functions and optimal cutting scores. It may be wise, therefore, to use these techniques only for a prior chosen mathematical form of the utility function. In addition to threshold loss function, however, other more useful functions have been adopted in decision theory. One such function will be considered below.

A Linear Utility Function

An obvious disadvantage of the threshold utility function is that it assumes constant utility for students to the left or to the right of π_0 , no matter how large their distance from π_0 is. For instance, a misclassified true master with a true level of functioning just above π_0 gives the same utility as a misclassified true master with a true level far above π_0 . It seems more realistic to suppose that for misclassified true masters the utility is a monotonically decreasing function of the variable π .

Moreover, as can be seen in Table 1, the threshold utility function shows a "threshold" at the point $\pi = \pi_0$, and this also seems unrealistic in many cases. In the neighbourhood of this point, the utilities for correct and incorrect decisions frequently change smoothly rather than abruptly.

In view of this, van der Linden and Mellenbergh (1977) propose a linear utility function:

$$u(a_i, \pi) = \begin{cases} b_0(\pi_0 - \pi) + d_0 & \text{for retain } (a_0) \\ b_1(\pi - \pi_0) + d_1 & \text{for advance } (a_1) \end{cases} \quad b_0, b_1 > 0$$

The above defined function consists of a constant term and a term proportional to the difference between the true

level of functioning π and the specified criterion level π_0 . The constant amounts of utility, d_j ($j=0,1$), can, for example, represent the utility of testing, which will be mostly negative because costs of testing are involved. The condition $b_0, b_1 > 0$ is equivalent to the statement that for actions a_0 and a_1 , utility is a strictly decreasing and increasing function of the variable π , respectively. The parameters $b_0, b_1, d_0,$ and d_1 have to be assessed empirically. Figure 1 displays an example of this function.

Insert Figure 1 about here

As the general linear utility function now stands, we need to determine the four constants $b_0, b_1, d_0,$ and d_1 before it can be applied. However, if we use the fact that a utility function needs to be determined only up to a positive multiplicative constant and an additive constant (e.g., Luce & Raiffa, 1957), we can reduce the number of unknown constants to two. Thus, since $b_1 > 0$, we may redefine $u(a_i, \pi)$ by making the positive linear transformation $u^*(a_i, \pi) = [u(a_i, \pi) - d_1] / b_1$. And so

$$u^*(a_i, \pi) = \begin{cases} b^*(\pi_0 - \pi) + d^* & , i = 0 \\ \pi - \pi_0 & , i = 1 \end{cases}$$

where $b^* = b_0/b_1$ and $d^* = (d_0-d_1)/b_1$.

We turn now to an illustration of one of the most direct methods available for determining the constants b^* and d^* . In order to make the method work, the decision-maker must be able to specify two ordered pairs (π_i, π_j) and (π'_i, π'_j) such that

$$u^*(a_0, \pi_i) = u^*(a_1, \pi_j)$$

and

$$u^*(a_0, \pi'_i) = u^*(a_1, \pi'_j).$$

Solving this system of equations, we find that

$$b^* = (\pi_i - \pi'_j) / (\pi_i - \pi'_i) \text{ and } d^* = \pi_j - \pi_0 - b^*(\pi_0 - \pi_i).$$

Analogous to the minimization of posterior expected loss, decision theory with a utility function requires us to select that action which will maximize the posterior expected utility. So, the decision rule that maximizes posterior expected utility in the case of a linear utility function is to advance a student with test score x for which

$$E[(\pi - \pi_0) | x, n] \geq E[(b^*(\pi_0 - \pi) + d^* | x, n)],$$

and to retain him/her otherwise. Since $(1+b^*) > 0$, this is equivalent to advancing a student if

$$E[\pi|x,n] \geq \pi_0 + d^*/(1+b^*),$$

and retaining him/her otherwise. In other words, with linear utility, the action taken depends only upon the expectation of the posterior distribution of π , other attributes of the distribution are irrelevant for decision purposes.

Using the fact that the expectation of a beta distribution $B(\alpha, \beta)$ is equal to $\alpha/(\alpha+\beta)$, and, thus, the posterior expectation equals $(\alpha+x)/(\alpha+x+\beta+n-x)$, it follows that a student is advanced if his/her test score x is such that

$$x \geq [\alpha+\beta+n][\pi_0+d^*/(1+b^*)]-\alpha,$$

and retained otherwise.

Putting $u^*(a_0, \pi)$ and $u^*(a_1, \pi)$ equal to each other, it appears that the π -coordinate of the intersection of both utility lines is equal to $\pi_0 + d^*/(1+b^*)$. Therefore, the decision rule can be viewed as advancing a student if his/her expectation of the posterior distribution of π is to the right of the intersection point, and retaining him/her otherwise.

When $d^* = 0$, that is $d_0 = d_1$, both utility lines intersect at $\pi = \pi_0$ and an interesting case arises. Then, all utility function parameters vanish from the decision rule, and thus, takes the form of advancing a student if

$$E[\pi|x,n] \geq \pi_0.$$

and retaining him/her otherwise. In other words, if the amounts of constant utility, d_j , for both decisions are equal or if there are no constant utilities at all, there is no need to assess the parameters d^* and b^* in adapting the number of interrogatory examples. In that case, the decision rule can even be simplified to advance a student if his/her expectation of the posterior distribution of π is greater than or equal to the specified criterion level π_0 , and to retain him/her otherwise.

New Lines of Research

There are a few new lines of research arising from the application of decision theory to the decision component in MAIS. The first is the extension of the work of Tennyson and his associates to situations where guessing and carelessness are incorporated. Morgan (1979) has developed a model with corrections for guessing and carelessness within a Bayesian decision-theoretic framework. The results of a computer simulation of the model indicate that guessing and carelessness may markedly affect the determination of cutting scores, and hence the accuracy of decisions about mastery.

The second line is research into other prior distributions about π (for example, the standard normal distribution) than the beta prior assumed in MAIS. It might also be assumed that no prior distribution about π is

available, because specifying such a distribution is too difficult a job to accomplish. In these circumstances, the minimax procedure may be an appropriate framework (e.g., Huynh, 1980; van der Linden, 1981) which requires no prior distribution regarding the true level of functioning. In this case, the optimum cutting score is obtained by minimizing the maximum risk which would be incurred by misclassifications.

Finally, an interesting new line of research seems to be an extension of the action space Ω . In MAIS, two actions were available to the decision-maker, namely advancing (a_1) or retaining (a_0) a student. However, it might also be hypothesized that there are three (or any finite number) of actions open to the decision-maker.

For example, in the three-action problem the student may be provided with additional instructional materials both of the present and the previous concept (a_2); (s)he may be provided only with additional instructional materials of the present concept (a_0); or, (s)he may advance to the next concept (a_1).

We might think of this problem in terms of specifying two cutting test scores c_0 and c_1 on the observed test score scale X , where $c_0 < c_1$. Then for observed test score $x < c_0$, action a_2 will be taken; for $c_0 < x < c_1$, action a_0 will be taken; and, for $x > c_1$, action a_1 will be taken.

Davis, Hickman and Novick (1973) have given a solution to the three-action problem by using natural extensions of the threshold loss function. Although the notation becomes more complex and the computation a bit more tedious, there

are no fundamentally new ideas in the multiple action problem.

Some Concluding Remarks

In this paper it was indicated how the MAIS decision procedure can be formalized within a Bayesian decision-theoretic framework by applying the basic elements of the theory to the instructional decision of determining the optimal number of interrogatory examples in MAIS. In fact, it turned out that this decision can be considered as a sequential mastery decision.

Moreover, it was the purpose of this paper to show how two features of MAIS can be improved as a consequence of this formalization by using other results from decision theory. The first feature is the determination of the loss ratio R . A lottery method has been discussed for assessing this ratio empirically. The second feature is the threshold loss function. It was argued that in many situations this is an unrealistic representation of the loss actually incurred. Instead, a linear utility function was proposed to meet the objections to threshold loss.

Whether or not the determination of the loss ratio by the described lottery method and the proposed linear utility function instead of the assumed threshold loss function, however, are really improvements of the present decision component in MAIS (in terms of student performance on

posttests, learning time, and amount of instruction) must be decided on the basis of experiments. Research projects in these areas have already been planned.

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Authors' Note

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Table 1

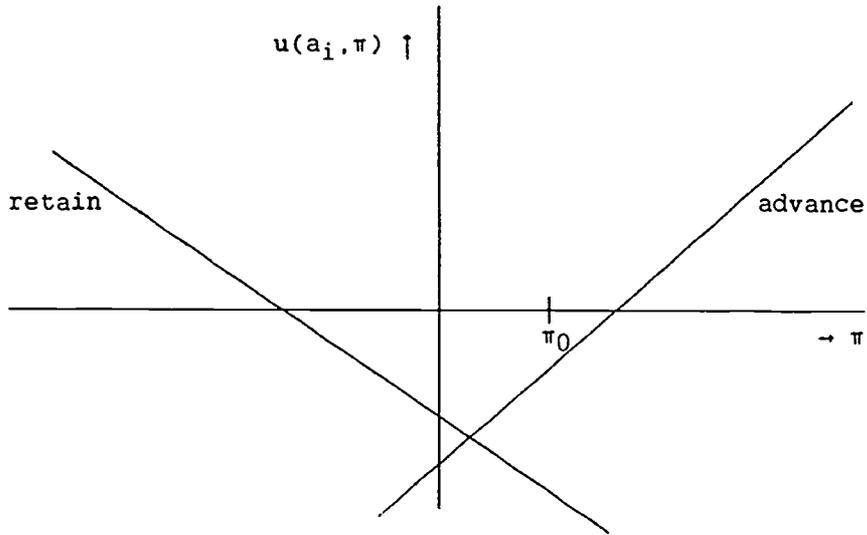
Twofold Table for Threshold Loss Function

		True level	
		$\pi \geq \pi_0$ (true master)	$\pi < \pi_0$ (true nonmaster)
Decision	Advance	0	110
	Retain	101	0

Figure Caption

Figure 1. An Example of a Linear Utility Function.

($b_0 \neq b_1$, $d_0 \neq d_1$)



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