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ABSTRACT

This study was undertaken to compare non-metric multidimensional scaling (MDS) and factor analysis (FA) as means of assessing dimensionality in relation to item response theory (IRT). FA assesses correlation matrices, while MDS performs an analysis of proximity measures. Seven data sets were generated; each differed from the others with respect to dimensionality. Each set consisted of 28 items and was either one- or two-dimensional, with two-dimensional data sets having interdimensional correlation coefficients of 0.01, 0.10, and 0.60. All sets were generated according to G. Masters' partial credit model (1982)--a Rasch model; 14 item parameters were taken from Masters' article and duplicated to produce the 28-item pool. Assessment of dimensionality through MDS was performed by stress by dimensionality graphs alone and in conjunction with plots of stress by dimensionality containing varying degrees of error. For the FA of each correlation matrix, the initial determination of dimensionality was performed through an examination of the matrix's scree plot. Results indicate that MDS may be a viable methodology for IRT researchers to use in assessing the dimensionality of ordered test or attitude data. Four data tables and 20 graphs are provided. (TJH)

A Comparison of Methods for Assessing Dimensionality for use in Item Response Theory[1]

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Objectives

The majority of individuals using item response theory (IRT) for modeling examinee performance assume that examinee performance is a function of a single latent trait (θ). The comparatively new multidimensional IRT models also require knowledge of the data's dimensionality in order to correctly specify the model's structure. Although no one technique has proven to be completely satisfactory for assessing the dimensionality of a data set, two commonly used methods are principal axis (FA) and component analysis.

FA and component analysis typically analyze either a phi or tetrachoric correlation matrix. However, for these correlation coefficients to be appropriate, the data must meet certain assumptions (e.g., the latent distribution of the variables is bivariate normal and the variables are measured at the interval level). The tenability of some of these assumptions with certain types of data has been questioned. Further, problems with the actual use of these coefficients have been identified. For instance, the use of phi coefficients has been found to sometimes lead to the identification of spurious "difficulty" factors related to the characteristics of the items rather than to true underlying relationships (Guilford, 1941). In addition, non-Gramian matrices may result when tetrachorics are factor analyzed. In general, FA has been found to overestimate the number of underlying dimensions in a data set (Hambleton & Rovinelli, 1986).

An alternative to FA and component analysis is McDonald's (1967) non-linear factor analysis (NLFA). NLFA is intuitively appealing to researchers in IRT because its principal assumption specifies a nonlinear relationship between item performance and ability (as do the IRT models). In a study comparing various

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methods for assessing dimensionality, Hambleton and Rovinelli (1986) found that NLFA correctly determined the dimensionality of various data sets. However, they encountered a problem determining the appropriate number of factors and polynomial terms to retain in their solution. Hambleton and Rovinelli addressed this issue by comparing the residual results from a "satisfactory" FA to those of NLFA.

Given NLFA's nonlinearity assumption and Hambleton and Rovinelli's results, a technique which simply assumes a monotonic relationship, such as nonmetric multidimensional scaling (MDS), would appear to be useful for assessing dimensionality. A more detailed presentation of MDS may be found in Davison (1983), Schiffman, Reynolds and Young (1981), or Kruskal and Wish (1978). This study's objective was to compare MDS with FA for the assessment of dimensionality. Additional factors investigated were the degree of intercorrelation between the dimensions and the number of items measuring a dimension.

Method

Data

Seven data sets were generated which differed from one another with respect to dimensionality, intercorrelation among dimensions, and number of items defining a dimension. The data sets consisted of 28 items and were either one- or two-dimensional, with the two-dimensional data sets having interdimensional correlation coefficients of 0.01, 0.10, and 0.60. For one set of data, the number of items used to create dimensions one and two were 14 and 14, and for a second set of data 18 items were used for defining the first dimension, whereas the second dimension was composed of 10 items. All data sets were generated according to Masters' (1982) partial credit (PC) model; fourteen PC item parameters were taken from Masters' (1982) article and duplicated to produce the 28 item pool. For these polychotomous data sets each item consisted of four alternatives. A dichotomous form of each of the seven data sets was obtained by using the fourth alternative as the correct answer and scoring each item as correct and incorrect. Therefore, each of the seven combinations of dimensionality, interdimensional correlation, and number of items defining a dimension existed in both a dichotomous and polychotomous form. Table 1 summarizes the study's

design.

Insert Table 1 about here

Because MDS assumes that the variables are measured at least at an ordinal level, data generation according to the PC model resulted in data compatible for use with MDS. This is analogous to utilizing examinees' raw responses to an attitude questionnaire or ability or achievement tests ir which responses have been evaluated according to degree of correctness.

The number of items selected for the study was based on restrictions imposed in part by the use of the program M-SPACE (to be discussed below); M-SPACE can only be used for 12-36 stimuli (Spence & Graef, 1973). Further, in order to minimize the effect on stress of the number of items and the number of dimensions, the minimum number of items defining a dimension needs to be greater than four times the number of dimensions (Kruskal & Wish, 1978). Therefore, for a two-dimensional solution the mimimum number of items is 8, and it was decided to be slightly more conservative and use a minimum of 10 items to define a dimension.

The one-dimensional data set was generated by sampling 500 examinees from a normal distribution (0,1); the z-values were considered to be true θs. These true θs plus the item difficulty parameters were used to generate the response strings. Specifically, for each θ the generation of the polychotomous response string was accomplished by calculating the probability of responding to each alternative of an item given the item's difficulty parameters and θ. Based on the probability for each alternative, cumulative probabilities were obtained for each alternative. To create the random error component for a response (in order to make the data more realistic), a random number was selected from a uniform distribution between 0 and 1 and compared to the cumulative probabilities to obtain the polychotomous response.

The two-dimensional data were generated according to a method which was based on a technique used by Hambleton and Rovinelli (1986). For each of the 500 simulees, two random numbers (X and Z) were generated from a normal distribution (0,1). Using Hoffman's (1959) formula :

$$Y = X + (k\sigma_X / r\sigma_Z)Z \qquad (1),$$

a third variable, Y, was generated; where $k=(1-r^2)^{1/2}$, r is the desired intercorrelation between Y and X, and $\sigma_X = \sigma_Z = 1.0$. X and Y served as two θs (θ_1, θ_2) used in the data generation. Using the technique outlined above, the polychotomous responses for the first fourteen items were generated using θ_1 (dimension 1), whereas θ_2 was used for the remaining items (dimension 2).

The nomenclature for specifying the various data sets was PC + number of dimensions + dimensional intercorrelation (first significant digit) + number of items on dimension 1 + number of items on dimension 2. For example, PC10280 was the one-dimensional data set with the dimension consisting of 28 items, PC211414 was a two-dimensional data set with an interdimensional correlation of 0.10 and 14 items defining each dimension.

Techniques

Whereas FA analyzes correlation matrices, MDS performs an analysis of proximity measures. Reckase (1982) found that the use of different proximity measures in the MDS of dichotomous data led to different configurations. Therefore, different proximity measures (i.e., Euclidean, cosine, squared Euclidean, block, and Chebychev) were used with MDS. The package ALSCAL (SPSS, 1988) was used for nonmetric MDS analysis. Further, the program M-SPACE (Spence & Graef, 1974) was used as an aid in the determination of the dimensionality of the MDS solutions.

For comparison with standard approaches to assessing dimensionality, FA of matrices of tetrachoric and phi coefficients were performed; squared multiple correlations were used as initial h^2 estimates. Additional proximity measures, Pearson product-moment and polychoric correlation coefficients, were obtained on the polychotomous data and factor analyzed. PRELIS (Joreskog & Sorbom, 1986) was used for estimating the tetrachoric and polychoric correlation coefficients; the polychoric correlation coefficient is a generalization of the tetrachoric correlation coefficient when both variables are ordered categorical (Olson, 1979).

To summarize, a polychotomous and dichotomous format of each of seven data sets were generated. Within each format (e.g., for the polychotomous version) six of the seven were two-dimensional, with the remaining one containing only one

dimension. Of these six data sets, there were three pairs which differed from one another in their interdimensional correlation (i.e., 0.01. 0.10, and 0.60) and each member of the pair differed from the other member in the number of items defining a dimension (i.e., 14 items per dimension or 18 items defining dimension 1 with dimension 2 consisting of 10 items). For each of the seven polychotomous data sets five proximity measures were calculated, each of which was subjected to MDS analysis. Further, for each data set four types of correlation matrices were calculated (two for each format) and each of these matrices was subsequently factor analyzed.

Analysis

Assessment of dimensionality through MDS was performed by stress by dimensionality graphs alone and in conjunction with plots of stress by dimensionality containing varying degrees of error (a.k.a., error plots). Stress is a measure of "badness-of-fit", where larger values indicate poorer fit to the data; stress formula one was used in the analyses. The error plots were created through the M-SPACE[2] program. In short, the M-SPACE program attempts to find the dimensionality and error level that best characterize the empirical stress values (Spence, 1983). A least squares loss function is employed, and the minimum for each of four generated dimensions is found. The appropriate dimensionality is assumed to be that which yields the lowest residual sum of squares over the four dimensions (Spence & Graef, 1974).

For the FA of each correlation matrix, the initial determination of dimensionality was performed through an examination of the matrix's scree plot. After examination of the initial solution, if a one-factor solution seemed likely, one- and two-factor solutions were obtained for comparison. However, if a two-factor solution appeared plausible, then one-, two-, and three-factor solutions were obtained. The final determination of dimensionality was made after considering : (a) percent of common variance accounted for by a factor, (b) percent of variance accounted for by the factor solution, (c) approximation to simple structure (VARIMAX and OBLIMIN rotations), (d) magnitude of loadings, and (e) number of residuals greater than 0.05. An additional criterion used for determining dimensionality was the number of eigenvalues greater than the largest

eigenvalue of a FA of a tetrachoric matrix based on random data; this criterion will be referred to as λ_{random}. The random data were generated by randomly sampling from a normal distribution (0,1) for each examinee and for each item. Normal deviates greater than or equal to 0 were coded as correct, whereas those below 0 were considered incorrect.

Results and Conclusions

MDS Results

The use of the Chebychev proximity measure resulted in extremely poor fit to the data and was eliminated from further analysis. The cosine measure overestimated the number of dimensions for the one-dimensional data set and consistently underestimated the number of dimensions for the two-dimensional data sets. However, the use of Euclidean and squared Euclidean proximities produced virtually identical results; the block measure resulted in an underestimation of the number of dimensions for the data sets with an interdimensional correlation of 0.60.

Table 2 presents the stress values for the various proximity measures for one- to five-dimensional solutions for each polychotomous data set. As can be seen, for all dimensional solutions stress values stayed relatively constant and low across data sets. Figure 1 presents the stress by number of dimensions plots for the one dimensional data set as well as two two-dimensional data sets, PC201414 and PC261810, based on the Euclidean proximity measure. Unlike FA's scree plots where the elbow (beginning of the scree) is not included in the number of factors (Cattell, 1979), MDS does include the elbow in determining the number of dimensions (Davison, 1983). According to Kruskal and Wish (1978), an elbow should seldom be accepted if the stress at the elbow is above 0.10, however, if the stress at dimension 1 is less than 0.15, then a one-dimensional solution is suggested. Given these guidelines, it can be seen that a one-dimensional solution was suggested for PC10280 (Figure 1a) and that a clear elbow can be seen at two dimensions in Figures 1b and 1c (data sets PC201414 and PC261810, respectively).

Insert Table 2 and Figure 1 about here

The identification of an elbow is subjective and may sometimes be problematic. In this regard, the M-SPACE program provides an objective means for identifying the dimensionality of a data set. Table 3 summarizes some of M-SPACE's output. As can be seen, for each of the proximities this table presents the solution's fit to the data, the number of latent dimension(s) identified as the correct solution, and an approximate error level for the data. For all data sets the Euclidean and squared Euclidean measures led to a correct determination of the number of dimensions in the data. The block proximity measure resulted in an incorrect identification of dimensionality for the two-dimensional data sets with an interdimensional correlation of 0.60.

Insert Table 3 about here

Additional output produced by M-SPACE is a graphical presentation of the relationship between the empirically obtained stress values from one- to five-dimensional solutions and five recovered dimensions based on monte carlo simulation data; this relationship is depicted for generated solutions in one- to four-dimensions (Spence & Graef, 1974; Spence, 1983). The generated solution which most closely matches the empirical stress values is identified as the correct dimensionality. Figures 2 and 3 present these graphs for the one-dimensional data and for the two-dimensional data with an interdimensional correlation of 0.60 and 18 items defining the first dimension. Clearly, although not perfect fits, the recovered dimensions fit the empirical stress values more closely in the appropriate dimensional solution.

Insert Figures 2 and 3 about here

FA Results

FA of the tetrachoric matrix for the random data produced a $\lambda_{random} = 1.109$. For the data sets of interest, FA of the polychoric and tetrachoric matrices resulted in non-Gramian matrices. In addition, the analysis of the tetrachoric matrix produced $h^2 > 1.0$. However, SPSS-X (SPSS Inc., 1988) performed the requested FAs (presumably after setting the matrices' determinants to some, albeit small, value)

and issued a warning that the results may not be valid; the technique used produced negative λs. Although in this study the validity of the results could be assessed given the data's known dimensionality, in an actual application this knowledge would not be available and the interpretation of the FA would be problematic.

Results of the FA of product-moment correlation matrices are presented in Table 4. The use of the λ_{random} criterion led to the incorrect retention of two factors for the one-dimensional data set, but the correct determination of factors for the two-dimensional data sets. Scree plots for PC10280, PC201414, and PC261810 are presented in Figure 4. As can be seen from Figure 4a as well as from Table 4, the FA of the phi correlation matrix for PC10280 suggested a one-factor solution. To support the scree plot interpretation further analyses, involving the criteria mentioned above, were undertaken. These analyses provided evidence for a two-factor solution (e.g., the two-factor had better fit to the matrix than did the one-factor solution); the additional factor appeared to be a difficulty factor.

For the uncorrelated two dimensional data it can be seen from Table 4 and Figures 4b and 4c that a two factor solution is clearly suggested for both the dichotomous and polychotomous data. The follow-up analyses supported this conclusion. A comparison of Figures 4b-4c and 4d-4e as well as an inspection of Table 4 showed that with greater interdimensional correlation, the scree plots appeared to indicate a one-factor solution, particularly for the polychotomous data. However, an analysis of the loadings with respect to simple structure, number of residuals with values greater than 0.05, and percentage of variance accounted for by the solution showed that a two-factor solution was clearly better than a one-factor solution. In all cases, items which defined the first dimension loaded on one factor, whereas items related to the second dimension loaded on the other factor. None of the three-factor solutions were preferable to the two-factor solutions.

Insert Table 4 and Figure 4 about here

Discussion

For the FA no one method of factor number determination led to the correct solution for all data sets. For instance, for the highly correlated interdimensional data sets (PC261414 and PC261810), the scree plot presented evidence that a single factor existed, whereas follow-up analyses showed that the two-factor solution provided better fit than the one-factor solution. However, for the one-dimensional data, the scree plot for PC10280 correctly identified a one-factor solution, whereas λ_{random} and the follow-up analyses suggested a two-factor solution. It is interesting to note that for corresponding data sets, the eigenvalues for the primary factor(s) were larger for the Pearson product-moment correlation calculated on polychotomous data than on dichotomous data. In addition, the percentage of variance accounted for by the solution was higher for the Pearson product-moment correlation than for phi coefficient for all data sets. Although, it appears that the use of Pearson correlations with polychotomous data may have desirable properties, previous research (Muthen, 1989) has shown that these coefficients calculated on polychotomous data tend to underestimate true factor loadings and in some cases may result in an overestimation of the number of factors; this latter situation did not occur in this study.

For all data sets, the MDS of the Euclidean or squared Euclidean proximity matrix resulted in the correct identification of the data's underlying dimensionality. Further, in regard to the determination of the number of underlying dimensions (through stress by dimensionality plots or M-SPACE analysis), MDS did not appear to be as affected by large interdimensional association and uneven number of items defining each factor as was the FA; this is not necessarily true with respect to "interpretability" of solution.

Although an additional consideration in the determination of dimensionality is the interpretability of the solution, given that simulation data were used interpretability of the stimulus configuration may not be all that meaningful. However, the stimulus configuration plots were examined. An interpretation of the one-dimensional solution for the one-dimensional data showed that the items from the first 14 items had scale values which were comparable to their corresponding item's scale value in the second 14 items (i.e., items 1 and 15, 2 and 16, 3

and 17, etc. were all corresponding items). This pattern of scaled values was, most likely, an artifact of the data generation methodology. However, it was found that the items' scale values were highly related to the average step difficulties for the items (r = 0.913 for all 28 items, r =0.917 for first 14 items). This latter relationship has also been observed with the simple Rasch model (Fitzpatrick, 1989). A two-dimensional solution for the one-dimensional data did not result in a clear, interpretable pattern of stimuli (Figure 5).

Insert Figure 5 about here

Examination of the two-dimensional configuration for PC201414 (Figure 6) showed two clear clusters of items. The two clusters differed from one another along dimension 2 with the first 14 items scaled with positive dimension 2 coordinates and the second set of 14 items given negative dimension 2 values. Items which were, in general, more difficult (based on their average step difficulties) were scaled with negative dimension 1 values, and items which were, in general, comparatively easier were given positive dimension 1 coordinates. As was the case for the one-dimensional data, corresponding items from each set of 14 items were given approximately equal stimulus coordinates along dimension 1. In short, MDS had correctly identified, on the basis of examinee responses, that the first fourteen items were more related or similar to one another than they were to the second set of fourteen items.

Insert Figure 6 about here

The two-dimensional stimuli configuration for PC261414 and PC261810 are presented in Figures 7 and 8. As can be seen from Figure 7, the same pattern found with Figure 6 was evident for in this case. The higher interdimensional association in PC261414 was reflected in dimension 2 scale values which were not as extreme as those for PC201414.

Insert Figure 7 about here

Examination of the two-dimensional configuration for the data with an

uneven representation of each dimension and an interdimensional correlation of 0.60 (Figure 8) showed that the interaction of the uneven distribution of items and the hig! interdimensional correlation obscured an interpretable pattern of items It should be noted that for PC201810 and PC211810 the cluster pattern exemplified in Figures 6 and 7 was observed; one cluster consisted of 18 stimuli, whereas the other cluster was composed of 10 item. Although no (apparent) interpetable pattern was found with PC261810, it should be recalled that on the basis of stress by dimensions plot and M-SPACE analysis (Euclidean and squared Euclidean metric), it was possible to correctly identify the number of latent dimensions in the data.

Insert Figure 8 about here

Given that the benefits of IRT (e.g., sample-free person and item parameters, construction of tests with known properties, etc.) may only be realized when the assumptions of the IRT model used (e.g., unidimensionality) are met by the data, the identification of whether the data contain more than one dimension is necessary before the fitting of a unidimensional IRT model may be done. For multidimensional IRT models (e.g., McKinley & Reckase, 1983) the identification of the correct number of dimensions is also necessary. From this study it appears that MDS may be a viable methodology for IRT researchers to use in the assessment of the dimensionality of ordered test or attitude data (i.e., data for which Samejima's (1969) graded response model, Andrich's (1978) rating scale model, or the PC model are appropriate). Future research will evaluate whether a rank ordering method may be applied to nominal level response data which can then be analyzed for dimensionality through MDS.

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Table 1 : Summary of study's design.

Number of dimensions	r	Number of items per dimension: dimension 1	dimension 2	Data Set Name[1]
1	-	28	0	PC10280
2	0.01	14	14	PC201414
2	0.10	14	14	PC211414
2	0.60	14	14	PC261414
2	0.01	18	10	PC201810
2	0.10	18	10	PC211810
2	0.60	18	10	PC261810

r : intercorrelation between dimensions.

[1]Each data set existed in a dichotomous and polychotomous format.

Table 2 : Stress Formula 1 Values[1] for Five Dimensions

Data Set	I	II	III	IV	V
PC10280					
Euclid	0.142	0.079	0.065	0.055	0.045
Seuclid	0.142	0.080	0.066	0.054	0.042
Block	0.133	0.074	0.060	0.050	0.041
Cosine	0.251	0.092	0.066	0.052	0.042
PC201414					
Euclid	0.299	0.060	0.045	0.038	0.031
Seuclid	0.299	0.060	0.046	0.036	0.030
Block	0.274	0.078	0.047	0.038	0.030
Cosine	0.123	0.054	0.045	0.039	0.031
PC201810					
Euclid	0.289	0.066	0.051	0.041	0.036
Seuclid	0.290	0.066	0.050	0.041	0.034
Block	0.267	0.078	0.052	0.041	0.036
Cosine	0.126	0.059	0.046	0.039	0.034
PC211414					
Euclid	0.287	0.064	0.049	0.041	0.033
Seuclid	0.287	0.064	0.050	0.040	0.033
Block	0.263	0.080	0.047	0.041	0.030
Cosine	0.124	0.056	0.047	0.038	0.029
PC211810					
Euclid	0.279	0.066	0.050	0.042	0.035
Seuclid	0.279	0.067	0.054	0.042	0.035
Block	0.256	0.080	0.051	0.043	0.036
Cosine	0.128	0.060	0.048	0.039	0.032
PC261414					
Euclid	0.217	0.077	0.052	0.043	0.034
Seuclid	0.217	0.078	0.052	0.042	0.035
Block	0.195	0.099	0.046	0.034	0.030
Cosine	0.168	0.107	0.062	0.045	0.038
PC261810					
Euclid	0.208	0.079	0.055	0.045	0.040
Seuclid	0.209	0.080	0.056	0.047	0.038
Block	0.189	0.094	0.053	0.042	0.036
Cosine	0.159	0.101	0.060	0.045	0.039

Note :[1]Chebychev stress values not presented
Seuclid - Squared Euclidean metric

Table 3 : M-SPACE results[1]

Assessed Data Set	Minimum Fit	Dimensionality	Error Level[2]
PC10280			
Euclid	21.3	1	17
Seuclid	21.7	1	17
Block	21.0	1	15
Cosine	27.6	2	14
PC201414			
Euclid	9.6	2	12
Seuclid	10.0	2	11
Block	19.9	2	14
Cosine	24.5	1	12
PC201810			
Euclid	11.6	2	13
Seuclid	11.7	2	13
Block	20.5	2	14
Cosine	24.6	1	12
PC211414			
Euclid	13.0	2	13
Seuclid	13.0	2	13
Block	23.6	2	14
Cosine	24.9	1	12
PC211810			
Euclid	15.6	2	13
Seuclid	15.3	2	14
Block	25.0	2	14
Cosine	25.3	1	12
PC261414			
Euclid	41.5	2	14
Seuclid	41.5	2	14
Block	46.5	1	20
Cosine	31.7	1	19
PC261810			
Euclid	45.0	2	14
Seuclid	44.6	2	14
Block	41.0	1	20
Cosine	29.2	118	

[1]Analysis of Chebychev stress not performed; Seuclid - Squared Euclidean

[2]Error level - Level where stress values most closely fit the monte carlo data with known error. Provides a basis for evaluating the worth of the solution.

Error Level Guidelines :

Error Percentage	Interpretation
0-10	Extremely low error
10-30	Low Error
30-70	Moderate Error
70-90	High Error
90-100	Possibly Excessive Error

Table 4 : FA of phi and Pearson product-moment correlation coefficients (λ_{random} = 1.109).

Data Set	Phi Coefficients I	II	III	Pearson Coefficients I	II	III
PC10280						
λ	6.486	1.501	1.025	2.893	1.736	-
% of σ^2	61.1	14.2	9.7	88.1	11.9	
PC201414						
λ	3.903	3.503	1.050	7.240	6.390	0.891
% of σ^2	37.1	33.3	9.9	47.4	41.8	1.6
PC201810						
λ	4.723	2.824	1.012	8.694	4.820	0.911
% of σ^2	45.8	27.4	9.8	57.5	31.8	1.7
PC211414						
λ	4.113	3.337	1.020	7.78	5.840	0.900
% of σ^2	38.5	31.1	9.4	50.9	38.3	1.7
PC211810						
λ	4.764	2.797	1.019	8.854	4.669	0.905
% of σ^2	46.3	27.3	9.8	58.3	30.8	1.7
PC261414						
λ	5.724	1.707	0.942	10.800	2.759	1.045
% of σ^2	55.7	16.7	9.3	71.0	18.2	6.8
PC261810						
λ	5.810	1.765	1.013	10.965	2.454	1.032
% of σ^2	55.5	16.9	9.7	73.0	16.4	6.9

Note : σ^2 represents common variance

Figure 1a

Stress by dimensionality
Data : One dimensional (PC10280)

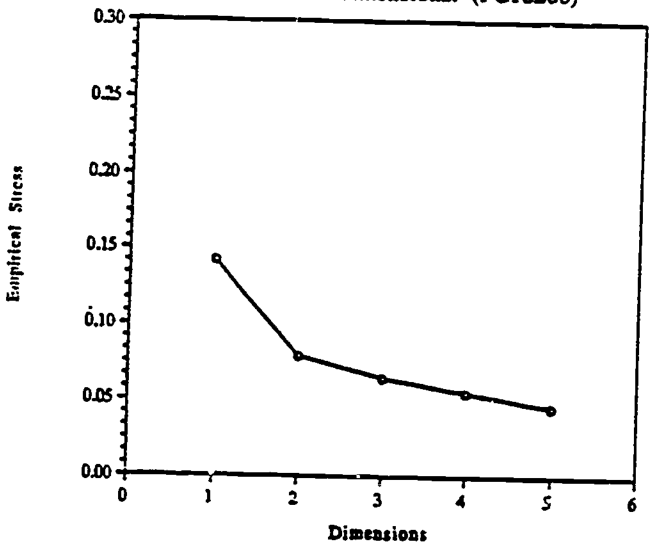


Figure 1b

Stress by dimensionality
Data : Two dimensional, 14 & 14 Items, r=0.01 (PC201414)

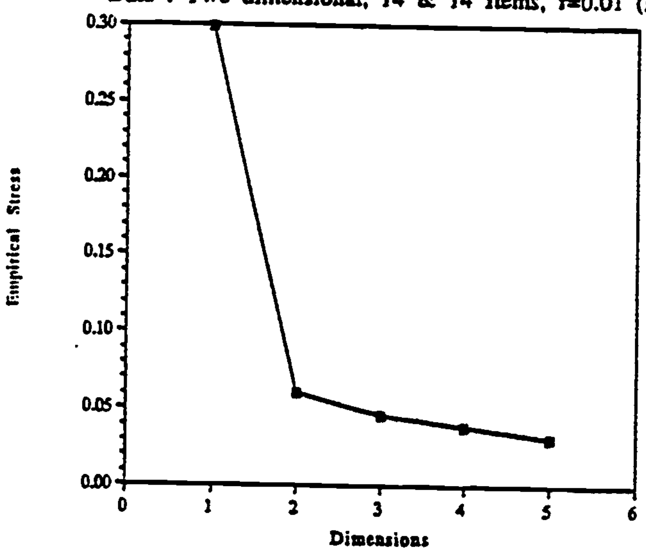


Figure 1c

Stress by dimensionality
Data : Two dimensional, 18 & 10 items, r=0.60 (PC261810)

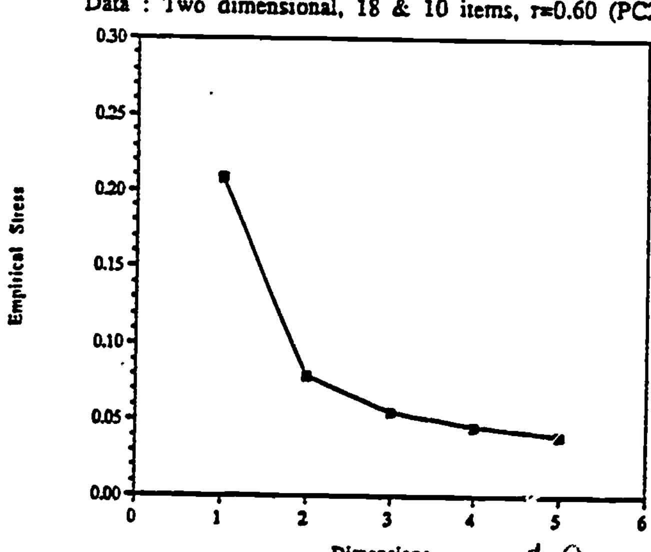


Figure 2a

Data : One dimensional (PC10280)
Solution : 1 dimension

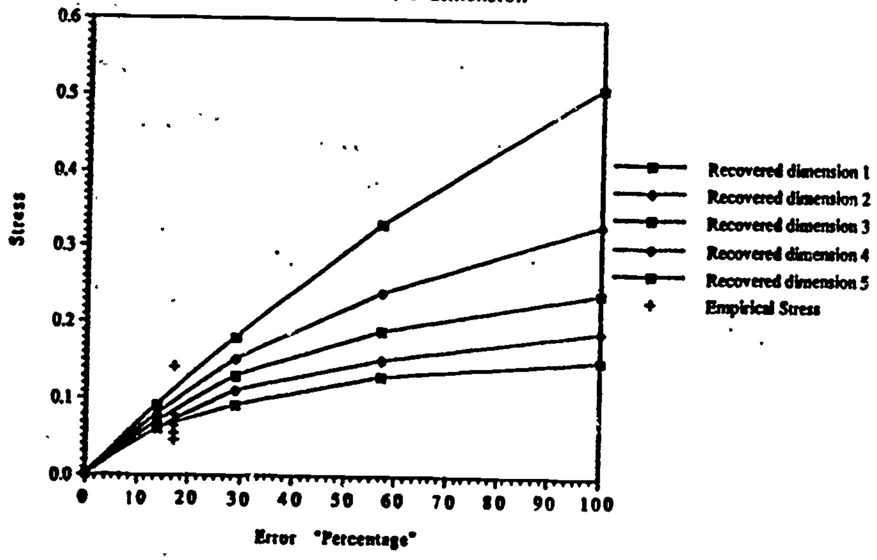


Figure 2b

Data : One dimensional (PC10280)
Solution : 2 dimensions

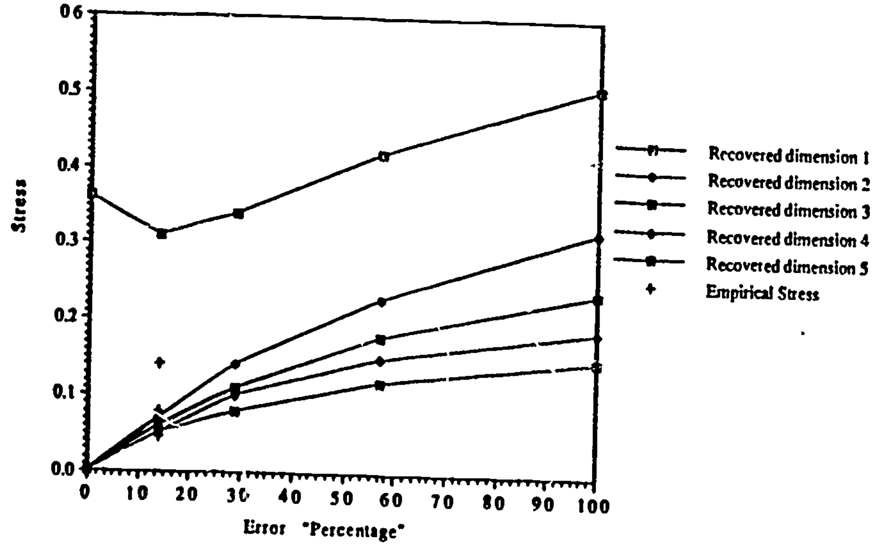


Figure 2c

Data : One dimensional (PC10280)
Solution : 3 dimensions

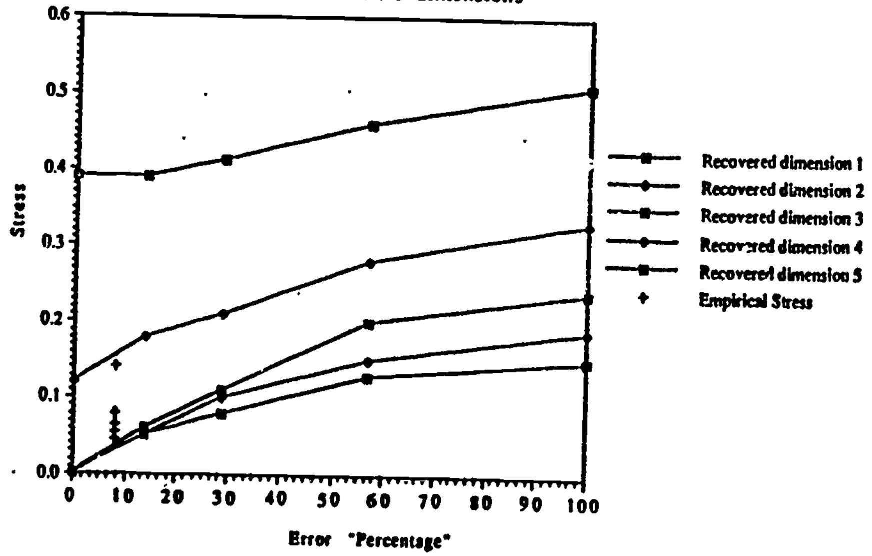


Figure 2d

Data : One dimensional (PC10280)
Solution : 4 dimensions

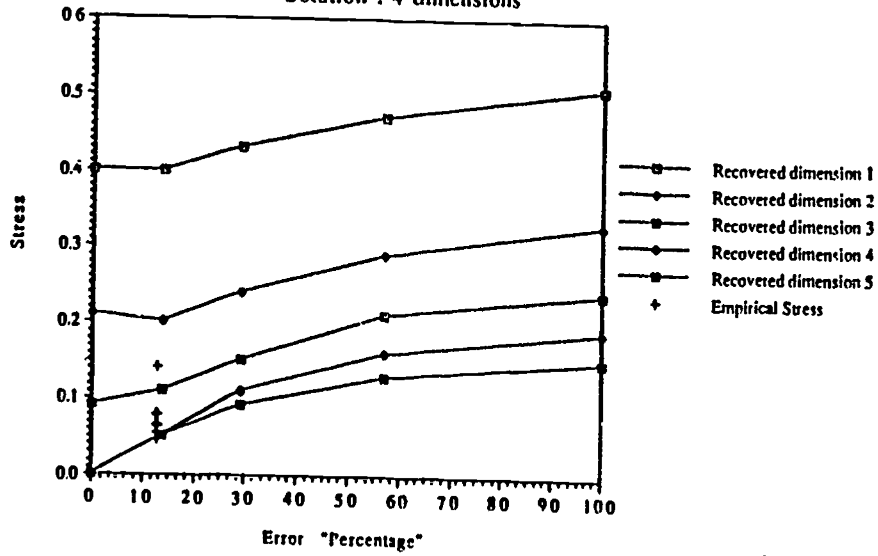


Figure 3a

Data : Two dimensional, 18 & 10 items, r=0.60 (PC261810)
Solution : 1 dimension

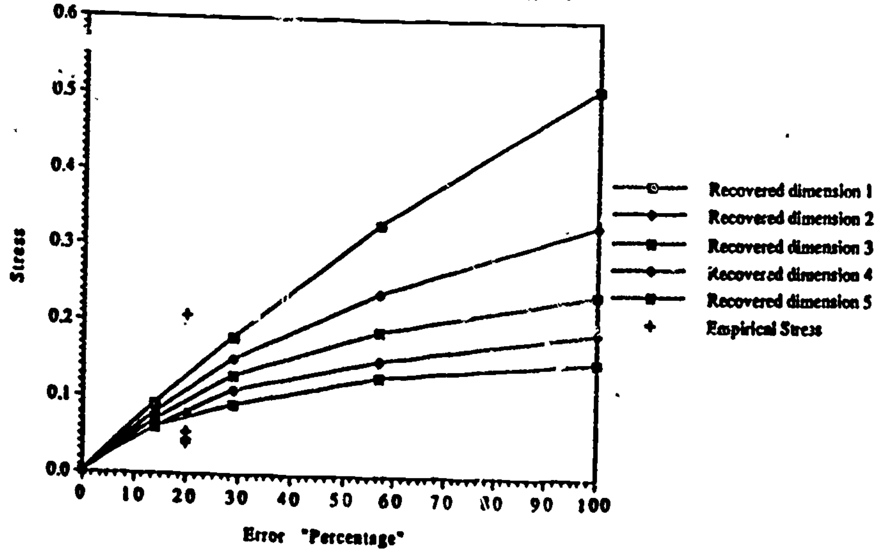


Figure 3b

Data : Two dimensional, 18 & 10 items, r=0.60 (PC261810)
Solution : 2 dimensions

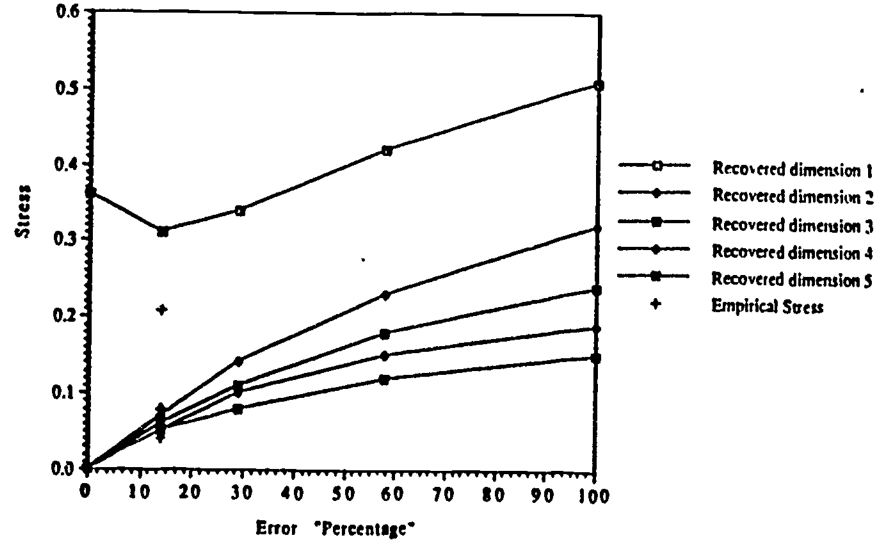


Figure 3c

Data : Two dimensional, 18 & 10 items, r=0.60 (PC261810)
Solution : 3 dimensions

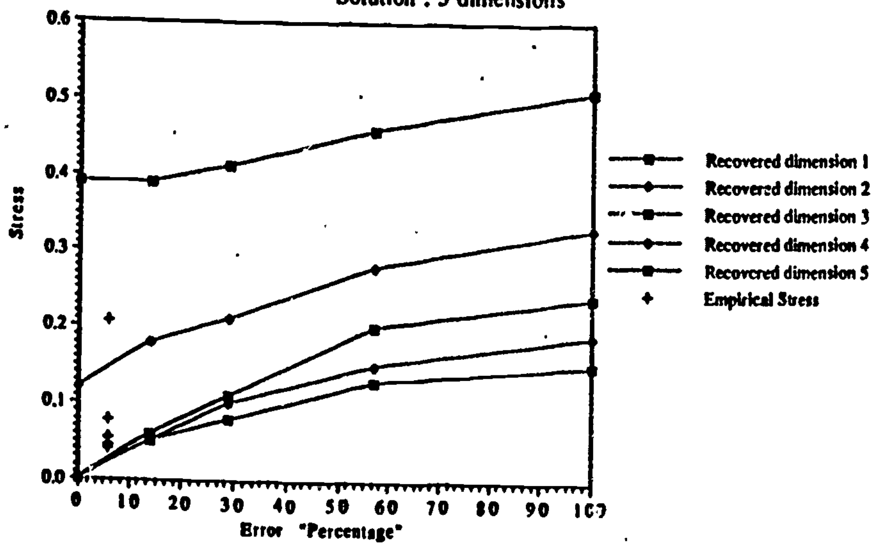


Figure 3d

Data : Two dimensional, 18 & 10 items, r=0.60 (PC261810)
Solution : 4 dimensions

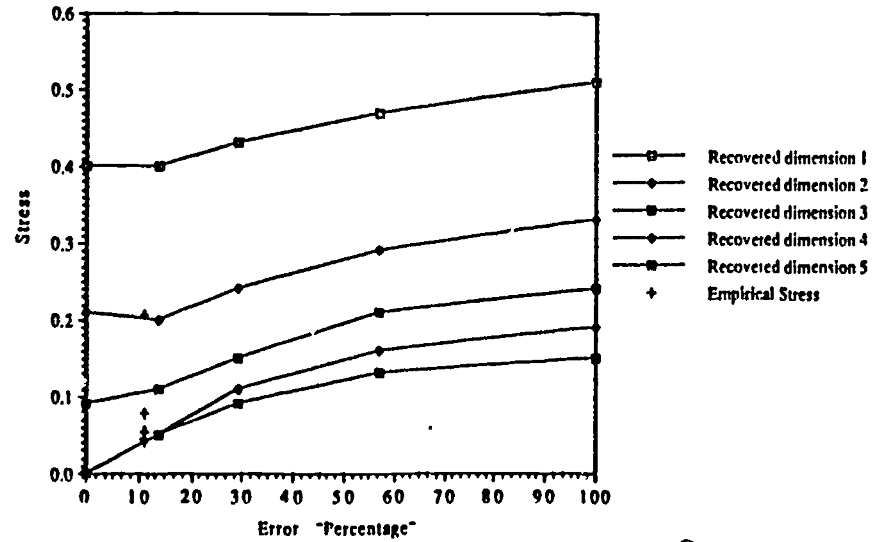


Figure 4a

Scree Plot for One Dimensional Data
Dichotomous data (PC10280)

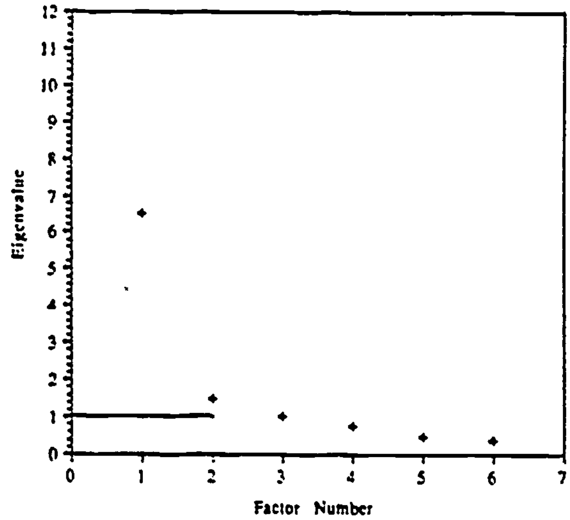


Figure 4b

Scree Plot for Two Dimensional Data, 14 & 14 items, r=0.01
Dichotomous data (PC201414)

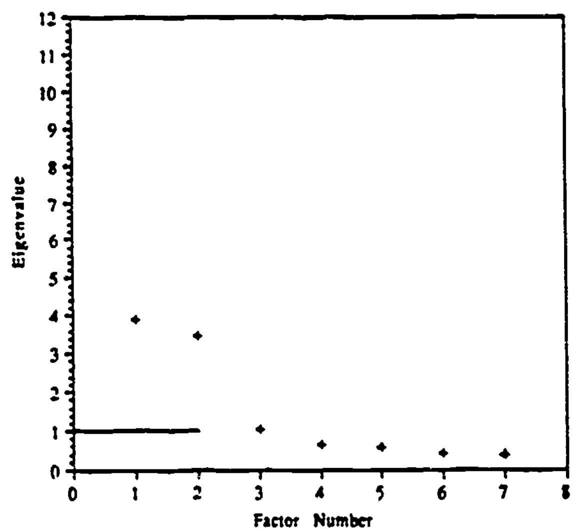


Figure 4c

Scree Plot for Two Dimensional Data, 14 & 14 items, r=0.01
Polychotomous data (PC201414)

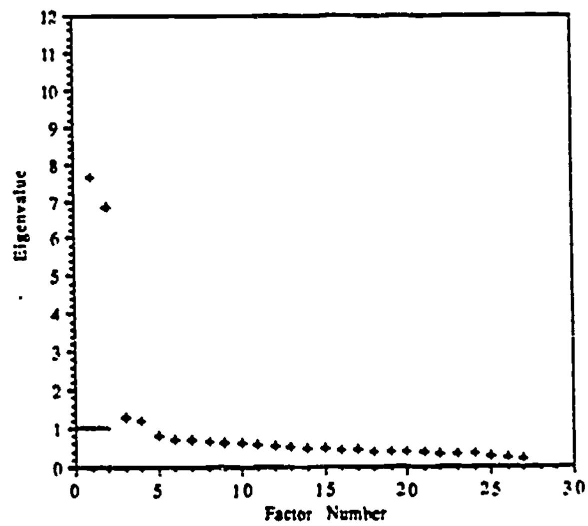


Figure 4d

Scree Plot for Two Dimensional Data, 18 & 10 items, r=0.60
Dichotomous data (PC261810)

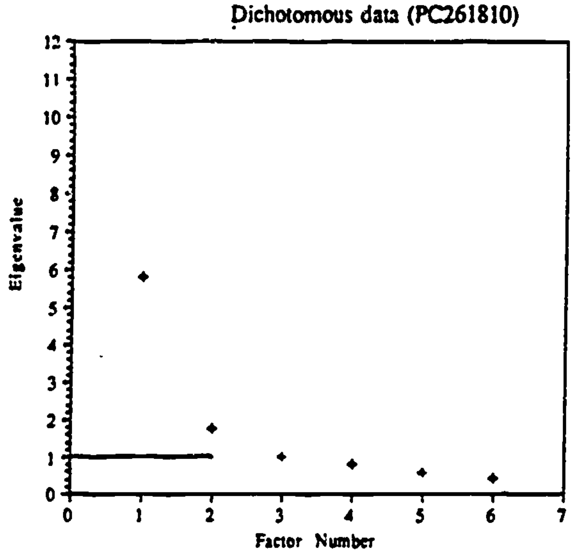


Figure 4e

Scree Plot for Two Dimensional Data, 18 & 10 items, r=0.60
Polychotomous data (PC261810)

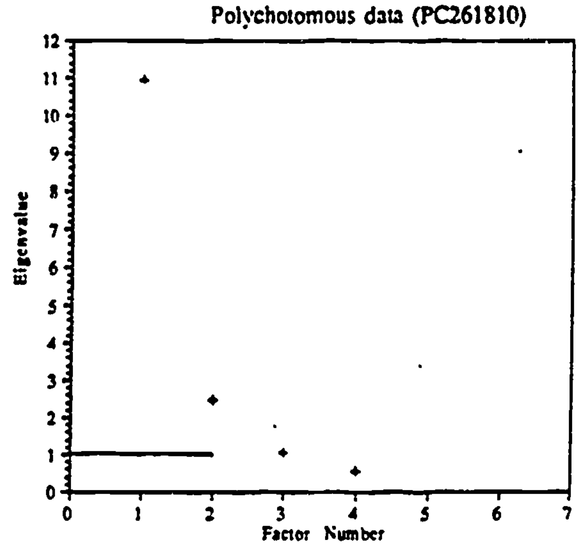


Figure 5

Two Dimensional Configuration for
One Dimensional data (PC10280)

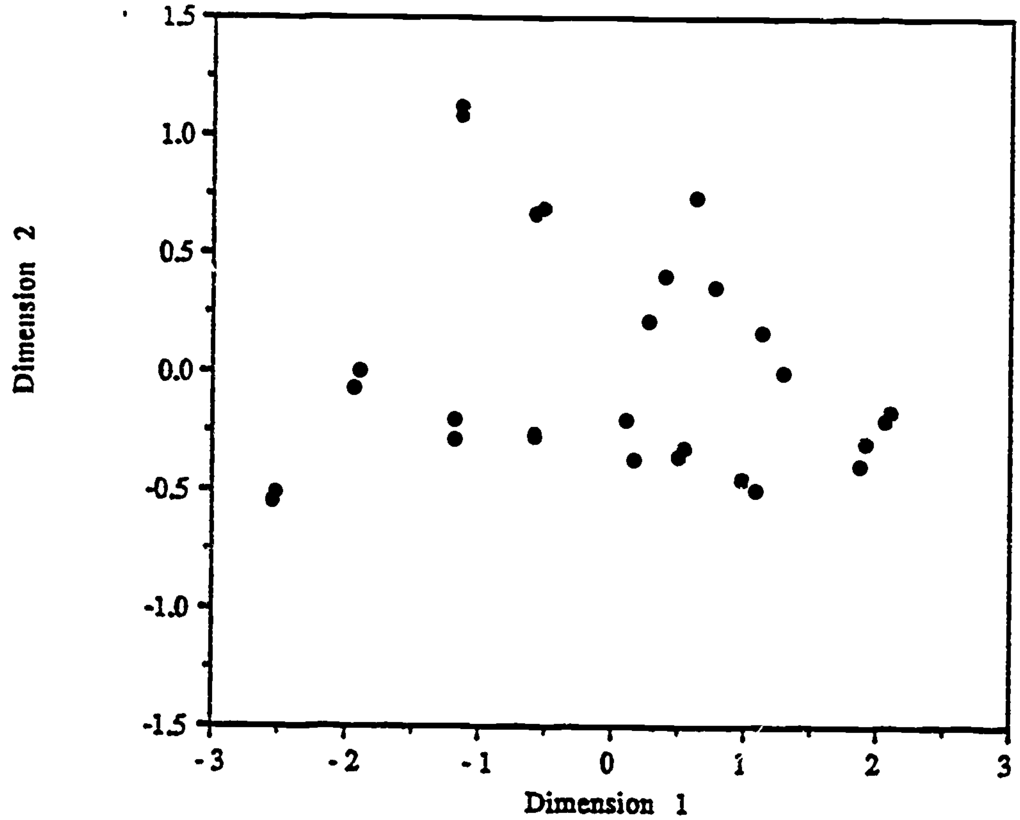


Figure 6

Two Dimensional Configuration for Two Dimensional data, 14 & 14 items, r = 0.01 (PC201414)

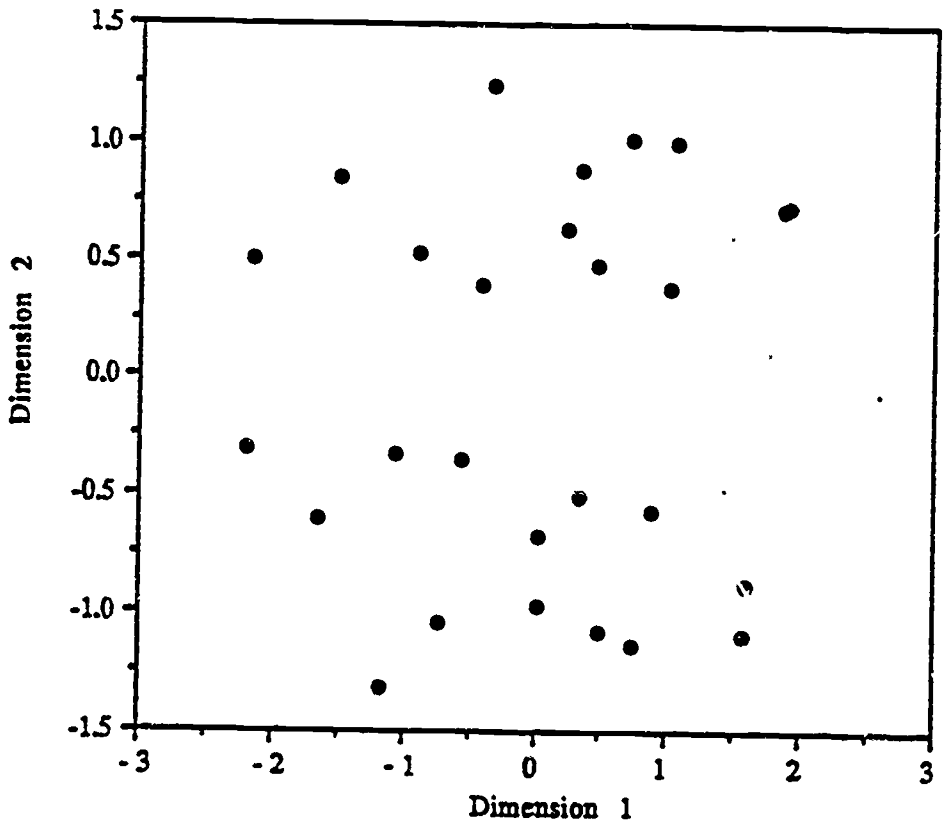


Figure 7

Two Dimensional Configuration for Two Dimensional data, 14 & 14 items, r = 0.60 (PC261414)

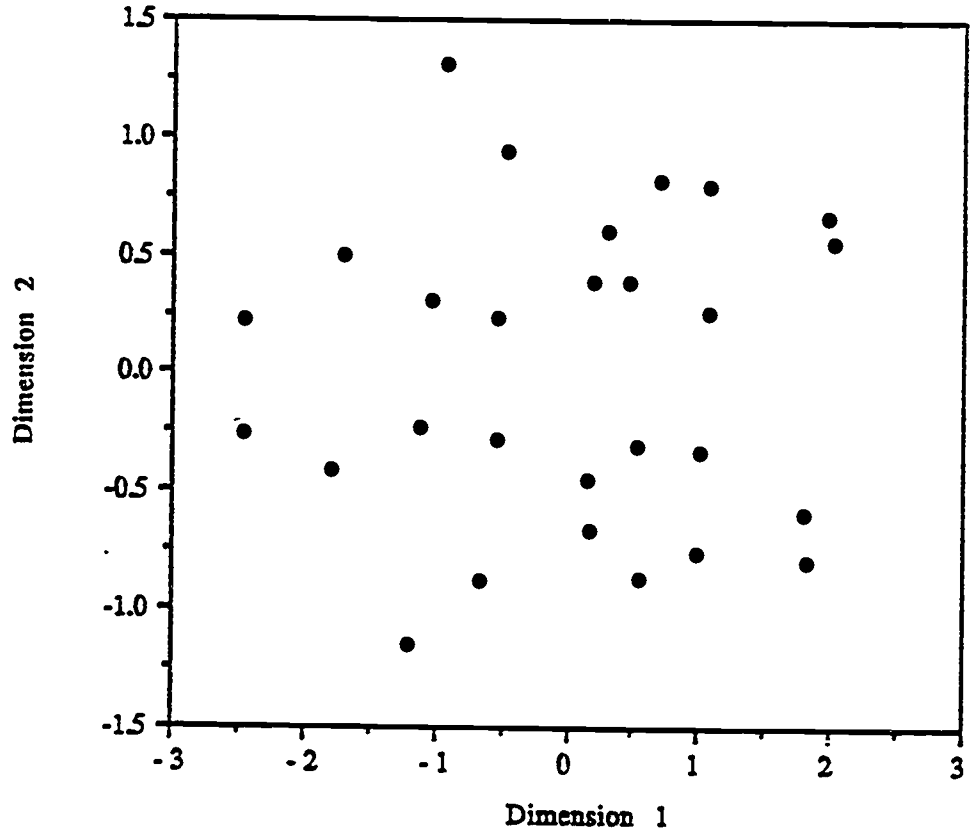
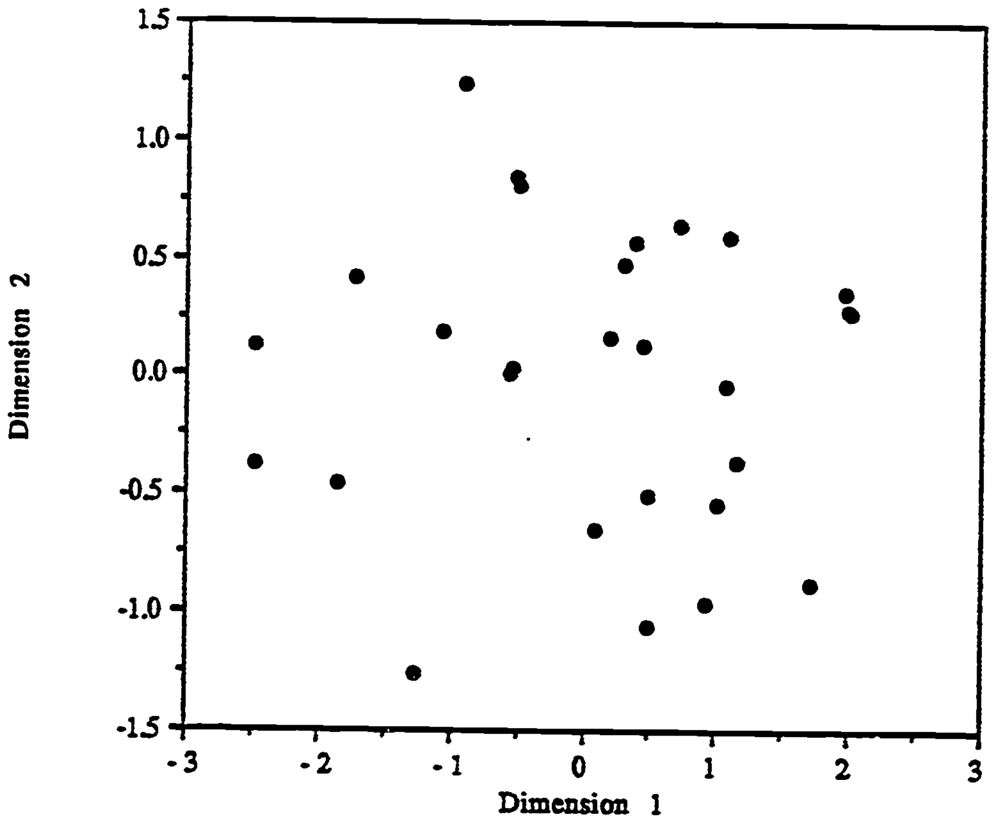


Figure 8

Two Dimensional Configuration for Two Dimensional data, 18 & 10 items, r = 0.60 (PC261810)



[2]The authors would like to thank Dr. Spence for granting permission to use M-SPACE.